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On the Optimal Taxation of Capital Income*

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We show that in models in which labor services are supplied jointly with human capital, the Chamley and Judd result on zero capital income taxation in the limit extends to labor taxes as long as accumulation technologies are constant returns to scale. Moreover, for a class of widely used preferences, consumption taxes are zero in the limit as well. However, we show by the construction of two examples that these results no longer hold for certains types of restrictions on tax rates or if there are profits generated. *Journal of Economic Literature* Classification Numbers: E62, H21.

1. INTRODUCTION

One of the most interesting and relevant topics in public finance concerns the optimal choice of tax rates. This question has a long history in economics beginning with the seminal work of Ramsey [24]. In that paper, Ramsey characterized the optimal levels for a system of excise taxes on consumption goods. He assumed that the government's goal was to choose these taxes to maximize social welfare subject to the constraints it faced.

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These constraints were assumed to be of two types. First, a given amount of revenue was to be raised. Second, Ramsey understood that whatever tax system the government adopted, consumers and firms in the economy would react in their own interest through a system of (assumed competitive) markets. This observation gives rise to a second type of constraint on the behavior of the government—it must take into account the equilibrium reactions by firms and consumers to the chosen tax policies. This gives what has become known as a "Ramsey Problem": Maximize social welfare through the choice of taxes subject to the constraints that final allocations must be consistent with a competitive equilibrium with distortionary taxes and that the given tax system raises a pre-specified amount of revenue.

Ramsey's insights have been developed extensively in the last few years (see the excellent survey in Auerbach [2]) as applied to optimal commodity taxation. A parallel literature has concentrated on optimal taxation of factor income in dynamic settings. Contributions to this literature include Atkinson and Sandmo [1], Chamley [6, 7], Judd [14, 16], Stiglitz [26, 27], Barro [4], King [17], Lucas [20], Yuen [28], Chari, Christiano and Kehoe [8], Zhu [29], Bull [5] and Jones, Manuelli, and Rossi [13]. Most of this literature discusses the setting of income taxes so as to maximize the utility of an infinitely lived representative consumer with perfect foresight subject to competitive equilibrium behavior and the need to fund a fixed stream of government expenditures. (Exceptions are the OLG group.)

The most startling finding of the literature on factor income taxation is that the optimal tax rate on capital income—a stock—is zero in the long run, while the optimal tax rate on labor—a pure flow—is positive. This was first exposited in Chamley and Judd in the context of simple single sector models of exogenous growth and has been shown to hold in cases with steady state, endogenous growth as well. Refinements to the stochastic case have been explored in King [17], Chari, Christiano and Kehoe [8], and Zhu [29].

Realistically, labor services are not a flow, however. In practice, labor is a combination of human capital, a stock, and worker's time, a flow. Thus, the relevant factor, termed effective labor in the growth literature, has both a stock and a flow component. Moreover, from the point of view of the theory of optimal taxation it is reasonable to assume that the government cannot impose separate tax rates on these two inputs. This means that existing results on optimal taxation that apply to the pure stock and pure flow cases are of limited value. Thus, our first goal in this paper is to determine how the inclusion of this new composite factor (part stock and part flow) affects optimal taxation. A second goal is to explore the sensitivity of the optimal long- run tax rate on capital income to changes in both the set of feasible instruments and technology. First, we ask why it is that capital income is treated so differently than other sources of income (e.g., labor income) in the Chamley–Judd set-up. When we introduce effective labor in a setting in which human capital is accumulated with a constant returns to scale technology, we see that there is nothing special about capital income—labor taxes (taxes on effective labor) are zero in the long-run as well. Intuitively, with constant returns to scale capital accumulation technologies, no-arbitrage conditions insure the absence of profits for the planner to tax in the long-run. Moreover, under a restricted, but widely used, class of preferences, all taxes (capital, labor and consumption) can be chosen to be zero in the limit.

We construct two examples to show the limits of the Chamley–Judd result. In the first, to confirm the intuition above concerning the role of profits in determining optimal long-run tax rates, we consider an example in which inelastic labor supply gives rise to profits which the planner cannot fully tax. We show that in this model, capital income tax rates are positive in the limit.

The qualitative nature of the Chamley–Judd results is changed not only by altering the technological assumptions but also by altering the feasible set of tax policies. All dynamic factor taxation exercises impose constraints both on the sources of income that can be taxed and on the path of permissible tax rates. Without these restrictions, optimal taxation degenerates to lump-sum taxes where the government confiscates the initial capital stock. Presumably, these restrictions are based on informational or political constraints which are not explicitly modeled. Other plausible sets of restrictions on tax policies can result in optimal non-zero limiting tax rates. In a second example, we show that if the planner cannot distinguish between different qualities of labor (i.e., it is forced to use the same tax rate for both skilled and unskilled labor income), then the optimal capital income tax rate is positive in the limit. To explore whether the qualitative result of positive limiting taxes is quantitatively significant, we solve a numerical version of this second example using the empirical work of Kwag and McMahon [18] to choose parameter values. We find that the limiting tax rate on capital income is significantly different from zero (it is about 7% for a variety of settings), although lower than the tax rate on pure labor (which is about 22%).

Taken as a whole, our findings force a reconsideration of the Chamley–Judd result in two important respects. First, our result that all taxes can be chosen to be zero for plausible preferences and technologies suggests that the fundamental characteristic of optimal dynamic policies is the timing of taxes and not differences among types of factor taxation. In a broad class of models, optimal taxation problems have the characteristic that the government taxes at a high rate in the initial periods to build up a surplus which it then lives off forever. Second, realistic changes in either the constraints on tax policies or technology can result in positive long-run tax rates.

Throughout, we follow Chamley and Judd and analyze models in which the economy converges to a steady state. Extensions of our results to settings in which the growth rate is positive (either because of exogenous technological change or endogenous growth) are straightforward.

The remainder of the paper is organized as follows. In section 2, we give the extension of the Chamley–Judd result to models including human capital and show that in reasonable cases, both labor income taxes and consumption taxes are also zero. In Section 3, we develop a general formulation for studying the limiting behavior of the tax rate on capital income and give the examples—both the theory as well as the numerical analysis—described above to show how they result in a non-zero tax rate on capital income in the long run. Finally, section 4 discusses extensions and offers some concluding comments.

2. IS PHYSICAL CAPITAL SPECIAL?

We start by describing a generalized version of the model analyzed by Judd [14] and Chamley [7]. To explore the question of whether capital is special, it is necessary to expand the model. We do this by adding human capital that—when combined with raw labor and market goods—is used to supply effective labor. In this formulation, effective labor has both a stock and a flow component. Because of this, the standard intuition that limiting tax rates on stocks should be zero while those on flows should be non-zero does not apply.[1]

This expanded setting allows us to address one of the questions posed in the introduction, namely whether there is something special about capital income. Specifically, we show that if three conditions are satisfied:

(i) there are no profits from accumulating either capital stock,

(ii) the tax code is sufficiently rich, and,

(iii) there is no role for relative prices to reduce the value of fixed sources of income,

then, both capital and labor income taxes can be chosen to be zero in the steady state. Moreover, if preferences satisfy an additional (but standard) condition, *all* taxes can be chosen to be asymptotically zero.

