Unemployment Insurance and Credit Frictions

(Job Market Paper)

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Abstract. This paper extends the existing literature on optimal unemployment insurance by allowing for self-insurance; individuals may save using a one-period riskless asset, but their access to the credit market is restricted. I show that under this market arrangement, an asset based unemployment insurance scheme implements the optimal allocations. The optimal benefit payments *policy* shows no duration dependence, and relies exclusively upon an individual's current asset position. Benefit payments are decreasing in wealth and, as a consequence, peaks at a constant level when the liquidity constraint is binding. Over the course of unemployment, individuals decumulate assets and the sequence of benefit payments is thus observationally non-decreasing; a result that stands in sharp contrast with the previous literature. In a quantitative exercise it is shown that the US unemployment insurance programme is surprisingly close to optimal for the asset poor, but too generous for wealthier individuals. The potential cost-savings of switching to the optimal program ranges from roughly 33% of the present value insurance budget for the affluent, to 7% for the less fortunate.

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1. Introduction

This paper examines how partial self-insurance and limited access to the credit market affect the properties of optimal unemployment insurance provisions in a model of job-search and moral hazard. The constrained Pareto-optimal allocations are implemented in a decentralized economy in which individuals may freely save using a one-period riskless asset, but face liquidity constraints that restricts borrowing. I show that the optimal unemployment insurance scheme is recursive in an agent's asset position and her current employment transition, and that unemployment benefits are decreasing in an agent's wealth level. Taxes upon reemployment, as well as unemployment benefit payments, are constant whenever the liquidity constraint is binding - arguably a relevant situation for many of the unemployed. Over the course of unemployment, agents decumulate assets and the sequence of benefit payments is thus observationally non-decreasing; a result that stands in sharp contrast with the previous literature, in which benefit payments displays a declining pattern along the duration of the unemployment spell (e.g. Shavell and Weiss (1979), and Hopenhayn and Nicolini (1997)). In a quantitative exercise it is shown that the US unemployment insurance programme is surprisingly close to optimal for the asset poor, but too generous for wealthier individuals. The potential cost-savings of switching to the optimal program ranges from roughly 33% of the present value insurance budget for the affluent, to 7% for the less fortunate.

In their seminal study, Shavell and Weiss (1979) consider a model of optimal unemployment insurance in which taxes upon reemployment are exogenously fixed at a constant level. They establish a celebrated result that benefit payments should be a decreasing function of the duration of unemployment, as this declining profile provides incentives for agents to undertake job search while still maintaining a considerable degree of consumption smoothing. Hopenhayn and Nicolini (1997) extend the result of Shavell and Weiss by allowing reemployment taxes to be set optimally. They show that taxes upon employment should increase along the unemployment duration, and that the welfare gain of this additional policy instrument is non negligible.

While these findings have repeatedly been confirmed in several studies (e.g. Pavoni (2007), Pavoni and Violante (2007), and Alvarez-Parra and Sanchez (2006)), they hinge upon a crucial assumption; individuals are precluded from any asset markets, and thus behave according to a *hand-to-mouth* principle. In contrast, several empirical studies have documented that (partial) self-insurance, and indeed, liquidity constraints, are relevant factors to consider when thinking about unemployment insurance. Browning and Crossley (2001) show that nearly half of job losers in the United States report zero liquid wealth at the time of job loss, suggesting that liquidity is a concern for many of the unemployed. Gruber

(1997) finds that the consumption smoothing effect of benefit payments is particularly high at late stages of the unemployment spell, arguing that this occurs when liquid wealth is depleted. In a recent study, Chetty (2007) divides unemployed individuals in the United States into subgroups based on how likely they are to be liquidity constrained. He finds that while the effect of unemployment benefits on the hazard rate of reemployment is extremely small for the unconstrained, the corresponding measure for the constrained group is severe. Chetty therefore concludes that access to liquid funds (or lack thereof) is an important aspect to consider when designing an unemployment benefits programme.

Motivated by these issues, this paper develops a theoretical model in order to characterize an optimal unemployment insurance programme. The economic environment follows closely Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997), extended with partial self-insurance and limited credit access. In particular, I allow agents to freely save using a one-period riskless asset. I model the credit limitation as a restriction on short sales of future expected labor income. While the qualitative results in the paper are robust to many other alternative liquidity constraints, it seems a reasonable hypothesis that consumers cannot borrow, on net, against non-traded assets such as future labor income.[1]

The utilitarian government has information on the agents' consumption level and preferences, but not on their search effort. The government's policy must therefore be *incentive compatible*. Without any loss of generality, I proceed in two distinct steps: The first step characterizes the incentive compatible Pareto-optimal consumption and search effort allocations. The second step implements these allocations through a tax system in a decentralized economy in which individuals may save using a one-period riskless asset and in which access to the credit market is limited.

The key assumption in identifying the unique role of the government's policy vis-à-vis the agent's asset position, is an imposed presumption that all *intertemporal* transfers of resources are done by the agent, and not by the government. This identifying assumption is commonly used in the dynamic public finance literature (Kocherlakota, 2005; Albanesi and Sleet, 2006), and is sometimes referred to as *affordability*; if the agent could buy the optimal allocations, she would period-by-period afford so. As a consequence, the structure of the optimal policy resembles current unemployment insurance programmes adopted in many modern economies, at actuarially fair prices; employed individuals pay a premium, unemployed individuals receive a benefit payment, and the premium paid equals the expected cost of the benefit payment.

[1] Short-selling constraints on expected future labor income is a standard ingredient in most models of consumption and saving (e.g. Zeldes (1989) and Deaton (1991)).

Under the proposed implementation, I show that unemployment benefits depend exclusively on an individual's current level of wealth and employment transition. This result stands in contrast to previous recursive schemes in the literature, in which unemployment benefits depend on the state variable "promised utility" (e.g. Hopenhayn and Nicolini (1997), Pavoni (2007)). The result is not trivial; incentive compatibility constraints restrict both current *and* future consumption choices, which usually renders wealth an insufficient, or even irrelevant, state variable (e.g. Marcet and Marimon (1998); Spear and Srivastava (1987)). Yet, I show that wealth entirely encodes an agent's complete employment history. As a corollary, benefit payments as well as reemployment taxes are constant whenever the liquidity constraint is binding.

While taxes and benefit payments are constant only when the liquidity constraint is binding, this is arguably a relevant part of the state space for many of the unemployed; Gruber (1997) reports that only 18.6% of the unemployed in the PSID have savings of more than two months of labor income, and Engen and Gruber (2001) show that the median unemployed has gross financial assets equivalent to less than three weeks of income (an average unemployment spell lasts for 13.1 weeks). Thus, for the sub-group of liquidity constrained individuals, these results conform much better to many modern economies' unemployment insurance programmes than previous results in the literature, in which unemployment benefits and taxes displays significant and complicated duration dependence.[2]

Unemployment benefits are negatively related to an agent's wealth level. Thus, as assets decline, benefits rise and peak when the liquidity constraint binds. During the course of unemployment, the agent decumulates assets, and the sequence of benefit payments is increasing. This result is of course in marked contrast with the existing literature, in which benefit payments is a decreasing function of the duration of unemployment. The underlying reason is the presence of self-insurance: First, wealth has a first order insurance effect. The higher is an agent's wealth, the less she needs to worry about loss of consumption if she loses her job. Second, in order to provide incentives to exert search effort, the government wishes to generate a positive correlation between consumption and employment. When the agent's utility function is concave, a higher level of savings makes it costlier for the government to induce such a correlation and the agent's search effort decreases. By generating a negative correlation between savings and unemployment benefits, the government manages to mitigate the distortionary effect of savings on search.

[2]In the OECD countries, benefits payments are typically given as a constant replacement rate for a fixed amount of time. Thereafter, unemployment- or social-assistance takes over, following the same constant pattern, although at a lower level. Taxes, or contributions to the unemployment insurance fund, are duration independent and constant (OECD, 2004; Pavoni, 2007).

In a calibrated version of the model I characterize the optimal benefit payments in relation to an individuals wealth level. The replacement rate for constrained agents with zero liquid wealth is optimal at 60%, markedly in line with the average current US level at 66% (Meyer, 1990; Hopenhayn and Nicolini, 1997). As a consequence, the potential cost-savings of switching to the optimal programme is for this group small. For the more affluent however, the current system is far too generous; the replacement rate is optimal at a mere 1.5%. The reason behind this swift increase in benefit payments as wealth decreases, is due to a "liquidity effect" (Chetty, 2007): Low benefit payments to individuals for which credit is a concern generates a higher than socially optimal search pressure. Increasing the insurance level compensates for the missing credit market, reduces search effort, and boosts present value utility. Switching to the optimal programme, affluent agents would be prepared to give up roughly 33% of their expected value of future benefit payments under the current system.

In a sequence of papers, Shimer and Werning (2006; 2007a; 2007b) consider a problem closely related to the question explored in this paper. As here, they stress the importance of self-insurance in considering unemployment insurance programmes. In particular, in Shimer and Werning (2007a) they reach the conclusion that both reemployment taxes and unemployment benefits should be constant at all elements of the state space, contrasting to this paper in which the equivalent result hold for the sub-group of liquidity constrained individuals. However, their results are derived under fundamentally different assumptions than mine. Departing from the main literature on unemployment insurance, Shimer and Werning consider versions of McCall's (1970) search model with hidden reservation wages. This paper closely follows the existing literature by considering hidden search effort decisions. More importantly, all qualitative properties explored by Shimer and Werning hinge on the assumption of CARA utility; on potentially negative consumption levels; and thus on the absence of credit frictions. Their results do not extend to utility functions that preclude negative consumption levels, such as CRRA utility. Notwithstanding some standard regulatory conditions, this paper puts no restrictions on the specific functional form of the agents' momentary utility function, and I do consider credit frictions. Needless to say, this paper builds upon, and I believe complements, the pioneering work of Shimer and Werning.

2. Structure of the economy

The economy is populated by a utilitarian government and a continuum of risk-averse agents. The planning horizon is infinite. Time is discrete and denoted by $t = 0, 1, \ldots$ At any given period t, an agent can either be employed or unemployed and the agent's employment status is publicly observable.

When an agent is employed, she earns a gross wage, w. There is no on-the-job search and the probability of being fired is exogenously given at the constant hazard rate $1-\gamma$.

