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## Keeping Up with the Joneses: Consumption Externalities, Portfolio Choice, and Asset Prices

THE PRESENT PAPER TRIES TO formalize the notion of *investment in financial assets as a social activity* by introducing a particular type of consumption externalities in otherwise standard portfolio and asset pricing models.

The “social aspects” of portfolio decisions, generally ignored in traditional financial models, have been stressed by authors like Shiller (1984), but have seldom been formally modeled in a way consistent with rational behavior.<sup>1</sup> The social nature of portfolio decisions in the models studied in this paper arises from the presence of *consumption externalities*: agents have preferences defined on their own consumption, as well as average (or per capita) consumption in the economy.<sup>2</sup> This allows for the idea that households care about their relative standard of living or, as the saying goes, they want “to keep up with the Joneses.”<sup>3</sup> Such consumption externalities are introduced in two otherwise standard models—a static CAPM model, and a multiperiod asset pricing model—and their impact on optimal portfolio decisions and equilibrium asset prices are analyzed.

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1. An exception, further discussed below, is given by Abel (1990).

2. Such a hypothesis may be given an alternative interpretation: agents in the model can be thought of as professional “portfolio managers” whose performance is evaluated in terms of the return on their portfolio relative to the rest of managers and/or the market. This interpretation is particularly appealing given the widespread perception of portfolio managers’ behavior as underlying much of the volatility observed in financial markets (for example, Sharfstein and Stein 1988).

3. Consumption externalities (also referred to as interdependent preferences) were first introduced by Duesenberry (1952) in order to reconcile cross-sectional and time series evidence on consumption.

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The main results can be summarized as follows. In the context of the CAPM model, the presence of consumption externalities has two related effects. First, the optimal risky share can be either larger or smaller than in the standard model, depending on the sign of the externalities. Second, a change in the risk-adjusted equity premium is associated with a larger (smaller) adjustment of investors' portfolios, relative to the no-externalities case.

Under our assumptions, the presence of consumption externalities in the multi-period economy yields a basic equivalence result: equilibrium asset prices and returns in such an economy are identical to those of an externality-free economy with a properly adjusted degree of risk aversion. That result is compared to Abel (1990), in which agents' preferences depend on *lagged*—but not current—per capita consumption.

The paper is structured as follows. Section 1 examines the implications of consumption externalities in the static CAPM model. Section 2 extends the analysis to a multiperiod model. Section 3 discusses the results and concludes.

## 1. CONSUMPTION EXTERNALITIES IN THE CAPM MODEL

Consumers have an initial endowment that is to be allocated between two assets: a risky asset (“equity”) yielding a random (gross) return  $Z$ , and a riskless asset (“debt”) with (gross) return  $R$ . At the end of the period all portfolio payoffs are consumed.

The representative household solves the following problem:

$$\max_{\lambda} E U(c, C) \tag{1}$$

subject to

$$c = w(R + \lambda x)$$

where  $c$  denotes the household's own consumption level at the end of the period, and  $C$  is the average (or per capita) consumption level in the economy. The distribution for the latter variable is taken as given by each household.  $w$  denotes initial wealth,  $\lambda$  is the “risky share” (that is, the fraction of wealth allocated to equity), and  $x \equiv Z - R$  is the (ex-post) difference between equity and debt returns. We assume  $x$  is an exogenous random variable with a distribution function  $F(x)$ .

The first-order condition for the problem above is

$$E U_1( w(R + \lambda x), W(R + \Lambda x) ) x = 0 \tag{2}$$

where  $\Lambda$  denotes the *aggregate* risky share, and  $W$  is per capita wealth. From now on we assume  $w = W$ .

Given  $W$  and  $R$ , (2) implicitly determines the consumer's optimal risky share as a function of  $\Lambda$  and  $F(x)$ . We use  $\lambda = \Phi [ \Lambda ; F(x) ]$  to represent that mapping. In a symmetric equilibrium, and for a given distribution  $F(x)$ , the risky share chosen by each household (and thus the aggregate risky share) is given by a fixed point  $\lambda^*[F(x)]$  of the  $\Phi$  functional, that is, a  $\lambda$  value satisfying  $\lambda^* = \Phi [ \lambda^* ; F(x) ]$ .

