Culture and Control: Should There Be Large Subsidies to Culture?*

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March 18, 2005

Abstract

The studies of culture are motivated by the special treatment of culture consumption in most modern societies: there are usually large, government-provided subsidies, the aim of which is to stimulate both the production and the consumption of culture. The purpose of the present work is to explore reasons for this special treatment. Using a stylized theoretical framework, the essays contrast culture with another, generic, good or activity. Culture is thus regarded as an "experience good": previous consumption of the good enhances the current appreciation of the good. The generic good is one where experience is assumed not to be at all relevant for the appreciation of the good. For experience goods, decisions made today will influence future utility and future choices. This makes the intertemporal preferences essential. If, in particular, consumers have time-inconsistent preferences of the type that can be characterized as a present-bias—modeled with "multiple selves" using quasi-geometric discounting—as opposed to standard, time-consistent preferences, there will be a case for government subsidies. The present paper studies the circumstances under which public support for culture is warranted. A policy example is designed to illustrate important aspects of public support systems currently in place, and is calibrated to Swedish data. The essay concludes that, given present-biased agents with self-control problems, public support of culture can work as a commitment device and improve long-run welfare. Furthermore, it is demonstrated that welfare-maximizing subsidies to culture can be substantial if the present-bias is profound and the taste-cultivation property of culture consumption is pronounced.

JEL Classification

Keywords: Culture, Time Inconsistency

*I thank Per Krusell, Henrik Horn, Harry Flam, Jonas Björnerstedt, Daria Finocchiaro, John Hassler and participants at the IIES workshop for valuable comments and suggestions. I also thank Christina Lönnblad for excellent editorial assistance. Usual disclaimer applies. Financial support from the Jan Wallander and Tom Hedelius Foundation is gratefully acknowledged.

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1 Introduction

In most Western countries, public support to culture is substantial, including subsidies to the performing arts, maintenance of museums and, in some countries, salaries paid directly to creative artists and tax exemptions for private donations to the higher arts. The form of cultural expression that appears to be receiving the bulk of public support is sometimes referred to as highbrow culture, including opera, drama, ballet and classical music. The public subsidies are frequently debated and questioned, especially since redistribution often favors a relatively wealthy and well-educated minority.

To some debaters, it seems self-evident that the fine arts constitute a “special case”, meriting special treatment and special encouragement from the government, whereas others, among them many economists, do not see any intrinsic qualities making cultural goods more special than any other good.

Over the years, there have been some concerns about underconsumption, or even the survival, of highbrow culture. Economists such as William Baumol and Staffan Burenstam-Linder have addressed these issues. They have examined potential extinction threats of highbrow culture by focusing on its production side. Various empirical studies have tried to establish trends in the performing arts markets, with mixed results. Attempts at finding a rationale for public support systems have been extensive. The justifications for support for culture have been based on various grounds, such as various inefficiencies or forms of market failure, externalities, the “market for lemons”-problems, or meritocracy; none of these arguments seem to have gained full acceptance, however, neither theoretically nor empirically. Some of the most popular arguments seem to be based on paternalism and irrationality of agents, which is certainly not in accordance with the standard analytical frameworks used in economics. Moreover, the arguments based on paternalism and irrationality are also not systematically founded either on theory or on empirical analysis.

The overall objective of this study is to put forward a theoretical framework specifying how the demand for culture (i) is qualitatively different than the demand for other goods and (ii) might be subject to a form of inefficiency not previously been proposed in this context. As part of the study, a policy analysis is performed and various policy instruments are discussed. An example is designed to illustrate some substantively important aspects of public support systems in practice. To assess the quantitative magnitudes, the model is simulated and calibrated to match Swedish data.

Cultural goods and services are often claimed to possess a number of properties distinguishing them from other goods. The role of "experience" in consuming culture is often mentioned, as are "intrinsic values" in consumption such as option and bequest values. Moreover, there are arguments for there being large economies of scale in culture production, externalities in the form of "spin-off effects", network externalities, public good characteristics, and so on. In the present study, the focus is on culture demand; I entirely abstract from the supply side. The focus chosen is that of viewing cultural goods as experience goods. The idea that culture is "hard to appreciate" without training/prior consumption is rather commonly accepted, and though there are also other goods with similar characteristics that might not be labeled culture goods, it seems useful to use the experience aspect to distinguish culture goods from other goods.

In the model, there are two kinds of goods or activities: culture goods and generic goods. They can also be considered as "highbrow" culture goods and "mass culture", or "lowbrow" culture goods. Significant for the cultural good is the taste cultivation property, which means that past consumption has a positive effect on current and future consumption decisions. Put differently, a refined/experienced mind is needed to grasp culture. The generic good, in contrast, gives instant utility but generates no intertemporal value-added. The taste cultivation assumption adds an investment aspect to cultural consumption: current consumption of culture is beneficial for current utility but also gives the byproduct of enhancing the future appreciation of culture. Being forward-looking is of importance for the appreciation of this second aspect.

The idea of culture as an experience good is not new. In *Principles of Economics*, Alfred Marshall writes

> It is therefore no exception to the Law (of diminishing marginal utility) that the more good music a man hears, the stronger is his taste for it likely to become.

In an article in the New York Times, Ernest van den Haag sought to distinguish lowbrow culture from highbrow culture.[1] He suggested that lowbrow culture provides entertainment rather than the sort

[1] van den Haag labels them "popular culture" and "high culture".

of gratification provided by genuine art. Highbrow culture is "aesthetically superior" in that it "reveals and changes one's experience of reality and possibility", while popular culture provides entertainment which "diverts the tired businessman or worker from reality".[2] A number of empirical studies have been undertaken to confirm this property of culture.[3] These studies, which use a wide variety of data, in most cases compiled from audience surveys, have in general affirmed that early exposure to the arts results in later participation.

George Stigler and Gary Becker (1977) tried to interpret and model Marshall's idea by adopting Becker's household production model to study the cultivation of taste, where the household maximizes a utility function of commodities (interpreted as the "appreciation" of a good) that is produced with market goods, time, skills, training and other human capital. The consumption of certain "addictive" goods leads to the accumulation of "consumption capital", which generates changes in taste over time by changing marginal utilities. Becker and Kevin M. Murphy (1988) develop a theory of rational addiction related to the Stigler and Becker model, according to which past consumption of an addictive good directly affects current utility through the consumption capital which, in this case, is an argument in the utility function, and does not have any effect via a household production function. These two studies are often cited as possible model frameworks for studies of culture.

The assumption of culture as an experience good produces intertemporal effects, since current consumption decisions will affect past and future utility. A recent literature points to certain "intrapersonal frictions" in making forward-looking decisions: consumers' attitudes toward the future are argued to be "time-inconsistent". In short, time inconsistency means that a consumer heavily discounts the future, but does not apply this heavy discounting between adjacent future dates. That is, in the eyes of a consumer today, the comparison of today and tomorrow—with a strong bias for today—is very different than the comparison between tomorrow and the day after tomorrow—where tomorrow is only viewed as marginally more important. Thus, a plan made currently for accumulation decisions tomorrow—deciding on allo-

[2]Cited from David, Cwi, "Public Support of the Arts: Three Arguments Examined", Chapter 31, Cultural Economics II. Original reference in van den Haag, Ernest, "Should We Subsidize Popular Arts? No - An Elitist View", New York Times, February 9, 1975, Section II,1,17.

[3]For example, Abbé-Decarroux and Grin (1992), Cameron (1999), Dobson and West (1988), Ekelund and Ritenour (1999), Gray (1998), Kurabayashi and Ito (1992), Lévy-Garbou and Montmarquette (1996), McCain (1995), Morrisson et al (1996), Prieto-Rodriquez and Fernández-Blanco (2000), Smith (1998) and West and McKee (1983).

cations between tomorrow and the times after that—would be abandoned tomorrow, when present-bias once more takes over. There is a recent literature, motivated by psychological and experimental evidence, dealing with "quasi-hyperbolic" discounting, which is a straightforward way of modeling preferences with this kind of time inconsistency. This form of discounting sets up a conflict between the preferences of different "intertemporal selves", and introduces a need for commitment. It seems reasonable to deduce that time-inconsistency might have important implications on the consumption of experience goods such as culture goods since, unlike standard goods, these goods give intertemporal utility effects. Intuitively, this phenomenon can be represented by the guilty conscience people tend to show when asked about their recent record of attending the theatre or the opera: "Not as often as I should" seems to be the prevalent answer, at least in my own experience.

Jonathan Gruber and Botond Köszegi (2001) integrate the Becker-Murphy framework with the Laibson version of hyperbolic discounting in order to study the implications of incorporating time-inconsistent preferences into models of addiction. They find that although the forward-looking behavior generated by time-inconsistent agents is consistent with the standard version of the model with time-consistent preferences, the policy implications can radically diverge. Instead of the benchmark result that the optimal tax on cigarettes only depends on the associated externalities of this product, they find that governmental intervention should also internalize the "internalities" caused by lacking self-control.

The theoretical approach of the present model is closely related to the Gruber and Köszegi model. Although cultural consumption and negative addiction can both be studied within a Becker-Murphy framework, they are two distinct goods, with crucial differences in partial derivatives which give significant qualitative and quantitative differences in economic behavior. Here, there is no direct effect on future utility of present culture consumption: it only appears if culture is consumed in the future. Thus, the intertemporal effects of culture consumption are rather narrow and precise here, with the entire focus on changing the marginal utilities of consuming culture over time.

There could be direct effects on future utility of current culture consumption. However, interpreting direct utility effects of the capital stock is not as straightforward as in the case of a negative addiction, where it can be explained as long-run negative health effects. Here, positive utility from previous consumption of culture could, for example, be considered as accumulated human capital or some sort of "life knowledge", i.e., insights about how to live a better life. It would be interesting to formalize these effects, which in themselves form an additional hypothesis, but this is an issue better considered for a separate paper.

The present study specifies how a lower degree of forward-looking, or patience, as well as a present-bias reduce the steady-state level of consumption of culture and increase the consumption of the generic good, if the utility function is characterized by intertemporal linkages such as taste cultivation properties. Conversely, if the utility function lacks taste cultivation properties, the intertemporal aspects of preferences have no effect on consumption behavior. In the case where preferences are time-consistent, the model can be used to analyze how culture demand depends on parameters of the model. However, in that case, no government intervention is called for. In contrast, when there is a time-inconsistency, consumers can be viewed as "underconsuming" culture. In particular, from the perspective of the current self of the consumer, the future selves do not consume enough culture, since they are less forward-looking than the current self. By implication, the current self underconsumes culture from the perspective of the past selves. In a model where the consumer is modelled as consisting of different selves with conflicting preferences, it is not clear how to evaluate welfare. However, several alternatives have been suggested in the literature, and they are discussed and compared in the paper.

Due to the intrapersonal friction, various forms of government interventions could help the consumer, if these interventions help induce more culture consumption. In particular, a constant subsidy to culture consumption would be beneficial; the current consumer may be induced to consume more culture than he/she wants, but the increase in future culture consumption is definitely appreciated. In other words, a constant subsidy to some extent plays the role of a commitment mechanism for setting culture consumption at a higher level in the future than what would materialize without the intervention.

Thus, the main purpose of the present study is to advance and formalize the hypothesis that subsidies to culture can be motivated using rather standard economic analysis: the argument relies on accepting the two key assumptions, namely that culture is an experience good (more than are other goods) and that individuals tend to have a present-bias. Both these assumptions are rather well-documented separately, but their interrelation has not previously been examined in the literature. However, a second and not less important purpose of the paper is to attempt a quantitative assessment of the size of the government subsidies needed to encourage culture consumption in an optimal way. This assessment, which is here performed using calibration analysis, involves several important inputs: an assessment of the degree of substitutability between the culture good and other goods, measuring the extent of forward-looking of consumers, and an assessment of how large is the present-bias. These key factors are not estimated here but the associated parameter values are borrowed from the existing literature.

The calibration exercise shows that large subsidies—in the order of those observed—might be optimal, if the level of time-inconsistency is extreme and the taste-cultivation properties of the utility function are pronounced. It is, however, doubtful that the present-bias is as strong as needed to give these result; for a standard calibration of the present-bias, the optimal subsidy is significantly lower than at least in the Swedish data. Moreover, although tax-financed subsidies appear to be a potent device in restoring consumption to time-consistent levels, the actual welfare gains from not implementing the subsidies are in general small.

