WEAKLY NONSEPARABLE PREFERENCES AND DISTORTIONARY TAXES IN A SMALL OPEN ECONOMY

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This paper examines the dynamic effects of distortionary taxes in a small open economy. The employed utility function implies both endogenous rates of time preference and a tractable form of weak nonseparability between consumption and leisure. Weak nonseparability induces novel long-run welfare and wealth effects of taxes and generates very different current account movements. Endogenous rates of time preference facilitate the examination of a tax on international borrowing and lending.

1. INTRODUCTION

It is now common to examine the dynamic effects of distortionary taxes in a utility-maximizing framework.2 Surprisingly, however, most dynamic analyses have restricted individuals' preferences to weakly separable and more accurately time-additive preferences. For two reasons relevant to the issue of taxation, the time-additive preferences should be extended. First, as Barro and King (1984) have shown, time additivity ties together the substitution and wealth effects of taxes in the sense that the relative responses of consumption and leisure to changes in any future real wage or interest rate must equal their relative responses to a wealth change. Such entangled wealth and substitution effects are a general restriction imposed by weakly separable preferences. Second, time additivity implies a constant rate of time preference. In a small open economy, the existence of equilibrium requires that this rate of time preference equal the exogenously given, after-tax world interest rate. Taxes which affect the after-tax world interest rate, such as a tax on international borrowing and lending, destroy such equality and lead to instability (Epstein and Hynes 1983). Only under severe restrictions on capital mobility can such taxes be examined with additive preferences. Even for taxes which do not affect the after-tax world interest rate, time additivity generates the dependence of the steady state on the initial conditions of the economy, making tax evaluation sensitive to those initial conditions (e.g. Sen and Turnovsky 1989). A simple solution to this problem is to endogenize the rates of time preference.

This paper employs a minimal extension of the Uzawa (1968) utility function to

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incorporate both weak nonseparability and endogenous rates of time preference. The utility function was described by Epstein, Ham, and Zin (1988). Preferences are weakly nonseparable in the sense that the marginal rate of substitution between leisure and consumption depends upon future but not past consumption and leisure. This dependence is entirely through a utility index and hence is tractable. Consumption is said to be more welfare stabilizing if an increase in the utility index shifts preferences toward current leisure and away from current consumption.

In a small open economy with perfect capital mobility, the utility function is used to examine the dynamic effects of permanent changes in a variety of distortionary taxes. The taxes considered are a tax on international borrowing and lending, and domestically based taxes which include taxes on consumption, labor income and domestic capital income. It is shown that weak nonseparability generates dynamic effects of taxes significantly different from weakly separable preferences.

The most significant difference is the long-run welfare and wealth effects. In particular, domestically based taxes increase long-run welfare and wealth if and only if consumption is more welfare stabilizing. A tax on international borrowing and lending generates an additional negative wealth effect by lowering the long-run rate of time preference and creates an ambiguous overall wealth effect. In contrast, weakly separable preferences imply no long-run welfare or wealth effect for domestically based taxes and an unambiguously negative wealth effect for a tax on international borrowing and lending. The presence of a wealth effect implies that capital taxation changes not only the composition of a country’s portfolio of assets but also the size of the portfolio.

Weak nonseparability creates the long-run welfare and wealth effects for a simple reason. It generates a discrepancy between the marginal rate of substitution between consumption and leisure and the rate of substitution implied by the requirement that the long-run rate of time preference equal the after-tax world interest rate. For the two rates to equal, welfare must change accordingly. So must wealth. The long-run welfare and wealth effects are novel in the sense that they do not depend on adjustment cost—an alternative channel for those effects in small open economies with time-additive preferences (Sen and Turnovsky 1989). More importantly, our model provides an example to show that adjustment cost is insufficient for the long-run welfare effect.

