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The Theoretical Limits to Redistribution

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GEM

1. INTRODUCTION

An analysis of what is achievable by government interference in the economic system is one of the most fundamental issues in welfare economics. Of particular interest is the potential for redistribution that exists. One view of the possibilities and costs of redistribution has been given by the "new public economics" literature of the last decade and the picture that emerges is one of the disincentive effects of taxation producing large efficiency costs and this then thwarting a "caring" government in its attempt to redistribute among individuals (Mirrlees (1971), Sah (1983)). Of course, these results depend upon the magnitude of disincentive effects that are presumed to exist and empirical work is required to settle the issue. The problem here is not simply the accurate measurement of labour supply and goods demand responses, but also the discovery of where responses occur. For instance, work effort in its most general sense is represented very inadequately by hours of work over a short period in the life-cycle. As well as issues like participation and retirement, there are also the less tangible issues of striving for promotion, etc. The failure to include all responses leads both to a bias in estimates of disincentive effects and to the worry that policy based upon such estimates is at best sub-optimal and at worst seriously damaging.

Given this as background, it is natural to ask whether or not one limits the redistribution that is easily achievable by limiting oneself to the tools of income and commodity taxes; this issue is the topic of the present paper. It is natural to separate the problem into two parts. Firstly, there is the question of whether taxes are the best tools to effect redistribution. The second fundamental theorem of welfare economics tells us that any first-best and, *a fortiori*, any optimal distribution of income, can be obtained through the use of appropriate lump-sum taxes. As is now well-known, the reason for thinking of this result as unhelpful is that a government does not possess the necessary information about individuals to implement this arrangement. It seems obvious that income and commodity taxes pose less of a problem in this respect. If this is the rationale for non-lump-sum taxes then the use of tools other than taxes must also face this informational problem. We are then into what has become known as the implementability problem (see, for instance, the papers in Laffont (1979), and the special issues of the *Review of Economic Studies* (1979) and the *Journal of Mathematical Economics* (1982)). The literature on this problem has dealt largely with the attainment of Pareto efficiency;

however, if an optimal distribution of income is unattainable, the usual second-best argument suggests that the sole pursuit of Pareto efficiency may be undesirable and most of the literature on implementability becomes irrelevant. It is therefore of some interest to know whether, by using the general approach taken in the implementability literature, methods exist which dominate income and commodity taxation (see also Hammond (1979), Maskin (1979) and Guesnerie (1981); for related issues, see Champsaur and Laroque (1981), (1982)). This and other issues connected with implementability are examined in Section 2.

The second way of viewing income and commodity taxes as a restriction comes from the idea of basing taxes on other observable characteristics. This idea has been broached by many and given explicit emphasis by Hahn (1973). The simplest characteristic is one that is immune to manipulation; age, colour, and sex tend to be put in this category. If a characteristic is correlated with an unknown that is relevant in matters of redistribution then its usefulness is obvious and this is true even when behaviour is affected. In Section 3, a simple model is presented of how far redistribution should be based upon a characteristic that can be manipulated. The purpose of the section is to give some view of how the result depends upon the difficulty of manipulation and the relationship of the characteristic with redistribution-relevant unknowns.

Many characteristics that are related to relevant unknowns are ones that were subject to manipulation in the past but are non-manipulable in the present. Here, one could think of educational attainment, occupation, social class and, to some extent, wealth-holding. As social class, say, is likely to give useful information about preferences, a tax partially based upon social class could be expected to increase redistributive possibilities. The tax will be lump-sum if it will not be applied to future generations (so that the present generation are not discouraged from striving to improve the lot of their children) and if the tax was not anticipated by past generations. A once-and-for-all capital levy has similar features.

Taxes based upon these characteristics are in the short run useful because there has been a belief that such tax schemes would not be implemented. Whether they could provide a useful base for taxation if the taxes were fully anticipated is another matter. In particular, because personal decisions about such matters are in some ways irreversible, there could be concern that a government might effect extensive redistribution using such characteristics. This belief could then lead to important distortions in the economic system.

The purpose of Section 4 is to investigate these issues. A model is presented where behaviour over time helps give better information about an individual and, at any time, the government chooses taxes optimally given the information then available. To see the problems that can arise, the extreme assumption is made that individuals have a zero discount rate. With this degree of concern for the future, quite weak assumptions lead to the result that *no* redistribution is possible. The model points to the inherent difficulty of improving possibilities by widening the tax base to include this type of characteristic.

Concluding remarks are presented in Section 5.

2. THE IMPLEMENTATION OF REDISTRIBUTIVE SCHEMES

The purpose of this section is to consider whether, in a world where income and commodity taxes are used because the information required to determine optimal lump-sum taxes is not available, there exist redistribution schemes superior in their performance to income and commodity taxes. A model that will provide the basis for the paper is first laid out and then the problem is posed more formally. Although the intention is not to survey the literature on the implementation problem, it is hoped that by considering how redistribution can be effected, a useful perspective is given.

As in the optimal tax literature, we consider a situation where the government or planner does not have sufficient information to determine optimal lump-sum taxes. Society is assumed to contain N individuals who, in terms of directly observable characteristics, appear to be identical (if people can be distinguished by direct observation then the N individuals are a sub-group of the population who appear to be identical). Each individual i has a utility function $u(x_i, \theta_i)$ where x_i is a vector of goods consumed (or supplied) and θ_i is a taste parameter than can vary over individuals and is unobserved by the government. The support of θ is Θ. Two individuals can have the same preferences but different tastes, i.e. the interpersonally comparable cardinalization of preferences can be taste dependent. If the marginal utility of income depends upon θ then the utilitarian optimal lump-sum tax rates depend upon the taste parameters of the N people.

The government has views (captured for instance, by a social welfare function) concerning how goods are allocated to individuals with different tastes. An allocation mechanism x describes the vector of goods received by each individual as a function of tastes, i.e. $x=\{x_i(\boldsymbol{\theta})\}_{i=1,\dots,N}$ where $\boldsymbol{\theta}=(\theta_1,\dots,\theta_N)$. x may be a correspondence.

The allociation mechanism that operates determines the social value of a scheme under which it is operating. First, allocation mechanisms for tax schemes can be described:

Tax implementation. An allocation mechanism is tax implementable if there exists a (budget set) B such that

$$x_i(\theta_1,\dots,\theta_i,\dots,\theta_N) \quad \text{maximises } u(\cdot,\theta_i) \quad \text{s.t. } x\in B.$$

The idea behind this is obvious. A tax scheme changes the budget set of an individual. With individuals facing the same prices and the same tax scheme (a feature which is taken to be a characteristic of taxes), individuals face the same budget for net trades. Three properties of allocation mechanisms that are tax implementable are

T.1 (*Decentralization*). x_i is independent of $\boldsymbol{\theta}_{-i}=(\theta_1,\dots,\theta_{i-1},\theta_{i+1},\dots,\theta_N)$.

