Lecture Notes
Natural Science:2

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Carbon circulation - block 3 in IAM

- Externality is created from carbon emission.
- For policy analysis as well as for forecasts, we need to now the dynamic mapping from path of emissions to path of CO_2 concentrations.
- We will look at two approaches:
  1. stock-flow approach. Idea; different reservoirs of carbon. A continuos flow between these. Stable system always tending towards a steady state.
  2. Non-structural (reduced form) – define a depreciation function that specifies how much of deviation or of an emitted unit remains in atmosphere over time.

Stocks and flows

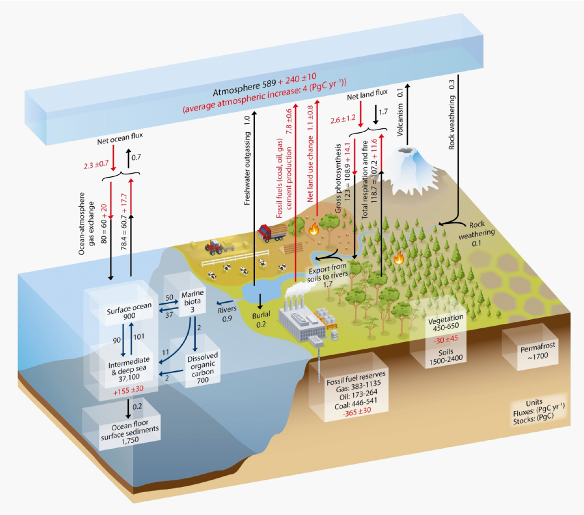


Figure: Global carbon cycle. Stocks in GtC (PgC) and flows GtC/year. Source: IPCC (2013), Figure 6.1

Easiest stock-flow case in continuous time

- Assume 2 reservoirs S_t, S_T^L. S_t represents the atmosphere in period t and S_t^L represents the deep oceans.
- Flow from S_t to S_t^L proportional to S_t, with proportionality factor ϕ_1.
- Flow from S_t^L to S_t proportional to S^L, with proportionality factor ϕ_2.
- Inflow to S_t also from emissions E_t.
- Change in stocks equal to net flows (in minus out), gives

$$
\begin{aligned}
\frac{dS_t}{dt} &= -\phi_1 S_t + \phi_2 S_t^L + E_t \\
\frac{dS_t^L}{dt} &= -\phi_2 S_t^L + \phi_1 S_t
\end{aligned}
$$

with $E_t = 0$, steady state satisfies

$$
\begin{aligned}
0 &= -\phi_1 S + \phi_2 S^L \\
0 &= \phi_1 S - \phi_2 S^L
\end{aligned}
$$

which cannot be uniquely solved, all solutions satisfy $S = \frac{\phi_2}{\phi_1} S^L$. Why?

Easiest case - discrete time approximation

- $$\begin{aligned} S_t - S_{t-1} &= -\phi_1 S_{t-1} + \phi_2 S^L_{t-1} + E_{t-1}. \\ S^L_t - S^L_{t-1} &= \phi_1 S_{t-1} - \phi_2 S^L_{t-1} \end{aligned}$$
- Same steady state and approximately the same dynamics.
- Such linear systems (in discrete or continuos time) can be solved analytically.
- Suppose emissions stop at t, then deviation from steady state $S_t = \frac{\phi_{21}}{\phi_{12}} S^L$ vanish over time as determined by the factor
$$(1 - \phi_1 - \phi_2)^{t+s}$$
- Specifically the law-of-motion for the stocks follow for $s \geq 0$ is given by;
$$\begin{aligned} S_{t+s} &= \tfrac{\phi_2}{\phi_1+\phi_2}\left(S_t + S^L_t\right) - \tfrac{\phi_2 S^L_t - \phi_1 S_t}{\phi_1+\phi_2}\left(1 - \phi_1 - \phi_2\right)^s \\ S^L_{t+s} &= \tfrac{\phi_1}{\phi_1+\phi_2}\left(S_t + S^L_t\right) + \tfrac{\phi_2 S^L_t - \phi_1 S_t}{\phi_1+\phi_2}\left(1 - \phi_1 - \phi_2\right)^s. \end{aligned}$$

A three reservoir system

- S_t represents the atmosphere in period t, S_t^U is the surface ocean, and finally S_t^L, which represents the deep oceans.
- Flows still assumed to be proportional to stocks and change is a reservoir is equal to net flow.
- We then have

$$
\begin{aligned}
S_t - S_{t-1} &= -\phi_{12} S_{t-1} + \phi_{21} S_{t-1}^U + E_{t-1} \\
S_t^U - S_{t-1}^U &= \phi_{12} S_{t-1} - (\phi_{21} + \phi_{23}) S_{t-1}^U + \phi_{32} S_{t-1}^L \\
S_t^L - S_{t-1}^L &= \phi_{23} S_{t-1}^U - \phi_{32} S_{t-1}^L .
\end{aligned}
$$

Calibration

- Two ways;
  - Try to choose the parameters to make model dynamics match as close as possible dynamics of more complicated models.
  - Take linear model seriously and use measured flows.
- Let's use the pre-industrial flows and stocks for the calibration.
  - Before industrialization we had 589 GtC in atmosphere and a flow to surface ocean of 60 GtC, implies $\phi_{12} = \frac{60}{589} \approx 0.102$.
  - The flow from the surface ocean to the atmospere gives $\phi_{21} = \frac{60.7}{900} \approx 0.067$
  - Use flow to deep ocean, giving $\phi_{23} = \frac{90}{900} = 0.100$.
  - Finally, the flow from the deep ocean to the surface ocean is set to the same value, giving $\phi_{32} = \frac{90}{37100} \approx 0.00243$.

