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Optimal Multiperiod Contracts and the Gain from Enduring Relationships under Private Information

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Informational asymmetries can play a key role in explaining the existence and nature of multiperiod contracts. In an illustrative risk-sharing model even relatively short (two-period) contracts can be mutually beneficial if there is private information, though one-period contracts suffice otherwise. Further, contracts which are Pareto optimal relative to the environment and the information structure are defined and partially characterized. An example illustrates how otherwise inefficient intertemporal tie-ins can be used optimally to mitigate incentive problems. The obvious borrowing-lending schemes are not private-information Pareto optimal; in these, period-by-period actions are not sufficiently constrained. Discounting affects the form of the optimal contract but none of these qualitative conclusions.

I. Introduction

The existence and nature of multiperiod contracts is a subject receiving renewed attention from authors interested in aggregative phenomena and industrial organization. Responding in part to the striking conclusions of Sargent and Wallace (1975), Fischer (1977*b*)

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and Phelps and Taylor (1977) have developed models in which price or wage (contractual) rigidities give real consequences to the systematic component of monetary policy. More recently, Taylor (1980) has produced a model in which staggered, fixed nominal-wage contracts play a key role in explaining persistence and momentum. Of course, this and earlier macroeconomic literature is closely associated with studies on the (cyclical) behavior of industrial prices, and their apparent inflexibility, beginning with Means (1935). Stigler and Kindahl (1970) outline these studies in providing evidence counter to Means's administered-price thesis. But more to the point, Stigler and Kindahl find that many prices are set by an explicit contract. This finding and the divergent movement of the BLS (spot) price index relative to the Stigler-Kindahl (contract) index motivates the work of Carlton (1979).

Most models with multiperiod contracts begin at the level of "descriptive realism," imposing directly (exogenously) the form of observed contracts. It is not the intent of such models to explain the existence and nature of multiperiod contracts. Thus Fischer (1977*a*) finds common ground with Barro's (1977) criticism of Gray (1976) and Fischer (1977*a*), noting that " . . . there is indeed a missing theoretical link in this area . . . and that is the *explanation* of the form that labor contracts take" (emphasis added). And thus Carlton (1979, p. 1037) argues "that any theory purporting to *explain* price movements must delve into (*a*) the incentives for contracting, (*b*) the function performed by contracts, (*c*) the incentives for contracting for different durations, and (*d*) the pricing and interrelationship of different-duration contracts" (emphasis added). This paper seeks to develop a theory one might use to begin to explain the existence and nature of some multiperiod agreements. In this theory informational asymmetries play a key role.[1]

Recent papers have made important advances in explanations of multiperiod agreements, concentrating on contract length. The dynamic labor contract paper of Dye (1979), in a microeconomic context, building on the implicit labor contract paradigm of Gordon (1974), Azariadis (1975), and Baily (1977), is one such advance. The variable contract length paper of Gray (1978), in a macroeconomic context, is another. Still, the results of these papers turn on the imposition of arrangements which are incomplete relative to those suggested by the Arrow (1953) and Debreu (1959) treatment of uncertainty—arrangements made possible by indexing commodities

[1] There is of course some literature on multiperiod contracts which does not exploit informational asymmetries: e.g., the matching models of Diamond and Maskin (1979); the labor-turnover model of Jovanovic (1979); the limited labor mobility model of Baily (1977), Barro (1977), and Bryant (1979); and the contingency cost models of Riddle (1979) and Dye (1980).

by all conceivable states of nature. In Dye (1979), for example, the labor force is fixed for the duration of the contract, independent of the state of demand, and in Gray (1978) wages are indexed to prices but not to real and monetary shocks. In fact Barro (1977) argues that the crucial element in the Keynesian conclusions of the models of Gray (1976) and Fischer (1977*c*) " . . . is the nonexecution of some perceived mutually advantageous trades."

This paper seeks to develop a theory which explains the existence and nature of multiperiod contracts without the imposition of exogenous restrictions—restrictions other than those implied by the economic environment and the information structure. That is, contracts are viewed as Pareto-optimal agreements between agents so that no mutually advantageous trades remain. The explanation is accomplished here at the cost of considerable abstraction; the model contains neither money nor competitive markets (prices), and thus it can address neither the policy issues of the macroeconomic literature nor the empirical observations of Stigler and Kindahl (1970). But it is hoped that this paper does take a modest conceptual step forward.

The motive for multiperiod contracts in this paper is the idea that agents attempt to circumvent the incentive information difficulties of single-period, one-shot agreements. Under single-period agreements, contingent exchanges that are mutually beneficial when information is complete can be impossible when information is private. If, for example, only one party to a two-party contract observes the outcome of an event upon which the contract depends, then he may be expected always to claim the outcome which is most favorable to himself. That is, the ultimate exchange is noncontingent after all.[2] But even if certain events are observed by only one party period by period, the well-informed agent can be given an incentive to report more honestly than in any one-shot agreement, by making future exchanges contingent on present claims.

The idea that mutually beneficial arrangements are possible in the context of private information, with future payments tied to present claims, is closely related to a result in the literature on supergames, that agents can achieve a cooperative solution if a game is played repeatedly. In supergames each agent can agree to play his cooperative strategy until the other agent deviates. In retaliation the one-shot Nash equilibrium strategy may be employed. This idea has been applied successfully by Friedman (1977), Green (1980), and Radner (1980) to dynamic oligopoly theory to explain how collusion can be maintained as a noncooperative solution: Each firm agrees to restrict

[2] Occasionally these considerations are so severe as to rule out any trade or contract whatever (e.g., Radner 1968; Arrow 1974; Townsend 1979*b*).

output to the collusion solution until at least one firm dumps, in which case they all dump. Thus the key idea of supergames is that *future payoffs to the decision maker are tied to present actions of the decision maker.* Similarly, in this paper future transfers to the informed agent depend not only on losses (unobserved by others) claimed in the future but also on present claims.[3] With a limited number of potential claims overall the informed agent will report more honestly than in any single-period agreement. This insight, that conditioning future payoffs on present claims or actions enables agents to do good things in repeated games with private information, is basic. Indeed this author has recently become aware of some work by Rubinstein and Yaari (1980) and Radner (1981) which makes use of the same idea.