T.2 (*Horizontal Equity*). If $\theta_i=\theta_j$, $x_i=x_j$.

T.3 (*Vertical Equity*). $u(x_i(\boldsymbol{\theta}),\theta_i)\geqq u(x_j(\boldsymbol{\theta}),\theta_i)$ for all i, h, $\boldsymbol{\theta}$.

When x is a correspondence, T.3—which could also be termed absence of envy—is interpreted as holding for each element of the correspondence. Following almost directly from this is

Proposition 1. *An allocation mechanism is tax implementable if and only if it satisfies* T.1, T.2 *and* T.3.

Proof. Tax implementability clearly implies T.1–T.3. For the converse, let x satisfy T.1–T.3 and define B by $B=\{x_j(\boldsymbol{\theta})$ for all $\boldsymbol{\theta}\}$ where j is any individual. By T.1, $x_j(\boldsymbol{\theta})$ is independent of $\boldsymbol{\theta}_{-j}$ and so may be written $x_j(\theta_j)$; by T.2, $x_j(\cdot)$ is independent *of j* and so may be written $x_j(\theta_j)=\underset{\sim}{x}(\theta_j)$. Thus $B=\{\underset{\sim}{x}(\theta)$ for all $\theta\in\Theta\}$ is independent of j. T.3

then gives

$$u(x_i(\boldsymbol{\theta}), \theta_i) \geqq u(\underline{x}(\theta), \theta_i) \quad \text{for all } \theta \in \Theta$$

which implies tax implementability under budget set B. ||

Before proceeding, it is useful to note that the notion of tax implementability can be given wider scope by redefining what is an allocation. In particular, allocations could be random variables and the utility function in the definition of tax implementability would then be expected utility. As Stiglitz (1982) and Weiss (1976) have shown, in terms of expected social welfare, random taxes can dominate deterministic tax schemes. With allocations being random variables, Proposition 1 continues to apply, u in T.3 being expected utility. Mechanisms with this property will be called random tax implementable mechanisms. As it will be used later, the result may be stated as:

Proposition 2 *A random allocation mechansim is random tax implementable if and only if it satisfies* T.1, T.2 and T.3 (*u in* T.3 *being expected utility*).

If taxation cannot bring about significant redistribution without considerable cost, then this is the same as saying that no allocation mechanism satisfying T.1–T.3 performs well. To decide whether there exist schemes superior to taxation, one must ask the reasons for working within the confines dictated by T.1–T.3. There seem to be two such reasons. Firstly, it is a matter of choice. Condition T.2 and, to a lesser extent, T.3 could be considered as conditions that one would wish to impose upon an acceptable allocation mechanism and this is the motivation for applying the same tax system to different people. Secondly, there is the idea, mentioned earlier, that the implementation of tax schemes requires the government to possess less information than that required to implement other allocation mechanisms. To understand the strength of this constraint it is necessary to look at implementability in greater detail.

The appeal of tax schemes in particular and the use of decentralized price systems in general depends in major part upon the idea that the bundle of goods an individual receives is the best that is available to him and so there is no incentive to "cheat". To make this more formal, the allocation mechanism can be considered as the result of a game where individuals choose strategies $\boldsymbol{s} = (s_1, \dots s_N)$ from some abstract strategy space and the strategies chosen determine, through some function $x^g(\cdot)$—a game-form—the allocations that individuals receive. If $\boldsymbol{s}(\boldsymbol{\theta})$ describes the strategies chosen when preferences are $\boldsymbol{\theta}$ then the allocation mechanism is given by $x(\cdot) = x^g(\boldsymbol{s}(\cdot))$.

Allocation mechanisms that can be implemented by the choice of game form depend upon the behavioural postulate which determines strategy choice. The three most discussed postulates are:

(1) *Dominant Strategies.* $\boldsymbol{s}(\boldsymbol{\theta}) = \{s_i(\theta_i)\}$ satisfies $u(x_i^g(s_i(\theta_i), \boldsymbol{s}_{-i}), \theta_i) \geqq u(x_i^g(s_i, \boldsymbol{s}_{-i}), \theta_i)$ for all i, $\boldsymbol{s}$.

(2) *Bayesian Strategies.* $\boldsymbol{s}(\boldsymbol{\theta}) = \{s_i(\theta_i)\}$ satisfies

$$E^i_{s-i}[u(x_i^g(s_i(\theta_i), \boldsymbol{s}_{-i}), \theta_i)] \geqq E^i_{\boldsymbol{s}_{-i}}[u(x_i^g(s_i, \boldsymbol{s}_{-i}), \theta_i] \text{ for all } i, s_i.$$

Here $E^i_{\boldsymbol{s}_{-i}}$ is the expectation operator applied using i's belief about $\boldsymbol{s}_{-i}$, the expectation operating over $\boldsymbol{s}_{-i}$.

(3) *Nash Strategies.* $s(\boldsymbol{\theta})$ satisfies $u(x_i^g(s_i(\boldsymbol{\theta}), s_{-i}(\boldsymbol{\theta})), \theta_i) \geqq u(x_i^g(s_i, s_{-i}(\boldsymbol{\theta})), \theta_i)$ for all i, s_i.

To define Bayesian strategies, it has been necessary to postulate beliefs that individuals possess concerning the strategies that will be chosen by others. This may be derived from a belief concerning the tastes of others or may be more direct. Whatever, it will be convenient to assume that all individuals *and* the government always have such beliefs and, with regard to the strategy that will be chosen by any individual i, all individuals other than i and the government have identical beliefs.

The implementation possibilities under each of these postulates have been much considered (see, for example, Dasgupta, Hammond and Maskin (D–H–M) (1979)). Dominant strategies derive from the weakest behavioural postulate but will often fail to exist (the other two include the dominant strategies when dominant strategies exist). Bayesian strategies are a natural extension to describe behaviour when dominant strategies fail to exist. Given the lack of knowledge of other individuals' preferences, Nash strategies are difficult to justify unless recontracting is permitted. It is our purpose now to investigate redistribution possibilities from implementable mechanisms where these behavioural postulates are utilized.

2.1. *Redistribution with dominant strategies*

The reason for restricting attention to tax implementable mechanisms is that they are dominant strategy implementable (Hammond (1979)). To see it is sufficient to note that if x is a tax implementable allocation mechanism then with strategy spaces for individuals being Θ, truth-telling is a dominant strategy if x is used as the game-form. Thus x is implemented.

