Rotten Parents and Disciplined Children: A Politico-Economic Theory of Public Expenditure and Debt.*

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Abstract

This paper proposes a dynamic politico-economic theory of debt, government finance and expenditure. Agents have preferences over a private and a government-provided public good, financed through labor taxation. Subsequent generations of voters choose taxation, government expenditure and debt accumulation through repeated elections. Debt introduces a conflict of interest between young and old voters: the young want more fiscal discipline. We characterize the Markov Perfect Equilibrium of the dynamic voting game. If taxes do not distort labor supply, the economy progressively depletes its resources through debt accumulation, leaving future generations "enslaved". However, if tax distortions are sufficiently large, the economy converges to a stationary debt level which is bounded away from the endogenous debt limit. The current fiscal policy is disciplined by the concern of young voters for the ability of future government to provide public goods. The steady-state and dynamics of debt depend on the voters' taste for public consumption. The stronger the preference for public consumption, the less debt is accumulates. We extend the analysis to redistributive policies and political shocks. The theory predicts government debt to be mean reverting and debt growth to be larger under right-wing than under left-wing governments. Data from the US and from a panel of 21 OECD countries confirm these theoretical predictions.

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1 Introduction

There are large differences in fiscal policies and government debt across countries and over time. Budgetary policies are the subject of major political disputes, and different governments appear to pursue very diverse debt strategies. For instance, under the Republican administrations of Reagan and Bush senior, the debt-GDP ratio in the US grew uninterruptedly from 26% to 49%. Clinton's administrations reversed this trend, and brought the ratio down to 35%. Thereafter, the debt has been again rising under George W. Bush. Despite the strong public interest in these controversial changes in fiscal policy, we still have a limited theoretical understanding of the politico-economic forces determining public debt.

Public debt breaks the link between taxation and expenditure, allowing governments to shift the fiscal burden to future generations. In a world where Ricardian equivalence does not hold, this raises a conflict of interest between current and future generations. As future generations are naturally under-represented in democratic decision making, there is a politico-economic force pushing towards debt accumulation. A fundamental question is, then: what prevents the current generations from passing the entire bill for current spending to the future generations?

Financial markets could be part of the explanation; markets must believe that government liabilities will be honored. Yet, debt remains significantly below levels threatening solvency in industrialized countries. Moreover, despite the large cross-country heterogeneity in debt-GDP ratios, local interest rates respond little to the size of debt, at least among OECD countries.[1] In this paper, we abstract from effects working through changes in interest rates, and explore a complementary explanation based on the dynamics of an intergenerational conflict between voters of different ages. We model this conflict as a dynamic voting game over the provision of public goods and its financing over time. More specifically, we assume that fiscal policy is set through repeated elections, so that current governments cannot bind the policies of future governments'. The theory shows that the intergenerational conflict combined with lack of commitment (dynamic voting) lead to an endogenous discipline in fiscal policy, even in a world where agents have no concern for future generations. The strength of this discipline depends on the intensity of voters' and governments' preference for public good provision or redistribution.

[1] For instance, the interest rate is almost uniform within the Euro area, although the debt ratios are very different across member countries (from less than 30% in Ireland, to close or above 100% in Belgium, Greece and Italy). In the same vein, in the last decade Japan has been the OECD country with the highest debt-to-GDP ratio and the lowest interest rate.

To describe the theoretical mechanism, we model a small open economy populated by two-period-lived overlapping generations of agents who work when young and consume a private and a government-provided public good in both periods of their lives. The government can issue debt up to the natural borrowing constraint and is committed to repay it. Every period agents vote on public good provision, distortive labor taxation, and debt accumulation. The intergenerational conflict plays out as follows. The old voters wish to maximize current public good consumption, and thus – due to their imperfect altruism – support a high deficit. Young voters, however, are more averse to debt, because they care directly about next period's public good provision. In particular, they anticipate that future governments inheriting a large debt will cut spending on public goods. The political process, represented as a probabilistic-voting model *à la* Lindbeck and Weibull (1987), generates a compromise between these two desired policies.

The forward-looking political behavior of young voters is key. When voting on the current budget they contemplate its implications on future public good provision. Leaving a large debt to the next generation triggers three adjustments in the next period: higher taxes, lower expenditure, and further debt expansion. When the lion's share of the response is a cut in expenditure, young voters support a strongly disciplined fiscal policy today. Conversely, when future governments are expected to respond by increasing taxes and debt, the young are prepared to accept a laxer fiscal policy. Thus, it is expectations about the response of future governments to debt that shape the current fiscal policy. We embed such expectations into a dynamic-voting Markov-perfect equilibrium where the strategies of current voters are conditioned only on pay-off-relevant state variables. In our model, the only such state variable is the debt level, which greatly simplifies the analysis. The equilibrium conduct of future governments turns out to depend crucially on the extent of tax distortions. Intuitively, the more distortionary future taxation, the less future governments will be tempted to increase taxes, and the more they will instead cut public good provision in response to inherited debt. Therefore, the fiscal discipline becomes stronger when taxes are more distortionary, i.e., when the Laffer curve is more concave.[2]

We show that, in the absence of labor supply distortions, the economy would deplete resources through a progressive debt accumulation. In the long run, future generations are "enslaved", i.e., their labor earnings are fully taxed away to pay the service of the outstanding debt, and their consumption, both private and public, tends to zero. Instead, if tax distortions

[2] For example, suppose that, at some high level of taxation, labor supply becomes infinitely elastic due to international tax competition. Then, future governments cannot increase taxes, and any increase in debt must be matched by a future reduction in expenditure. In this case, tax competition strengthens fiscal discipline.

are sufficiently large, the economy converges to an "interior" debt level which is bounded away from the natural borrowing constraint. In this steady-state, both private and public good consumption are positive. Thus, tax distortions provide future generations with a credible threat that prevents fiscal abuse from their parents.[3]

The fiscal discipline hinges on the lack of commitment. In contrast, in a Ramsey problem where the first generation of voters can commit the entire future fiscal policy, debt is systematically larger than under repeated voting and converges asymptotically to the natural borrowing constraint. Thus, in our theory, on the one hand the lack of commitment reduces the welfare of the first generation of voters compared with the Ramsey allocation. On the other hand, future generations are better off in the political equilibrium than under Ramsey. In this sense, our time inconsistency has a *benign* nature; it redistributes resources from earlier to later generations.[4]

Our political equilibrium features a determinate debt level. An unexpected fiscal shock, such as a war, is financed partly by a short-term increase in debt, and partly by a temporary increase in taxation and a temporary reduction in (non-military) public good provision. When the war shock is over, debt, taxes, and public goods revert back smoothly to their steady state levels. This prediction contrasts with the tax-smoothing implication of Barro (1979). He shows that if the distortionary costs of taxation are convex, governments should use debt to absorb fiscal shocks, and spread the tax burden evenly over future periods. Thus, debt should not be mean reverting; after the war, there is no reason to reduce debt unless an opposite shock occurs. As in Aiyagari et al. (2002), the same result holds in our model under commitment. The data support this prediction of our model. Bohn (1998) shows that a short-lived increase in US government expenditures implies an increase in debt with a subsequent reversion in debt. We find that this stylized fact holds up for a panel data set of OECD countries. Moreover, as noted by Barro (1986), non-military spending is crowded out during wars in the US– exactly as our model predicts.

In the second part of the paper, we incorporate intra-generational conflict into the theory. We assume cross-sectional wage heterogeneity (persistent over cohorts) and progressive taxation. Public good provision entails then a redistributive component: the *poor* want more government expenditure than the *rich.* In equilibrium, the level of government expenditure

[3] The point that, in the presence of commitment problems, government expenditure may be higher when the tax base is more elastic echoes the analysis of Krusell, Quadrini and Rios-Rull (1997).

[4] In standard formulations, the planner only attaches a positive weight to the welfare of the first generations, while future generations enter the planner's preferences indirectly through the altruism of the first generation. For an exception, see Farhi and Werning (2005) where the planner attaches a positive weight on the welfare of all generations, resulting in an effective social discount factors exceeding the private one.

depends on the political clout of the poor relative to the rich. This is assumed to be a state variable evolving according to a two-state first-order Markov process. "Leftist" times are periods when the poor have more political influence and can impose higher taxes and expenditure relative to "rightist" times. Interestingly, governments that attach a higher weight to the interests of poor voters ("left-wing governments") will be less prone to expand debt than "right-wing governments". The rich and the poor have a different trade off between taxation and public good provision. Increasing debt today will partially finance a current tax break at the cost of crowding out future public good provision. Thus leftist governments are less eager to increase the debt.[5] Changes in the color of governments lead to changes in fiscal policy: right-wing governments run larger deficits and accumulate more debt, in spite of no difference in intergenerational altruism between left-wing and right-wing voters.

The predictions of the theory conform with the evidence from both US time series and OECD panel data that debt expansion is positively correlated with the right-wing orientation of governments. For instance, we find that in the US a shift from a democrat president to a republican one is associated with an average increase in the debt-output ratio of about 2% per year. The difference is statistically significant and robust to a number of control variables. Similar results obtain in a panel of 21 OECD countries using various alternative measures of the political orientation of governments.

Our paper contributes to a broad literature on the politico-economic determinants of government debt. A closely related literature is that on the strategic use of debt. Two important forebears are Persson and Svensson (1989) and Alesina and Tabellini (1990), who were among the first to emphasize political conflict as a driving factor for public debt. Different from us, these papers focus on two-period models without any intergenerational conflict. They therefore miss the dynamic game between generations, which in our model gives rise to fiscal discipline and limits the debt accumulation.[6]

Our paper is not the first one predicting autoregressive debt dynamics following a fiscal shock. In particular, Aiyagari *et al.* (2002) find that when the government has the ability to commit to future policies and only issues non-contingent debt, debt is stationary, albeit with a high persistence. An important recent contribution which is methodologically more similar to our paper is Battaglini and Coate (2006). They analyze fiscal policy and government debt with shocks to government policy in a legislative-bargaining model. In their model infinitely-lived

[5] Our argument is reminiscent of the right-wing "starve-the-beast" argument, i.e., that budget deficits can be used to force reductions in future social expenditure.

[6] Several authors have tested the implications of the strategic debt models, albeit with mixed success (for example, Lambertini, 2003, find little support in OECD panel data, while Pettersson-Lidbom (2001) find significant support in data on Swedish municipalities).

agents would like to commit to large government savings when the value of the public good is low (i.e., what we label "peace") and debt accumulation and public-good provision when the value of the public good is high ("war"). However, legislators can also divert resources to pork-barrel transfers to geographically-defined districts. Due to this political conflict, legislators opt for inefficient transfers instead of government savings when the debt is too low. Consequently, the equilibrium features too much debt, too little public-good provision, and stationary debt dynamics. Battaglini and Coate (2006) focus on a different mechanism from ours. While we emphasize that debt is restrained due to an intergenerational conflict, they focus on how cross-district political conflict induce excessive debt accumulation. Finally, Yared (2007) argue that debt should be persistent but stationary, due to voters trying to discipline a self-interested government. We view our paper as complementary to these papers, emphasizing a quite different mechanism for mean reversion of debt in the absence of commitment.

A growing related politico-economic literature on time-consistent dynamic fiscal policy, where heterogeneous agents vote repeatedly on redistribution and taxation, includes Krusell *et al.* (1996), Krusell and Ríos-Rull (1999), Hassler *et al.* (2003), Hassler *et al.* (2005), Song (2005a, 2005b), and Azzimonti Renzo (2005). These papers are also methodologically similar to ours, although they assume balanced government budget. One exception is Krusell *et al.* (2005), who investigate debt policies in a representative-agent Lucas-Stokey model without commitment. They find that the time-consistent policy resembles closely the time-inconsistent Ramsey plan where the debt is used to manipulate the interest rate.

Future pension liabilities are a form of government debt. Several authors have examined the political economy of pensions. The paper most closely related to ours is Tabellini (1990), who argues that pensions are driven by a coalition between young poor voters who want redistribution and retirees who want transfers. A large literature focus on the politico-economic forces that would create and sustain the pension system.[7] The focus of these papers is different insofar they assume that there is no guarantee that the debt implicit in pension systems be honored. In our model we abstract from this and from other debt repudiation issues (e.g., sovereign debt), in order ot narrow the focus on the intergenerational conflict about the timing of public-good consumption and taxation.

The paper is organized as follows. In Section 2 we describe the model environment and derive the Generalized Euler Equation which is key to the characterization of the political equi-

[7] For example, Chen and Song (2005) and Gonzalez Eiras and Niepelt (2004) show that the pension system can be sustained as a Markov equilibrium where young voters stick to a pension system in order to lower aggregate savings and thereby increase the interest rate. Other authors focus on explanations based on implicit contracts between generations, i.e., history-dependent (trigger) strategies in infinite-horizon games (see e.g. Cooley and Soares, 1999, and references therein).

librium. Section 3 provides two examples that admit an analytical solution. Section 4 analyzes the general case. Section 5 documents some stylized facts about debt dynamics in response to shocks. In the following two section we extend our theory to incorporate shocks and show that we can account for the stylized facts on debt dynamics. Sections 5.2 explores the adjustment to fiscal shocks and Section 5.3 analyzes changes in the political color of government. Section 6 concludes. The proof of all Lemmas and Propositions are in Appendix 8 (unless indicated otherwise).

2 Model Economy

The model economy is populated by overlapping generations of two-period lived agents who work in the first period and live off their savings in the second period. The population size is constant. Agents consume two goods: a private good (c) and a public good (g), provided by the government.

Private goods can be produced via two technologies – market and household production. Market production is subject to constant returns, and agents earn an hourly wage w. The household production technology is represented by the following production function;

$$y_H = F(1-h), \; F'(\cdot) > 0, \; F''(\cdot) \leq 0,$$

where the total time endowment is 1, h is the market labor supply, and $1-h \geq 0$ is the time for household production. Since the government cannot tax household production, taxation distorts the time agents work in the market. Agents choose the allocation of their time so as to maximize total after-tax labor income, denoted by $A(\tau)$, where

$$A(\tau) \equiv \max_h \{(1-\tau) wh + F(1-h)\}. \quad (1)$$

This program defines the optimal market labor supply as a function of the tax rate, τ;

$$h = H(\tau), \; H'(\cdot) \leq 0. \quad (2)$$

Consider the preferences of a young agent in dynasty i, born in period t;

$$U_{Y,i,t} = \log(c_{Y,i,t}) + \theta \log(g_t) + \beta (\log(c_{O,i,t+1}) + \theta \log(g_{t+1}) + \lambda U_{Y,i,t+1}), \quad (3)$$

where the subscript Y and O stand for "young" and "old", respectively. β is the discount rate, θ is a parameter describing the intensity of preferences for public good consumption, and $\lambda \geq 0$ is the altruistic weight on the utility of the agent's child (denoted by $U_{Y,i,t+1}$). In the rest of the paper, we omit time and dynasty subscripts when there is no source of confusion.

We assume throughout λ to be insufficiently large to induce private bequests.[8] This implies that the Ricardian equivalence does not hold, and that there exists an inter-generational conflict about the timing of taxation and public debt policy. Given labor supply $H(\tau)$, agents choose private consumption to maximize utility, (3), subject to their lifetime budget constraint;

$$c_{Y,i} + c_{O,i}/R = A(\tau), \tag{4}$$

where R is the gross interest rate, and τ is the tax rate prevailing in the first period of the agent's life. This yields

$$c_{Y,i} = c_Y = \frac{A(\tau)}{1+\beta},\ c_{O,i} = c_O = \frac{\beta R A(\tau)}{1+\beta}. \tag{5}$$

Fiscal policy is determined every period through repeated elections. We model electoral competition as a two-candidate political model of probabilistic voting *à la* Lindbeck and Weibull (1987), which is extensively discussed in Persson and Tabellini (2000). In this model, agents cast their votes on one of two office-seeking candidates. Voters' preferences may differ not only over fiscal policy, but also over other orthogonal policy dimensions about which the candidates cannot make binding commitments. In a probabilistic voting equilibrium, both candidates propose the same fiscal policy, which turns out to maximize a weighted sum of individual utilities where the weights may differ between young and old agents.[9] Thus, the equilibrium policy maximizes a "political objective function" that is a weighted average utility for all voters.

Given an inherited debt b, the elected government chooses the tax rate ($\tau \in [0,1]$), the public good provision ($g \geq 0$) and the debt accumulation (b'), subject to the following dynamic budget constraint[10]

$$b' = g + Rb - \tau w H(\tau). \tag{6}$$

Both private agents and governments have access to an international capital market providing borrowing and lending at the gross interest rate $R > 1$. The government is committed to not

[8] A number of studies documents that the bequest motive is modest and circumscribed to a limited fraction of the population (see, e.g., Hurd, 1989). For instance, in the PSID 64% of the households declare they have not received any inheritance (Leitner and Ohlsson, 2001), and part of the bequests of the remaining population may be of involuntary nature.

It is in general not possible to provide an analytical expression for an upper bound on λ. We have, however, checked numerically that in all equilibria for the calibrated economies we consider, agents choose not to leave any bequest along the equilibrium path. Moreover, there are obviously no bequests when $\lambda = 0$ (a case which is encompassed by our analysis).

[9] The weights can differ due to differences (between young and old) in their focus on fiscal policy relative to the orthogonal issues. The political clout of each group reflects the relative proportion of "swing voters", or the ability of the group to organize lobbies (see Persson and Tabellini, 2000).

[10] Hereafter, we switch to a recursive notation with primes denoting next-period variables.

repudiate the debt. This implies that debt cannot exceed the present discounted value of the maximum tax revenue that can be collected;

$$b \leq \frac{\max_{\tau} \{\tau w H(\tau)\}}{R-1} \equiv \bar{b}, \tag{7}$$

where $\bar{b}$ denotes the endogenous debt ceiling. This constraint rules out government Ponzi schemes.

Since agents vote twice in their life, the first step to characterize the political equilibrium is to compute the indirect utility of young and old agents. In the case of the young, substituting (1) and (5) into (3), and ignoring irrelevant constant terms yields:

$$U_Y(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g}) = (1+\beta)\log A(\tau) + \theta \log g + \beta\left(\theta \log g' + \lambda U_Y\left(\boldsymbol{b}', \boldsymbol{\tau}', \boldsymbol{g}'\right)\right), \tag{8}$$

where the primes denote next period's variables and boldface variables are vectors, defined as follows:

$$\boldsymbol{x} = \begin{bmatrix} x \\ x' \\ x'' \\ \ldots \end{bmatrix} = \begin{bmatrix} x \\ \boldsymbol{x}' \end{bmatrix}.$$

Similarly, after ignoring again irrelevant constant terms, the indirect utility of old voters can be expressed as[11]

$$U_O(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g}) = \log\left(A\left(1-\tau_{-1}\right)\right) + \theta \log g + \lambda U_Y(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g}), \tag{9}$$

where τ_{-1} denotes the tax rate in the period when the current old were young. Note that the old care about their children who are alive with them, so the children's utility, U_Y, is not discounted.

The equilibrium of a probabilistic voting model can be represented as the choice over time of τ, g and b' maximizing a weighted average indirect utility of young and old households, given b. We denote the weights of the old and young as ω and $1-\omega$, respectively. Then the "political objective function" which is maximized by both political candidates is

$$U(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g}) = (1-\omega) U_Y(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g}) + \omega U_O(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g}), \tag{10}$$

subject to (6) and (7).

[11] The term $A(1-\tau_{-1})$ captures the wealth of the old. Note that due to log-utility there is no interaction between the wealth of the old and any political choice variable. We focus on Markov equilibria so τ_{-1} should be irrelevant. With some abuse of notation, we therefore write $U_O(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g})$ instead of $U_O(\boldsymbol{b}, \tau_{-1}, \tau, \boldsymbol{g})$.

2.1 The commitment solution

In our model, fiscal policy is not, in general, time consistent. The source of time inconsistency is, to the best of our knowledge, novel. It stems from the fact that each agent votes more than once and can influence the fiscal policy choice at different stages of his life. We start by characterizing the policy sequence that would be chosen by the first generation of voters if they could commit the entire future path of fiscal policy.

We consider, first, a particular case in which there is no time inconsistency. Suppose that the first generation of old agents can dictate its preferred policy ($\omega = 1$). Using equations (8)-(9), the problem admits the following recursive formulation;

$$V_O^{comm}(b) = \max_{\{\tau,g,b'\}} \left\{ v(\tau, g) + \beta\lambda V_O^{comm}(b') \right\} \tag{11}$$

subject to (6) and (7), where

$$v(\tau, g) \equiv (1+\lambda)\,\theta \log g + (1+\beta)\,\lambda \log A(\tau) \tag{12}$$

is the flow utility accruing to the initially old agents from the current public and private consumption, either directly or through their altruism for their children.