Under weakly nonseparable preferences, the immediate impacts of taxes and the implied comovement among variables are also different from those under weakly separable preferences. For example, when leisure is more welfare stabilizing, capital income taxation induces a current account deficit rather than a surplus as suggested by conventional wisdom. Nevertheless, the result of Barro and King (1984), that taxes create negative comovement between consumption and labor employment, is still valid.

The Uzawa function has been increasingly applied in economic modelling, but in most applications leisure and consumption have been made weakly separable. An exception is Judd (1985). Using a similar specification in a closed economy, Judd

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has shown that nonseparability and endogenous rates of time preferences do not change the result in the time-additive framework that the second-best capital income taxation is zero asymptotically. Although this result can also be validated in an open economy, weak nonseparability and endogenous rates of time preference do generate significant differences in other aspects of taxation.

The remainder of this paper is organized as follows. Section 2 describes the preferences and the economy. Section 3 and Section 4 examine, respectively, the long-run and the transitional effects of taxes. Section 5 concludes the paper. All proofs are collected in the Appendix.

2. A SMALL OPEN ECONOMY

2.1. Intertemporal Utility and Weak Nonseparability. A program $E$ assigns consumption $c(t)$ and leisure $e(t)$ to each time $t \geq 0$ over an infinite horizon. It permits partitions into $(E_T, \tau E)$ for all $T > 0$, where $E_T$ and $\tau E$ are the programs up to time $T$ and after time $T$ respectively. The intertemporal utility function is defined over such programs,

\begin{equation}
U(E) = \int_0^\infty v(c(t), e(t)) \exp \left[ - \int_0^t \beta(c(\tau), e(\tau)) \, d\tau \right] \, dt.
\end{equation}

This is the Uzawa utility function extended by Epstein, Ham, and Zin (1988) to include leisure. It is generated by the following differential equation for the utility index $\phi(T) = U(\tau E)$:

\begin{equation}
\dot{\phi} = -q(c, e, \phi), \quad q(c, e, \phi) = v(c, e) - \phi \beta(c, e)
\end{equation}

\[ \lim_{t \to \infty} \phi(t) \exp \left[ - \int_0^t \beta(c(\tau), e(\tau)) \, d\tau \right] = 0. \]

The preferences in (2.1) are weakly nonseparable in two aspects. First, consumption (or leisure) at different dates is weakly nonseparable. To see this, compute the marginal utility of consumption near time $T$, $\partial U/\partial c(T)$, via the Volterra derivative (see Ryder and Heal 1973 for a reference):

\begin{equation}
\frac{\partial U}{\partial c(T)} = q_c \exp \left[ - \int_0^T \beta(\tau) \, d\tau \right].
\end{equation}

Because $q_c = v_c - \phi \beta_c$, the marginal rate of substitution between consumption at different dates depends on future consumption and leisure through the utility index $\phi$ if $\beta_c \neq 0$. The same nonseparability exists for leisure if $\beta_e \neq 0$.

Second, consumption and leisure are weakly nonseparable: the marginal rate of substitution between the two depends upon future consumption and leisure.
Throughout the paper, weak nonseparability refers to this meaning. It can be verified that the dependence on future variables is entirely summarized by the utility index $\phi$. This simple dependence makes the form of weak nonseparability tractable and facilitates the following definition. Define an index of weak nonseparability, $\lambda$, by

$$
\lambda(T) = \frac{\partial}{\partial \phi(T)} \log(MRS(T)), \quad MRS(T) = \frac{\partial U}{\partial e(T)} \left/ \frac{\partial U}{\partial c(T)} \right.
$$

Preferences are weakly separable if $\lambda(T) = 0$ for all $T$. Computation yields

(2.4) \[ \lambda = (v_e \beta - v_e \beta_e)/(q_e q_e). \]

Therefore, $\lambda(T) = 0$ if functions $v$ and $\beta$ are transformations of the same function. Special cases of weakly separable preferences include the time-additive preferences (where $\beta = $ constant), the original Uzawa preferences (where $\beta = \beta(v)$) and the one in Epstein and Hynes (1983) (where $v = $ constant).