Before proceeding with our analysis, we specialize the utility function to be of the form

$$U(c, C) = (1 - \alpha)^{-1} c^{(1-\alpha)} C^{\gamma\alpha} \quad \alpha > 0, \gamma < 1 \tag{3}$$

which satisfies the following properties:

$$U_1(c, C) > 0, \text{ all } c, C \geq 0; \tag{A1}$$

$$U_{11}(c, C) \leq 0, \text{ all } c, C \geq 0; \tag{A2}$$

$$- U_{11}(c, c) c / U_1(c, c) = \alpha, \text{ all } c \geq 0; \tag{A3}$$

$$U_{12}(c, c) c / U_1(c, c) = \alpha \gamma, \text{ all } c \geq 0; \tag{A4}$$

(A1) and (A2) are standard properties, implying that  $U$  is an increasing and concave function of own-consumption  $c$ , for any given level of average consumption. The left-hand-side term in (A3) can be thought of as a measure of relative risk aversion (around a symmetric equilibrium). Finally, (A4) implies that the *average-consumption* elasticity of marginal utility (around a symmetric equilibrium) is constant, and given (without loss of generality) by a multiple  $\gamma$  of  $\alpha$ .

The sign of  $\gamma$  plays a crucial role in determining the effects of consumption externalities. When  $\gamma > 0$ , an increase in average consumption raises the marginal utility of an individual household's own consumption: any given addition to his current level of consumption becomes more valuable, for it is needed to "keep up with the Joneses." For convenience, we refer to this type of externalities as *positive consumption externalities*. Alternatively, if  $\gamma < 0$ , other households' consumption behaves as a "substitute" for each household's own consumption: increases in  $C$  lower the marginal utility of own consumption. In that case we can think of the economy's single good as showing some "public good" features. We use the expression *negative consumption externalities* to refer to this type of externalities. Notice that the intensity of any eventual positive externalities (relative to the degree of risk aversion) is bounded above by the assumption  $\gamma < 1$ , in order to guarantee the existence of a symmetric equilibrium.

For small values of  $Ex$ ,  $\Phi [ \Lambda ; F(x) ]$  and  $\lambda^*[F(x)]$  can be approximated<sup>4</sup> as a

4. Note that  $U_1(c, C) \cong U_1(W, W) + U_{11}(W, W)W(R - 1 + \lambda x) + U_{12}(W, W)W(R - 1 + \Lambda x) = U_1(W, W) [ -\alpha(R - 1 + \lambda x) + \alpha\gamma(R - 1 + \Lambda x) ]$ . An approximation to the first-order condition is then obtained by multiplying the above expression by  $x$ , taking expectations, and equating the resulting expression to zero. The linear version of  $\Phi$  follows trivially by solving for  $\lambda$ , and using the fact that  $\sigma_x^2 \cong Ex^2$  and  $(R - 1)Ex \cong 0$  for small values of  $Ex$  and the (net) riskless rate.

function of  $\alpha$ ,  $\gamma$ , and the risk-adjusted equity premium  $\Omega \equiv Ex/\sigma_x^2$ , where  $Ex$  and  $\sigma_x^2$  are, respectively, the mean and variance of  $x$ :

$$\Phi [ \Lambda ; F(x) ] \cong \gamma \Lambda + ( \Omega / \alpha ) ;$$

$$\lambda^*(F(x)) \cong \Omega / \alpha ( 1 - \gamma ) .$$

Note that when  $\gamma = 0$  the optimal risky share is independent of  $\Lambda$ , and given by  $\Omega/\alpha$ , as in the standard mean-variance model (for example, Friend and Blume 1975). When  $0 < \gamma < 1$  we have  $\lambda^* > \Omega/\alpha$ , whereas  $\lambda^* < \Omega/\alpha$  obtains whenever  $\gamma < 0$ . Thus, in a symmetric equilibrium, the presence of positive consumption externalities tends to increase the risky share in the optimal portfolio, whereas negative externalities tend to reduce it. Furthermore, we see that  $\partial\lambda^*/\partial\Omega > (1/\alpha)$  if  $0 < \gamma < 1$ , while  $\partial\lambda^*/\partial\Omega < (1/\alpha)$  if  $\gamma < 0$ . In words, when consumption externalities are positive (negative), a change in the risk-adjusted equity premium is associated—in a comparative statics sense—with a larger (smaller) adjustment of investors' portfolios, relative to the no-externalities case.