It might be considered that the mechanism proposed here—that culture consumption is too low because it is a good involving "capital accumulation" and preferences are present-biased—would also have some bearing on related goods with long-term effects, such as durable goods. Should such goods also be subsidized? Indeed, if undersaving is a general phenomenon due to present-biased preferences, the answer ought to be yes. However, this conclusion seems unwarranted. Markets allow most consumers to avoid this outcome, because durable goods are usually offered along with a loan. A typical example is car loans offered by car dealers, which encourage the individual to enjoy the capital in advance and repay the seller later. Here, the consumer does not need to forgo consumption early on to enjoy the capital good later. In fact, it rather seems that durable goods of this sort instead ought to be taxed, because it is likely that consumers with a present-bias are tempted to sign up for the loan in order to enjoy the good now, thus accepting to pay, and reducing other forms of consumption, only later. The fundamental difference between durable goods and experience goods, like experience, or more generally human capital, is that the latter are intangible and cannot be transferred in advance: they must be painfully accumulated.

The rest of the paper is divided into four sections. Section 2 outlines the theoretical framework of cultural consumption and investigates steady states and dynamic implications for forward-looking and present-biased agents. In section 3, various policy instruments are discussed and a numerical example calibrated to Swedish data is presented. Section 4 concludes.

2 A theory of cultural consumption

In this section, a basic model framework of cultural consumption will be outlined, equilibrium will be defined and the characteristics of the model examined.

2.1 The basic model

2.1.1 The case with time-consistent preferences

The analysis will start with a characterization of equilibrium in a standard setting, where the representative agent has standard time-consistent preferences. Time is discrete and infinite. As already discussed, the agent can consume two types of cultural goods, a highbrow good, y, or a generic good, x. The utility she receives from consuming culture depends on the previous consumption of the good, summarized by the stock of consumption capital, k. The preferences of this agent are given by $\sum_{t=0}^{\infty} \delta^t u(x_t, y_t, k_t)$. There is no source of uncertainty in the economy, so consumption will be a function of k. The current period utility function is assumed to have the following properties; $u_x > u_y > 0$, $u_k > 0$, u_{xx}, u_{yy}, $u_{kk} \leq 0$, $u_{xy} \lesseqgtr 0$, $u_{xk} \leq 0$ and $u_{yk} > 0$. The last assumption, i.e., setting the cross derivative of the state and control variable as positive, is the essence of the taste formation models. By letting the marginal utility of capital, u_k, be positive, this model diverges from the models of addiction.

The agent maximizes utility subject to a constraint in each period which can be regarded as a time or budget constraint. The endowment in each period is unity and prices are constant over time, so that feasible allocations are those satisfying

$$x + y = 1. \tag{1}$$

The stock of consumption capital is accumulating according to the investment equation

$$k' = h(y,k). \tag{2}$$

This expression is supposed to capture depreciation and addition to the capital stock through current consumption of y. The idea here is that k' increases in y and k such that $h_y, h_k \in (0,1)$.

Since the preferences are time-consistent, the individual's dynamic program is a standard recursion. The consumer's planning problem, with k^0 given, is

$$\begin{aligned} V(k) &= \max_{x,y} \{u(x,y,k) + \delta V(k')\} \qquad s.t. \\ x + y &= 1 \\ k' &= h(y,k). \end{aligned} \tag{3}$$

The optimal choice, given this recursion, is given by control variable y with the policy function $y = g(k)$, such that

$$y = g(k) \in \arg\max \{u(1-y,y,k) + \delta V(h(y,k))\}, \tag{4}$$

which solves the problem for all k. Unlike the standard savings-capital model, the propensity to consume, $g'(k)$, is not necessarily positive. For a sufficiently large capital stock, the wealth effect could be dominating, such that the agent prefers to consume the generic good x rather than maintaining the stock of consumption capital. Here, $g(k)$ will typically be increasing, which is the case for adequate values of u_{yk}.

The current period utility function is assumed to be strictly increasing in both arguments and strictly concave and the set $\{(k',k) : k' = h(y,k), x = 1-y, y \in R^n\}$ is convex and compact. To solve the maximization problem, substitute the constraint (1) and technology (2) in the Bellman equation (3) and maximize with respect to y. The first-order condition can be written as

$$-u_x\left(1-y,y,k\right)+u_y\left(1-y,y,k\right)-\delta h_y V_k\left(h(y,k)\right)=0. \tag{5}$$

$V_k\left(h\left(y,k\right)\right)$ can be derived using the envelope theorem. In short, the total derivative of the value function V with respect to a parameter k is equal to the partial derivative of the objective function with respect to the parameter evaluated at the optimal policy $g\left(k\right)$. Using this result,

$$V_k\left(k\right)=u_k\left(1-y,y,k\right)+\delta h_k V_k\left(h(y,k)\right). \tag{6}$$

Next, use the first-order condition (5) to solve for $V_k\left(h(y,k)\right)$

$$V_k\left(h(y,k)\right)=\frac{1}{\delta h_y}\left(-u_x\left(1-y,y,k\right)+u_y\left(1-y,y,k\right)\right).$$

Substitute this expression in equation (6), transpose one period ahead and substitute into the first-order condition. First, define

$$\Delta\left(k\right)=u_x(1-g(k),g(k),k)-u_y(1-g(k),g(k),k).$$

This is the difference between marginal utility of x and marginal utility of y. In a statical model, this difference is zero, while in a model with intertemporal links, it is in general different from zero. Here, the agent experiences instant utility from consuming x and instant as well as future utility from consuming y. $\Delta\left(k\right)$ can be interpreted in terms of the future utility from accumulated capital, which follows from the consumption of y.

Using this notation, the Euler equation takes the form

$$\Delta(k)=\delta h_y(k,g(k))\left(u_k(1-g(k'),g(k'),k')+\Delta(k')\frac{h_k(k',g(k'))}{h_y(k',g(k'))}\right).$$

The interpretation of the Euler equation is that the cost of consuming y instead of x in time period t, in terms of lost utility, must equal the discounted direct effect of the increased capital stock on utility in time period $t+1$ and the discounted change of marginal utilities that will follow from the consumption of y in the previous period. More intuitively, the agent will choose her consumption bundle so that the instant utility loss from consuming culture instead of the generic good, will equate the discounted future gains of the investment in culture today. Put differently, the Euler equation is the relation between Δ today and Δ tomorrow. Clearly, in this model framework, current consumption is not only history dependent but is also affected by future consumption decisions. Changes in the past affect current consumption by changing the current stock of capital, whereas changes in the future affect current consumption by changing current shadow prices through the effects on future stocks and consumption.

2.1.2 Time-inconsistent preferences

If the agent is present-biased and places a larger value on current consumption, preferences are time-inconsistent. Time-inconsistent preferences mean that a single individual can be viewed as a collection of selves, each in a different time period and each with a different set of preferences. Here, time-inconsistency will be modeled in terms of quasi-hyperbolic, or quasi-geometric, discounting, as labeled by Laibson (1997) or Krusell and Smith (2003).[4] With these kinds of preferences, the current one-period discount rate is higher than the future ones. This means that the agent will, in general, deviate from the policy rules derived at any earlier time period. More specifically, self t and self $t+1$ agree on the discounting between all time-periods, except for periods $t+1$ and $t+2$, and so on. Here, individuals are assumed to be aware of this feature of their own behavior and to choose economic behavior taking into account the behavior of their future selves.

I will consider intrapersonal games and use a Markov perfect equilibrium concept with k_t as the state variable, i.e., all decisions made today will depend on the initial capital stock, k_t, and no other aspects of the past.[5] Formally stated in a recursive setting, a consumer foreseeing that her future selves have

[4] Laibson refers to experimental and psychological evidence as the motivation for quasi-hyperbolic discounting. See e.g. Laibson (1997).

[5] Reputational equilibrium where current actions depend on additional information from the past, not captured in k_t, can also exist in this framework.

different preferences, solves the problem

$$W(k) \equiv \max_{x,y} \{u(x, y, k) + \beta\delta V(k')\} \tag{7}$$

subject to (1) and the dynamic equation (2). The value function V is the indirect utility of capital from the next period and onwards and must satisfy the functional equation,

$$V(k) \equiv u(1 - g(k), g(k), k) + \delta V(h(g(k), k)). \tag{8}$$

Thus,

$$V(k_t) = u(x_t, y_t, k_t) + \delta u(x_{t+1}, y_{t+1}, k_{t+1}) + \delta^2 u(x_{t+2}, y_{t+2}, k_{t+2}) + \cdots,$$

so that $\delta V(k_{t+1})$ captures the indirect utility for self t of leaving a stock of k_{t+1} to self $t+1$. Given V, the consumer chooses control variable y with the policy function $y = g(k)$.

The β parameter represents the prospect of present-biased agents with control problems and lies between 0 and 1. With $\beta = 1$, the model nests the standard time-consistent case with geometric preferences and sophisticated behavior. In this case, $V(k)$ would coincide with $W(k)$. If $\beta \neq 1$, there is time-inconsistency: a consumer disagrees with his past and future selves about how to consume and save, and solving the problem is now non-trivial.

The resulting Euler equation takes the form

$$\Delta(k) = \beta\delta h_y(k, g(k)) \left(u_k(1 - g(k'), g(k'), k') + \Delta(k') \left((\frac{1}{\beta} - 1) g_k(k') + \frac{1}{\beta} \frac{h_k(k', g(k'))}{h_y(k', g(k'))} \right) \right), \tag{9}$$

where $\Delta(k)$ is defined as before, see A.1 for derivations. Comparing the Euler equation in this time-inconsistent setting with the standard case above, two new effects appear. First, we have a direct effect on the shadow price of consumption capital, insofar as the discounting is stronger in the latter set-up and hence, the valuation of future effects of capital on utility is smaller. Second, there are indirect effects on the shadow price of capital. A new term appears in the Euler equation; $\Delta(k') \left(\frac{1}{\beta} - 1 \right) g_k(k')$. The term consists of three parts; the marginal utility gain of larger k, the degree of disagreement between selves and the response of the next self, i.e., her propensity to invest. Since $\Delta(k')$ and $\left(\frac{1}{\beta}-1\right)$ are positive and $g_k(k')$ is assumed to be positive, the new term is positive. The intuition is as follows: the present self considers that the next self accumulates too little capital, since the next self is present-biased. Therefore, it is in the interest of the current self to leave somewhat more capital than she would otherwise prefer, to indirectly influence the actions of the next self.

2.1.3 The parametric example

To more closely examine and illustrate the characteristics of the model, let the utility function take the following form

$$u(x,y,k) = \alpha_0 + \alpha_x x + \alpha_y y + \alpha_k k + \frac{\alpha_{xx}}{2}x^2 + \frac{\alpha_{yy}}{2}y^2 + \frac{\alpha_{kk}}{2}k^2 + \alpha_{xy}xy + \alpha_{xk}xk + \alpha_{yk}yk.$$

This is a quadratic utility function with parameter values set to satisfy the above assumptions: α_x, $\alpha_y > 0$, α_{xx}, $\alpha_{yy} \leq 0$, $\alpha_{yk} > 0$ and, for simplicity, α_0, α_{xk}, α_{xy}, α_k, $\alpha_{kk} = 0$. A quadratic utility function apprehends the previously discussed features in a straightforward way. Cross-effects and curvature are explicitly visible in the functional form and closed-form solutions can be derived. In addition, the linear-quadratic specification has the benefit of allowing the easy inclusion of stochastic shocks to the setup, since certainty equivalence holds. I briefly discuss a stochastic environment in section 2.6.

Adding a cubic term in a standard consumption-savings model without taste-cultivation properties has no major qualitative consequences. In a model with habits, however, it can have drastic consequences, such as multiple steady states and thus, heterogeneity in consumption. In addition, it can lead to discontinuous solutions for $g(k)$ and a large number of steady states if preferences are time-inconsistent. These issues are investigated in Stavlöt (2005).

The form of the capital accumulation equation remains to be established. Letting $h(y_t, k_t)$ be linear such that

$$k_{t+1} = h(y_t, k_t) = c_0 + c_y y_t + c_k k_t,$$

the model will be linear-quadratic.

2.2 Equilibrium

The equilibrium in the intrapersonal game consists of a decision rule $g(k)$ and value functions $V(k)$, $W(k)$, such that these functions solve the dynamic program for the individual:

- given $g(k)$, $V(k)$ satisfies equation (8)
- given $V(k)$, $W(k)$ and $g(k)$ satisfy equation (4).