The nature of weak nonseparability is governed by the sign of $\lambda$. If $\lambda > 0$, the amount of consumption which the consumer is willing to give up for a marginal unit of leisure increases with future utility $\phi$. Hence an increase in future utility shifts preferences away from current consumption and toward current leisure. In this case, consumption increases more slowly (than leisure) on an increasing path of utility and decreases more slowly on a decreasing path of utility. For this reason, consumption is said to be more welfare stabilizing (than leisure) if $\lambda > 0$ and less welfare stabilizing if $\lambda < 0$.

For an alternative interpretation of weak nonseparability and for the dynamic analysis, we link weak nonseparability to the rates of time preference. Following Epstein and Hynes (1983), define the (local) rate of time preference for consumption as

$$
\rho_c(T) = -\frac{d}{dT} \log \left. \frac{\partial U}{\partial c(T)} \right|_{\dot{e} = \ddot{e} = 0}.
$$

$\rho_c$ measures the proportional decrease of marginal utility of $c$ caused by a postponement of the program. It can be computed that

(2.5) \[ \rho_c = (\beta v_c - v \beta_e)/q_c. \]

The rate of time preference for leisure, $\rho_\dot{e}$, can be defined and computed similarly.

Along any constant programs, $\phi = v/\beta$ and the two rates of time preferences equal, $\rho_c = \rho_\dot{e} = \beta$. We refer to $\beta$ as the long-run (or steady-state) rate of time preference. Along any nonconstant programs, however, the two rates of time preference are distinct. The difference between them is

(2.6) \[ \rho_c - \rho_\dot{e} = \lambda \phi. \]

4 Weak nonseparability apparently differs from nonseparability between $c$ and $e$ at a given time. The latter is represented by $q_{ce} \neq 0$, which we assume away in the following analysis.
Therefore, consumption and leisure are weakly separable if and only if \( \rho_c(T) = \rho_e(T) \) for all \( T \). Consumption is more welfare stabilizing if consumers are more patient toward consumption than leisure when utility is falling but more impatient when utility is rising.

It is important to note that the rates of time preference are functions of future consumption and leisure only through the utility index \( \phi \). \( \rho_c(\rho_e) \) is increasing in \( \phi \) if and only if \( \beta \) is increasing in \( c(e) \). It is well known from Epstein (1987) and Lucas and Stokey (1984) that local stability requires increasing rates of time preferences. Thus we assume that \( \beta \) is an increasing function of \( c \) and \( e \). Although the assumption of increasing rates of time preference is controversial, we refer to Epstein (1987) and Obstfeld (1990) for supportive arguments. The assumption and others which guarantee monotonicity and local concavity of the intertemporal utility function \( U \) are given in Assumption 1 below (see the Appendix for a proof).

**Assumption 1.** \( v \) and \( \beta \) are twice continuously differentiable and

(i) \( q_c > 0, q_e > 0 \);
(ii) \( \beta > 0, \beta_c > 0, \beta_e > 0 \);
(iii) \( q_{cc} < 0, q_{ee} < 0, q_{ce} = 0 ; \lambda \in (\beta q_{cc}/q_c^2, -\beta q_{ee}/q_e^2) \).

Condition (i) is necessary and sufficient for the marginal utility of \( c \) and \( e \) to be positive and hence is intuitively linked to monotonicity of \( U \). The Appendix shows that (i) and (ii) imply \( \lambda \in (-\beta_c/q_c, \beta_e/q_e) \), and that \( \lambda > 0 \) if and only if \( MRS > \beta_e/\beta_c \). These features will be used in Section 3. Condition (iii) is sufficient for \( U \) to be locally concave. The restriction \( q_{ce} = 0 \) is imposed for specificity.

