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Optimal Fiscal Policy in a Business Cycle Model

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This paper develops the quantitative implications of optimal fiscal policy in a business cycle model. In a stationary equilibrium, the ex ante tax rate on capital income is approximately zero. There is an equivalence class of ex post capital income tax rates and bond policies that support a given allocation. Within this class, the optimal ex post capital tax rates can range from close to independently and identically distributed to close to a random walk. The tax rate on labor income fluctuates very little and inherits the persistence properties of the exogenous shocks; thus there is no presumption that optimal labor tax rates follow a random walk. Most of the welfare gains realized by switching from a tax system like that of the United States to the Ramsey system come from an initial period of high taxation on capital income.

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I. Introduction

A fundamental question in macroeconomics is, How should fiscal policy be set over the business cycle? Standard Keynesian models imply that fiscal policy should be countercyclical. This means, for example, cutting taxes during recessions. The tax-smoothing models of Barro (1979) and others imply that tax rates should change only when unanticipated shocks affect the government budget constraint. Thus, when output declines unexpectedly—and, hence, so do tax revenues—tax rates should be raised enough to meet the government's expected present-value budget constraint. In this paper, we use standard neoclassical theory to answer this fiscal policy question. In particular, we analyze welfare-maximizing policy using a quantitative version of the standard neoclassical growth model with distorting taxes with parameter values and stochastic processes for shocks chosen to be similar to those in the real business cycle literature. Under the optimal policies, there is one period of transition, during which labor income taxes are negative and capital income taxes are large; after that, (*a*) tax rates on labor income are essentially constant, (*b*) expected tax rates on capital income are roughly zero in each period, and (*c*) the return on debt and the ex post tax on capital income absorb most of the shocks to the government budget constraint. In terms of welfare, we find that most of the welfare gains come from high capital income taxation in the one period of transition.

Our finding that optimal labor taxes should not respond to unanticipated shocks is quite different from the results in tax-smoothing models. In particular, such models imply that tax rates should follow a random walk regardless of the stochastic processes for the underlying shocks. In contrast, we find that optimal labor tax rates should fluctuate very little, and to the extent that they do fluctuate, their serial correlation inherits the serial correlation properties of the shocks. Our finding that the ex ante tax rate on capital income should be roughly zero is reminiscent of Judd (1985) and Chamley's (1986) result in the deterministic literature that in a steady state, the optimal capital income tax rate is zero.[1]

In terms of our shock absorber finding, Lucas and Stokey (1983) show in a model without capital that state-contingent returns on gov-

[1] The public finance literature on various aspects of optimal capital income taxes is voluminous. It includes Atkinson (1971), Diamond (1973), Pestieau (1974), and Atkinson and Sandmo (1980). (See also Auerbach and Feldstein [1985, chap. 2] and the references cited there.) These analyses primarily deal with overlapping generations models, whereas we use a model with infinitely lived agents. For analyses in an infinite-lived agents context, with human and physical capital, see Bull (1990), Lucas (1990), and Jones, Manuelli, and Rossi (1993).

ernment debt can play a role in smoothing tax distortions across states of nature. In our model, tax distortions across states of nature can be smoothed by state-contingent taxes on capital as well as state-contingent returns on debt. We find that these smoothing devices are quantitatively important: When there is an innovation in government spending, over 80 percent of the resulting change in the present value of government spending is financed through the state-contingent instruments.

In terms of welfare, we consider the welfare gain starting with a benchmark tax system that is a crude approximation to the U.S. system and switching to the Ramsey system. We decompose this total gain into the gains from the transition period and those from smoothing labor tax rates and making ex ante capital income tax rates zero. We find that at most 20 percent of the total gain comes from smoothing labor tax rates and making expected capital income tax rates zero. The lion's share of the gain comes from the high capital income taxation in the transitional period.

We emphasize that our findings are quantitative. In some interesting theoretical work, Zhu (1992) shows that there is no theoretical presumption that labor tax rates should be constant or that ex ante capital income tax rates should be zero. Our contribution is to examine the quantitative significance of these features. We find that there is a quantitative presumption that labor tax rates should be constant and that ex ante capital income tax rates should be zero.

In reporting our results, we focus on three policy variables pinned down by the model. One is the tax rate on labor income. Another is the ex ante tax rate on capital income, which is defined as the ratio of the value of tax revenues across states of nature in a given period to the value of capital income across states of nature in that period. The third policy variable is the revenues from the state-contingent capital income taxes and the state-contingent debt. One interpretation of these revenues is that they are raised by taxing the return on debt as well as the return on capital. Since capital and debt are the assets available to private agents, we call these revenues the *taxes on private assets*. State-by-state capital income taxes and state-by-state returns on debt are not uniquely determined in our model. Both instruments play similar roles in smoothing tax distortions across states of nature. Arbitrage conditions require that the returns on both types of assets weighted by the intertemporal marginal rates of substitution be equalized. However, the pattern of tax rates on these assets can be structured in a variety of ways that meet the arbitrage conditions and raise the same revenue in each state of nature. We show that there is an equivalence class of tax rates on capital income and rates of return on government debt that can be used to support the Ramsey allocations. (For an independent derivation of this result, see Zhu [1992]. For some related work, see King [1990].) Indeed, from a quantitative standpoint, we find that, depending on the way in which policies are chosen from this equivalence class, the tax rate on capital can range from close to independently and identically distributed (i.i.d.) to close to a random walk. This finding contrasts with the results of Judd (1989), who argues that the ex post capital income tax rates should be i.i.d.

II. The Economy

Consider a production economy populated by a large number of identical, infinitely lived consumers. In each period $t = 0, 1, \ldots,$ the economy experiences one of finitely many events s_t. We denote by $s^t = (s_0, \ldots, s_t)$ the history of events up through and including period t. The probability, as of period 0, of any particular history s^t is $\mu(s^t)$. The initial realization s_0 is given. This suggests a natural commodity space in which goods are differentiated by histories.

In each period t, the economy has two goods: labor and a consumption-capital good. A constant-returns-to-scale technology is available to transform labor $l(s^t)$ and capital $k(s^{t-1})$ into output via the production function $F(k(s^{t-1}), l(s^t), s_t)$. Notice that this function incorporates a stochastic shock. The output can be used for private consumption $c(s^t)$, government consumption $g(s^t)$, and new capital $k(s^t)$. Throughout, we shall assume that government consumption is exogenously specified. Feasibility requires that

$$c(s^t) + g(s^t) + k(s^t) = F(k(s^{t-1}), l(s^t), s_t) + (1 - \delta)k(s^{t-1}), \tag{1}$$

where δ is the depreciation rate on capital. The preferences of each consumer are given by

$$\sum_{t,s^t} \beta^t \mu(s^t) U(c(s^t), l(s^t)), \tag{2}$$

where the parameter $0 < \beta < 1$ and the utility function U is increasing in consumption, decreasing in labor, and strictly concave and satisfies the Inada conditions.

In this economy, government consumption is financed by proportional taxes on the income from labor and capital and by debt. Let $\tau(s^t)$ and $\theta(s^t)$ denote the tax rates on the income from labor and capital. Government debt has a one-period maturity and a state-contingent return. Let $b(s^t)$ denote the number of units of debt issued at state s^t and $R_b(s^{t+1})b(s^t)$ denote the payoff at any state $s^{t+1} =$

(s^t, s_{t+1}). The consumer budget constraint is

$$c(s^t) + k(s^t) + b(s^t) \leq [1 - \tau(s^t)]w(s^t)l(s^t) + R_b(s^t)b(s^{t-1}) + R_k(s^t)k(s^{t-1}), \tag{3}$$

where $R_k(s^t) = 1 + [1 - \theta(s^t)][r(s^t) - \delta]$ is the gross return on capital after taxes and depreciation and $r(s^t)$ and $w(s^t)$ are the before-tax returns on capital and labor. Competitive pricing ensures that these returns equal their marginal products, namely, that

$$r(s^t) = F_k(k(s^{t-1}), l(s^t), s_t) \tag{4}$$

and

$$w(s^t) = F_l(k(s^{t-1}), l(s^t), s_t). \tag{5}$$

Consumer purchases of capital are constrained to be nonnegative, and the purchases of government debt are bounded above and below by some arbitrarily large constants. Let $x(s^t) = (c(s^t), l(s^t), k(s^t), b(s^t))$ denote an allocation for consumers at s^t, and let $x = (x(s^t))$ denote an allocation for all s^t.

The government sets tax rates on labor and capital income and returns for government debt in order to finance the exogenous sequence of government consumption. The government budget constraint is

$$b(s^t) = R_b(s^t)b(s^{t-1}) + g(s^t) - \tau(s^t)w(s^t)l(s^t) - \theta(s^t)[r(s^t) - \delta]k(s^{t-1}). \tag{6}$$

Let $\pi(s^t) = (\tau(s^t), \theta(s^t), R_b(s^t))$ denote the government policy at s^t, and let $\pi = (\pi(s^t))$ denote the policy for all s^t. The initial stock of debt b_{-1} and the initial stock of capital k_{-1} are given.

Notice that, for notational simplicity, markets in private claims are not explicitly included in this economy. Since consumers are identical, such claims will not be traded in equilibrium; hence, their absence will not affect the equilibrium. Thus the current model can always be interpreted as having complete contingent private claims markets.

III. The Ramsey Equilibrium

Consider now the policy problem faced by the government. Suppose that there is an institution or commitment technology through which the government can bind itself to a particular sequence of policies once and for all in period 0. We model this by having the government choose a policy $\pi = (\pi(s^t))$ at the beginning of time and then having consumers choose their allocations. Since the government needs to predict how consumer allocations and prices will respond to its policies, we describe consumer allocations and prices by rules that associate government policies with allocations. Formally, allocation rules are sequences of functions $x(\pi) = (x(s^t|\pi))$ that map policies π into allocations $x(\pi)$. Price rules are sequences of functions $w(\pi) = (w(s^t|\pi))$ and $r(\pi) = (r(s^t|\pi))$ that map policies π into price systems $w(\pi)$ and $r(\pi)$.