This is a standard recursive program whose solution is unique and independent of whether the entire sequence is dictated by the initial generation of old agents or is chosen sequentially through elections in which only the old participate. To solve the program, note that the intra-temporal first-order condition linking g and τ in problem (11) is;[12]

$$\frac{1+\beta}{\left(1+\frac{1}{\lambda}\right)\theta} g = A(\tau)\left(1 - e(\tau)\right), \tag{13}$$

where $e(\tau) \equiv -\left(dH(\tau)/d\tau\right)\left(\tau/H(\tau)\right)$ is the elasticity of labor supply. The intertemporal first-order condition leads to a standard Euler equation for public consumption;

$$\frac{g'}{g} = \beta\lambda R. \tag{14}$$

If $\beta\lambda R = 1$, the solution is stationary, so debt, taxes, and consumption remain constant at their initial levels. Moreover, an unexpected temporary fiscal shock (e.g., a war) would trigger a permanent increase of debt, financed by a permanent increase in future taxes and a permanent decline in public goods, along the lines of Barro (1979). This paper focuses on the case when λ

[12] The first-order conditions with respect to τ and g are;

$$\frac{(1+\beta)\lambda}{A(\tau)} \frac{A'(\tau)}{\left(wH(\tau) + \tau w H'(\tau)\right)} = -\beta\lambda \frac{\partial}{\partial b}\left(V_O^{comm}(b')\right) \quad \text{and} \quad -\frac{(1+\lambda)\theta}{g} = -\beta\lambda\frac{\partial}{\partial b}\left(V_O^{comm}(b')\right),$$

These equations, plus the fact that $A'(\tau) = -wH(\tau)$, lead to (13).

is small enough to ensure $\beta\lambda R < 1$. In this case, public good provision declines asymptotically to zero, while debt accumulates progressively, converging asymptotically to the natural limit, $\bar{b}$.

Next, we generalize the commitment solution to the case where the policy maximizes the weighted average discounted utility of all agents who are alive in the initial period, with $\omega < 1$ being the weight of the initial young. In this case, a standard recursive formulation does not exist. However, the program admits a two-stage recursive formulation formalized in the following lemma;

Lemma 1 *The commitment problem admits a two-stage recursive formulation where;*

(i) In the initial period, policies are such that

$$\{\tau_0, g_0, b_1\} = \arg \max_{\{\tau_0, g_0, b_1\}} \{v(\tau_0, g_0) - (1 - \psi\lambda)\theta \log g_0 + \beta\lambda V_O^{comm}(b_1)\}, \tag{15}$$

subject to (6) and (7), where the function $V_O^{comm}(.)$ is given by (11), and the constant ψ is

$$\psi \equiv \frac{\omega}{1 - \omega(1 - \lambda)} \in \left(0, \frac{1}{\lambda}\right).$$

(ii) After the first period, the problem is equivalent to (11).

Comparing (15) with (11) shows that a positive weight on the initially young implies less concern for current public good provision (g_0) relative to current taxation (τ_0) and debt accumulation (b_1).[13] While this force is present only in the first period in the commitment problem, it will operate repeatedly over time in the political-economy game.

Lemma 1 implies that the first-period policy is different from the policy rule in the subsequent periods. Thus, the commitment solution is time inconsistent, except in the particular case when $\omega = 1$.[14] However, after the first period, the solution features the same dynamics, irrespective of ω : equation (14) governs the government expenditure dynamics from the second period onwards. In particular, the allocation features tax smoothing as in Barro (1979), and whether debt increases, decreases or remains constant over time depends only on the product between the rate of return (R) and the effective discount factor ($\beta\lambda$).

[13] Equation (15) is derived from writing the maximization problem as

$$\arg \max_{\{b_1, g_0, \tau_0\}} \{(1 + \lambda\omega)\theta \log g_0 + (1 - \omega + \lambda\omega)(1 + \beta)\log A(\tau_0) + (1 - \omega + \lambda\omega)\beta V_O^{comm}(b_1)\}. \tag{16}$$

Here, both private consumption, $\log A(\tau_0)$, and the discounted continuation utility, $\beta V_O^{comm}(b_1)$, are weighted by $1-\omega$ (the weight of the young) plus $\lambda\omega$ (the altruistic preference of the old), whereas public-good consumption, $\theta \log g_0$, is weighted by one (the sum of the weights of the young and of the old) plus $\lambda\omega$ (the altruistic preference of the old). Multiplying each term by $\lambda/(1 - \omega + \lambda\omega)$ and rearranging terms yields (15).

[14] When $\omega = 1$, then $\lambda\psi = 1$ and there is no difference between the first-period policy and the continuation policy rule.

Proposition 1 *The "commitment" solution is such that (i) if* $\beta\lambda R < 1$*, then* $\lim_{t\to\infty} b_t = \bar{b}$*, (ii) If* $\beta\lambda R > 1$*, then* $\lim_{t\to\infty} b_t = -\infty$*, (iii) if* $\beta\lambda R = 1$*, then* $b_{t+1} = b_t$ *for* $t \geq 1$*.*

Proof in Appendix 8.

2.2 The political equilibrium

We now characterize the political equilibrium without commitment. This is the main contribution of our paper. In general, a dynamic game between successive generations of voters arises, and the set of equilibria is potentially large. We restrict attention to Markov-perfect equilibria where agents condition their choices on only pay-off-relevant state variables. In principle, consecutive periods are linked by two state variables: the government debt, b, and the private wealth of the old. However, since preferences are separable between private and public goods consumption, the wealth of the old does not affect their preference over fiscal policies.[15] Therefore, b is the only pay-off-relevant state variable. Our Markov equilibria thus feature policy rules as functions of b only.

Definition 1 *A (Markov perfect) political equilibrium is defined as a 3-tuple of functions* $\langle B, G, T\rangle$*, where* $B : (-\infty, \bar{b}] \to (-\infty, \bar{b}]$ *is a debt rule,* $b' = B(b)$*,* $G : (-\infty, \bar{b}] \to R^+$ *is a government expenditure rule,* $g = G(b)$*, and* $T : (-\infty, \bar{b}] \to [0, 1]$ *is a tax rule, such that the following functional equations hold:*

1. $\langle B(b), G(b), T(b)\rangle = \arg\max_{\{b' \leq \bar{b}, g \geq 0, \tau \in [0,1]\}} U(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g})$*, subject to (6) and (7), where*

$$\boldsymbol{\tau} = \begin{bmatrix} \tau \\ T(b') \\ T(B(b')) \\ T(B(B(b'))) \\ \dots \end{bmatrix}, \boldsymbol{g} = \begin{bmatrix} g \\ G(b') \\ G(B(b')) \\ G(B(B(b'))) \\ \dots \end{bmatrix} \text{ and } \boldsymbol{b} = \begin{bmatrix} b \\ b' \\ B(b') \\ B(B(b')) \\ \dots \end{bmatrix}$$

   and $U(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g})$ *is defined as in (10).*

2. $B(b) = G(b) + Rb - T(b) \cdot H(T(b))$*.*

In words, the government chooses the current fiscal policy (taxation, expenditure and debt accumulation) subject to the budget constraint, under the expectation that future fiscal policies will follow the equilibrium policy rules, $\langle B(b), G(b), T(b)\rangle$. Furthermore, the vector of policy functions must be a fixed point of the system of functional equations in part 1 and 2 of the

[15] Recall that taxes are only levied on labor income and that the old do not work.

definition, where part 2 requires the equilibrium policy to be consistent with the resource constraint.

The following Lemma is a useful step to characterize the Markov equilibrium.

Lemma 2 *The first functional equation in Definition 1 admits the following two-stage recursive formulation:*

$$\langle B(b), G(b), T(b)\rangle = \arg \max_{\left\{b' \leq \bar{b}, g \geq 0, \tau \in [0,1]\right\}} \left\{ v(\tau, g) - (1 - \psi\lambda)\,\theta \log g + \beta\lambda V_O\left(b'\right)\right\}, \qquad (17)$$

where $v(.)$ is defined as in (12), subject to (6) and (7), and where V_O satisfies the following functional equation;

$$V_O\left(b'\right) = v\left(T\left(b'\right), G\left(b'\right)\right) + \beta\lambda V_O\left(B\left(b'\right)\right). \qquad (18)$$

The difference between the commitment solution and the political equilibrium can be seen by comparing the expressions of V_O^{comm} in (11) and that of V_O in (18). In the political equilibrium, the first generation of voters cannot choose the entire future policy sequence, but take the mapping from the state variable into the (future) policy choices as given. For this reason, there is no max operator in the definition of V_O. However, the two programs are identical when $\omega = 1$ (only the old vote), as in this case fiscal policy is time consistent.

What is the source of time inconsistency? When $\omega < 1$, the young, who care directly (i.e., not only through their altruism) about next-period public expenditure, want more public savings than the old. Hence, the young want more fiscal discipline than their parents. In the commitment solution, the effect of the conflict between "rotten parents" and "disciplined children" is limited to the first-period fiscal policy. Since the altruistic preferences of the initial parents and children are aligned, they agree on the continuation fiscal policy rule from the second period onwards. In contrast, the conflict is persistent in the political equilibrium, as a new generation of young voters enters the stage in each election. Since the young want more fiscal discipline, the political equilibrium features, as we shall see, less debt accumulation.

We characterize the political equilibrium as follows. First, the intra-temporal first-order condition linking g and τ in problem (17) is;

$$\frac{1+\beta}{(1+\psi)\,\theta} g = A(\tau)\,(1 - e(\tau)). \qquad (19)$$

The only difference between (19) and (13) in the commitment solution lies in the denominator of the term on the left-hand side, where λ^{-1} is replaced by ψ.

Next, applying standard recursive methods to the first-order conditions of (17)-(18), together with (19), leads to the following key result.

Proposition 2 *The politico-economic equilibrium dynamics of public good provision satisfies the following Generalized Euler Equation (GEE)*

$$\frac{G(B(b))}{G(b)} = \beta\lambda R - \underbrace{\beta\lambda G'(B(b))\left(\frac{1+\lambda^{-1}}{1+\psi} - 1\right)}_{\text{the disciplining effect}}. \tag{20}$$

Compare equation (20) with its counterpart in the commitment solution, (14). The "disciplining" effect is absent in the commitment solution. When all power lies in the hands of the old ($\omega = 1$), the two GEEs coincide, since in this case $\psi = \lambda^{-1}$ and the disciplining effect is also absent in the political equilibrium.

As we showed above, in the commitment solution the dynamics of government expenditure are linear. In contrast, the GEE in the political equilibrium imply that the dynamics of g (and, hence, of b) may be non-linear. Nevertheless, it is still possible that the GEE admits a linear equilibrium solution. In the next section, we study a particular case where the political equilibrium is linear and can be fully characterized analytically.

Some additional properties can be inferred from the GEE. Suppose that a steady-state debt level b^* exists and that G and B are continuously differentiable in a neighborhood around b^*. Since, in steady state, $G(B(b^*)) = G(b^*)$, then

$$G'(b^*) = -\frac{(1+\psi)(1-\beta\lambda R)}{\beta(1-\lambda\psi)} \equiv \zeta < 0, \tag{21}$$

which is constant and independent of the value of b^*. Thus, in the neighborhood of any such steady state $G'(.)$ must be negative; higher debt is associated with lower public spending. Plugging in $G'(b^*)$ into (20) shows that in the neighborhood of b^*, the growth rate of public spending is higher than it would be under commitment. The difference is proportional to ζ. In addition, if an interior steady state ($b^* < \bar{b}$) exists and b converges monotonically to b^* in a neighborhood of b^*, then $G(b)$ must be concave around b^*.[16]

3 Two Analytical Examples

In the rest of the paper we parameterize the household production technology as follows:

$$F(1-h) = X(1-h)^{\xi},$$

[16]Intuitively, when debt is above (below) the steady state, the fiscal discipline must be stronger (laxer) in order to reduce (increase) public consumption and move debt back towards steady state.

The formal argument for the concavity of G is as follows. Consider a small perturbation of debt from the steady state; $\tilde{b} = b^* + \varepsilon$, $\varepsilon > 0$. The monotone convergence implies that $B(\tilde{b}) \in (b^*, \tilde{b})$. Due to the negative slope of $G(b)$ around b^*, $G(B(\tilde{b})) > G(\tilde{b})$, which implies that $G'(B(\tilde{b})) < \zeta$ according to (20). Since $B(\tilde{b}) > b^*$, this establishes that $G'(b) < \zeta$ for $b > b^*$. A similar argument establishes that $G'(b) > \zeta$ for $b > b^*$, by letting $\varepsilon < 0$. So, $G(b)$ must be concave around b^*.

where $\xi \in [0,1]$ and we assume that $X < w$. In this section we study two special cases that we can solve analytically. In the first case, we set $\xi = 0$, implying that agents cannot substitute market hours with household activity. Due to the logarithmic preferences, labor taxation does not distort labor supply. We will see that in this case, a linear equilibrium exists, and the dynamics of debt resemble qualitatively the commitment solution. In the second case, we set $\xi = 1$. This implies that market hours are supplied inelastically as long as $\tau \leq \bar{\tau} \equiv 1 - X/w$. However, if taxation exceeds $\bar{\tau}$, market hours and tax revenue fall to zero. In this case, the equilibrium expenditure function G is concave, and a stable interior steady state with positive public good provision may exist.

3.1 Example I: $\xi = 0$

With $\xi = 0$, market hours are $H = 1$, irrespective of taxes. Hence, $A(\tau) = (1-\tau)w$ and $e(\tau) = 0$. Furthermore, tax revenue is maximized as $\tau \to 1$, so the maximum debt is $\bar{b} = w/(R-1)$. The FOC (19) can be expressed as

$$1 - \tau = \frac{1+\beta}{(1+\psi)\,\theta w} g. \tag{22}$$

Substituting (22) into the government budget constraint (6) yields;

$$b' = \left(1 + \frac{1+\beta}{\theta(1+\psi)}\right) g + Rb - w. \tag{23}$$

To obtain a solution, we guess that G is linear; $G(b) = \gamma\left(\bar{b} - b\right)$. Then, the GEE, (20), yields:

$$\frac{\gamma\left(\bar{b} - B(b)\right)}{\gamma\left(\bar{b} - b\right)} = \beta\lambda R - \beta\lambda\gamma\left(\frac{1+\lambda^{-1}}{1+\psi} - 1\right). \tag{24}$$

Next, using (24), the budget constraint, (23), the equilibrium condition $b' = B(b)$, and the expression for $\bar{b}$ given above, yields the following solution for γ;

$$\gamma = \frac{(1-\beta\lambda)\theta(1+\psi)R}{(1+\theta)(1+\beta) + (1-\beta\lambda)\theta\psi}.$$

Finally, substituting g by its equilibrium expression, $g = \gamma\left(\bar{b} - b\right)$, into (22) and (23), yields a complete analytical characterization, summarized in the following Proposition (proof in the text).[17]

Proposition 3 *Assume that $\xi = 0$. Then, the time-consistent equilibrium is given by the following policy functions*

$$\tau = T(b) = 1 - \frac{1}{w}\frac{(1-\beta\lambda)(1+\beta)R}{(1+\theta)(1+\beta) + (1-\beta\lambda)\theta\psi}\left(\bar{b} - b\right), \tag{25}$$

[17] The results of Proposition 3 extend to economies with population growth and technical change. Details are available upon request.

$$g = G(b) = \frac{(1-\beta\lambda)\theta(1+\psi)R}{(1+\theta)(1+\beta)+(1-\beta\lambda)\theta\psi}\left(\bar{b}-b\right), \tag{26}$$

$$b' = B(b) = \bar{b} - \frac{\theta+\lambda(1+\beta+\theta)}{(1+\theta)(1+\beta)+(1-\beta\lambda)\theta\psi}\beta R\left(\bar{b}-b\right), \tag{27}$$

where $\bar{b} \equiv w/(R-1)$.

Note that $G'(.) = -\gamma < 0$, implying that the disciplining effect in (20) increases the growth rate of public spending, as discussed above. Due to the linearity of $G(.)$, however, the disciplining effect does not change with the debt level. For this reason, the dynamics cannot lead to a stable interior steady state. If the interest rate is sufficiently low, the economy converges asymptotically to the maximum debt level $\bar{b}$. Else, the government surplus will be ever increasing and the economy will accumulate foreign assets.

The slope of the debt function $B(b)$ is always steeper in the political equilibrium than under commitment, so that debt accumulation is slower in the political equilibrium. In fact, there exists a range of parameters such that, under commitment, the economy would accumulate debt till the maximum level ($b \to \bar{b}$), while the political equilibrium leads to an ever-growing surplus ($b \to -\infty$). This illustrates that future generations benefit from political empowerment.

Figure 1 illustrates a political equilibrium when debt converges to $\bar{b}$. Panel *a* shows that the equilibrium tax rate increases linearly with debt. Panel *b* shows that the equilibrium public good provision declines linearly with debt. Finally, Panel *c* shows the law of motion of debt converging to $\bar{b}$ (in the figure, the parameters imply that $\bar{b} = 0.7$). Panel *d* shows the time path of b starting out with $b_0 = 0$. As the figure shows, the economy progressively depletes its resources over time. Generation after generation, agents find their private and public consumption progressively crowded out by debt repayment to foreign lenders.

FIGURE 1 (FOUR PANELS) HERE

The immiseration occurs gradually, even in a model without altruism ($\lambda = 0$). Under commitment and no altruism, debt converges to $\bar{b}$ in only two periods. In contrast, the political equilibrium features

$$\bar{b} - b' = \bar{b} - B(b) = \frac{\theta}{(1+\theta)(1+\beta)+\theta\psi}\beta R(\bar{b}-b),$$

where $\psi = \omega/(1-\omega)$. In spite of the lack of concern for future generations, voters do not support a "big party" which would consume the present value of the entire future income stream. Such big party would be supported by the old, but is opposed by the young since it crowds out public expenditure when they become old. The concern for *public* consumption is crucial to prevent the big party; if $\theta = 0$, the initial young and old voters would agree to set $b = \bar{b}$, and the young would secure their private consumption in old age through savings.

As the discipline on fiscal policy stems from the young voters, a larger political influence of the old (i.e., larger ω) increases debt accumulation and taxes and decreases current public good provision in every period. If the young had no influence on the political process ($\omega = 1$), the maximum debt would be attained in the first period.

Finally, we note that the political equilibrium and the commitment solution are identical in the first period (proof available upon request). Namely, the disciplining effect in the political equilibrium is of the same size as in the first period of the commitment solution, despite the fact that the first generation of young voters anticipates different future levels of public expenditure across the two regimes. This surprising result is due to cancellation of an income and a substitution effect that occurs under logarithmic preferences, given that future public goods are linear in $(\bar{b} - b)$. If public funds were to be spent more lavishly in future, the return on public savings – in terms of next-period public expenditures – would be higher. This substitution effect implies more public saving, i.e. less debt. However, with a large return it is not necessary to save as much, so the income effect suggests more debt.[18]

3.2 Example II: $\xi = 1$

We now present our second tractable case, assuming constant returns to labor in the household production technology, i.e., $\xi = 1$. In this case, taxation does not distort labor supply as long as $\tau \leq \bar{\tau} \equiv 1 - X/w$, namely, agents only work in the market. If $\tau > \bar{\tau}$, however, agents stop working in the market, and the tax revenue falls to zero. Thus, $\bar{\tau}$ is the top of the Laffer curve and the Markov-perfect political equilibrium necessarily features $\tau \leq \bar{\tau}$.[19]

Under a parametric condition, the equilibrium is qualitatively different from the linear case of Section 3.1; an economy starting from low initial debt converges in finite time to a steady state where steady-state taxes are maximized ($\tau = \bar{\tau}$) but steady-state debt is strictly below

[18] To see this result technically, note that whenever the policy rule is on the following form $G(b) = \gamma(\bar{b} - b)$ for some γ, the cross derivative $\frac{\partial^2 V_Y(b)}{\partial b \partial \gamma}$ is always equal to zero. This means that the future lavishness, i.e. γ, will not impact on current political decisions.