When unemployed, the agent receives unemployment benefits and searches for a job with effort e. The probability of finding a job, conditional on search effort, is denoted $p(e)$. Search effort - and thus the probability of finding a job - is considered private information, not observable by the government or by any other agent in the economy.[3] The wage distribution is degenerate, and a job offer is, consequently, always accepted.

The agents save using a riskless asset that pays pre-tax return equal to $r > 0$. The intertemporal price of consumption, $1/(1+r)$, is denoted q. Borrowing is restricted by a liquidity constraint that prevents short-sales of future labor income.

2.1. **Model.** An agent's employment status in any period t is given by $\theta_t \in \Theta = \{0,1\}$. Let $\theta_t = 1$ denote employment. The history of employment status up to period t is given by $\theta^t = (\theta_0, \dots, \theta_t) \in \Theta^t$, where $\Theta^t = \{(\theta_0, \dots, \theta_t) | \theta_i \in \Theta, i = 0, \dots, t\}$, represent all possible histories up to period t.

At time zero, each agent is born as either employed or unemployed, and she is entitled some level of initial resources, b_0. The initial entitlement/employment status-pair, (b_0, θ_0), is taken as given by each agent in the economy (the government included). The joint distribution of (b_0, θ_0) is given by $\psi(b_0, \theta_0)$, with support on $B \times \Theta$, where B is some subset of the real numbers, $B \subseteq \mathbb{R}$. Thus, at every date, t, each agent is distinguished by her initial entitlements and history of employment status, (b_0, θ^t).

Without any loss of generality, I will henceforth formulate the problem such the agents choose p - the probability of finding a job - rather than effort e. The agent then ranks contemporaneous consumption and search effort allocations according an additively separable felicity function, $\{u(c) - (1-\theta)v(p)\}$. The function u is strictly concave, strictly increasing, and once continuously differentiable. The function v is strictly convex, strictly increasing, and twice continuously differentiable. In addition, $u(0) = -\infty$, $\lim_{p \downarrow 0} v'(p) = 0$ and $\lim_{p \uparrow 1} v'(p) = \infty$. There is no disutility from working.[4]

An *allocation* in this economy is denoted $\boldsymbol{\sigma} = \{c_t, p_t\}_{t=0}^{\infty}$, where

$$c_t : B \times \Theta^t \to \mathbb{R}_+$$
$$p_t : B \times \Theta^t \to [0,1]$$

Here, $c_t(b_0, \theta^t)$ is the amount of consumption an (b_0, θ_0)-agent is assigned under history θ^t. The contemporaneous probability of finding a job, $p_t(b_0, \theta^t)$, is defined equivalently.

[3] This is the source of moral hazard in the model; if benefit payments would be made contingent upon search effort, the economy would reach its first best allocation.

[4] Including disutility from working would not change any of the results in the paper.

Let $\lambda(b_0, \theta^{t+1})$ denote the probability measure for history θ^{t+1}, conditional on (b_0, θ_0). For notational convenience let $p_t(b_0, \theta^t)$ be defined as γ if and only if $\theta_t = 1$. $\lambda(b_0, \theta^{t+1})$ is then recursively given by

$$\lambda(b_0, \theta^{t+1}) = \begin{cases} p_t(b_0, \theta^t)\lambda(b_0, \theta^t), & \theta_{t+1} = 1 \\ (1 - p_t(b_0, \theta^t))\lambda(b_0, \theta^t), & \theta_{t+1} = 0 \end{cases}$$

An agent's net present value utility of an allocation $\boldsymbol{\sigma}$ is given as

$$V(\boldsymbol{\sigma}, b_0, \theta_0) = \sum_{t=0}^{\infty} \beta^t \int_{\Theta^t} \{u(c_t(b_0, \theta^t)) - (1 - \theta_t)v(p_t(b_0, \theta^t))\}\lambda(b_0, \theta^t)d\theta^t \tag{1}$$

Given the joint initial distribution of (b_0, θ_0)-pairs, ψ, the utilitarian government wishes to find $\boldsymbol{\sigma}$ that maximizes the sum of net present value utilities

$$\hat{V}(\psi) = \max_{\boldsymbol{\sigma}} \int_{B\times\Theta} \{V(\boldsymbol{\sigma}, b_0, \theta_0)\}d\psi \tag{2}$$

subject to each agent's present value budget constraint

$$b_0 \geq \sum_{t=0}^{\infty} q^t \int_{\Theta^t} \{c_t(b_0, \theta^t) - \theta_t w\}\lambda(b_0, \theta^t)d\theta^t, \quad \forall\ (b_0, \theta_0) \in B \times \Theta \tag{3}$$

Furthermore, since the search effort allocation is private information, the optimal allocation must respect incentive compatibility

$$\{p_t\}_{t=0}^{\infty} = \operatorname{argmax}\{V(\boldsymbol{\sigma}, b_0, \theta_0)\}, \quad \forall\ (b_0, \theta_0) \in B \times \Theta \tag{4}$$

The motivation behind the *incentive compatibility constraint* is simple: Each agent takes the consumption allocation as given and chooses search effort to maximize her private utility. Without any loss of generality, the problem is formulated such that the government directly proposes a search effort allocation that coincides with the agent's private optimal choice.

Additionally, the government faces the following sequence of liquidity constraints

$$\sum_{s=1}^{\infty} q^{s-1} \int_{\Theta^{t+s}} c_{t+s}(b_0, \theta^{t+s}) \frac{\lambda(b_0, \theta^{t+s})}{\lambda(b_0, \theta^t)} d\theta^{t+s} \tag{5}$$
$$\geq \sum_{s=1}^{\infty} q^{s-1} \int_{\Theta^{t+s}} \theta_{t+s} w \frac{\lambda(b_0, \theta^{t+s})}{\lambda(b_0, \theta^t)} d\theta^{t+s}, \quad \forall\ (b_0, \theta^t) \in B \times \Theta^t$$

The left-hand side in (5) represent the period t expected net present value of consumption allocations, conditional on history θ^t. The right-hand side is the equivalent expected net present value of labor income. The constraint states that the value of future consumption allocations must be greater than the value of future wages. In other words, short-selling expected future labor income is not permitted.

It should be noted that constraint (3) together with the liquidity constraint ensures feasibility. Constraint (3) will always hold as an equality; if it did not, the government could simply increase the agent's period zero consumption without interfering with neither incentive compatibility nor the liquidity constraint. An allocation that is both incentive compatible and feasible will be referred to as *incentive feasible*.

The following lemma states that maximizing (1) subject to the individual budget constraint, incentive compatibility and the liquidity constraint, is equal to solving the more complicated problem given in (2). The result is standard and the proof is merely included for completeness.

Lemma 1. *Define* $\boldsymbol{\sigma}^*$ *as the allocation that maximizes* (1) *for each* $(b_0, \theta_0) \in B \times \Theta$*, subject to individual incentive compatibility, feasibility and the liquidity constraint. Define* $\hat{\boldsymbol{\sigma}}^*$ *as the allocation that solves* (2). *Then*

$$\hat{V}(\psi) = \int_{B\times\Theta} V(\boldsymbol{\sigma}^*, b_0, \theta_0) d\psi$$

Proof. By construction, $\hat{V}(\psi) \geq \int_{B\times\Theta} V(\boldsymbol{\sigma}^*, b_0, \theta_0)d\psi$. If the inequality was strict, then there exist some (b_0, θ_0) such that $V(\hat{\boldsymbol{\sigma}}^*, b_0, \theta_0) > V(\boldsymbol{\sigma}^*, b_0, \theta_0)$. Since $\hat{\boldsymbol{\sigma}}^*$ is incentive compatible, delivers b_0, and satisfies the liquidity constraint, $\boldsymbol{\sigma}^*$ could not have attained the maximum in (1). □

Remarks. There are several reasons to why I adopt the specific liquidity constraint in (5). Constraining allocations allows me to restrict borrowing without explicitly stating the government's policy vis-á-vis the agents' asset positions. In particular, it stipulates an underlying *procedure* of the credit sector in which the storage technology available to agents and the policy conducted by the government are unimportant; what is important is the total amount of resources outstanding. In this respect, the constraint is not subject to the Lucas Critique on optimal policy - for any policy and asset structure, any "decentralized" liquidity constraint must endogenously respond in order for (5) to hold. Additionally it should be noted that adding an arbitrary constant to either side of (5), virtually leaving open for a continuum of different constraints, will not change any of the qualitative results derived the the subsequent sections. However, I proceed under the, quite reasonable, hypothesis that consumers cannot borrow, on net, against non-traded assets such as future labor income.

It should be noted that the liquidity constraint facing consumers also restricts the allocations attainable by the government. An alternative, and welfare improving policy, would thus be for the government to borrow in an agent's name, effectively relieving the agent any impediment caused by the liquidity constraint. However, such a *liquidity providing policy* is an additional instrument, unrelated to what we generally perceive as unemployment insurance. Liquidity providing policies, such as *"unemployment insurance savings accounts"* (Feldstein and Altman, 1998), are thus ruled out a priori.

2.2. **A recursive formulation.** Following the insights provided by Lemma 1, the problem of interest is given by

$$V(b_0,\theta_0) = \max_{\boldsymbol{\sigma}} \sum_{t=0}^{\infty} \beta^t \int_{\Theta^t} \{u(c_t(b_0,\theta^t)) - (1-\theta_t)v(p_t(b_0,\theta^t))\}\lambda(b_0,\theta^t)d\theta^t \tag{6}$$

$$\text{s.t.} \quad \{p_t\}_{t=0}^{\infty} = \text{argmax}\{V(\boldsymbol{\sigma}, b_0, \theta_0)\} \tag{7}$$

$$b_0 = \sum_{t=0}^{\infty} q^t \int_{\Theta^t} \{c_t(b_0,\theta^t) - \theta_t w\}\lambda(b_0,\theta^t)d\theta^t \tag{8}$$

$$0 \geq -\sum_{s=1}^{\infty} q^{s-1} \int_{\Theta^{t+s}} \{c_{t+s}(b_0,\theta^{t+s}) - \theta_{t+s}w\}\frac{\lambda(b_0,\theta^{t+s})}{\lambda(b_0,\theta^t)}d\theta^{t+s}, \quad \forall\, t \tag{9}$$

Under an optimal allocation, $\boldsymbol{\sigma^*}$, equations (6), (8) and (9) can be written as

$$V(b_0,\theta_0) = u(c_0^*(b_0,\theta_0)) - (1-\theta_0)v(p_0^*(b_0,\theta_0)) + \beta \int_{\Theta^1} V(\boldsymbol{\sigma^*}, b^*(\theta_1),\theta_1)\lambda(b_0,\theta^1)d\theta^1 \tag{10}$$

$$b_0 = c_0^*(b_0,\theta_0) - \theta_0 w + q\int_{\Theta^1} b^*(\theta_1)\lambda(b_0,\theta^1)d\theta^1 \tag{11}$$

$$0 \geq -q\int_{\Theta^1} b^*(\theta_1)\lambda(b_0,\theta^1)d\theta^1 \tag{12}$$

The following lemma asserts that, given the budget $b^*(\theta_1)$, re-optimizing the problem in period one, does not alter period zero present value utility.