As noted, this paper seeks to develop a theory which one might use to begin to explain the existence and nature of some multiperiod contracts; it establishes the gain to enduring relationships in a setting with private information. To explain why contracts are of finite length, this gain must be balanced against a cost, some sense in which the contract becomes less suitable over time. This can be accomplished formally by imagining that agents are traveling about on Hotelling's (1929) one-dimensional space. Maintaining a match or pairing can be supposed to become more costly over time; this would capture the cost associated with changing circumstances. To explain why contracts are negotiated at various times, rather than in a planning period, it can be imagined that communication is impossible between agents not in the same location or Phelps's (1970) island. In this setting one might well imagine that the greater is the gain from enduring relationships the greater is the tendency for longer agreements. But a deeper explanation of contract length is not pursued in this paper.

The paper proceeds as follows. Section II describes the basic environment. There are two agents, one risk averse with a random endowment of the single consumption good of the model which is independent and identically distributed over time and one risk neutral with a constant endowment.[4] A full-information utility pos-

[3] A difference between the approach of this paper and the approach of supergames should be pointed out, however. In oligopoly contexts, e.g., explicit collusion, i.e., an agreement to fix outputs, is presumed to be inherently unenforceable even if there is full information. In contrast, in the spirit of this paper any game or contract which restricts actions is viable if it is consistent with the information structure. But with private information, this is not enough to ensure a full-information optimal allocation. And that is where the notion of future retaliation comes into play.

[4] The idea here is to produce a model in which there are obvious gains to trade, at least under full information: the risk-neutral agent can absorb all risk. Less extreme (more general) hypotheses could be tolerated in much of what follows but at the cost of an increase in the complexity of notation, techniques, and exposition. In a way the entire paper can be viewed as an illustrative example, not an attempt at maximum generality.

sibilities frontier is derived, and it is established that points on the frontier can be achieved by a sequence of one-period contracts; with full information, multiperiod contracts are not needed. Private information is introduced in Section III; it is assumed that the risk-averse agent alone observes his endowment realizations. In the context of this private information there are open issues as to how to define achievable allocations; this is done in this paper by the derivation and imposition of certain incentive-compatibility constraints which arise naturally in a consideration of resource allocation mechanisms.[5] It is then shown that with private information a restriction to one-period contracts is quite damning—there can be no mutually beneficial exchange. But Section IV establishes that mutually beneficial trade is possible in a simple, two-period, borrowing-lending scheme; thus with private information, multiperiod contracts are a good thing. Private-information optimal two-period schemes are also defined, and it is shown that the borrowing-lending scheme, though beneficial, is nonoptimal. An example is given which illustrates the optimal use of otherwise inefficient intertemporal tie-ins. It is established generally that in a private-information optimum at least one incentive-compatibility condition must be binding; these results parallel some results on the inconsistency of optimal plans of Kydland and Prescott (1977). Finally, Section V argues that every major result of the paper is valid when agents discount the future (at the same rate), though asymptotic results based on an average-utility criterion, while illustrative of the gain to enduring relationships, are vulnerable.

II. A Two-Agent Economy and Full-Information Optimal Allocations

Consider an economy with just two infinitely lived agents, one risk averse with an endowment sequence of the single consumption good of the model that is random and one risk neutral with a constant endowment sequence. Preferences of the risk-averse agent (labeled agent a) over consumption c in any period are described by a utility function $U(c)$, which is strictly concave, strictly increasing, and continuously differentiable with $U'(0) = \infty$ and $U(0) = 0$. Let y_t denote the number of units of the consumption good of agent a in period t, where y_t is a random variable which takes on the values y' and y'', $0 < y' < y''$, with probabilities Π and $1 - \Pi$, respectively. Thus, the sequence

[5] Thus this paper extends to a simple dynamic model some earlier results on private information for essentially static environments, namely, Harris and Townsend (1977, 1981) and Myerson (1979). More generally, this paper builds on the seminal work of Hurwicz (1972).

$\{y_t\}_{t=1}^{\infty}$ is a very special sequence of independent and identically distributed random variables. Preferences of the risk-neutral agent (labeled agent b) are described by the utility function $V(c) = c$. Let $W > 0$ denote the (constant) number of units of the consumption good of agent b in each period t.

In what follows attention will be limited to resource allocation schemes over finite (but arbitrary) horizons T; of course, these schemes can be spliced together to form an infinite path. As was indicated in the introduction, finite duration schemes arise endogenously if there is a cost of changing circumstances. Attention will be limited also to paths which are stationary; that is, if a particular resource allocation scheme is employed from periods 1 through T, then that same scheme is employed from periods $T + 1$ through $2T$, and so on. Despite these two restrictions, there occasionally arises the need to assign utility values to infinite-horizon paths. To facilitate the exposition this assignment will be made using the average-utility criterion, but, as indicated in Section V, apart from an asymptotic result none of the major conclusions of this paper depends on the absence of discounting or the average-utility criterion.[6]

Now suppose for the remainder of this section that realizations of the sequence $\{y_t\}_{t=1}^{\infty}$ are fully observed by both agents. Then it is clear that mutually beneficial trade is possible; the risk-neutral agent can absorb all risk. More formally, let $F_t(y_1, y_2, \ldots, y_t)$ denote an endowment-contingent transfer, that is, the number of units of the consumption good transferred from agent b to agent a as a function of the realizations $y^t \equiv (y_1, y_2, \ldots, y_t)$. Suppose also that the horizon over which such contingent trades are possible is fixed at some (finite) real number T. Then a full-information optimal exchange relative to T can be found as a solution to

Problem 1:

$$\max_{\{F_t(y^t)\}_{t=1}^{T}} E \sum_{t=1}^{T} U[y_t + F_t(y^t)]$$

subject to

$$E \sum_{t=1}^{T} [W - F_t(y^t)] \geqslant KT, \tag{1}$$

$$y_t + F_t(y^t) \geqslant 0, W - F_t(y^t) \geqslant 0, \qquad t = 1, 2, \ldots, T. \tag{2}$$

Here the objective function is the expected utility of the risk-averse agent; (1) imposes a lower bound on the expected utility of the

[6] Typically, the average utility criterion is used in the literature without discounting.

risk-neutral agent, where K is some constant and T is fixed, and (2) is the nonnegativity constraint on consumption.