[1] It is easy to extend the results presented here to models in which the growth is endogenous. (See Jones, Manuelli, and Rossi [13] and Bull [5].)

We consider the simplest infinitely lived agent model consistent with the presence of both human and physical capital. We assume that there is one representative family that takes prices and tax rates as given. Their utility maximization problem is given by

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t, 1-n_{mt}-n_{ht})$$

$$\begin{aligned} \text{s.t. } & \sum_{t=0}^{\infty} [(1+\tau_t^c)c_t + x_{ht} + (1+\tau_t^m)x_{mt} + x_{kt} - (1-\tau_t^n)w_t z_t \\ & \qquad - (1-\tau_t^k)r_t k_t - (1+\tau_t^c)T_t]p_t \leqslant b_0, \\ & k_{t+1} \leqslant (1-\delta_k)k_t + x_{kt}, \\ & h_{t+1} \leqslant (1-\delta_h)h_t + G(x_{ht}, h_t, n_{ht}), \\ & \quad z_t \leqslant M(x_{mt}, h_t, n_{mt}),^2 \end{aligned} \tag{P.1}$$

where c_t is consumption, n_{jt}, $j=h, m$, is the number of hours allocated to human capital formation and market activities respectively, x_{ht} is the amount of market goods used in the production of human capital, x_{kt} represents market goods used in producing new capital goods, x_{mt} is the amount of market goods used in the provision of 'effective labor,' k_t and h_t are the stocks of physical and human capital available at the beginning of period t, z_t is effective labor allocated to the production of market goods, T_t are transfers received from the government that the household takes as given, and b_0 is the initial stock of government debt. We assume that both G and M are homogeneous of degree one in market goods ($x_{jt}, j=h, m$) and human capital, and C^2 with strictly decreasing (but everywhere positive) marginal products of all factors. The household takes as given the price of consumption at time t in terms of numeraire, p_t, as well as the tax rates, $\tau_t^j, j=k, n, m, c$. The standard non-negativity constraints apply.

The idea that the accumulation of human capital is an internal activity that uses market goods as well as human capital and labor appears in Heckman [12] and is relatively standard in the labor economics literature. Our formulation has some popular specifications as special cases. For example, Heckman assumes $G(x, h, n)=F(x, hn)$ with F homogeneous of degree one, while the specification $M=hm(n)$ and $G=hg(n)$ is used in a large number of papers in the endogenous growth literature (see Lucas [19] and Bull [5] for example).

[2] To make explicit the sense in which this model is a generalization of the Cass-Koopmans model, simply note that by setting $G\equiv 0$, $M(x, h, n_m)=n_m$, and $h_0=0$, we obtain the standard growth model underlying the analysis in Judd and Chamley.

The necessary conditions for an interior solution of the consumer's maximization problem are given by

$$p_t = \beta^t \frac{u_c(t)}{1+\tau_t^c} \frac{(1+\tau_0^c)}{u_c(0)} \qquad t=0, 1, ... \tag{1.a}$$

$$u_l(t) = (1-\tau_t^n)\, w_t M_n(t) \frac{u_c(t)}{1+\tau_t^c} \qquad t=0, 1, ... \tag{1.b}$$

$$u_l(t) = \frac{u_c(t)}{1+\tau_t^c} \frac{G_n(t)}{G_x(t)} \qquad t=0, 1, ... \tag{1.c}$$

$$p_t = p_{t+1}[1-\delta_k + (1-\tau_{t+1}^k) r_{t+1}] \qquad t=0, 1, ... \tag{1.d}$$

$$(1+\tau_t^m) = (1-\tau_t^n) w_t M_x(t) \qquad t=0, 1, ... \tag{1.e}$$

$$p_t/G_x(t) = p_{t+1} \left\{ \frac{(1-\delta_h + G_h(t+1))}{G_x(t+1)} + (1-\tau_{t+1}^n) w_{t+1} M_h(t+1) \right\}$$
$$t=0, 1, ..., \tag{1.f}$$

in addition to the constraints on problem (P.1). Using the first order conditions and the assumption that G and M are homogeneous of degree one it is possible to show that in "equilibrium" the consumer's budget constraint can be greatly simplified.

Specifically, consider the term $\sum_{t=0}^{\infty} p_t [x_{kt} - (1-\tau_t^k) r_t k_t]$. Using the law of motion for k_t we can rewrite this sum as

$$\sum_{t=0}^{\infty} p_t [x_{kt} - (1-\tau_t^k) r_t k_t] = p_0[(1-\delta_k) + (1-\tau_0^k) r_0] k_0$$
$$+ \sum_{t=1}^{\infty} k_t \{ p_t [1-\delta_k + (1-\tau_t^k) r_t] - p_{t-1} \}.$$

However, the second term on the right hand side is zero given (1.d). Next, consider the term $\sum_{t=0}^{\infty} p_t [x_{ht} + (1+\tau_t^m) x_{mt} - (1-\tau_t^n)\, w_t M(x_{mt}, h_t, n_{mt})]$. Given the law of motion for the stock of human capital (which holds as an equality) and the assumption that G and M are linearly homogeneous in (x, h), it follows that this term is given by

$$\sum_{t=0}^{\infty} p_t \Bigg[((1+\tau_t^m) - (1-\tau_t^n)\, w_t M_x(t)) x_{mt}$$
$$+ \frac{h_{t+1} - (1-\delta_h + G_h(t))}{G_x(t)} h_t - (1-\tau_t^n)\, w_t M_h(t) h_t \Bigg].$$

Using (1.e) and rearranging we obtain that the infinite sum is given by

$$-p_0\left[\frac{1-\delta_h+G_h(0)}{G_x(0)}+[(1-\tau_0^n)\,w_0 M_h(0)]\,h_{0t}\right]$$
$$+\sum_{t=1}^{\infty} h_t\left\{\frac{p_{t-1}}{G_x(t-1)}-p_t\left[\frac{(1-\delta_h+G_h(t))}{G_x(t)}+(1-\tau_t^n)\,w_t M_h(t)\right]\right\}.$$

The second term in this expression equals zero from (1.f). Thus, in equilibrium, the consumer's budget constraint is given by

$$\sum_{t=0}^{\infty} p_t[(1+\tau_t^c)\{c_t-T_t\}]=p_0\left\{[1-\delta_k+(1-\tau_0^k)r_0]k_0\right.$$
$$\left.+\left[\frac{1-\delta_h+G_h(0)}{G_x(0)}+(1-\tau_0^n)\,w_0 M_h(0)\right]h_0\right\}+b_0. \tag{2}$$

The right hand side is simply the value of wealth at time zero while the left hand side does not include any terms that depend on x_{jt}, k_t or h_t, $j=m, h, k$. The reason for this is simple: Since the activity "capital income" and the activity "labor income" display constant returns to scale in reproducible factors, their "profits" cannot enter the budget constraint in equilibrium. If they are positive, the scale of this activity can be increased without any cost to the consumer and, conversely, if "profits" are negative the activity should be eliminated.