2.2. **Competitive Equilibrium.** The representative consumer with utility function (2.1) has perfect access to international goods and capital markets, where the interest rate is given at \( \tau \). The consumer solves the following maximizing problem:

\[
(P) \quad \max U \quad \text{s.t.} \quad (2.7) \quad (1 + \tau_c)c + I + F \leq r(1 - \tau_F)F + r_d(1 - \tau_d)k + \omega(1 - \tau_w)n - \tau_u
\]

\[
k = I - \delta k, \quad F(0) + k(0) = F_0 + k_0 \quad \text{given,}
\]

where \( \tau_u, \tau_c, \tau_d, \tau_w \) and \( \tau_F \) are respectively the lump-sum tax and rates of taxes on consumption, domestic capital income, labor income and on international borrowing and lending. \( k \) is domestic capital stock, \( I \) gross investment, \( F \) the claims on foreign assets, \( n = 1 - e \) the amount of labor supply, \( r_d \) the before-tax rate of return to domestic investment and \( \omega \) the wage rate. The total nonhuman wealth of the agent is \( a = k + F \). The initial value of \( a \) is given at \( a(0) = a_0 \), but its composition, \( k(0) \) and \( F(0) \), can instantaneously change without adjustment cost.

Competitive firms are endowed with a constant returns to scale production function \( f(k, n) \) with \( f_{kk} < 0 \) and \( f_{nn} < 0 \). Firms maximize profit. The

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5 Blanchard and Fischer (1989) present the arguments against increasing time preference. Appealing stability properties can also be achieved in Blanchard's (1985) version of the overlapping generations model, though his assumption of an age-independent probability of death may seem unrealistic.
maximization conditions are \( r_d = f_k(k, n) \) and \( w = f_w(k, n) \). The domestic government collects taxes, issues bonds and maintains the following budget constraint:\(^6\)

\[
g \leq \tau_c c + \tau_w w + \tau_d d + \tau_f F + \tau_r r_d k + \tau_n w n + \tau_u u.
\]

In the following discussion, we assume that all tax changes are permanent and hence \( \tau = 0 \). Furthermore, we assume that government spending is constant. Thus the revenues from distortionary taxes are rebated through lump-sum transfers.

A competitive equilibrium is a convergent path of \( (c, n, k, I, w, r_d, r_f, \tau) \) such that (i) given \( \tau \) and \( (w, r_d) \), \( (c, n, k, I) \) solve \( (P) \); (ii) \( (k, n, w, r_d) \) conform with firm's profit maximization; and (iii) \( \tau \) balances the government budget. From the existence of the steady state in Section 3 and local stability in Section 4 one can show that an equilibrium exists. To explore the equilibrium, combine (i) and (ii) to derive

\[
f_k = e_1, \quad e_1 = \frac{r(1 - \tau_F) + \delta}{1 - \tau_k},
\]

(2.9)

\[
MRS(c, e, \phi) = e_2 f_n, \quad e_2 = \frac{1 - \tau_w}{1 + \tau_c}.
\]

Equation (2.9) requires that the after-tax rates of return to investment abroad and domestically be equal; (2.10) requires that the marginal rate of substitution of leisure for consumption equal the tax-adjusted wage rate. (2.9) and (2.10) solve for \( k \) and \( c \) as functions of \( n, \phi \) and the tax rates:

\[
k = K(n, e_1), \quad c = \psi(n, \phi, e_1, e_2).
\]

(2.11)

Under the assumption of constant returns to scale, \( K_n > 0 \) and \( \psi_n < 0 \). Also, \( \psi_\phi > 0 \) if and only if \( \lambda < 0 \).

The law of motion of labor employment can be derived from (2.11), the maximization conditions and the assumption \( \tau = 0 \):

\[
h = -\frac{q_e}{q_c} [\rho_c - r(1 - \tau_F)].
\]

(2.12)

The law of motion of consumption can be recovered by (2.11):

\[
c = \frac{q_c}{q_c} [\rho_c - r(1 - \tau_F)].
\]

(2.13)

(2.12) and (2.13) mimic the corresponding equations under time-additive preferences.\(^7\)

Combining (2.7) and (2.8) obtains the motion of the total assets.