If we write out the necessary first-order conditions for an interior maximum (and ignore [2]), it becomes apparent that

$$U'[y_t + F_t(y^t)] - \lambda = 0 \tag{3}$$

at each time t and for each possible y^t, where λ is a positive Lagrange multiplier. That is, *the consumption of the risk-averse agent should be a constant, say* c, *over both time and endowment realizations.* Thus a specification of c determines completely the amount to be transferred as a function of the realization y_t. If $y_t = y'$, so that agent a suffers a loss, then agent a receives a payment d from agent b. If $y_t = y''$ then agent a pays out p. This is so for all periods t, $t = 1, 2, \ldots, T$.

A utility possibilities frontier may now be derived. First, imposing constant consumption in problem 1 and noting that the number T enters as a multiplicative constant in the objective function and in constraint (1) (and is, therefore, inessential) one obtains
Problem 2:

$$\max_c U(c)$$

subject to

$$W + E(y_t) - c \geqslant K, \tag{4}$$

$$c \geqslant 0, W - c + y' \geqslant 0, \tag{5}$$

which is the obvious, one-period, full-information Pareto problem. Of course, (4) may be presumed to hold as an equality. Here then the parameter K is the average utility of agent b over the horizon T. If we ignore (5), solutions to problem 2 depend entirely on this parameter. Using superscripts to denote this dependence, we see that (4) as an equality yields

$$c^K = E(y) - K + W, \tag{6}$$

where $E(y) = E(y_t)$ for all t, so that

$$\bar{U}^K = U(c^K) = U[E(y) - K + W], \tag{7}$$

$$\bar{V}^K = K \tag{8}$$

(let the bars denote average utility). Thus, a typical concave utility possibilities frontier may be traced out as one varies the parameter K. Nonnegativity constraint (5) implies

$$E(y) - y' \leqslant K \leqslant E(y) + W. \tag{9}$$

Hereafter K is restricted to lie in this interval so that the construction above is valid. Finally, it is straightforward to establish that the utility possibilities frontier described above is, in fact, the full-information (average) utility possibilities frontier for the given model, with the horizon infinite.

The implication of the discussion above is obvious: *Any point on the full-information, utility possibilities frontier can be obtained by a sequence of one-period agreements.* Any such point on the frontier is associated with constant consumption c for the risk-averse agent, which in turn is associated with a transfer of $+d$ or $-p$ for y_t equal to y' or y'', respectively. There is no sense in which this contingent exchange depends on past exchanges. There is no gain to enduring relationships.

III. Private Information and a Class of Incentive-compatible Contracts

The model discussed thus far, the one with full information, has the property that multiperiod contracts are not needed. It is now shown how this conclusion is altered with the introduction of private information. This will make the point that private information can be crucial in an explanation of the existence of multiperiod contracts.

Thus suppose that the endowment y_t of agent a in each period t is never observed by any agent other than a. In this private-information context then we are confronted with an obvious question: How are we to define achievable allocations or exchanges, so that we may discover the implication of private information for multiperiod contracts?

To begin the discussion, suppose that we are intent on achieving some endowment-contingent transfer $F_t(y_1, y_2, \ldots, y_t)$ if the sequence of realized endowments is $y^t \equiv (y_1, y_2, \ldots, y_t)$. We imagine that we might do this by asking agent a to name a value for y_t in each period t, hoping that he might tell the truth. More formally, let $M_t = \{y', y''\}$ be the message space in period t, possible announcements of y_t, and let $F_t(m_1, m_2, \ldots, m_t)$ be the outcome in period t, a transfer which is a function of previous and current messages $m^t \equiv (m_1, m_2, \ldots, m_t)$. This outcome function F_t is the same as the endowment-contingent transfer function F_t, but its arguments are messages. Now in the last period of a T-period arrangement we want agent a to tell the truth about endowment realizations, that is, to send the message $m_T = y'$ if indeed the endowment is $y_T = y'$ instead of message $m_T = y''$. Clearly he will have an incentive to do this, no matter what message he has sent in the past, if his utility is greater, that is, if the outcome function is such that[7]

$$U[y' + F_T(m^{T-1}, y')] \geqslant U[y' + F_T(m^{T-1}, y'')] \qquad (10)$$

[7] It is assumed, in the case of indifference, that agent a continues to tell the truth. The argument here and below ignores nonnegativity constraints on consumption.

for all previous messages m^{T-1}. (A similar expression must hold if $y_T = y''$.) Similarly, in period $T - 1$ we want agent a to tell the truth, given that he will tell the truth in the future no matter what he does now. That is, let

$$\begin{aligned} &U[y' + F_{T-1}(m^{T-2}, y')] + EU[y_T + F_T(m^{T-2}, y', y_T)] \\ &\geqslant U[y' + F_{T-1}(m^{T-2}, y'')] + EU[y_T + F_T(m^{T-2}, y'', y_T)] \end{aligned} \tag{11}$$

for all previous messages m^{T-2}, and so on for $y_{T-1} = y''$. (Here the expectation is over y_T.) Working backward in this way, one derives a series of so-called incentive-compatibility conditions which ensure truth telling in each period t, $t = 1, 2, \ldots, T$. That is, if these conditions are satisfied, agent a will tell the truth in periods one through T, and the sequence of endowment realizations $(y_1, y_2, \ldots, y_T)$ will be identical with the sequence of messages $(m_1, m_2, \ldots, m_T)$. Thus the transfer $F_t(y_1, y_2, \ldots, y_t)$ is achieved in each period t. This was our objective.