To understand the mechanism underlying the previous results it is useful to consider the effect of an exogenous increase in  $\Lambda$  on the individual investor's problem: in the presence of positive (negative) externalities,  $U_1$  will be higher (lower) in "good times"—corresponding to large realizations of  $x$ —, and lower (higher) in bad times—that is, for small realized values of  $x$ . Accordingly, the optimal individual risky share will increase (decrease) when the aggregate risky share is higher.

Assuming that riskless debt is in zero net supply,  $\lambda^* = 1$  must hold in equilibrium. Accordingly, the equity premium  $Ex$  will be approximately given by

$$Ex = \alpha ( 1 - \gamma ) \sigma_x^2 .$$

Given  $\alpha$  and  $\sigma_x^2$ , positive externalities ( $0 < \gamma < 1$ ) tend to reduce the size of the equity premium investors require to absorb the entire equity supply. In contrast, negative externalities ( $\gamma < 0$ ) tend to increase that equity premium. Those results follow from the effective shift in the share of risky assets demanded for each level of  $\Omega$ , caused by the presence of consumption externalities (as discussed above).

The next section generalizes some of the previous results by examining the role of consumption externalities in a more general asset pricing model.

## 2. CONSUMPTION EXTERNALITIES IN A MULTIPERIOD ASSET PRICING MODEL

Consider a Lucas (1978) economy with a single perishable good, and a number of identical infinite-lived consumers with separable preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, C_t) \tag{4}$$

where  $c$  and  $C$  are, respectively, own-consumption and per capita consumption,  $0 < \beta < 1$  is a constant discount factor, and  $U$  is the utility functional specified in (3). There exist  $K$  assets indexed by  $k = 1, 2, \dots, K$ . Asset  $k$  yields a (possibly random) payoff sequence  $\{d_{k,t}\}_{t=0}^{\infty}$ , and trades at an (ex-dividend) price  $p_{k,t}$  in period  $t$ . Given a sequence of random vectors  $\{(d_{1,t}, \dots, d_{K,t})\}'_{t=0}^{\infty}$ , the following first-order condition must be satisfied in a symmetric equilibrium:

$$U_1(c_t, c_t) p_{k,t} = \beta E_t [U_1(c_{t+1}, c_{t+1}) (p_{k,t+1} + d_{k,t+1})] . \tag{5}$$

Ruling out speculative bubbles,<sup>5</sup> (5) can be solved recursively to yield the equilibrium pricing equation:

$$p_{k,t} = E_t \sum_{j=0}^{\infty} \beta^j [U_1(c_{t+j}, c_{t+j})/U_1(c_t, c_t)] d_{k,t+j} \tag{6}$$

for  $k = 1, 2, \dots, K$ , and  $t = 0, 1, 2, \dots$ .

Consider next an economy without externalities, but otherwise identical to the economy just described. The representative consumer in the externality-free economy maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t V(c_t) \tag{7}$$

where  $V(c) = (1 - \sigma)^{-1} c^{(1-\sigma)}$ , that is,  $V$  is a standard CRRA utility function with relative risk aversion coefficient  $\sigma$ . Letting  $p_{k,t}^*$  denote the price of asset  $k$  at time  $t$  in the externality-free economy, the following equilibrium pricing equation holds<sup>6</sup> (for example, Lucas 1978):

$$p_{k,t}^* = E_t \sum_{j=0}^{\infty} \beta^j [V'(c_{t+j})/V'(c_t)] d_{k,t+j} . \tag{8}$$

The following proposition formalizes a simple equivalence result concerning the behavior of asset prices in the two economies introduced above.

**PROPOSITION:** *Consider two multiperiod economies, with preferences represented, respectively, by (4) and (7). Given a random sequence  $\{d_{1,t}, \dots, d_{K,t}\}'_{t=0}^{\infty}$  common to both economies  $p_{k,t} = p_{k,t}^*$  holds for  $k = 1, 2, \dots, K$ , and  $t = 0, 1, 2, \dots$ , if and only if  $\sigma = \alpha(1 - \gamma)$ .*

5. To be precise, and given the assumption of a finite number of identical consumers, speculative bubbles can be ruled out for finite-lived assets and for perpetual assets in positive net supply, but not for perpetual assets in zero net supply. Our discussion thus excludes the latter.