Lemma 2. *$V(\boldsymbol{\sigma^*}, b^*(\theta_1), \theta_1)$ maximizes the agent's utility subject to the budget $b^*(\theta_1)$, the liquidity constraint, and incentive compatibility. That is, $V(\boldsymbol{\sigma^*}, b^*(\theta_1), \theta_1) = V(b^*(\theta_1), \theta_1)$.*

Proof. See Appendix A. □

The result is nontrivial. If $V(b^*(\theta_1),\theta_1) > V(\boldsymbol{\sigma^*}, b^*(\theta_1),\theta_1)$ for some θ_1, period zero incentive compatibility is violated. The idea behind the proof lies in the fact that $V(b_0,\theta_0)$ is strictly increasing in b_0, and that $b^*(\theta_1)$ must therefore be resource minimizing given utility $V(\boldsymbol{\sigma^*}, b^*(\theta_1),\theta_1)$. The proof then proceed by showing that duality holds: If $b^*(\theta_1)$ is resource minimizing under utility $V(\boldsymbol{\sigma^*}, b^*(\theta_1),\theta_1)$, $V(\boldsymbol{\sigma^*}, b^*(\theta_1),\theta_1)$ must be utility maximizing under the budget $b^*(\theta_1)$.

Let b_e and b_u denote period $t+1$ contingent claims in the employed and unemployed state, respectively. Then - by exploiting the insights provided by Lemma 2 and following the arguments outlined in Spear and Srivastava (1987) - problem (6) can be made recursive as

$$V(b,\theta) = \max_{c,p,b_e,b_u} \{u(c) - (1-\theta)v(p) + \beta(pV(b_e) + (1-p)V(b_u))\} \tag{13}$$

subject to

$$p = \text{argmax}_p\{u(c) - \theta v(p) + \beta(pV(b_e) + (1-p)V(b_u))\} \tag{14}$$

and

$$b = c - \theta w + q(pb_e + (1-p)b_u) \tag{15}$$

and

$$0 \geq -pb_e - (1-p)b_u \tag{16}$$

in which (16) is the recursive representation of the liquidity constraint in (9).

Since the function v is differentiable, strictly convex, and fulfills the Inada conditions, the incentive compatibility constraint (14) can be replaced by its first order condition

$$v'(p) = \beta(V(b_e) - V(b_u))$$

The solution to (13)-(16) yields a value function, $V(b, \theta)$, associated with policy functions $c(b, \theta)$, $p(b, \theta)$, $b_e(b, \theta)$ and $b_u(b, \theta)$.[5] When there is no confusion regarding the agent's employment status, the policy functions will be addressed by their respective initial letter, and reliance on b will be left implicit.

Previous studies on optimal unemployment insurance adopt a dual formulation to the problem in (13)-(16). Specifically, the literature has, without exception, followed the cost-minimization framework commonly employed in the repeated-agency literature. Fundamentally, this approach amounts to minimize (3) such that the agent receives a pre-specified level of present value utility, and subject to incentive compatibility. Due to Spear and Srivastava (1987), Thomas and Worrall (1988), Abreu, Pearce and Stacchetti (1990), and Phelan and Townsend (1991), this dual formulation lends itself straightforwardly to a recursive representation. In contrast, this paper adopts a *primal* approach. The reason for this is twofold: First, the primal formulation simplifies the subsequent analysis and provides an intuitive recursive representation in terms of (non-labor) cash-on-hand, b. Second, this way of formulating the problem has a quite appealing and natural interpretation: Akin to a *social planner*, the government maximizes the agent's utility by choosing current consumption, search effort, and one period ahead Arrow securities at prices qp and $q(1-p)$. By respecting incentive compatibility, moral hazard is internalized through *individually* and *quantity contingently* priced assets.

3. Analysis

Consistent with the formulation of the problem in (13)-(16), the government chooses *allocations* rather than *policies*. While it facilitates the analysis of the governments optimal policy problem, it also restricts the subsequent analysis to proceed in two separate steps.

[5]Although these policy functions may well be correspondences, I will consistently *refer* to them as policy *functions*.

The first step concerns the optimal allocations. The second step considers the tax functions that implement these allocations in a decentralized economy.

Although the two steps presented above may appear distinctly separate, they are, in effect, intimately related. Thus, as a third, clarifying, step, Section 3.3 will show how the shape of the derived tax functions are closely tied to the incentive compatibility constraint, and how a quite esoteric optimality condition, commonly known as the inverse Euler equation, relate to a more familiar form of the standard Euler equation.

3.1. **Allocations.** Analogous to the definition of b_e and b_u, let c_e and c_u denote period $t+1$ consumption at the associated employment states. During employment, moral hazard is absent and the first order necessary conditions from (13) (together with the envelope condition) gives

$$u'(c) - \mu = \frac{\beta}{q} u'(c_e) = \frac{\beta}{q} u'(c_u), \quad \mu \geq 0 \tag{17}$$

Where μ is the Lagrange multiplier on the liquidity constraint (16). When $\beta = q$ and $\mu = 0$, condition (17) implies that consumption is constant for any two consecutive periods; on a period-by-period basis, the agent is fully insured. This result is hardly surprising in the current setting; when agents are employed, moral hazard is absent, and first-best is attainable. When $\mu > 0$, consumption at $t+1$ is higher than consumption at t. However, consumption is still constant across states, $c_e = c_u$.

The equivalent optimality conditions for an unemployed agent gives

$$\frac{1}{u'(c) - \mu} = \frac{q}{\beta} \left(p \frac{1}{u'(c_e)} + (1-p) \frac{1}{u'(c_u)} \right), \quad \mu \geq 0 \tag{18}$$

$$\lambda_2 v''(p) = \lambda_1 q (b_e - b_u) \tag{19}$$

$$\frac{\lambda_2}{\lambda_1} = p(1-p) \left(\frac{1}{u'(c_u)} - \frac{1}{u'(c_e)} \right) \tag{20}$$

Where λ_1 and λ_2 are the Lagrange multipliers on the budget- and the incentive compatibility constraint, respectively.

When $\mu = 0$, equation (18) is commonly known as the "inverse Euler equation" (Rogerson, 1985). When $c_e \neq c_u$, Jensen's inequality implies (Golosov, Kocherlakota and Tsyvinski, 2003)

$$u'(c) < \frac{\beta}{q} (p u'(c_e) + (1-p) u'(c_u)) \tag{21}$$

Rearranging terms, equation (21) infers that there is a wedge between the agent's marginal rate of substitution and the economy's marginal rate of transformation. In particular, (21) implies that current marginal utility of consumption is lower than the expected future marginal utility (the interest rate). In other words, the agent is savings constrained relative to an economy with no private information. Golosov et al. (2003) interpret this wedge as an "implicit tax".

According to the *standard* Euler equation, an optimal intertemporal plan has the property that any marginal, temporary and feasible change in behavior equates marginal benefits to marginal costs in the present and in the future. The inverse Euler equation *appears* to violate this logic. For a given value of p, consider the choice of reallocating resources from period t to period $t+1$. If an increase in savings would bring about a proportional increase in b_e as well as b_u, equation (21) reveals that, at least on the margin, such a policy would increase overall utility. However, the incentive compatibility constraint in (14) does generally not permit a proportional increase in b_e and b_u. To keep the choice of p unaltered - which is without loss of generality if we consider marginal changes - the incentive compatibility constraint forces the increase in resources to be relatively low in future states where the marginal utility of resources is relatively high, and vice versa.[6] Period $t+1$ marginal utilities will thus be "weighted" by their respective incentive compatible inflow of state contingent resources. In contrast, utility maximization implies relatively high weights of resource inflow to states in which the marginal benefit of resources is relatively high. Since incentive compatibility inflicts with period $t+1$ resources only, it is thus optimal to relegate a high degree of resources to period t consumption. As a result, the agent appears savings constrained. The inverse Euler equation is simply the resulting expression when these conflicting forces are internalized. Section 3.3 will more algebraically confirm the validity of this interpretation of the inverse Euler equation.

The following lemma extends the results of Hopenhayn and Nicolini (1997) to a model with multiple employment spells.

Lemma 3. *If $V(b, \theta)$ is concave, $q = \beta$ and $\mu = 0$, then*

(i) $c_e(b, 0) > c(b, 0) > c_u(b, 0)$.

(ii) $c(b, 1) > c(b, 0)$.

(iii) $b > b_u(b, 0) > b_e(b, 0)$ *and* $b_u(b, 1) > b = b_e(b, 1)$.

Proof. (i) Assume that $c_u(b, 0) \geq c_e(b, 0)$. Then from equation (19), $b_e(b, 0) \geq b_u(b, 0)$. From (18) it is immediate that $c \in (c_e, c_u)$ and thus that $b_u(b, 0) \geq b$. By concavity of V, $c(b, \theta)$ is non-decreasing, and thus $c(b, 0) \geq c_e(b, 0) \geq c(b, 1)$, where the last inequality follows from $b_e(b, 0) \geq b_u(b, 0) \geq b$. When $\theta = 1$, we have that $b = b_e(b, 1)$. Moreover, since $c(b, 0) \geq c(b, 1) = c_u(b, 1)$, $b \geq b_u(b, 1)$. Collecting inequalities yield

$$b_e(b, 0) \geq b_u(b, 0) \geq b = b_e(b, 1) \geq b_u(b, 1)$$

[6] One may see this from the incentive compatibility constraint: $v'(p) = \beta(V(b_e) - V(b_u))$. How should a given inflow of period $t+1$ resources be divided between b_e and b_u such that p is kept constant?