2.3 Solution of the model

Substituting out k' and x with constraints (1) and (2), and replacing y with the guessed decision rule $y = a + bk$, the Euler equation can be rewritten in terms of the state variable k, in the form $A + Bk = 0$.[6] Since this condition must be met for all k, A and B must be zero, which produces two equations in the two unknown decision rule parameters, a and b. This system is non-linear but can easily be characterized. For the numerical examples, I solve for a and b with a typical non-linear equation-solving routine available in MATLAB. For the general linear-quadratic model, these two equations are specified in A.2.

2.4 Steady states

A steady state is a stationary point, $\{\bar{x}, \bar{y}, \bar{k}\}$, of the policy function, such that $x' = x \equiv \bar{x}$, $y' = y \equiv \bar{y}$ and $k' = k \equiv \bar{k}$. The steady state can thus be solved from $\bar{k} = h\left(g\left(\bar{k}\right), \bar{k}\right)$. In the linear-quadratic case, this becomes a simple linear equation, $\bar{k} = c_0 + c_y\left(a + b\bar{k}\right) + c_k\bar{k}$, i.e., $\bar{k} = (c_0 + c_y a) / (1 - c_y b - c_k)$. Naturally, recall that a and b are endogenous. Steady-state consumption is easily found, since we have the policy rule and the steady-state consumption capital.

Depending on the parameter values of the utility function and the dynamic equation of consumption capital, the steady states demonstrate different properties. This steady state, which is typically unique, can be stable or unstable, depending on the parameter values. Comparative statics confirm that the model behaves as expected when the parameter values are changed. I do not perform general comparative statics, but for the ranges of parameter values I have studied based on the calibration, I find the consumption

[6] It is a well-known result that quadratic utility functions give linear decision rules, which can readily be proven by backward induction.

capital level of a stable steady state to increase in α_k, α_y, α_{yk}, c_k, c_y, $|\alpha_{xx}|$ and δ and decrease in α_x, $|\alpha_{yy}|$, $|\alpha_{kk}|$, $|\alpha_{xy}|$ and $|\alpha_{xk}|$. Time-inconsistency decreases the steady-state level as long as the utility function encompasses intertemporal linkages. This means that for $\alpha_k = \alpha_{kk} = \alpha_{yk} = \alpha_{xk} = 0$, time-inconsistency has no effect on steady-state consumption or capital levels.

2.5 Dynamics

The dynamics in the basic linear-quadratic model are fully described by the slope coefficient in the reduced form capital accumulation equation, $k' = c_0 + c_y (a + bk) + c_k k$, i.e., $c_y b + c_k$, which depends non-trivially on all primitive parameters in the model. If the absolute value of the slope coefficient is less than 1, the steady states are stable, while if its absolute value is larger than 1, the steady states are unstable. Only stable dynamics is associated with optimizing behavior. In a typical case, the slope is positive, giving rise to monotonous dynamics. The flatter the slope, the more rapid is the convergence to steady state. The speed of convergence is captured in how close to zero the coefficient is. If zero, the convergence is immediate.

In one interesting case which I study later, the dynamics are given by figure 1.[7] It is apparent that in this case, consumption of culture will be homogenous since all initial levels of consumption capital give the same long-run outcome.

By changing each parameter in the model while keeping the others constant, the dynamic qualities of this model are examined. As stated in the section above, no general comparative statics are performed, but the following inference can be made for ranges of parameters that seem reasonable. Starting with the utility function, we see that the coefficients of the linear piece of the quadratic utility function, α_x, α_y and α_k, generate a shift in the decision rule, while the slope remains constant. This result is easily grasped since these coefficients operate as constants in optimum, which should be clear after differentiating the utility function.

The second derivatives of the utility function will have an impact on the slope of the policy function, which is likewise apparent from the form of the utility function. As in the standard consumption-

[7] The following parameter values are used: $\alpha_x = 1$, $\alpha_y = 0.04$, $\alpha_k = 0$, $\alpha_{xx} = -1.008$, $\alpha_{yy} = -1.008$, $\alpha_{kk} = 0$, $\alpha_{xy} = 0$, $\alpha_0 = 0$, $\alpha_{xk} = 0$, $\alpha_{yk} = 0.25, c_0 = 0$, $c_y = 0.15$, $c_k = 0.85$, $\beta = 1$ and $\delta = 0.96$. The values are chosen to make the utility function concave and marginal utilities positive.

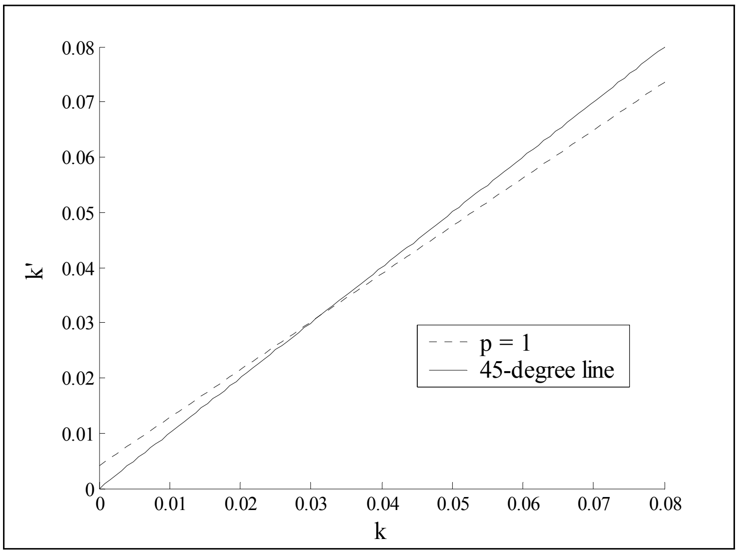

Figure 1: Decision Rule

smoothing model, the curvature of the utility function with regard to consumption goods x and y slow down convergence. This is captured in the model as high absolute values on α_{xx} and α_{yy}. Intuitively, high curvature requires consumption smoothing and hence, slows down convergence.

Since this model excludes monetary saving, the intertemporal linkages solely evolve from consumption capital. This means that parameters α_{yk}, α_{xk}, and α_{kk} are determinants of the dynamics in terms of consumption. If all three parameters were zero, the convergence to steady-state consumption would be immediate, since the decision rule would be independent of consumption capital. While a high absolute value on α_{kk} makes the consumption capital less important in the utility function, the other two parameters, α_{yk} and α_{xk}, reinforce the intertemporal linkages and more dynamics are introduced.

Parameter α_{yx} affects the dynamics, depending on whether culture and the generic good are substitutes or complements, such that the stronger is the substitutability in consumption, the more dynamics, whereas the stronger is the complementarity in consumption, the less dynamics.

Next, examine the two remaining preference parameters, i.e., discount factors β and δ. Low discount factors, β and δ, both decrease the slope and increase the speed of convergence. The motivation behind this is that low discount rates imply that the agent puts less value on the future. The model then resembles a static model and convergence will thus be faster.

The parameters determining consumption capital accumulation, c_y and c_k, have opposite effects on dynamics. A high value of c_k has a similar effect as a high discount factor. The closer to 1, the more dynamics and the slower the speed of convergence. Conversely, when the weight on y is high, the internal dynamics of k becomes less important than the immediate effect of y on k and the speed of convergence is increasing.

It should be noted that most parameters both shift and turn the curve, which means that the time to convergence is not only dependent on the slope b, but also on the constant a and obviously, on the initial capital, k_0.

2.6 Introducing shocks in the economy

Observations of cultural consumption in real life reveal significant differences in the level of consumption among individuals. There are alternative explanations to this heterogeneity of cultural consumption. One obvious, although not very sophisticated, theory is that individuals have different tastes, interpreted as dissimilar utility functions. Various long- or short-run shocks to the individual could also explain consumption patterns in culture. By solving the model with shocks, alternative explanations of heterogeneity can be compared. In terms of the theoretical framework outlined above, there are two alternative shocks to the demand side that seem reasonable. First, there could be an additive shock to technology, such that $k' = c_0 + c_y y + c_k k + \varepsilon$. In a way, this shock could also be regarded as a change in tastes, since a larger capital stock involves altered marginal utilities. Possible stories for explaining this kind of shock in real life could be a new opera loving acquaintance enthusiastically promoting her interest, or a governmental information campaign with the aim of fostering the cultural education of the population.

Second, there could be shocks to time or income that cannot be predicted by the consumer, such that $x + y = 1 + \varepsilon$. The time series properties of ε are important in discriminating between different models and identifying the taste cultivation effects.

Various shocks and heterogeneity in preferences can explain changing consumption patterns in culture and the question is how these can be separated. Without formally showing this, the following can intuitively be argued.

Heterogeneity in preferences would certainly explain low or high consumption of culture, but would generate constant consumption patterns over time, i.e., level effects.

Permanent shocks in demand would affect prices and quantities, but the change would be constant over time, i.e., a level effect. A temporary shock would change consumption as long as the shock lasts, but it would immediately jump back to the old level when the shock is reversed, since there are no saving opportunities.

With taste cultivation properties, dynamics and inertia are introduced. Assume that the quantity demanded increased after a positive demand or supply shock. In case of a permanent shock, consumption would increase further in the long run and, in the case of a temporary shock, it would not immediately return to its previous size, since it will take some time for the consumption capital, which was build up during the temporary demand peak, to depreciate to the old level.

In the next section, the time-series properties of the basic model will be contrasted with those of a standard model, both solved with shocks.

2.7 Autocorrelation

As already stated, the inclusion of past consumption in the utility function gives intertemporal effects and the model features inherent serial correlation in consumption. By eliminating the effects of second derivatives related to k, i.e., by letting $\alpha_{yk} = \alpha_{xk} = \alpha_{kk} = 0$, the serial correlation can be extinguished. However, correlation over time can also develop from shocks in the economic environment. Technology shocks will obviously have no effect if second derivatives are set to zero, but shocks to the constraint could generate serial correlation. Consider an additive shock to the time constraint, $x + y = 1 + \varepsilon$, with ε following a Markov process of the form $\varepsilon' = \rho\varepsilon + u'$, where u is iid normal with mean zero and variance σ^2. The computational details of how the model is solved with shocks are reported in A.5. The dotted line in figure (2) depicts the autocorrelation generated from an AR(1) shock with $\rho = 0.5$ and $\sigma_\varepsilon^2 = 1.0e-5$.[8,9]

In contrast, if serial correlation is generated from taste cultivation properties instead of a Markov shock, the autocorrelation process will follow an ARMA(1,1) process. To see this property, consider a shock to the time constraint as above, but let ε be iid normal with the mean zero and the variance σ_ε^2. The decision rule can be written as

$$y_t = \lambda_0 + \lambda_1 k_t + \lambda_2 \varepsilon_t$$

and, hence, the capital accumulation equation

$$\begin{aligned} k_{t+1} &= c_0 + c_y \left(\lambda_0 + \lambda_1 k_t + \lambda_2 \varepsilon_t\right) + c_k k_t \\ &= c_0 + c_y \lambda_0 + \left(c_y \lambda_1 + c_k\right) k_t + c_y \lambda_2 \varepsilon_t \\ &= A + B k_t + C \varepsilon_t \end{aligned}$$

where

$$\begin{aligned} A &= \left(c_0 + c_y \lambda_0\right) \\ B &= \left(c_y \lambda_1 + c_k\right) \\ C &= c_y \lambda_2 . \end{aligned}$$

Substitute the expression for k_t in the decision rule and iterate

[8] The following parameter values are used: $\alpha_x = 1$, $\alpha_y = 0.03$, $\alpha_k = 0$, $\alpha_{xx} = -1.005$, $\alpha_{yy} = -1.005$, $\alpha_{kk} = 0$, $\alpha_{xy} = 0$, $\alpha_0 = 0$, $\alpha_{xk} = 0$, $\alpha_{yk} = 0, c_0 = 0$, $c_y = 0.15$, $c_k = 0.85$, $\beta = 1$ and $\delta = 0.96$.