It must now be emphasized that arbitrary endowment-contingent transfers are not always achievable. The incentive-compatibility conditions (10), (11), and so forth impose restrictions on any $F_t(y_1, y_2, \ldots, y_t)$ we might hope to achieve in this way. Stated differently, only those transfers $F_t(y_1, y_2, \ldots, y_t)$ that satisfy these incentive-compatibility conditions can be achieved in the class of announcement games with truth telling, those discussed above.

Before I discuss the generality of this result, a rather striking implication of the incentive-compatibility conditions should be noted. As more is preferred to less and there is only one commodity, condition (10) and its analogue for $y_T = y''$ will be satisfied if and only if

$$\begin{aligned} F_T(m^{T-1}, y') &\geqslant F_T(m^{T-1}, y''), \\ F_T(m^{T-1}, y'') &\geqslant F_T(m^{T-1}, y') \end{aligned} \tag{12}$$

for all previous messages m^{T-1}. Thus

$$F_T(m^{T-1}, y') = F_T(m^{T-1}, y''), \tag{13}$$

and the last-period transfer cannot depend on the last-period announcement. That is, the transfer is at most some constant in the last period. This is a severe restriction on trade. In fact, with $T = 1$, so that the last period is the only period, it is clear that mutually beneficial exchange becomes impossible. The implication is summarized: *Under private information, a sequence of one-period agreements implies no trade.* Thus the introduction of private information into this model has fairly damning consequences.[8]

[8] Needless to say, this implication need not be so severe; trade may be possible if there is more than one commodity. The point to be made here is that private information can make a difference in the consideration of contract length.

With this result the issue of the generality of the incentive-compatibility conditions takes on added importance. That is, these incentive-compatibility conditions were derived from the requirement of truth telling in an announcement game. Perhaps these conditions can be circumvented by other schemes. Consider then a more general class of schemes. Let $M_t(m^{t-1})$ be the message space of agent a in period t, now as a function of previous messages $m^{t-1} \equiv (m_1, m_2, \ldots, m_{t-1})$. Here also the message space is arbitrary. Also let $F_t(m^{t-1}, m_t)$ be the outcome as a function of current and previous messages as before. Here then any scheme is completely defined by the message spaces $\{M_t(m^{t-1})\}_{t=1}^T$ and outcome functions $\{F_t(m^{t-1}, m_t)\}_{t=1}^T$.[9] Such a scheme may be thought of as a contract of duration T periods.

Under any such contract, agent a is confronted with
Problem 3:

$$\max_{\{m_t\}_{t=1}^T} E \sum_{t=1}^{T} U[y_t + F_t(m^{t-1}, m_t)]$$

subject to $m_t \in M_t(m^{t-1})$, $t = 1, 2, \ldots, T$. Here the expectation is taken over y_t, $t = 1, 2, \ldots, T$. A solution to this problem (assuming its existence) may be taken to be a sequence of decision rules $\sigma_t(m^{t-1}, y_t)$, $t = 1, 2, \ldots, T$, mappings from previous messages and the current endowment realization to a current message. A sequence of decision rules generates a sequence of messages which in turn generates a sequence of transfers, as a function of the realized endowments. The issue of generality can now be resolved: Given any contract $\{M_t(m^{t-1})\}_{t=1}^T$, $\{F_t(m^{t-1}, m_t)\}_{t=1}^T$, and associated optimal decision rules $\{\sigma_t(m^{t-1}, y_t)\}_{t=1}^T$, there exists an alternative contract with message space $\hat{M}_t = \{y', y''\}$ and outcome functions $\hat{F}_t(m^{t-1}, m_t)$ under which the truth-telling strategy $\hat{\sigma}_t(m^{t-1}, y_t) = y_t$ is maximal and which achieves the same allocation of resources (see App. for proof).[10] In this sense the incentive-compatibility conditions derived earlier were without loss of generality.[11] They provide natural constraints on the space of achievable allocations in this environment with private information.

[9] Here m_0 is some fixed number so that $M_1(m^0)$ denotes a fixed message space; similar notation applies at $t = 1$ in what follows.

[10] There is some obvious intuition behind the truth-telling theorem for the class of announcement games in a single-period context. Suppose y' occurs and agent a says y'' and is transferred A, while when y'' occurs agent a says y' and is transferred B. Then just change the payoff so that A is the transfer if y' is reported and B is the transfer if y'' is reported. For additional discussion and general proofs of the truth-telling theorem in static contexts see Harris and Townsend (1977, 1981) and Myerson (1979).

[11] Of course, this result is limited by the class of games under consideration. Prescott and Townsend (1979), among others, have shown in other contexts that random exchanges may be beneficial. Whether or not they are here is left as an open question.

IV. Two-Period Contracts, the Gains to Trade, and Private-Information Optima

This section seeks to illustrate the gain to multiperiod relationships, even if such relationships are relatively short. Though mutually beneficial trade is impossible with single-period, one-shot agreements, mutually beneficial trade is possible with two-period agreements. Moreover, the two-period, private-information optima are partially characterized.