From the budget constraint, and using the fact that $w > 0$, this implies that $c(b, 1) > c(b, 0)$, which contradicts $c(b, 1) \leq c(b, 0)$. Since $c(b, 1) \leq c(b, 0)$ was a corollary of $c_u(b, 0) \geq c_e(b, 0)$, we must have $c_u(b, 0) < c_e(b, 0)$.

Claims *(ii)* and *(iii)* are immediate consequences of the proof of *(i)*. □

The mechanisms underlying the proof can be seen from equation (20), in which the utility gain/cost from a marginal increase in p is equalized. If $c_u > c_e$, the left-hand side in equation (20) states the utility *gained* through a marginal increase in p. It is a gain since a small increase in c_e, accompanied with a decrease in c_u, attains the marginal change in the right-hand side of the incentive compatibility constraint (14) necessary to accompany the change in p. Such a change provides more insurance and thus *increases* utility. However, due to interiority, there is an associated utility *cost*; from equation (19), b_e must be larger than b_u, and an increase in p thus increase the share of the budget spent on period $t + 1$ resources. The proof then proceeds by showing that $c_u > c_e$ together with $b_u < b_e$, cannot be budget feasible since the wage when employed is strictly positive.

In a two period setting, the terms b_e and b_u in equation (19) may be replaced by $c_e - w$ and c_u, respectively. The intuition behind the result in Lemma 3 is then straightforward: To provide incentives to exert search effort, the government generates a positive correlation between employment and consumption, $c_e > c_u$. Insurance is provided by a low intertemporal variance, $c_e > c > c_u$. Concavity then ensures that this logic extends to a setting with an infinite planning horizon.

Remarks. In Lemma 3, concavity of $V(b, \theta)$ is assumed.[7] The assumption is common in the literature and is indispensable for the analysis (Hopenhayn and Nicolini, 1997; Ljungqvist and Sargent, 2004). The difficulty in proving concavity lies in the fact that the choice set in (13) is not necessarily convex, and that (functions of) some choice variables do not enter the Bellman equation additively.[8]

Previous studies on optimal unemployment insurance abstract from self-insurance (e.g. Shavell and Weiss (1979), Hopenhayn and Nicolini (1997) and Pavoni (2007)). In the absence of savings, the policy implication from Lemma 3 is lucid; the tax/subsidy policy is defined as the difference between consumption and labor income, and benefit payments should therefore decrease along the duration of an unemployment spell. While Lemma 3

[7]Indeed, conditions (17)-(20) are derived using Benveniste and Scheinkman's (1979) envelope theorem - a theorem that requires concavity.

[8]Note that these are sufficient, but not necessary conditions for concavity. All numerical solutions in this paper and, for instance, in Hopenhayn and Nicolini (1997) and Ljungqvist and Sargent (2004) display a strictly concave value function (or, equivalently, a strictly convex cost function).

reveals that the consumption pattern remains unaltered in the current setting with self-insurance (given $\mu = 0$), there are strong a priori reasons to believe that the unemployment benefit policy does not: Most theoretical models of self-insurance and unemployment risk (e.g. Aiyagari (1994)) display a decreasing consumption profile even in the absence of *any* unemployment benefit programme. It is thus the aim of the next section to characterize the policy that can implement the optimal allocations in an economy with self-insurance.

3.2. **Decentralization.** The previous section characterized the constrained Pareto-optimal allocations attainable in the economy. This section will demonstrate *how* these allocations may be attained in a setting in which *the agents* choose consumption, search effort, and savings, taking the government's policy as given. The ultimate task of this section is thus to find the a policy such that the agents' private choices corresponds to the optimal allocations derived above.

This section is divided into two parts. The first part will show that there exist a decentralized unemployment insurance policy that implements the optimal allocations. Moreover, it is shown that this policy is recursive in an agent's wealth level, and that benefit payments, as well as any reemployment taxes, are constant when the liquidity constraint is binding. The aim of the second part is then to characterize the shape of the optimal policy functions, and the sequence of benefit payments along the duration of an employment spell. It is shown that benefits payments are decreasing in an agent's asset position and peaks when the liquidity constraint is binding. Over the course of unemployment, the agent decumulates assets and the sequence of benefit payments is thus non-decreasing.

3.2.1. *A fiscal implementation.* The agents in the decentralized economy have access to a riskless bond, a, that pays net (pre-tax) return equal to r. Borrowing is subject to a liquidity constraint, ϕ, such that $a_t \geq \phi$ for all t. At time zero, the agents enter a market economy with a given level of cash-on-hand equal to b_0. For a given tax policy, the agents maximize their utility by choosing consumption, savings, and search processes that fulfill their intertemporal budget constraint. If there is a one-to-one correspondence between the chosen processes and the optimal allocation, $\boldsymbol{\sigma}^*$, the tax allocation is called a *fiscal implementation* of $\boldsymbol{\sigma}^*$.

Formally,

Definition 1. *Let* $b_0 = a_0 - T_0$ *be given. If there exist a tax allocation* $\hat{T} = \{T_t\}_{t=0}^{\infty}$, $T_t : \Theta^t \times \mathbb{R}^t \to \mathbb{R}$, *such that* $\{c_t, a_{t+1}, p_t\}_{t=0}^{\infty}$ *solves*

$$V(b_0, \theta_0) = \max_{\{c_t, a_{t+1}, p_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \int_{\Theta^{t+1}} \{u(c_t(b_0, \theta^t)) - (1-\theta_t)v(p_t(b_0, \theta^t))\}\lambda(b_0, \theta^t)d\theta^t \quad (22)$$

subject to

$$w\theta_t + a_t(b_0, \theta^{t-1}) - T_t(\theta^t, a(b_0, \theta^t)) = c_t(b_0, \theta^t) + qa_{t+1}(b_0, \theta^t) \tag{23}$$

and

$$a_{t+1} \geq \phi, \qquad \text{for } t = 0, 1, \dots \tag{24}$$

and $\{c_t, p_t\}_{t=0}^{\infty}$ *equals the optimal allocation* $\boldsymbol{\sigma^*}$*, then* $\hat{T}$ *is said to be a* ***fiscal implementation*** *of* $\boldsymbol{\sigma^*}$*.*

Note that the tax allocation has a very general form. Taxes in any period t may depend on the full history of employment as well on the full history of asset positions. The motivation underlying this formulation is not obvious; since the agents choose $t+1$ assets using information available up to period t, it is plausible to conjecture that taxes in $t+1$ will themselves only depend on information available up to period t. However, as shown by Kocherlakota (2005), this intuition may fail; when actions are hidden there might not exist a fiscal implementation limited to this information set. Section 3.3 will explore the underlying reasons behind this conclusion further.

The following proposition shows that a fiscal implementation exists and that the resulting tax functions are *simple*: The tax level is recursive and contingent on the agent's current transition and her level of wealth. The proof of the proposition provides some important insights and is therefore included in the main text.

Proposition 1. *There exist a time invariant tax function,* $T_t = T(a_t, \theta_t, \theta_{t-1})$*, that implements* $\boldsymbol{\sigma^*}$*.*

The proof is direct and establishes a one-to-one relationship between the government's and the agent's problem.

By Bellman's Principle of Optimality, the government's problem in (13)-(16) can be split up as

$$V(b, \theta) = \max_{c,\zeta}\{u(c) + X(\zeta, \theta)\} \tag{25}$$

$$\text{s.t.} \quad b = c - \theta w + q\zeta \tag{26}$$

$$\zeta \geq 0 \tag{27}$$

$$X(\zeta, \theta) = \max_{p,b_e,b_u}\{-(1-\theta)v(p) + \beta(pV(b_e) + (1-p)V(b_u))\} \tag{28}$$

$$\text{s.t.} \quad v'(p) = \beta(V(b_e) - V(b_u)) \tag{29}$$

$$\zeta = pb_e + (1-p)b_u \tag{30}$$

Define functions T_e and T_u as $T_e(\zeta,\theta) = \zeta - b_e(\zeta,\theta)$ and $T_u(\zeta,\theta) = \zeta - b_u(\zeta,\theta)$, respectively. By definition,

$$X(\zeta,\theta) = \max_p\{-(1-\theta)v(p) + \beta(pV(\zeta - T_e(\zeta,\theta)) + (1-p)V(\zeta - T_u(\zeta,\theta)))\}$$

Thus,

$$\begin{aligned} V(b,\theta) &= \max_{c,\zeta}\{u(c) + \max_p\{-(1-\theta)v(p) + \beta(pV(\zeta - T_e(\zeta,\theta)) + (1-p)V(\zeta - T_u(\zeta,\theta)))\}\} \\ &= \max_{c,\zeta,p}\{u(c) - (1-\theta)v(p) + \beta(pV(\zeta - T_e(\zeta,\theta)) + (1-p)V(\zeta - T_u(\zeta,\theta)))\} \\ \text{s.t.}\quad & b = c - \theta w + q\zeta \\ & \zeta \geq 0 \end{aligned}$$

Where the last equality follows, again, from the Principle of Optimality. By construction, if $a' = \zeta$, the above Bellman equation is the recursive formulation of the decentralized problem given in Definition 1. □

The above proposition hinges upon an important assumption: As in Kocherlakota (2005) and Albanesi and Sleet (2006), I assume that the fiscal implementation is such that the optimal allocation is *"affordable"*. Affordability means that if the agent had the possibility to buy the optimal allocation, she would period-by-period afford it. That is,

$$w\theta_t + a_t - T_t = c_t + q(p_t b_{e,t+1} + (1-p_t)b_{u,t+1})$$

This restriction is crucial for separating the effect of savings and taxes on consumption. Affordability implies that the government's state variable, b_t, must equal the agent's non-labor cash-on-hand $a_t - T_t$. As a consequence, taxes are strictly redistributive

$$a_{t+1} = (p_t(a_{t+1} - T_{e,t+1}) + (1-p_t)(a_{t+1} - T_{u,t+1})) \tag{31}$$

Or, said differently, *the insurance system is actuarially fair*: The premium paid, $p_t T_{e,t+1}$, equals the expected cost, $-(1-p_t)T_{u,t+1}$.

By Lemma 3, it is thus immediate that $b_{u,t+1} > a_{t+1} > b_{e,t+1}$. The agent is consequently positively taxed when employed and negatively taxed when unemployed (or equivalently, receiving an unemployment benefit).