[9] For computational details, see the Appendix. The correlogram is calculated as

$$\begin{aligned} \rho &= \frac{\frac{\sum_{t=1}^{T}(y_t-\overline{y})(y_{t-1}-\overline{y})}{T}}{\widehat{\sigma}_y^2} \quad \text{where} \\ \overline{y} &= \frac{\sum_{t=1}^{T} y_t}{T} \quad \text{and} \\ \widehat{\sigma}_y^2 &= \frac{\sum \left(y_t - \overline{y}\right)^2}{T-1}. \end{aligned}$$

$$
\begin{aligned}
y_t &= \lambda_0 + \lambda_1 \left(A + Bk_{t-1} + C\varepsilon_{t-1}\right) + \lambda_2 \varepsilon_t \\
&= \lambda_0 + \lambda_1 \left(A + B\left(\left(A + Bk_{t-2} + C\varepsilon_{t-2}\right)\right) + C\varepsilon_{t-1}\right) + \lambda_2 \varepsilon_t \\
&= \lambda_0 + \lambda_1 \left(A + B\left(\left(A + B\left(\left(A + Bk_{t-3} + C\varepsilon_{t-3}\right)\right) + C\varepsilon_{t-2}\right)\right) + C\varepsilon_{t-1}\right) + \lambda_2 \varepsilon_t \\
&= \lambda_0 + \lambda_1 A\left(1 + B + B^2 + \cdots\right) + \lambda_1 C\left(\varepsilon_{t-1} + B\varepsilon_{t-2} + B^2\varepsilon_{t-3} + \cdots\right) + \lambda_2 \varepsilon_t.
\end{aligned}
$$

Derive y_{t-1} in the same manner and multiply by B.

$$
By_{t-1} = B\lambda_0 + B\lambda_1 A\left(1 + B + B^2 + \cdots\right) + B\lambda_1 C\left(\varepsilon_{t-2} + B\varepsilon_{t-3} + B^2\varepsilon_{t-4} + \cdots\right) + B\lambda_2 \varepsilon_{t-1}
$$

Subtract By_{t-1} from y_t and rewrite in terms of lag operators.

$$
\begin{aligned}
y_t - By_{t-1} &= (1-B)\left(\lambda_0 + \frac{\lambda_1 A}{1-B}\right) + \lambda_1 C\varepsilon_{t-1} + \lambda_2\varepsilon_t - B\lambda_2\varepsilon_{t-1} \\
(1-BL)\,y_t &= D + \left(\lambda_2 + \left(\lambda_1 C - B\lambda_2\right)L\right)\varepsilon_t \\
(1-BL)\,y_t &= D + \lambda_2\left(1 + \left(\frac{\lambda_1}{\lambda_2}C - B\right)L\right)\varepsilon_t \\
(1-BL)\,\hat{y}_t &= \left(1 + \left(\frac{\lambda_1}{\lambda_2}C - B\right)L\right)\hat{\varepsilon}_t
\end{aligned}
$$

which can be written as

$$
(1-aL)\,\hat{y}_t = (1-bL)\,\hat{\varepsilon}_t
$$

where

$$
\begin{aligned}
a &= B = \left(c_y\lambda_1 + c_k\right) \\
b &= -\left(\frac{\lambda_1}{\lambda_2}C - B\right) = -\left(\frac{\lambda_1}{\lambda_2}c_y\lambda_2 - \left(c_y\lambda_1 + c_k\right)\right) = c_k.
\end{aligned}
$$

Thus, y follows an ARMA(1,1) process. A familiar result of the ARMA(1,1) process is that the

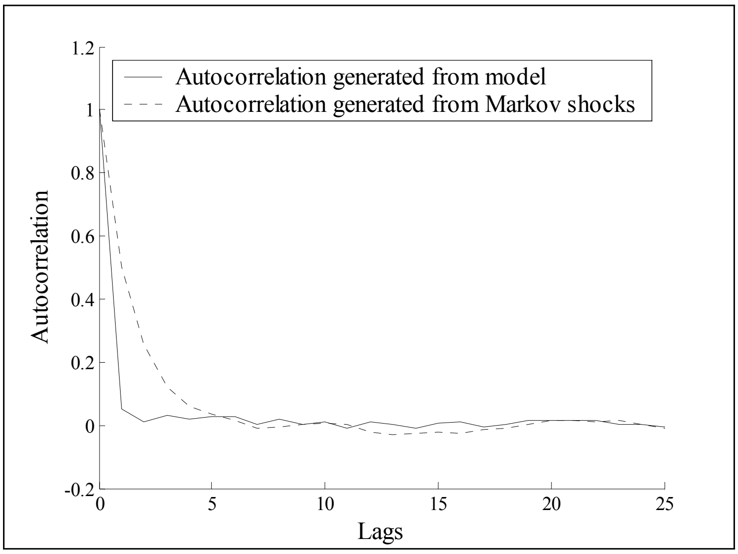

Figure 2: Correlogram

autocorrelation function will look similar to that of an MA(1) process between time 0 and 1, whereas from time 1 and onwards, the autocorrelations will display an AR(1) behavior. If the roots of the AR and MA polynominals are close to each other, the ARMA autocorrelations are very close to those of the white noise, i.e., the process is almost uncorrelated. A closer look at a and b reveals that the roots are identical except for the term $c_y\lambda_1$. This term is always less than one since $c_y < 1$ and $\lambda_1 < 1$. However, with the parameter values used in the calibration in the later analysis, this term will be negligible and the result will be close to white noise; see the solid line in figure 2.

Consequently, it is difficult to use the autocorrelation properties as a device for identifying or falsifying the model.

2.8 Welfare

From the preceding exercises, it seems as if the presence of time-inconsistency in a model with taste cultivation does not have any major effects on behavior.[10] Steady-state consumption is decreasing and the convergence to steady state is delayed. However, as will be shown, time-inconsistency will indeed have far-ranging implications on cultural consumption and public support. Whereas the previous section has

[10] This property called "observational equivalence" could easily be demonstrated in a simpler model of time-inconsistency. See Krusell, Kuruşçu and Smith, (2002), for further analysis.

primarily been concerned with characterizing behavior, the welfare implications will be discussed below.

Since the consumer's different selves disagree, there is a conceptual problem in terms of welfare comparisons. There is not obvious according to which self's preferences the evaluation should be done. In a standard model with time-consistent preferences, policy evaluation is straightforward: the model is solved with various policy alternatives, the computed utility levels can be ordinally compared and the welfare maximizing policy identified. In this economy with time-inconsistent preferences and lack of commitment, implemented policy instruments have different effects on the set of selves. In principle, the problem is captured by the two welfare measures derived above in equations (7) and (8). Function W is the welfare of the present self at the consumption capital level k. The function V is the indirect utility function used by the past consumer to evaluate the present self's welfare at the consumption capital level k. One point worth repeating is that whereas the entire future is discounted at the rate $\beta\delta$ from the perspective of the present self, V is computed at the discount rate δ. Welfare analysis in a time-inconsistent context is therefore ill-defined. In the literature, there exist two different approaches for welfare evaluation in a time-inconsistent context. According to Laibson (1997), the welfare effects for the entire set of selves should be computed and considered, i.e., $V(k_0)$ and, in addition, the range $W(k_0)$, $V(k_1)$, $W(k_1)$ ..., $W(k_{ss})$. Gul and Pesendorfer (2004) show axiomatically that $V(k_0)$ is the proper measure to use, and this is also the approach used in this paper, although $W(k_0)$ and $W(k_{ss})$ will, in general, also be presented.

3 Tax-financed subsidy

There is a broad international consensus on the necessity of public support of the arts and the bulk of public support seems to be directed towards highbrow culture. In Sweden, the total consumption of culture amounts to 53.2 billion kronor, 16.1 billion of which, i.e., approximately 1.2 percent of the government budget, originate from public consumption. The theatre and dance sector is supported with 1.9 billion kronor. Approximately 13 percent of the budgets of the national theatres consist of box office income. Other theatre groups finance approximately 30 percent of their budget by box office income.

Apart from direct monetary support to the production side of culture in order to suppress consumer prices to tolerable levels, there are various other forms of public support. The quality of the art experience is nourished by the maintenance of artistic schools or salaries paid directly to creative artists. Children's creative minds and abilities to appreciate culture are fostered by direct campaigns towards schools, in forms of easy access for school classes to the theater, teaching aids for studying plays etc.

All these mentioned policy instruments can be easily introduced into the model framework described above and it permits us to consider fiscal policies of practical interest. For whatever reasons governments choose to engage in public funding of the fine arts, the financially affluent and numerically small audience could present political problems, as discussed in the introduction. In those countries where the fine arts mainly are promoted by taxes forgone or forgiven, e.g., the United States, public support seems to be less controversial than when tax income is used to give grants to highbrow art institutions. This section will discuss an example of the latter type of policy instrument, chosen to illustrate some substantively important aspects of public support of the arts in Western Europe in general and Sweden in particular.

Consider an economy with a representative agent who lives forever and faces no uncertainty. Preferences are time-inconsistent and the period utility function is quadratic. Individuals live in an endowment economy and the endowment in every time period is given by I. The budget constraint faced by an individual is given by

$$x_t + p_t y_t = I_t - \tau_t, \tag{10}$$

where the price of x is normalized to 1, p is the relative price of y, including subsidies, and τ is a lump-sum tax paid by the individual. The idea in the latter policy experiment is that the subsidy is fully financed with a balanced budget, period by period. Moreover, I will consider constant subsidy rates, i.e., $p_t = p$ $\forall$ t.

Now, the changes in individual behavior due to the introduction of a constant subsidy will be studied. Following the recursive method from above, maximizing (7) subject to (2) and (10), the Euler equation can be derived as,

$$\begin{aligned}
\Delta(k) &= \beta\delta h_y(k,g(k))\Big(\ u_k(I-\tau\left(g\left(h\left(g\left(k\right),k\right)\right)\right)-p'g(k'),g(k'),k') \\
&\quad +\Delta(k')\Big(\ (\frac{1}{\beta}-1)g_k(k')+\frac{1}{\beta}\frac{h_k(k',g(k'))}{h_y(k',g(k'))}\Big) \ \Big)
\end{aligned} \tag{11}$$

$$\Delta(k) = pu_x(I-\tau\left(g\left(k\right)\right)-pg(k),g(k),k)-u_y(I-\tau\left(g\left(k\right)\right)-pg(k),g(k),k).$$

3.1 Equilibrium with a constant subsidy policy

The equilibrium in the intrapersonal game is a decision rule $g\left(k\right)$ and value functions $V\left(k\right)$, $W\left(k\right)$ where $g\left(k\right)$ satisfies Euler equation (11) with $\tau=\tau\left(g\left(k\right)\right)$ and $\tau'=\tau\left(g\left(h\left(g\left(k\right),k\right)\right)\right)$ and $V\left(k\right)$ and $W\left(k\right)$ satisfy the associated value function equations.

3.2 The government's problem

There is an infinitely lived government that gives subsidies $(1-p)$ to the cultural good y, and taxes endowment in every period with τ. There is no government debt. In every time period, the governmental budget constraint is satisfied such that

$$\left(1-p_t\right)y_t=\tau_t. \tag{12}$$

The benevolent government maximizes consumer welfare by choosing p subject to (11). Notice that this is a one-dimensional maximization problem which will be solved with numerical methods, see A.4 for computational details. As discussed in section 2.8, consumer welfare can be measured in different ways. In most of the analysis below, I will simply tabulate V and W for different values of p and discuss the differences in changing p on different selves.

3.3 Calibration

To obtain numerical solutions to the model, particular parameter values must be chosen. The model is calibrated under the assumption of the model period being one year. The discount parameter δ is set to 0.96, which approximately corresponds to an interest rate of 4 percent, as is standard in the macroeconomic literature. Various experimental studies suggest a coefficient of time-inconsistency around 0.5 to be a reasonable base case.

The growth of consumption capital accumulation, represented by parameters c_y and c_k, is taken from Ravn, Schmitt-Grohe and Uribe (2004) and set, according to their estimations, to 0.15 and 0.85.[11] Although the estimated coefficients in Ravn et al correspond to consumption habits in general, it seems to be a reasonable approximation for the consumption of culture. Endowment, I, is normalized to 1.