With this as background, it is natural to ask whether tax implementable schemes exhaust the class of dominant strategy implementable schemes. Equal treatment of equals (condition T.2) could obviously be dispensed with but, just as we restrict attention to an anonymous tax schemes, so it seems desirable to consider only mechanisms embodying T.2. Assume that an allocation mechanism satisfies T.1 and T.2 and is implementable in dominant strategies with game form x^g. If $s_i(\cdot)$ is i's dominant strategy function then

$$x_i(\theta_i) = x_i(\boldsymbol{\theta}) = x_i^g(s_i(\theta_i), s_{-i}) \quad \text{for all } s_{-i}.$$

Applying this, dominant strategy implementability gives

$$u(x_i(\boldsymbol{\theta}), \theta_i) \geqq u(x_j(\boldsymbol{\theta}), \theta_i)$$

if T.2 is invoked and thus condition T.3 is satisfied:

Proposition 3. *An allocation mechanism satisfying T.1 and T.2 is dominant strategy implementable if and only if it is tax implementable.*

The only way forward is to relax the requirement that an individual's allocation is independent of other individuals' tastes. Before considering this possibility, the question of *feasibility* of allocation mechanisms needs to be addressed. For most problems, it is natural to impose a requirement that aggregate consumption is feasible in the sense that it lies in some production set. However, if T.1 is imposed then the problems associated with ensuring feasibility are severe. For instance, in an exchange economy, the requirement that $\sum x_i \leqq 0$ imposes the restriction that $x_i \leqq 0$ if T.1 and T.2 are invoked, for otherwise all individuals may have the same tastes and it will be impossible to give a strictly positive amount of some good to all individuals; in this case, no trade is the best that is possible. Similarly, in the standard two-good model of income taxation where one unit of leisure can be converted into one unit of consumption, T.1 and T.2 impose the restriction of no redistribution, for if an individual with tastes θ receives more income than leisure foregone then if everybody has these tastes, infeasibility will arise.

Although the issue of feasibility is addressed in the literature on optimal taxation, it takes the form of saying that under some distribution of tastes, resource requirements are feasible. This is open to two interpretations:

1. *Assignment Uncertainty.* Although any one individual's tastes are unknown, the true distribution of tastes is known. Uncertainty by the government takes the form of not knowing who's who.

2. *Weak feasibility.* If the distribution of tastes is assumed to reflect the government's beliefs concerning tastes then the constraint is that expected resource demands are feasible.

Weak feasibility is the most appealing interpretation and becomes more reasonable in large economies when expected demands can, in a relative sense, approximate actual demands. It is important to realise that tax implementable allocations depend upon weak feasibility or assignment uncertainty; if full feasibility is required then dominant strategy implementable allocations take very restrictive forms (for preferences over an abstract space we are led to the Gibbard–Satterthwaite theorem stating the allocations are sensitive to the tastes of at most one individual, for preferences in economic environments, allocations are insensitive to one's own tastes; for formal results, see Sattherthwaite and Sonnenschein (1981)).

Accepting the weak feasibility condition, are there dominant strategy implementable allocation mechanisms that fail to be tax implementable? If the allocation received by an individual depends upon the tastes of others then, before these tastes become known, the individuals' allocation is a random variable. The question posed above is answered by

Proposition 4. *Under weak feasibility, for any dominant strategy implementable allocation mechanism satisfying* T.2, *there exists a random tax implementable mechanism giving each individual the same expected utility as the dominant strategy scheme.*

Proof. If x is dominant strategy implementable using x^g then to i, $x_i^g(s_i, \cdot)$ is a random variable depending upon the strategies chosen by others. Define $y_i^g(s_i)$ to be a random variable identical in distribution to $x_i^g(s_i, \cdot)$ but independent of s_{-i} (the distribution of s_{-i} being i's beliefs). Thus the realization of y is independent of the realization of s_{-i}. Now, it is clear that as there exists a dominant strategy under x_i^g, the dominant strategies under x_i^g and y_i^g coincide (there always exists a dominant strategy under y_i^g independently of whether such a strategy exists under x_i^g). As i possesses the same dominant strategy function, i has the same expected utility under x_i^g *and* y_i^g. Furthermore, as i's belief concerning s_{-i} coincides with that of the government, the expected value of x_i^g is identical to y_i^g and weak feasibility under x^g implies weak feasibility under y^g. Finally, y_i^g depends upon only s_i which in turn depends upon only θ_i. Thus y, implemented by y^g, is a random allocation mechanism satisfying weak feasibility and T.1, T.2 follows directly as x satisfies

T.2, T.3 follows by applying the definition of dominant strategy implementability. Applying Proposition 3, the result follows. ||

With regard to Proposition 4, it shoud be noted that, whilst a risk-averse individual would prefer to be offered the expectation of a random variable with certainty to the random variable, to do so would affect the way that the individual behaves. This explains how random taxation can often dominate the more commonplace non-random taxation (cf. the introduction to this section). What has been shown is that dominant strategy implementable allocation mechanisms cannot, on average, perform better than random taxes.

2.2. *Redistribution with Bayesian strategies*

The strongest requirement imposed by dominant strategy implementation is that under the game-form used, dominant strategies must always exist. In this regard, Bayesian strategies pose no problem and this can help explain why positive results on implementation are possible even when feasibility in a strict sense is imposed (d'Aspremont and Gerard-Varet (1979)).

If weak feasibility is all that is required then a similar idea to that just applied to dominant strategy procedures may be invoked. For if x^g is a game-form implementing x under Bayesian strategies then $x_i^g(s_i, \cdot)$ is a random variable to i; so by choosing $y_i^g(s_i)$ to be a random variable identical in distribution to $x_i^g(s_i, \cdot)$, i retains the same incentives and is indifferent between the two schemes. In fact, as $y_i^g(s_i)$ is independent of others's strategies, the Bayesian strategy function $s_i(\theta)$ under x^g becomes a dominant strategy function under y^g. So given that it is feasibility in an expected sense that is demanded, Bayesian strategies offer no improvement in terms of implementable allocation mechanisms over dominant strategies:

Proposition 5. *Under weak feasibility, for any Bayesian strategy implementable allocation mechanism satisfying* T.2, *there exists a random tax implementable mechanism giving each individual the same expected utility as the Bayesian strategy scheme.*

2.3. *Redistribution with Nash strategies*

A widley held view is that with Nash strategies, many allocation mechanisms are implementable. In general, investigation has centred on the implementation of Pareto efficient mechanisms and, more particularly, mechanisms giving rise to Walradian allocations (e.g. Hurwicz (1979)). But consider whether it is possible to implement a mechanism which always attains the first-best under some Bergson–Samuelson SWF. For almost all preferences that are possible the first-best allocation will be unique. As was shown in Roberts (1979), when preferences are defined over an abstract space, if an allocation mechanism implemented in Nash strategies is almost always unique then it is a dictatorship, i.e. under uniqueness or near uniqueness we return to the Gibbard–Satterthwaite result of dominant strategies. Thus first-best redistribution must usually be unattainable. This result does not necessarily hold when preferences are "well-behaved" orderings of commodity space, but there is a clear suggestion that Nash implementability may impose severe restrictions.