The representative firm rents capital and effective labor and it is subject to no taxes. Profit maximization implies

$$r_t=F_k(k_t, z_t) \tag{3.a}$$

$$w_t=F_z(k_t, z_t). \tag{3.b}$$

The Ramsey problem for this economy can be described as maximizing the welfare of the representative family given feasibility, the government's budget constraint and the first order conditions from both the household's and the firm's maximization problems as well as the household's budget constraint.[3] Using the method described in Lucas and Stokey [21], this problem can be considerably simplified. The basic idea is that—whenever it is possible—the first order conditions should be used as *defining* prices and tax rates given an allocation. Hence, these conditions, along with the

[3] In this case, given that for each sequence of tax rates, the problems faced by consumers and firms are concave, it is appropriate to impose the first order conditions. For nonconvex cases, see Mirrlees [22 and 23] and Grossman and Hart [11].

prices and tax rates as choice variables, need not be explicitly included in the planner's problem.

We first assume that taxes at time zero $(\tau_0^c, \tau_0^n, \tau_0^k, \tau_0^m)$ are given and equal to zero and normalize p_0 to equal one. Next, (1.a) defines p_t, (1.b) determines τ_t^n, (1.c) can be used to compute τ_t^c, (1.d) to calculate τ_t^k and (1.e) to set τ_t^m. This process leaves (1.f) as a condition that must be imposed. The reason for having to include this extra constraint is simple: We restricted the tax code to impose a tax on the output of the effective labor activity (τ_t^n) and this tax affects both the static choice of labor supply (n_{mt}) and the dynamic choice of human capital (h_t). It is then necessary to guarantee that—given an allocation—the tax τ_t^n from (1.b) and (1.f) coincide. Imposing this equality is equivalent to requiring $\phi(t)=0$, where

$$\begin{aligned}\phi(t) &= \phi(v_{t-1}, v_t)\\ &= u_l(t-1)\, G_n(t) - \beta u_l(t)\, G_n(t-1)\left\{1-\delta_h + G_h(t) + G_n(t)\frac{M_h(t)}{M_n(t)}\right\},\end{aligned}$$

where $v_t = (c_t, n_{ht}, n_{mt}, x_{ht}, h_t, x_{mt})$. In addition, it is necessary to impose (2). The planner's problem is then,

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t, 1-n_{ht}-n_{mt})$$

$$\begin{aligned}\text{s.t.} \sum_{t=0}^{\infty} \beta^t[u_c(t)(c_t - T_t)] - W_0 &= 0 \quad (\lambda)\\ F(k_t, M(x_{mt}, h_t, n_{mt})) + (1-\delta_k)\, k_t - k_{t+1} - x_{ht} - x_{mt} - c_t - g_t &= 0 \quad (\beta^t \mu_{1t})\\ (1-\delta_h) h_t + G(x_{ht}, h_t, n_{ht}) - h_{t+1} &= 0 \quad (\beta^t \mu_{2t})\\ \phi(v_{t-1}, v_t) &= 0 \quad (\beta^t \eta_t).\end{aligned} \tag{P.2}$$

where the first constraint is equation (2) after all prices have been substituted out using (1) and (3),

$$W_0 = u_c(0)\left\{b_0 + (1-\delta_k + F_k(0))k_0 + \left[\frac{(1-\delta_h + G_h(0))}{G_x(0)} + F_z(0)\, M_h(0)\right] h_0\right\},$$

and the symbols in parentheses indicate the Lagrange multipliers for each constraint. Note that the first constraint is the consumer's budget constraint after prices have been substituted out. This constraint and feasibility guarantee that the government's budget constraint is satisfied.

The first constraint is similar to the objective function in the sense that they are both discounted infinite sums of terms. Thus, given the Lagrange multiplier, λ,—which, of course, is endogenous—it is possible to rewrite (P.2) as

$$\max \sum_{t=0}^{\infty} \beta^t W(c_t, n_{ht}, n_{mt}, T_t; \lambda) - W_0 \tag{P.3}$$

subject to the "flow" constraints from (P.2), where

$$W(c, n_h, n_m, T; \lambda) \equiv u(c, 1 - n_h - n_m) + \lambda u_c(c - T).$$

The first order conditions for this problem evaluated at the steady state are

$$W_c^* = \mu_1^* - \eta^*(\phi_c^* + \beta\phi_{\underline{c}}^*) \tag{4.a}$$

$$W_{n_h}^* = -\mu_2^* G_n - \eta^*(\phi_{n_h}^* + \beta\phi_{\underline{n}_h}^*) \tag{4.b}$$

$$W_{n_m}^* = -\mu_1^* F_z^* M_n^* - \eta^*(\phi_{n_m}^* + \beta\phi_{\underline{n}_m}^*) \tag{4.c}$$

$$0 = \mu_1^* - \mu_2^* G_x^* - \eta^*(\phi_{x_h}^* + \beta\phi_{\underline{x}_h}^*) \tag{4.d}$$

$$\mu_1^*(F_z^* M_x^* - 1) + \eta^*\phi_{x_m}^* = 0 \tag{4.e}$$

$$1 = \beta(1 - \delta_k + F_k^*) \tag{4.f}$$

$$1 = \beta(1 - \delta_h + G_h^*) + \beta\frac{\mu_1^*}{\mu_2^*} F_z^* M_h^* + \frac{\beta\eta^*}{\mu_2^*}(\phi_h^* + \beta\phi_{\underline{h}}^*) \tag{4.g}$$

along with the constraints from (P.2). To simplify notation we use the shorthand notation

$$\phi_x \equiv \frac{\partial\phi}{\partial x_t}(t) \qquad \text{and} \qquad \phi_{\underline{x}} \equiv \frac{\partial\phi}{\partial x_t}(t+1).$$

(a) *Asymptotic Labor Taxes*

The result of Judd and Chamley that capital taxes are zero in the limit follows directly from an evaluation of these first order conditions. Consider (1.d) and (4.f). From (1.d) evaluated at the steady state, it follows that $1 = \beta[1 - \delta_k + (1 - \tau_\infty^k) F_k^*]$. This condition and (4.f) directly imply that $\tau_\infty^k = 0$. Next we show that labor income taxes are zero as well.

PROPOSITION 1. *Let* (P.2′) *be the maximization problem* (P.2) *without the constraint* $\phi(t) = 0$. *Assume that both the solution to* (P.2) *and* (P.2′) *converge to* a *unique steady state. Then,* $\tau_\infty^n = 0$ *and* $\tau_\infty^m = 0$.