[19] It is straightforward to analyze the cases with sufficiently high or sufficiently low interest rates. We have omitted them since they yield debt dynamics qualitatively similar to the linear case of Section 3.1. With R sufficiently low, debt converges asymptotically to its maximum level, $\bar{b} = \bar{\tau}w/(R-1)$, and the economy features public poverty in the long run, i.e. $\lim_{t\to\infty} g_t = 0$. However, since taxes are bounded from above by $\bar{\tau}$, private consumption does not fall to zero, but converges to $(1-\bar{\tau})\, w > 0$. Second, when the interest rate is sufficiently high, the equilibrium is, after the first period, identical to the linear case above.

$\bar{b}$ and public good provision is strictly positive. In a neighborhood of the steady state, the equilibrium dynamics of the fiscal variables and the steady-state debt level are given by[20]

$$b' = B(b) = b_0^* \equiv \bar{b}\left(1 - \frac{\theta(1+\psi)(1-\bar{\tau})}{\bar{\tau}(1+\beta)}\right) \tag{28}$$

$$\tau = T(b) = \bar{\tau} - \frac{R(1+\beta)}{w(1+\beta+\theta(1+\psi))}(b_0^* - b) \tag{29}$$

$$g = G(b) = \frac{w\theta(1+\psi)(1-\bar{\tau})}{1+\beta} + \frac{\theta(1+\psi)R}{1+\beta+\theta(1+\psi)}(b_0^* - b) \tag{30}$$

FIGURE 2 (FOUR PANELS) HERE

Figure 2 plots the equilibrium functions for an economy where $\bar{\tau} = 0.6$ and $\bar{b} = 0.42$. The parameters of the simulation imply a steady-state debt of $b_0^* = 0.12$. Panel a shows the equilibrium tax policy: taxes increase linearly with the debt level as long as $b \leq b_0^*$. Thereafter, T is flat at $\tau = \bar{\tau}$. Panel b shows the equilibrium expenditure: public good provision declines linearly with the debt level as $b \leq b_0^*$. To the right of b_0^*, the government loses the ability to adjust taxes, and thus the government expenditure function becomes steeper. Panel c shows that the debt policy is flat around b_0^*. Therefore, if the initial debt level is sufficiently close to b_0^*, debt converges to b_0^* in one period and remains there thereafter. The figure also shows that the debt and expenditure policy function feature discontinuous dynamics for high initial debt levels.[21] Moreover, there are multiple steady states. The multiple steady states are a fragile feature of this particular example which vanishes once one considers a smooth labor supply distortion (i.e., $\xi < 1$). However, the most important feature of this equilibrium is robust; as we shall see in Section 4, there may exist an internal and locally stable steady-state debt level even when the labor distortion is smoother ($\xi < 1$). Finally, panel d shows the time path of b starting out with $b = 0$. Convergence occurs in the fist period.

We now discuss the intuition for the dynamics in the neighborhood of b_0^* focusing, for simplicity, on the case of no altruism ($\lambda = 0$). In the linear equilibrium of example I, the concern of young voters for next period's public good provision did not prevent the debt from increasing in every period, progressively impoverishing future generations. Why? Because it is not

[20] See the Appendix for a formal Proposition with a complete characterization of the equilibrium. Its proof is provided in the Appendix C, available upon request.

[21] In order to visualize better the region around b_0^*, the figure only reports the policy functions for $b \leq 0.2$. The behavior of the policy function in the omitted region is as expected: T(b) is flat at $\bar{\tau}$, while G(b) and B(b) are piece wise linear functions, such that G($\bar{\text{b}}$)=0 and B($\bar{\text{b}}$)=$\bar{\text{b}}$.

rational to believe that future generations would cut public good provision drastically should they inherit a large debt. To the contrary; along the linear equilibrium path, current voters know that the next government will respond to a larger debt by not only cutting expenditure, but also by increasing taxes and debt proportionally. To the current young voters this is a small cost to pay, and as a result, each generation of voters "passes the bill" to the next generation by only suffering a partial sacrifice of public consumption. Passing the bill to future generations becomes harder, however, when taxation is increasingly distortionary. In example II, this effect is particularly stark. As the debt approaches b_0^* and taxes approach $\bar{\tau}$, voters anticipate that future generations will not be able to increase taxes over $\bar{\tau}$. The expenditure response to a larger debt is then sharper, and the disciplining effect is stronger. Note that $G(.)$ is concave around the steady state b_0^*. To the right of b_0^*, the disciplining effect is so strong that debt falls and reverts to b_0^* in just one period. In contrast, to the left to b_0^*, $G(b)$ is less steep, implying a smaller disciplining effect. Consequently, voters support an increasing debt, and b_0^* is a steady state.[22]

4 The General Case: $\xi \in (0, 1)$

The intuition behind the result of example II carries over to the general case with $\xi \in (0, 1)$, with smooth labor supply distortions. In this case, however, the equilibrium policy functions are non-linear, and the model does not admit an analytical solution. We must therefore resort to numerical analysis. To this end, we use a standard projection method with Chebyshev collocation (Judd, 1992) to approximate T and G, exploiting the first-order conditions (19) and (20).

We calibrate the parameters as follows. Since agents live for two periods, we let a period correspond to thirty years. Accordingly, we set $\beta = 0.98^{30}$ and $R = 1.025^{30}$. implying a 2% annual discount rate and a 2.5% annual interest rate. This value of β is standard in the macroeconomics literature, and the value of R is consistent with the average real long-term U.S. government bond yields (2.5%) between 1960 and 1990. We do not have a strong prior on ω, so we simply assume equal political weights on the young and old ($\omega = 0.5$). The wage is set equal to unity (a normalization).

Four parameters remain to be calibrated; θ, λ, ξ, and X. We calibrate these parameters

[22] A related intuition explains why there is no internal steady state when the interest rate is low. In that case $G' > \zeta$ everywhere, so the GEE (20) implies an ever-decreasing sequence of public goods. Hence, with a low interest rate, the disciplining effect is not strong enough to generate falling debt for any $b \leq \bar{b}$, so $b \to \bar{b}$, irrespectively of the initial b.

to match four empirical observations:[23]

1. The ratio of hours worked in the market to hours worked at home is on average 2 in the US (Aguiar and Hurst, 2006), which implies a steady-state labor input of $H = 2/3$.

2. In the US, the ratio of explicit federal debt to GDP has been around 40% over the last decades. However, the government has also significant pension liabilities. The estimated size of the pension liabilities that have already accrued is 60-90% (van den Noord and Herd, 1993). This puts the total US debt-output ratio to 100-130%. One period in our model corresponds to 30 years. Our notion of aggregate production abstracts from capital. With an empirical labor's share of output of, say, 2/3, our notion of "output" should be 30*2/3=20 times larger than the empirical annual GDP. Therefore, a plausible quantitative target is a steady-state level of b/wH equal to 120%/20=6%, which implies b=4%.

3. The average tax on labor income in the US in the last two decades has been about 27%.[24] So we set $\tau^* = 0.27$.

4. The elasticity of the tax revenue to changes in the after-tax rate, $\chi(\tau) \equiv \frac{\partial(wH)}{\partial(1-\tau)} \frac{1-\tau}{wH}$ is set equal to 0.6 in the steady-state. In our model $\chi(\tau)$ coincides with the Frisch elasticity of labor supply. The estimates of these elasticities have a wide range. Micro estimates of the Frisch elasticity along the intensive margin – based on people who remain employed – indicate an elasticity close to zero for men and somewhat higher for women (see e.g. Altonji, 1986). Macro estimates tend to be higher, as they include adjustments along the extensive margin. For example, the Real-Business-Cycle literature often assumes an elasticity of unity (Cooley and Prescott, 1995). However, in our stylized model labor supply is the only margin of distortion, and in the theory it is the size of the fiscal distortion rather than its channel (labor supply) that matters. Estimates of the elasticity of the total tax revenue to changes in the after-tax rate vary, again, over a wide range. For instance, Feldstein (1987) argues that the elasticity is between one and two. In contrast, a micro literature based on the marginal-cost-of-funds approach that the elasticity is significantly lower (see e.g. Ballard and Fullerton, 1992, and Kleven and Kreiner, 2006).

[23] Given H, τ, and the labor elasticity, the expressions for labor supply and the Frisch elasticity pin down the parameters ξ and X. θ and λ are then jointly determined by debt-to-output ratio and the tax-to-output ratio.

[24] In the period 1979-2004, the average personal income tax as percentage of the gross earnings in the US was 18.7%. However, this increases to 26.1% and 31.4% if one adds, respectively, the employees'social security contributions (net of transfer payments), and in addition the employer's social security contributions (see Source OECD). Klein and Rios-Rull (2003) report an average income tax rate of 24% for the period 1947-90.

Given the lack of consensus, we choose an intermediate value ($\chi(\tau^*) = 0.6$). This yields a marginal cost of funds of about two, slightly above the preferred estimate of Browning (1987). We discuss robustness to changes of this elasticity below.

Table 1 summarizes the parameters.

Table 1: Calibration

Target observation		*Parameter*	
Annual discount rate	2%	β	0.98^{30}
Annual interest rate	2.5%	R	1.025^{30}
Average tax on labor	27%	θ	0.09
Market-household hours ratio	2	X	1.75
Elasticity of the tax revenue to changes in the after-tax rate	0.6	ξ	0.17
Debt-GDP ratio (including Social Security liabilities)	120%	λ	0.75
Relative political weight young-old	equal	ω	0.5

Figure 3 plots the equilibrium functions of our calibrated economy.[25] As in example I of section 3.1, taxes are increasing in b (panel a) and public expenditure is decreasing in b (panel b). The debt policy, however, is now a strictly convex function of b which crosses the 45-degree line twice: first at an interior steady-state level ($b = 0.04$), and then at the maximum debt. Only the interior steady-state is stable. Thus, for any initial debt level $b < \bar{b}$, the economy converges to the internal steady state with no public poverty (see panel d). The steady-state level of government expenditure is $g^* = 0.14$, implying a ratio of public expenditure to private market consumption of 21%. Panel d provides information about the speed of convergence of debt towards the steady-state. For example, it takes about four periods (i.e., 120 years) to get from $b_0 = 0$ to $b = 0.02$, i.e., to close half the gap between zero debt and the steady state, with an implied annual rate of convergence of 0.6%. Namely, debt is mean reverting, but with a high persistence. We will show that this is also a feature of the data. Finally, we note that the altruism in the calibrated economy is sufficiently low that along the equilibrium path, agents do not want to bequeath to their children. Hence, the no-bequest constraint is not binding in

[25] Although numerical solutions do not establish that these Markov equilibria are unique, we have run many simulations and never found more than one equilibrium for each parameter configuration, qualitatively similar to those displayed in the figure.

equilibrium.

FIGURE 3 (FOUR PANELS) HERE

To gain intuition for our main result – the internal stable steady state – it is useful to compare the calibrated economy with the analytical examples. In all cases, the tax function is non-decreasing and concave (strictly concave if $\xi > 0$), while the expenditure function is decreasing and concave (strictly concave if $\xi > 0$). In example I ($\xi = 0$), where taxation is not distortionary, an increasing debt causes a proportional increase in taxation and cut in expenditure, so as to keep c/g constant. In example II, the policy functions are piece-wise linear with a kink at the steady state. This is because taxation is non-distortionary to the left of $\bar{\tau}$ and infinitely distortionary to the right of it. Accordingly, the c/g ratio is constant for $b \leq b^*$, and increasing thereafter. In the general case of $\xi \in (0, 1)$, as b increases, the tax function, $T(b)$ becomes less steep, whereas the expenditure function, $G(b)$, becomes steeper. Namely, at high debt levels, the government responds to debt accumulation by cutting expenditure more than by increasing taxes. Hence, the ratio of public-to-private consumption falls as b increases. This fall in relative government expenditure is what deters young voters from demanding debt increases in steady state.

The qualitative findings of an internal steady state are robust to a large range of all parameter values. The most critical one is χ. Clearly, an internal steady-state hinges on the presence of significant tax distortions. In the calibrated economy any tax elasticity χ larger than 0.52 are consistent with an internal steady state such as that in Figure 3, when all other calibration targets are held constant (provided, of course, that the top of the Laffer curve is larger than τ^*). The range can be expanded if we allow a larger labor supply. For example, $\chi = 0.2$ and $H = 0.85$ will still generate an internal steady state. As far as altruism is concerned, it is important that λ be not too small (in particular $\lambda > 0.66$), or else the B(b) function continues to be strictly convex, but only crosses the 45-degree line at $\bar{b}$. In this case, the economy converges to the maximum debt.[26]

4.1 Markov vs. Ramsey

This subsection compares the Markov equilibrium with the commitment solution (Ramsey) in the calibrated economy. We perform the following experiment: start out with an initial debt

[26] We should stress, however, that an equilibrium with an interior steady state can be sustained even for economies with no altruism ($\lambda = 0$). However, this requires either a higher interest rate, or a higher tax elasticity.

of zero, and simulate subsequent equilibrium paths. The results are shown in figure 4.

FIGURE 4 (THREE PANELS) HERE

Recall that in the commitment case agents vote over the entire fiscal policy sequence in period zero. Young and old vote with equal weight. We have already shown theoretically that under commitment debt converges to $\bar{b}$, implying that g converges to zero. We can now illustrate the dynamics in more detail. In the first period, the Ramsey solution features lower taxes (τ_0) and a slightly larger government spending (g_0) than the Markov equilibrium. Consequently, b_1 is higher in the Ramsey case. Government expenditure is also larger in the following period (g_1) – recall that g_1 enters directly (i.e., not only through altruism) the utility of the first generation of voters –. This comes at the expense of a further increase in the debt left to agents born in period two. Later generations do not influence the fiscal policy path, and cannot discipline fiscal policy. Since the altruism of the first generation of voters is imperfect, the subsequent Ramsey path is increasingly unfavorable to future generations over time. Debt accumulates at a higher rate and converges to $\bar{b}$ (panel a); tax rates approaches the top of the Laffer curve (panel b) and public spending declines to zero (panel c).

All generations born in period two or after are strictly worse off in the commitment solution, while agents born before period one are better off. This is intuitive, as in the Ramsey allocation the first generation dictates the entire fiscal policy and passes the bill of their high private and public consumption to the future generations. In contrast, in the political equilibrium all future generations are sequentially empowered and discipline period-by-period the fiscal policy. The difference in the long-run outcome is striking: even generations that can only exercise their political power in the far future inherit low debt and can enjoy public good consumption.

5 Shocks and debt dynamics

So far, we have developed a politico-economic theory of government debt. In the rest of the paper we extend this theory by introducing shocks to fiscal policy and shocks to political preferences.

5.1 Empirical evidence

We start by documenting some salient features of debt dynamics in response to fiscal and political shocks and then show that our theory can account for these stylized facts. In Appendix 7 we document our empirical analysis in detail. Here we summarize our main findings.

Fiscal shocks: How does government debt respond to fiscal shocks? Bohn (1998) analyzes the effects of short-lived increases in US government expenditures on the debt-to-output ratio. He uses data for the years 1917-1990, a period encompassing the two world wars. He finds that the debt-to-output ratio is mean reverting. Namely, a short-lived expenditure increase induces an increase in the debt-to-output ratio on impact and a subsequent reversion towards its initial level. This finding is robust to different time periods and to controlling for cyclical components of fiscal policy. According to his estimates, the annual rate of convergence is about 0.065, which implies that shocks to the debt-to-output ratio have a half-life of about 10 years.

In Appendix 7 we first replicate and update Bohn's findings for the US (see Table 2). We then show that this stylized fact holds up for a panel data set of 21 OECD countries over the period 1960-2005. In particular, Table 3 in Appendix 7 documents that debt is mean reverting, albeit on average more persistent than for the US.

Debt dynamics after political shocks: We then analyze whether debt policy is correlated with the political inclination (left vs. right) of governments. One observation that motivated the work of Persson and Svensson (1989) was that Republican US administrations in the 1980's tended to accumulate more debt. Here, we ask whether this a general feature of the data, in both the US and in a panel of OECD countries. To address this question, we augment Bohn's specification with political dummies so as to allow different debt growth (and, hence, different long-run debt levels) first across Republican vs. Democrat administrations in the US, and then across governments of different ideologies in a set of countries.

We find that the growth in the debt-to-output ratio is significantly correlated with the party in power in the US over the 1948-2005 period (see the regressions in Table 2 in Appendix 7).[27] This conclusion is robust to using different time periods (before and after 1980) and to the inclusion of various control variables. Moreover, the estimated magnitudes are large: the debt-to-output ratio is increasing about two percentage points per year under a Republican president than under a Democrat president.

The same conclusion holds up for a broader set of countries. The cross-country analysis is less straightforward and subject to larger measurement error, due to the large heterogeneity in political and electoral systems across countries. Our preferred political measure is a classi-

[27] We focus on the post-war period because in this period the identification of Republicans with"right-wing" and that of Democrats with "left-wing" is not controversial, while this becomes more contentious in earlier periods.

If one is prepared to give a causal interpretation to the results, one can infer from the estimates the long-run debt level under Republican and Democrat administrations. An infinite sequence of Republican (Democrat) administrations would yield a steady-state debt-GDP ratio of 42.7% (19.3%). The estimated difference is both large and statistically significant.

fication of governments on a left-right scale taken from the World Bank Database of Political Institutions (see Beck et al. 2001), which is discussed in more details in Appendix 7. We find that in a panel regression including country fixed effects and time dummies debt accumulation is significantly positively correlated with right-wing governments, although the estimated difference between right and left is quantitatively smaller than for the US (see the regressions in Table 3 in Appendix 7). This conclusion is robust to various classifications of political parties in OECD countries on the left-right scale, and to including various control variables.[28]

In conclusion, we find that the debt-GDP ratio is mean-reverting after temporary fiscal shocks (albeit with a large autocorrelation coefficient), and that and that the growth in the debt-to-output ratio is positively correlated with right-wing parties being in government. This evidence motivates us to extend our model to include shocks so as to examine if the theory can account for these stylized facts. Section 5.2 investigates the response of debt to fiscal shocks, while Section 5.3 introduces intra-generational conflict along a left-right scale so as to be able to address the effects of political shifts.

5.2 Fiscal Shocks

This section analyzes fiscal policy adjustment after a "surprise" fiscal shock in our model. To fix ideas, assume that the country is forced to fight a one-period "war" requiring an exogenous spending of Z units. During the war, the government's budget constraint (6) changes to

$$b' = g + Rb - \tau w H(\tau) + Z, \tag{31}$$

and then, as peace returns, it reverts to (6). The shock occurs at the beginning of the period, before the government sets g, τ and b', and hits an economy when this is in a steady state.[29]

Consider, first, the economy of example II in Section 3.2 ($\xi = 1$). Suppose that the economy starts out with a debt level b_0^* and is hit by a fiscal shock. The adjustment dynamics are equivalent to those triggered by an exogenous increase of debt from b_0^* to $b_0^* + Z/R$. As one can see from panel c of figure 2, as long as Z is not too large, debt does not increase.[30]

[28] There is an empirical literature focusing on strategic use of debt driven by ideological differences across parties (see e.g. Pettersson-Lidbom, 2001). Lambertini (2003) examines if the color of government affects the budget deficits in OECD countries but does not find significant effects. However, she uses a shorter data sample than us and does not include the current level of debt as a control variable.

[29] In the example I of Section 3.2, an economy would be unable to finance a surprise war in steady state ($b = \bar{b}$). This case can be analyzed by either assuming that the economy is not initially in the steady state, or considering a benign fiscal shock ($Z < 0$) such as a windfall oil discovery.

[30] When fiscal policy shock is "small", the economy returns to the original steady after the "war" is finished (see panel c of figure 2). Larger shocks generate qualitatively similar responses for taxes and expenditure, except that the economy converges to a higher debt level and to a lower level of public-good provision than in the initial steady state. We emphasize the small-shock case because multiple steady states is a non-robust feature of example II that disappears in the general case analyzed below.

Nor do taxes increase, as the tax constraint ($\tau \leq \bar{\tau}$) was binding already before the war. Since both debt and taxes remain unchanged, the war must be financed entirely via a reduction in non-war expenditure, such that total government spending (war expenditure plus public good provision) stays constant. In particular, during war time public-good provision falls to $g = G(b_0^*) - Z/R$. Then, in the following period, the economy moves back to the steady state. The impulse-responses to a fiscal shock equal to 1% of the GDP is shown in figure 5 (panel b).

The case of $\xi = 1$ is extreme insofar as the government does not use at all debt and taxes to smooth non-war expenditure. Panel a of figure 5 illustrates the general case, with the aid of the calibrated economy of section 4. Even in this case non-war expenditure falls albeit less than in panel b.[31] However, some of the cost of the war is financed by increases in taxes and debt, smoothing the effects on public good provision. After the shock, debt reverts slowly to the original steady-state level.