Some theoretical restrictions on the preference coefficients follow directly from the model: To capture the essence of the model that the effect of increasing marginal utility in past consumption is positive, i.e., $\alpha_{yk} > 0$, and all remaining coefficients separating this model from a standard time-separable model are set to zero such that $\alpha_k = \alpha_{kk} = 0$. Per definition, x and k have no reciprocal effect and thus, $\alpha_{xk} = 0$. The conventional coefficients of the quadratic utility function are constrained to be as unbiased as possible, in the sense of both cultural goods being treated equally. This means that the second-order effects are neutralized by letting $\alpha_{xx} = \alpha_{yy}$. The first-order coefficient α_x is normalized to 1, whereas α_y is used to regulate the consumption shares. There is no inherent assumption in the model on the substitutability between x and y and thus, α_{yx} is set free to vary. In addition, the parameters must be set so that the fundamental qualities of the utility function and solution are met: the marginal utilities are positive, i.e., $u_x, u_y, u_k > 0$, the consumption of y is increasing in k, i.e., $g_k(k) = b > 0$, the utility function is concave and the steady states are stable. The remaining preference parameters α_y, α_{yy}, α_{yx} and α_{yk}, which correspond to four degrees of freedom, are set so as to match the following target statistics:

Average share of consumption of highbrow culture relative to lowbrow culture, $\frac{y}{x}$. The data on the time spent on entertainment[12] relative to the time spent on watching TV is used to calibrate this ratio to 0.05. The statistics is taken from The Swedish Time Use Survey, SCB.

Price elasticity of demand, $\varepsilon_p = \frac{\delta(y/x)}{\delta p}\frac{p}{(y/x)}$. A number of empirical studies have estimated this variable on various cultural goods and have delivered broadly consistent elasticities. The elasticities of highbrow culture are mainly inelastic and the statistics is set to be approximately -0.5.[13] In the

[11] c_k corresponds to their parameter ρ and c_y corresponds to $(1-\rho)$. Using U.S. quarterly data spanning the period 1967 - 2003 on consumer expenditure on durables, Ravn et al, estimate ρ to be 0.85.

[12] Entertainment is defined as attendance at concerts, the theatre, the cinema, exhibitions, libraries, sports events etc.

[13] See e.g. Abbé-Decarroux (1994), Bille Hansen (1991), Corning-Levy (2002), Frey et al (1989), Gapinsky (1984) (1986), Moore (1966), O'Hagan (1994), Schimmelpfenning (1997), Throsby (1983), Urrutiaguer (2002) and Withers (1980)

model, two types of elasticities can be derived. Long-term elasticity depicts the response of steady-state consumption at a permanent price change, whereas short-term elasticity captures the change in consumption between two periods at a one-period price change. In general, the empirical studies assess the former type of elasticity and the calibration will therefore be set to match long-term elasticity. However, both long-term and short-term elasticities will be reported in the exercises below.

Box office coverage ratio, p. The remaining parameters are chosen to match the box office coverage ratio of Swedish theatre, which is $0.13 - 0.30$ percent.

The three target statistics above are not sufficient to pin down the values of the four remaining parameters. However, a sensitivity analysis will be performed to control for the effects of individual coefficients.

In all exercises below, the model is recalibrated according to the above target statistics.

3.4 Results

The section is introduced by the examination of some properties of an economy where the agents exhibit time-consistent preferences, i.e., $\beta = 1$. This economy will serve as a point of comparison in the later analysis of public support of the arts in economies with time-inconsistent agents.

3.4.1 Time-consistent preferences

The time-consistent economy is, as pointed out above, calibrated to match the empirically observed highbrow/lowbrow consumption ratio of approximately 0.05 at a price subsidy of 70 percent.[14] The long-term and short-term elasticities in the model are inelastic and equal to -0.4902 and -0.3418.

Table (1) shows the steady-state properties of this economy at various government subsidies of culture. With a 70 percent subsidy, the steady-state consumption of y is 0.0490 and the y/x consumption ratio is 0.0516.[15] As the price is decreasing, a monotonic increase in the consumption of culture and consumption capital can be observed. Subsidizing the price of culture by 70 percent, steady-state consumption is

[14] The parameters are set to $\alpha_x = 1$, $\alpha_y = 0.04$, $\alpha_k = 0$, $\alpha_{xx} = -1.008$, $\alpha_{yy} = -1.008$, $\alpha_{kk} = 0$, $\alpha_{xy} = 0$, $\alpha_0 = 0$, $\alpha_{xk} = 0$, $\alpha_{yk} = 0.25$, $c_0 = 0$, $c_y = 0.15$, $c_k = 0.85$ and $\delta = 0.96$.

[15] Note that for $c_y + c_k = 1$, it is trivial to show that $y = k$ in steady state. Use the capital accumulation equation and solve for k^{ss}.

increasing by 60 percent.

Table 1: Effects of Public Subsidies with Time-Consistent Preferences

p	$\frac{y_{ss}}{x_{ss}}$	k_{ss}	$V(k_0)$	$W(k_0)$	$V(k_{ss}), W(k_{ss})$
1.0	0.0315	0.0306	12.4190	12.4190	12.4190
0.9	0.0332	0.0321	12.4189	12.4189	12.4190
0.8	0.0351	0.0339	12.4188	12.4187	12.4189
0.7	0.0373	0.0360	12.4184	12.4184	12.4186
0.6	0.0399	0.0384	12.4178	12.4178	12.4181
0.5	0.0430	0.0413	12.4168	12.4168	12.4173
0.4	0.0468	0.0447	12.4153	12.4152	12.4158
0.3	0.0516	0.0490	12.4127	12.4127	12.4133
0.2	0.0576	0.0545	12.4086	12.4086	12.4092
0.1	0.0656	0.0615	12.4018	12.4018	12.4021

The fourth and fifth columns of Table 1 examine the welfare of past and present selves, as defined in section 2.8, if the government introduces a subsidy; see A.3 for computational details. As shown above, welfare is maximized at a zero subsidy and is monotonically decreasing in price. The key to this outcome is the lack of externalities or internalities in the consumer's optimization problem, which means that the government can never improve the agent's allocation. The last column shows the welfare for past and present selves, not considering the utility loss during the transition, which is the case in the formerly reported welfare measures. Thus, the welfare measures are based on different capital stocks. The former is based on original capital stock, k_0, whereas the latter is based on the final steady-state capital stock, k_{ss}. The utility loss induced during the transition, can be regarded as the effort cost in terms of lost consumption to build up the capital stock. In this case, welfare is maximized at a minor subsidy. However, this welfare criterion is not relevant, unless the new steady-state capital stock is costlessly transferred to the agent in the time period when the subsidy is introduced.

In the next section, the behavioral and welfare effects of public subsidies of culture when preferences are time-inconsistent are examined. Finally, some sensitivity analysis is performed.

3.4.2 Time-inconsistent preferences

Consider behavior in an economy populated by agents with time-inconsistent preferences. Since there is a conflict between the preferences of different intertemporal selves, a need for self-control is introduced. A public support system could be viewed as a commitment device with the purpose of constraining the agents' future choices with time-inconsistent preferences.

The benchmark case Before reporting the results of the policy experiment, a standard is establish against which the effects of public subsidies can be compared. The time-inconsistent economy is recalibrated to match the empirically observed highbrow/lowbrow consumption ratio of approximately 0.05 at a price subsidy of 70 percent, and a time-inconsistency level corresponding to $\beta = 0.5$.[16] Table 2 summarizes the steady-state consumption ratio, the level of capital, the welfare of past and present selves and long and short-run elasticities at different levels of time-inconsistency for $p = 1$, and constant parameter values.

Table 2: Default Case

β	$\frac{y_{ss}}{x_{ss}}$	k_{ss}	$V(k_0)$	$W(k_0)$	$V(k_{ss})$	$W(k_{ss})$	$\varepsilon^y_{p,LR}$	$\varepsilon^y_{p,SR}$
1.0	0.0277	0.0270	12.4501	12.4501	12.4501	12.4501	−0.6496	−0.3695
0.5	0.0249	0.0243	12.4498	6.4739	12.4498	6.4739	−0.5806	−0.3596
0.1	0.0227	0.0222	12.4495	1.6931	12.4495	1.6931	−0.5110	−0.3506

The last two columns show that long-run elasticities are increasing in time-inconsistency, whereas short-run elasticities persist on the same level. Present-biased consumers tend to put less weight on the future, which generates a shift in the demand function as well as a tilt. The effect is of a similar nature as when the discount rate, δ, is decreasing. Nesting the model to a standard model with no intertemporal linkages, confirms this claim: the elasticities are constant, irrespective of time-inconsistency.

Next, for the same parameter values as those underlining table 2, let us examine the effects on behavior and welfare when introducing the public support system described above, in the form of subsidies to the cultural good, y. Table 3 summarizes the economic effects when the subsidized price is altered.

Table 3: Effects of Public Support with Time-Inconsistent Preferences

p	$\frac{y_{ss}}{x_{ss}}$	k_{ss}	$V(k_0)$	$W(k_0)$	$V(k_{ss})$	$W(k_{ss})$
1.0	0.0249	0.0243	12.44983	6.473913	12.44983	6.47391
0.9	0.0265	0.0258	12.44992	6.473949	12.45001	6.47400
0.8	0.0283	0.0275	12.44993	6.473945	12.45013	6.47407
0.7	0.0305	0.0296	12.44983	6.473879	12.45015	6.47408
0.6	0.0331	0.0321	12.44953	6.473711	12.45001	6.47400
0.5	0.0364	0.0351	12.44891	6.473381	12.44958	6.47378
0.4	0.0404	0.0389	12.44777	6.472780	12.44866	6.47330
0.3	0.0457	0.0437	12.44571	6.471712	12.44684	6.47236
0.2	0.0528	0.0502	12.44197	6.469799	12.44332	6.47053
0.1	0.0628	0.0591	12.43494	6.466247	12.43636	6.46691

[16] The following parameter values are used; $\alpha_x = 1$, $\alpha_y = 0.03$, $\alpha_k = 0$, $\alpha_{xx} = -1.005$, $\alpha_{yy} = -1.005$, $\alpha_{kk} = 0$, $\alpha_{xy} = 0$, $\alpha_0 = 0$, $\alpha_{xk} = 0$, $\alpha_{yk} = 0.4, c_0 = 0$, $c_y = 0.15$, $c_k = 0.85$ and $\delta = 0.96$.

Particularly noticeable in table 3 is that public subsidies of culture are actually welfare increasing for an agent with time-inconsistent preferences. In this explicit calibration, maximum welfare for the present self is achieved at $p = 0.9$, which corresponds to a subsidy of 10 percent. The past self prefers more subsidies, in which case the welfare maximizing price is 0.8. It can be shown that the past self always prefers a lower price and thus, more subsidies than the present self. This should be expected, since time-inconsistency implicates that different selves may disagree in the ranking of policy arrangements. The future selves prefer a higher subsidy of 30 percent at the completion of the transition path, since they need not consider the accumulation of consumption capital. Although the welfare levels diverge, the welfare maximizing subsidy will always coincide.

The results from this exercise demonstrate that the steady-state consumption of culture and the steady-state capital stock are monotonically decreasing in price, just as in the time-consistent case, see table 1. The changes in cultural consumption are fairly substantial: a 70 percent subsidy, as in the Swedish case, will increase consumption by almost 80 percent.

A further observation to be made is that although the increase in the steady-state level of consumption of y is considerable and the level is more than 60 percent higher than the time-consistent solution, public policy does not have any large effects on welfare levels. With this parameterization, the welfare gains for the past and present selves are almost negligible, when subsidies are sized to maximize welfare, and negative when subsidies correspond to the Swedish level at 70 percent of the price. In order to establish the importance of the calibration on results, I will more closely examine the set of coefficients in the next section.

3.4.3 Sensitivity analysis

In this section, the specific impact of the parameters of the utility function, capital accumulation and discounting on economic behavior, public policy and welfare are examined. The first row in table 4 once more shows the default case with a finer grid on p to make it possible to see subtle effects from the comparative statics. Now, to test the taste cultivation properties of the utility function, nest the model to a standard time-separable model with $\alpha_{yk} = 0$, and, as before, $\alpha_k = \alpha_{kk} = 0$. The rest of the parameter values are the same as those underlining table 3.

The second row in the table displays a consumption ratio of 0.0177 and a consumption capital level at 0.0174. As expected, the agent chooses to consume a lower amount of y, which is in line with the steady-state analysis in section 2.4. Particularly interesting is the finding that the optimal policy is now not to subsidize prices.

Before making any more profound comments on the latter finding, continue the exercise and let α_{xx} and α_{yy} vary, i.e., introduce additional decreases in marginal utilities. Thus, by increasing the absolute value of the two parameters and keeping the constraint $\alpha_{xx} = \alpha_{yy}$, the consumption level of y is higher as compared to the default case. The welfare maximizing subsidy is now larger and the long-run price elasticity smaller.