Remark. The assumption of convergence to a unique steady state is standard in the literature. It was implicitly used in our proof of the result that capital income taxes are zero asymptotically.

Proof. We will show that there is a solution to the set of equations (4) with $\eta^* = 0$ and that at that solution, $\tau^n_\infty = 0$.

The solution to the steady state of (P.2) when $\eta^* = 0$ is given by

$$
\begin{aligned}
W_n^* + W_c^* F_z^* M_n^* &= 0 \\
G_x^* &= \frac{G_n^*}{F_z^* M_n^*} \\
1 &= F_z^* M_x^* \\
1 &= \beta(1 - \delta_k + F_k^*) \\
1 &= \beta(1 - \delta_h + G_h^* + G_x^* F_z^* M_h^*) \\
\delta_k k^* &= x_k^* \\
\delta_h h^* &= G(x_h^*, h^*, n_h^*) \\
c^* + x_m^* + x_k^* + x_h^* + g &= F(k^*, M(x_m^*, h^*, n_m^*)),
\end{aligned}
$$

where we use the result that $W^*_{n_m} = W^*_{n_h} = W_n$, *and* $\mu_1^*/\mu_2^* = G_x^*$. To show that this is the solution to (P.2), note the above system of equations characterize the steady state conditions for (P.2′). The key property is that the steady state version of the condition $\phi(t) = 0$ (in this case it is given by $1 = \beta(1 - \delta_h + G_h^* + G_x^* F_z^* M_h^*)$ is automatically satisfied—even though it is not imposed—by the solution to (P.2′). This follows from the Euler equation for the optimal choice of human capital in (P.2′). Thus, the steady state solution of (P.2′) solves the system of equations (4) when coupled with $\eta^* = 0$. Since the solution is assumed unique, it follows that this is the unique steady state corresponding to (P.2).

From (4.e) and (1.e) it follows that $1 + \tau^m_\infty = 1 - \tau^n_\infty$. Hence, if $\tau^n_\infty = 0$, then, $\tau^m_\infty = 0$ as well. From the steady state equations, it follows that in any solution to the planner's problem, it must be the case that, $G_x^* = G_n^*/(F_z^* M_n^*)$. On the other hand, (1.b) and (1.c) imply that $(1 - \tau^n_\infty)\, G_x^* = G_n^*/(F_z^* M_n^*)$. These two conditions imply that $\tau^n_\infty = 0$. ∎

(b) *Asymptotic Properties of Consumption Taxes*

The previous proposition shows that $\tau^n_\infty = \tau^m_\infty = \tau^k_\infty = 0$. However, in general, this implies $\tau^c_\infty \neq 0$. To see this use (1.b) and the first order conditions in the proof above to get

$$1 + \tau^c_\infty = \frac{u_c^*}{u_l^*} F_z^* M_n^*.$$

From the planner's problem,

$$F_z^* M_n^* = -W_n^* / W_c^* = \frac{u_l^* + \lambda u_{lc}^* c^*}{u_c^*(1+\lambda) + \lambda u_{cc}^* c^*}.$$

Hence,

$$1 + \tau_\infty^c = \frac{u_l^* u_c^* + \lambda u_{lc}^* u_c c^*}{u_c^* u_l^* + \lambda[u_c^* u_l^* + u_{cc}^* u_l^* c^*]}$$

There are two possibilities: either $\lambda = 0$ in which case, $\tau_t^c = 0$ and the solution is first best, or $\lambda \neq 0$ in which case, $\tau_\infty^c = 0$ if and only if

$$u_{lc}^* u_c^* c^* = u_c^* u_l^* + u_{cc}^* u_l^* c^*. \tag{5}$$

In general, (5) will not be satisfied. However, there is an interesting class of functions that is consistent with this condition. It is straightforward to verify that if $u(\mathrm{c}, l)$ is given by

$$u(c, l) = \begin{cases} \dfrac{c^{1-\sigma}}{1-\sigma} v(l) & \text{if} \quad \sigma > 0 \quad \sigma \neq 1, \\ \ln(c) + v(l) & (\text{if} \quad \sigma = 1), \end{cases}$$

(5) holds.

Although a narrow class of functions from a theoretical point of view, this class includes many of the functional forms used in applied work on optimal taxation as a special case[4]. In addition, a subset of this class contains the class of functions that are necessary for the economy to have a balanced growth path. It follows that in endogenous growth models that satisfy our technological constant-returns-to-scale assumption and have a balanced growth path all taxes must be zero in the long run.

Note that in this case, since all taxes are zero in the long run, it follows that the government must raise revenue in excess of expenditures in the initial periods. More precisely, since the long run interest rate is $(\beta^{-1} - 1)$ the steady state level of government assets (net claims on private income) is b, where b is defined by $(\beta^{-1} - 1)b = g + T$.

As can be seen from the proof, our zero tax results are driven by zero profit conditions. Zero profits follow from the assumption of linearity in the accumulation technologies. In particular, if either G or M violate this assumption, $\tau_\infty^n \neq 0$. (Additionally, if physical capital accumulation is subject to decreasing returns, $\tau_\infty^k \neq 0$.)

[4] Exceptions include Judd [15] and Auerbach and Kotlikoff [3].

In addition, there are two other features of the model that are essential for the result that taxes vanish asymptotically. The reader can verify that if transfers had been fixed in "before tax" levels of consumption (i.e., T_t enters the budget constraint rather than $(1+\tau_t^c)T_t$), Proposition 1 does not hold unless the limiting value of transfers is zero. The reason for this is simple: If transfers are not fixed in terms of consumption, it is possible for the planner to affect their value at time zero (the planner would like to make transfers as small as possible) by manipulating relative prices. In this example it is possible to show that $\tau_\infty^n \neq 0$. Thus, the first essential condition is that transfers are fixed in terms of after tax consumption. The second essential condition is that the tax code is sufficiently rich. It can be verified that if (5) does not hold and the planner is constrained to set $\tau_t^c = 0$ for all t, then $\tau_\infty^n \neq 0$.

(c) *Alternative Choices of Tax Instruments*

Our results were derived under a particular specification of the set of tax instruments available to the planner. Alternative specifications of this set of instruments are possible. At one extreme we could add taxes on all goods to the list of instruments. That is, we could add taxes on both investment in physical capital and market goods used for the production of human capital, x_{kt} and x_{ht}, respectively. As it turns out, the addition of these extra taxes is superfluous since, even with this expanded set of instruments, the planner's problem and the resulting optimal allocation remain unchanged. It is also true that other, smaller sets of instruments give rise to the same supportable allocations. For example, starting from the set of taxes used in (b), we can drop the tax on x_{mt}, market goods used in the production of effective labor, and replace it by a tax on x_{ht}, market goods used in the production of human capital, and none of the results except the obvious ones about the two taxes changes. Moreover, the set of sustainable allocations—not just the optimal allocation—is invariant to this change. This is evidence of a degree of indeterminacy in the choice of the tax instruments that are used to support the optimal allocation in these types of problems.