A *Ramsey equilibrium* is a policy π, an allocation rule $x(\cdot)$, and price rules $w(\cdot)$ and $r(\cdot)$ such that (i) the policy π maximizes $\sum_{t,s^t} \beta^t \mu(s^t) U(c(s^t|\pi), l(s^t|\pi))$ subject to (6) with allocations and prices given by $x(\pi)$, $w(\pi)$, and $r(\pi)$; (ii) for every π', the allocation $x(\pi')$ maximizes (2) subject to (3) evaluated at the policy π' and the prices $w(\pi')$ and $r(\pi')$; and (iii) for every π', the prices satisfy

$$w(s^t|\pi') = F_l(k(s^{t-1}|\pi'), l(s^t|\pi'), s_t) \tag{7}$$

and

$$r(s^t|\pi') = F_k(k(s^{t-1}|\pi'), l(s^t|\pi'), s_t). \tag{8}$$

The allocations in a Ramsey equilibrium solve a simple programming problem called the *Ramsey allocation problem.* Now, it is well known that in a Ramsey equilibrium the government has an incentive to set the initial tax rate on capital income as large as possible. To make the problem interesting, we adopt the convention that the initial capital tax rate $\theta(s_0)$ and the initial return on debt $R_b(s_0)$ are fixed. We place no other restrictions on the tax rates for capital and labor income. In terms of notation, for convenience, here and throughout the paper, let $U_c(s^t)$ and $U_l(s^t)$ denote the marginal utilities of consumption and leisure at state s^t and let $F_k(s^t)$ and $F_l(s^t)$ denote the marginal products of capital and labor at state s^t. We have, then, the following proposition.

PROPOSITION 1. *The Ramsey allocations.*—The consumption, labor, and capital allocations in a Ramsey equilibrium solve the Ramsey allocation problem

$$\max \sum_{t,s^t} \beta^t \mu(s^t) U(c(s^t), l(s^t)) \tag{9}$$

subject to

$$c(s^t) + g(s^t) + k(s^t) = F(k(s^{t-1}), l(s^t), s_t) + (1 - \delta)k(s^{t-1}) \tag{10}$$

and

$$\sum_{t,s^t} \beta^t \mu(s^t)[U_c(s^t)c(s^t) + U_l(s^t)l(s^t)] = U_c(s_0)[R_k(s_0)k_{-1} + R_b(s_0)b_{-1}]. \tag{11}$$

Proof. In the Ramsey equilibrium, the government must satisfy its budget constraint taking as given the allocation rule $x(\pi)$ and the pricing rules $w(\pi)$ and $r(\pi)$. These requirements impose restrictions on the set of allocations the government can achieve by varying its policies. We claim that these restrictions are summarized by constraints (10) and (11). To demonstrate that, we first show that the restrictions imply (10) and (11). To see this, note that (3) and (6) can be added to get (10); thus feasibility is satisfied in equilibrium. Next, consider the allocation rule $x(\pi)$. For any policy π, we describe the necessary and sufficient conditions for c, l, b, and k to solve the consumer's problem. Let $p(s^t)$ denote the Lagrange multiplier on the consumer budget constraint (3). Then by Weitzman's (1973) theorem, these conditions are constraint (3) together with the first-order conditions for consumption and labor:

$$\beta^t \mu(s^t) U_c(s^t) \leq p(s^t), \quad \text{with equality if } c(s^t) > 0 \tag{12}$$

and

$$\beta^t \mu(s^t) U_l(s^t) \leq -p(s^t)[1 - \tau(s^t)]w(s^t), \quad \text{with equality if } l(s^t) > 0; \tag{13}$$

first-order conditions for capital and bonds:

$$\left[p(s^t) - \sum_{s_{t+1}} p(s^{t+1}) R_b(s^{t+1})\right] b(s^t) = 0 \tag{14}$$

and

$$\left[p(s^t) - \sum_{s_{t+1}} p(s^{t+1}) R_k(s^{t+1})\right] k(s^t) = 0; \tag{15}$$

and the two transversality conditions, which specify that, for any infinite history s^∞,

$$\lim p(s^t) b(s^t) = 0 \tag{16}$$

and

$$\lim p(s^t) k(s^t) = 0, \tag{17}$$

where the limits are taken over sequences of histories s^t contained in the infinite history s^∞.

We claim that any allocation that satisfies (3) and (12)–(17) must satisfy (11). To see this, multiply (3) by $p(s^t)$, sum over t and s^t, and use (14)–(17) to give

$$\begin{aligned} \sum_{t,s^t} p(s^t)\{c(s^t) - [1 - \tau(s^t)] w(s^t) l(s^t)\} \\ = p(s_0)[R_k(s_0)k_{-1} + R_b(s_0)b_{-1}]. \end{aligned} \tag{18}$$

Using (12) and (13) and noting that interiority follows from the Inada conditions, we can rewrite (18) as

$$\sum_{t,s^t} \beta^t \mu(s^t)[c(s^t)U_c(s^t) + l(s^t)U_l(s^t)] = U_c(s_0)[R_k(s_0)k_{-1} + R_b(s_0)b_{-1}]. \tag{19}$$

Thus (10) and (11) are necessary conditions that any Ramsey equilibrium must satisfy. Next, given any allocation that satisfies (10) and (11), we can construct sequences of bond holdings and sequences of policies such that these allocations satisfy (3), (6), and (12)–(17). Therefore, the restrictions on the set of allocations achievable by the government are equivalent to (10) and (11); thus the proposition follows. Q.E.D.

Proposition 1 describes the consumption, labor, and capital allocations. Using these allocations, we construct the bond allocation $b(s^r)$ as follows. Multiply (3) by $p(s^t)$, and sum over all periods and states following s^r. Use (12)–(15) to obtain

$$b(s^r) = \sum_{t=r+1}\sum_{s^t} \frac{\beta^{t-r}\mu(s^t|s^r)[U_c(s^t)c(s^t) + U_l(s^t)l(s^t)]}{U_c(s^r)} - k(s^r). \tag{20}$$

Now, for convenience later, write the Ramsey allocation problem in Lagrangian form:

$$\max \sum_{t,s^t} \beta^t \mu(s^t) W(c(s^t), l(s^t), \lambda) - \lambda U_c(s_0)[R_k(s_0)k_{-1} + R_b(s_0)b_{-1}] \tag{21}$$

subject to (10). The function W simply incorporates the implementability constraint into the maximand. Let

$$W(c(s^t), l(s^t), \lambda) = U(c(s^t), l(s^t)) + \lambda[U_c(s^t)c(s^t) + U_l(s^t)l(s^t)]. \tag{22}$$

Here, λ is the Lagrange multiplier on the implementability constraint (11). The first-order conditions for this problem imply that, for $t \geq 1$,

$$-\frac{W_l(s^t)}{W_c(s^t)} = F_l(s^t) \tag{23}$$

and

$$W_c(s^t) - \sum_{s^{t+1}} \beta\mu(s^{t+1}|s^t)W_c(s^{t+1})[1 - \delta + F_k(s^{t+1})] = 0. \tag{24}$$

For $t = 0$, these conditions are

$$-\frac{W_l(s_0) - \lambda\{U_{cl}(s_0)[R_k(s_0)k_{-1} + R_b(s_0)b_{-1}] + U_c(s_0)[1 - \theta(s_0)]F_{kl}(s_0)\}}{W_c(s_0) - \lambda U_{cc}[R_k(s_0)k_{-1} + R_b(s_0)b_{-1}]} = F_l(s_0) \tag{25}$$

and

$$\begin{aligned} &W_c(s_0) - \lambda U_{cc}[R_k(s_0)k_{-1} + R_b(s_0)b_{-1}] \\ &- \sum_{s^1} \beta\mu(s^1|s_0)W_c(s^1)[1 - \delta + F_k(s^1)] = 0. \end{aligned} \tag{26}$$

A useful property of the Ramsey allocations is the following. If the stochastic process on s follows a Markov process, then (23) and (24) imply that the allocations from period 1 onward can be described by time-invariant allocation rules $c(k, s; \lambda)$, $l(k, s; \lambda)$, $k'(k, s; \lambda)$, and $b(k, s; \lambda)$. The period 0 first-order conditions (25) and (26) include terms related to the initial stocks of capital and bonds and are, therefore, different from the other first-order conditions. The period 0 allocation rules are, thus, different from the stationary allocation rules that govern behavior from period 1 on.

IV. The Ramsey Policies

Proposition 1 describes the Ramsey allocations, or the allocations that actually occur in a Ramsey equilibrium. We are also interested in describing the set of policies and prices that may arise in a Ramsey equilibrium. That is, for some given allocations that solve the Ramsey allocation problem, we construct policies and prices that decentralize it.

We pose the problem as follows. Given a Ramsey allocation c, l, and k and a b given by (20), find the set of prices w and r, returns R_b, and tax rates τ and θ that satisfy the marginal product conditions, the consumer first-order conditions, and the budget constraints of the consumers and the government. Now since the Ramsey allocations satisfy feasibility, any policies and prices that satisfy the consumer budget constraint must also satisfy the government budget constraint. The wage rate and the rental rate on capital are obtained from the marginal product conditions. Substituting these prices into consumer first-order conditions gives an intratemporal condition

$$-\frac{U_l(s^t)}{U_c(s^t)} = [1 - \tau(s^t)]F_l(s^t) \tag{27}$$

as well as two intertemporal conditions

$$U_c(s^t) = \sum_{s^{t+1}} \beta\mu(s^{t+1}|s^t)U_c(s^{t+1})R_b(s^{t+1}) \tag{28}$$

and

$$U_c(s^t) = \sum_{s^{t+1}} \beta\mu(s^{t+1}|s^t)U_c(s^{t+1})R_k(s^{t+1}), \tag{29}$$

where $R_k(s^{t+1}) = 1 + [1 - \theta(s^{t+1})][F_k(s^{t+1}) - \delta]$. The consumer budget constraint is

$$\begin{aligned} c(s^{t+1}) + b(s^{t+1}) + k(s^{t+1}) &= [1 - \tau(s^{t+1})]w(s^{t+1})l(s^{t+1}) \\ &\quad + R_b(s^{t+1})b(s^t) + R_k(s^{t+1})k(s^t). \end{aligned} \tag{30}$$

The tax rate on labor is determined from (27). Consider next the determination of bond returns R_b and the capital income tax rate θ. We shall use (28)–(30) to show that they are indeterminate. Suppose that, in some period t, s_{t+1} can take on N values. Then counting equations and unknowns in (28)–(30) gives $2N$ unknowns but only $N + 2$ equations. Actually, however, there is one linear dependency across these equations. To see this, multiply (30) by $\beta\mu(s^{t+1}|s^t)U_c(s^{t+1})$. Then summing across the states in period $t + 1$ and using (28), (29), and (20) yield an equation that does not depend on R_b and θ. Thus there are $N - 1$ degrees of indeterminacy. We have proved the following proposition.