Again, it will be assumed that we wish to implement mechanisms satisfying the horizontal equity condition T.2. To help understand the restrictions imposed upon Nash implementable mechanisms, consider Figure 1 which is concerned with the case where

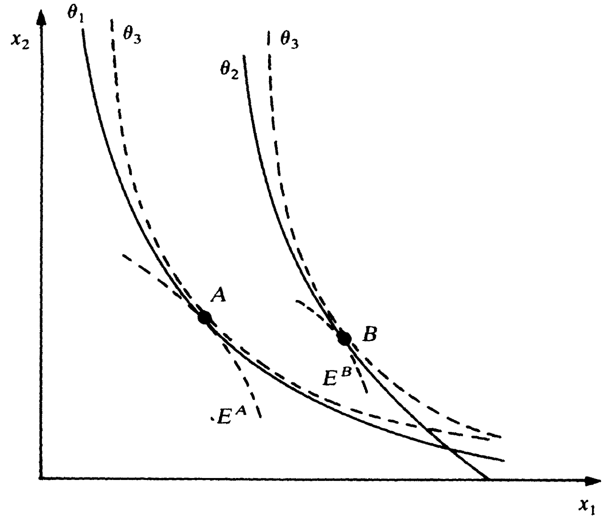

FIGURE 1

allocations are two-dimensional vectors, and there are two individuals (generalization of the argument to be presented is straightforward). Assume that when the two individuals have preferences $\{\theta_1, \theta_2\}$ the allocation $\{A, B\}$ is a member of the allocation mechanism. If this is implemented using strategies $\{s_1, s_2\}$ then the allocations that individual 1 can achieve by choosing a different strategy—the set E^A in the figure—must nowhere lie above the θ_1 indifference curve through A. Similarly, the same is true of E^B for individual 2. Now assume that both individuals have the same preferences θ_3 as in the figure. If 2 chooses s_2 then s_1 is the best strategy for 1 and *vice versa.* Thus the allocation $\{A, B\}$ under $\{\theta_3, \theta_3\}$ is implemented and condition T.2 is violated. To obtain this result, one needs to be able to construct the preference map θ_3. This is always possible if B is preferred to A by an individual with θ_1 preferences. Thus Nash implementability together with T.2 imposes the constraint that

$$u(x_i(\boldsymbol{\theta}), \theta_i) \geqq u(x_j(\boldsymbol{\theta}), \theta_i) \quad \text{for all } i, j, \boldsymbol{\theta},$$

i.e. if horizontal equity is imposed then vertical equity (condition T.3) must be satisfied. This allows the following result to be stated:

Proposition 6. *If $y(\boldsymbol{\theta})$ is an allocation forming part of a Nash implementable allocation mechanism $(y(\boldsymbol{\theta}) \in x(\boldsymbol{\theta}))$ satisfying* T.2 *then there exists a $B(\boldsymbol{\theta})$ such that $y_i(\boldsymbol{\theta})$ maximizes $u(x_i, \theta_i)$ s.t. $x_i \in B(\boldsymbol{\theta})$.*

Proof. Given that T.3 has been shown to hold, $B(\boldsymbol{\theta})$ can be defined to be $\{y_1(\boldsymbol{\theta}), y_2(\boldsymbol{\theta}), \ldots, y_N(\boldsymbol{\theta})\}$. ||

The interpretation of Proposition 6 is that Nash implementable allocation mechanisms can achieve no more than tax implementable allocations *if* the government is operating under assignment uncertainty as defined in Section 2.1. As the optimal tax literature can be interpreted as dealing with situations of assignment uncertainty, *the redistributive limits suggested in the optimal tax literature define the redistributive limits of Nash implementable allocation mechanisms.*

Assignment uncertainty lacks appeal as a working assumption and it may be possible for Nash implementable mechanisms to achieve under full uncertainty what taxes can achieve only under assignment uncertainty. Consider what restrictions are imposed by Nash implementability. Returning to Figure 1, if A is a Nash equilibrium position for an individual with θ_1 preferences then it is an equilibrium under all preferences θ_3 with the property that the indifference curve touching A never lies below the θ_1 indifference curve—we can say that θ_3 cannot be screened from (θ_1, A) if there exists no z such that $u(z, \theta_3) > u(A, \theta_3)$ and $u(z, \theta_1) \leqq u(A, \theta_1)$. Nash implementability imposes the restriction, often called monotonicity (e.g. D–H–M (1979)) which may be stated as:

Monotonicity. If θ_i^* cannot be screened from (θ_i^{**}, z_i) where $(z_1, \ldots, z_N) \in x(\theta_i^{**}, \boldsymbol{\theta}_{-i})$ then $(z_1, \ldots, z_N) \in x(\theta_i^*, \boldsymbol{\theta}_{-i})$.

Maskin (1977) has shown that any mechanism satisfying monotonicity and a condition which he terms no veto power can be Nash implemented. In economic environments, the condition of no veto power utilized in the proof of Maskin's result is always satisfied and, in the Appendix, a game-form is constructed to implement any mechanism satisfying monotonicity. The game-form constructed has the virtues of being both direct and naturally appealing in terms of the strategies that agents must use and the allocations that result from strategies.

Proposition 6 states that, in one sense, Nash implementable allocation mechanisms must be capable of decentralization. On the other hand not all allocations derivable from a budget function $B(\boldsymbol{\theta})$ satisfy monotonicity and are implementable. In this respect, the following proposition, whilst only a partial result, is of some interest:

Proposition 7. *Let $B(x)$ be a budget set defined as a function of N consumption bundles. Define $x(\boldsymbol{\theta})$ as follows:*

$$x(\boldsymbol{\theta}) = \{x : x_i \max u(z, \theta_i) \text{ s.t. } z \in B(x)\}.$$

If $x(\boldsymbol{\theta})$ is non-empty for all $\boldsymbol{\theta}$ then $x(\boldsymbol{\theta})$ is a Nash implementable allocation mechanism.

Proof. Assume that $(z_1, \ldots, z_N) \in x(\theta_i^{**}, \boldsymbol{\theta}_{-i})$. Thus z_j maximizes $u(\cdot, \theta_j)$ over $\{z_1, \ldots, z_N\}$, $j \neq i$, and z_i maximizes $u(\cdot, \theta_i^{**})$ over $\{z_1, \ldots, z_N\}$. If θ_i^* cannot be screened from (θ_i^{**}, z_i) then z_i maximizes $u(\cdot, \theta_i^*)$ over $\{z_1, \ldots, z_N\}$ and thus $(z_1, \ldots, z_N) \in x(\theta_i^*, \boldsymbol{\theta}_{-i})$ which proves monotonicity of $x(\boldsymbol{\theta})$. ||

The relevance of this result is that $x(\boldsymbol{\theta})$ can be interpreted as the equilibrium of a *tatonnement* adjustment process where the "auctioneer" adjusts the budget set available to individuals in response to their demands. This budget set is $B(x)$. Notice also that as in the standard *tatonnement* adjustment mechanism of Walrasian theory, individuals announce what is the best allocation in the budget set B, their decision being myopic and in ignorance of the fact that this will imply a change in the budget set on offer. The important caveat that must be made to this interpretation is that Nash implementable allocations cannot be implemented *using* this tatonnement mechanism because, with Nash strategies, individuals would recognize the dependence of the budget set on their announced consumption bundle.