Next, see how the parameters of the capital accumulation equation affect the results. Let the depreciation rate of consumption capital decrease so that c_k increases from 0.85 to 0.90, followed by an increase in the "saving" rate c_y from 0.15 to 0.20. Both parameters have a similar effect on the steady-state behavior, public policy and welfare: the levels of consumption and capital, the long-run elasticities and the welfare maximizing subsidies are larger than in the benchmark case.

The next exercise is to drop the constraints on α_k and α_{kk} and allow these parameters to affect the outcome. To isolate the effects of each parameter, let $\alpha_{yk} = 0$ and start by letting $\alpha_k = 0.01$, i.e., let the agent experience utility from the consumption capital. Compared to the new default case in the second row in table 4, steady-state consumption of y and consumption capital are larger. Although α_{yk} is zero, there is a case for welfare improving subsidies. Next, let $\alpha_{kk} = -0.1$, i.e., introduce a decreasing marginal utility of cultural capital. This makes the consumption of y smaller than the default case and the agent prefers to tax consumption of y, which depends on the negative marginal utility of k.

The exercise has shown that the parameters separating this model from standard models of consumption, α_k, α_{kk}, and α_{yk}, have important effects on the result. There is a case for welfare improving subsidies with α_k, $\alpha_{yk} > 0$.

Not only the taste cultivation properties of the utility function have been demonstrated to be of importance for public policy decisions. Time-inconsistency, and thus the β parameter, also affect optimal policy. Keep the calibration from above when $\alpha_{yk} \neq 0$ and decrease β to 0.4. The results clearly

Table 4: Effects of Taste-Cultivation Coefficients

Exercise	$\frac{y_{ss}}{x_{ss}}\Big\|_{p=1}$	$k_{ss}\|_{p=1}$	$V(k_0)\|_{p=1}$	$W(k_0)\|_{p=1}$	p^*[17]	t
Default	0.0249	0.0243	12.4498	6.4739	0.83	0.0046
$\alpha_{yk} = 0$	0.0177	0.0174	12.4451	6.4715	1.00	0.0000
$\alpha_{xx} = -1.01$	0.0284	0.0276	12.3910	6.4433	0.80	0.0061
$c_k = 0.9$	0.0308	0.0448	12.4546	6.4764	0.82	0.0062
$c_y = 0.2$	0.0290	0.0376	12.4528	6.4755	0.81	0.0062
$\alpha_k = 0.01$	0.0187	0.0184	12.4474	6.4726	0.86	0.0027
$\alpha_{kk} = -0.1$	0.0174	0.0171	12.4448	6.4713	1.06	-0.0010
$\beta = 0.4$	0.0256	0.0250	12.4535	5.2803	0.7200	0.0084
$I = 1.01$	0.0335	0.0328	12.4603	6.4794	0.71	0.0109
cont.	$\frac{y^*}{x^*}$	k^*	$V^*(k_0)$	$W^*(k_0)$	$\varepsilon^y_{p,LR}$	$\varepsilon^y_{p,SR}$
	0.0277	0.0270	12.4499	6.4740	-0.5806	-0.3596
	0.0177	0.0174	12.4451	6.4715	-0.3635	-0.3319
	0.0315	0.0305	12.3911	6.4434	-0.4707	-0.2932
	0.0359	0.0520	12.4549	6.4765	-0.7674	-0.3783
	0.0337	0.0435	12.4531	6.4756	-0.7123	-0.3695
	0.0198	0.0194	12.4474	6.4727	-0.3715	-0.3390
	0.0170	0.0167	12.4448	6.4713	-0.3537	-0.3292
	0.0308	0.0299	12.4539	5.2804	-0.5773	-0.4082
	0.0386	0.0375	12.4607	6.4796	-0.4078	-0.2904

demonstrate that the magnitude of time-inconsistency is highly significant in determining the size of public subsidies. The optimal price is monotonously decreasing in time-inconsistency as represented by β.

So far, I have not commented on variable I, which is normalized to 1 in all previous analyses. What is the effect of income on consumption and preferred public policy? Comparative statics of I shows the consumption ratio of culture to the generic good to be increasing in income. Moreover, the preferred subsidies are increasing the larger is income.

The above findings can be summarized into a central conclusion of this study: time-inconsistent preferences and taste cultivation properties of the utility function are necessary and sufficient conditions for welfare enhancing funded subsidies. However, quantitatively large effects are hard to come by, without drastic changes in the model or unrealistic parameter values.

3.4.4 Could huge subsidies be optimal in the case of Sweden?

The results from the previous exercises suggest that the size of α_{yk}, α_{xx}, α_{yy} and β are crucial in designing an optimal support system for culture. To examine the sensitivity of welfare maximizing subsidies, I experiment with various parameter combinations with the purpose of matching available moments from Swedish data. As before, I will try to match the following targets; a consumption ratio of approximately 0.05 and a price elasticity of demand, approximately around -0.5. It is also necessary for the utility function to be concave, marginal utilities to be positive, the steady state stable and cultural consumption increasing in capital, i.e., $g_k(k) > 0$. The first exercise is to find the set of parameters that will motivate a 70 percent subsidy on the price of the arts, under the above conditions. First, the search will be restricted to $\beta = 0.5$, since this size of the coefficient appears to be standard in the literature.

The simplest approach for finding a set of parameters to match the desired data is the trial-and-error approach. This method is obviously time-consuming and all possible combinations cannot be controlled for. Despite a thorough search, I have not found any parameter combination yielding the optimal price of 0.3, under the condition that marginal utilities are positive. The best hit gave a welfare-maximizing price of 0.80 for past and present selves, see the result in table 5[18]. Since this result far from matches Swedish data, the price elasticity is now allowed to deviate from the default value fixed at around -0.5. For twice as inelastic demand, the optimal price can be reduced to 0.65; the results are noted in the second row.[19]

Relaxing the restriction on β by assuming the agent's preferences to be heavily time-inconsistent, i.e., letting $\beta = 0.1$, and slightly adjusting the coefficients, a welfare maximizing price of 0.50 is obtained.[20]

Although the theoretical and intuitive meaning of a positive direct utility of consumption capital, $\alpha_k > 0$, has only been briefly discussed earlier in the paper, I will manipulate this coefficient to see whether it is possible to match an optimal subsidy of 70 percent.[21] Keeping the strong time-inconsistent preferences and letting $\alpha_k = 0.08$ and $\alpha_{kk} = -0.20$, the welfare maximizing price is 0.15 for the past and

[18] The following parameter values are used; $\alpha_x = 1, \alpha_y = 0.025, \alpha_k = 0, \alpha_{xx} = -1.01, \alpha_{yy} = -1.01, \alpha_{kk} = 0, \alpha_{xy} = 0, \alpha_{xk} = 0, \alpha_{yk} = 0.45, c_y = 0.15, c_k = 0.85$ and $\delta = 0.96$.

[19] The following parameter values are used; $\alpha_x = 1$, $\alpha_y = 0.02$, $\alpha_k = 0$, $\alpha_{xx} = -1.024$, $\alpha_{yy} = -1.024$, $\alpha_{kk} = 0$, $\alpha_{xy} = 0$, $\alpha_{xk} = 0$, $\alpha_{yk} = 0.5$, $c_y = 0.15$, $c_k = 0.85$ and $\delta = 0.96$.

[20] The following parameter values are used; $\alpha_x = 1, \alpha_y = 0.025, \alpha_k = 0, \alpha_{xx} = -1.01, \alpha_{yy} = -1.01, \alpha_{kk} = 0, \alpha_{xy} = 0, \alpha_{xk} = 0, \alpha_{yk} = 0.6, c_y = 0.15, c_k = 0.85$ and $\delta = 0.96$

[21] The following parameter values are used; $\alpha_x = 1$, $\alpha_y = 0.06$, $\alpha_k = 0.08$, $\alpha_{xx} = -1.0$, $\alpha_{yy} = -1.0$, $\alpha_{kk} = -0.20$, $\alpha_{xy} = 0$, $\alpha_{xk} = 0$, $\alpha_{yk} = 0.03$, $c_y = 0.15$, $c_k = 0.85$ and $\delta = 0.96$

0.35 for the present self.

This exercise has demonstrated the sensitivity of parameterization on outcome. However, the results appear to confirm that the level of time-inconsistency and intertemporal linkages, in the form of taste formation properties, is positively correlated to the magnitude of public subsidies, which leads me next to the main conclusion of this paper: With high levels of time-inconsistency and strong taste cultivation properties of the utility function, a public subsidy of more than 70 percent could be optimal.

Consequently, whether public subsidies are welfare enhancing for present-biased agents is ultimately a quantitative question.

Table 5: Sensitivity Analysis

β	$\left.\frac{y_{ss}}{x_{ss}}\right\|_{p=1}$	$\left.k_{ss}\right\|_{p=1}$	$\left.V(k_0)\right\|_{p=1}$	$\left.W(k_0)\right\|_{p=1}$	p^*	t
0.5	0.0261	0.0254	12.3882	6.44186	0.80	0.0056
0.5	0.0342	0.0331	12.2221	6.3555	0.65	0.0130
0.1	0.0264	0.0257	12.3907	1.6851	0.50	0.0178
0.1	0.0346	0.0334	12.5871	1.7118	0.15	0.0498
cont.	$\frac{y^*}{x^*}$	k^*	$V^*(k_0)$	$W^*(k_0)$	$\varepsilon^y_{p,LR}$	$\varepsilon^y_{p,SR}$
	0.0290	0.0282	12.3883	6.44192	−0.4757	−0.2781
	0.0385	0.0371	12.2224	6.3556	−0.2527	−0.1355
	0.0369	0.0356	12.3918	1.6852	−0.4808	−0.2768
	0.0622	0.0585	12.6034	1.7127	−0.5223	−0.4645

4 Conclusions

This study has focused on public support systems of culture. The analysis may contribute to a better understanding of consumption behavior and the welfare implications of cultural consumption and has offered a plausible motivation for public subsidies. A number of key features have been explored, such as forward-looking but present-biased consumers and taste cultivation properties of preferences. A theoretical model has been outlined which depicts relevant qualities of cultural consumption and serves as a framework for policy analysis. A policy example is designed to illustrate some substantially important aspects of public support systems of culture in practice and is calibrated to Swedish data.

The central findings of this study can be summarized as follows: consumption of culture is repressed and there is a substantial welfare cost if preferences are significantly present-biased and the utility function has taste cultivation properties. These two latter properties, time-inconsistency and intertemporal linkages in the utility function, are necessary and sufficient conditions for tax-financed subsidies to be welfare improving. If the level of time-inconsistency is extreme and the taste cultivation properties of the utility function are pronounced, large subsidies could be optimal. However, although tax-financed subsidies appear to be a potent device in restoring consumption to time-consistent levels, the welfare gains seem to be rather small in general, irrespective of the calibration.

Whereas the direct impact on utility of past consumption in the form of consumption capital is a central assumption in the addiction literature, this topic has only been briefly discussed here. Attempts at incorporating this effect in the model, without formalization, have shown important quantitative effects on the design of public support systems. In fact, when adding a positive direct utility of past consumption, subsidies of 70 percent or more of the gross price could be optimal, which matches the Swedish data. Exploring this channel more deeply is an interesting task which, however, must be left for future work.

