

Journal of Public Economics 60 (1996) 147-151



# Should capital income be taxed in the steady state?

Isabel H. Correia<sup>a,b,\*</sup>

<sup>\*</sup>Portuguese Catholic University, FCEE, Palma de Cima, 1600 Lisboa, Portugal <sup>b</sup>Banco de Portugal DEE, R. Francisco Ribeiro, 1100 Lisboa, Portugal

Received March 1993; final version received December 1994

## Abstract

This paper provides a new economic interpretation of the well-known dynamic optimal taxation principle that capital income should not be taxed in the steady state. We show that the result is related to the minimization of distortions at the intratemporal margin. When every factor of production can be taxed at the optimal rate, capital income should not be taxed in the steady state. But when there are restrictions on the taxation of production factors, the tax rate on capital income in the steady state is different from zero.

Keywords: Capital taxation; Optimal taxation

JEL classification: H2

## 1. Introduction

Chamley's (1986) paper is a fundamental mark in the literature on optimal capital taxation. The main finding of that work, which I refer to as the 'Chamley theorem', is that capital income should not be taxed in the steady state. This paper clarifies the economic reasoning that underpins this important result by constructing an example where it is optimal to have a positive steady-state tax on capital.

In the model that underpins Chamley's analysis - the neoclassical model -

\* Correspondence address: Banco de Portugal DEE, R. Francisco Ribeiro, 1100 Lisboa, Portugal.

the tax policy *cannot* affect the steady-state after-tax real interest rate. In the absence of technological progress, this rate can take a single value that depends on the discount factor ( $\beta$ ) but is independent of tax policy. Therefore the growth rate of consumption and consequently the steady-state intertemporal marginal rate of substitution are also independent of the tax policy. It is exactly when capital tax policy cannot affect the intertemporal margin that it is optimal to set the capital tax rate to zero. Since the government cannot affect the intertemporal margin, the optimality of zero capital taxation has to be related to the minimization of distortions at the intratemporal margins. The long-run burden of capital taxation is shifted to other factors, and it is more efficient to tax directly these other factors of production. When these other factors cannot be taxed directly the optimality of the zero tax rate on capital income disappears. The following section gives one counter-example of the Chamley theorem, which improves our understanding of that result.

# 2. The role of restrictions on taxation

To clarify the underpinnings of Chamley's result it is useful to consider a neoclassical growth model in which, besides capital (K) and labor (N), there is a third production factor which cannot be taxed. The production function,  $F(K, N, \mathcal{T})$ , is homogeneous of degree one, concave, twice continuously differentiable and satisfies the Inada conditions. Alternatively, we can think of a setting in which profits are not taxed and the production function exhibits decreasing returns to scale. We consider that the supply of this additional production factor,  $\mathcal{T}$ , is elastic.<sup>1</sup> The competitive remuneration of factor  $\mathcal{T}$  will be denoted by  $s_i$ .

The competitive equilibrium under perfect foresight for this economy can be found by combining the solution to the problem described below with the competitive firm's profit-maximization conditions:

$$\max U = \sum_{i=0}^{\infty} \beta^{i} U(C_{i}, L - N_{i}, \overline{\mathcal{T}} - \mathcal{T}_{i}), \quad 0 < \beta < 1^{2},$$

subject to

$$(R_{t} - \delta)(1 - \tau_{K_{t}})K_{t} + w_{t}(1 - \tau_{N_{t}})N_{t} + s_{t}\mathcal{T}_{t} \ge C_{t} + K_{t+1} - K_{t} + B_{t+1}^{g} - (1 + r_{t})B_{t}^{g}, \qquad (1)$$

<sup>1</sup> We define preferences over consumption, C, leisure, L - N, and  $\overline{\mathcal{F}} - \mathcal{T}$ .

 $^2$  We assume that momentary utility is concave, twice continuously differentiable, and satisfies the Inada conditions.

where  $B_0$  and  $K_0$  are given and  $\{w_i, R_i, r_i, \tau_{N_l}, \tau_{N_l}\}$  are exogenous sequences of the wage rate, the rental price of capital, the after-tax real rate of return on public debt, the tax rate on labor income and the tax rate on capital income, respectively.  $C_i$  is private consumption and  $B_i^g$  is the stock of public debt. To simplify, we assume that U is strongly separable between C and  $\overline{\mathcal{T}} - \mathcal{T}$ , and isoelastic in C and in  $\overline{\mathcal{T}} - \mathcal{T}$ , with the constant elasticities designated, respectively, by  $\sigma$  and by  $\eta$ .

The competitive firm's profit maximization conditions are<sup>3</sup>

$$F_{1,t} = R_t , \qquad (2)$$

$$F_{2,t} = w_t , \tag{3}$$

$$F_{3,t} = s_t . (4)$$

In the absence of restrictions on taxation, in addition to the impossibility of using lump-sum taxation, the second-best tax policy can be obtained by solving the following problem:

$$\max U = \sum_{t=0}^{\infty} \beta^{t} U(C_{t}, L - N_{t}, \bar{\mathcal{T}} - \mathcal{T}_{t})$$

subject to

$$F(K_{t}, N_{t}, \mathcal{T}_{t}) + (1 - \delta)K_{t} \ge C_{t} + G_{t} + K_{t+1}, \qquad (5)$$

$$\sum_{t=0}^{\infty} \beta' [U_{1,t}C_{t} - U_{2,t}N_{t} - U_{3,t}\mathcal{T}_{t}] = U_{1,0}[K_{0} + (F_{1,0} - \delta)(1 - \tau_{K0})K_{0}], \qquad (6)$$

where G denotes the exogenous sequence of public expenditures and Eq. (6) is the implentability constraint (Lucas and Stokey, 1983). The existence of restrictions on taxation implies that this formalization is not complete. From the first-order conditions of the second-best problem we obtain the following condition:

$$\frac{U_{3,t}}{U_{1,t}} \frac{1 + \psi[1 - \sigma]}{1 + \psi[1 + \eta \mathcal{T}t/(\bar{\mathcal{T}} - \mathcal{T}_t)]} = F_{3,t} .$$
(7)