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\(^6\) Following a referee's suggestion, we abstract from government bonds. Introducing government bonds does not change the results if the initial value of bonds is zero, as assumed in typical analysis of taxation such as Fudoli (1987).

\(^7\) If \( q_{ee} \neq 0 \) the law of motion of \( c \) depends on both \( \rho_c - r(1 - \tau_F) \) and \( \rho_c - r(1 - \tau_F) \).
\( \dot{a} = ra + f(k, n) - (r + \delta)k - c - g. \)

The dynamic system consists of (2.2), (2.12) and (2.14) with the initial condition, \( \phi(0) = \phi_0 \), and proper transversality conditions. It is a three dimensional system, governing the motion of \( \phi, n \) and \( a \). The laws of motion of \( c \) and \( k \) can be recovered by (2.11). Finally, the current account, \( \tilde{F} \), is recovered by the relation \( F = a - k \).

3. LONG-RUN EFFECTS OF DISTORTIONARY TAXES

Tax changes are specified as follows. The economy is in the steady state at time 0 with tax rates \( \tau_c, \tau_w, \tau_k, \tau_F, \tau_a \). Taxes change unexpectedly at time 0, from the initial values to \( \tau_0 + h\tau_1 \), where \( h \) is a constant. All tax changes are marginal so that \( h \) is infinitesimal. The changes in a specific tax alone or in several taxes simultaneously can be modeled by choosing the corresponding elements in \( \tau_1 \) to be nonzero. According to our earlier assumption, \( \tau_1 \) is constant.

To begin, we characterize the steady state of the dynamic system. The steady state, denoted \((\phi^*, n^*, a^*)\), is given by

\[
\begin{align*}
\phi^* &= \psi(c^*, 1 - n^*)/\beta(c^*, 1 - n^*) \\
\rho^*_c &= \beta(c^*, 1 - n^*) = r(1 - \tau_F) \\
a^* &= [c^* + g - f(k^*, n^*) + (r + \delta)k^*]/r,
\end{align*}
\]

where \( c^* \) and \( k^* \) satisfy (2.11). In the steady state the two rates of time preference equal the after-tax international interest rate. To solve for the steady state, note that if \( c^* \) and \( n^* \) are solved then other steady-state variables are uniquely determined by (3.1a), (3.1c) and (2.11). \( c^* \) and \( n^* \), in turn, satisfy conditions (3.1b) and (2.10). Rewrite (2.10) by substituting (3.1a):

\[
MRS(c^*, 1 - n^*, \psi(c^*, 1 - n^*)/\beta(c^*, 1 - n^*)) = \varepsilon_1 f_\varepsilon(K(n^*, \varepsilon_1), n^*).
\]

\( c^* \) and \( n^* \) can be solved from (3.2) and (3.1b) independently of other variables. Note that for constant returns to scale technology, the right-hand side of (3.2) is independent of \( n^* \).

The determination of \((c^*, n^*)\) is depicted in Figure 1. The MRS curve, representing (3.2), is downward sloping under (iii) of Assumption 1. The \( \beta \) curve, representing (3.1b), is upward sloping under the assumptions \( \beta_c > 0 \) and \( \beta_n > 0 \). Thus the steady state exists and is unique (assuming interior solutions).

The long-run effects of taxes can be obtained by replacing \( \tau_0 + h\tau_1 \), differentiating the steady state with respect to \( h \) and evaluating the results at \( h = 0 \). Using a subscript \( h \) to indicate the changes of variables with respect to tax changes, the Appendix gives the results for \( \phi_h^*, n_h^* \) and \( a_h^* \). The effects of taxes on other variables can be determined accordingly.