PROPOSITION 2. *The indeterminacy of capital tax rates.*—If R_b and θ satisfy (28)–(30), then so do any $\bar{R}_b$ and $\bar{\theta}$, where

$$\sum_{s^{t+1}} \mu(s^{t+1}|s^t)U_c(s^{t+1})R_b(s^{t+1}) = \sum_{s^{t+1}} \mu(s^{t+1}|s^t)U_c(s^{t+1})\bar{R}_b(s^{t+1}), \tag{31}$$

$$\begin{aligned} &\sum_{s^{t+1}} \mu(s^{t+1}|s^t)U_c(s^{t+1})\theta(s^{t+1})[F_k(s^{t+1}) - \delta] \\ &= \sum_{s^{t+1}} \mu(s^{t+1}|s^t)U_c(s^{t+1})\bar{\theta}(s^{t+1})[F_k(s^{t+1}) - \delta], \end{aligned} \tag{32}$$

and

$$\begin{aligned} &\theta(s^{t+1})[F_k(s^{t+1}) - \delta]k(s^t) - R_b(s^{t+1})b(s^t) \\ &= \bar{\theta}(s^{t+1})[F_k(s^{t+1}) - \delta]k(s^t) - \bar{R}_b(s^{t+1})b(s^t). \end{aligned} \tag{33}$$

To get some feel for the different possibilities, consider two extreme cases. First suppose that the government is restricted to making capital taxes not contingent on the realization of the current state. That is, suppose that, for each s^t,

$$\theta(s^t, s_{t+1}) = \bar{\theta}(s^t), \quad \text{for all } s_{t+1}. \tag{34}$$

These conditions add $N - 1$ restrictions in each period and state and lead to a unique policy. The capital tax rate is pinned down by the first-order condition for capital, and the bond returns are then pinned down by the consumer budget constraint and the first-order condition for bonds. For another extreme, suppose that the government is restricted to making the returns on debt not contingent on the current state. That is, suppose that, for each s^t,

$$R_b(s^t, s_{t+1}) = \overline{R}_b(s^t), \quad \text{for all } s_{t+1}. \tag{35}$$

These conditions also add $N - 1$ restrictions in each period and state and lead to a unique policy. The return on bonds is pinned down by the first-order condition for bonds, and the capital tax rates are pinned down by the consumer budget constraint and the first-order conditions for capital. More generally, at each node s^t, there are $N - 1$ degrees of freedom in determining the debt and capital tax policies. Each of these two extremes adds $N - 1$ restrictions at each node and leads to a unique policy. Of course, any other set of restrictions across capital tax rates and returns to debt that leads to $N - 1$ restrictions at each node will also lead to a unique policy.

In summary, we have shown that the policies for debt and capital taxes are not uniquely determined by the Ramsey allocations. If the government has either state-contingent capital taxes or state-contingent debt, it can support the Ramsey allocations. If the capital taxes are restricted to not depend on the current state, the government can vary the returns to bonds in exactly the right way to support the optimal allocations. Alternatively, if the returns to debt are restricted to not depend on the current state, the government can vary the returns to capital in exactly the right way to support the same allocation. In particular, notice that restricting the government from issuing state-contingent debt has no effect on either optimal allocations or welfare. Note, however, that if the government has neither state-contingent capital taxes nor state-contingent debt, there are more equations than unknowns, and it cannot support the Ramsey allocations. Indeed, if the instruments available to the government are so restricted, then the Ramsey problem must be modified to include extra constraints that capture the effect of these restrictions.

For later analysis, let us now isolate certain fiscal variables that are uniquely determined by the theory. First, as we have mentioned, the tax rate on labor income is determined. Second, while the state-by-state capital tax rates are not pinned down, (32) establishes that the value of the tax payments across states of nature is determined. To turn this value into a rate, consider the ratio of the value of tax payments across states to the value of net revenues from capital across states; namely,

$$\theta^e(s^t) = \frac{\Sigma\, q(s^{t+1})\,\theta(s^{t+1})[F_k(s^{t+1}) - \delta]}{\Sigma\, q(s^t)[F_k(s^{t+1}) - \delta]}, \tag{36}$$

where $q(s^{t+1}) = \beta\mu(s^{t+1}|s^t)\,U_c(s^{t+1})/U_c(s^t)$ is the Arrow-Debreu price of a unit of consumption at state s^{t+1} in units of consumption at s^t. We call $\theta^e(s^t)$ the *ex ante tax rate on capital income*. Conceptually, this rate corresponds to what Jorgenson (1963) and Hall and Jorgenson (1967) call the *effective capital tax rate.* The capital tax rate $\theta(s^t)$ corresponds to what Judd (1989) calls the *ex post capital tax rate.*

The third fiscal variable that is determined by the theory is given in (33), namely, the revenues from capital taxation minus the value of outstanding debt. For ease of comparison with the labor tax rate and the ex ante capital tax rate, we transform these revenues into a rate. One way of doing so is to imagine that the government achieves the desired state contingency in debt returns by promising a state-noncontingent rate of return on government debt $\bar{r}(s^t)$ that satisfies

$$\sum_{s_{t+1}} q(s^{t+1})R_b(s^{t+1}) = \sum_{s_{t+1}} q(s^{t+1})[1 + \bar{r}(s^t)] \tag{37}$$

and by levying a state-contingent tax $\nu(s^{t+1})$ on interest payments from government debt that satisfies

$$R_b(s^{t+1}) = 1 + \bar{r}(s^t)[1 - \nu(s^{t+1})]. \tag{38}$$

Notice that $\Sigma\, q(s^{t+1})\nu(s^{t+1}) = 0$, and thus the present value of revenues raised from taxation of interest on debt is zero. Next note from (31) and (37) that $\bar{r}(s^t)$ is pinned down and from (20) that $b(s^t)$ is pinned down. Thus (33) can be thought of as pinning down the sum of the tax revenues from the capital income tax and the debt income tax given by

$$\theta(s^{t+1})[F_k(s^{t+1}) - \delta]k(s^t) + \nu(s^{t+1})\bar{r}(s^t)b(s^t). \tag{39}$$

We transform these revenues into a rate by dividing them by the income from capital and debt to obtain

$$\eta(s^{t+1}) = \frac{\theta(s^{t+1})[F_k(s^{t+1}) - \delta]k(s^t) + \nu(s^{t+1})\bar{r}(s^t)b(s^t)}{[F_k(s^{t+1}) - \delta]k(s^t) + \bar{r}(s^t)b(s^t)}. \tag{40}$$

A useful property of the Ramsey policies is the following. The three tax rates can be described by time-invariant policy rules of the form $\tau(k, s; \lambda)$, $\theta^e(k, s; \lambda)$, and $\eta(k, s; \lambda)$ from period 1 on. The policy rules for period 0 are different from these time-invariant rules. To see this, recall from the discussion after equations (23)–(26) that the allocations follow time-invariant rules from period 1 on. Inspection of (27), (36), and (40) establishes that the policy rules do also.

Notice that a subtle asymmetry exists between the ex ante capital tax rate and the other two tax rates. Specifically, the tax rates on labor $\tau(s^t)$ and the tax rate on private assets $\eta(s^t)$ are levied on income received in period t, and the ex ante tax rate on capital $\theta^e(s^t)$ is a weighted average of the tax rates on capital income received in period $t + 1$. Thus, under the Ramsey policies, the income from labor and private assets is taxed differently than under the stationary policies only in period 0, and the income from capital is taxed differently than under the stationary policies in period 1. Of course, the income from capital in period 0 is also taxed differently because the tax rate there is fixed at some rate.

V. A Class of Utilities

Now we examine the nature of the Ramsey taxes for a class of utility functions. We show that for such a class it is optimal not to distort the consumer capital accumulation decision made in period 1 or thereafter. To motivate the result, we write the consumer's first-order condition for capital as

$$1 - \sum_{s_{t+1}} q(s^{t+1})[1 - \delta + F_k(s^{t+1})] = \sum_{s_{t+1}} q(s^{t+1})\theta(s^{t+1})[F_k(s^{t+1}) - \delta]. \tag{41}$$

In an undistorted equilibrium, the consumer's first-order condition has the same left side as (41), but the right side equals zero. Thus the right side of (41) measures the size of the wedge between the distorted and undistorted first-order conditions for capital accumulation in period t. Note that the right side of (41) is the market value at t of claims to the revenues from capital taxation at $t + 1$. Since the right side of (41) is the numerator of (36), the capital accumulation decision is undistorted if and only if the ex ante rate on capital income is zero.

Consider utility functions of the form

$$U(c, l) = \frac{c^{1-\sigma}}{1-\sigma} + V(l). \tag{42}$$

We then have the following proposition.