Finally, if preferences are sufficiently restricted then the monotonicity restriction will have no power. In particular, a common assumption in the optimal tax literature (e.g. Mirrlees (1971), (1976)) is based upon the idea that individuals differ in one parameter n and an individual with a higher n requires less reward to work harder. In this case, individuals with different preferences can always be screened from each other and the first-best can be Nash implemented. The game-form constructed in the Appendix can be used to implement any allocation in this case. Although this result is of some interest, it would be difficult to maintain that it has wide applicability.

2.4. *Summary*

In view of the disparate arguments and results that have been brought together in this section, it may be useful to offer a brief summary.

Our purpose has been to investigate whether, in those circumstances where an informational parsimony rules out the use of first-best lump-sum taxation, there exist implementable allocation mechanisms superior to ones that are (income and commodity) tax implementable. Tax implementable mechanisms can be interpreted as an example of what is achievable under either assignment uncertainty or, more plausibly, weak feasibility. The results of Sections 2.1 and 2.2 show that if weak feasibility is required, dominant and Bayesian strategy implementable mechanisms cannot improve upon the utilities that individuals expect to achieve under tax implementable mechanisms. The results of the last section show that under assignment uncertainty, Nash implementability can achieve no more than tax implementability. Thus, the extent of redistribution that is acheivable through dominant, Bayesian or Nash implementable mechanisms is no greater than what is achievable through taxation.

This answers the basic question that we set out to consider. However, the analysis has thrown up another issue of importance. Tax implementability has been taken as a benchmark but without a weakened feasibility condition or an assumption like assignment uncertainty, tax implementable mechanisms are no longer feasible. Proposition 7 shows that with Nash implementability, say, one may be able to get close under full uncertainty to what tax implementable mechanisms can achieve under assignment uncertainty. The possibilities using taxes may usefully define the limits to redistribution.

3. THE TAXATION OF AN INDEX FOR REDISTRIBUTIVE PURPOSES

So far, we have considered the problem of whether the possibilities for redistribution are limited by restricting attention to taxes. In the rest of the paper the intention is to consider the desirability of altering the tax base.

Consider a situation like the one of the last section where a taste parameter θ_i is unknown to the government. Observing the behaviour of individuals provides some information concerning taste parameters and this is the idea behind the use of commodity and income taxes for redistributive purposes. In this section, a simple—perhaps the simplest—example of this problem is examined. Assume that an index r possessed by individuals is observed, perhaps after manipulation, by the government. The usefulness of basing taxation for redistributive purposes on this index depends upon two factors: (i) the extent to which an individual can manipulate r when there is an incentive to do so; (ii) the extent to which r is redistribution-relevant, e.g. is r a good indicator of an individual's marginal utility of income?

To give an idea of the degree to which some abstract index r should be used for redistributive purposes, a simple model will be examined. The model may be considered as a quadratic approximation to a general differentiable model where index manipulation has a zero income effect. Assume that an indivdual with true index r and an observed index of $\hat{r}$ receives utility

$$u = c(\hat{r}) - \alpha m(r)(r - \hat{r})^2 \tag{1}$$

$c(\hat{r})$ is the consumption level of the individual, its dependence on $\hat{r}$ being because the government bases taxes on $\hat{r}$. α is a parameter which measures the difficulty of manipulation of the index r. Assume that, by a suitable transformation, r if correctly observed is the expected social marginal utility of income at the optimum. The government's objective is

$$W = \int_{\underline{r}}^{\bar{r}} U(v(r), r)f(r)\, dr \tag{2}$$

where $f(r)$ is the known density of r in the population (with suport $[\underline{r}, \bar{r}]$),

$$v(r) = \max_{\hat{r}}[c(\hat{r}) - \alpha m(r)(r - \hat{r})^2], \tag{3}$$

and $\partial U/\partial v = r$ at the optimum. If $\hat{r}(r)$ solves (3) then the implied first-order conditions are equivalent to

$$\frac{dv}{dr} = -2\alpha m(r)(r - \hat{r}) - \alpha \frac{dm}{dr}(r - \hat{r})^2. \tag{4}$$

Finally, if total consumption is fixed then this constraint may be written as

$$\int_{\underline{r}}^{\bar{r}} [v(r) + \alpha m(r)(r - \hat{r})^2]f(r)\, dr = \bar{C}. \tag{5}$$

The government wishes to choose $c(\hat{r})$ directly, or $v(r)$ and $\hat{r}(r)$ indirectly, to maximise (2) subject to (4) and (5). Introducing multipliers $\lambda(r)$ and μ, Pontryagin's maximum principle gives first-order conditions

$$\frac{d\lambda}{dr} = (\mu + r)f(r) \tag{6}$$

$$\lambda(r) + \lambda(r)(r - \hat{r})\frac{dm/dr}{m} + \mu(r - \hat{r})f(r) = 0 \tag{7}$$

$$\lambda(\underline{r}) = \lambda(\bar{r}) = 0 \quad \text{(transversality)}. \tag{8}$$

The most important observation is that these equations do not involve α. Equations (6) and (8) taken together allow the multipliers $\lambda(r)$ and μ to be determined and then $r - \hat{r}$ is given by (7). The upshot of this is that the degree to which individuals should be encouraged to manipulate an index is independent of the cost of manipulation as measured by α. If the costs double, the rate of change of c with r doubles but the degree of manipulation is unchanged.

To consider how the redistribution-relevance of an index enters the picture, it is useful to generalize the model to allow for a number of indices. Thus, assume that the taxation of indices $r_1, \ldots, r_K$ is being considered. Normalizing as before, let r_k be measured so that the expected social marginal utility of income of an individual conditional on r_k is given by $\bar{r}+\beta_k r_k$ at the optimum and assume that all the indices are identically and independently distributed with mean zero and density f. Thus, the distributional relevance of an index is captured solely by β_k. Looking across indices, the parameter β is proportional to the correlation between r_k and the social marginal utility of income. $\bar{r}$ is the average social marginal utility of income. Equation (1) is replaced by

$$u=c(\hat{r}_1,\ldots,\hat{r}_K)-\sum_{k=1}^K \alpha_k m(r_k)(r_k-\hat{r}_k)^2 \tag{9}$$

and by a simple generalization of the single index case, first-order conditions are

$$\sum_{k=1}^K \frac{\partial\lambda_k}{\partial r_k}=(\rho(r_1,\ldots,r_K)+\mu)\prod_{k=1}^K f(r_k) \tag{10}$$

$$\lambda_k(r_k)+\lambda_k(r_k)(r_k-\hat{r}_k)\frac{dm/dr_k}{m}+\mu(r_k-\hat{r}_k)f(r_k)=0 \qquad k=1,\ldots,K. \tag{11}$$

$$\lambda_k(r_1,\ldots,\underline{r}_k,\ldots,r_K)=\lambda_k(r_1,\ldots,\bar{r}_k,\ldots,r_K)=0 \qquad k=1,\ldots,K. \tag{12}$$