FIGURE 5 (TWO PANELS) HERE

In conclusion, our theory predicts that a fiscal shock is absorbed by a combination of cuts in non-war expenditure and increases in debt and taxation. Moreover, after the war debt, taxes and expenditure revert slowly to their original steady-state levels. These results are consistent with the empirical evidence, and stand in contrast with the implications of the tax-smoothing model of Barro (1979), as well as the commitment version of our model. There, the lion's share of the current cost of the war would be financed by debt and, following the principle of tax smoothing, taxes and non-war expenditure would only be adjusted so as to guarantee a smooth repayment of the excess debt. Therefore, the immediate effects on fiscal policy are small but permanent.

We have extended the analysis to recurrent wars, assuming that the state of the economy (war or peace) evolves following a first-order stationary Markov process. Details are available upon request. The results are similar to those of a surprise war. However, the positive probability of future wars induces an additional precautionary motive for public savings during peacetime.[32]

[31] Barro (1986) notes that non-military spending is crowded out during wars in the US, consistently with the prediction of our model.

[32] Interestingly, such motive is also present in the commitment solution, and it turns out that with recurrent wars, even the commitment solution features mean-reverting debt dynamics. Aiyagari et al. (2002) makes a similar point. They study a calibrated version of a representative-agent neoclassical growth model with exogenous stochastic government expenditures financed with debt and distortionary taxation. They show that under commitment, the debt dynamics are stationary, albeit highly persistent.

5.3 Political Shocks and Intra-generational Conflict

In the theory discussed so far, there was political conflict only between generations. In this section, we introduce cross-sectional wage heterogeneity and intra-generational political conflict. The purpose of the extension is to analyze political shifts and to show that our theory can account for the stylized fact that right-wing governments tend to accumulate more government debt.[33]

Suppose that there are two types of dynasties, with high and low productivity (*rich* and *poor*), respectively. Poor agents do not pay taxes, and thus their labor earnings are independent of τ.[34] This captures in a simplified way the notion that taxes are progressive. Each cohort consists of a unit measure of rich and of a measure $\tilde{p}$ of poor. For the sake of simplicity, we assume that productivity is perfectly correlated within dynasties. This is not essential: our argument only relies on *some* degree of inter-generational persistence in income, i.e. imperfect social mobility. The labor income process of the rich is the same as described in section 2.[35]

We will first show that this model is identical (up to a reinterpretation of the parameter θ) to the benchmark model of section 2. Ignoring constants and irrelevant terms, the indirect utility of the young and old poor can be written, respectively, as

$$\begin{aligned} U_{YP}(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g}) &= \log(g) + \beta \log(g') + \beta\lambda U_{YP}\left(\boldsymbol{b}', \boldsymbol{\tau}', \boldsymbol{g}'\right), \\ U_{OP}(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g}) &= \log(g) + \lambda U_{YP}(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g}). \end{aligned}$$

Consider, now, the probabilistic voting equilibrium. Denote by p the political weight of poor dynasties (when the poor and the rich have the same clout, $p = \tilde{p}$). The political objective function can then be written as

$$U(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g}; \boldsymbol{p}) = (1-\omega)\left(pU_{YP}(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g}) + U_{YR}(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g})\right) + \omega\left(pU_{OP}(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g}) + U_{OR}(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g})\right) \tag{32}$$

Appendix 8 (see the proof of Proposition 4) shows that a version of Lemma 2 applies to this model, with the only modification that the weight on public good consumption in (17) is $\theta(1+p)$ instead of θ.[36] Namely, the political preference for public goods increases with the

[33] In our probabilistic voting model, there are no explicit parties, and candidates always converge in equilibrium to the same fiscal policy platform. However, in leftist times, the winning platform is more favorable to the poor. In the discussion of stylized facts of Section 5, we proxied leftist times by leftist governments. To make this mapping explicit in the theory, one could introduce elements of imperfect commitment and partisan politics (such as in the citizen-candidate model of Besley and Coate, 1997).

[34] Alternatively, we could assume that the poor have zero productivity in market activity and hence spend all their time in home production which is not taxed.

[35] An alternative model delivering similar empirical predictions assumes instead of financing public goods, the government spends the tax revenue in transfers to the poor.

[36] More precisely, equation (17) is replaced by

political clout of the poor. All of our previous results extend to this alternative model.

We assume that the political clout of the poor, p, changes over time. Formally, we let p follow a two-state Markov first-order process, with realizations $p \in \{p_r, p_l\}$, where $p_r < p_l$ (R and L stand for right-wing and left-wing, respectively). The leftist wave of the 1960's and the neo-conservative revolution of the 1980's are examples of such political shifts. We denote by π_{ij} the probability that, conditional on the current state being j, next-period state will be i.

The equilibrium definition must be generalized to include p as an additional state variable. We denote by $T(b,p), G(b,p)$ and $B(b,p)$ the equilibrium policy functions conditional on the debt level b and on the political state p. The following generalization of Proposition 2 can be proved

Proposition 4 *In the model with political shocks, the politico-economic equilibrium dynamics of public good provision satisfies the following stochastic GEE*

$$\frac{1}{G(b,p)} = \beta\lambda R \cdot E_p \frac{1}{G(B(b,p),p')} - \beta\lambda\left(\frac{1+\lambda^{-1}}{1+\psi} - 1\right) E_p \frac{G'(B(b,p),p')}{G(B(b,p),p')}. \tag{33}$$

where E_p is a conditional expectation operator (e.g., $E_{p_l} G(B(b,p_l),p') = \pi_{ll} G(B(b,p_l),p_l) + \pi_{rl} G(B(b,p_l),p_r)$).

The stochastic GEE has a similar interpretation to (20) in the deterministic model. The first term on the right hand-side is the standard Euler equation term. The second term arises from dynamic voting and captures the disciplining effect of the young voters.

We start by characterizing the equilibrium in the tractable case of $\xi = 0$ (example I of Section 3), when a linear equilibrium obtains. We focus on the particular case of no altruism ($\lambda = 0$). Although a linear equilibrium with similar comparative statics also exists when $\lambda > 0$, the expressions are more involved and we do not report them.

Proposition 5 *Assume that $\xi = 0$ and $\lambda = 0$. Then, the equilibrium with political shocks is given by the following policy functions.*

$$T(b,p) = 1 - \frac{(1-\omega) R (1+\beta)}{w((1-\omega)(1+\theta(1+p))(1+\beta) + \omega\theta(1+p))}\left(\bar{b} - b\right),$$

$$G(b,p) = \frac{\theta(1+p) R}{\omega\theta(1+p) + (1-\omega)(1+\theta(1+p))(1+\beta)}\left(\bar{b} - b\right),$$

$$\langle B(b), G(b), T(b)\rangle = \arg\max_{\left\{b' \leq \bar{b}, g \geq 0, \tau \in [0,1]\right\}} \left\{v(\tau, g) - (1-\psi\lambda)(1+p)\theta\log g + \beta\lambda V_O(b')\right\},$$

where $v(\tau,g) \equiv (1+\lambda)(1+p)\theta\log g + (1+\beta)\lambda\log A(\tau)$.

$$B(b,p) = \bar{b} - \frac{(1-\omega)\,\theta\,(1+p)\,\beta R}{\omega\theta\,(1+p) + (1-\omega)\,(1+\theta\,(1+p))\,(1+\beta)}\left(\bar{b} - b\right),$$

where $\bar{b} \equiv wh/(R-1)$, *and* $p \in \{p_r, p_l\}$.

Proposition 5 implies that a shift to the right ($p_l \to p_r$) leads to lower taxes, lower government expenditure and larger debt. More formally, the tax policy, T, shifts downwards, the policy function G shifts downwards, and the policy function B shifts upwards. Thus: $T(b,p_r) < T(b,p_l)$, $G(b,p_r) < G(b,p_l)$, and $B(b,p_r) > B(b,p_l)$. This result extends to the case with positive altruism ($\lambda > 0$). Intuitively, a larger clout of leftist voters is observationally equivalent to an increase in the public appreciation of public good consumption relative to private consumption. The reason is that for the rich debt has a positive value insofar as it reduces current taxation, while this motive is absent for the poor whose private consumption is invariant to taxes. Hence, leftist voters are more averse to debt.

In the linear case with inelastic labor supply, it is straightforward that right-wing governments have lower taxes and lower spending, given the level of debt. However, in the general case with labor distortion, the short-run effects are less sharp.[37] The empirical evidence suggests that left-wing governments are indeed associated with higher spending and taxes. For example, Perotti and Kontopoulos (2002) find that changes in transfers and changes in expenditure are higher under left-wing governments in OECD countries, although the effects are not always significant. Moreover, they do not find any effect on taxes. Pettersson-Lidbom (2003) find that both taxes and spending are significantly higher for left-wing municipal governments for a panel of Swedish municipalities.

An interesting observation is that the probabilities $\pi_{j,i}$ do not enter the equilibrium functions $T(.)$, $G(.)$ and $B(.)$.[38] This implies that neither the variance nor the persistence of political shocks have any effect on the equilibrium. In particular, a permanent shift has the same effect as a temporary one. This surprising result – which is not robust to the introduction of altruism – depends on the cancellation of an income and a substitution effect. Suppose, for example, that in a leftist period voters anticipate a shift to the right. On the one hand, a disciplined fiscal policy today has a lower return to leftist voters since the next generation is rightist and will spend a smaller share of $\bar{b} - b'$ on public goods. Therefore, the substitution

[37] For example, in Example II of Section 3 the predictions for debt are as in the linear case, while predictions about expenditure and taxes are different. For example, if there is a permanent change to a left-wing government starting from an initial right-wing steady state, the economy would go to a lower steady-state level of debt. Due to the tax distortion there would be no change in tax revenue, so the transition would have to be financed by an initial fall in public expenditure after the left get to power (followed, of course, by an increase in subsequent periods).

[38] Note that this is formally identical to the equilibrium of Proposition 3 in the case of $\lambda = 0$, although there were no political shocks there.

effect increases the desire to accumulate debt. On the other hand, the marginal utility of future government expenditure is larger precisely because the next government has a lower propensity to spend. Thus, the income effect induces more fiscal discipline from the leftist government. Under logarithmic preferences and no altruism the two effects cancel out exactly.

This result is of independent interest. In an influential article, Persson and Svensson (1989) argued that when governments are subject to a positive non-reelection probability debt policy is affected by strategic considerations. For instance, a right-wing government issues more debt when it anticipates to be replaced by a left-wing government with a stronger taste for public expenditure. They derive their results in a two-period model. In our environment, the sign of the strategic effects is ambiguous, being exactly zero under logarithmic preferences, no altruism and non-distortionary taxation. Our finding may explain why the empirical literature has found mixed support to this prediction.

Similar results obtain when the labor supply is elastic, although in this case the shocks also affect the steady-state debt level. To illustrate this case, we calibrate the model as in Table 1, letting in addition, $p_l = 0.11$ and $p_r = 0$. In this example, the poor are totally unrepresented under the right-wing regime. Thus, $\theta(1 + p_r) = 0.37$, as in Table 1. We consider three alternative levels of persistence of political shocks: i.i.d. shocks ($\pi_{ll} = \pi_{rr} = 0.5$), persistent shocks ($\pi_{ll} = \pi_{rr} = 0.9$) and permanent shocks ($\pi_{ll} = \pi_{rr} = 1$).[39] Figure 6 plots the equilibrium policy rules and the debt dynamics for the two realizations of the shock (p_l and p_r) in the case of zero persistence.[40] Dotted lines are for the left-wing regime, whereas solid lines are for the right-wing regime. The figure shows that for a given level of debt a right-wing government delivers lower taxes and public good provision, and more debt accumulation.

FIGURES 6 (Four Panels) HERE

Figure 7 plots the time-series dynamics of g, τ and b under the political regime shift in the three cases. The solid, dashed and dotted lines corresponds to permanent shocks, persistent shocks and i.i.d. shocks, respectively. All three cases feature similar qualitative dynamics; public spending decreases monotonically, public debt increases monotonically, while the tax rate falls in the first period, and increases thereafter. The figures reveal that in this calibration

[39] In the data, if we assume the transition matrix to be symmetric, the estimated annual rate of persistence is 0.89 (allowing non-symmetric matrix gives very similar results for the left and the right). In our model, one period is calibrated to be thirty years. Over a thirty-period horizon, there is almost no persistence and i.i.d. is the best approximation. Clearly, the two period-model is a major limitation for this exercise.

[40] To aid the visualization, we zoom on the region of the state space where $b \in [0, 0.1]$.

of the model the size of all changes in fiscal policy is decreasing with the persistence of the shock. Namely, a lower probability of re-election makes governments behave more extremely in the sense that right-wing (left-wing) governments accumulate more (less) debt the higher is their probability of reelection. Thus, the implications are in this case in line with those of Persson and Svensson (1989) (contrary to the linear case, where the reelection probabilities did not influence debt policies).

FIGURES 7 (Three Panels) HERE.

6 Conclusion

In this paper, we have proposed a positive theory of fiscal policy under repeated voting. In the absence of commitment, which is a natural assumption in a politico-economic environment, the concern of voters for future public good provision can offset the desire of voters to pass the bill of their expenditure to future generations, and drive the economy to an interior steady-state debt level. This result holds even for economies in which agents have no altruistic concerns for future generations' welfare, local interest rates do not respond to the fiscal policy, and the commitment solution would converge to the endogenous debt limit with zero public-good consumption. Tax distortions are crucial for the survival of the welfare state, as they make it credible that accumulating high debt will induce future governments to make large expenditure cuts. Thus, distortions discipline current voters. Somewhat paradoxically, an increase in the elasticity of the tax base, due, e.g., to tax competition may ultimately increase public good provision.

The model can alternatively be interpreted as a standard rich-poor redistributive conflict. In times where the poor have a stronger influence on the political process (leftist periods) governments accumulate less debt than when the rich have a tighter control on political power (rightist periods). We document empirical support for this prediction.

Our analysis is subject to a number of caveats. For instance, both left-wing and right-wing populism may reflect a decrease in the current voters' altruism to future generations, whereas altruism has been kept constant across political regimes in our analysis. Nor does our theory deals with the determination of public debt under coalition governments.

While our analysis aims to explain the effects of within-country shifts in political preferences, we do not view it as an explanation of cross-country differences (e.g., why Italy and

Belgium have a larger debt than Switzerland or Sweden) which is left to future research. We conjecture that differences in the efficiency of public good provision may affect voters' preferences for public savings. For instance, it is often argued that Italy, a country with one of the largest public debts, has an inefficient public administration, while the public sector is more efficient in Scandinavian countries which have a lower propensity to indebtedness.

Finally, we have maintained throughout that governments are committed to repay their debt and ruled out government Ponzi schemes. The analysis could be enriched by endogenizing the incentive of government to repay debt. For instance, in equilibria with immiseration there would be incentives for voters to support international default. Integrating our analysis with the insights of the sovereign debt literature may give rise to novel insights but requires non-trivial extensions which are also left to further research.

References

Aiyagari, Rao, Albert Marcet, Thomas Sargent, and Juha Seppälä "Optimal Taxation without State-Contingent Debt," *Journal of Political Economy*, 110 (2002), 1220-1254.

Alesina, Alberto, and Guido Tabellini "Why is Fiscal Policy Often Procyclical?" *Review of Economic Studies*, 57 (1990), 403-414.

Alesina, Alberto, and Guido Tabellini "Voting on the Budget Deficit," *American Economic Review*, 80 (1990), 37-49.

Alesina, Alberto, and Guido Tabellini "A Positive Theory of Fiscal Deficits and Government Debt," NBER Working Paper 11600 (2005).

Altonji, Joseph G., "Intertemporal substitution in labor supply: Evidence from micro data," *Journal of Political Economy*, 94 (1986), S176–S215.

Azzimonti Renzo, Marina "On the Dynamic Inefficiency of Governments," Mimeo, University of Iowa (2005).

Ballard, C. L. and D. Fullerton, "Distortionary Taxes and the Provision of Public Goods." *Journal of Economic Perspectives* 6 (1992), 117-131.

Barro, Robert "U.S. Deficits Since World War I," *Scandinavian Journal of Economics*, 88 (1986), 195-222.

Barro, Robert "On the Determination of the Public Debt," *Journal of Political Economy*, 87 (1979), 940-971.

Battaglini, Marco and Stephen Coate, "A Dynamic Theory of Public Spending, Taxation and Debt," NBER Working Paper 12100, 2006.

Beck, Thorsten, George Clarke, Alberto Groff, Philip Keefer, and Patrick Walsh. "New tools in comparative political economy: The Database of Political Institutions." 15 (2001), 165-176, *World Bank Economic Review.*

Besley, Tim, and Stephen Coate "An Economic Model of Representative Democracy," *Quarterly Journal of Economics*, 112 (1997), 85-114.

Bohn, Henning "The Behavior of U.S. Public Debt and Deficits," *Quarterly Journal of Economics*, 113 (1998), 949-963.

Chen, Kaiji and Zheng Song "A Markovian Social Contract of Social Security," Mimeo, IIES Stockholm University (2005).

Cooley, Thomas F., and Edward C. Prescott, "Economic Growth and Business Cycles," in Thomas F. Cooley (ed.) *Frontiers of Business Cycle Research*, Princeton University Press (1995).

Cooley, Thomas, and Jorge Soares "A Positive Theory of Social Security Based on Reputation," *Journal of Political Economy*, 107 (1999), 135-160.

Farhi, Emmanuel, and Iván Werning "Inequality, Social Discounting and Estate Taxation," NBER Working Paper 11408 (1998).

Franzese, Robert "Macroeconomic Policy of Developed Democracies," *Cambridge University Press* (2002).

Gonzalez-Eiras, Martín, and Dirk Niepelt "Sustaining Social Security," CESifo Working Paper 1494 (1995)

Hassler, John, Per Krusell, Kjetil Storesletten, and Fabrizio Zilibotti "The Dynamics of Government," *Journal of Monetary Economics*, 52 (2005), 1331-1358.

Hassler, John, José Rodríguez Mora, Kjetil Storesletten, Fabrizio Zilibotti "The Survival of the Welfare State," *American Economic Review*, 93 (2003), 87-112.

Hassler, John, Kjetil Storesletten, and Fabrizio Zilibotti "Democratic Public Good Provision," *Journal of Economic Theory*, forthcoming (2006).

Hurd, Michael "Mortality Risk and Bequests", *Econometrica*, 57 (1989), 779-813.

Judd, Kenneth "Projection Methods for Solving Aggregate Growth Models," *Journal of Economic Theory*, 58 (1992), 410-452.

Judson, Ruth A. and Ann L. Owen, "Estimating Dynamic Panel Data Models: A Guide for Macroeconomists," *Economic Letters*, 65 (1999), 9-15.

Kleven, Henrik J., and Claus T. Kreiner, "The marginal cost of public funds: Hours of work versus labor force participation," *Journal of Public Economics*, 90 (2006), 1955-1973.

Perotti, Roberto and Yianos Kontopoulos, "Fragmented fiscal policy," *Journal of Public Economics* 86 (2002), 191-222.

Krusell, Per, Fernando Martin, and José-Víctor Ríos-Rull “Time-Consistent Debt,” Mimeo (2006).

Krusell, Per, Vincenco Quadrini, and José-Víctor Ríos-Rull “Are Consumption Taxes Really Better than Income Taxes?” *Journal of Monetary Economics*, 37 (1996), 475-503.

Krusell, Per, Vincenco Quadrini, and José-Víctor Ríos-Rull “Politico-Economic Equilibrium and Economic Growth,” *Journal of Economic Dynamics and Control*, 21 (1997), 243-272.

Krusell, Per, and José-Víctor Ríos-Rull “On the Size of the U.S. Government: Political Economy in the Neoclassical Growth Model,” *American Economic Review*, 89 (1999), 1156-1181.

Krusell, Per, and Anthony Smith “Consumption-Savings Decisions with Quasi-Geometric Discounting,” *Econometrica*, 71 (2003), 365–375.

Lambertini, Luisa “Are Budget Deficits Used Strategically?” Boston College Working Paper 578 (2003).

Leitner, John, and Henry Ohlsson “Bequest motives: a comparison of Sweden and the United States”, *Journal of Public Economics,* 79 (2001) 205–236

Lindbeck, Assar, and Jörgen Weibull “Balanced-Budget Redistribution as the Outcome of Political Competition,” *Public Choice*, 52 (1987), 273-297.