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A Appendix

A.1 Derivation of the Euler equation in the model with time-inconsistent preferences

In order to derive the Euler equation in the model with time-inconsistent preferences, the following algorithm is used:

With a recursive formulation, the consumer's maximization problem is

$$W(k) = \max_{x,y} \left\{ \begin{array}{c} u(x,y,k) + \beta\delta V(k') \\ x + y = 1 \\ k' = h(y,k) \end{array} \right\},$$

given

$$\begin{aligned} V(k) &\equiv u(1-g(k), g(k), k) + \delta V(h(g(k),k)) \\ &= u_t + \delta u_{t+1} + \delta^2 u_{t+2} + \ldots \qquad . \end{aligned}$$

The agent chooses control variable y with policy function $y^* = g(k)$. Substitute the restrictions into the welfare function

$$\max_y u(1-y,y,k) + \beta\delta V(h(y,k))$$

and derive the first-order condition

$$-u_x + u_y + \beta\delta V_k' h_y = 0, \tag{13}$$

where a prime denotes the function evaluated in the next period. Next, find V_k'. Start by deriving V_k.

$$V_k = (-u_x + u_y + \delta V_k' h_y) g_k + u_k + \delta V_k' h_k.$$

Since the same is true in the next period, transpose one period ahead

$$V_k' = (-u_x' + u_y' + \delta V_k'' h_y') g_k' + u_k' + \delta V_k'' h_k'. \tag{14}$$

Transpose the first-order condition in equation (13) and solve for V_k'', which gives

$$V_k'' = \frac{1}{\beta\delta h_y'}(u_x' - u_y'). \tag{15}$$

Use equations (15) and (14) and V_k' can be written as

$$\begin{aligned} V_k' &= (-u_x' + u_y' + \delta \left(\frac{1}{\beta\delta h_y'}(u_x' - u_y') \right) h_y') g_k' + u_k' + \delta \left(\frac{1}{\beta\delta h_y'}(u_x' - u_y') \right) h_k' \\ &= (u_x' - u_y')(\frac{1}{\beta} - 1) g_k' + u_k' + (u_x' - u_y') \frac{1}{\beta} \frac{h_k'}{h_y'}. \end{aligned} \tag{16}$$

Substitute this expression into the first-order condition

$$-u_x + u_y + \beta\delta \left((u_x' - u_y')(\frac{1}{\beta} - 1) g_k' + u_k' + (u_x' - u_y') \frac{1}{\beta} \frac{h_k'}{h_y'} \right) h_y = 0$$

and with the complete notation, the Euler equation can be written

$$\begin{aligned} \Delta(k) &= \beta\delta h_y(k, g(k)) \left(\begin{array}{c} u_k(1 - g(k'), g(k'), k') \\ +\Delta(k') \left((\frac{1}{\beta} - 1) g_k(k') + \frac{1}{\beta} \frac{h_k(k', g(k'))}{h_y(k', g(k'))} \right) \end{array} \right) \\ \Delta(k) &= u_x(1 - g(k), g(k), k) - u_y(1 - g(k), g(k), k). \end{aligned} \tag{17}$$

A.2 Solution of the model with time-inconsistent preferences

Solve the decision rule, $g(k)$, for a linear-quadratic case when the instant utility function is given by

$$u(x, y, k) = a_0 + a_x x + a_y y + a_k k + \frac{a_{xx}}{2} x^2 + \frac{a_{yy}}{2} y^2 + \frac{a_{kk}}{2} k^2 + a_{xy} xy + a_{xk} xk + a_{yk} yk$$

and the capital accumulation function is

$$k_{t+1} = h(y_t, k_t) = c_0 + c_y y_t + c_k k_t.$$

Since the model is linear-quadratic, it is easily shown that the policy function is linear in k, such that $y = a + bk$. Derive marginal utilities and first-order derivatives of the capital accumulation function, solve for x from the resource constraint (1), and substitute in the Euler equation (**??**). Rewrite the first-order condition as $A + Bk = 0$, and we will have

$$
\begin{aligned}
A &= \alpha_x - \alpha_y - a\alpha_{yy} - \alpha_{xy}(1-2a) + \alpha_{xx}(1-a) \\
&-\beta\delta c_y \Big(\alpha_k + \alpha_{xk} + (\alpha_{yk} - \alpha_{xk})(a + b(c_0 + ac_y)) + (c_0 + ac_y)\alpha_{kk} + \\
&\left(b\left(\frac{1}{\beta} - 1\right) + \frac{1}{\beta}\frac{c_k}{c_y}\right) \times \Big(\alpha_x - \alpha_y + (\alpha_{xk} - \alpha_{yk})(c_0 + ac_y) \\
&+ (2\alpha_{xy} - \alpha_{yy} - \alpha_{xx})(a + b(c_0 + ac_y)) - \alpha_{xy} + \alpha_{xx}\Big) \Big) \\
B &= (\alpha_{xk} - \alpha_{yk} + b(2\alpha_{xy} - \alpha_{xx} - \alpha_{yy})) \\
&-\beta\delta c_y \Big((c_k + bc_y)(\alpha_{kk} - b(\alpha_{xk} - \alpha_{yk})) \\
&+ \left(b\left(\frac{1}{\beta} - 1\right) + \frac{1}{\beta}\frac{c_k}{c_y}\right) \times \Big(b(2\alpha_{xy} - \alpha_{xx} - \alpha_{yy}) + \alpha_{xk} \\
&-\alpha_{yk}\Big) (c_k + bc_y)\Big) .
\end{aligned}
$$

Since $A + Bk = 0$ must be met for all k, A and B must be zero. This produces a non-linear equation system in the two unknown decision rule parameters a and b.

A.3 Derivation of welfare over the transition path

To evaluate welfare by implementing a subsidy, solve

$$\max W(k) = u(x, y, k) + \delta\beta V(h(y, k)),$$

under the budget constraint and capital accumulation equation for $p = 1$ and $p < 1$. Find the initial steady-state consumption capital, k_0 and the subsidy funded tax, τ. Find decision rules

$$
\begin{aligned}
g_x(k, (p, t)) &= a_x + b_x k \\
g_y(k, (p, t)) &= a_y + b_y k \\
g_k(k, (p, t)) &= a_k + b_k k
\end{aligned}
$$

such that

$$\begin{aligned} a_k &= c_0 + c_y a_y, \\ b_k &= c_y b_y + c_k \quad \text{and} \\ a_x &= I - t - p a_y, \\ b_x &= -p b_y. \end{aligned}$$

Evaluate

$$V(k) = u\left(g_x\left(k, (p,t)\right), g_y\left(k, (p,t)\right), k\right) + \delta V\left(g_k\left(k, (p,t)\right)\right),$$

which can be rewritten as

$$V(k) = \gamma_0 + \gamma_1 k + \gamma_2 k^2,$$

and in the next period as

$$V(k') = \gamma_0 + \gamma_1 \left(a_k + b_k k\right) + \gamma_2 \left(a_k + b_k k\right)^2.$$

Evaluate the value function

$$\begin{aligned} V(k) &= u(x, y, k) + \delta V(k') \\ &= \alpha_0 + \alpha_x x + \alpha_y y + \alpha_k k + \frac{\alpha_{xx}}{2} x^2 + \frac{\alpha_{yy}}{2} y^2 + \frac{\alpha_{kk}}{2} k^2 + \alpha_{xy} xy + \alpha_{xk} xk + \alpha_{yk} yk \\ &\quad + \delta \left[\gamma_0 + \gamma_1 k' + \gamma_2 \left(k'\right)^2\right] \end{aligned}$$

in the decision rules and $V(k')$, which gives

$$\begin{aligned}
LHS &= \gamma_0 + \gamma_1 k + \gamma_2 k^2 \\
RHS &= \alpha_0 + \alpha_x (a_x + b_x k) + \alpha_y (a_y + b_y k) + \alpha_k (k) + \frac{\alpha_{xx}}{2} (a_x + b_x k)^2 \\
&+ \frac{\alpha_{yy}}{2} (a_y + b_y k)^2 + \frac{\alpha_{kk}}{2} (k)^2 + \alpha_{xy} (a_x + b_x k)(a_y + b_y k) \\
&+ \alpha_{xk} (a_x + b_x k)(k) + \alpha_{yk} (a_y + b_y k)(k) \\
&+ \delta \left[\gamma_0 + \gamma_1 (a_k + b_k k) + \gamma_2 (a_k + b_k k)^2 \right].
\end{aligned}$$

This can be written as

$$A + Bk + Ck^2 = 0$$

where

$$\begin{aligned}
A &= \gamma_0 - \alpha_0 - a_x \alpha_x - a_y \alpha_y - \frac{1}{2} a_x^2 \alpha_{xx} - \frac{1}{2} a_y^2 \alpha_{yy} - a_x a_y \alpha_{xy} - \delta \left(\gamma_0 + \gamma_1 a_k + \gamma_2 a_k^2 \right) \\
B &= \gamma_1 - \alpha_k - b_x \alpha_x - b_y \alpha_y - a_x b_x \alpha_{xx} - \alpha_{xy} (a_x b_y + a_y b_x) - a_y b_y \alpha_{yy} - a_x \alpha_{kx} \\
&- a_y \alpha_{ky} - \delta (\gamma_1 b_k + 2\gamma_2 a_k b_k) \\
C &= \gamma_2 - \frac{1}{2} b_x^2 \alpha_{xx} - b_x b_y \alpha_{xy} - \frac{1}{2} \alpha_{kk} - \frac{1}{2} b_y^2 \alpha_{yy} - b_x \alpha_{kx} - b_y \alpha_{ky} - \delta \gamma_2 b_k^2.
\end{aligned}$$

Solve for γ_0, γ_1, and γ_2: using the fact that $A + Bk + Ck^2 = 0$, we obtain $A = B = C = 0$, and

$$\begin{aligned}
\gamma_2 &= \frac{1}{1 - \delta b_k^2} \left(b_x \alpha_{kx} + b_y \alpha_{ky} + b_x b_y \alpha_{xy} + \frac{1}{2} \alpha_{kk} + \frac{1}{2} b_x^2 \alpha_{xx} + \frac{1}{2} b_y^2 \alpha_{yy} \right) \\
\gamma_1 &= \frac{1}{1 - \delta b_k} \left(\begin{array}{c} \alpha_k + b_x \alpha_x + b_y \alpha_y + a_x \alpha_{kx} + a_y \alpha_{ky} + \alpha_{xy} (a_x b_y + a_y b_x) \\ + a_x b_x \alpha_{xx} + a_y b_y \alpha_{yy} + \delta 2 \gamma_2 a_k b_k \end{array} \right) \\
\gamma_0 &= \frac{1}{1 - \delta} \left(\alpha_0 + a_x \alpha_x + a_y \alpha_y + a_x a_y \alpha_{xy} + \frac{1}{2} a_x^2 \alpha_{xx} + \frac{1}{2} a_y^2 \alpha_{yy} + \delta \left(\gamma_1 a_k + \gamma_2 a_k^2 \right) \right).
\end{aligned}$$

Finally, evaluate $V(k)$ at k_0.

A.4 Solving the model with the governmental budget constraint holding for all t

To solve the model when the governmental budget constraint is required to hold in every time period, the following algorithm is used.

Calculate the steady state for $p = 1 : \left\{\bar{k}_1, V(k)\right\}$. Let $p < 1$ and solve for the new steady state and endogenous tax, τ. Next, solve the transition path for $k_0 = \bar{k}_1$, p and τ_0 with standard dynamic programming and derive the sequences for k and y. Let T be the sequence $\{(1-p)\,y_0, (1-p)\,y_1, \cdots, \tau_0\}$. For a transition length of 10 periods, let the $10th$ element be τ_0 and then the $9th < \tau_0$ and so forth. The $11th$ element is τ_0, the $12th$ element is τ_0 and so on. Solve for period 10 and forward with standard dynamic programming, i.e., use the solution derived above. Now, solve for $y_9 = \arg\max u(x, y, k) + \beta\delta V_{10}$, where $V_{10} = \gamma_{0,10} + \gamma_{1,10} k_{10} + \gamma_{2,10} (k_{10})^2$ is the solution derived earlier. Continue and find $V_8 \cdots$, and so forth. With the computed decision rules, update the sequences for y over the transition and find T. If the old and the new sequences T are the same, repeat the procedure with a new p along a grid. Otherwise, find a new sequence of τs and iterate. When the solution has converged, compare welfare derived at the initial capital stock for different p, $V_{10}\left(\bar{k}_1\right)$, and find the welfare-optimizing subsidy.

A.5 Solving the model with shocks

First, guess that the value function has the same functional form as the period utility function,

$$V(k) = \gamma + \gamma_k k + \gamma_\varepsilon \varepsilon + \frac{\gamma_{kk}}{2} k^2 + \frac{\gamma_{\varepsilon\varepsilon}}{2} \varepsilon^2 + \gamma_{k\varepsilon} k\varepsilon,$$

where the γ coefficients are unknown and to be determined in the maximization process.