As a further example of the effects of this redundancy, our derivation of the zero tax result on capital income assumes that the tax rate on investment in physical capital is zero. In fact, the planner's problem uniquely determines the asymptotic rate of return on investment in physical capital. In the steady state, the rate of return is $[1-\delta_k+(1-\tau^k)F_k/(1+\tau^{x_k})]$, where τ^{x_k} is the tax rate on market goods used in the production of new capital, i.e., investment. From the planner's problem, it follows that this rate of return is given by $[1-\delta_k+F_k]$. Thus, any combination of τ^k and τ^{x_k} satisfying $(1-\tau^k)/(1+\tau^{x_k})=1$ will implement the planner's allocation. One possible combination, used by Chamley–Judd, is $\tau^k=\tau^{x_k}=0$. Of course, any positive tax rate on capital income can also be supported by choosing the appropriate subsidy on investment. This should not be interpreted as implying that the zero taxation result is vacuous. Rather, the rate of return argument given above implies that the *net* rate of taxation on capital income must be zero in the limit and any combination where the inputs are subsidized and the outputs are taxed appropriately will satisfy this.

Another, more complicated, example of this phenomenon concerns the asymptotic tax rate on labor in the model as considered in (b). We have assumed that the tax rate on x_h is zero. Given this assumption, it follows that the limiting value of τ^n is zero. In fact, the planner's problem solution uniquely determines the rate of return to investing in human capital. It can be shown that the private rate of return depends on the ratio, $(1-\tau^n)/(1+\tau^{x_h})$ and that any tax system that implements the planner's solution must have $(1-\tau^n)/(1+\tau^{x_h})=1$. Analogously to the discussion of physical capital, it is possible to subsidize x_h and tax labor income. Here, however, an additional compensating change must be made to the tax on consumption. Again, the only combinations of these taxes which are possible are ones in which the net tax on labor income is zero. Thus, the simple intuition given above for the case of physical capital extends directly to this case as well.

To sum up, the particular choice of tax code that we emphasize is special in the sense that some results about specific taxes do depend on our choice. However, the results are quite general in that the allocations are invariant to the choice of instruments (of course within the class that does not include lump sum taxes). This emphasizes that it is the "effective tax" on activities that matters for real decisions. In dynamic settings, effective taxes are a combination of many individual taxes. The model does not pin down these individual taxes, rather, it is the effective taxes that are determined. In the examples described above these effective taxes on labor and capital income are zero independently of the details of the tax code.

3. WHEN IS THE LIMITING TAX ON CAPITAL NON-ZERO?

In this section we discuss the other question raised in the introduction: What changes in the Chamley–Judd single sector framework would produce non-zero limiting taxes on capital income? We begin by describing an abstract framework that is useful in determining the asymptotic value of the capital income tax. This framework is based on a pseudo planner's problem that is comparable to the problem faced by the representative consumer. Using this framework we see that there are two types of changes that imply a non-zero tax rate. First, if the capital stock enters the objective function of the pseudo planner's problem, then the planner will tax capital income in the limit. One example of this, discussed below, occurs when pure rents appear in the consumer's budget constraint. Secondly, if the planner faces different constraints than the household which involve the capital stock, again capital is taxed in the limit. This is the focus of our second example in which there are two types of labor and the planner must tax them equally. Finally, to gauge the quantitative importance of these deviations, we solve for the optimal tax system in a parameterized version of the second example.

To keep the presentation as simple as possible, from now on we restrict attention to a one capital good growth model.

Consider a pseudo planner's problem given by

$$\max \sum_{t=0}^{\infty} \beta^t \{ W(c_t, n_t, k_t, \lambda)\} - m(c_0, n_0, k_0, \lambda, b_0)$$

$$\begin{aligned} c_t + x_t + g_t &\leqslant F(k_t, n_t) \\ k_{t+1} &\leqslant (1-\delta)\, k_t + x_t \\ \phi_i(c_t, n_t, k_t, c_{t+1}, n_{t+1}, k_{t+1}) &\leqslant 0 \qquad i = 1, 2, ..., I, \end{aligned} \tag{P.4}$$

where the interpretations of all of the variables are as in Section 2, except that in this section, we will consider an example in which n is a vector.

This problem is general enough so that one special case is the one capital good version of (P.3). More precisely, consider a standard one sector growth model where preferences are given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t, 1-n_t)$$

and the resource constraints are

$$\begin{aligned} c_t + g_t + x_t &\leqslant F(k_t, n_t) \\ k_{t+1} &\leqslant (1-\delta) k_t + x_t. \end{aligned}$$

Then, following the steps described in the previous section, it is possible to show that the Ramsey problem for this economy when the planner can choose τ_t^k and τ_t^n, $t = 1, 2, ...$, can be found as the solution to (P.4) where

$$W(c, n, k; \lambda) \equiv u(c, 1-n) + \lambda[u_c c - u_l n],$$

$$m(c_0, n_0, k_0; \lambda) \equiv \lambda u_c(c_0, 1-n_0)[F_k(k_0, n_0)(1-\tau_0^k) + 1 - \delta_k + b_0],$$

and the constraints, $\phi_i(t)$ correspond, for example, to bounds on the feasible tax rates.

Letting μ_t be the Lagrange multiplier corresponding to the resource constraint (once again the law of motion for capital has been substituted in) and η_{it} the Lagrange multiplier for the ϕ_i constraint in period t, the first order conditions for (P.4) are

$$c: \qquad W_c(t) - \mu_t + \sum_{i=1}^{I} \eta_{it}\phi_{ic}(t) + \beta \sum_{i=1}^{I} \eta_{it+1}\phi_{i\underline{c}}(t+1) = 0$$

$$n: \qquad W_n(t) + \mu_t F_n(t) + \sum_{i=1}^{I} \eta_{it}\phi_{in}(t) + \beta \sum_{i=1}^{I} \eta_{it+1}\phi_{i\underline{n}}(t+1) = 0$$

$$k': \qquad -\mu_t + \beta\mu_{t+1}[1-\delta+F_k(t+1)] + \beta W_k(t+1) + \beta \sum_{i=1}^{I} \eta_{it}\phi_{ik}(t+1) + \beta^2 \sum_{i=1}^{I} \eta_{it+1}\phi_{i\underline{k}}(t+2) = 0,$$

where, as before, for any variable x_t we use the following convention:

$$\phi_{ix}(t) = \frac{\partial\phi_i}{\partial x_t}(x_{t-1}, x_t)$$

$$\phi_{i\underline{x}}(t) = \frac{\partial\phi_i}{\partial x_{t-1}}(x_{t-1}, x_t).$$