Lucas, Robert, and Nancy Stokey “Optimal Fiscal and Monetary Policy in an Economy Without Capital,” *Journal of Monetary Economics*, 12 (1983), 55-93.

Persson, Torsten, and Lars Svensson “Why a Stubborn Conservative Would Run a Deficit: Policy with Time-Inconsistent Preferences,” *Quarterly Journal of Economics*, 104 (1989), 325-345.

Persson, Torsten, and Guido Tabellini “Political Economics: Explaining Economic Policy,” *MIT Press* (2002).

Persson, Torsten, and Guido Tabellini “Political Economics and Public Finance,” in: A. J. Auerbach and M. Feldstein (ed.), *Handbook of Public Economics*, edition 1, volume 3, chapter 24, 1549-1659, *Elsevier* (2002).

Pettersson-Lidbom, Per “An Empirical Investigation of the Strategic Use of Debt,” *Journal of Political Economy*, 109 (2001), 570-84.

Pettersson-Lidbom, Per "Do Parties Matter for Economic Outcomes? A Regression-Discontinuity Approach," Research Papers in Economics 2003:15, University of Stockholm (2003).

Song, Zheng "Ideology and the Determination of Public Policies Over Time," Mimeo, IIES Stockholm University (2005a).

Song, Zheng "The Dynamics of Inequality and Social Security," Mimeo, IIES Stockholm University (2005b).

Van den Noord, Paul, and Richard Herd, "Pension Liabilities in the Seven Major Economies", OECD Economics Department Working Papers No. 142 (1993).

Woldendorp, Jaap, Hans Keman, and Ian Budge "Party Government in 20 Democracies: An Update (1990-1995)," *European Journal of Political Research*, 33 (1998), 125-164.

Yared, Pierre, "Politicians, Taxes, and Debt," Mimeo, MIT (2007).

7 Appendix A: Empirical Analysis of Debt Dynamics

This appendix documents our empirical analysis of debt dynamics in two data sets: data for the US and a panel of 21 OECD countries.

For the US, we use annual data for the period 1948-2005 from the Economic Report of the President. We run the following regression

$$\Delta d_t = \alpha_0 + \alpha_1 DEM_t + \alpha_2 d_t + \alpha_3 \left(U_t - \bar{U}\right) + \varepsilon_t.$$

The dependent variable, Δd_t, is the annual change in the debt-GDP ratio. Coherently with the timing of our theory, we define $\Delta d_t \equiv D_{t+1}/Y_{t+1} - D_t/Y_t$, namely, the government in office at t sets (through its budget law) the surplus or deficit in the following year.[41] As explanatory variables we include the debt-GDP ratio ($d_t = D_t/Y_t$), intended to capture the autoregressive component of debt (see Bohn, 1998); an indicator of the party affiliation of the president in office, and unemployment. The latter is intended to capture cyclical components of debt policy that are independent of politics.[42] We net unemployment of its sample average in order to ease the interpretation of the coefficients.

[41] Our simple empirical analysis ignores the possibility of feedback from debt accumulation to the probability of election of different governments.

[42] One might argue that the ideology of governments may affect their response to business cycle fluctuations. However, an interaction between unemployment and the political measure has an insignificant effect in the regression.

The main variables of interest are DEM_t and d_t. DEM_t is a dummy variable that takes on the value one when the president is a Democrat and the value zero when the president is a Republican. Our theory says that α_1 will be negative; debt growth should be lower under Democrat administrations. Note that α_0 measures the conditional mean of debt growth under Republicans, whereas $\alpha_0 + \alpha_1$ measures the conditional mean of debt growth under Democrats. Our theory says that α_2 will be negative; debt growth should decrease with the level of debt, so that d_t becomes autoregressive.

TABLE 2 HERE

Table 2 summarizes the results. The baseline regression (column 1) shows that Republican administrations, controlling for the autoregressive component only, are associated an average increase in the debt-GDP ratio of 3.8 percentage points per year. Given the autocorrelation coefficient (-0.088), an infinite sequence of Republican governments would yield a steady-state debt-GDP ratio of 42.7%. In contrast, Democrat administrations are associated with an average increase in the debt-GDP ratio of 1.7 percentage points, implying a steady-state debt-GDP ratio of 19.3%. The estimated difference is both large and statistically significant.

The autoregressive coefficient α_2 is negative, as predicted. The coefficient is significant in the first regression. The point estimates imply an annual rate of convergence of about 0.08, so a debt shock has a half-life of 8-10 years. Bohn (1998) uses a longer sample for the US and finds point estimates of about the same magnitude. This rate of convergence is faster than in our calibrated economy.

Controlling for unemployment (column 2) has no major effects on the results. The difference between Republicans and Democrats remains highly significant (well above 99%). Moreover, in this case the steady-state debt-GDP ratios become, respectively, 40.7% (Republicans) and 15.1% (Democrats).[43]. We also checked the sample stability by allowing the effect of Democrats to be different before and after 1980 (column 3), and found no significant difference between the early and late part of the sample (the test that the two coefficients are identical is not rejected). In both subperiods Republican administrations accumulate debt at a higher rate.

[43] The autoregressive coefficient remains negative but drops to -0.067, becoming marginally insignificant. Interestingly, this estimate is very similar to that of Bohn (1998) who finds – after controlling for cyclical components in output and government expenditures – an autoregressive coefficient of -0.064 for the period 1948-95 (see Table II, p. 956).

Adding a linear-quadratic time trend to the regression does not change the result of interest: the difference between Democrat and Republican administrations remain significant above 99%.

We next extend the analysis to a panel of 21 OECD countries for the period 1960-2005.[44] The major issue concerns measuring the political color of governments across countries and over time. Problems of cross-country comparability between governments' political ideologies are avoided by including country-specific fixed effects in the regressions. In addition, we filter out shocks common to all countries by including time effects. It is well known that the estimates are biased when using a Least Square Dummy Variable (LSDV) estimator. However, for sample sizes of $T \geq 30$ and $N = 20$, the bias is small and the LSDV estimator generally perform better than the Arellano-Bond estimator or the Anderson-Hsiao estimator (see Judson and Owen, 1999).

Our political measure (POL_{WB}) is taken from the World Bank Database of Political Institutions (see Beck et al. 2001) which measures, ranging from -1 to 1, the government's position in the left-right spectrum by classifying the political inclination of the chief executive's party inclination. Since the dataset only starts in 1975, we extend it backwards in time using the same criteria. To check the robustness of the results, we also considered two alternative political measures constructed by Franzese (2003) and Woldendorp, Keman and Budge (1998), respectively.[45]

We run the following basic specification for the panel regressions

$$\Delta d_{ct} = f_c + f_t + \alpha_1 POL_{ct} + \alpha_2 d_{ct} + \alpha_3 U_{ct} + \varepsilon_{ct},$$

where f_c and f_t are country and time fixed effects, respectively. In all regressions we exclude non-democratic governments. In some specifications, we run this regression with some additional control variables including GDP per capita, openness, and two measures of the age structure of the population (proportion below 14 and above 65).

[44] The data for debt-GDP ratio are from the OECD Dataset on Central Government Debt (http://stats.oecd.org/wbos/default.aspx?datasetcode=GOV_DEBT) for the period 1980-2006. For the previous years, this has been chained with the data provided by Franzese (2003), based on different sources. GDP per Capita and Openness are from the Penn World Tables 6.2. Unemployment is from OECD for the period 1980-2005, which has also been chained with Franzese (2003) for earlier years. Demographical variables are from the Demographic Yearbook of the United Nations (various issues), with missing observations filled by interpolation.

[45] Franzese (2003) codes all parties in government from 1948 to 1997 from far left (value 0) to far right (value 10). For consistency with the other measures, we re-scaled this variable so that it ranges between -1 (far right) to +1 (far left). We did not extend this measure after 1997 since the criteria for extending the measure are complect and at instances ambiguous. Woldendorp, Keman and Budge (1998) assign scores for government and parliament from "right-wing dominance" (value 1) to "left-wing dominance" (value 5). The criterion for "dominance" is set by the share of seats in government and parliament. We extended and simplified their data assigning the value -1 for RIGHT, 0 for CENTER and 1 for LEFT.

TABLE 3 HERE

Table 3 summarizes the results. Columns 1-6 use the World Bank measure. In the baseline specification (column 1) the coefficient of interest (POL_{WB}) is negative and significant. Since this measure is increasing as governments move to the left, the regression confirms the theoretical implication that right-wing governments run larger debt as it was the case for the US. The quantitative effect is sizeable: a shift from a left-wing (+1) to a right-wing (-1) government increases the debt-GDP ratio by ca. 0.6 percentage points per year. The effect is smaller than that estimated for the US alone. We should note however that common political shocks are absorbed by the time dummies in the panel regressions, and this could explain the smaller effects.

The autoregressive coefficient (d_t) is negative but insignificant in columns 1 and 2, where no control variables other than unemployment are included. However, the apparent lack of mean reversion is driven by an outlier, Japan, whose debt has risen sharply in recent years. If we introduce an interaction between d_t and a dummy variable for Japan (namely, we allow the autoregressive coefficient of Japan to be different), the process is significantly mean reverting (see column 4 and 5), and the Japanese dummy is positive and highly significant. Moreover, once the full set of control variables is included, the autoregressive coefficient is again negative and highly significant both with and without the Japanese dummy. The point estimates for the annual rate of convergence of debt is -1.8% to -3.5% (in columns 3-8). This implies a half-life of a debt shock of 20-37 years. We note that the OECD data imply higher persistence of debt than the US data.

Unemployment has in all cases the expected positive effect on debt accumulation. Finally, column 7 and 8 show that the results are also in accordance with the theory when one uses each of the alternative political measures.[46]

[46] As discussed in the previous footnote, POL_{FR} (column 7) ranges from -1 to +1. However, most observations are between -0.4 and 0.1, namely, the range of variation of this political variable is about one fourth as that of the other political variables. This complicates the comparison of the coefficients. If one divides the estimated coefficient (-0.0138) by four, one obtains a quantitative effect which is similar to that in the other columns.

8 Appendix B: proofs of Lemmas and Propositions

8.1 Proof of Lemma 1

We rewrite the political objective function (10) as:

$$\begin{aligned}
& \frac{\lambda}{1-\omega+\omega\lambda} U(\boldsymbol{b},\boldsymbol{\tau},\boldsymbol{g}) \\
= & \frac{\lambda}{1-\omega+\omega\lambda}\left((1-\omega)U_Y(\boldsymbol{b},\boldsymbol{\tau},\boldsymbol{g})+\omega\left(\theta\log(g_0)+\lambda U_Y(\boldsymbol{b},\boldsymbol{\tau},\boldsymbol{g})\right)\right) \\
= & \frac{\lambda}{1-\omega+\omega\lambda}\left(\omega\theta\log(g_0)+(1-\omega+\lambda\omega)U_Y(\boldsymbol{b},\boldsymbol{\tau},\boldsymbol{g})\right) \\
= & \frac{\omega\lambda}{1-\omega+\omega\lambda}\theta\log(g_0)+\lambda\sum_{t=0}^{\infty}(\lambda\beta)^t\left((1+\beta)\log(A(\tau_t))+\theta\log(g_t)+\beta\theta\log(g_{t+1})\right) \\
= & \lambda(1+\beta)\log(A(\tau_0))+(1+\psi)\lambda\theta\log(g_0)+\sum_{t=1}^{\infty}(\lambda\beta)^t\left((1+\beta)\lambda\log(A(\tau_t))+(1+\lambda)\theta\log(g_t)\right) \\
= & (1+\beta)\lambda\log(A(\tau_0))+\lambda(1+\psi)\theta\log(g_0)+\sum_{t=1}^{\infty}(\lambda\beta)^t v(g_t,\tau_t)\,.
\end{aligned}$$

It follows that:

$$\begin{aligned}
& \max_{\{\tau_t,g_t,b_{t+1}\}_{t=0}^{\infty}}\left\{\frac{\lambda}{1-\omega+\omega\lambda}U(...)\right\}|_{b_0} \\
= & \max_{\{\tau_0,g_0,b_1\}}\left\{(1+\beta)\lambda\log(A(\tau_0))+\lambda(1+\psi)\theta\log(g_0)+\max_{\{\tau_t,g_t,b_{t+1}\}_{t=1}^{\infty}|_{b_1}}\left\{\sum_{t=1}^{\infty}(\lambda\beta)^t v(g_t,\tau_t)\right\}|_{b_1}\right\}|_{b_0} \\
= & \max_{\{\tau_0,g_0,b_1\}_{t=0}^{\infty}}\left\{(1+\beta)\lambda\log(A(\tau_0))+\lambda(1+\psi)\theta\log(g_0)+\beta\lambda V_O^{comm}(b_1)\right\}|_{b_0},
\end{aligned}$$

where all maximization is subject to (6), and the last step follows from equation (11). Given the definition of $v(\tau, g)$, the last expression is identical to (15). Hence, we have proven part (i) of the Lemma. Part (ii) of the Lemma follows from equations (11)-(14) in the text.

8.2 Proof Proposition 1

The intertemporal government budget constraint after the first period can be written as:

$$Rb_1+\sum_{t=1}^{\infty}\frac{g_t}{R^{t-1}}=\sum_{t=1}^{\infty}\frac{w\tau_t H(\tau_t)}{R^{t-1}}. \tag{34}$$

First, consider an economy where $\beta\lambda R = 1$. In this economy, (14) implies that g is constant. If the elasticity of labor supply $e(\tau)$ is an increasing function, a constant g and (13) imply a unique constant τ over time. Therefore, (34) establishes that

$$(R-1)\,b_1=\tau^* wH(\tau^*)-g^*,$$

where τ^* and g^* denote constant solutions of τ and g, respectively. Substituting the above equation into (6), we obtain $b_t = b_1$ for $t \geq 2$.

Now consider the case in which $\beta\lambda R < 1$. (14) implies that $\lim_{t\to\infty} g_t = 0$, from which in turn it follows, by (13), that $\lim_{t\to\infty} e(\tau_t) = 1$. Hence, the long-run tax rate attains the top of the Laffer curve. These two facts establish that $b_t \to \bar{b}$ as $t \to \infty$.

Finally, if $\beta\lambda R > 1$, (14) implies that $g_t \to \infty$ as $t \to \infty$. Since the tax base is bounded, this is only feasible if $b_t \to -\infty$ as $t \to \infty$.

8.3 Proof Lemma 2

The proof of Lemma 1 shows that the political objective function can be written as

$$\begin{aligned} & \frac{\lambda}{1-\omega+\omega\lambda} U(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g}) \\ = \; & (1+\beta)\lambda \log(A(\tau_0)) + \lambda(1+\psi)\theta\log(g_0) + \sum_{t=1}^{\infty} (\lambda\beta)^t v(g_t, \tau_t). \end{aligned}$$

Given the policy rules $T(b)$, $G(b)$ and $B(b)$, we define

$$V_O(b) \equiv \sum_{t=0}^{\infty} (\lambda\beta)^t v\left(G\left(B^t(b)\right), T\left(B^t(b)\right)\right),$$

representing the discounted utility of the old.[47] $V_O(b)$ admits a recursive expression:

$$V_O(b) = v(G(b), T(b)) + \beta\lambda V_O(B(b)). \tag{35}$$

Therefore, the political choices can be rewritten as:

$$\max_{\{g,\tau,b'\}} \left\{ v(\tau, g) - (1-\psi\lambda)\theta\log g + \beta\lambda V_O\left(b'\right) \right\},$$

subject to (6), and the function $V_O(b')$ solving (35).

8.4 Proof of Proposition 2

The FOCs of the program (17) yield:

$$\begin{aligned} \frac{(1+\beta)\lambda A'(\tau)}{A(\tau)} - \beta\lambda V_O'\left(b'\right)\left(wH(\tau) + \tau w H'(\tau)\right) &= 0, \\ \frac{\lambda\theta(1+\psi)}{g} + \beta\lambda\hat{V}'\left(b'\right) &= 0. \end{aligned}$$

[47] It can be shown that $V_O(b) = \theta\log G(b) + \lambda V_Y(b)$.

Using the definition of $e(\tau)$ and the fact that $A'(\tau) = -wh_M(\tau)$, these can be rewritten as:

$$-\frac{(1+\beta)\lambda}{A(\tau)} - \beta\lambda V_O'(b')(1-e(\tau)) = 0, \tag{36}$$

$$\frac{\lambda\theta(1+\psi)}{g} + \beta\lambda V_O'(b') = 0. \tag{37}$$

Combining two FOCs yields (19):

$$\frac{1+\beta}{(1+\psi)\theta} g = A(\tau)(1-e(\tau)).$$

Then we can rewrite (18) and the government budget constraint (6) as:

$$\begin{aligned} V_O(b) &= ((1+\beta)\lambda + (1+\lambda)\theta)\log(G(b)) - (1+\beta)\lambda\log(1-e(T(b))) + \beta\lambda V_O(B(b)), \\ B(b) &= G(b) + Rb - T(b)wH(T(b)). \end{aligned}$$

Differentiating $V_O(b)$ and $B(b)$ yields:

$$V_O'(b) = ((1+\beta)\lambda + (1+\lambda)\theta)\frac{G'(b)}{G(b)} - \frac{-(1+\beta)\lambda e'(T(b))T'(b)}{1-e(T(b))} + \beta\lambda V_O'(B(b))B'(b) \tag{38}$$

$$\begin{aligned} B'(b) &= G'(b) + R - T'(b)wH(T(b))(1-e(T(b))) \\ &= \left(1+\frac{1+\beta}{\theta(1+\psi)}\right)G'(b) + R + e'(T(b))T'(b)A(T(b)). \end{aligned} \tag{39}$$

The last equality follows from the fact that

$$-T'(b)wh_M(T(b))(1-e(T(b))) - e'(T(b))T'(b)A(T(b)) = \frac{1+\beta}{\theta(1+\psi)}G'(b),$$

as implied by (19) and $A'(\tau) = -wH(\tau)$. Leading by one period equation (38) yields an expression for $V_O'(b')$ which can be used, together with (37), to eliminate $V_O'(b')$ and $V_O'(B(b))$. The resulting expression is:

$$\frac{1}{G(b)} = \frac{\beta\lambda}{G(B(b))}\left(B'(B(b)) - \frac{1+\beta+\left(1+\frac{1}{\lambda}\right)\theta}{\theta(1+\psi)}G'(B(b)) - \frac{(1+\beta)G(B(b))e'(T(B(b)))T'(B(b))}{\theta(1+\psi)(1-e(T(B(b))))}\right).$$

Next, using (39) to eliminate $B'(B(b))$ leads to:

$$\frac{1}{G(b)} = \frac{\beta\lambda}{G(B(b))}\left(\begin{array}{c} R + \left(1-\frac{1+\frac{1}{\lambda}}{1+\psi}\right)G'(B(b)) + e'(T(B(b)))T'(B(b))A(T(B(b))) \\ -\frac{(1+\beta)G(B(b))e'(T(B(b)))T'(B(b))}{\theta(1+\psi)(1-e(T(B(b))))} \end{array}\right).$$

Finally, note the FOC (19) implies:

$$A(T(B(b)))(1-e(T(B(b)))) = \frac{1+\beta}{\theta(1+\psi)}G(B(b)).$$

Then, the generalized Euler equation simplifies to:

$$\frac{1}{G(b)} = \frac{\beta\lambda R}{G(B(b))} - \frac{\beta\lambda G'(B(b))}{G(B(b))}\left(\frac{1+\frac{1}{\lambda}}{1+\psi} - 1\right),$$

that is, as in equation (20).