When savings and taxes are identified as above, the intuition underlying Proposition 1 is quite straightforward. Bellman's Principle of Optimality reveals that savings, a', is a sufficient state variable for the choice of b_e, b_u and p. The tax functions are then defined as the difference between savings and the optimal $t+1$ non-labor cash-on-hand, b_e and b_u. By the design of the tax function, the agent can always choose the assigned allocation. Any other (privately) feasible choice amounts to imitating the $t+1$ allocation of some other agent. By construction, imitating someone else is incentive feasible. Thus, since the allocation is optimal under incentive compatibility and budget feasibility, imitation cannot be optimal.

The tax functions in Proposition 1 are recursive in an agent's wealth, her current and previous employment state. Akin to the tax functions that map *savings* to state contingent cash-on-hand, functions $b_e(b, \theta)$ and $b_u(b, \theta)$ map period t resources to period $t + 1$ state contingent cash-on-hand. Why, then, could the tax functions not be recursive in (b, θ)? Inasmuch the optimal allocation still would be attainable for an agent operating in the decentralized economy, choosing the allocation would no longer be optimal: Imitating someone else is feasible, but *not* incentive compatible with respect to the imitatee. By the same logic underlying the inverse Euler equation, the agent would, then, increase savings to equalize equation (21), violating the incentive compatibility of the optimal allocation.

The following corollary states that unemployment benefits as well as any reemployment taxes must be constant whenever the liquidity constraint is binding.

Corollary 1. *For all b such that the liquidity constraint is binding, benefit payments and taxes are independent of b.*

Proof. By Proposition 1, savings is a sufficient state for the choice of T_e and T_u. Thus, conditional on $a' = 0$, T_e and T_u are independent of b. □

Remarks. It is important to note that the results in Proposition 1 and Corollary 1 are derived under quite general conditions. In particular there are no assumptions on the economy's interest rate or concavity/convexity assumptions on equilibrium functionals.

There is a continuum of tax systems that may implement any incentive feasible allocation. To appreciate this, consider an arbitrary incentive feasible allocation at time t. The agent consumes c and she exerted search effort in the previous period inducing p_{-1}. Her asset position and unemployment benefit handouts equal a and τ, respectively. Then another allocation with $a' = a + \epsilon$, $\tau' = \tau - \epsilon$ and $c' = c$, is still incentive compatible, feasible, and generates the same level of utility to the agent *for any real value of* ε.[9] At one extreme, 100% wealth- and labor taxes with lump-sum transfers equal to consumption, would indeed implement *any* allocation. Arguably, such a tax system is quite draconian and does not resemble the combined usage of taxes and markets to reallocate resources observed in most current economies. At another extreme, zero taxes and individually and quantitative-contingently priced Arrow securities could be designed to exactly mimic the problem in

[9]Note that since c is left unchanged, the new allocation also respects the liquidity constraint.

(13)-(16). While perhaps elegant, and by construction optimal, such a market arrangement requires an elaborate pricing system relying on common knowledge of individual asset positions and preferences.

Ruling out such elaborate asset structures and focusing on the one bond scenario, one may, alternatively, view the problem of indeterminacy as a question regarding savings. Specifically, it is a question regarding whether it is the government, or the agent (or any combination of the two), that carries out the intertemporal allocations of resources. Of course, inasmuch there are a continuum of possible storage arrangements, one may legitimately wonder on what basis one can rationally chose between those arrangements. As in Kocherlakota (2005) and Albanesi and Sleet (2006), this paper imposes two assumptions in order to identify the effect of self-insurance from taxes/benefits on consumption. First, agents save using a riskless bond. The presence of a riskless bond can be thought of as a parsimonious representation of a more elaborate underlying diversified portfolio choice of assets uncorrelated with private employment status. Second, the optimal allocation is assumed to be period-by-period affordable. Fundamentally this assumes that all intertemporal transfers of resources are actualized by the agents' savings. As a consequence, the optimal policy closely resembles a non-profit insurance program with actuarially fair prices; the premium of the insurance equals the expect cost of the benefit payment. This identification scheme guarantees to attain the optimal allocation with minimal governmental interference.[10]

3.2.2. *The shape of the benefit function.* While taxes has been shown to have a simple recursive representation, so far little has been shown regarding their properties. Examining the qualitative properties of the tax function T corresponds to examine how $T = a - b$ responds to a change in a. To this end, I will derive and exploit the properties of the *marginal tax functions.*

This section will state the main results, supported by brief comments. In the following section, I will relate the results presented here to properties of a "weighted" Euler equation, and, in turn, relate this equation to the inverse Euler equation. Since taxes when employed are lump-sum and constant, focus is put on the case of interest at $\theta = 0$. To facilitate notation, let $T_e(a')$ and $T_u(a')$ denote period $t + 1$ taxes at the associated employment states at $\theta = 0$.

[10]Allowing the government to intertemporally allocate resources using her own storage technology, however subject to some "iceberg cost", would endogenously identify savings, and thus taxes, as in the current setting.

Proposition 2. *If $V(b,\theta)$ is concave, there exist marginal tax functions given by*

$$T_e'(a') = 1 - \frac{u'(c_u)}{pu'(c_u) + (1-p)u'(c_e)}, \quad T_u'(a') = 1 - \frac{u'(c_e)}{pu'(c_u) + (1-p)u'(c_e)}$$

Proof. See Appendix A. □

The idea behind the proof is to consider an infinitesimal change in a'. The resulting marginal change in taxes must be such that the government's first order conditions hold, incentive compatibility is preserved and the budget balances. In addition, the agent's decentralized first order condition must hold

$$u'(c) - \mu = \frac{\beta}{q}(pu'(c_e)(1 - T_e'(a')) + (1-p)u'(c_u)(1 - T_u'(a')))$$

Corollary 2. *If $V(b,\theta)$ is concave, $\beta = q$, and $\mu = 0$, then both unemployment benefits and "reemployment taxes" are decreasing with the agents asset position.*

Proof. Combining the marginal taxes in Proposition 2 with the inverse Euler equation in (18) gives

$$T_e'(a') = 1 - \frac{q}{\beta}\frac{u'(c) - \mu}{u'(c_e)}, \quad T_u'(a') = 1 - \frac{q}{\beta}\frac{u'(c) - \mu}{u'(c_u)}$$

If $\beta = q$, $\mu = 0$, and since $c_e > c > c_u$, it is evident that $T_e'(a') < 0$ and $1 > T_u'(a') > 0$.[11] □

The intuition behind this result is simple, yet subtle, and is relegated to Section 3.3.

3.2.3. *Benefit payments and the duration of unemployment.* The main part of the literature on optimal unemployment insurance has concluded that benefit payments ought to decrease along the duration of unemployment. The result is intuitive; in the absence of savings, a decreasing benefit profile induces a decreasing consumption profile, providing both insurance as well as sufficient search effort incentives. Abstracting from savings, Lemma 3 confirms this result. Nevertheless, Proposition 1 shows that this result does not immediately generalize to a setting in which partial self-insurance is present: The tax policy is time-invariant and thus independent of the duration of the unemployment spell. In addition, the following two propositions reveal that the intuition supporting a decreasing benefit profile fails in the current setting. Indeed, along the duration of the unemployment spell, the agent will decumulate assets and the sequence of unemployment benefits will observationally be *non-decreasing.*

Proposition 3. *If $V(b,\theta)$ is concave, $\mu = 0$, and $\beta = q$, then (i) $a > a'$, (ii) $T_u(a) > T_u(a')$, and (iii) $T_e(a) < T_e(a')$.*

[11] Note that the assumption of $\mu = 0$ is without any loss of generality. As was shown in Corollary 1, benefit payments are constant when $\mu > 0$.

Proof. By Proposition 2, $1 > T_u'(a') > 0$. Thus for any a_1 and a_2, such that $a_1 > a_2$, $T_u(a_1) > T_u(a_2)$. If $a' \geq a$, $1 > T_u'(a')$ implies that $b_u \geq b$, which contradicts Lemma 3, part *(iii)*. Thus $a > a'$, $T_u(a) > T_u(a')$ and, by Proposition 2, $T_e(a) < T_e(a')$. □

The result is intuitive. During unemployment, the agent exploits the insurance effect of savings by decumulating assets. Proposition 2 infers that unemployment taxes are positively related to the agent's asset position. Thus, as the agent's level of assets decline, so does the level of the tax. Since unemployment taxes are negative this implies that unemployment benefits will increase.

Proposition 4. *If $X(a', \theta)$ in (28) is concave, and $q = \beta$, there exist an interval $[\bar{b}, \underline{b}]$, such that for any $b \in [\bar{b}, \underline{b}]$, $a'(b, 0) = 0$ and $b_u(0, 0) = \underline{b}$.*

Proof. Let $\theta = 0$ be implicit throughout the proof. Note that concavity of X implies strict concavity of V. From the first order conditions of (25)-(27), $a'(b)$ is strictly increasing in b when $\mu = 0$. By the Maximum Theorem, $a'(b)$ is a continuous function (Stokey, Lucas and Prescott, 1989). Thus there exist a $\bar{b}$ such that $a'(\bar{b}) = 0$ and $\mu = 0$. By Lemma 3, $\bar{b} > b_u(0)$. Now, consider a $b \in (\bar{b}, b_u(0)]$. The proposition claims that $a'(b) = 0$ and that $\mu > 0$. Assume the opposite; that is, $a'(b) \geq 0$ and $\mu = 0$. Since concavity of X implies strict concavity of V, $c(b) \geq c(\bar{b})$. By the budget constraint (26), this implies that $b \geq \bar{b}$ which contradicts $b \in (\bar{b}, b_u(0)]$. Thus for any two $b, b' \in [\bar{b}, b_u(0)]$, $a'(b) = a'(b') = 0$ and $\underline{b} = b_u(0)$. □

The intuition underlying the proposition is straightforward: If the constraint is binding at a certain b, then it is binding for any $b' < b$. The policy function from (28)-(30) is denoted $b_u(a')$. Since for any binding b, a' is by definition equal to 0. As long as b is a binding value, b_u is independent of b (see Corollary 1). Thus, $b_u(0)$ is the lowest possible value of b, and $a'(b) = 0$ at $b = b_u(0)$.