Next, write the right-hand side of the Bellman equation as

$$\max_y \alpha_x x + \alpha_y y + \alpha_k k + \frac{\alpha_{xx}}{2} x^2 + \frac{\alpha_{yy}}{2} y^2 + \frac{\alpha_{kk}}{2} k^2 + \alpha_{xy} xy + \alpha_{xk} xk + \alpha_{yk} yk +$$

$$\beta\delta E\left[\gamma + \gamma_k k' + \gamma_\varepsilon \varepsilon' + \frac{\gamma_{kk}}{2} (k')^2 + \frac{\gamma_{\varepsilon\varepsilon}}{2} (\varepsilon')^2 + \gamma_{k\varepsilon} k' \varepsilon' | \varepsilon\right],$$

subject to $x+y=1+\varepsilon$ and $k'=c_0+c_y y+c_k k$. The first-order condition can be derived as

$$\begin{aligned}-\alpha_x+\alpha_y-\alpha_{xx}(1+\varepsilon-y)+\alpha_{yy}y+\alpha_{xy}(1+\varepsilon-2y)-\alpha_{xk}k+\alpha_{yk}k\\+\beta\delta E\left[\gamma_k c_y+\gamma_{kk}c_y(c_0+c_y y+c_k k)+\gamma_{k\varepsilon}\varepsilon' c_y|\varepsilon\right] &= 0.\end{aligned}$$

Take expectations with respect to ε, which gives

$$\begin{aligned}-\alpha_x+\alpha_y-\alpha_{xx}(1+\varepsilon-y)+\alpha_{yy}y+\alpha_{xy}(1+\varepsilon-2y)-\alpha_{xk}k+\alpha_{yk}k\\+\beta\delta\left[\gamma_k c_y+\gamma_{kk}c_y(c_0+c_y y+c_k k)+\gamma_{k\varepsilon}c_y\rho\varepsilon\right] &= 0.\end{aligned}$$

Solve for y

$$\begin{aligned}y &= \begin{pmatrix}\alpha_x-\alpha_y+\alpha_{xx}-\alpha_{xy}-\beta\delta\left(c_y\gamma_k+c_0c_y\gamma_{kk}\right)\\ -k\left(\alpha_{ky}-\alpha_{kx}+\beta\delta c_k c_y\gamma_{kk}\right)\\ -\varepsilon\left(\alpha_{xy}+\rho\beta\delta c_y\gamma_{k\varepsilon}-\alpha_{xx}\right)\end{pmatrix}\times\frac{1}{\left(\alpha_{xx}-2\alpha_{xy}+\alpha_{yy}+\beta\delta c_y^2\gamma_{kk}\right)}\\ &= \lambda_0+\lambda_1 k+\lambda_2\varepsilon\end{aligned}$$

where

$$\begin{aligned}\lambda_0 &= \frac{\alpha_x-\alpha_y+\alpha_{xx}-\alpha_{xy}-\beta\delta\left(c_y\gamma_k+c_0c_y\gamma_{kk}\right)}{\left(\alpha_{xx}+\alpha_{yy}-2\alpha_{xy}+\beta\delta\gamma_{kk}c_y^2\right)}\\ \lambda_1 &= \left(-\frac{\left(\alpha_{ky}-\alpha_{kx}+\beta\delta c_k c_y\gamma_{kk}\right)}{\left(\alpha_{xx}+\alpha_{yy}-2\alpha_{xy}+\beta\delta\gamma_{kk}c_y^2\right)}\right)\\ \lambda_2 &= \left(-\frac{\left(\alpha_{xy}+\rho\beta\delta c_y\gamma_{k\varepsilon}-\alpha_{xx}\right)}{\left(\alpha_{xx}+\alpha_{yy}-2\alpha_{xy}+\beta\delta\gamma_{kk}c_y^2\right)}\right).\end{aligned}$$

Next, the unknown coefficients, γ, must be determined. For this purpose, take $V(k,\epsilon)=u(1+\varepsilon-y,y,k)+\delta E\left[V(k',\varepsilon'|\varepsilon)\right]$, and use the linear expression for y and the expression for k'. Notice that this functional equation has δ, not $\beta\delta$, which is the time inconsistency.

$$\begin{aligned} V(k,\epsilon) &= u(1+\varepsilon-y,y,k)+\delta E\left[V(k',\varepsilon'|\varepsilon)\right] \\ &= \alpha_x(1+\varepsilon-y)+\alpha_y y+\alpha_k k+\frac{\alpha_{xx}}{2}(1+\varepsilon-y)^2+\frac{\alpha_{yy}}{2}y^2 \\ &\quad +\frac{\alpha_{kk}}{2}k^2+\alpha_{xy}(1+\varepsilon-y)y+\alpha_{xk}(1+\varepsilon-y)k+\alpha_{yk}yk \\ &\quad + \left[\begin{array}{c} \delta\left(\gamma+\gamma_k\left(c_0+c_y y+c_k k\right)\right)+\delta\gamma_\varepsilon\rho\varepsilon+\delta\frac{\gamma_{kk}}{2}\left(c_0+c_y y+c_k k\right)^2 \\ +\delta\frac{\gamma_{\varepsilon\varepsilon}}{2}\sigma_\varepsilon^2+\delta\frac{\gamma_{\varepsilon\varepsilon}}{2}\left(\rho\varepsilon\right)^2+\delta\gamma_{k\varepsilon}\left(c_0+c_y y+c_k k\right)\rho\varepsilon \end{array}\right] \end{aligned}$$

where

$$E\left[\delta\frac{\gamma_{\varepsilon\varepsilon}}{2}(\varepsilon')^2 \mid \varepsilon\right]=\delta\frac{\gamma_{\varepsilon\varepsilon}}{2}\sigma_\varepsilon^2+\delta\frac{\gamma_{\varepsilon\varepsilon}}{2}\left(\rho\varepsilon\right)^2.$$

Substitute for y and rewrite as

$$\begin{aligned} V(k,\epsilon) &= \alpha_x(1+\varepsilon-(\lambda_0+\lambda_1 k+\lambda_2\varepsilon))+\alpha_y\left(\lambda_0+\lambda_1 k+\lambda_2\varepsilon\right)+\alpha_k k \\ &\quad +\frac{\alpha_{xx}}{2}(1+\varepsilon-(\lambda_0+\lambda_1 k+\lambda_2\varepsilon))^2+\frac{\alpha_{yy}}{2}\left(\lambda_0+\lambda_1 k+\lambda_2\varepsilon\right)^2 \\ &\quad +\frac{\alpha_{kk}}{2}k^2+\alpha_{xy}(1+\varepsilon-(\lambda_0+\lambda_1 k+\lambda_2\varepsilon))\left(\lambda_0+\lambda_1 k+\lambda_2\varepsilon\right) \\ &\quad +\alpha_{xk}(1+\varepsilon-(\lambda_0+\lambda_1 k+\lambda_2\varepsilon))k+\alpha_{yk}\left(\lambda_0+\lambda_1 k+\lambda_2\varepsilon\right)k \\ &\quad + \left(\begin{array}{c} \delta\left(\gamma+\gamma_k\left(c_0+c_y\left(\lambda_0+\lambda_1 k+\lambda_2\varepsilon\right)+c_k k\right)\right)+\delta\gamma_\varepsilon\rho\varepsilon \\ +\delta\frac{\gamma_{kk}}{2}\left(c_0+c_y\left(\lambda_0+\lambda_1 k+\lambda_2\varepsilon\right)+c_k k\right)^2 \\ +\delta\frac{\gamma_{\varepsilon\varepsilon}}{2}\sigma_\varepsilon^2+\delta\frac{\gamma_{\varepsilon\varepsilon}}{2}\left(\rho\varepsilon\right)^2+\delta\gamma_{k\varepsilon}\left(c_0+c_y\left(\lambda_0+\lambda_1 k+\lambda_2\varepsilon\right)+c_k k\right)\rho\varepsilon \end{array}\right). \end{aligned}$$

Use $V\left(k,\varepsilon\right)=\gamma+\gamma_k k+\gamma_\varepsilon\varepsilon+\frac{\gamma_{kk}}{2}k^2+\frac{\gamma_{\varepsilon\varepsilon}}{2}\varepsilon^2+\gamma_{k\varepsilon}k\varepsilon$, and rewrite the equation as:

$$A+Bk+C\varepsilon+Dk^2+E\varepsilon^2+Fk\varepsilon=0.$$

Since $A,B,C,D,E,$ and F must each be zero for the equation to hold for all values of k and ε, the equation system can be used to solve for γ. We have

$$\begin{aligned} A &= \lambda_0^2 \left(\alpha_{xy} - \frac{1}{2}\alpha_{x^2} - \frac{1}{2}\alpha_{y^2} - \frac{1}{2}\delta c_y^2 \gamma_{k^2} \right) \\ &+\lambda_0 \left(\alpha_x - \alpha_y - \alpha_{xy} - \delta c_y \gamma_k + \alpha_{x^2} - \delta c_0 c_y \gamma_{k^2} \right) \\ &+\gamma - \alpha_x - \delta \left(\gamma + c_0 \gamma_k \right) - \frac{1}{2}\alpha_{x^2} - \frac{1}{2}\delta c_0^2 \gamma_{k^2} - \frac{1}{2}\delta \sigma_\varepsilon^2 \gamma_{\varepsilon^2} \end{aligned}$$

$$\begin{aligned} B &= \lambda_0 \lambda_1 \left(2\alpha_{xy} - \alpha_{x^2} - \alpha_{y^2} - \delta c_y^2 \gamma_{k^2} \right) \\ &+\lambda_1 \left(\alpha_x - \alpha_y - \alpha_{xy} - \delta c_y \gamma_k + \alpha_{x^2} - \delta c_0 c_y \gamma_{k^2} \right) \\ &+\lambda_0 \left(\alpha_{kx} - \alpha_{ky} - \delta c_k c_y \gamma_{k^2} \right) \\ &+\gamma_k - \alpha_k - \alpha_{kx} - \delta c_k \gamma_k - \delta c_0 c_k \gamma_{k^2} \end{aligned}$$

$$\begin{aligned} C &= \lambda_0 \lambda_2 \left(2\alpha_{xy} - \alpha_{x^2} - \alpha_{y^2} - \delta c_y^2 \gamma_{k^2} \right) \\ &+\lambda_2 \left(\alpha_x - \alpha_y - \alpha_{xy} - \delta c_y \gamma_k + \alpha_{x^2} - \delta c_0 c_y \gamma_{k^2} \right) \\ &+\lambda_0 \left(\alpha_{x^2} - \rho \delta c_y \gamma_{k\varepsilon} - \alpha_{xy} \right) \\ &+\gamma_\varepsilon - \alpha_x - \rho \delta \gamma_\varepsilon - \rho \delta c_0 \gamma_{k\varepsilon} - \alpha_{x^2} \end{aligned}$$

$$\begin{aligned} D &= \lambda_1^2 \left(\alpha_{xy} - \frac{1}{2}\alpha_{x^2} - \frac{1}{2}\alpha_{y^2} - \frac{1}{2}\delta c_y^2 \gamma_{k^2} \right) \\ &+\lambda_1 \left(\alpha_{kx} - \alpha_{ky} - \delta c_k c_y \gamma_{k^2} \right) \\ &+\frac{1}{2}\gamma_{k^2} - \frac{1}{2}\alpha_{k^2} - \frac{1}{2}\delta c_k^2 \gamma_{k^2} \end{aligned}$$

$$\begin{aligned} E &= \lambda_2^2 \left(\alpha_{xy} - \frac{1}{2}\alpha_{x^2} - \frac{1}{2}\alpha_{y^2} - \frac{1}{2}\delta c_y^2 \gamma_{k^2} \right) \\ &+\lambda_2 \left(\alpha_{x^2} - \rho \delta c_y \gamma_{k\varepsilon} - \alpha_{xy} \right) \\ &+\frac{1}{2}\gamma_{\varepsilon^2} - \frac{1}{2}\alpha_{x^2} - \frac{1}{2}\rho^2 \delta \gamma_{\varepsilon^2} \end{aligned}$$

$$
\begin{aligned}
F &= \lambda_1\lambda_2\left(2\alpha_{xy} - \alpha_{x^2} - \alpha_{y^2} - \delta c_y^2\gamma_{k^2}\right) \\
&\quad + \lambda_2\left(\alpha_{kx} - \alpha_{ky} - \delta c_k c_y \gamma_{k^2}\right) \\
&\quad + \lambda_1\left(\alpha_{x^2} - \rho\delta c_y \gamma_{k\varepsilon} - \alpha_{xy}\right) \\
&\quad + \gamma_{k\varepsilon} - \alpha_{kx} - \rho\delta c_k \gamma_{k\varepsilon}.
\end{aligned}
$$