8.5 Analysis of Example II, Section 3

Proposition 6 *Suppose* $R \in [1+(1+\psi)/\zeta, R_h]$*, and let the initial debt level be* $b = b^0 \in [\underline{b}, \bar{b}])$*, where* $\bar{b} \equiv \bar{\tau}w/(R-1)$ *and* $\zeta \equiv (1+\lambda)\beta/(1-\beta\lambda)$*, and* R_h *and* $\underline{b}$ *are defined in Section* **??**. *Then, the equilibrium is given by the following policy functions*

$$\tau = T(b) \equiv \begin{cases} \bar{\tau} - \frac{R(1+\beta)}{w(1+\beta+\theta(1+\psi))}(b_0^* - b) & \text{if } b \in [\underline{b}, b_0^*) \\ \bar{\tau} & \text{otherwise} \end{cases}, \tag{40}$$

$$g = G(b) \equiv \begin{cases} g_0^* + \frac{\theta(1+\psi)R}{1+\beta+\theta(1+\psi)}(b_0^* - b) & \text{if } b \in [\underline{b}, b_0^*) \\ b_n^* + \bar{\tau}w - Rb & \text{if } b \in [b_n^*, b_{n+1}^*) \end{cases}, \tag{41}$$

$$b' = B(b) \equiv \begin{cases} b_0^* \equiv \bar{b}\left(1 - \frac{\theta(1+\psi)(1-\bar{\tau})}{\bar{\tau}(1+\beta)}\right) & \text{if } b \in [\underline{b}, b_1^*) \\ b_n^* & \text{if } b \in [b_n^*, b_{n+1}^*] \end{cases}, \tag{42}$$

where $g_0^* \equiv w\theta(1+\psi)(1-\bar{\tau})/(1+\beta) > 0$*, and the sequence* $\{b_n^*\}_{n=0,1,2,..,\infty}$ *is the unique solution to the difference equation*

$$\left(b_n^* - b_{n+1}^* + \bar{\tau}w\right)^{1+\psi}\left(b_n^* - Rb_n^* + \bar{\tau}w\right)^{\zeta} = \left(b_{n+1}^* - Rb_{n+1}^* + \bar{\tau}w\right)^{1+\psi+\zeta}, \tag{43}$$

given b_0^*. *The sequence* $\{b_n^*\}_{n=0,1,2,..,\infty}$ *is monotonically increasing in* n *and* $\lim_{n\to\infty} b_n^* = \bar{b}$.

Proof: See Appendix C, available upon request.

8.6 Proof of Proposition 4

We first show that the deterministic version of the model with intragenerational transfers yields the same exact formulation of the political equilibrium as in Lemma 2. The political objective function can be written as

$$\begin{aligned} & \frac{\lambda}{1-\omega+\omega\lambda} U(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g}) \\ = & \frac{\lambda}{1-\omega+\omega\lambda}\left((1-\omega)\left(pU_{YP}(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g}) + U_{YR}(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g})\right) + \omega\left(pU_{OP}(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g}) + U_{OR}(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g})\right)\right) \\ = & \frac{\lambda}{1-\omega+\omega\lambda}\left(\omega(1+p)\theta\log(g_0) + (1-\omega+\lambda\omega)\left(pU_{YP}(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g}) + U_{YR}(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g})\right)\right) \\ = & \frac{\omega\lambda(1+p)}{1-\omega+\omega\lambda}\theta\log(g_0) + \lambda\sum_{t=0}^{\infty}(\lambda\beta)^t\left((1+\beta)\log(A(\tau_t)) + \theta(1+p)\log(g_t) + \beta\theta(1+p)\log(g_{t+1})\right), \end{aligned}$$

which is exactly the same as the political objective function in the proof of Lemma 1, with the slight modification that the taste for the public good becomes $\hat{\theta} \equiv (1+p)\theta$. Therefore, the results in Lemma 1 and 2 carry over unchanged to the model with intragenerational transfers.

When p is stochastic and follows a Markov process, the state vector consists of the level of debt and the political state, p. It is straightforward to extend Lemma 2 to the stochastic case. The political objective function can be expressed as

$$
\begin{aligned}
& \frac{\lambda}{(1-\omega+\omega\lambda)} E_p U(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g}, p) \\
= \; & \lambda\psi(1+p)\theta\log g + \lambda E_p \{pU_{YP}(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g}) + U_{YR}(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g})\} \\
= \; & (1+\beta)\lambda\log(A(\tau)) + \lambda(1+\psi)(1+p)\theta\log g + \sum_{t=1}^{\infty}\sum_{p_t}(\lambda\beta)^t \pi_t(p_t, p) v(g_t, \tau_t, p),
\end{aligned}
$$

where p_t denotes the political weight at time t and $\pi_t(p_t, p)$ is the probability of p_t in period t, conditional on the initial state p, and

$$v(g_t, \tau_t, p) \equiv (1+\beta)\lambda\log A(\tau_t) + (1+\lambda)(1+p)\theta\log(g_t).$$

Therefore, the equilibrium must satisfy:

$$\left\langle \begin{array}{c} B(b,p), \\ G(b,p), \\ T(b,p) \end{array} \right\rangle = \arg \max_{\{b' \leq \bar{b}, g \geq 0, \tau \in [0,1]\}} \left\{ \begin{array}{c} v(\tau, g, p) - (1-\psi\lambda)(1+p)\theta\log g \\ +\beta\lambda E_p V_O(B(b,p), p', p) \end{array} \right\}, \tag{44}$$

subject to (6), (7), and

$$V_O(b, p, p) = v(G(b,p), T(b,p), p) + \beta\lambda E_p V_O\left(B(b,p), p', p\right),$$

where E_p is a conditional expectation operator. The second argument of the function V_O stands for the current p, while the third argument refers to the initial p. These two arguments are identical for the initial period.

We now proceed to solve the program and to derive the GEE. If all policy functions are continuous and differentiable, the solution must satisfy the following First Order Conditions

$$-\frac{(1+\beta)\lambda}{A(\tau)(1-e(\tau))} = \beta\lambda E_p V_O'\left(B(b,p), p', p\right) \tag{45}$$

$$-\frac{\lambda(1+\psi)(1+p)\theta}{g} = \beta\lambda E_p V_O'\left(B(b,p), p', p\right) \tag{46}$$

where $V_O'(b', p', p)$ denotes the derivative of V_O with respect to b'. The two equations, (45)-(46), together with the equilibrium conditions $g = G(b,p)$ and $\tau = T(b,p)$ imply, for all p, the intra-temporal condition

$$\frac{1+\beta}{(1+\psi)(1+p)\theta} G(b,p) = A(\tau)(1-e(T(b,p))), \tag{47}$$

which is the analogue of equation (19). This leads to

$$\begin{aligned} V_O(b,p,p) &= ((1+\beta)\lambda + (1+\lambda)(1+p)\theta)\log(G(b,p)) - (1+\beta)\lambda\log(1-e(T(b,p))) \\ &\quad +\beta\lambda E_p V_O\left(B(b,p),p',p\right), \end{aligned} \tag{48}$$

$$B(b,p) = G(b,p) + Rb - T(b,p)\,wH(T(b,p)) \tag{49}$$

Differentiating $V_O(b,p)$ and $B(b,p)$ with respect to b yields, then,

$$\begin{aligned} V_O'(b,p,p) &= ((1+\beta)\lambda + (1+p)\theta(1+\lambda))\frac{G'(b,p)}{G(b,p)} + \frac{(1+\beta)\lambda e'(T(b,p))\,T'(b,p)}{1-e(T(b,p))} \\ &\quad +\beta\lambda\cdot E_p V_O'\left(B(b,p),p',p\right)\cdot B'(b,p), \end{aligned} \tag{50}$$

$$\begin{aligned} B'(b,p) &= G'(b,p) + R - T'(b,p)\,wh_M(T(b,p))(1-e(T(b,p))) \\ &= \left(1+\frac{1+\beta}{\theta(1+p)(1+\psi)}\right)G'(b,p) + R + e'(T(b,p))\,T'(b,p)\,A(T(b,p)), \end{aligned} \tag{51}$$

where the last equality is derived as in the proof of deterministic case. Recalling that the First Order Condition, (46), implies that

$$-\frac{\lambda(1+\psi)(1+p)\theta}{\beta\lambda g} = E_p V_O'\left(B(b,p),p',p\right), \tag{52}$$

we can rewrite (50) as

$$\begin{aligned} V_O'(b,p,p) &= ((1+\beta)\lambda + \theta(1+p)(1+\lambda))\frac{G'(b,p)}{G(b,p)} + \frac{(1+\beta)\lambda e'(T(b,p))\,T'(b,p)}{1-e(T(b,p))} - \\ &\quad \frac{\lambda(1+\psi)(1+p)\theta}{g}\cdot \\ &\quad \left(\left(1+\frac{1+\beta}{(1+p)\theta(1+\psi)}\right)G'(b,p) + R + e'(T(b,p))\,T'(b,p)\,A(T(b,p))\right), \end{aligned}$$

Taking one-period lead expectations,

$$\begin{aligned} E_p V_O'\left(b',p',p\right) = E_p\Bigg[&((1+\beta)\lambda + (1+p)\theta(1+\lambda))\frac{G'(B(b,p),p')}{G(B(b,p),p')} + \\ &\frac{(1+\beta)\lambda e'(T(B(b,p),p'))\,T'(B(b,p),p')}{1-e(T(B(b,p),p'))} - \\ &\frac{\lambda(1+\psi)(1+p)\theta}{G(B(b,p),p')}\cdot\left(\left(1+\frac{1+\beta}{\theta(1+p)(1+\psi)}\right)G'\left(B(b,p),p'\right) + R + \right. \\ &e'\left(T\left(B(b,p),p'\right)\right)T'\left(B(b,p),p'\right)A\left(T\left(B(b,p),p'\right)\right)\Bigg]. \end{aligned}$$

Hence,

$$
\begin{aligned}
-\frac{\lambda(1+\psi)(1+p)\theta}{\beta\lambda G(b,p)} &= E_p\left((1+\beta)\lambda+\theta(1+p)(1+\lambda)\right)\frac{G'(B(b,p),p')}{G(B(b,p),p')} \\
&\quad -E_p\frac{\lambda(1+\psi)(1+p)\theta}{G(B(b,p),p')}\cdot\left(\left(1+\frac{1+\beta}{\theta(1+p)(1+\psi)}\right)G'\left(B(b,p),p'\right)+R\right)
\end{aligned}
$$

where the term on the left-hand side has been replaced using again (52), while the simplification on the right hand-side follows from (47). Finally, after rearranging terms, we obtain

$$
\frac{1}{G(b,p)}=\beta\lambda R\cdot E_p\frac{1}{G(B(b,p),p')}-\beta\lambda E_p\left(\frac{1+\lambda^{-1}}{1+\psi}-1\right)\frac{G'(B(b,p),p')}{G(B(b,p),p')},
$$

that is, the GEE (33) in the text. This concludes the proof of Proposition 4..

8.7 Proof of Proposition 5

When $\xi=0$ and $\lambda=0$, the GEE, (33), simplifies to:

$$
\frac{1}{G(b,p)}=-\frac{\beta}{1+\psi}\cdot E_p\left[\frac{G'(B(b,p),p')}{G(B(b,p),p')}\right], \tag{53}
$$

We guess that

$$
G(b,p)=\gamma(p)\left(\bar{b}-b\right). \tag{54}
$$

Combining equations (47), (49), and (54) imply that

$$
\bar{b}-B(b,p)=\left(R-\left(1+\frac{1+\beta}{(1+p)\theta(1+\psi)}\right)\gamma(p)\right)\left(\bar{b}-b\right). \tag{55}
$$

Combining equations (53)-(55) and rearranging terms yield;

$$
\gamma(p)=\frac{(1+\psi)\theta(1+p)R}{\psi\theta(1+p)+(1+\beta)(1+\theta(1+p))}
$$

and

$$
G(b,p)=\frac{(1+\psi)\theta(1+p)R}{\psi\theta(1+p)+(1+\beta)(1+\theta(1+p))}\left(\bar{b}-b\right). \tag{56}
$$

Hence, substituting the expression of $\gamma(p)$ into (55) leads to

$$
B(b,p)=\bar{b}-\frac{\beta R\theta(1+p)}{\psi\theta(1+p)+(1+\beta)(1+\theta(1+p))}\left(\bar{b}-b\right). \tag{57}
$$

Additionally, in the case of $\lambda=\xi=0$ the intra-temporal condition, (47), simplifies to

$$
1-T(b,p)=\frac{1+\beta}{(1+\psi)\theta(1+p)wh}G(b,p). \tag{58}
$$

Finally, recall that, when $\lambda=0$, then $\psi=\omega/(1-\omega)$. Then, equations (56), (57) and (58) yield the policy functions in Proposition 5. This concludes the proof.

9 Appendix C: Proof of Proposition 6

The proof is based on several lemmas.

Lemma 3 *The following equation.*

$$(1+\psi) R^{1+\frac{2\zeta}{1+\psi}} \zeta^{\frac{\zeta}{1+\psi}} = ((1+\psi+\zeta) R - (1+\psi))^{1+\frac{\zeta}{1+\psi}}. \tag{59}$$

has two roots.

Proof. Any root of (59), denoted by $\hat{R}$, must satisfy $\Delta\left(\hat{R}\right) = 0$, where

$$\Delta\left(\hat{R}\right) \equiv \left(1+\frac{2\zeta}{1+\psi}\right)\log \hat{R} + \frac{\zeta}{1+\psi}\log \zeta + \log(1+\zeta) - \left(1+\frac{\zeta}{1+\psi}\right)\log\left(\hat{R}(1+\psi+\zeta) - (1+\psi)\right) = 0.$$

Since $\Delta(1) > 0$, $\Delta(\infty) > 0$ and $\Delta' < 0$ for $R < (1+\psi+2\zeta)/(\zeta(1+\psi+\zeta))$, $\Delta' > 0$ for $R > (1+\psi+2\zeta)/(\zeta(1+\psi+\zeta))$, it is sufficient to show that $\Delta\left(1+\frac{1+\psi}{\zeta}\right) < 0$, or equivalently,

$$\Pi(\zeta,\psi) \equiv \left(1+\frac{2\zeta}{1+\psi}\right)\log(1+\psi+\zeta) + \log(1+\psi) - \left(1+\frac{\zeta}{1+\psi}\right)\log\left((1+\psi+\zeta)^2 - \zeta(1+\psi)\right) < 0.$$

This is easy to be confirmed by the facts that $\zeta \geq 0$, $\Pi(0,\psi) = 0$, $\left.\frac{\partial \Pi(\zeta,\psi)}{\partial \zeta}\right|_{\zeta=0} = 0$ and

$$\frac{\partial^2 \Pi(\zeta,\psi)}{\partial \zeta^2} = \frac{\zeta\left(-3(1+\psi)^3 - 2\zeta(1+\psi)^2 + \zeta^2(1+\psi) + \zeta^3\right)}{(1+\psi+\zeta)^2\left(\zeta^2+\zeta(1+\psi)+(1+\psi)^2\right)^2} < 0$$

for $\zeta \geq$ and any $\psi \geq 0$. ■

Definition 2 R_h *is the larger root of equation (59).*

Lemma 4 *For any* $R \in \left[1+\frac{1+\psi}{\zeta}, R_h\right]$, *we have*

$$(1+\psi) R^{1+\frac{2\zeta}{1+\psi}} \zeta^{\frac{\zeta}{1+\psi}} \leq ((1+\psi+\zeta) R - (1+\psi))^{1+\frac{\zeta}{1+\psi}}.$$

Proof. By the proof of Lemma 3. ■

Lemma 5 *The sequence* $\{b_n^*\}_{n=0}^{\infty}$ *converges to* $\bar{b}$ *along an increasing path.*

Proof. (43) gives an implicit difference equation of b_n^*. Rearranging (43) and using the fact that $\bar{\tau}wh = (R-1)\bar{b}$, we obtain

$$y_n = (x_n)^{-\frac{\zeta}{1+\psi}}, \tag{60}$$

where $y_n \equiv \frac{(R-1)\bar{b}+b_n^*-Rb_{n+1}^*}{(R-1)\left(\bar{b}-b_{n+1}^*\right)}$ and $x_n \equiv \frac{\bar{b}-b_n^*}{\bar{b}-b_{n+1}^*}$. Linearizing (60) around $b' = b$ yields

$$y_n - 1 = -\frac{\zeta}{1+\psi}\left(x_n - 1\right),$$

or equivalently

$$\frac{b_n^* - b_{n+1}^*}{(R-1)\left(\bar{b} - b_{n+1}^*\right)} = \frac{\zeta}{1+\psi}\frac{\left(b_n^* - b_{n+1}^*\right)}{\bar{b} - b_n^*}.$$

This establishes

$$b_{n+1}^* = \bar{b} - \frac{1+\psi}{\zeta\left(R-1\right)}\left(\bar{b} - b_n^*\right). \tag{61}$$

It is immediate that if $\frac{1+\psi}{\zeta(R-1)} < 1$ (or equivalently $R > 1 + \frac{1+\psi}{\zeta}$), b_n^* is converging to the maximum debt level $\bar{b}$ along an increasing path. ■

Lemma 6 *For any* $R \in \left[1 + \frac{1+\psi}{\zeta}, R_h\right]$, *we have*

$$x_0 < \frac{\zeta R^2}{\zeta R + (1+\psi)(R-1)}. \tag{62}$$

Proof. First note that (43) can be rewritten as follows.

$$S\left(x_n\right) \equiv R - x_n - (R-1)\left(x_n\right)^{-\frac{\zeta}{1+\psi}} = 0. \tag{63}$$

Since $x_n \geq 1$, it is easy to show that for $x_n > 1$, there is a unique $\hat{x}_n$ such that $S\left(\hat{x}_n\right) = 0$. Moreover, $S'\left(\hat{x}_n\right) < 0$. Hence, for (62) to hold, we need to show

$$S\left(\frac{\zeta R^2}{\zeta R + (1+\psi)(R-1)}\right) \leq 0,$$

which implies

$$\frac{(1+\psi)R}{\zeta R + (1+\psi)(R-1)} \leq \left(\frac{\zeta R + (1+\psi)(R-1)}{\zeta R^2}\right)^{\frac{\zeta}{1+\psi}}.$$

This is ensured by Lemma 4. ■

Lemma 7 *Suppose that future policy outcomes follow (40), (41) and (42). Then, the current government's objective function is*

$$V\left(b'; b\right) = (1+\psi)\,\theta\log\left(b' - Rb + \tau wh\right) + (1+\beta)\log\left(1-\tau\right) + \theta\zeta\log\left(b_n^* - Rb' + \bar{\tau}wh\right). \tag{64}$$

Proof. (40), (41) and (42) establish that $\tau_{t+i} = \bar{\tau}$ and $b_{t+1+i} = b_n^*$ for $i \geq 1$. Therefore, (18) implies

$$V_O\left(b_{t+i}\right) = (1+\lambda)\,\theta\log\left(b_n^* + \bar{\tau}wh - Rb_{t+i}\right) + (1+\beta)\,\lambda\log\left(1-\bar{\tau}\right) + \beta\lambda V_O\left(b_{t+i+1}\right).$$

Denoting b' as the current government's choice of public debt and ignoring constant terms, we have

$$V_O\left(b'\right) = \frac{(1+\lambda)\,\theta\log\left(b_n^* - Rb' + \bar{\tau}wh\right)}{1-\beta\lambda}.$$

Substituting the above equation into (17) leads to (64). ■

Lemma 8 *Suppose that future policy outcomes follow (40), (41) and (42). Then,* $\tau = 1 - \frac{1+\beta}{\theta(1+\psi)wh}g \leq \bar{\tau}$ *if* $b \leq b_0^*$ *and* $\tau = \bar{\tau}$ *otherwise.*

Proof. By Lemma 7, we know that the government's objective function follows (64). The first-order condition establishes

$$\tau = \begin{cases} 1 - \frac{1+\beta}{\theta(1+\psi)wh}g & \text{if } g \geq \frac{(1-\bar{\tau})\theta(1+\psi)wh}{1+\beta} \\ \bar{\tau} & \text{otherwise} \end{cases}.$$

Given $B(b)$, the above equality leads to $G(b)$. Replacing g with $G(b)$, we obtain an equivalence between $g \geq \frac{(1-\bar{\tau})\theta(1+\psi)wh}{1+\beta}$ and $b \leq b_0^*$. ■

Lemma 9 *Suppose that future policy outcomes follow (40), (41) and (42). Then, for* $b \in [b_0^*, b_n^*]$*, any choice* $b' \in \left(b_n^*, b_{n+1}^*\right)$ *can be improved by* $b' = b_n^*$*.*

Proof. Since the tax rate is constrained when $b \geq b_0^*$, the government's objective function can be written as

$$V\left(b';b\right) = (1+\psi)\,\theta\log\left(b' - Rb + \bar{\tau}wh\right) + \theta\zeta\log\left(b_n^* - Rb' + \bar{\tau}wh\right) \text{ for } b' \in \left[b_n^*, b_{n+1}^*\right).$$

Differentiating V with respect to b' yields

$$\begin{aligned} \frac{\partial V}{\partial b'} &= \frac{(1+\psi)\,\theta}{b' - Rb + \bar{\tau}wh} - \frac{\theta\zeta R}{b_n^* - Rb' + \bar{\tau}wh} \\ &= \frac{(1+\psi)\,\theta}{b' - Rb + \bar{\tau}wh} + \frac{\theta\zeta R}{-b_n^* + Rb' - \bar{\tau}wh} \end{aligned}$$