Properties of steady state

- If emissions stop, this system also asymptotically approach a steady state. Solve

$$\begin{aligned}
0 &= -\phi_{12}S + \phi_{21}S^U \\
0 &= \phi_{12}S - (\phi_{21} + \phi_{23})\, S^U + \phi_{32}S^L \\
0 &= \phi_{23}S^U - \phi_{32}S^L
\end{aligned}$$

again no unique solution, but all solutions satisfy

$$\begin{aligned}
S &= \frac{\phi_{21}}{\phi_{12}}\frac{\phi_{32}}{\phi_{23}}S^L \\
S^U &= \frac{\phi_{32}}{\phi_{23}}S^L
\end{aligned}$$

i.e., proportions between stocks are always restored.

Excel implementation

Table 2. Three stock carbon circulation

Year	S_t	S_t^U	S_t^L	M_t
=2011	=589+240	=900+550	=37100+155	=7.8+1.1
=1+A2	=B2-0.102*B2+0.0667*C2	=C2+0.102*B2-(0.0667+0.100)*C2	=D2+0.100*C2	=7.8+1.1
	+E2	+0.00243*D2	-0.00243*D2	
=1+A2	=B3-0.102*B3+0.0667*C3	=C3+0.102*B3-(0.0667+0.100)*C3	=D2+0.100*C3	=7.8+1.1
	+E3	+0.00243*D3	-0.00243*D3	

Non-structural carbon circulation models

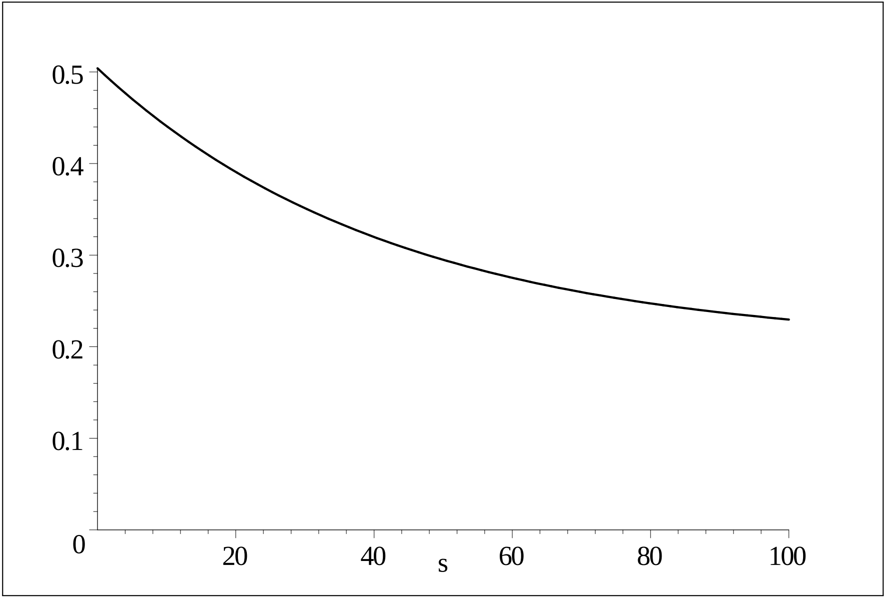
- Structural model may anyway be to simplified. Misses non-linearities, and other relevant variables.
- Could then instead try to match key characteristics directly; (IPCC and Archer 2005).
  - a share (ca 50%) is removed quite quickly (a few years to a few decades)
  - another share (ca 20-25%) stays very long (thousands of years) until CO_2 acidification has been buffered
  - remainder decays with a half-life of a few centuries.
- These features can be modeled directly by a depreciation function (rather remainder function), $d(s)$ that says how much remains of an emitted unit after s period.

$$d\left(s\right) = \varphi_L + \left(1 - \varphi_L\right) \varphi_0 \left(1 - \varphi\right)^s$$

Calibration

- Use decades Let's set $\varphi_L = 0.2$.
- $d(1) = 0.5$
- and $(1-\varphi)^{30} = \frac{1}{2}$.
- Gives $\ln(1-\varphi) \approx -\varphi = \frac{\ln\frac{1}{2}}{30} = -0.023$.
- $d(1) = 0.5 = 0.2 + (1-0.2)\,\varphi_0\,(1-0.023)^1, \Rightarrow \varphi_0 = 0.38$

$$\left[\varphi_L + (1-\varphi_L)\,\varphi_0\,(1-\varphi)^s\right]_{\varphi_L=0.2,\varphi=0.023,\varphi_0=0.38}$$



Non-linearities

- The parameters in the models we have presented are likely to be affected by the emission scenario.
- For example, more emissions reduce the capacity of oceans to store carbon (temperature and chemistry).
- Implies that more than 20-25% stays in atmosphere for thousands of years if cumulated emissions are large.
- With 10 times current cumulated emissions a twice as big share is likely to remain.

CCR – Carbon Climate Response

- Climate system and carbon circulation are dynamic and non-linear.
- An increase in forcing has a delayed impact (increasing over time) on temperature and is concave (logarithmic).
- Emission of carbon has a decaying impact (decreasing over time) on atmospheric CO_2 concentration and the relation is convex since other sinks storage capacity decreases when emissions have been large.
- Surprisingly, these non-linearities seem to cancel each other in most advanced climate models. The global mean temperature is linear in cumulative emissions.
- Increase in GMT is between 1 and 2.1 degrees Celsius per 1000 GtC both in short and long run. This constant is called CCR.