Here, ρ is expected social marginal utility of income conditional on $r_1,\ldots,r_K$. Consider integrating (10) over r_j, $j\neq k$. Making use of (12), this gives

$$\frac{\partial\lambda_k}{\partial r_k}=(\bar{r}+\beta_k r_k+\mu)f(r_k). \tag{13}$$

Combining (12) and (13) shows that λ_k is independent of r_j, $j\neq k$. Furthermore, $\mu=-\bar{r}$ and (13) solves to give

$$\lambda_k(r_k)=\beta_k\int_{\underline{r}}^{r_k}\tilde{r}f(\tilde{r})\,d\tilde{r}. \tag{14}$$

To interpret these equations, it is useful to note first that eliminating λ_k from (11) and (14) gives an equation for the distortion of index k, $r_k-\hat{r}_k$, which is independent of any other index. Thus the marginal gain in consumption from changing $\hat{r}_k$ is independent of the levels announced for other indices—in this world of separable utility functions and independent indices, optimal taxation takes the form of a separate tax schedule for each index.

Applying (14) to (11) shows that, as in the single index case, the degree to which individuals should be encouraged to manipulate an index is independent of the cost of manipulation α. This suggests a rule directly akin to the Ramsey tax rule: *the taxation of indices for redistributive purposes should, other things being equal, lead individuals to manipulate each index by the same amount.* Here, the "other things" in "other things being equal" are the spread of manipulation costs m and the redistribution-relevance parameter β.

As, from (14), λ_k is proportional to β_k, (11) only gives a simple answer to the question of the effect of β_k on the extent of redistribution brought about by taxing index k when $dm/dr_k=0$. In this case, the degree of manipulation, $r_k-\hat{r}_k$, under the optimal tax is proportional to β_k. However, when $dm/dr_k\neq 0$, the sign of dm/dr determines whether the degree of manipulation increases by more or less than in proportion to β_k. However, whatever the value of dm/dr, applying second-order conditions allows the conclusion to be reached that the degree of manipulation at every r_k increases with an increase in β_k. The extent of redistribution is unambiguously greater for an index with a higher β_k parameter.

Finally, what happens when two indices differ in both their α and β parameters? Assume that $m \equiv 1$ so that $r_k - \hat{r}_k$ is proportional to β_k. In this case, the rate of change of utility across r is proportional to $\alpha_k \beta_k$ (see (4) above) and the welfare gain in redistribution is proportional to β_k. Thus, it is not surprising that the welfare gain from the optimal taxation of an index is proportional to $\alpha_k \beta_k^2$. As β_k is proportional to the correlation between r_k and the social marginal utility of income, if index k has half the correlation of index l, it is necessary for k to have manipulation costs in excess of four times those of l for it to be the case that the tax on index k is redistributionally more effective than the tax on index l.

4. THE TAXATION OF AN INDEX BASED UPON PAST ECONOMIC CHOICES

As we have seen, for redistributive purposes it is desirable to tax those indices most correlated with social marginal utilities of income. But the manipulability problem is also important. In particular, one could have a situation where untaxed indices provide perfect information but only continue to do so at no social cost as long as they remain untaxed. Therefore, it is insufficient to say that redistribution is limited because informationally relevant indices are ignored.

Despite this, there do seem to be relevant indices that are by their nature, non-manipulable. Anything which has been chosen in the past is now a fixed datum of history and non-manipulable. A once-and-for-all tax on such an index would, if unanticipated, induce no distortion and act like a lump-sum tax. If the index is correlated with the social marginal utility of income then a tax on the index would be desirable. Examples of indices of this sort are social class, educational attainment and wealth-holding.

The purpose of this section is to provide some analysis of the taxation of such indices. If the taxation of an index was anticipated then, just like the taxation of goods and income, distortions would be induced. The fact that decisions about such indices may be irreversible means that individuals could be "locked in" to a considerable tax burden. In this case, large distortions could result even if the probability of such tax measures being enacted is small.

To understand some of these issues, a simple model will be examined. Infinitely lived individuals have a constant taste parameter θ which is unknown to the government. From decisions made over time, the government updates its beliefs and chooses policies to maximize expected welfare given those beliefs. Individuals are rational in the sense that they know how the government will treat people and individuals act to maximize their average lifetime utility. This no discounting assumption is made to simplify the problem and to give clear-cut results.

Proceeding more formally, assume that individuals have preferences given by the twice-differentiable utility function $u(c, y, \theta)$ where θ—a real number—is the taste parameter, y is pre-tax income which may be thought of as work effort ($u_y < 0$) and c is post-tax income which may be thought of as consumption ($u_c > 0$). The support of θ, Θ, is assumed to be an interval. The utility function is defined over a set C of consumption/income pairs which is assumed to be compact for simplicity. The utility function is also taken to be the "correct" cardinalization of preferences, the government being interested in maximizing the expected sum of average lifetime utilities. Notice that social welfare is assumed to be additive in utilities

(i) at different dates,
(ii) in different states,
(iii) for different individuals.

Four assumptions imposed upon u will be

U.1. $u_{c\theta} \leqq 0$.

U.2. $\partial/\partial\theta(-u_y/u_c) < 0$.

U.3. $u_{cy} - u_{cc}(u_y/u_c) < 0$ (leisure is normal).

U.4. u is strictly concave.

Condition 1 states that, *ceteris paribus*, individuals with a higher θ have a lower social marginal utility of consumption; condition 2 implies that under any budget set, individuals with a higher θ will prefer a bundle of goods with a higher y argument; conditions 3 and 4 are obvious. All four conditions are standard in the optimal tax literature—see Mirrlees (1971). In the Mirrlees model, θ is interpreted as a skill parameter.

The choice by an individual of a stream of consumption/income bundles over time gives the government indirect information concerning the tastes of that individual. If taxation can be related to past economic choices then, at date t, the government offers a budget set B to individual i which can depend upon the history of this individual $H_i^t = \{(c_i^\tau, y_i^\tau)\}_{\tau<t}$ (in principle, the budget set offered to i could depend upon the past choices of j, $j \neq i$, just as, in Section 2, the possibility of relating taxes paid by i to the choices made by j was considered). Starting at 0, the government has a prior belief, captured by a probability density function f, of any one individual's taste parameter (assumed the same for everybody). As history accumulates, the government updates its beliefs and the budget set offered conditional on past history maximizes expected social welfare conditional on the government's beliefs concerning the θ values of individuals with the given history. Thus no precommitment by the government is assumed possible—if an individual by his actions reveals his taste parameter, this information will then be used by the government. More importantly for the working of the model, individuals are assumed to know that the government will act in this way.