PROPOSITION 3. *Ramsey policies for specific utilities.*—For utility functions of the form (42), it is not optimal to distort the capital accumulation decision in period 1 or thereafter. Namely, the ex ante rate on capital income received in period t is zero for period $t \geq 2$ or, equivalently,

$$\sum_{s_{t+1}} q(s^{t+1})\theta(s^{t+1})[F_k(s^{t+1}) - \delta] = 0, \quad \text{for } t \geq 1. \tag{43}$$

Proof. For $t \geq 1$, the first-order conditions for the Ramsey problem imply that

$$1 = \sum_{s^{t+1}} \beta\mu(s^{t+1}|s^t)\frac{W_c(s^{t+1})}{W_c(s^t)}[1 - \delta + F_k(s^{t+1})]. \tag{44}$$

For $t \geq 1$, the consumer's first-order conditions for capital imply that

$$1 = \sum_{s^{t+1}} \beta\mu(s^{t+1}|s^t)\frac{U_c(s^{t+1})}{U_c(s^t)}\{1 + [1 - \theta(s^{t+1})][F_k(s^{t+1}) - \delta]\}. \quad (45)$$

Now, for any utility function of the form (42), it is easy to show that

$$\frac{W_c(s^{t+1})}{W_c(s^t)} = \frac{U_c(s^{t+1})}{U_c(s^t)}. \quad (46)$$

Substituting (46) into (44) and subtracting it from (45) give the result. Q.E.D.

Note that for a deterministic version of the model, proposition 3 implies that the tax rate on capital income received in period t is zero for $t \geq 2$ and is typically different from zero in period 1. In period 0, of course, the tax rate is fixed. Now recall that in a continuous-time version of the deterministic model with instantaneous preferences given by (42), Chamley (1986) shows that the tax rate on capital income is constant for a finite length of time and zero thereafter. The reason for the difference is that Chamley imposes an exogenous upper bound on the tax rate on capital income. If we impose such an upper bound, the Ramsey problem must be amended to include an extra constraint to capture the restrictions imposed by this upper bound. In the deterministic version of the model, with preferences given by (42), the tax rate would be constant at this upper bound for a finite number of periods, there would be one period of transition, and thereafter the tax rate would be zero.

In the stochastic version of the model, constraints of this kind can also be imposed. The motivation for such an exogenous upper bound, however, is not clear. We find it more interesting to derive an endogenous upper bound. Consider the following scenario. At the end of each period t, consumers can rent capital to firms for use in period $t + 1$ and pay taxes on the rental income from capital in period $t + 1$. Or consumers can choose to hide the capital, say, in their basements. If they hide it, the capital depreciates and is not available for use at $t + 1$. Thus, if they hide it, they get no capital income and pay zero capital taxes. It is easy to show that this extra option leads to the following constraint on the Ramsey problem:

$$U_c(s^t) \geq \sum_{s^{t+1}} \beta\mu(s^{t+1}|s^t)U_c(s^{t+1})(1 - \delta). \quad (47)$$

For a special case of the preferences in (42), which we use in the baseline model in our computations, this constraint binds for a finite number of periods; then there is one period of transition, and thereafter the capital tax rate is zero. A proof of this result is available on request.

VI. Computation and Parameterization

Theoretically characterizing the Ramsey policies for more general utility functions than those considered in Section V turns out to be difficult. Therefore, we characterize these policies quantitatively. We are particularly interested in the quantitative properties of optimal tax rates in the class of economies similar to those studied in the business cycle literature (see, e.g., Kydland and Prescott 1982). In this literature, the preferences are described by utility functions of the form

$$U(c, l) = \frac{[c^{1-\gamma}(1-l)^{\gamma}]^{\psi}}{\psi}. \tag{48}$$

The technology is described by a production function of the form

$$F(k, l, z, t) = k^{\alpha}(e^{\rho t + z} l)^{1-\alpha}. \tag{49}$$

We incorporate two kinds of labor-augmenting technological change into the production technology. The variable ρ captures deterministic growth in this technological change. The variable z is a zero mean technology shock that follows a symmetric two-state Markov chain. Let government consumption be given by $g_t = Ge^{\rho t + \tilde{g}}$, where G is a constant, ρ is the deterministic growth rate, and the zero mean process $\tilde{g}$ follows a symmetric two-state Markov chain. Notice that without technology shocks, the economy has a balanced-growth path along which consumption, capital, and government spending grow at rate ρ and labor is constant. This formulation assumes that the economy grows over time. It is straightforward to modify the theoretical models of the previous sections to allow for exogenous growth.

We consider several parameterizations of this model. Our baseline model has $\psi = 0$ and, thus, logarithmic preferences. The parameters for preferences and technology are chosen using the same procedures as in Christiano and Eichenbaum (1992) but modified appropriately to take account of distorting taxes. Briefly, this procedure involves choosing parameters so that along the nonstochastic, balanced-growth path of an economy with distorting taxes, the capital/output ratio, the fraction of available time worked, the ratio of government spending to output, and the debt/output ratio are the same as those in U.S. data. We choose the capital and labor tax rates so that their ratio matches the ratio of the mean of Barro and Sahasakul's (1983) estimate of the average marginal tax rate to the mean of Jorgenson and Sullivan's (1981) estimate of the effective corporate tax rate. Our empirical measures of the capital/output ratio, the fraction of available time worked, the ratio of government spending to output, and the debt/output ratio are 2.71, 0.23, 0.18, and 0.51, respectively. The values of the capital and labor tax rates determined by our procedure are 27.1 and 23.7 percent, respectively. We refer to these policies as the *nonstochastic benchmark policies.* These tax numbers are lower than most estimates of tax rates on labor and capital because the model does not have transfer payments. Our procedure also determines β, γ, and G.

We choose the two parameters of the Markov chain for $\tilde{g}$ so that the autocorrelation ρ_g and the standard deviation σ_g are the same as the annualized versions of the corresponding statistics in Christiano and Eichenbaum (1992). We choose the two parameters of the Markov chain for the technology shock so that the autocorrelation ρ_z and the standard deviation σ_z are the same as the annualized versions of the corresponding statistics in Prescott (1986).

We also consider a model with high risk aversion by setting $\psi = -8$, a model with i.i.d. shocks, a model with only technology shocks, and a model with only government spending shocks. In the high risk aversion model, we adjust the discount factor to keep the capital/output ratio the same as before along the balanced-growth path of an economy with the nonstochastic benchmark policies. We also consider models with a range of risk aversion parameters, and for each we adjust the discount factor in a similar way. The initial conditions for our experiments, unless explicitly stated otherwise, are given by the balanced-growth path of a deterministic economy with the nonstochastic benchmark policies, $k_{-1} = 1.05$ and $R_b(s_0)b_{-1} = 0.20$. Our parameter values for that economy are reported in table 1.

We also consider a model with a high level of initial debt. This model is an attempt to capture some of the consequences of including transfers in our setup. If transfers are thought of as obligations by the government to pay a fixed amount in present-value terms, then they are equivalent to government debt. In that vein, we calculate the present value of transfer payments assuming that along the balanced-growth path transfers are 12 percent of gross national product, which is approximately their value in 1985. We then add this value to the initial government debt.

We briefly describe our computational procedure. We use the standard procedure of transforming our economy with growth into one without growth. This transformation affects only the discount factor

TABLE 1

Baseline Model Parameter Values

Preferences	$\gamma = .75$	$\psi = 0$	$\beta = .98$
Technology	$\alpha = .34$	$\delta = .08$	$\rho = .016$
Stochastic process for government consumption	$G = .07$	$\rho_g = .89$	$\sigma_g = .07$
Stochastic process for technology shock	$\rho_z = .81$	$\sigma_z = .04$	

and the depreciation rate (see Christiano and Eichenbaum 1992). Let s denote a pair of shocks (z, g), and let $\mu(s'|s)$ denote the associated transition probabilities. We begin by fixing an initial value for the Lagrange multiplier λ on the implementability constraint. Given this value of λ, the solutions to the Ramsey allocation problem for $t \geq 1$ are stationary functions of (k, s). We use the resource constraint and (23) to express $c(k, s)$ and $l(k, s)$ in terms of k, s, and the capital accumulation rule $k'(k, s)$. We use the Euler equation (24) to determine $k'(k, s)$.[2] We use the resource constraint and (25) to express period 0 consumption and employment in terms of the end-of-period capital stock and the Euler equation (26) to determine the end-of-period capital stock.

We solve for λ as follows. Since, for $t \geq 1$, consumption, labor, and capital are stationary functions of (k, s) for each λ, (20) establishes that the bond allocation rule is also a stationary function of (k, s) for each λ. From (20), the bond allocation can be recursively written as

$$\begin{aligned} U_c(k,s)b(k,s) = \sum_{s'} \beta\mu(s'|s)[&U_c(k',s')c(k',s') \\ &+ U_l(k',s')l(k',s') + U_c(k',s')b(k',s') \\ &+ U_c(k',s')k'(k',s')] - U_c(k,s)k', \end{aligned} \tag{50}$$

where $k' = k'(k, s)$ and $U_c(k, s)$ and $U_l(k, s)$ are the marginal utilities of consumption and labor. Notice that (50) defines a linear operator

[2] Finding a function $k'(k, s)$ that satisfies (24) for all k, s is computationally infeasible. Thus we limit ourselves to a finite-parameter class of decision rules,

$$k'(k, s; a) = \exp\left[\sum_{i=0}^{n-1} a_i(s) T_i(\psi(\log(k)))\right],$$

where $T_i(\cdot)$ is the ith Choleski polynomial (Press et al. 1988) and $a_i(s)$, for $i = 0, \ldots, n-1$, is a set of coefficients for each of the four possible values of s. The $4n$-element vector, $\mathbf{a}$, denotes these coefficients. The function $\psi(\cdot)$ maps an interval containing the ergodic set for $\log(k)$ into the interval $[-1, 1]$. We choose values for $\mathbf{a}$ to get the expression to the left of the equality in (24) to be close to zero. For this, we use the following version of the Galerkin method discussed in Judd (1992). Let k_j, for $j = 1, \ldots, m$, denote the values of k satisfying $T_m(\psi(\log(k))) = 0$, where $m \geq n$. Let $\mathbf{A}$ denote the $n \times m$ matrix with components $A_{ij} = T_{i-1}(\psi(\log(k_j)))$, for $i = 1, \ldots, n$ and $j = 1, \ldots, m$. Let $\mathbf{R}(s, a)$ denote the $m \times 1$ vector formed by evaluating the expression on the left of the equality in (24) using the decision rule $k'(k, s; a)$ at the m values of k, for each s. Then select the $4n$ parameters $\mathbf{a}$ so that the $4n$ equations, $AR(s, a)$ for all s, equal zero. For this, we use a standard nonlinear equation solver. We obtain a starting value for these calculations by finding the nonexplosive, log-linear capital decision rule that solves a version of (24) in which the function for which the expectation is taken is log-linearized about the nonstochastic steady-state capital stock. We find that $n = 10$ and $m = 41$ works well in the sense that larger values for these parameters result in no noticeable change in our results. For further details, see Chari, Christiano, and Kehoe (1991).

that maps bond allocation rules into themselves. The stationary bond allocation rule is the fixed point of that mapping, where marginal utilities and quantities are computed using the stationary quantity allocation rules.[3] We substitute the period 0 consumption and employment rules into the marginal utility of consumption on the left side of (50) and the stationary rules on the right side to derive the end-of-period 0 bond allocation rule.