To illustrate that mutually beneficial trade is possible with two-period agreements attention is restricted to a simple, intuitively appealing scheme. Suppose agent a may ask for whatever transfers he likes in each of two periods, subsequent to his endowment realizations, but that these transfers must sum to zero. This is a type of borrowing-lending scheme in which the risk-neutral agent acts as a banker with zero interest rate. In effect we have a general equilibrium model which rationalizes a two-period version of the consumer savings problem discussed by Schechtman (1976), Yaari (1976), and Bewley (1977) in a partial equilibrium context. Letting $d(y_1)$ denote the first-period transfer to agent a as a function of the endowment realization y_1, we see that agent a is confronted with
Problem 4:

$$\max_{d(y_1)} U[y_1 + d(y_1)] + EU[y_2 - d(y_1)],$$

which yields the necessary first-order condition for an interior maximum

$$U'[y_1 + d(y_1)] = EU'[y_2 - d(y_1)], y_1 \in \{y', y''\}. \tag{14}$$

Condition (14) states that under a solution the marginal utility of current consumption should equal the expected marginal utility of future consumption. In general then the solution $d(y_1)$ is nonzero. (For example, suppose $y_1 = y'$ though Π is close to zero.) Thus, in general, agent a is better off than in autarky. As for agent b, he is clearly no worse off; his utility function is linear, and he is therefore indifferent to the timing of receipts. (It is assumed that the environment is such that the nonnegativity constraint on agent b's consumption is not binding.) To reiterate, this two-period scheme allows for mutually beneficial trade because future transfers are tied to present claims; in this case they are equal except for sign.

Granting that this two-period contract is mutually beneficial, one may well ask whether it is not Pareto optimal. That is, does there exist another two-period contract which is better for at least one agent without being worse for the other? This question can be answered affirmatively by making use of the results of the previous section

where it was established that the incentive-compatibility conditions are natural and unavoidable constraints in the space of endowment-contingent transfers. Thus two-period, private-information optimal, endowment-contingent transfers may be found as solutions to
Problem 5:

$$\max \Pi\{U[y' + F_1(y')] + EU[y_2 + F_2(y')]\} + (1 - \Pi)\{U[y'' + F_1(y'')] + EU[y_2 + F_2(y'')]\}$$

subject to

$$U[y' + F_1(y')] + EU[y_2 + F_2(y')] \geqslant U[y' + F_1(y'')] + EU[y_2 + F_2(y'')], \tag{15}$$

$$U[y'' + F_1(y'')] + EU[y_2 + F_2(y'')] \geqslant U[y'' + F_1(y')] + EU[y_2 + F_2(y')], \tag{16}$$

$$\Pi[W - F_1(y') + W - F_2(y')] + (1 - \Pi)[W - F_1(y'') + W - F_2(y'')] \geqslant 2K, \tag{17}$$

by choice of first-period transfers $F_1(y')$ and $F_1(y'')$ and second-period transfers $F_2(y')$ and $F_2(y'')$, both as functions of first-period realizations y' and y''. Here the objective function is the expected utility of agent a, constraints (15) and (16) are the incentive-compatibility constraints implied by truth telling in period one, and (17) is the constraint that the expected utility of agent b be no lower than some constant. (The incentive-compatibility condition that $F_2[y_1, y_2]$ not depend on y_2 is implicit in the notation.) Among the necessary first-order conditions for an (interior) solution to this problem are

$$\Pi U'[y' + F_1(y')] + \Psi_1 U'[y' + F_1(y')] - \Psi_2 U'[y'' + F_1(y')] - \Pi\Psi_3 = 0, \tag{18}$$

$$(1 - \Pi)U'[y'' + F_1(y'')] - \Psi_1 U'[y' + F_1(y'')] + \Psi_2 U'[y'' + F_1(y'')] - (1 - \Pi)\Psi_3 = 0, \tag{19}$$

$$\Pi EU'[y_2 + F_2(y')] + \Psi_1 EU'[y_2 + F_2(y')] - \Psi_2 EU'[y_2 + F_2(y')] - \Pi\Psi_3 = 0, \tag{20}$$

$$(1 - \Pi)EU'[y_2 + F_2(y'')] - \Psi_1 EU'[y_2 + F_2(y'')] + \Psi_2 EU'[y_2 + F_2(y'')] - (1 - \Pi)\Psi_3 = 0, \tag{21}$$

where $\Psi_1 \geqslant 0$, $\Psi_2 \geqslant 0$, $\Psi_3 > 0$.

It is not obvious from inspection of (18)–(21) that a private-information optimum can be easily characterized. But it is readily verified that at least one of the incentive-compatibility constraints (15) or (16) must be binding in an optimum. For suppose $\Psi_1 = \Psi_2 = 0$. Then (20) and (21) yield $F_2(y') = F_2(y'')$, and (18) and (19) yield

$$U'[y' + F_1(y')] = U'[y'' + F_1(y'')], \tag{22}$$

so that $F_1(y') > F_1(y'')$. But this specification will violate (16).[12]

It is now apparent that the two-period, borrowing-lending scheme analyzed at the outset of this section is nonoptimal: From the strict concavity of $U(\cdot)$ and the fact that $d(y')$ and $d(y'')$ are maximal for realizations y' and y'', respectively, $y' \neq y''$,

$$\begin{aligned} &U[y' + d(y')] + EU[y_2 - d(y')] \\ &> U[y' + d(y'')] + EU[y_2 - d(y'')], \end{aligned} \tag{23}$$

$$\begin{aligned} &U[y'' + d(y'')] + EU[y_2 - d(y'')] \\ &> U[y'' + d(y')] + EU[y_2 - d(y')]. \end{aligned} \tag{24}$$

But if we let $F_1(y') = d(y')$, $F_2(y') = -d(y')$, $F_1(y'') = d(y'')$, and $F_2(y'') = -d(y'')$, this specification satisfies both incentive-compatibility constraints (15) and (16) as inequalities.