The above remarks are relevant whether or not there is discounting. But when there is no discounting, the sole interest of individuals is the limiting consumption/income pair that can be obtained from a possible history. Conceivably, a history may lead to many limit points with a certain proportion of time being spent in each limit point—limit points are assured by the compactness of C. For this reason it is useful to look to the choice of history producing a random variable b over the consumption set. Let $\mathscr{B}$ be the set of all (measurable) random variables taking on values in C. From the set of b that can be attained through a suitable choice of history, individuals choose b to maximize their expected utility. Notice that as the variation is over time rather than states of nature, this is true even if u is not an individual's Von Neumann–Morgenstern utility function.

Given this as background, on equilibrium is defined as follows:

Definition. A *no commitment tax equilibrium* is a set $B \subseteq \mathscr{B}$ of random variables defined over C, and a function $g(b, \theta)$ defined over $B \times \Theta$, such that

E.1. $g(b, \theta) = 0$ if $U(b, \theta) < U(b', \theta)$, $b' \in B$, where $U(b, \theta)$ is the expected utility of a θ-type individual receiving b.

E.2. $\int_B g(b, \theta)\,db = f(\theta)$.

E.3. With beliefs

$$h(b, \theta) = \frac{g(b, \theta)}{\int_\Theta g(b, \theta')\,d\theta'}$$

b maximizes expected welfare over all $b' \in \mathscr{B}$ such that $E(b) = E(b')$(E being the expectation operator).

E.4. If $\rho(b, \theta)$ is the expected marginal utility of consumption of a θ-type individual under b then $\int_\Theta \rho(b, \theta)h(b, \theta)\,d\theta$ is constant over all $b \in B$ such that $\int_\Theta g(b, \theta)\,d\theta \neq 0$.

This definition is a minimal requirement for equlibrium. $g(b, \theta)$ is to be viewed as the probability that an agent is a θ-type individual and he chooses b. For any $b \in B$, $h(b, \theta)$ is the posterior density function of θ given b. E.1. requires that, in updating its beliefs, the government takes into account the fact that agents will not make inferior choices. E.3 demands that with beliefs $h(b, \theta)$, b should not be inferior to some other random variable which is also feasible (using the weakened feasibility condition of Section 2). E.4 requires that there be no incentive to transfer resources from individuals making one choice to individuals making another choice. Looking at E.3, what is not ruled out is the possibility that there is a budget *set* which can be expected to be superior to b—there may exist tax equilibria that one would wish to rule out on the grounds of optimizing behaviour. In spite of this, it is possible to prove the following result.

Proposition 8. *Under assumptions* U.1–U.4 *on preferences, for any no commitment tax equilibrium there exists a* $b^* \in B$ *such that*

(i) $\int_\Theta g(b, \theta)\,d\theta = 0$ *for all* $b \in B/\{b^*\}$.
(ii) b^* *takes on some value* (c^*, y^*) *with probability unity.*

This result says that the only no commitment tax equlibrium involves the choice of some consumption/income profile (c^*, y^*) with probability unity by everybody. Thus, in terms of expected welfare, there is no loss in generality in only allowing individuals to choose this allocation.

Proof. Consider any $b \in B$ such that $\int_\Theta g(b, \theta)\,d\theta \neq 0$. Applying U.4, $U(E(b), \theta) > U(b, \theta)$ if b is random and E.3 is then violated (by letting b' be the non-random variable $E(b)$). Thus b must be non-random.

Next, assume that there exists b, b', $b \neq b'$ such that $\int_\Theta g(b, \theta)\,d\theta \neq 0$ and $\int_\Theta g(b', \theta)\,d\theta \neq 0$: b is the pair (c, y), say, and b' is the pair (c', y'). Without loss of generality, assume that $y' > y$ (if $y = y'$, $c = c'$ from E.1 and $b = b'$). E.1 and continuity of u in θ imply that there exists a $\theta^* \in \Theta$ such that $u(c, y, \theta^*) = u(c', y', \theta^*) = u^*$. Applying U.2 and E.1:

If $\theta > \theta^*$:

$$u(c', y', \theta) > u(c, y, \theta) \Rightarrow g((c, y), \theta) = 0.$$

If $\theta < \theta^*$:

$$u(c', y', \theta) < u(c, y, \theta) \Rightarrow g((c', y'), \theta) = 0.$$

Now, $\rho((c, y), \theta) = u_c(c, y, \theta)$ so that U.1 gives

$$\begin{aligned} &\int_\Theta \rho(b, \theta) h(b, \theta) d\theta \geqq u_c(c, y, \theta^*), \\ &\int_\Theta \rho(b', \theta) h(b', \theta) d\theta \leqq u_c(c', y', \theta^*). \end{aligned} \tag{15}$$

Next, define the function $c(\cdot)$—θ^*'s indifference curve—by

$$u(c(\tilde{y}), \tilde{y}, \theta^*) = u^*$$

(note that $c(y) = c$, $c(y') = c'$). Integrating around the indifference curve:

$$\begin{aligned} u_c(c', y', \theta^*) - u_c(c, y, \theta^*) &= \int_y^{y'} \frac{d}{d\tilde{y}} u_c(c(\tilde{y}), \tilde{y}, \theta^*) d\tilde{y} \\ &= \int_y^{y'} \left(u_{cc} \frac{dc}{dy} + u_{cy} \right) d\tilde{y} \\ &= \int_y^{y'} \left(u_{cy} - u_{cc} \frac{u_y}{u_c} \right) d\tilde{y} \\ &< 0 \end{aligned}$$

from U.3 (as $dc/dy = -u_y/u_c$). Using this in (15) gives

$$\int_\Theta \rho(b', \theta) h(b', \theta) d\theta < \int_\Theta \rho(b, \theta) h(b, \theta) d\theta$$

which violates E.4. ||

Most directly, this proposition shows the cost to a government of not being able to precommit itself to a tax policy of its choice. The model illustrates the idea that in a government/society problem or, more generally, in a principal/agent problem, the government may gain by being restrained in its actions. This idea has been developed by Kydland and Prescott (1977) and is investigated in the context of repeated principal/agent problems in Roberts (1982).

The resulting situation described in the proposition is inefficient in the extreme—if some individuals find it difficult to supply work effort then everybody works a small amount—and it is inferior to a situation where the government "narrows" the tax base so that taxes based upon past choices are ruled out. In this case, a common unvarying budget set would be offered each period and the model would be reduced to the standard optimal income tax problem (Mirrlees (1971)). Notice that as in the no commitment equilibrium feasibility requires that $c^* = y^* - R$ where R is the *per capita* revenue requirement of the government, one (feasible) budget set which is Pareto superior to the no commitment equilibrium is the set $\{(y-R, y) \text{ for all } y\}$, i.e. a constant poll tax produces an outcome superior to that produced by a government that insists on changing taxes in the light of all information obtained!