Finally, we substitute the end-of-period 0 bond allocation rule together with the other period 0 allocation rules into the consumer budget constraint (3) evaluated in period 0. Using the equality between the marginal rate of substitution between consumption and labor and the after-tax wage rate and setting $\theta(s_0)$ to our initial rate of 27.1 percent, we calculate a value for $R_b(s_0)b_{-1}$. We iterate on λ until the initial value for $R_b(s_0)b_{-1}$ is 0.20, which is the steady-state value of government obligations in a deterministic economy with the nonstochastic benchmark policies. We compute the tax rate on labor income, the ex ante tax on capital income, and the tax on private assets by substituting the allocation rules into (27), (36), and (40), respectively.

VII. Findings

In this section, we report on the statistical properties of the allocations and policies of our theoretical economies. For each setting of the parameter values, we simulate our economy for 4,500 periods. As discussed in Sections IV and V, the optimal labor tax rate in period 0 and the optimal ex ante capital tax rate on capital income received in period 1 are different from the stationary policies. We find that the period 0 labor tax rate is -36 percent for the baseline model and -17 percent for the high risk aversion model. The period 1 ex ante capital income tax rate is 796 percent for the baseline model and 1,326 percent for the high risk aversion model. In terms of the properties of the stationary policies, we drop the first 100 periods of our simulations to ensure that the allocations and policies are drawn from their stationary distributions. Then we compute a variety of statistics of the policies and the allocations.[4]

[3] Finding a function $b(k, s)$ that satisfies (50) for all k, s is not computationally feasible. Instead, we restrict the bond rule to be continuous and piecewise linear in k for each fixed s and require that (50) be satisfied at a finite set of points. For each s, the nodes of our bond function occur at the values of k in the m-dimensional capital grid discussed in n. 2. The values of the debt rule at these node points define its parameters. The requirement that (50) be satisfied at the $4m$ node points defines a linear map from the $4m$-dimensional space of bond rule parameters into itself. We find the fixed point of this mapping by solving a system of $4m$ linear equations.

[4] Recall that the properties of the stationary decision rules depend on initial condi-

A. *Cyclical Properties*

In table 2 we report on some properties of the fiscal variables for our models. This table illustrates three of our main findings. First, in all the models, the labor tax rate fluctuates very little. Second, as is to be expected from proposition 3, in all the models with log utility, the ex ante capital tax rate is identically zero. Both of these findings hold even for the high risk aversion model, which has nonseparable utility: the ex ante capital tax rate is close to zero on average and fluctuates very little. Third, the tax rate on private assets is close to zero on average and fluctuates a great deal. These three findings are further illustrated in figure 1. There we plot histograms of these three tax rates for our high risk aversion model.

To get a feel for the sensitivity of these results, we vary a number of parameters. We start by varying the risk aversion parameter ψ from zero to -20. While adjusting the discount factor appropriately, in figures 2 and 3 we plot the means and the standard deviations of the optimal tax rates against the risk aversion parameter. These figures reinforce our basic findings. The mean labor tax rate declines as ψ becomes more negative because a lower intertemporal elasticity of substitution makes increasing the tax on capital in the transition period optimal. This reduces revenue requirements in the steady state. The mean of the ex ante tax rate on capital is less than 1 percent even for values of the risk aversion parameter as extreme as -20.

An interesting feature of figure 3 is that the standard deviation of the labor tax rate is not monotone in the risk aversion parameter. This finding is connected to a result we discuss below, namely, that the correlation between the labor tax rate and the underlying shocks changes sign near $\psi = -4$. The mean tax rate on private assets decreases with the risk aversion parameter for large negative values of ψ. This occurs because the steady-state value of the debt becomes large and negative since the tax on capital in the transition phase rises as ψ falls. The standard deviation of the ex ante capital tax rate rises as ψ falls. This occurs because the lower intertemporal substitutability of consumption makes smaller the welfare costs of varying capital tax rates over time.

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tions only through their effect on the Lagrange multiplier λ. We have done some experiments by varying λ and have found that some properties of the stationary distribution, such as average tax rates, are sensitive to initial conditions whereas others, such as the second-moment properties, are less so. Note, too, that constraints on the Ramsey problem that bind only for a finite number of periods also affect the properties of the stationary policies and allocations only through λ. Therefore, upper bounds on capital income tax rates that bind only for a finite number of periods do not much affect the second-moment properties of the allocations and policies. Of course, differences in initial conditions or constraints may have large effects on welfare.

TABLE 2

PROPERTIES OF TAX RATES FOR MODEL ECONOMIES

	Baseline Model	High Risk Aversion Model	Alternative Shock Processes: Only Technology	Alternative Shock Processes: Only Government Consumption	Alternative Shock Processes: I.I.D.
			Labor Tax Rate		
Mean	23.87	20.69	23.80	23.87	23.84
Standard deviation	.10	.04	.08	.06	.15
Autocorrelation	.80	.85	.71	.90	−.04
Correlation with government consumption	.65	−.59	NA	1.00	.10
Correlation with technology shock	.55	−.84	.64	NA	.95
			Ex Ante Capital Tax Rate		
Mean	.00	−.06	.00	.00	.00
Standard deviation	.00	4.06	.00	.00	.00
Autocorrelation	NA	.83	NA	NA	NA
Correlation with government consumption	NA	.33	NA	NA	NA
Correlation with technology shock	NA	.96	NA	NA	NA
			Private Assets Tax Rate		
Mean	1.10	−.88	2.10	−.89	−.51
Standard deviation	53.86	78.56	23.71	47.20	15.93
Autocorrelation	−.01	.02	.01	.01	−.02
Correlation with government consumption	.39	.46	NA	.45	.93
Correlation with technology shock	−.23	.02	−.55	NA	−.31
			Capital Tax Rate with Uncontingent Debt		
Mean	.55	−.42	1.19	−.59	.23
Standard deviation	40.93	30.35	17.67	36.22	12.03
Autocorrelation	−.01	.02	.01	.01	−.02
Correlation with government consumption	.40	.47	NA	.46	.94
Correlation with technology shock	−.24	−.02	−.56	NA	.33

NOTE.—To compute the statistics, we simulated a realization of 4,500 periods and then dropped the first 100 periods. The means and standard deviations are in percentage terms. The NA indicates that the relevant statistic is not well defined.

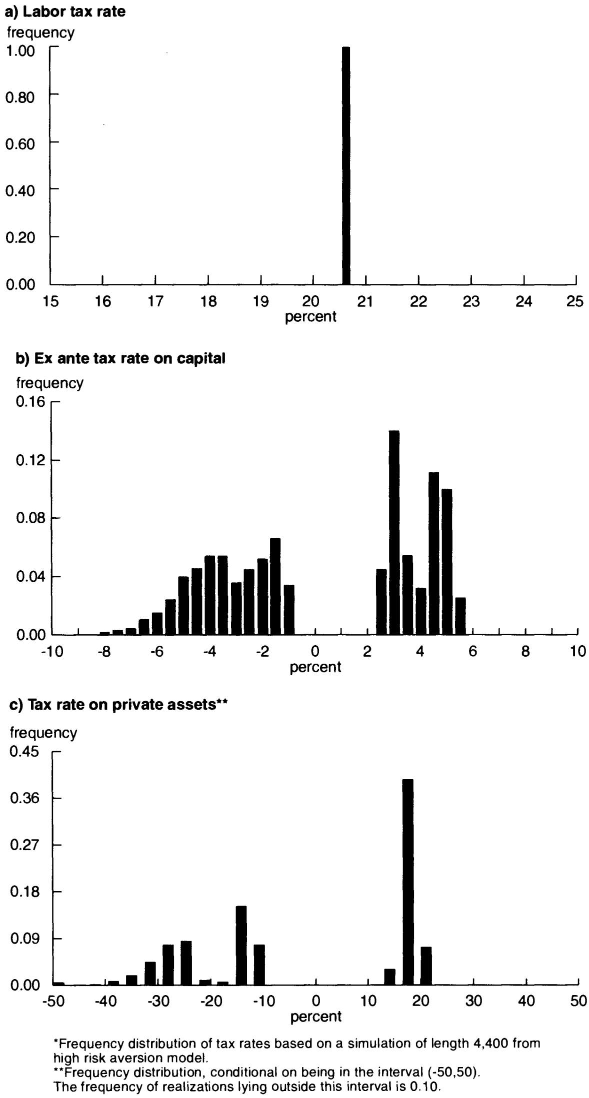
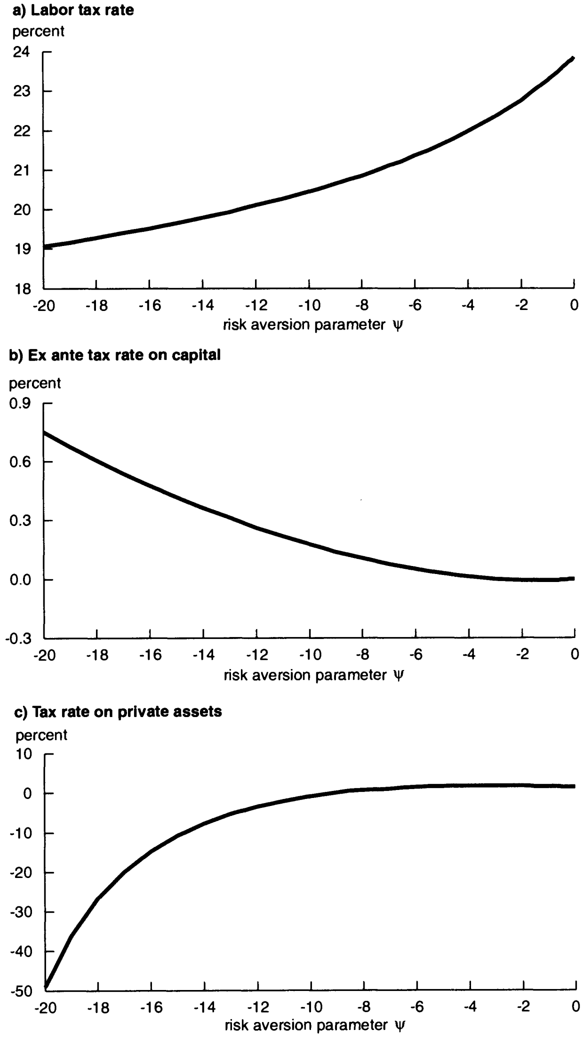