At the steady state, we can summarize the restrictions that the model imposes in the following set of equations:

$$W_c^* - \mu^* + \sum_{i=1}^{I} \eta_i^*(\phi_{ic}^* + \beta\phi_{i\underline{c}}^*) = 0 \tag{6.a}$$

$$W_n^* + \mu^* F_n^* + \sum_{i=1}^{I} \eta_i^*(\phi_{in}^* + \beta\phi_{i\underline{n}}^*) = 0 \tag{6.b}$$

$$-1 + \beta(1-\delta+F_k^*) + \frac{\beta W_k^*}{\mu^*} + (\mu^*)^{-1}\beta \sum_{i=1}^{I} \eta_i^*(\phi_{ik}^* + \beta\phi_{i\underline{k}}^*) = 0 \tag{6.c}$$

$$c^* + g = F(k^*, n^*) - \delta k^*. \tag{6.d}$$

In the interpretation of (P.4) as the Ramsey problem faced by a planner in the standard one sector growth model, it is possible to show that (1.a) holds in any interior equilibrium. The steady state version of this condition is

$$1 = \beta[1 - \delta_k + (1 - \tau_\infty^k)F_k^*].$$

Given the definition of W (in this case), it follows that $W_k^* = 0$. If there are no binding ϕ_i constraints (i.e., if the tax bounds are not binding) then (6.c) directly implies $\tau_\infty^k = 0$.[5]

Although for the standard case the formulation in (6) is unnecessarily cumbersome it will prove useful in studying deviations from the basic setup that are both economically interesting and that yield non-zero limiting tax rates on capital income. In terms of this more general setting it follows that $\tau_\infty^k \neq 0$ if and only if

$$\beta W_k^* + \beta \sum_{i=1}^{I} \eta_i^* (\phi_{ik}^* + \beta \phi_{\underline{ik}}^*) \neq 0.$$

In what follows, we will describe two economic settings that result in this expression being non-zero. Our examples will highlight the fact that either rents or restrictions on tax codes will result in $W_k^* \neq 0$ or $\eta_i^* \neq 0$. Conversely, if either the stock of capital does not enter the planner's utility function (i.e., there are no fixed factors), or the shadow cost, from the point of view of the planner, of all additional constraints is zero (i.e., $\eta_i^* = 0$, for all i) we are back to the traditional result.

(a) *Pure Rents*: *Inelastic Labor Supply*

In this example we study a model that differs from the simple one capital good version of the setting of section 2 in only one dimension: labor is supplied inelastically. (In independent work, Correia [9] also explores the implications of fixed factors in a system of territorial taxation and concludes that the limiting tax on capital is not zero in the optimal system.) The model then resembles the standard one sector growth model studied by Cass and Koopmans. We assume that there is a bound on the rate at which labor income (a "pure rent") can be taxed. This bound might arise, for example, due to political or other types of constraints that we do not explicitly model. In the absence of such a constraint it is possible to show that the solution to the problem is similar to that obtained whenever lump sum taxes are available. It is to prevent this uninteresting (from the point of view of this paper) outcome that we impose an exogenously given bound on taxation of a factor in fixed supply. In this case the wage rate will be given by $w_t = F_n(k_t, 1)$. However, the marginal condition that determines the marginal rate of substitution between consumption and leisure as a

[5] Although this formulation takes λ as given, in the true problem λ is such that the appropriate version of (2) (the budget constraint) is satisfied. Since our arguments do not depend on the value of λ (only the fact that it is positive), we can study (P.4) to determine the properties of optimal steady state taxes.

function of the after-tax real wage, equation (1.b), no longer applies. It can be shown that, the relevant version of (2) is

$$\sum_{t=0}^{\infty} \beta^t u_c(t)[c_t - (1-\tau_t^n) F_n(t)] = u_c(0)\{[F_k(0)(1-\tau_0^k) + 1 - \delta]\, k_0 + b_0\}. \quad (7)$$

Since taxation of labor income generates only income effects it is clear that the optimal tax rate is $\tau_t^n = 1$. If this is the bound on labor taxes, it can be shown that the result in section 2 holds—the limiting capital income tax rate is zero. To highlight the consequences of less than full taxation of profits—we will make two additional assumptions. First, that there is an upper bound on the tax rate on labor income given by $\bar{\tau}^n < 1$. Second, we assume that the present value of labor income evaluated using the prices $\{p_t\}$ induced by the solution to the *unconstrained* planner's problem (described in Proposition 2) and the initial revenue from capital taxation falls short of the value of government expenditure. With these two assumptions it follows that the government will choose $\tau_t^n = \bar{\tau}^n$. To simplify the presentation—and since we are interested in asymptotic results this is without loss of generality—from now on we set the initial stock of government debt equal to zero.

For this problem, it follows that the relevant pseudo utility function W is

$$W(c, n, k, \lambda) = u(c) + \lambda[u_c(c)(c - (1-\bar{\tau}^n)\, F_n(k, 1))]$$

and $W_k = -\lambda u_c(c)(1-\bar{\tau}^n)\, F_{kn}$ is different from zero if $F_{kn} \neq 0$.

The first order conditions for this problem in the steady state are then given by (6) with the $\eta_i^* = 0$, $i = 1, ..., \mathrm{I}$, since there are no constraints other than feasibility that are binding in the steady state. The relevant conditions are

$$W_c^* = \mu^*$$

$$1 = \beta[F_k^* + 1 - \delta] + \frac{\beta W_k^*}{\mu^*}.$$

Since $\mu^* > 0$ (this is the multiplier corresponding to the resource constraint) it suffices to show that $W_k^* < 0$ to prove that $\tau_\infty^k > 0$. Note that given $F_{kn} > 0$ (from our assumption that F is concave and homogeneous of degree one and $F_{kn} \neq 0$), $W_k^* < 0$ if and only if $\lambda > 0$. We now show that $\lambda > 0$.

PROPOSITION 2. *Let $(\hat{c}, \hat{x}, \hat{k})$ be the solution to the unconstrained planner's problem*

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t)$$
$$\text{s.t. } c_t + k_{t+1} + g_t \leqslant F(k_t, 1) + (1-\delta) k_t$$
$$k_0 > 0 \textit{ given}$$

If

$$\hat{u}_c(0)\, \tau_0^k F_k(\hat{k}_0, 1) k_0 + \sum_{t=0}^{\infty} \beta^t \hat{u}_c(t)\, \bar{\tau}^n F_n(t) < \sum_{t=0}^{\infty} \beta^t \hat{u}_c(t) g_t, \tag{8}$$

then the solution to the planner's problem is such that $\lambda > 0$.

Before we present the proof of the proposition it is useful to interpret (8). The left hand side is revenue from taxation of capital at time zero plus the present value of labor income taxation. Since both k_0 and labor are in fixed supply, taxation of these two factors is equivalent to lump sum taxation. The right hand side is the present value of government spending. Thus, (8) says that, at the prices implied by the first best allocation, revenue from lump sum taxes falls short of expenditures. This is of course an arbitrary assumption. It is equivalent to ruling out lump sum taxation and without it the problem becomes uninteresting as no distortionary taxes are used. It seems a reasonable assumption in real world applications.