Finally, Proposition 8 has been derived under the assumption of no discounting. However, if saving and borrowing can occur then similar results emerge under discounting. For assume that (infinitely lived) individuals and the government can save and borrow at interest rate r and assume that individuals have a utility function of the form

$$u = u(c) + v(y, \theta)$$

and common utility discount rate r. In this case, the marginal utility of consumption at each date will be equated across individuals if total discounted earnings net of tax are equated across individuals. Now assume that taxes can be based upon past decisions. When the government has reached the limiting state in terms of information acquisition then, knowing this, it is optimal to impose taxes to equalize wealth (discounted back to time zero) across individuals. However, individuals knowing this will occur never have an incentive to increase work effort in return for extra consumption. It is clear that the solution reached is again one of extreme inefficiency.

5. CONCLUDING REMARKS

Once we move away from the assumption of complete information which is the crucial underpinning of first-best welfare economics, the issue of how much redistribution is possible is of central concern. The vast literature on optimal taxation has given many useful answers to questions relating to the exact structure of taxes on goods and income which achieve redistribution whilst minimising efficiency costs. This paper has attempted to take a step away from this literature and ask the more basic question of whether, in a world of imperfect information, there exist tools for achieving redistribution which are superior to taxes on goods and income. In Section 2 it was shown that, subject to the conditions laid down there, no other mechanism, no matter how abstract and/or complicated, could expect to perform better than a tax system. With this result in mind, it is natural to look to what should be taxed. In Section 3 an attempt was made to throw light on desirable candidates for inclusion in the tax base. Althouth there is no reason why only economic activities should be taxed, there is little that is not a derivative of past and present economic activity. Accepting this, the taxation of goods and income is the taxation of the present flow of activities and ignores the possibility of taxing stocks determined by past economic activities or past economic activities themselves. The "no discounting" assumption of Proposition 8 is undoubtedly strong but the form of the proposition raises doubts concerning the efficiency of widening the tax base to include past economic activities. In particular, restricting the tax base to present economic activity can result in a Pareto superior situation.

Although a purely theoretical investigation has been conducted, there is policy relevance to such analyses. By asking the questions addressed in Section 2, one gains insights into the alternatives to straightforward tax schemes. The results of Section 3 provide a pointer to guide one in the search for a better tax base and the analysis of Section 4 points to disincentive effects that have been neglected in the literature. Theoretical work is capable of providing an insight which empirical analysis can turn into a clear guide to policy.

APPENDIX

The purpose of the Appendix is to show that if an allocation mechanism satisfies the condition of monotonicity then it is implementable in Nash strategies.

The result will be shown constructively, the game-form used having similarities to that in Maskin (1977). Unlike Maskin's construction which makes individuals say as part of their strategy the taste parameter of each person in society, the individual reports the *anonymous* distribution of taste parameters for society.

For each individual i, x_i is a vector of goods, different x_i being ranked using $u(\cdot, \theta_i)$ which increases with an increase in all arguments and is continuous (so that a "best" element exists in a compact set). The consumption set X is taken to be the positive orthant and this is known to the government. Let $x(\boldsymbol{\theta})$ be the allocation mechanism to be implemented. It is assumed that $x_i(\boldsymbol{\theta}) \geqq \bar{x} \gg 0$ (allocations are in the interior of the consumption set) and that x satisfies monotonicity: for all θ_i^*, θ_i^{**}, $\boldsymbol{\theta}_{-i}$ if θ_i^* cannot be screened from (θ_i^{**}, z_i) where $(z_1, \ldots, z_N) \in x(\theta_i^{**}, \boldsymbol{\theta}_{-i})$ then $(z_1, \ldots, z_N) \in x(\theta_i^*, \boldsymbol{\theta}_{-i})$.

The game-form is constructed as follows:

1. *Strategies.* $s_i = (\boldsymbol{\theta}, \theta_i, z) \in \Theta^N \times \Theta \times X$.

2. *The Game-Form*.

2.1. If the number of different distributions (treating permutations as equivalent) $\boldsymbol{\theta}$ reported is $k \geqq 2$ and the most commonly reported $\boldsymbol{\theta}$ is reported r times then

$$x_i^g(s) = \frac{r}{Nk}\bar{x} \quad \text{for all } i.$$

2.2. If the same $\boldsymbol{\theta}$ is reported by everybody but $j \geqq 2$ of the reported θ_i would need to be changed to ensure that the distribution of reported $\{\theta_i\}$ coincides with the common $\boldsymbol{\theta}$ then

$$x_i^g(s) = \left(1 - \frac{j}{2N}\right)\bar{x} \quad \text{for all } i.$$

2.3. If the same $\boldsymbol{\theta}$ is reported by everybody and only one θ_i would require changing to ensure compatibility of $\{\theta_i\}$ with $\boldsymbol{\theta}$ then the number saying θ^*, say, is one less than occurs in the distribution $\boldsymbol{\theta}$. If $I^* \subseteq N$ is the group of individuals who, by changing the reported θ value to θ^* would ensure compatibility let $i^* = \min_i I^*$. Then

$$x_i^g(s) = \begin{cases} z & \text{if } u(z, \theta^*) < u(x_{i^*}(\{\theta_j\} - \theta_{i^*} + \theta_{i^*}^*)) \\ \left(1 - \dfrac{1}{2N}\right)\bar{x} & \text{otherwise.} \end{cases}$$

Here, $x_{i^*}(\{\theta_i\} - \theta_{i^*} + \theta_{i^*}^*)$ is the allocation going to i^* under the distribution $\{\theta_j\}$ except that i^*'s taste parameter is θ^*.

2.4. If the same $\boldsymbol{\theta}$ is reported by everybody and the reported $\{\theta_i\}$ is compatible with this distribution then

$$x_i^g(s) = x_i(\{\theta_i\}).$$

The rules 2.1–2.4 defining the game-form define it for all strategies. Now consider the structure of Nash equilibria. If 2.1 applies then individuals have an incentive to change their $\boldsymbol{\theta}$ to the one most commonly reported. Thus a Nash equilibrium cannot occur under 2.1. Under 2.2 individuals have an incentive to change the θ reported to improve compatibility with $\boldsymbol{\theta}$ so that a Nash equilibrium under 2.2 cannot occur. Under 2.3 it is always optimal to choose a z which lies in the defined set but, as this is an *open* set, no Nash equilibrium can exist. Thus a Nash equilibrium can arise only in case 2.4. The mechanism will be implemented if each individual is reporting his true taste parameter θ and, because of the construction in 2.3, truth-telling is a Nash equilibrium. Assume that i with preferences θ_i^* is reporting θ_i^{**} and receiving z_i where $\{z_j\} \in x(\theta_i^{**}, \boldsymbol{\theta}_{-i})$. By changing what he reports, 2.3 will apply and he can choose a z to make himself better-off *unless* θ_i^* cannot be screened from (θ_i^{**}, z_i). But as monotonicity is satisfied by the allocation mechanism, it is still implemented by x^g. Thus, all allocations occur as truth-telling Nash equilibria and no other allocations are supported as Nash equilibria.

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