Accompanied with the inverse Euler equation, Proposition 3 has an intuitive explanation. First, wealth has a first order insurance effect. The higher is an agent's wealth, the less she needs to worry about loss of consumption if she loses her job. Second, in order to provide incentives to exert search effort, the government wishes to generate a positive correlation between consumption and employment. When the agent's utility function is concave, a higher level of savings makes it costlier for the government to induce such a correlation and the agent's search effort decreases. By generating a negative correlation between savings and unemployment benefits, the government manages to mitigate the distortionary effect of savings on search.

3.3. **The Euler equation, taxes, and the inverse Euler equation.** I now provide a deeper intuition underlying some of the results presented in the preceding sections. To this end I will consider an equivalent version of the government's problem in which the sole choice is strictly intertemporal, and not state contingent. For expositional clarity, it is assumed throughout that $\mu = 0$. It will be shown how this problem formulation leads to a "weighted Euler equation", and further how these weights relate to marginal taxes. At the optimum, the weighted Euler equation implies the inverse Euler equation.

As noted in Section 3.1, the inverse Euler equation can be thought of as the outcome when savings are chosen to balance two conflicting forces: To maximize utility, resources should be allocated to where the marginal benefit of resources is relatively high. For incentive compatibility, resources should be allocated to states in which the marginal benefit of resources is relatively low. Since incentive compatibility inflicts with period $t+1$ resources only, it is thus optimal relegate a relatively high degree of resources to period t consumption. As a result, the agent *appears* savings constrained.

For a given value of savings, it is instructive to think of the optimal division of period $t+1$ resources across employment states as *functions* fulfilling two restrictions: The incentive compatibility constraint and the budget constraint. Similar to the tax functions explored in the previous section, these functions then allocate, for a given level of savings, resources to the different employment states. Let the government choose savings, a', and let the functions $\delta_e(a')$ and $\delta_u(a')$ allocate resources between employment states such that the budget is balanced and incentive compatibility holds. That is, for a given p, $a' = p\delta_e(a') + (1-p)\delta_u(a')$ and $v'(p) = \beta(V(\delta_e(a')) - V(\delta_u(a')))$.

The government then faces the following intertemporal maximization problem

$$V(b) = \max_{a'}\{u(b - qa') + \beta(pV(\delta_e(a')) + (1-p)V(\delta_u(a')))\}$$

The first order condition to the above problem, evaluated at the optimal solution, is given by

$$u'(c) = \frac{\beta}{q}(pV'(b_e)\delta_e'(a') + (1-p)V'(b_u)\delta_u'(a')) \tag{32}$$

Equation (32) resembles a standard Euler equation, and has an interpretation in terms of marginal intertemporal trade-offs: The utility cost of an marginal increase in savings (left-hand side) equals its feasible marginal utility gain (right-hand side). As with standard intertemporal problems, the $t+1$ feasible marginal utility gain is determined by the feasible inflow of resources in period $t+1$ - a marginal decrease of period t consumption is accompanied by a proportional marginal increase of period $t+1$ resources, weighted by the interest rate: $1 = p\delta_e'(a') + (1-p)\delta_u'(a')$. In addition, however, there is a further restriction on how the period $t+1$ resources must be divided between employment states. In order to leave p unaltered, a *marginal* incentive compatibility constraint must hold

$$V'(\delta_e(a'))\delta_e'(a') = V'(\delta_u(a'))\delta_u'(a') \tag{33}$$

One can combine this marginal incentive compatibility constraint with the "marginal budget constraint" above, to solve for the weights $\delta'(a')$

$$\delta'_e(a') = \frac{V'(b_u)}{pV'(b_u) + (1-p)V'(b_e)}, \quad \delta'_u(a') = \frac{V'(b_e)}{pV'(b_u) + (1-p)V'(b_e)} \tag{34}$$

The expressions above reveals an important feature: Whenever $V'(b_u) > V'(b_e)$, $\delta'_e(a') > \delta'_u(a')$, and vice versa. That is, for states in which the marginal value of resources is relatively high, the marginal inflow of resources should be relatively low. Substituting the relationship in (34) into (32) gives the inverse Euler equation.

It is important to note that the functions in (34) are directly related to the marginal taxes derived in Proposition 2. In particular, $\delta'(a') = 1 - T'(a')$. The intuition underlying the shape of the tax function then becomes evident: For a certain choice of p to remain incentive compatible, an increase in savings must be divided between employment states such that the incentive compatibility constraint holds. That is, the inflow of resources must be relatively high at states in which the marginal value of resources is relatively low. By Lemma 3, the marginal value of resources is high in the unemployed state, and the additional inflow must therefore be low. Since the optimal policy is recursive in an agent's wealth, a higher level of assets must induce a lower level of unemployment benefits.

Additionally, the marginal incentive compatibility constraint in (33) illuminates the answer to a further inquiry explored in the literature (e.g. Kocherlakota (2005), Section 3): As savings are chosen on the basis of information available in period t, could period $t+1$ taxes be a function of period t information only? That is, could $\delta'_e(a')$ equal $\delta'_u(a')$? From equation (33) it is straightforward to see that this cannot be the case. In order for incentive compatibility to hold, period $t+1$ taxes can only be a function of period t information if (and only if) $V'(b_e) = V'(b_u)$, or, equivalently, if $c_e = c_u$. Under all other circumstances, a tax contingent on period t information only would, with certainty, violate the incentive compatibility constraint.

4. Quantitative Analysis

To shed further light on the properties of the optimal unemployment insurance program, I turn to a calibrated version of the model. The aim of this section is to quantitatively characterize the optimal unmployment insurance programme, and to calculate the potential cost-savings of the optimal versus the current insurance system in the United States.

As will be shown, benefit payments equal roughly 60% of the preunemployment wage for liquidity constrained individuals, and swiftly declines to - and levels off at - around 1.5%

for the affluent. As percentage of the total expected cost of the current US insurance programme, the potential cost-savings of the optimal programme ranges from 7% for liquidity constrained individuals with zero liquid wealth, to approximately 33% for the wealthy.

4.1. **Calibration.** Following the main macroeconomic literature the function u is chosen to be of the type constant relative risk aversion

$$u(c) = \lim_{\rho \to \sigma} \frac{c^{1-\rho}}{1-\rho}$$

The coefficient of risk-aversion σ is set to 2.[12] As in Pavoni (2007) and Pavoni and Violante (2007), the length of each period is assumed to be one month. The yearly interest rate is set at 5% and the intertemporal discount factor β is thus $1.05^{-1/12}$. In order for the results to be comparable with the previous (contractual) literature on unemployment insurance, the hazard rate of unemployment, $1-\gamma$, is set to zero.[13] Employment is thus an absorbing state. The net wage, w, is normalized to unity.

The function governing the disutility of search effort, v, is assumed to have the following functional form

$$v(p) = -\frac{\ln(1-p)}{\alpha} - \frac{p}{\alpha}$$

Note that v is strictly convex on $[0,1]$ and that $v(0) = 0$, $v'(0) = 0$ $v(1) = \infty$ and $\lim_{p \uparrow 1} v'(1) = \infty$. Several articles on optimal unemployment insurance (e.g Hopenhayn and Nicolini (1997), Young (2004) and Wang and Williamson (2002)), assume that $p(e) = 1 - \exp(-\alpha e)$ and that the disutility of search is linear and equals e. A choice consistent with the literature would thus be $v(p) = -\ln(1-p)/\alpha$. To simplify computation, however, the above simple modification to the standard function is employed.[14]

To calibrate the parameter α in the function v - the parameter determining the degree of moral hazard in the model - an auxiliary economy is used. The auxiliary economy is given as the problem in equations (22)-(24), but in which the government's policy, T, is exogenously specified. I describe this economy in detail in Appendix B. The exogenous replacement rate T is constant for perpetuity and is set to 66% (Meyer, 1990; Hopenhayn and Nicolini, 1997).[15] I choose α such that the elasticity of the hazard rate of reemployment with respect to T for a liquidity constrained individual, matches the estimate obtained for

[12] Empirical estimates show that this parameter roughly ranges from 1 to 3. (Mehra and Prescott, 1985).

[13] With the previous literature, I refer to Shavell and Weiss (1979), Hopenhayn and Nicolini (1997), Pavoni (2007) and Pavoni and Violante (2007).

[14] Note that without this modification $v'(0) = \frac{1}{\alpha} \neq 0$, and interiority is not guaranteed.

[15] Although unemployment benefits in the United States lapses after six months, I consider as an approximation a perpetually lasting replacement rate as a benchmark for calibration.

this group by Chetty (2007). In particular, α is set to 1.4 which generates an elasticity of $-.72$.

Table 1 summarizes the baseline parameter calibration.

TABLE 1. Calibration of Parameters

Parameter	β	σ	r	γ	w	α
Value	.996	2	.41%	1	1	1.4

It should be noted that I target the elasticity of the hazard rate of reemployment with respect to this replacement rate *for a liquidity constrained individual.* The underlying reason is simple: Since the interest rate is set exogenously, the auxiliary economy displays a partial equilibrium. As such, the economy does not generate a realistic endogenous distribution of wealth holdings, and terms such as *average agent*, or *median wealth holdings* become meaningless. On the other hand, the constraint restricting access to liquid funds is exogenously given and constant, and for *any* endogenous distribution there will be an atom of agents at the binding level of the liquidity constraint. Thus, a calibration targeted to match features of the data at this part of the distribution is still a consistent practice.

4.2. **Computational Procedure.** I compute the model by jointly solving the three equilibrium functionals in (18)-(20), together with the incentive compatibility constraint (14), the budget constraint (15), and the liquidity constraint (16). The numerical method used is time-, or policy function, iteration as described in Coleman (1990). The main advantage of this procedure is its (virtually) global convergence properties for Pareto-optimal problems, even in the presence of inequality constraints, such as restricted borrowing (Rendahl, 2006). Time iteration also does not rely on discretization of the state space, but instead requires interpolation techniques that preserve the continuous nature of the state space. This is a salient feature since it dramatically increases accuracy (Judd, 1998). In order to simplify the numerical computations, I follow Carroll (2006) and create a grid in savings, a', rather than in non-labor cash-on-hand b. I use 200 logarithmically spaced gridnodes for savings, ranging from value of the liquidity constraint at zero to 12. Linear interpolation is employed to evaluate functions at values in between nodes. Further increasing the number of nodes, or altering the interpolation procedure, does not lead to any changes in the results.