Eq. (7) shows that, as long as  $\psi$ , the multiplier of Eq. (6), is different from zero, the condition of the competitive equilibrium,

$$\frac{U_{3,t}}{U_{1,t}} = F_{3,t},$$
(8)

<sup>3</sup> We use the notation  $f_{j,t}$  to represent the *j* partial derivative of *f*, evaluated at *t*. Higher order derivatives are represented by  $f_{ij,t}$ .

does not hold and hence this solution to the second-best problem cannot be decentralized. This means that condition (8) has to be imposed explicitly in the Ramsey problem. Let  $\theta_i$  denote the shadow price associated with this condition.

The optimal steady-state tax rate on capital income can be derived from the following first-order conditions of this problem, which characterize respectively the optimal choice of  $C_t$  and  $K_{t+1}$ :

$$\beta^{t} U_{1,t} + \psi \beta^{t} [U_{1,t} + U_{11,t} C_{t} - U_{12,t} N_{t}] - \theta_{t} U_{11,t} U_{3,t} / (U_{1,t})^{2} = \lambda_{t} , \quad t \ge 1 ,$$
(9)

$$\lambda_{t} = \lambda_{t+1} (1 + F_{1,t+1} - \delta) - \theta_{t+1} F_{31,t+1} , \qquad (10)$$

where  $\lambda$  is the multiplier of Eq. (5). The term in Eq. (10),  $\theta_{t+1}F_{31,t+1}$ , is different from zero whenever financing government expenditures requires the use of distorting taxes ( $\psi > 0$ ) and whenever changes in  $K_t$  affect the marginal product of  $\mathcal{T}$  ( $F_{31} \neq 0$ ). Under these conditions Eq. (10) implies that the social return to capital is not equal to the marginal productivity of capital net of depreciation. This fact determines the following result:

Proposition. When there exist factors of production that cannot be taxed at the optimal rate the steady-state optimal tax rate on capital income is generally not zero; if these factors are complements to capital, the tax is positive, and it is negative in cases where the factors are substitutes to capital. It is only when the production function exhibits strong separability between capital and the non-optimally taxed factor that we obtain Chamley's result.

Proof. In the steady state, condition (9) implies that

$$\frac{1}{\beta} = \frac{\lambda_i}{\lambda_{i+1}}.$$
(11)

Using conditions (11), (10) and the equation that states that the net rate of return of capital at the steady state in the competitive equilibrium is equal to  $1/\beta - 1$ , we obtain the following equation:

$$(1 - \tau_K)(F_1 - \delta) = F_1 - \delta - (\theta_{t+1}F_{31})/\lambda_{t+1}.$$
(12)

Since restriction (8) is always binding,  $\theta_i$  is always positive. This implies that the optimal value of  $\tau_K$  in the steady state is positive, zero or negative depending on the value of  $F_{31}$ .  $\Box$ 

This result can be generalized to the case where  $\mathcal{T}$  can be taxed at an arbitrary rate which, for some reason, is different from the optimal one. Also it is easy to see that if we interpret  $\mathcal{T}$  as labor, the optimal solution

150

when labor cannot be taxed is to tax capital income at a positive rate in the steady state.

Jones et al. (1993) study an example in which the optimal steady-state tax on capital is positive. Their result is a consequence of the general principle summarized in the proposition above: in their model there is one untaxed factor (government expenditures) that enhances the productivity of private capital.

## 3. Conclusions

We show that the Chamley theorem should be interpreted as an extension to a dynamic environment of the static results presented in Munk (1980). Munk shows that whether the first-best rules apply in the Ramsey problem (e.g. productive efficiency) depends on the restrictions imposed on the taxes that can be used. In our case the second best is characterized by an equalization of the intertemporal rate of substitution and the marginal productivity of capital when the tax system is not restricted. This first-best rule of the second-best problem disappears when some restrictions are imposed on the capacity to tax. The introduction of a third factor of production in the neoclassical growth model, which cannot (in practice or legally) be optimally taxed, implies an optimal steady-state tax rate on capital income that is different from zero.

## Acknowledgements

I wish to thank Christophe Chamley and two anonymous referees for their helpful comments. The errors that remain are those of the author.

## References

- Chamley, C., 1986, Optimal taxation of capital income in general equilibrium with infinite lives, Econometrica 54, 607-622.
- Jones, L., J. Manuelli and P. Rossi, 1993, Optimal taxation in models of endogenous growth, Journal of Political Economy 101, 485-517.
- Lucas, R. and N. Stokey, 1983, Optimal fiscal and monetary theory in a world without capital, Journal of Monetary Economics 12, 55–93.
- Munk, K.J., 1980, Optimal taxation with some non-taxable commodities, Review of Economic Studies 67, 755–765.