First, note that $\frac{\partial V}{\partial b'}$ is decreasing in b'. So, it is sufficient to prove that $\frac{\partial V}{\partial b'} \leq 0$ at $b' = b_n^*$.

$$\left.\frac{\partial V}{\partial b'}\right|_{b'=b_n^*} = \frac{(1+\psi)\,\theta}{b_n^* - Rb + \bar{\tau}wh} - \frac{\theta\zeta R}{b_n^* - Rb_n + \bar{\tau}wh}$$

We show that

$$\begin{aligned} \frac{(1+\psi)}{b_n^* - Rb + \bar{\tau}wh} &\leq \frac{\zeta R}{b_n^* - Rb_n^* + \bar{\tau}wh} \\ (1+\psi)\left(b_n^* + \bar{\tau}wh\right) - (1+\psi)\,Rb_n^* &\leq \zeta R\left(b_n^* + \bar{\tau}wh\right) - \zeta R^2 b \\ (1+\psi-\zeta R)\left(b_n^* + \bar{\tau}wh\right) &\leq \left(1+\psi - \frac{(1+\lambda)\,\beta}{1-\beta\lambda}R\right)Rb_n - \frac{(1+\lambda)\,\beta}{1-\beta\lambda}R^2\left(b - b_n^*\right) \\ (\zeta R - 1 - \psi)\left(b_n^* - Rb_n^* + \bar{\tau}wh\right) &\geq \zeta R^2\left(b - b_n^*\right) \end{aligned}$$

This is always true if $b \leq b_n^*$. ■

Lemma 10 *Suppose that future policy outcomes follow (40), (41) and (42). Then, for* $b \in [\underline{b}, b_0^*]$*, any choice* $b' \in (\underline{b}, b_0^*)$ *can be improved by* $b' = b_0^*$ *and any choice* $b' \in \left(b_n^*, b_{n+1}^*\right)$ *with* $n \geq 0$ *can be improved by* $b' = b_n^*$*.*

Proof. For $b \in [\underline{b}, b_0^*]$, using the first-order condition $\tau = 1 - \frac{1+\beta}{(1+\psi)\theta wh} g$, the government's objective function can be written as

$$V\left(b'; b\right) = \begin{cases} (1+\beta+\theta(1+\psi))\log\left(b' + wh - Rb\right) \\ \quad +\theta\zeta\log\left(b_0^* - Rb' + \bar{\tau}wh\right) & \text{if } b' \in [\underline{b}, b_0^*] \\ (1+\beta+\theta(1+\psi))\log\left(b' + wh - Rb\right) \\ \quad +\theta\zeta\log\left(b_n^* + \bar{\tau}wh - Rb'\right) & \text{if } b' \in \left[b_n^*, b_{n+1}^*\right] \end{cases}. \tag{65}$$

For $b' \in [\underline{b}, b_0^*]$, it is sufficient to show that

$$\left.\frac{\partial V}{\partial b'}\right|_{b=\underline{b}, b'=b_0^*} = \frac{1+\beta+\theta(1+\psi)}{b_0^* + wh - R\underline{b}} - \frac{\theta\zeta R}{b_0^* + wh - Rb_0} \geq 0.$$

This can directly be confirmed by

$$\frac{1+\beta+\theta(1+\psi)}{b_0^* + wh - R\underline{b}} = \frac{\theta\zeta R}{b_0^* + wh - Rb_0^*}, \tag{66}$$

which is implied by the fact that $B(\underline{b}) = b_0^*$. Hence, any choice $b' \in [\underline{b}, b_0^*]$ can be improved by $b' = b_0^*$.

For $b' \in \left[b_n^*, b_{n+1}^*\right]$ with $n \geq 0$, it is sufficient to prove that

$$\left.\frac{\partial V}{\partial b'}\right|_{b=b_0^*, b'=b_n^*} = \frac{1+\beta+\theta(1+\psi)}{b_n^* + wh - Rb_0^*} - \frac{\theta\zeta R}{b_n^* + \bar{\tau}wh - Rb_n^*} \leq 0.$$

Since $\left.\frac{\partial V}{\partial b'}\right|_{b=b_0^*, b'=b_n^*}$ is decreasing in b_n^*, we only need to show

$$\frac{1+\beta+\theta(1+\psi)}{b_0^* + wh - Rb_0^*} \leq \frac{\theta\zeta R}{b_0^* + \bar{\tau}wh - Rb_0^*}.$$

The above inequality implies that

$$\begin{aligned} (1+\beta+\theta(1+\psi))\bar{\tau} - \theta\zeta R &\leq (1+\beta+\theta(1+\psi) - \theta\zeta R)(R-1)\frac{b_0^*}{wh} \\ &= (1+\beta+\theta(1+\psi) - \theta\zeta R)\left(\frac{(1+\beta+\theta(1+\psi))\bar{\tau} - \theta(1+\psi)}{1+\beta}\right) \end{aligned}$$

First note that when $\bar{\tau} = 1$, LHS is equal to RHS. For the inequality to hold for $\bar{\tau} \in [0, 1)$, we need to show that the slope of LHS $1+\beta+\theta(1+\psi)$ is greater than the slope of RHS $\frac{(1+\beta+\theta(1+\psi)-\theta\zeta R)(1+\beta+\theta(1+\psi))}{1+\beta}$. This is ensured by the fact that $R > 1 + \frac{1+\psi}{\zeta}$. Hence, any $b' \in \left(b_n^*, b_{n+1}^*\right)$ with $n \geq 0$ can be improved by b_n^*.[48] ■

[48] First note that when $\bar{\tau} = 1$, LHS is equal to RHS. For the inequality to hold for $\bar{\tau} \in [0, 1)$, we need to show that the slope of LHS $1+\beta+\theta(1+\omega)$ is greater than the slope of RHS $\frac{(1+\beta+\theta(1+\omega)-\theta\beta R)(1+\beta+\theta(1+\omega))}{1+\beta}$. This is ensured by the fact that $\beta R > 1 + \omega$.

Lemma 11 *Suppose that future policy outcomes follow (40), (41) and (42). Then, for any* $b \in \left[b_n^*, b_{n+1}^*\right)$, $b' = b_{n+s}$ *for any* $s > 0$ *is dominated by* $b' = b_n^*$.

Proof. We need to show that for any $b \in \left[b_n^*, b_{n+1}^*\right)$,

$$\begin{aligned} & \theta\left(1+\psi\right)\log\left(b_n^* - Rb + \bar{\tau}wh\right) + \theta\zeta\log\left(b_n^* - Rb_n^* + \bar{\tau}wh\right) \\ > \; & \theta\left(1+\psi\right)\log\left(b_{n+s}^* - Rb + \bar{\tau}wh\right) + \theta\zeta\log\left(b_{n+s} - Rb_{n+s}^* + \bar{\tau}wh\right). \end{aligned} \tag{67}$$

Rearrange

$$\begin{aligned} & \theta\left(1+\psi\right)\left(\log\left(b_n^* - Rb + \bar{\tau}wh\right) - \log\left(b_{n+s}^* - Rb + \bar{\tau}wh\right)\right) \\ > \; & \theta\zeta\left(\log\left(b_{n+s}^* - Rb_{n+s}^* + \bar{\tau}wh\right) - \log\left(b_n^* - Rb_n^* + \bar{\tau}wh\right)\right) \end{aligned}$$

The LHS and RHS of this expression can be written as

$$\sum_{i=0}^{i=s-1} \theta\left(1+\psi\right)\left(\log\left(b_{n+i}^* - Rb + \bar{\tau}wh\right) - \log\left(b_{n+i+1}^* - Rb + \bar{\tau}wh\right)\right) \tag{68}$$

$$\sum_{i=0}^{i=s-1} \theta\zeta\left(\log\left(b_{n+i+1}^* - Rb_{n+i+1}^* + \bar{\tau}wh\right) - \log\left(b_{n+i}^* - Rb_{n+i}^* + \bar{\tau}wh\right)\right). \tag{69}$$

According to the difference equation

$$\left(b_n^* - Rb_{n+1}^* + \bar{\tau}wh\right)^{1+\psi}\left(b_n^* - Rb_n^* + \bar{\tau}wh\right)^{\zeta} = \left(b_{n+1}^* - Rb_{n+1}^* + \bar{\tau}wh\right)^{1+\psi+\zeta}.$$

(69) is equal to

$$\sum_{i=0}^{i=s-1} \left(1+\psi\right)\left(\log\left(b_{n+i}^* - Rb_{n+i+1}^* + \bar{\tau}wh\right) - \log\left(b_{n+i}^* - Rb_{n+i}^* + \bar{\tau}wh\right)\right)$$

Due to the strict concavity of log utility and the increasing b_n^*,

$$\begin{aligned} & \log\left(b_{n+i}^* - Rb + \bar{\tau}wh\right) - \log\left(b_{n+i+1}^* - Rb + \bar{\tau}wh\right) \\ > \; & \log\left(b_{n+i}^* - Rb_{n+i+1}^* + \bar{\tau}wh\right) - \log\left(b_{n+i+1}^* - Rb_{n+i+1}^* + \bar{\tau}wh\right) \\ > \; & \log\left(b_{n+i}^* - Rb_{n+i+1}^* + \bar{\tau}wh\right) - \log\left(b_{n+i}^* - Rb_{n+i}^* + \bar{\tau}wh\right) \end{aligned}$$

for any $b \in \left[b_n^*, b_{n+1}^*\right)$. This establishes that the LHS of (67) is indeed larger than the RHS of (67). ■

Lemma 12 *Suppose that future policy outcomes follow (40), (41) and (42). Then, for* $b \in [\underline{b}, b_0^*]$, $b' = b_s$ *for any* $s > 0$ *is dominated by* $b' = b_0^*$.

Proof. We need to show that for any $b \in \left[b_n^*, b_{n+1}^*\right)$,

$$\begin{aligned} & (1+\beta+\theta(1+\psi)) \log \left(b_0^*-Rb+wh\right)+\theta\zeta \log \left(b_0^*-Rb_0^*+\bar{\tau}wh\right) \\ > \; & (1+\beta+\theta(1+\psi)) \log \left(b_s^*-Rb+wh\right)+\theta\zeta \log \left(b_s^*-Rb_s^*+\bar{\tau}wh\right). \end{aligned}$$

The rest of the proof simply follows the same procedure as in Lemma 11. ■

Lemma 13 *Suppose $z_H > z_L$ and $b_H > b_L$. Then,*

$$\begin{aligned} & \theta(1+\psi) \log \left(z_H-Rb_L+\bar{\tau}wh\right)+\theta\zeta \log \left(B\left(z_H\right)-Rz_H+\bar{\tau}wh\right) \\ > \; & \theta(1+\psi) \log \left(z_L-Rb_L+\bar{\tau}wh\right)+\theta\zeta \log \left(B\left(z_L\right)-Rz_L+\bar{\tau}wh\right) \end{aligned}$$

implies that

$$\begin{aligned} & \theta(1+\psi) \log \left(z_H-Rb_H+\bar{\tau}wh\right)+\theta\zeta \log \left(B\left(z_H\right)-Rz_H+\bar{\tau}wh\right) \\ > \; & \theta(1+\psi) \log \left(z_L-Rb_H+\bar{\tau}wh\right)+\theta\zeta \log \left(B\left(z_L\right)-Rz_L+\bar{\tau}wh\right). \end{aligned}$$

Proof. We need to show that

$$\begin{aligned} \log \left(z_H-Rb_L+\bar{\tau}wh\right) & > \log \left(z_L-Rb_L+\bar{\tau}wh\right) \Rightarrow \\ \log \left(z_H-Rb_H+\bar{\tau}wh\right) & > \log \left(z_L-Rb_H+\bar{\tau}wh\right) \end{aligned}$$

Define

$$F(b) \equiv \log \left(z_H-Rb+\bar{\tau}wh\right)-\log \left(z_L-Rb+\bar{\tau}wh\right)$$

It is straightforward that F is increasing in b since $z_L < z_H$. Hence, if $F(b_L) > 0$, $F(b_H)$ must be positive. ■

Now we can prove the proposition. We start with $b \in [\underline{b}, b_0^*]$, Lemma 10 and 12 imply that the optimal $b' = b_0^*$. For $b \in [b_0^*, b_1^*]$, Lemma 9 and 11 establish that $b' = b_1^*$ is better than any $b' \geq b_1^*$. For $b' \in [b_0^*, b_1^*]$, we have

$$V\left(b'; b\right)=\theta(1+\psi) \log \left(b'-Rb+\bar{\tau}wh\right)+\theta\zeta \log \left(b_0^*-Rb'+\bar{\tau}wh\right).$$

The optimal $b' = b_0^*$ if $\frac{\partial V}{\partial b'} < 0$ for any $b \in [b_0^*, b_1^*]$ and $b' \in [b_0^*, b_1^*]$. Since $\frac{\partial V}{\partial b'}$ is increasing in b and decreasing in b', the corner solution is obtained if

$$\frac{1+\psi}{b_0^*-Rb_1^*+\bar{\tau}wh} < \frac{\zeta R}{b_0^*-Rb_0^*+\bar{\tau}wh}.$$

Rearrange

$$b_1^* < \bar{b}-\frac{\zeta R+(1+\psi)(R-1)}{\zeta R^2}\left(\bar{b}-b_0^*\right)$$

or equivalently

$$x_0 < \frac{\zeta R^2}{\zeta R + (1+\psi)(R-1)}. \tag{70}$$

This must hold according to Lemma 6.

Then we move to $b \in [b_1^*, b_2^*]$. Lemma 9 and 11 establish that $b' = b_2^*$ is better than any $b' \geq b_2^*$. Moreover, Lemma 13 establishes that any choice $b' < b_1^*$ cannot be optimal. For $b' \in [b_1^*, b_2^*]$, it can easily be shown that the optimal $b' = b_1^*$ if

$$x_1 < \frac{\zeta R^2}{\zeta R + (1+\psi)(R-1)}. \tag{71}$$

This must hold according to Lemma 6.

The proof is completed by following the same procedure for any $n > 1$.

Table 2: Regression for U.S. Data

Dep. Variable	change in the debt-GDP ratio Δd_t		
	(1)	(2)	(3)
constant	0.0378**	0.0274	0.0272
	(2.51)	(1.61)	(1.62)
d_t	-0.0885**	-0.0675	-0.0670
	(-2.34)	(-1.60)	(-1.61)
DEMO	-0.0207***	-0.0172***	-
	(-4.02)	(-3.27)	
UNEMPL	-	0.0064***	0.0064***
		(2.92)	(2.84)
DEMO_PRE1980	-	-	-0.0182**
			(-2.61)
DEMO_POST1980	-	-	-0.0156***
			(-2.78)
Obs.	57	57	57
R^2	0.3974	0.4934	0.4942

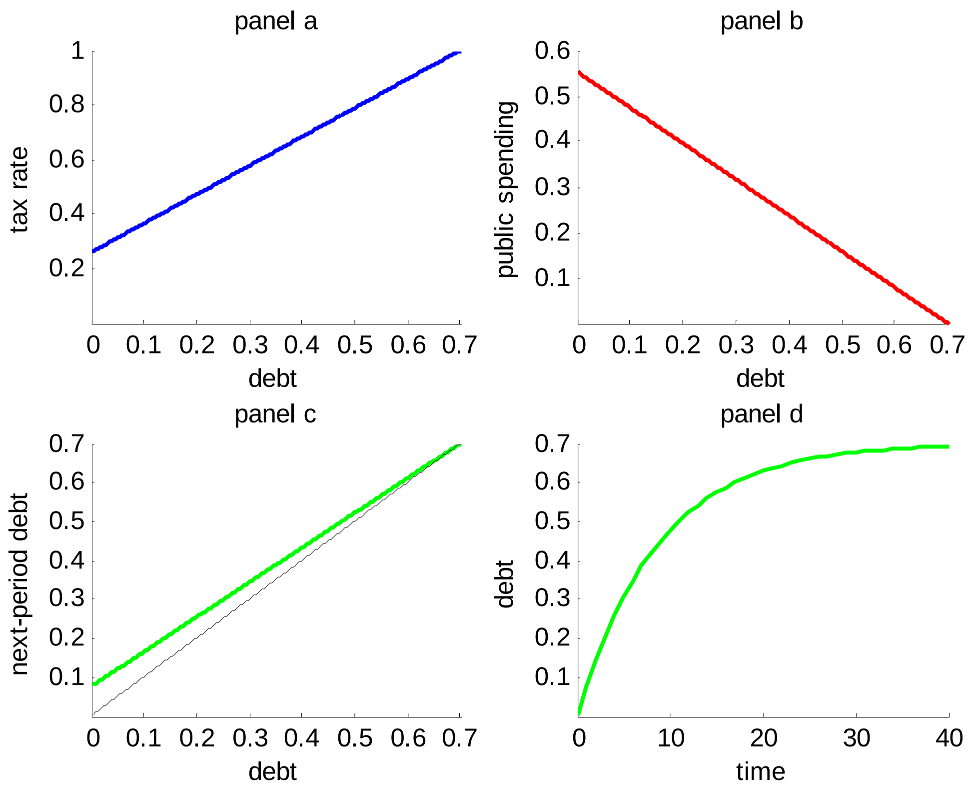
Notes: DEMO is a dummy variable which equals one or zero when the president is a Democrat or Republican, respectively. UNEMPL stands for the unemployment rate subtracted by the mean of the unemployment rate. DEMO_PRE1980 is set equal to DEMO before 1980 and zero afterwards, while DEMO_POST1980 equals DEMO after 1980 and zero otherwise. Robust *t* statistics is in brackets. ***, ** and * is significant at 1%, 5% and 10%, respectively.

Table 3: Panel Regression

Dep. Variable	change in the debt-GDP ratio Δd_t							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
d_t	0.0061	0.0104	-0.0184*	-0.0196**	-0.0264***	-0.0346***	-0.0206*	-0.0208**
	(0.59)	(0.90)	(-1.79)	(-2.53)	(-2.94)	(-3.50)	(-1.79)	(-2.01)
d_t*JPN	-	-	-	0.1135***	0.1252***	0.0991***	-	-
				(6.77)	(7.14)	(5.20)		
POL_WB	-0.0028**	0.0029**	-0.0028**	-0.0029**	-0.0030**	-0.0031**	-	-
	(2.28)	(-2.37)	(-2.28)	(-2.40)	(-2.51)	(-2.52)		
POL_FR	-	-	-	-	-	-	-0.0138***	-
							(-2.76)	
POL_SSZ	-	-	-	-	-	-	-	-0.0041***
								(-2.86)
UNEMPL	-	0.0012*	0.0023***	-	0.0025***	0.0026***	0.0034***	0.0025***
		(1.84)	(3.63)		(3.97)	(4.13)	(4.61)	(4.10)
Control Variables	No	No	Yes	No	No	Yes	Yes	Yes
obs.	951	894	877	951	894	877	754	931
Ad. R^2	0.3089	0.2836	0.3415	0.3594	0.3441	0.3697	0.3207	0.3320

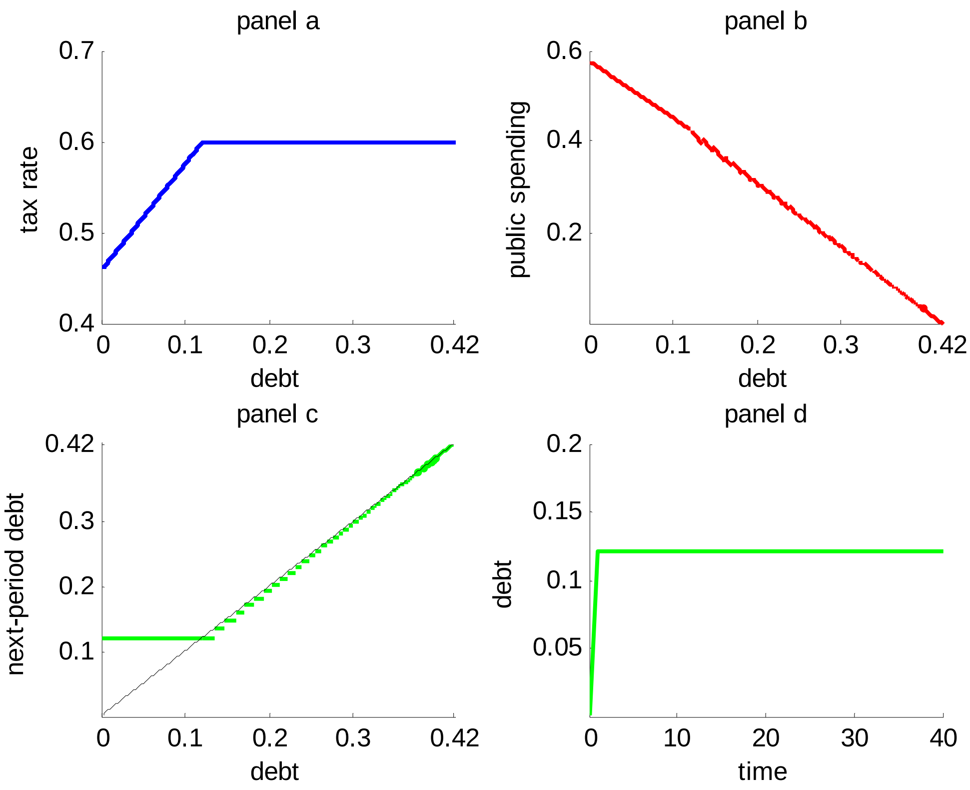
Notes: Country dummies and year dummies are included to control for the fixed effects and time effects. JPN is a dummy variable which equals one for Japan and zero otherwise. POL codes left-right positions of government through a three-point scale: -1 for the right-wing government, 0 for the coalition government and 1 for the left-wing government. POL_FR codes left-right positions of government at far left to 1 and at far right to -1. POL_SSZ assigns scores through a three-point scale: -1 for the right-wing government, 0 for the coalition government and 1 for the left-wing government. UNEMPL stands for the unemployment rate. Control variables are the log of real GDP per capita, openness, the sizes of population over 65 and below 14. Robust *t* statistics is in brackets. ***, ** and * is significant at 1%, 5% and 10%, respectively.