FIG. 1.—Frequency distribution of tax rates in the high risk aversion model*

We have also done a variety of experiments in which we vary other parameters of preferences, technology, and the stochastic processes for shocks. With one notable exception, all the experiments confirm our findings on the mean and variability of optimal tax rates. The exception occurs when shocks are i.i.d., risk aversion is large, and initial debt is at its baseline level. Under these conditions, we find that while the mean of the ex ante capital tax rate is close to zero, its

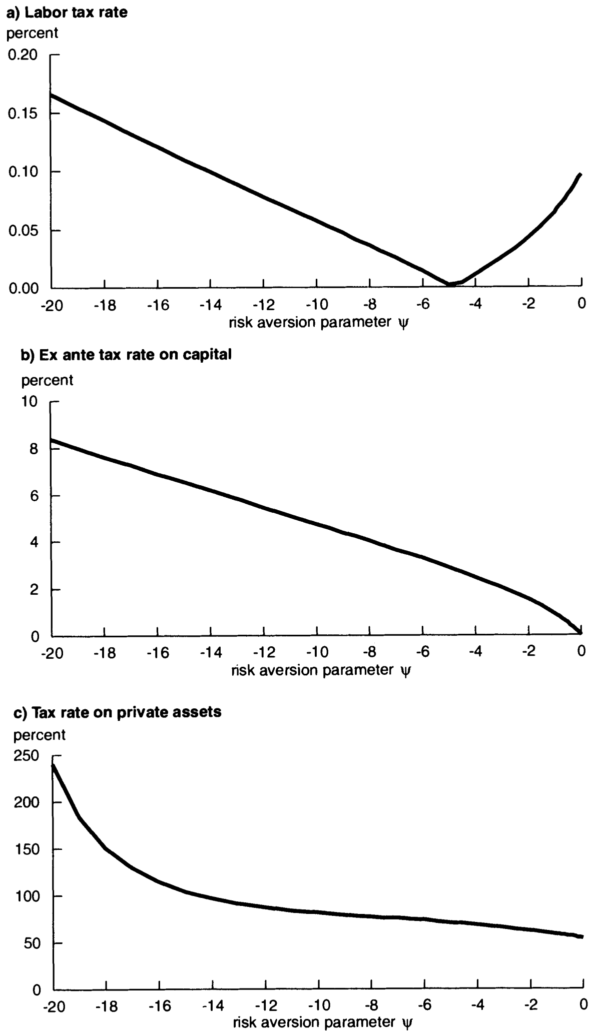


*Mean tax rates computed from simulation of length 4,400 for each value of risk aversion ψ.

FIG. 2.—Mean tax rates*

standard deviation is quite different from zero. For example, when $\psi = -8$, the standard deviation is about 25 percent; when $\psi = -20$, the mean is only 2.7 percent, but the standard deviation is about 70 percent.

Table 2 also illustrates two other features of optimal policies. The labor tax rate is highly persistent when the shocks are highly persistent and close to i.i.d. when the shocks are close to i.i.d. Thus the



*Standard deviation, expressed in percentage terms, computed from simulation of length 4,400 for each value of risk aversion ψ.

FIG. 3.—Standard deviation of tax rates*

labor tax rate inherits the persistence properties of the exogenous shocks. To investigate the robustness of this result, we vary the autocorrelation of the technology and government spending shocks and compute the optimal policies. In figure 4, we plot the autocorrelation of the labor tax rate against the autocorrelations of the technology and the government spending shocks; in each case we fix the other shock at its mean level. Figure 4 shows that, for both the log utility

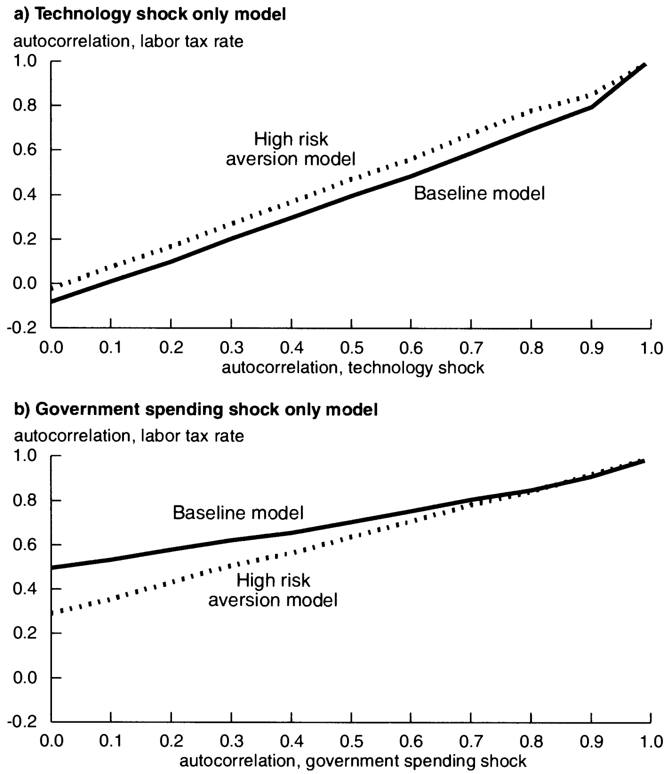


FIG. 4.—Autocorrelation of labor tax rate

and high risk aversion models, the autocorrelation of the labor tax rate rises with the persistence of the shocks. Thus there is no presumption that the Ramsey tax rates on labor should follow a random walk.

We note that this result can be established analytically for the version of our model without capital. Then, from the analogue of (23) and the resource constraint, we see that consumption and employment—and, thus, the tax rate on labor—depend only on the current realization of the exogenous shocks. Thus we can prove that the tax rate on labor inherits the persistence properties of the exogenous shocks. In our quantitative model with capital, the labor tax rate also inherits the persistence properties of the shocks.

Table 2 also illustrates that the properties of the tax rate on capital depend critically on how the Ramsey allocations are decentralized. We report the properties of the capital tax rate under two decentralizations. In one, the capital tax rates are not state-contingent and thus are simply the ex ante tax rates. In the other decentralization, the return on debt is not state-contingent. The statistical properties of the capital tax rates under these two decentralizations are obviously quite different. For example, for the high risk aversion model, the ex ante tax rate is highly persistent and the tax rate with uncontingent debt is serially uncorrelated. Thus our model suggests that, depending on the particular decentralization, the stochastic process for capital tax rates can range anywhere from i.i.d. to nearly a random walk.

Next we study the properties of optimal policies in more detail in figures 5 and 6. In these figures, we plot a segment of two simulations of the high risk aversion model, one with technology shocks only (fig. 5) and one with government spending shocks only (fig. 6). Notice that all the tax rates jump when the underlying shocks change value and are relatively constant otherwise. Notice that the labor tax rate rises both when technology drops and when government spending drops (figs. 5*a* and 6*a*). We find that, for the baseline model, these patterns are reversed, and the reversal in the patterns occurs approximately at $\psi = -4$. This finding suggests that the variance of the labor tax rate should be zero at approximately $\psi = -4$. Recall from figure 3*a* that this is indeed what we found. In figures 5*c* and 6*c*, notice also that when there is a negative innovation to the technology shock or a positive innovation to government consumption, there is a positive innovation in the tax on private assets. The reason is that the tax on private assets performs a shock absorber role. A negative innovation to the technology shock or a positive innovation to government consumption implies a negative shock to the government budget constraint. It is efficient for these shocks to be absorbed mainly by the tax on private assets rather than by changes in the labor tax rate.