Thus far we have a general proof that the borrowing-lending scheme is nonoptimal. But there is as yet little intuition as to the cause of nonoptimality or the nature of an optimal contract that Pareto dominates. To provide some intuition, further simplifying assumptions are made, namely, that the probability of loss parameter Π equals one-half and that the utility function of agent a is quadratic; that is, $U(c) = c - bc^2$.[13] Under these assumptions straightforward algebraic manipulation of condition (14) yields the solution to the borrowing-lending scheme, $d(y') = (y'' - y')/4$ and $d(y'') = (y' - y'')/4$. These transfers are half of the full-information optimal transfers and are equal except for sign, a kind of symmetry condition. To find a contract that Pareto dominates, return to problem 5 and impose there in addition these latter kinds of symmetry restrictions, that if the transfer to agent a in period one for $y_1 = y'$ is $+p_1$, then the transfer to agent a in period one for $y_1 = y''$ must be $-p_1$. Similarly, if the transfer to agent a in period two for $y_1 = y'$ is $-p_2$, then the transfer to agent a in period two for $y_1 = y''$ must be $+p_2$. Finally, let $K = W$ in (17) as an equality, consistent with the zero-profit condition of the borrowing-lending scheme. Also note that the symmetry restrictions require that

[12] This is a strong hint that only (16) will be binding in an optimum.

[13] I am much indebted to Gerald Faulhaber for suggesting the quadratic example which follows. It is assumed, of course, that the utility function is strictly increasing in the relevant range, and for purpose of the analysis which follows, the assumption that $U'(0) = \infty$ is dropped.

transfers be balanced across first-period states but not necessarily over time as in the borrowing-lending scheme. There is no proof given here that these symmetry restrictions are not binding in an optimum, a solution to problem 5; they may well be. But at least this procedure yields a contract that satisfies these restrictions and Pareto dominates the borrowing-lending scheme. Moreover, considerable intuition is provided for the nature of an (unrestricted) optimum.

As a matter of notation it is convenient to let $p_1 = \rho$ and $p_2 = \alpha\rho$. Then the incentive-compatibility conditions (15) and (16) can be written as

$$\begin{aligned} U(y' + \rho) - U(y' - \rho) &\geqslant EU(y_2 + \alpha\rho) - EU(y_2 - \alpha\rho) \\ &\geqslant U(y'' + \rho) - U(y'' - \rho) \end{aligned} \tag{25}$$

from the left and right inequalities, respectively. Under the simplifying assumptions (25) is equivalent with

$$\frac{U'(y')}{U'(\bar{y})} \geqslant \alpha \geqslant \frac{U'(y'')}{U'(\bar{y})}, \tag{26}$$

where $\bar{y} = (y'' + y')/2$. To be noted is that (26) is independent of ρ. Also, both sides of (26) cannot hold as equalities, and thus only one incentive-compatibility condition can be binding. Also note that under the symmetry restrictions (17) is satisfied identically and may be dropped as an explicit constraint in problem (5); though transfers need not be balanced over time, expected transfers are zero. Finally the objective function may be written

$$\begin{aligned} &\tfrac{1}{2}[U(y' + \rho) + \tfrac{1}{2}U(y' - \alpha\rho) + \tfrac{1}{2}U(y'' - \alpha\rho)] \\ &+ \tfrac{1}{2}[U(y'' - \rho) + \tfrac{1}{2}U(y' + \alpha\rho) + \tfrac{1}{2}U(y'' + \alpha\rho)]. \end{aligned} \tag{27}$$

Differentiating (27) with respect to α subject to both inequalities in (26) yields

$$\begin{aligned} &-\tfrac{1}{4}U'(y' - \alpha\rho)\rho - \tfrac{1}{4}U'(y'' - \alpha\rho)\rho + \tfrac{1}{4}U'(y' + \alpha\rho)\rho \\ &+ \tfrac{1}{4}U'(y'' + \alpha\rho)\rho + \mu - \lambda = 0. \end{aligned} \tag{28}$$

Under the simplifying assumptions the expression in marginal utilities on the left-hand side of (28) is negative; thus the nonnegative Lagrange multipliers λ and μ for the left and right inequalities in (26) must satisfy $\mu > 0$, $\lambda = 0$. (Recall both cannot be positive.) Thus the only binding incentive-compatibility constraint in this (restricted) optimum must be (16), at the high realization $y_1 = y''$. And thus from (26),

$$0 < \alpha = \frac{U'(y'')}{U'(\bar{y})} < 1. \tag{29}$$

The transfer ρ may be found by differentiating (27) with respect to ρ, which yields the solution

$$\rho = \frac{y'' - y'}{2(1 + \alpha^2)}. \tag{30}$$

It may now be clearer how the (restricted) optimum dominates the borrowing-lending scheme. The borrowing-lending scheme is of course optimal conditional on the realizations of y_1, but it fails to take into account the ex ante risk associated with uncertainty over y_1. Now an obvious first step in the direction of improving the allocation of risk is to maximize ex ante expected utility of the risk-averse agent subject to the restriction that $\alpha = 1$, that is, that the transfer be balanced over time as in the borrowing-lending scheme. It turns out, though, that here the solution to that maximization problem, which in general might be termed the ex ante modified borrowing-lending scheme, is just the borrowing-lending scheme itself. However, the borrowing-lending scheme is never optimal—(further) movement toward a full-information, first-period optimum is called for. That is, parameter ρ should be increased toward $(y'' - y')/2$ and parameter α decreased toward zero. Now this movement in α and ρ in a private-information optimum should not come as a surprise. After all, the borrowing-lending scheme was just a device to allow mutually beneficial trade in the context of private information. Borrowing-lending per se is not desirable; the full-information optimum is accomplished completely by risk-sharing arrangements. The movement toward complete first-period risk sharing in a private-information optimum is limited by the incentive of agent a to cheat at $y_1 = y''$. The greater is this incentive, as measured by values of $U'(y'')/U'(\bar{y})$, the greater is α, the more limited is first-period risk sharing, and the greater is the reliance on intertemporal tie-ins (of which borrowing-lending is an extreme case) to overcome incentive problems.