Proof. Consider the following less restrictive version of the Ramsey problem,

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t)$$
$$\text{s.t. } c_t + g_t + k_{t+1} \leqslant F(k_t, 1) + (1-\delta) k_t$$
$$\text{s.t. } \sum_{t=0}^{\infty} \beta^t u_c(t)[c_t - (1-\bar{\tau}^n)\, F_n(t)] - u_c(0)[F_k(0)(1-\tau_0^k) + 1 - \delta] k_0 \geqslant 0,$$

and let be the Lagrange multiplier associated with the second constraint. Suppose first that $\lambda = 0$. In this case the solution is "first best" and given by $(\hat{c}, \hat{k})$.

Note that,

$$\begin{aligned}
&\sum_{t=0}^{\infty} \beta^t \hat{u}_c(t)(\hat{c}_t - (1-\bar{\tau}^n)\, F_n(t)) \\
&\quad = \sum_{t=0}^{\infty} \beta^t \hat{u}^c(t)[(F_k(t) + 1 - \delta)\hat{k}_t + \bar{\tau}^n F_n(t) - \hat{k}_{t+1} - g_t] \\
&\quad = \hat{u}_c(0)(F_k(0) + 1 - \delta) k_0 + \sum_{t=0}^{\infty} \beta^t \hat{u}_c(t)\, \bar{\tau}^n F_n(t) - \sum_{t=0}^{\infty} \beta^t \hat{u}_c(t) g_t,
\end{aligned}$$

where we used the property that $\hat{u}_c(t)=\hat{u}_c(t+1)\,\beta[F_k(t+1)+1-\delta]$ for $t\geqslant 0$.

Using this equality in the second constraint of the less restrictive problem we get

$$\hat{u}_c(0)(F_k(0)+1-\delta)k_0+\sum_{t=0}^{\infty}\beta^t\hat{u}_c(t)\,\bar{\tau}^n F_n(t)$$

$$-\sum_{t=0}^{\infty}\beta^t\hat{u}_c(t)\,g_t-\hat{u}_c(0)[(1-\tau_0^k)\,F_k(0)+1-\delta]k_0\geqslant 0$$

or

$$\hat{u}_c(0)\,\tau_0^k F_k(0)k_0+\sum_{t=0}^{\infty}\beta^t\hat{u}_c(t)\,\bar{\tau}^n F_n(t)\geqslant\sum_{t=0}^{\infty}\beta^t\hat{u}_c(t)\,g_t.$$

This contradicts (8) and hence it shows that $\lambda>0$. ∎

An alternative, more intuitive argument that shows that $\lambda>0$ can be constructed as follows: Denote by V^* the maximized value of the planner's objective function, and note that the change in V^* due to an increase in τ_0^k is given by $\lambda u_c^*(0)\,F_k^*(0)k_0$. Since increases in τ_0^k are equivalent to increases in lump sum taxes and since the higher the level of lump sum taxes the higher the value of utility, it follows that λ must be positive.

Our findings for the case of factor income taxation are reminiscent of the results of Diamond and Mirrlees [10] and Stiglitz and Dasgupta [25] that also show that the existence of pure profits affects the optimal commodity tax schedule.

(b) *Two Types of Labor with Equal Taxes*

The previous example discussed a particular form of violation of the assumptions that give rise to the Chamley–Judd result: in terms of the pseudo-utility function W of the planner's problem, the term W_k is not zero. Here, we consider the alternative possibility that the constraints ϕ_i are binding in the steady state. A simple example of such a case is when the planner cannot distinguish between income from two types of labor. Because of this, we will require that the tax rate on the two types of labor be equal. This is a convenient way of modeling a more general type of constraint: restrictions on tax rates. The example we will consider features one household that sells two types of labor to the market, n_1 and n_2. This is in the spirit of Stiglitz [27] where an example is analyzed with two types of consumers each with a different type of labor. He analyzes a planner's problem with a weighted average of the two consumer's utility functions and shows that the limiting tax rate on capital income is not zero. He ascribes this finding to redistributional motives that arise due to the heterogeneity. What we show here is that this finding holds in some cases even with a representative household that supplies two different types of labor if the planner is constrained to choose the tax rates on the two types of labor equally. (An assumption implicit in Stiglitz's work.)

Here, the period utility function of the household is given by $u(c, 1-n_1, 1-n_2)$ and the production function is $F(k, n_1, n_2)$.

It can be shown that the W function for this example is given by:

$$W(c, n_1, n_2) = u(c, 1-n_1, 1-n_2) + \lambda[u_c c - u_{n_1} n_1 - u_{n_2} n_2]$$

and that the constraints of the planner's problem are

$$\begin{aligned} c_t + x_t + g_t &\leqslant F(k_t, n_{1t}, n_{2t}), \\ k_{t+1} &= (1-\delta)k_t + x_t, \\ \phi(t) &\equiv u_{n_1}(t)\, F_{n_1}(t) - u_{n_2}(t)\, F_{n_2}(t) = 0. \end{aligned}$$

The first two constraints describe feasibility while the third uses the first order conditions of the household to impose the constraint that the two labor tax rates should be equal, $\tau_{n_1} = \tau_{n_2}$ for all t. Letting the Lagrange multipliers on the first and third constraints in the steady state be given by μ^* and η^* respectively, the relevant steady state conditions are

$$\begin{aligned} W_c^* &= \mu^* - \eta^* \phi_c^* \\ W_{n_1}^* &= -\mu^* F_{n_1}^* + \eta_{n_1}^* \\ 1 &= \beta[F_k^* + 1 - \delta] + \beta\eta^*\phi_k^*/\mu^*. \end{aligned}$$

In section (c), below, we provide a numerical example in which $\tau_\infty^k \neq 0$. The equations above indicate when the limiting tax will be zero. If $\tau_\infty^k = 0$, it must be the case that either η^* or ϕ_n^* is zero. From the definition of ϕ, it follows that ϕ_k is given by

$$\phi_k = \frac{u_{n_1}}{F_{n_1}} \left(\frac{F_{n_1 k}}{F_{n_1}} - \frac{F_{n_2 k}}{F_{n_2}} \right).$$

It follows that $\phi_k = 0$ if and only if

$$\frac{F_{n_1 k}}{F_{n_1}} = \frac{F_{n_2 k}}{F_{n_2}}.$$

This condition defines a special class of production functions which include Cobb-Douglas but not the general CES case. To get our non-zero result we have to assume that our production function is not in this class.

Alternatively, $\tau^k_\infty = 0$ will hold when $\eta^* = 0$. If the solution has the property that the restriction that the two tax rates be equal is not binding this will be the case. This, however, also requires that the planner's marginal rate of substitution between consumption and the two types of labor (as given by the W function) be equal to the household's marginal rate of substitution. In general this will not be the case. The economic reason is that the planner's function includes—in addition to household's utility function—another term that captures the impact of policy decisions upon relative prices. Thus, it is only in cases in which this relative price effect is irrelevant that the constraint will not be binding and η^* will be zero. Examples of this exceptional case include the situation when both types of labor are perfect substitutes in utility and production, since this pins down their relative prices and there is no price effect.