6. Again, the statement refers to assets for which speculative bubbles can be ruled out (see previous footnote).

PROOF: See appendix.

The intuition for the previous result is straightforward: given the dividend process, asset prices are determined by the marginal rate of substitution (MRS) between current consumption and consumption at all future dates. In the no-externalities case that MRS is given by  $\beta^j(c_t/c_{t+j})^{-\sigma}$ ,  $j = 1, 2, \dots$ , whereas in the externalities case the same variable is given in the symmetric equilibrium by  $\beta^j(c_t/c_{t+j})^{-\alpha(1-\gamma)}$ . Thus, equilibrium asset prices in an economy with externalities and preferences satisfying (4) are identical to those in an economy with the same dividend process, no externalities, and CRRA preferences with a risk-aversion coefficient  $\sigma = \alpha(1 - \gamma)$ . Changes in the intensity of the externality will have the same effect on the time series properties of asset prices as a (proper) change in the degree of risk aversion in the externality-free economy.

The previous result has several implications. First, the introduction of consumption externalities is unlikely to account for the observed "excess volatility" in stock prices (for example, Shiller 1981), for changes in consumption-based discount rates brought about by a modification in the risk-aversion parameter fail to account for that volatility (Campbell and Shiller 1989, Cochrane 1989).<sup>7</sup> Second, given the degree of relative risk aversion  $\alpha$ , the externality parameter  $\gamma$  will be inversely related to the equity premium. This follows from the proposition above and the fact that the equity premium is positively related to the degree of risk aversion in the absence of externalities (for example, Mehra and Prescott 1985). Thus, under our specification of preferences, positive (negative) externalities ( $\gamma > 0$ ) would tend to reduce (increase) the equity premium. For any level of risk aversion, the large equity premium observed in the U.S. data could be generated by introducing large enough negative externalities. Unfortunately, that would only provide a partial solution to the "equity premium" puzzle (Mehra and Prescott 1985), for stronger negative externalities, having effects on asset prices equivalent to those of a higher level of risk aversion, would generate too high a riskless rate.<sup>8</sup>

At this point it may be useful to compare our model and results to those in Abel (1990). In the latter paper the utility function is assumed to depend on contemporaneous own consumption as well as *lagged* per capita consumption, with the latter feature motivating the use of the phrase "catching up with the Joneses" in the title. The utility function analyzed by Abel takes the form<sup>9</sup>  $U(c_t, C_{t-1}) = (1 - \alpha)^{-1} (c_t/C_{t-1})^{(1-\alpha)}$ . Under the assumption of i.i.d. consumption and a single stock with  $d_t = c_t$ , all  $t$ , Abel computes the average return on that stock and (zero net supply) riskless bonds, corresponding to different values of  $\alpha$ . Interestingly, under the Abel assumptions increases in  $\alpha$  can generate a higher equity premium without simul-

7. Given the portfolio-manager interpretation of our model's externalities suggested above, that result would question the view that blames the observed market volatility on the existing evaluation systems for portfolio managers.

8. The riskless rate in the economy with externalities is given by  $R_t = 1 / \{ \beta E_t(c_t/c_{t+1})^{-\alpha(1-\gamma)} \}$ . In the presence of positive average consumption growth, a low value of  $\gamma$  will correspond to a higher riskless rate.

9. Though a more general utility function is initially assumed by Abel, including "habit formation" effects, this is the function for which the results of the model with consumption externalities (and thus the model relevant to this paper) are finally reported and discussed.

taneously raising the riskless rate, thus providing a potential solution to the equity premium puzzle. Unfortunately, as acknowledged by Abel, a new “puzzle” arises: the standard deviation of the riskless rate implied by the “catching up” specification becomes implausibly large. A comparison of Abel’s results and ours suggests, in any event, that the effect of consumption externalities on asset price behavior seems to be highly sensitive to the specification of those externalities. This can be illustrated by recasting Abel’s utility function in a way consistent with our model above. This is done by setting  $U(c_t, C_t) = (1 - \alpha)^{-1} (c_t/C_t)^{(1-\alpha)}$ . Applying the proposition above one can easily show that asset prices in an economy with such preferences are identical to those in an externality-free economy with *log* utility ( $\sigma = 1$ ), independently of the value of  $\alpha$ . Changes in  $\alpha$  will thus affect neither stock nor bond returns.