The welfare conclusions above may bring to mind some work on the inconsistency of optimal plans, for example, Kydland and Prescott (1977). Suppose that in a dynamic context agents necessarily take the best possible action period by period conditioned on beginning-of-period state variables. Thus, a global (multiperiod) optimum which requires that agents take nonmaximizing (inconsistent) actions period by period may be viewed as unattainable. To circumvent this problem, at least partially, it may be optimal to devise schemes or institutions under which agents constrain future actions, in effect, tie themselves to the mast. Of course, these institutional constraints may well be binding as the future unfolds, but schemes without that property may not be ex ante (constrained) optima. Similarly, in this paper it is natural to impose constraints which require truth telling, and thus a full-information, Pareto-optimal allocation may be viewed as unattainable. In this context the optimal ex ante arrangement is for agents to enter into multiperiod contracts which, in effect, constrain future actions. That is, as the future unfolds, there may well arise an event under which all agents may wish to renege on the arrangement. Simple borrowing-lending schemes which do not have that property are not ex ante, private-information Pareto optimal.[14]

V. Multiperiod Contracts and the Discount Rate

It has been established thus far that multiperiod contracts are mutually beneficial relative to a sequence of single-period contracts in the context of private information; that is, there are a variety of two-period contracts which Pareto improve upon autarky. On the other hand, it is equally clear that no two-period contract, or T-period contract for that matter, can achieve the utility of a full-information optimum; recall the implication of the last-period incentive-compatibility condition, that the last-period trade be noncontingent. It is in this sense that agents cannot completely overcome the barriers to trade introduced by private information. These results do raise the question, though, of whether agents can get arbitrarily close to the utility of a full-information optimum as contract length is extended to infinity. It turns out that this question can be answered affirmatively if agents do not discount the future. To see this, take advantage of the trivial insight that as contract length T approaches infinity, approximately fraction Π of the realizations under the endowment sequence $\{y_t\}$ will be y' and $(1 - \Pi)$ will be y''. So, if the length T is such that ΠT is an integer, give agent a the option of claiming a low realization, y', ΠT times in total, each such claim effecting a transfer $(c - y') \equiv d > 0$. If no claim is filed, agent a is to pay $(y'' - c) \equiv p > 0$. (Here c is determined as the solution to the full-information Pareto problem, problem 2.) Under this T-period contract, agent a has an obvious strategy, which turns out to be maximal in this case (essentially by Jensen's inequality): Claim a low realization whenever it occurs until either there are no claims remaining (in which case no more claims are filed to the end of the contract) or there are as many claims remaining as time periods (in which case a claim is filed in each period to the end of the contract). It can be established formally (see Townsend 1979a) that the average utilities for agents a and b under

[14] Here, however, unlike Kydland and Prescott (1977), schemes requiring agents to take arbitrary actions period by period are perfectly enforceable if they are consistent with the information structure (see n. 3).

this strategy have the desired limits, namely $\overline{U} = \Pi U(y' + d) + (1 - \Pi)U(y'' - p)$ and $\overline{V} = W - \Pi d + (1 - \Pi)p$, respectively.[15]

But what if agents *do* discount the future? Though no formal proof is offered here, it seems likely that such strong asymptotic results are vulnerable. It is revealing to note, for example, that the specified strategy of agent a in the contract just described would not be maximizing for sufficiently large T; a better strategy would be to claim that the first ΠT realizations are low, effecting the transfer $(c - y') > 0$. (The gain or loss of utility in the last $[1 - \Pi]T$ periods would matter little.) Nor would agent b be indifferent to the timing of claims. More generally, the nonexistence of an asymptotically full-information optimal contract would not come as a surprise; in many private-information contexts, full-information optimal allocations cannot be achieved, and here as well discounting might push agents back toward the best one-shot arrangement. Asymptotic results aside, however, the theory of multiperiod contracts being developed here is not vulnerable to discounting. As is now indicated, every major result of the paper is valid when agents discount the future. In particular, private information can still serve to explain the existence of multiperiod contracts, though the nature of contracts can be affected.

To begin this discussion, suppose both agents discount future utility at rate β, $0 < \beta < 1$. (More technically, suppose that β^t enters as a multiplicative factor on the t-period utilities of agents a and b and that time begins at $t = 0$.) Then it is straightforward to establish that the properties of a full-information optimal allocation will remain unchanged: the consumption of the risk-averse agent should be a constant over both time and endowment realizations. Similarly, the full-information utility possibilities frontier will be unchanged essentially, with utilities scaled by the factor $1/(1 - \beta)$, and any point on (a restricted portion of) this frontier could be supported with the same sequence of one-period contracts. Again, the introduction of private information is damning: with one-period contracts there would be no trade (the incentive-compatibility conditions remain unchanged apart from the factors β^t—in particular [10] has a β^T on both sides of the equation). On the other hand, there would still exist gains to trade under a two-period borrowing-lending scheme with interest rate r determined by $(1 + r) = 1/\beta$; in this case

$$d(y') = \frac{(y'' - y')\beta}{2(1 + \beta)}, \; d(y'') = \frac{(y' - y'')\beta}{2(1 + \beta)}.$$

[15] The T-period version of the two-period borrowing-lending scheme discussed in the previous section is also asymptotically optimal; again see Townsend (1979*a*).