How general is the result that restrictions on tax rates result in non-zero limiting taxes on capital? There are a variety of other restrictions on tax codes that can be modeled along similar lines giving the same result. For example, in a standard growth model in which factor income from all sources (i.e., both capital and labor) is restricted to be taxed at the same rate, the common limiting rate is non-zero. Alternatively, if pure public goods generate private rents which are indistinguishable from payments to capital, then the limiting tax rate on capital is again non-zero. The common thread in all of these examples is a restriction on the planner's ability to independently tax income sources. For example, in the case of pure rents, this restriction is the assumed inability of the planner to completely tax away these rents.

(c) *Tax Calculations for the Two Labor Example*

While the examples above show that the tax rate can be non-zero, it remains an open question whether or not the magnitude of this tax rate is significant. To study this question, we calculate the solution to the optimal tax problem for a specific choice of the example in section (b). In particular, we set

$$u(c, 1-n_1, 1-n_2) = c^{1-\sigma}(1-n_1)^{\gamma_1}(1-n_2)^{\gamma_2}/(1-\sigma)$$

and

$$F(k, n_1, n_2) = A\{bk^{-\rho} + (1-b)(n_1)^{-\rho}\}^{-\psi/\rho}(n_2)^{1-\psi},$$

where we interpret n_1 as skilled labor and n_2 as unskilled labor. This utility function can be interpreted as that of a household with two members each of which is able to supply one unit of leisure to the market each period. Our specification of technology is similar to the nested CES model estimated by Kwag and McMahon [18]. They consider a setting in which the inner function is a CES between skilled labor and physical capital; this aggregate, in turn, is combined using another CES with unskilled capital. They estimate that the elasticity of substitution between physical capital and skilled labor is very low (i.e., 0.013), while that between the combined total capital and low skilled labor is in the range of 0.482 to 0.769. Guided by this we assumed that the elasticity of substitution between n_2 and the composite between n_1 and k is one to simplify the analysis, and we set $\rho = 5.0$ (which corresponds to an elasticity of 0.16 between capital and skilled labor). This is the rationale behind our interpretation of n_1 as skilled labor and n_2 as unskilled labor. We chose $\sigma = 2$, $\psi = 0.9$, $\gamma_1 = -0.2$, and $\gamma_2 = -0.8$ as a base case and chose g so that the budget would exactly balance with a flat income tax rate of 20% on all sources of income.

We calculated the solution to the planner's problem (see Jones, Manuelli and Rossi [13] for a detailed discussion of the methods) both from a wide range of initial capital stocks and with and without imposing bounds (of 100%) on the capital income tax rate along the path to the steady state. We find that the limiting value of the capital income tax rate is approximately 7 percent for all of these scenarios as shown in Table 1. In all solutions reported in table 1, we fixed the initial tax rate on capital income at 20 percent.

As is clear from the table, the limiting tax rate is not materially affected by changes in initial conditions or the imposition of tax bounds. Moreover, the solution path is well-behaved with rapid convergence to the steady state. In all examples considered, the labor tax rates are fairly stable after period 1 and have a steady state value of approximately 22 percent. The limiting value of the capital stock under the optimal tax system is approximately 0.91 which is slightly higher than the steady state capital stock under the initial (sub-optimal) tax code.

TABLE I

Asymptotic Capital Tax Rates[1]

Tax rate upper bound	Initial capital stock		
	0.5	0.88	1.1
100 %	7.47	7.21	7.19
No bound	7.47	7.17	7.12

[1] Rates are given as percentages.

4. EXTENSIONS AND CONCLUSIONS

Our analysis of optimal factor taxation in dynamic models suggests that simple intuitive arguments purporting to explain why the tax rate on capital income is zero in the limit (e.g., due to infinite supply elasticity) are not very useful. By useful we mean a type of economic intuition that is robust to relative minor changes in the structure of the model. All the different environments discussed in this paper are very close, yet, our conclusions about limiting behavior of tax rates are quite different.

A more promising route to understanding the basic forces that drive the asymptotic behavior of optimally chosen tax rates is to delineate features of the economy which account for the result. In this paper we showed that, in the context of a model with both human and physical capital, if there are constant returns to scale in the reproducible factors (no profits), a sufficiently rich tax code and no possibilities for relative prices to affect wealth then limiting tax rates on both capital and labor income are zero. Each of these three conditions is essential.

Consider the linearity or "no profits" condition first. In the model with an inelastic labor supply, the presence of rents result in positive limiting tax rates on capital. Our interpretation is that by distorting the choice of capital the planner can "tax" the pure rents.

The second element is the richness of the tax code. As the discussion in section 3 (b) shows (as well as the model in section 2 in the absence of consumption taxes), the presence of restrictions across tax rates results in non-zero taxes on capital income. More generally, a limited ability to set sufficiently many taxes independently gives the same result. We interpret this as a standard "second best" argument: the imposition of an additional restriction (e.g., a restriction across tax rates) calls for a change in how the unaffected policy instruments are chosen. In this case, the restrictions force a switch from zero to some positive level of capital income taxation in the long run.

The third element that seems essential is the absence of a role for a change in relative prices as a form of extracting rents from the private sector. In this respect, the model in section 2 in which transfers (a pure rent from the point of view of both the consumers and the planner) are not fixed in terms of consumption results in non-zero taxes on labor income.

The reader may wonder if the three features that we have identified overturn the zero limiting tax result only in the Chamley–Judd environment. Although it is impossible to give an exhaustive answer, we have explored other environments in which both the Chamley–Judd result obtains and the violation of one of the three conditions that we identified results in non-zero limiting tax rates. These more general environments allow for heterogeneity (see also Judd [14]), multiple consumption goods and types of labor and multi-sector settings in which the price of capital in the steady state is endogenous.

Finally, the model in section 2 suggests that given the linearity assumption and a rich tax code, the Ramsey problem has very strong implications about both the timing of tax revenues and the structure of the tax system: Under the optimal scheme, revenue is 'front-loaded' and all factors are treated symmetrically in the limit. This revenue front-loading is a disturbing but essential feature of the optimal tax code. Reasonable restrictions on the time path of deficits (e.g., period by period budget balance) can be shown to undo the zero limiting tax result. In essence, this is another type of restriction on tax codes not unlike that described in section 3 (b). This example highlights the interdependence of the initial behavior of the optimal tax code and its limiting properties.

Because of the delicacy of the mapping between the features of the economy and the structure of optimal tax codes, further progress will necessitate a detailed analysis of the entire time path of taxes as in section 3 (c). Even for relatively simple examples, this will require a reliance on numerical methods.

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