Although, on an a priori basis, we find our utility specification (using *contemporaneous* per capita consumption) more appealing than Abel’s, we see both this paper and Abel’s as just preliminary steps toward a more thorough understanding of the effects of consumption externalities on asset pricing and portfolio choice. An analysis of more general preference specifications involving those externalities, as well as the study of their effects in models of the business cycle are among the extensions currently in our agenda.

### 3. SUMMARY AND CONCLUSIONS

We have examined the effects of consumption externalities on portfolio decisions and equilibrium asset prices. In the CAPM model, the presence of consumption externalities makes the optimal risky share either larger or smaller than in the standard model, depending on the sign of the externalities. In addition, a change in the risk-adjusted equity premium is associated with a larger (smaller) adjustment of investors’ portfolios, relative to the no-externalities case.

The introduction of consumption externalities in a multiperiod asset pricing model yields, under our assumptions, a basic equivalence result: equilibrium asset prices and returns in an economy with externalities, are identical to those of an externality-free economy with a properly adjusted degree of risk aversion.

#### APPENDIX: PROOF OF PROPOSITION

(Necessity) Assume  $p_{k,t} = p_{k,t}^*$  holds for all  $t$  and  $k$ , conditional on any process  $\{(d_{1,t}, \dots, d_{k,t})\}_{t=0}^{\infty}$ . Then it must be the case that  $V'(c) = \xi U_1(c, c)$  for all  $c$ , with  $\xi$  being an indeterminate constant. Consequently,  $V''(c) = \xi [U_{11}(c, c) + U_{12}(c, c)]$ , all  $c$ . Multiplying both sides by  $-c/V'(c)$  the desired result,  $\sigma = \alpha(1 - \gamma)$ , follows trivially.

(Sufficiency) Conversely, assume  $\sigma = \alpha(1 - \gamma)$ . By definition, we thus have  $-V''(c)c/V'(c) = -[U_{11}(c, c) + U_{12}(c, c)] c/U_1(c, c)$ , for any  $c \geq 0$ . Dropping the  $c$  factor and integrating both sides of the equality we obtain  $\log V'(c) = \log U_1(c, c)$

+  $\xi'$ , where  $\xi'$  is an indeterminate constant. It follows that  $V'(c) = \xi U_1(c, c)$  for any  $c$ , with  $\xi \equiv \exp(\xi') > 0$ . Then (6) and (7) imply that  $p_{k,t} = p_{k,t}^*$  holds for all  $t$  and  $k$ , conditional on a given  $\{(d_{1,t}, \dots, d_{K,t})\}_{t=0}^{\infty}$  process. QED.

## LITERATURE CITED

- Abel, Andrew B. "Asset Prices under Habit Formation and Catching Up with the Joneses." *American Economic Review* 80 (May 1990), 43–47.
- Campbell, John Y., and Robert J. Shiller. "The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors." *The Review of Financial Studies* 1 (September 1989), 195–228.
- Cochrane, John H. "Explaining the Variance of Price Dividend Ratios." National Bureau of Economic Research Working Paper # 3157 (1989).
- Duesenberry, James S. *Income, Saving, and the Theory of Consumer Behavior*. Cambridge: Harvard University Press, 1952.
- Friend, Irwin, and Marshall E. Blume. "The Demand for Risky Assets." *American Economic Review* 65 (December 1975), 900–22.
- Lucas, Robert E. "Asset Prices in an Exchange Economy." *Econometrica* 46 (November 1978), 1429–45.
- Mehra, Rajnish, and Edward C. Prescott. "The Equity Premium: A Puzzle." *Journal of Monetary Economics* 15 (March 1985), 145–62.
- Sharfstein, David S., and Jeremy C. Stein. "Herd Behavior and Investment." Mimeo (1988).
- Shiller, Robert J. "Do Stock Prices Move Too Much to Be Justified by Subsequent Changes in Dividends?" *American Economic Review* 71 (June 1981), 421–36.
- . "Stock Prices and Social Dynamics." *Brookings Papers on Economic Activity* 2 (1984), 457–98.