Figure 1: Example I (ξ=0)



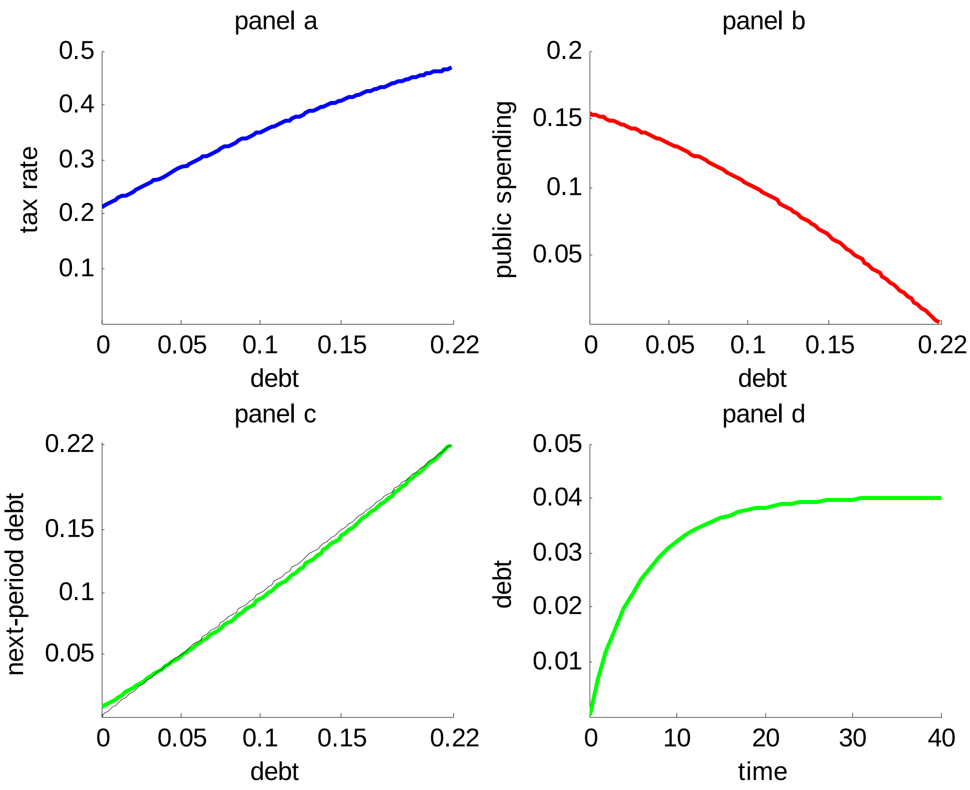
The figure shows equilibrium policy rules T(b) (panel a), G(b) (panel b), B(b) (panel c) and the equilibrium path of b (panel d). Parameter values are $\beta = \lambda = 0.98^{30}$, $R = 1.03^{30}$, $\omega = 0.50$, $\theta = 1.00$ and $w = 1$. The maximum debt level is $\bar{b} = 0.70$.

Figure 2: Example II (ξ=1)



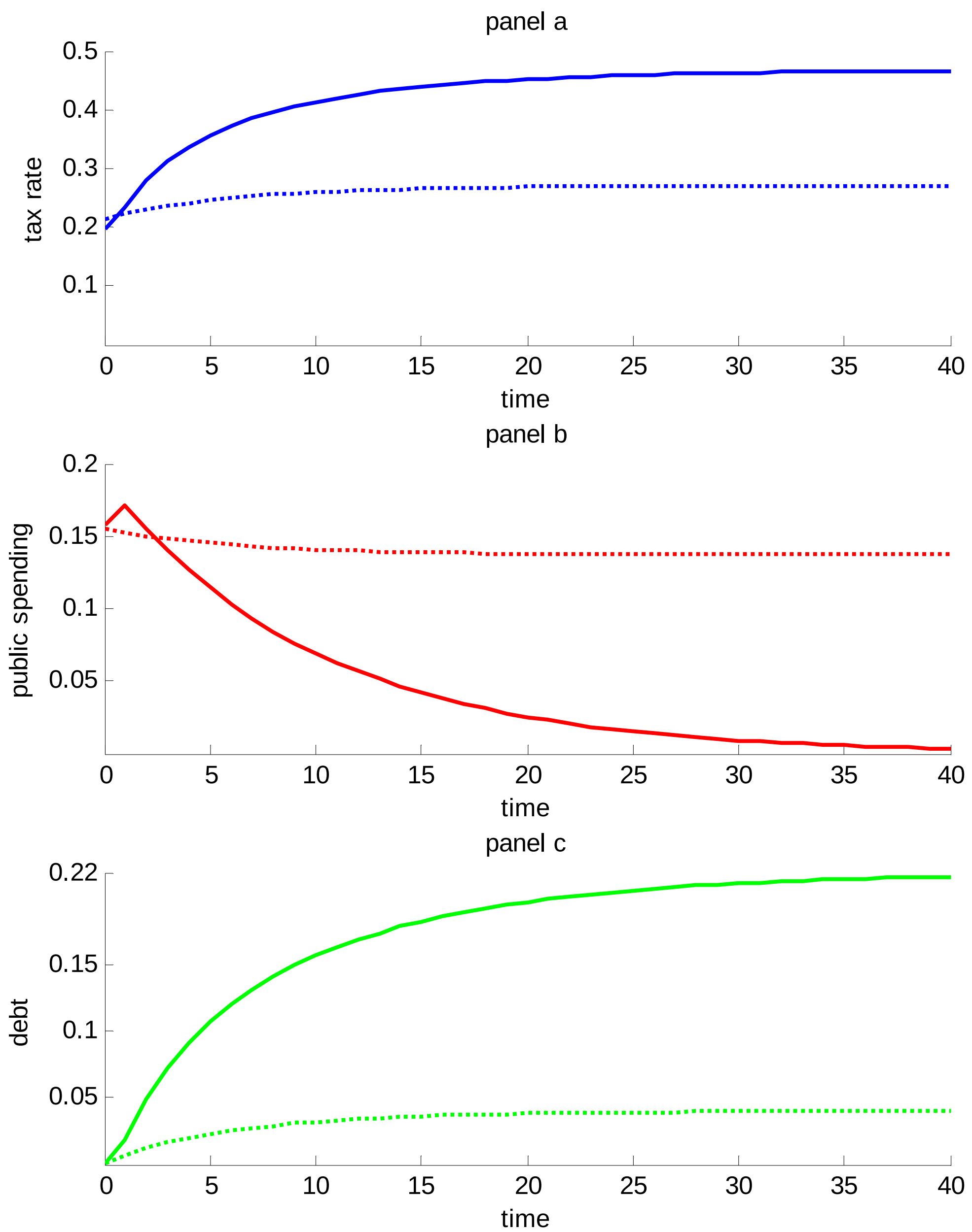
The figure shows equilibrium policy rules T(b) (panel a), G(b) (panel b), B(b) (panel c) and the equilibrium path of b (panel d). Parameter values are $\beta = \lambda = 0.98^{30}$, $R = 1.03^{30}$, $\omega = 0.50$, $\theta = 1.00$, $w = 1$ and $\bar{\tau} = 0.60$. The maximum debt level is $\bar{b} = 0.42$.

Figure 3: General case ($0<\xi<1$)



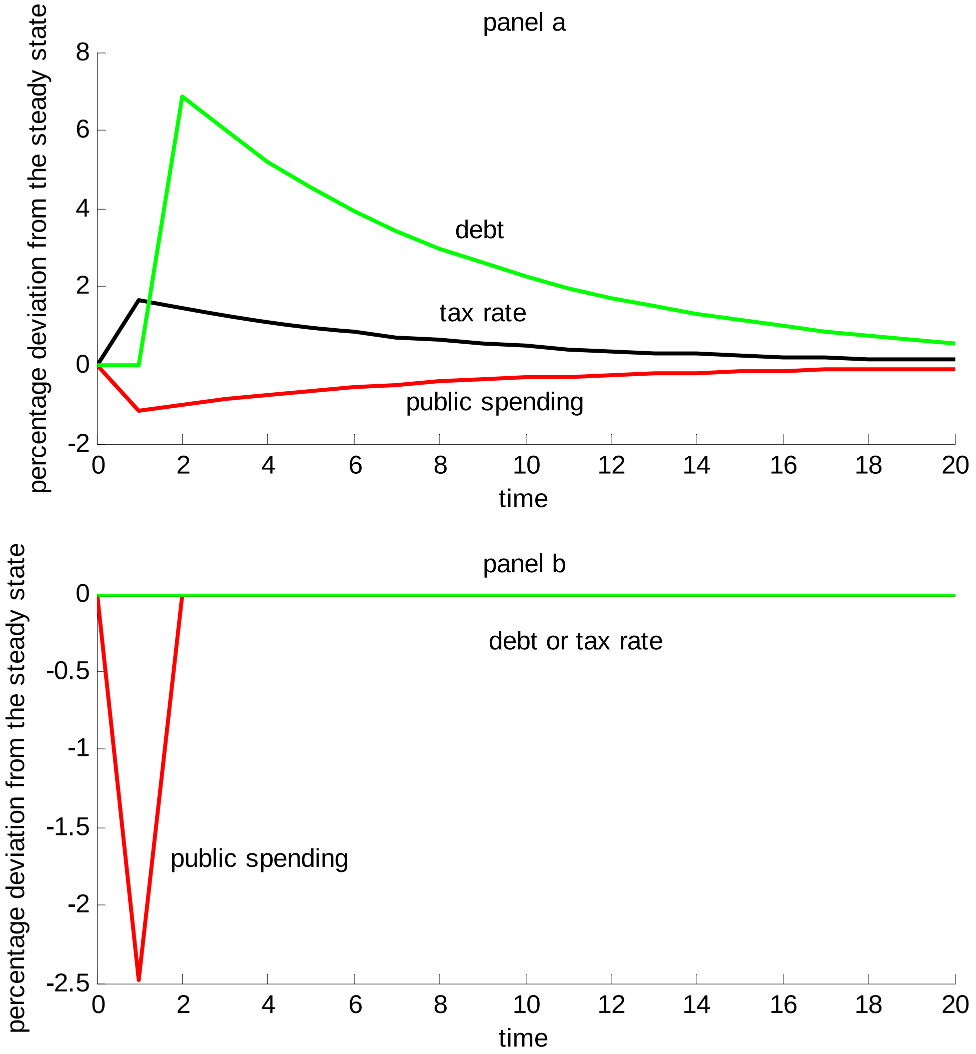
The figure shows equilibrium policy rules for the calibrated economy: T(b) (panel a), G(b) (panel b), B(b) (panel c) and the equilibrium path of b (panel d). Parameter values are $\beta = 0.98^{30}$, $\lambda = 0.79$, $R = 1.025^{30}$, $\omega = 0.50$, $\theta = 0.09$, $\xi = 0.17$, $X = 1.75$, and $w = 1$. The maximum debt level is $\bar{b} = 0.22$. The steady state levels are $\tau = 0.27, g = 0.14, b = 0.04$.

Figure 4: Ramsey versus Markov



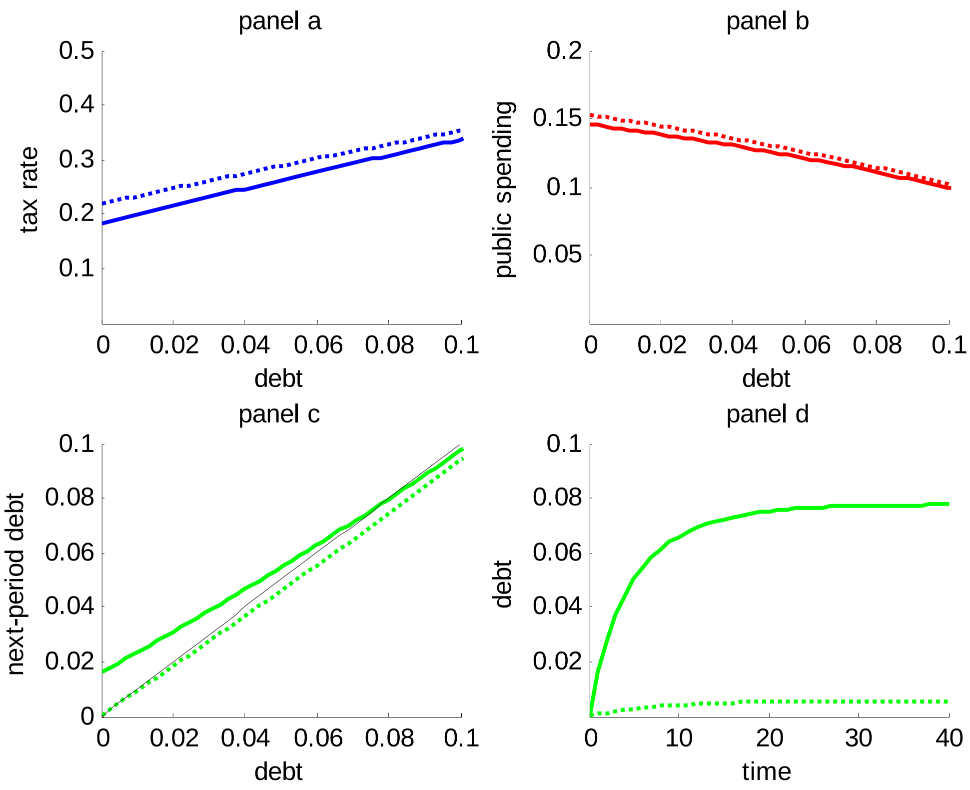
The figure shows the Ramsey paths (solid lines) and Markov equilibrium paths (dotted lines) of taxes (panel a), public spending (panel b) and debt (panel c). All parameter values are as in Figure 3 (calibrated economy).

Figure 5: Impulse Response Functions for an Unanticipated War



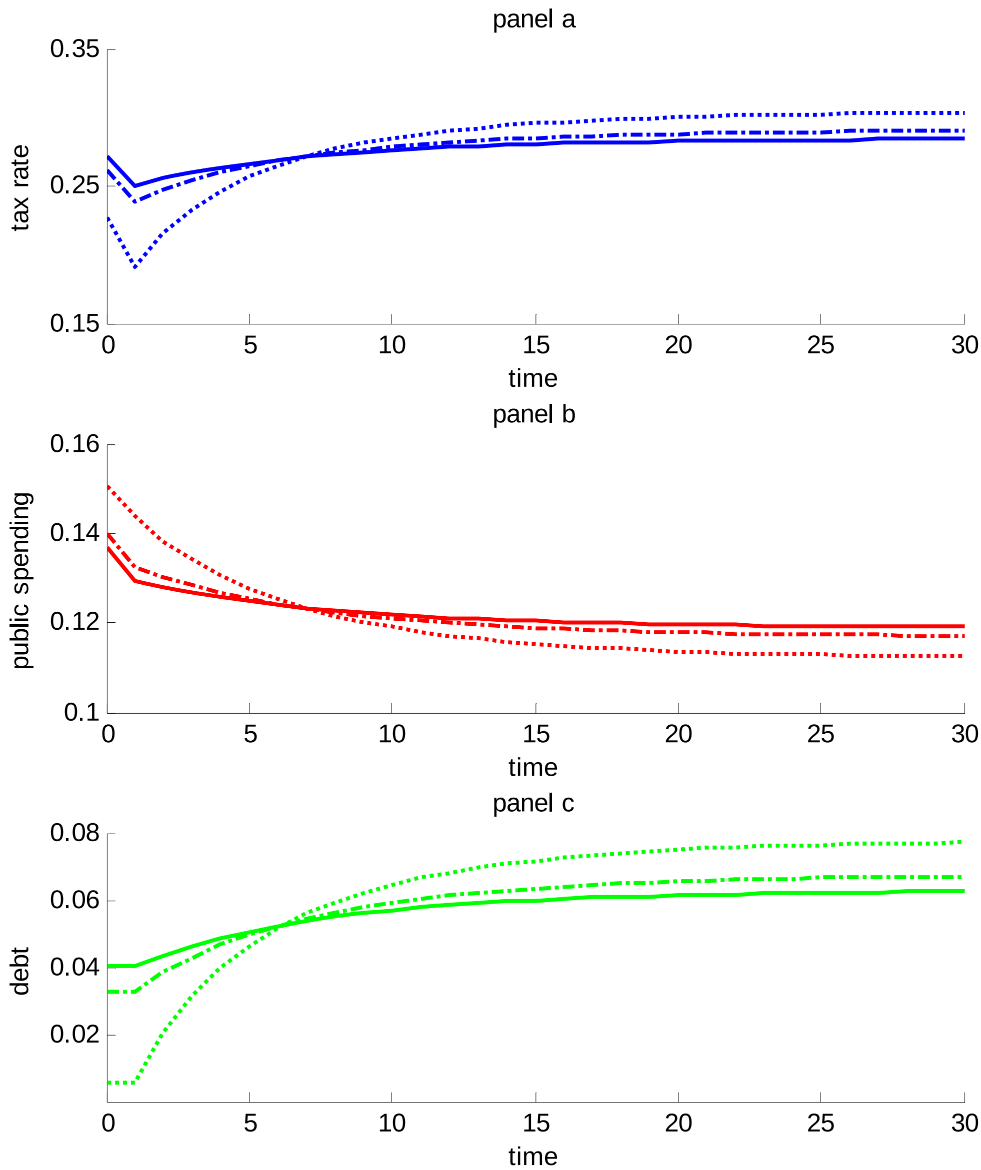
The figure shows the impulse-response functions of tax, government spending and debt for the calibrated economy (panel a), and for the economy in example II (panel b). Parameters are as in figure 3 and figure 2, respectively. The war expenditure Z is set equal to 1% of GDP.

Figure 6: Political Shocks



The figure shows equilibrium policy rules for the calibrated economy under left-wing (dotted lines) and right-wing (solid lines) governments: T(b) (panel a), G(b) (panel b), B(b) (panel c) and the equilibrium path of b (panel d). Parameter values are: $p_l = 0.11$, $p_r = 0$, $\pi_{ll} = \pi_{rr} = 0.5$. The other parameter values are as in Figure 3 (calibrated economy). Panel d plots the evolution of debt under *perpetual* right- and left-wing governments.

Figure 7: Response to a Right-wing Shift



The figure shows the equilibrium paths of taxes (panel a), public spending (panel b) and debt (panel c) for economies which are initially in the left-wing steady states and experience a persistent shift to the right. The three lines in each panel represent economies with different persistence of political color: $\pi_{ll} = \pi_{rr} = 1.0$ (solid lines), $\pi_{ll} = \pi_{rr} = 0.9$ (dashed lines), and $\pi_{ll} = \pi_{rr} = 0.5$ (dotted lines). Parameter values are as in Figure 6.