4.3. **Numerical Results.** The solid line in Figure 1 depicts how the level of unemployment benefits are related to an agent's asset position. An agent's wealth is featured on the horizontal axis. Wealth ranges from zero to the US median labor income to wealth ratio (which is, on yearly basis, equal to one). The vertical axis displays the level of unemployment benefits as a percentage of the monthly wage.

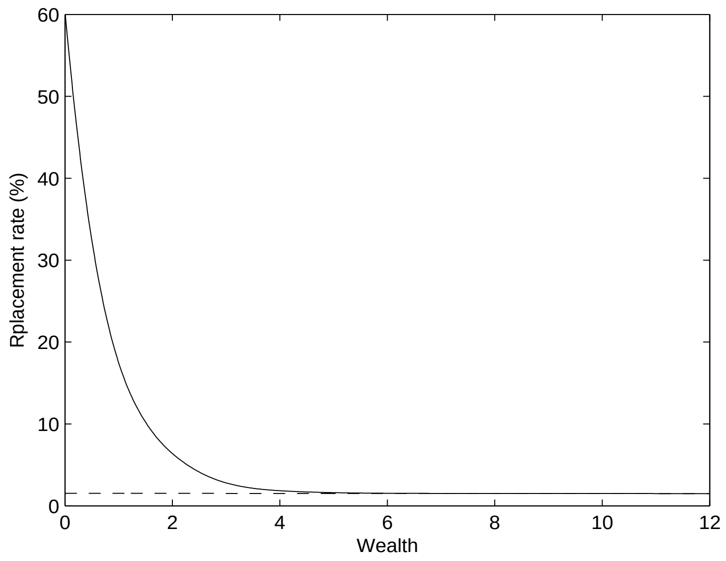

Figure 1. Unemployment benefits and wealth (solid line).

The figure reveals three illuminating patterns. *First*, the replacement rate for constrained agents with zero liquid wealth is optimal at 60%, markedly in line with the average current US level at 66%, which was used for calibration (Meyer, 1990; Hopenhayn and Nicolini, 1997).[16] Thus, at least for this subgroup, there are reasons to believe that the current level of the US replacement rate is close to optimal. *Second*, unemployment benefits for the asset poor ought to be orders of magnitude of that of a wealthy agent. For instance, unemployment benefits paid to an agent with wealth equal to three month labor income - wealth enough to sustain an average unemployment spell - is 5% of the level paid to a constrained agent with zero liquid wealth. *Third*, unemployment benefits for the affluent appears to be extremely close to constant. Taken together, these observations give support for an asset based means tested unemployment insurance programme.

It is tempting to conclude that the swift negative relationship between unemployment benefits and wealth is a consequence of the qualitative results proved in Proposition 2 and Corollary 2. This is not the case. The dashed line in Figure 1 represents the solution to the optimal problem *in the absence of credit restrictions*.[17] As is clear from the figure, the liquidity constraint is responsible for virtually the entire effect. The reason is due to what

[16]Re-calibrating to a replacement rate of 50% (as in Davidson and Woodbury (2002)), leaves the optimal level of 60% virtually unchanged. In fact, this level is surprisingly robust to changes in the parameter α.

[17]As Proposition 2 and Corollary 2 predicts, this line is strictly decreasing, although the scale in Figure 1 conceals this fact.

Chetty (2007) refers to as a "liquidity effect": Absent the ability to smooth consumption through borrowing, a relatively low level of benefit payments is accompanied by a very low present value utility of unemployment. A constrained agent thus faces severe pressure of finding a new job quickly, leading to a short duration of unemployment. However, this enhanced search intensity is driven by a market failure - incomplete credit and risk-sharing markets - and thus widely exceeds the social optimum. As a consequence, benefit payments should be increased in order to compensate for this missing market, and the search effort will then approach its socially optimal level.

4.3.1. *Potential cost-savings.* Figure 2 depicts the potential cost-savings gained by replacing the current US system with the optimal benefits programme. An agent's net-worth under the current US system, defined as current wealth plus the expected net present value of future benefit payments, is depicted on the horizontal axis. By construction of the liquidity constraint in Appendix B, net-worth ranges, as wealth in the optimal programme, from zero to twelve. The vertical axis displays the potential cost-savings of switching to the optimal programme as a percentage of the net present value of future benefit payments of the current programme.

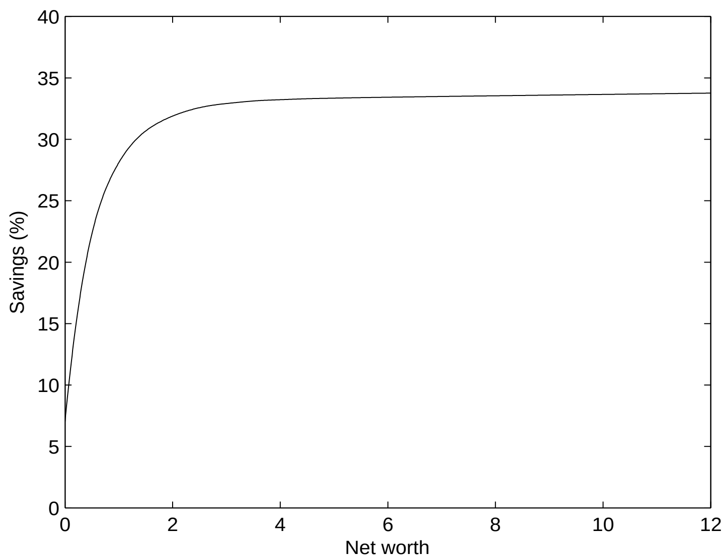

FIGURE 2. The potential cost-savings of an optimal unemployment insurance scheme as percentage of the expected cost of the current unemployment insurance programme.

Not surprisingly, the potential cost-savings are quite low for the group of constrained individuals. The reason is of course that the optimal-, in similarity to the current programme, are both constant and remarkably comparable in level. Cost-savings of around 7% can however be gained by reducing the replacement rate from 66% to 60%. For the more affluent however, potential cost-savings increases dramatically, and levels off at around 33%. As a consequence, for *any* initial distribution of wealth in the economy, cost-savings ranges from the minimum at 7% to the, perhaps more likely, maximum of around 33%. According to Green and Riddell (1993), roughly 0.4% of US GDP is devoted to the unemployment insurance budget. Putting the numbers in Figure 2 in context thus reveals that potential cost-savings range from 0.03% to 0.13% of total US GDP.

Considering Figure 1 and 2 together, it appears as if affluent individuals would be better off reducing their level of insurance. This conclusion is not correct. Affluent agents would be better off with a lower insurance level *accompanied* by a lump-sum transfer of resources equal to the reduction in the present value of future benefit payments. In fact, according to Figure 2 they are willing to forgo 33% of this lump-sum transfer in order to stay indifferent.

5. Concluding Remarks

This paper has studied a model of optimal redistribution policies in which the foremost risk in an agent's life is unemployment. Moral hazard arises as job search effort is unobservable. The model permits agents to self-insure by means of a riskless bond, but access to the credit market is limited.

In contrast with previous studies in the literature, the optimal benefit payments *policy* shows no duration dependence, and relies exclusively upon an individual's current asset position. Benefit payments are decreasing in wealth and, as a consequence, peaks at a constant level when the liquidity constraint is binding. Over the course of unemployment, individuals decumulate assets and the sequence of benefit payments is thus observationally non-decreasing. In a quantitative exercise it is shown that the US unemployment insurance programme is surprisingly close to optimal for the asset poor, but too generous for wealthier individuals. The potential cost-savings of switching to the optimal program ranges from roughly 33% of the present value insurance budget for the affluent, to 7% for the less fortunate.

The policy implications from the analysis are stark; unemployment benefits should be asset based and relate negatively to wealth. As wealth itself encodes insurance possibilities, the negative relation between wealth and unemployment benefits is intuitive. There are several ways in which an asset based unemployment insurance programme could be accomplished. As with Medicaid, food stamps, and until recently, Aid to Families with Dependent

Children (AFDC), to mention a few social policies in the United States, unemployment benefits may be asset based means tested; that is, unemployment benefits are paid only if an agent has assets below a specified maximum amount. Alternatively, and obviously, schemes may be more elaborate with a continuous decline in benefit payments as assets increases. However, the most favorable, simple, asset based scheme appears to build upon two distinct features: Benefits should be positive across the whole wealth distribution, but significantly higher for the asset poor, and affluent agents should receive a one-shot lump-sum transfer that compensates for their reduction in insurance.

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Appendix A. Proofs

A.1. **Lemma 2.**

Proof. By the Principle of Optimality, the problem in (6)-(9) can be split up as

$$V(b_0, \theta_0) = \max_{c_0, a_1} \{u(c_0) - \theta_0 v(p_0) + X(a_1, \theta_0)\} \tag{A1}$$

$$\text{s.t.} \quad b_0 = c_0 - \theta_0 w + q a_1 \tag{A2}$$

$$0 \geq \phi - a_1 \tag{A3}$$

$$X(a_1, \theta_0) = \max_{\boldsymbol{\sigma}(a_1)} \sum_{t=1}^{\infty} \beta^t \int_{\Theta^t} \{u(c_t(a_1, \theta^t)) - (1 - \theta_t) v(p_t(a_1, \theta^t))\} \lambda(a_1, \theta^t) d\theta^t \tag{A4}$$

$$\text{s.t.} \quad \{p_t\}_{t=0}^{\infty} = \operatorname{argmax}\{X(a_1, \theta_0)\} \tag{A5}$$

$$a_1 = \sum_{t=1}^{\infty} q^t \int_{\Theta^t} \{c_t(a_1, \theta^t) - \theta_t w\} \lambda(a_1, \theta^t) d\theta^t \tag{A6}$$

$$0 \geq \phi - \sum_{s=1}^{\infty} q^{s-1} \int_{\Theta^{t+s}} \{c_{t+s}(a_1, \theta^{t+s}) - \theta_{t+s} w\} \frac{\lambda(a_1, \theta^{t+s})}{\lambda(a_1, \theta^t)} d\theta^{t+s}, \quad \text{for } t = 1, 2, \ldots \tag{A7}$$

Under an optimal allocation, equations (A4) and (A6) can be written as

$$X(a_1, \theta_0) = -v(p_0^*(a_1, \theta_0)) + \beta \int_{\Theta^1} V(\boldsymbol{\sigma}^*, b^*(a_1, \theta_1), \theta_1) \lambda(a_1, \theta^1) d\theta^1 \tag{A8}$$

$$a_1 = q \int_{\Theta^1} b^*(a_1, \theta_1) \lambda(b_0, \theta^1) d\theta^1 \tag{A9}$$

The proof then proceeds in three steps. The first step shows that $X(a_1, \theta_0)$ is strictly increasing in a_1. By exploiting this fact, the second step will then proceed by showing that $b^*(a_1, \theta_1)$ must be resource minimizing under promised utility $V(\boldsymbol{\sigma}^*, b^*(a_1, \theta_1), \theta_1)$. Lastly, the third step then shows that duality holds: If $b^*(a_1, \theta_1)$ is resource minimizing under utility $V(\boldsymbol{\sigma}^*, b^*(a_1, \theta_1), \theta_1)$, then $V(\boldsymbol{\sigma}^*, b^*(a_1, \theta_1), \theta_1)$ must be utility maximizing under resources $b^*(a_1, \theta_1)$.