Also, by a proof virtually identical to the one in Section IV, such borrowing-lending schemes are not private-information Pareto optimal. In fact for the quadratic utility example in Section IV, a (restricted) private-information optimum, characterized by the parameters ρ and α, takes the form

$$0 < \alpha = \frac{U'(y'')}{\beta U'(\bar{y})} < \frac{1}{\beta}, \rho = \frac{(y'' - y')}{2(1 + \beta\alpha^2)}.$$

Here again the ex ante modified borrowing-lending scheme with $\alpha = 1/\beta$ does not produce an increase in welfare over the simple borrowing-lending scheme, but again less extreme intertemporal tie-ins and more complete first-period risk sharing are called for. It is interesting to note, however, that the more the future is discounted (the lower is β), the greater is the incentive to cheat at $y_1 = y''$, that is, the greater is α (though α remains a specified fraction of the rate $1/\beta$), and the less complete is first-period risk sharing, that is, the lower is ρ. Thus, the more the future is discounted, the less effective are intertemporal tie-ins in overcoming incentive problems. This does alter the nature of the optimal contract.

Appendix

Given some T-period contract, $\{M_t(m^{t-1})\}_{t=1}^T$, $\{F_t(m^{t-1}, m_t)\}_{t=1}^T$, let $\{\sigma_t(m^{t-1}, y_t)\}_{t=1}^T$ solve problem 3. Let $W_t(m^{t-1})$ denote the total expected utility of agent a from periods t through T, as of the beginning of period t, prior to the realization of y_t, under the maximizing decision rules $\{\sigma_s(m^{s-1}, y_s)\}_{s=t}^T$. The sequence $\{W_t(m^{t-1})\}_{t=1}^T$ must satisfy the functional equation

$$W_t(m^{t-1}) = \sum_{y_t} \Pr(y_t)\{ \max_{m_t \in M_t(m^{t-1})} U[y_t + F_t(m^{t-1}, m_t)] + W_{t+1}(m^{t-1}, m_t)\}$$

with $W_{T+1} \equiv 0$. Then since $\sigma_t(m^{t-1}, y_t)$ is maximizing given the realization y_t, it follows that

$$\begin{aligned} &U\{\tilde{y}_t + F_t[m^{t-1}, \sigma_t(m^{t-1}, \tilde{y}_t)]\} + W_{t+1}[m^{t-1}, \sigma_t(m^{t-1}, \tilde{y}_t)] \\ &\geq U\{\tilde{y}_t + F_t[m^{t-1}, \sigma_t(m^{t-1}, \hat{y}_t)]\} + W_{t+1}[m^{t-1}, \sigma_t(m^{t-1}, \hat{y}_t)] \end{aligned} \tag{A1}$$

for all m^{t-1} and for all $\tilde{y}_t, \hat{y}_t \in \{y', y''\}$. Note that on the right-hand side of (A1), the maximizing decision rule is evaluated at some counterfactual realization $\hat{y}_t$. Thus (A1) expresses a crucial incentive-compatibility condition: Prescribed decision rules for the agent with private information must be consistent with maximization.

The sequence of generated messages under the specified contract, $\{m_t^*(y^t)\}_{t=1}^T$, may be defined recursively by

$$m_t^*(y^t) = \sigma_t[m^{*t-1}(y^{t-1}), y_t]. \tag{A2}$$

The sequence of generated trades is then simply $\{F_t[m^{*t}(y^t)]\}_{t=1}^T$. Now (A1) may be evaluated along any of these generated paths to yield

$$\begin{aligned} &U(\tilde{y}_t + F_t\{m^{*t-1}(y^{t-1}), \sigma_t[m^{*t-1}(y^{t-1}), \tilde{y}_t]\}) \\ &\quad + W_{t+1}\{m^{*t-1}(y^{t-1}), \sigma_t[m^{*t-1}(y^{t-1}), \tilde{y}_t]\} \\ &\geq U(\tilde{y}_t + F_t\{m^{*t-1}(y^{t-1}), \sigma_t[m^{*t-1}(y^{t-1}), \hat{y}_t]\}) \\ &\quad + W_{t+1}\{m^{*t-1}(y^{t-1}), \sigma_t[m^{*t-1}(y^{t-1}), \hat{y}_t]\} \end{aligned} \tag{A3}$$

for all y^{t-1} and for all $\tilde{y}_t, \hat{y}_t \in \{y', y''\}$. Now consider an alternative game. Define $\hat{M}_t(m^{t-1}, m_t) \equiv \{y', y''\}$ and define

$$\hat{F}_t(y^{t-1}, y_t) \equiv F_t\{m^{*t-1}(y^{t-1}), \sigma_t[m^{*t-1}(y^{t-1}), y_t]\} \tag{A4}$$

for all possible named values y^{t-1}, y_t. Let $X_t(y^{t-1})$ denote the total expected utility of agent a in this alternative game as of the beginning of time t through time T, prior to the realization y_t, as a function of previous messages y^{t-1}, under the truth-telling decision rules $\hat{\sigma}_s(y^{s-1}, y_s) = y_s$, $s = t, t + 1, \ldots, T$, with $X_{T+1} \equiv 0$. Now it may be verified using the recursion formulas defining $m_t^*(y^t)$ and the definition (A4) that

$$W_{t+1}\{m^{*t-1}(y^{t-1}), \sigma_t[m^{*t-1}(y^{t-1}), \tilde{y}_t]\} = X_{t+1}(y^{t-1}, \tilde{y}_t), \tag{A5}$$

and similarly for $\hat{y}_t$. Substituting (A5) and (A4) into (A3) generates

$$U[\tilde{y}_t + \hat{F}_t(y^{t-1}, \tilde{y}_t)] + X_{t+1}(y^{t-1}, \tilde{y}_t) \geq U[\tilde{y}_t + \hat{F}_t(y^{t-1}, \hat{y}_t)] + X_{t+1}(y^{t-1}, \hat{y}_t) \tag{A6}$$

for all y^{t-1} and for all $\tilde{y}_t, \hat{y}_t \in \{y', y''\}$. That is, the truth-telling decision rule is maximizing for agent a in this alternative game (work backward from period T). Applying this decision rule from $t = 1$ onward, it is clear that the sequence of announced endowment values will equal the sequence of actual realizations. Relationship (A4) then guarantees the desired result.

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