We can get an idea of the magnitude of the shock absorber role of the tax on private assets from figure 6*c*. In the figure, we report government spending relative to output, where output comes from the economy with the nonstochastic benchmark policies. When government spending rises from 15.2 percent to 17.5 percent of output, the tax rate on private assets rises from 0 to 300 percent. To further understand the magnitude of the shock absorber role, we regress the innovation in the revenues from the tax on private assets on the innovations in government spending. The regression coefficient for the baseline model is 6.67, and that for the high risk aversion model is 5.49. For both economies, an increase in government spending of 1 percent of (steady-state) output implies that the expected present value of government spending increases approximately 8.06 percent. In the current period, the tax on private assets finances approximately 83 percent of the innovation in this expected value in the

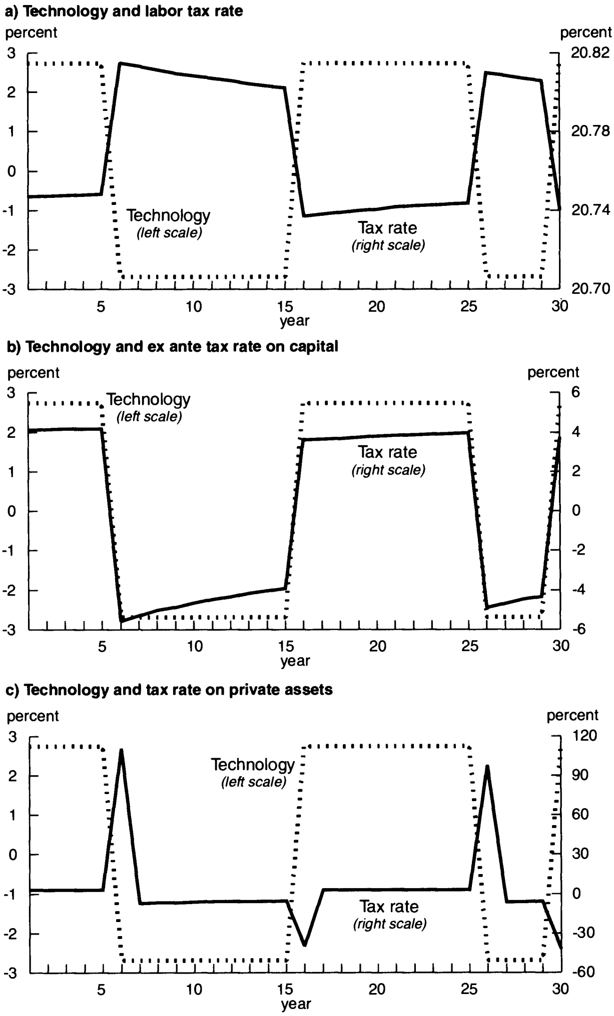


*Figure 5 displays 30 simulated observations from the high risk aversion model with no government spending shocks. Tax rates are in percentage terms, and the technology shock is expressed as $\{\exp[(1-\alpha)z_t]-1\}\times 100$.

FIG. 5.—Technology shock model simulation*

baseline model and finances 68 percent in the high risk aversion model.

Next we investigate the cyclical properties of the Ramsey allocations. We report on these properties for the baseline model; the results for the high risk aversion model are basically the same. We are particularly interested in how these properties compare to those in a benchmark economy in which the taxes are not optimally set. For a

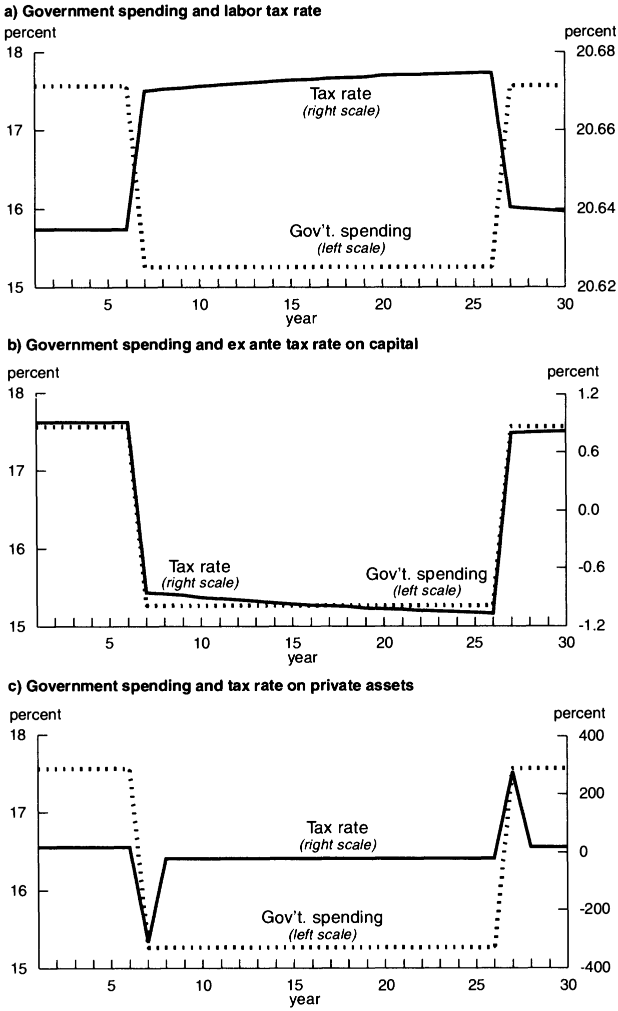


*Figure 6 displays 30 simulated observations from high risk aversion model with no technology shocks. Tax rates are expressed in percentage terms, and government spending is expressed as a percentage of nonstochastic steady-state output.

FIG. 6.—Government spending shock model simulation*

benchmark, we construct a crude approximation of the U.S. tax system. In doing so we have to be mindful of two issues. First, the U.S. tax system has a vast array of taxes as well as transfer payments, whereas our model has only taxes on capital and labor and no transfer payments. Second, in the U.S. economy, tax rates change in response to a variety of shocks, whereas our economy has only two shocks. We construct our crude approximation by considering stochastic processes for the tax rate on labor and the ex ante tax rate on capital of the form

$$\tau_t = a_0 + a_1 z_t + a_2 \tilde{g}_t \tag{51}$$

and

$$\theta_t^e = b_0 + b_1 z_t + b_2 \tilde{g}_t. \tag{52}$$

For the labor tax, we use Barro and Sahasakul's (1983) estimate of the average marginal tax rate. For the ex ante tax rate on capital income, we use Jorgenson and Sullivan's (1981) estimate of the effective corporate tax rate. For the technology shock and the government spending process, we use Christiano's (1988) data. We detrend all variables using a continuous, piecewise-linear trend with a single break in 1969 and obtain the coefficients (a_1, a_2, b_1, b_2) by ordinary least squares. We then set a_0 and b_0 to achieve two objectives. First, the ratio of the means of the tax rate on labor and the ex ante tax rate on capital equal those in the data. Second, in the model the tax revenues generated from the tax rate processes satisfy the government intertemporal budget constraint with an initial debt equal to that in the deterministic economy with the nonstochastic benchmark policies; that is, $R_b(s_0)b_{-1} = 0.20$ (for details, see Chari et al. [1991]). We obtain $a_1 = -0.027$, $a_2 = 0.11$, $b_1 = -0.71$, and $b_2 = 0.52$. The mean levels of the constructed labor tax rate and ex ante capital tax rate are 23.80 and 27.10 percent.

In tables 3 and 4, we report the standard business cycle statistics for our model economy with the Ramsey policies and with our estimated version of the U.S. tax system. (We solve the latter model using an appropriately modified version of the method used to solve the Ram-

TABLE 3

Cyclical Properties of the Baseline Model under the Ramsey Tax System

Variable	Standard Deviation: Percentage	Standard Deviation: Relative to Output	Cross-Correlation with Output at Lag k: $k=-2$	$k=-1$	$k=0$	$k=1$	$k=2$
Output	2.72	1.00	.42	.68	1.00	.68	.42
Consumption	1.69	.62	.66	.75	.78	.45	.20
Investment	6.89	2.54	.24	.55	.96	.68	.46
Hours	1.27	.47	.00	.32	.79	.61	.45
Government spending	3.97	1.46	.05	.09	.12	.08	.04
Productivity	1.87	.69	.60	.76	.91	.57	.30

Note.—Statistics pertain to Hodrick-Prescott filtered data.

TABLE 4

Cyclical Properties of the Baseline Model under the Estimated Tax System

	Standard Deviation		Cross-Correlation with Output at Lag k				
Variable	Percentage	Relative to Output	$k = -2$	$k = -1$	$k = 0$	$k = 1$	$k = 2$
Output	3.03	1.00	.44	.69	1.00	.69	.44
Consumption	1.79	.59	.72	.79	.80	.47	.22
Investment	9.23	3.05	.26	.55	.95	.69	.48
Hours	1.57	.52	.04	.37	.85	.66	.50
Government spending	3.97	1.31	−.03	−.06	−.10	−.09	−.08
Productivity	1.87	.62	.67	.80	.90	.56	.29

Note.—Statistics pertain to Hodrick-Prescott filtered data.

sey problem.) Comparing these tables, we see that the fluctuations in output, consumption, investment, and hours are smaller under the Ramsey policies, and the correlation between government spending and output is higher under the Ramsey policies. The reason for these features is that labor and ex ante capital tax rates under the Ramsey system are smoother than under the estimated system; thus allocations fluctuate less. Tables 3 and 4 also show that the correlation between output and government spending is positive under the Ramsey system but negative under the estimated system. Again, the reason is that the tax rate on labor is much less responsive to shocks under the Ramsey system than under the estimated system. In fact, under the estimated system, when government spending rises, the tax rate on labor rises by so much that employment, and therefore output, actually falls.