Step 1. Assume that there is an inflow of resources to the left-hand side of (A6) equal to $\varepsilon > 0$. For notational convenience, define c_1 and c_0 as period one consumption in the employed and unemployed state respectively. Pick an $\varepsilon_1 \geq 0$ and $\varepsilon_0 \geq 0$ such that

$$u(c_1 + \varepsilon_1) - u(c_0 + \varepsilon_0) = u(c_1) - u(c_0)$$

$$\varepsilon_1 + \varepsilon_0 = \varepsilon$$

Since the relative value between employment states are unaltered, $p_0^*(a_1, \theta_0)$ is still incentive compatible and period zero expected utility has increased by

$$\beta(p_0(u(c_1 + \varepsilon_1) - u(c_1)) + (1 - p_0)(u(c_0 + \varepsilon_0) - u(c_0))) > 0$$

Where $p_0 = p_0^*(a_1, \theta_0)$.

Step 2. Consider the following resource minimization problem:

$$b(V, \hat{\theta}_0) = \min_{\boldsymbol{\sigma}} \sum_{t=0}^{\infty} q^t \int_{\Theta^t} \{c_t(V, \hat{\theta}^t) - \hat{\theta}_t w\} \lambda(V, \hat{\theta}^t) d\hat{\theta}^t \tag{A10}$$

$$\text{s.t.} \quad V \leq \sum_{t=0}^{\infty} \beta^t \int_{\Theta^t} \{u(c_t(V, \hat{\theta}^t)) - (1 - \hat{\theta}_t) v(p_t(V, \hat{\theta}^t))\} \lambda(V, \hat{\theta}^t) d\hat{\theta}^t \tag{A11}$$

and subject to the incentive compatibility and liquidity constraint. A "hat" on the sequence θ_t is used to distinguish it from the values of θ_t in the original problem (6)-(9). It is important to note if constraint (A11) in problem (A10)-(A11) is slack, then $c_0(V, \hat{\theta}^0)$ is interior; if it was not, since $u(0) = -\infty$, the right-hand side in (A11) would equal minus infinity, and $V \geq -\infty$. It is then straightforward to see that constraint (A11) will hold as an equality. If it did not, period zero consumption could simply be reduced without interfering with neither incentive compatibility nor the liquidity constraint, reducing the objective function.

Now, consider the scenario in which $\hat{\theta}_0 = \theta_1$ and $V = V(\boldsymbol{\sigma^*}, b^*(a_1, \theta_1), \theta_1)$. Could $b(V, \hat{\theta}_0)$ in (A10) be smaller than $b^*(a_1, \theta_1)$, for at least one value of θ_1? Assume that it is. Define a_1' as $a_1' = p_0(a_1, \theta^0)b(V, 1) + (1 - p_0(a_1, \theta^0))b(V, 0)$, and note that $a_1 > a_1'$, and that a_1' is budget feasible, incentive compatible and delivers utility $V(b_0, \theta_0)$. a_1' might not, however, respect the time zero liquidity constraint. Pick an a_1'' such that $a_1 > a_1'' > a_1'$. Then, since $X(a_1', \theta_0)$ is strictly increasing and continuous (Aliprantis and Border, 1999), $X(a_1'', \theta_0) > X(a_1, \theta_0)$, which violates the optimality of $V(b_0, \theta_0)$. Thus, $b^*(a_1, \theta_1)$ is resource minimizing under promised utility $V(\boldsymbol{\sigma^*}, b^*(a_1, \theta_1), \theta_1)$.

Step 3. In order to complete the proof, it must be shown that $V(\boldsymbol{\sigma^*}, b^*(a_1, \theta_1), \theta_1)$ attains the maximum value under resources $b^*(a_1, \theta_1)$.

Assume that $V(b^*(a_1, \theta_1), \theta_1) > V(\boldsymbol{\sigma^*}, b^*(a_1, \theta_1), \theta_1)$. By Berge's Maximum Theorem (Aliprantis and Border, 1999), $V(b^*(a_1, \theta_1), \theta_1)$ is continuous in b. By the same argument as above, $c_1(b^*(a_1, \theta_1), \theta_1) > 0$ since $u(0) = -\infty$. Thus there exist a $b^{**}(a_1, \theta_1)$ arbitrarily close to $b^*(a_1, \theta_1)$ such that $b^*(a_1, \theta_1) > b^{**}(a_1, \theta_1)$ and $V(b^{**}(a_1, \theta_1), \theta_1) > V(\boldsymbol{\sigma^*}, b^*(a_1, \theta_1), \theta_1)$. This contradicts that $b^*(a_1, \theta_1)$ was resource minimizing for $V(\boldsymbol{\sigma^*}, b^*(a_1, \theta_1), \theta_1)$. Thus $V(b^*(a_1, \theta_1), \theta_1) = V(\boldsymbol{\sigma^*}, b^*(a_1, \theta_1), \theta_1)$. □

A.2. **Proposition 2.**

Proof. The proof is direct and derives the implied marginal taxes from an infinitesimal change in assets.

By construction, the equilibrium tax functions satisfies

$$a' = p(a')(a' - T_e(a')) + (1 - p(a))(a' - T_u(a'))$$

Thus, if the tax functions are differentiable, the following must hold for the marginal tax

$$p'(a')(T_u(a') - T_e(a')) = pT_e'(a') + (1 - p)T_u'(a') \tag{A12}$$

From the incentive compatibility constraint we have

$$v''(p)p'(a') = \beta(V_e'(a')(1 - T_e'(a')) - V_u'(a')(1 - T_u'(a'))) \tag{A13}$$

Substituting the relationships $b_e = a' - T_e(a')$ and $b_u = a' - T_u(a')$ into (19) (the government's first order condition for p) gives

$$q(T_u(a') - T_e(a')) = \zeta v''(p) \tag{A14}$$

Where ζ is the ratio of the multipliers on the budget and incentive compatibility constraint, respectively. Elementary algebra shows that $\zeta = p(1-p)(1/u'(c_u) - 1/u'(c_e))$ (see equations (19) and (20)). Substituting (A14) into (A12)

$$p'(a')v''(p)\zeta = pT_e'(a') + (1 - p)T_u'(a') \tag{A15}$$

Substituting (A13) into (A15)

$$\frac{\beta}{q}(V_e'(a')(1 - T_e'(a')) - V_u'(a')(1 - T_u'(a')))\zeta = pT_e'(a') + (1 - p)T_u'(a') \tag{A16}$$

In addition, the agent's decentralized first order condition must hold:

$$u'(c) - \mu = \frac{\beta}{q}(pu'(c_e)(1 - T_e'(a')) + (1-p)u'(c_u)(1 - T_u'(a'))) \tag{A17}$$

Using equation (18) and solving equations (A16) and (A17) yields

$$T_e'(a') = 1 - \frac{u'(c_u)}{pu'(c_u) + (1-p)u'(c_e)}, \quad T_u'(a') = 1 - \frac{u'(c_e)}{pu'(c_u) + (1-p)u'(c_e)} \qquad \square$$

Appendix B. An auxiliary economy

The auxiliary economy is characterized by the following Bellman equation

$$V(a,\theta,\delta)=\max_{c,a',p}\{u(c)-(1-\theta)v(p)+\beta(pV(a',1)+(1-p)V(a',0,\delta'))\} \tag{B1}$$

$$\text{s.t.}\quad qa'+c=\theta w+(1-\theta)T+a \tag{B2}$$

$$a'\geq -\delta \quad \text{for } \theta=0 \tag{B3}$$

$$\delta'=h(\delta) \tag{B4}$$

where of course $p=1$ if $\theta=1$. The parameter δ and its law of motion $h(\cdot)$ is taken as given by the agent, and equals the net present value of future benefit payments

$$\begin{aligned}\delta_0=\sum_{t=1}^{\infty}q^{t-1}(1-p_t)T&=E_0\sum_{t=1}^{\infty}q^{t-1}(1-\theta_t)\\&=(1-p_1)(T+q\delta_1)\end{aligned}$$

In order for the problem in (B1)-(B4) to be consistent with the government's problem (13)-(16), it has to be shown that the liquidity constraint in (B3) is equal to the liquidity constraint in (5).

Iterating the budget constraint (B2) forward, ruling out explosive paths, yields

$$a_0=E_0\sum_{t=0}^{\infty}q^t\{c_t-\theta_t w-(1-\theta_t)T\}$$

or, equivalently

$$\begin{aligned}a_0+(1-\theta_0)T+q\delta_0&=E_0\sum_{t=0}^{\infty}q^t\{c_t-\theta_t w\}\\&=c_0-\theta_0 w+qp_0E_1[\sum_{t=1}^{\infty}q^t\{c_t-\theta_t\}|\theta_1=1]+q(1-p_0)E_1[\sum_{t=1}^{\infty}q^t\{c_t-\theta_t\}|\theta_1=0]\\&=c_0-\theta_0 w+qp_0a_1+q(1-p_0)(a_1+T+q\delta_1)\\&=c_0-\theta_0 w+qa_1+q(1-p_0)(T+q\delta_1)\\&=c_0-\theta_0 w+qa_1+q\delta_0\end{aligned}$$

Thus as long as $qa_1+q\delta_0\geq 0$, equation (5) will hold. Constraint (B3) is sufficient to guarantee this.