B. *Welfare*

Next we compute welfare gains from alternative tax systems relative to benchmark tax systems for a deterministic version and a stochastic version of our model. Our welfare measure is that constant percentage amount by which consumption must be increased in all periods and states in the benchmark economy, while leaving employment unchanged, so as to yield the same utility as under the policy experiment. We begin by considering the balanced-growth path of a benchmark deterministic economy that has government spending and the technology shocks fixed at their mean values and tax rates on labor and capital constant. The ratios of the tax rates on labor and capital in the model are chosen as in our comparison of business cycle statis-

TABLE 5

Percentage Welfare Gains of Alternative Tax Systems Relative to Benchmark Economies

Benchmark Economies	Ramsey	Policy Experiments: Constant Tax Rates — Zero Capital Tax	High Capital Tax
		Log Utility	
Deterministic economy	1.00	.20	.00
Stochastic economy:			
Estimated policy	1.00	.20	.03
Variable policy	1.60	.80	.60
		High Risk Aversion	
Deterministic economy	1.30	−.06	.00
Stochastic economy:			
Estimated policy	1.20	−.02	.03
Variable policy	1.60	.33	.38
		High Initial Debt	
Deterministic economy	5.20	−.70	.00
Stochastic economy:			
Estimated policy	5.10	−.60	.05
Variable policy	6.10	.30	1.00

Note.—The welfare measure is that constant percentage amount by which consumption must be increased in all periods and states in a benchmark economy, while leaving employment unchanged, so as to yield the same utility as under the policy experiment. The benchmark deterministic economy has government spending and technology shocks set to their mean values and has constant tax rates. The benchmark stochastic economy with estimated policy has shocks as in the baseline model and policies estimated from postwar U.S. data. The benchmark stochastic economy with variable policy has shocks as in the baseline model and has policies five times more responsive to technology and government spending shocks than the estimated policies.

tics. That is, they equal the mean of those same ratios in the data, and the levels are chosen so that, along the balanced-growth path, the ratio of government debt to gross national product equals its average in the postwar U.S. economy: 51 percent.

We conduct several experiments and report the results in table 5. In each case, the initial conditions are given by the balanced-growth path of the deterministic economy. In the first set of experiments, we compute the welfare gains from adopting the Ramsey policy for deterministic economies with log utility, high risk aversion ($\psi = -8$), and high initial debt. (Recall that the economy with high initial debt is an attempt to capture some of the consequences of introducing transfers into our setup.) For the model with log utility, the welfare gains are 1 percent of consumption; for the model with high risk aversion, the gains are 1.3 percent; and for the economy with high initial debt, they are 5.2 percent.

We decompose the welfare gains into three sources. To motivate this decomposition, recall that the Ramsey policies can be reasonably characterized as having a negative labor tax rate in period 0 followed by a constant labor tax rate in all subsequent periods together with a large positive tax rate on capital in period 1, followed by a zero tax rate on capital in all subsequent periods. The benchmark deterministic economy has constant positive tax rates on both types of income. Thus, for a deterministic economy, the welfare gains come from three sources: the negative labor tax rate in period 0, the large positive capital tax rate in period 1, and the zero capital tax rate thereafter and in the steady state.

We decompose these welfare gains into these sources as follows. We compute the welfare gain relative to the benchmark policy by first computing welfare under a system in which labor tax rates are constant from period 0 on, keeping the capital tax rate as in the Ramsey system. The welfare gains are indistinguishable from those under the Ramsey system, so we do not report them. Thus the negative period 0 labor tax rate plays a very minor role in the Ramsey plan. Next we compute the welfare gains from a system under which tax rates on capital are zero in all periods from period 1 on and labor tax rates are constant. In table 5, we refer to this system as a *constant tax system with zero capital taxes.* From the table, we see that for our baseline model with log utility, the welfare gains are 0.2 percent. Thus 80 percent of the welfare gains of the Ramsey system come from the large initial tax on capital income, and only 20 percent come from the subsequent and steady-state elimination of capital income taxation.

The results are even more dramatic for the high risk aversion and high-debt economies. Here, switching to a system with zero capital taxes, in all periods including the first period of transition, actually lowers welfare. Of course, from a theoretical perspective, this should not be surprising since the optimal capital tax is nonstationary: a large initial tax and a zero rate thereafter. A single constant tax of zero in all periods misses the large initial tax and thus could be worse than a constant positive tax in all periods.

Next we investigate the welfare gains in stochastic economies. We consider two benchmarks. In both, the policies have the form in (51) and (52). In the estimated policy benchmark, the parameters are obtained from regressions on U.S. data as described above. In the variable policy benchmark, the policies are made more variable by multiplying a_1, a_2, b_1, and b_2 by a factor of five. In stochastic economies, there is a source of welfare gains from the Ramsey policy, in addition to the three sources mentioned above. This source stems from our finding that under the Ramsey system, the labor tax rates and the ex ante capital tax rates are essentially constant and not variable as in our benchmarks.

We find that here, as in the deterministic economy, the negative tax rate in period 0 has insignificant welfare effects. Next we investigate the welfare effects of the high capital tax rate in period 1. Recall that under the Ramsey system, both labor tax rates and capital tax rates are essentially constant after period 1. Thus the difference in the welfare gains between the Ramsey system and the constant tax system with zero capital tax is due to the initial capital tax. Here, as in the deterministic economy, we find that this source is sizable and accounts in the log utility case for 80 percent of the welfare gains from the Ramsey system.

Finally, we investigate the welfare gains from the smoothing of the labor and ex ante capital tax rates under the Ramsey system. One way of isolating these welfare gains is to consider a system that taxes capital and labor at high average rates as in the benchmark economies but does not permit them to fluctuate. In table 5, we refer to this system as one with *constant taxes* and *high capital taxes.* As can be seen from the table, the welfare gain from such a system is 0.03 percent for both the log utility and high risk aversion parameterizations. This welfare gain is small. One reason for this small welfare gain could be that the estimated policies are not that variable to start with. To investigate this possibility, we consider benchmark economies in which labor and ex ante capital tax rates are five times as volatile as the estimated policies. For such benchmarks, we find sizable welfare gains from eliminating fluctuations in their tax rates. For example, for the log utility case, the gain in welfare from this source accounts for almost 40 percent of the Ramsey gains.

VIII. Remarks on Scope

We have studied an economy in which the government uses capital and labor income taxation to raise revenues and have shown how the problem of solving the Ramsey equilibrium reduces to the simpler problem of solving for the Ramsey allocations. A wide variety of other tax systems lead to the same Ramsey allocation problem. For example, consider a tax system that includes consumption taxes as well as labor and capital income taxes. It can be shown that the Ramsey allocations can be supported by a tax system that uses any two of the three types of tax instruments. Thus, for example, the Ramsey allocations can be supported by consumption and capital income taxes only, consumption and labor income taxes only, or capital and labor income taxes only.

To illustrate this point, we consider an economy with consumption and labor income taxes. The consumer's intratemporal first-order condition is

$$-\frac{U_l(s^t)}{U_c(s^t)} = \frac{1 - \tau(s^t)}{1 + \gamma(s^t)} F_l(s^t), \tag{53}$$

where γ is the tax rate on consumption. The consumer's first-order condition for capital is

$$\sum_{s_{t+1}} q(s^{t+1}) \frac{1 + \gamma(s^t)}{1 + \gamma(s^{t+1})} [1 - \delta + F_k(s^{t+1})] = 1. \tag{54}$$

The analogue of proposition 3 for this economy is that, for $t \geq 1$,

$$\begin{aligned} &\sum_{s_{t+1}} q(s^{t+1}) \frac{1 + \gamma(s^t)}{1 + \gamma(s^{t+1})} [1 - \delta + F_k(s^{t+1})] \\ &\quad = \sum_{s_{t+1}} q(s^{t+1})[1 - \delta + F_k(s^{t+1})]. \end{aligned} \tag{55}$$

For reasons analogous to those in proposition 2, this economy has an indeterminacy in the consumption tax rates and the debt policy. One way of supporting the optimal allocations is to make the consumption taxes not contingent on the current state. For such a decentralization, (55) implies that, for $t \geq 1$, all the consumption tax rates are equal. This result is a generalization of well-known results on uniform taxation (see Atkinson and Stiglitz 1972).

Clearly, the detailed implications for tax rates depend on the particulars of the tax system chosen. In contrast, the theory has unambiguous implications about the relation between marginal rates of substitution and marginal rates of transformation. For example, the central implication of proposition 3 is that, for $t \geq 1$,

$$1 - \sum_{s_{t+1}} q(s^{t+1})[1 - \delta + F_k(s^{t+1})] = 0. \tag{56}$$

That is, distorting the consumer's intertemporal first-order condition is not optimal. In this paper, we have chosen to focus on capital and labor income taxation to make our work comparable to the literature.

IX. Conclusions

We have investigated here the quantitative properties of optimal fiscal policy in a standard business cycle model. We have found that the ex ante tax on capital income is approximately zero, that the labor tax rate fluctuates very little and inherits the serial correlation properties of the exogenous shocks, that the tax on private assets fluctuates a great deal, and that the welfare gains to optimal taxation come primarily from the transition phase of high capital income taxation.

In the model, the tax on private assets plays the role of a shock absorber. To see this, consider decentralizing a Ramsey allocation with state-uncontingent capital taxes. In such a decentralization, the fluctuations in the tax on private assets arise from the variations in the real payments on government debt. In a Ramsey equilibrium, the government structures these payments in order to insure itself from having to sharply change labor tax rates when the economy is hit by shocks. In this sense, state-contingent debt is a form of insurance purchased by the government from consumers.

One can imagine a variety of reasons why issuing and enforcing these types of insurance claims would be difficult. One can also imagine forces that limit the state contingency of capital tax rates. Extreme cases to study would be economies in which both real debt and capital tax rates are restricted to be state-uncontingent. We conjecture that in such economies, labor tax rates will be more persistent than the underlying shocks. Another avenue of research is the role of inflation in converting nominal uncontingent claims on the government into real state-contingent claims. Exploring this avenue may also lead to insights into the role of optimal monetary policy. We are currently exploring both of these avenues (see Chari, Christiano, and Kehoe 1993).

An interesting finding is that only a small fraction of the welfare gains come from smoothing tax rates and eliminating capital income taxation. Rather, most of the welfare gains come from the high taxation of capital in the transition period. In this sense, the temptation to renege on the previously chosen policies is large once the transition phase has passed. Thus the time inconsistency problem is quantitatively severe. Hence, implementing policies of the type described here without strong safeguards against reneging in the future is likely to prove counterproductive.

Finally, our model abstracts from a variety of issues including income distribution, heterogeneity, externalities, money, and growth. (For some recent work on fiscal policy in models with money and growth, see Cooley and Hansen [1992] and Jones et al. [1993].) Instead, the model focuses attention on intertemporal efficiency. We think that the forces driving our results will be present in more elaborate dynamic models.

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