We first examine domestically based taxes. An increase in domestically based taxes shifts the MRS curve down to the left but leaves the \( \beta \) curve intact. Therefore, long-run consumption and labor employment decrease. Given the complementarity between capital and labor implied by the production function,
domestic capital stock is also reduced. Since both $k^*$ and $n^*$ decline, the long-run output decreases.

Domestically based taxes have long-run welfare effects. They raise long-run welfare if and only if consumption is more welfare stabilizing in the sense defined in Section 2. To see this, use (3.1a) and (3.1b) to derive

\[
\frac{d\phi^*}{dn^*} = \frac{q_c}{\beta} \left( \frac{\beta_e}{\beta_c} - \frac{q_e}{q_c} \right) \frac{dn^*}{\beta_c}.
\]

Since $dn^* < 0$, $d\phi^* > 0$ if and only if $\beta_e/\beta_c < q_e/q_c$, which is equivalent to $\lambda > 0$ as noted in Section 2. Such a welfare result can be illustrated in Figure 1 by drawing the long-run utility curve $w/\beta$. Since the long-run utility curve has the slope $q_e/q_c$, it is steeper than the $\beta$ curve, whose slope is $\beta_e/\beta_c$, if and only if $\lambda > 0$. With domestically based taxes, the steady state moves from point $E$ to point $E'$. It is apparent that the long-run utility level is raised if and only if point $E'$ is above the long-run utility curve and hence if and only if $\lambda > 0$.

Figure 1 also provides a clear illustration of how weak nonseparability helps to disentangle the substitution and wealth effects of taxes. The total change in consumption and leisure can be decomposed into two. The first is the response to
the substitution effect of taxes, represented by the move from point $E$ to point $A$ along the same long-run utility curve; the second is the response to a wealth effect, represented by the move from point $A$ to point $E'$ along the new $MRS$ curve. If preferences are weakly separable, the second response disappears and hence domestically based taxes do not change long-run welfare.

To justify the association of the long-run welfare effect to the long-run wealth effect, examine the case without initial taxes. Differentiating (3.1c) and using (2.9) and (2.10), one can obtain

$$d a^* = \frac{1}{r} \left( \frac{d e^*}{d n^*} - MRS \right) d n^*. \tag{3.4}$$

Thus a domestically based tax, reducing labor supply, increases long-run wealth if and only if $d e^*/d n^* < MRS$. Since long-run welfare is increased under the same condition, the tax changes long-run wealth and welfare in the same direction if there are no initial taxes. By continuity, long-run welfare and wealth are positively associated when the initial taxes are nonzero but sufficiently small. We restrict the discussion below to the case of small initial taxes.\(^9\)

The nontrivial wealth effect generates new implications on how domestic capital income taxation affects the long-run credit position of the country. Capital income taxation reduces the long-run domestic capital stock but increases the country's long-run foreign assets. Since total nonhuman wealth decreases when leisure is more welfare stabilizing, the reduction in domestic capital stocks exceeds the increase in the country's foreign assets in this case. On the other hand, the reduction in domestic capital stocks is exceeded by the increase in foreign assets when consumption is more welfare stabilizing. In contrast, the standard infinite-horizon model suggests that the size of the portfolio in the steady state is unaffected by a capital income tax.\(^10\) This latter result occurs here only when preferences are weakly separable.

To understand the long-run welfare effects in this model, it is important to compare them with those in two other types of models: one is overlapping-generations models, the other is infinite-horizon models with adjustment cost. In both types of models, a capital income tax can raise long-run welfare under suitable conditions, but the mechanism involved is quite different from the current model.\(^11\)

First, we compare with Diamond's (1965) overlapping generations models. In a two-country version of the model, Sibert (1990, p. 308) shows the possibility that a capital income tax in a country raises the long-run welfare of that country by affecting the world interest rate. However, when the country has no influence on

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\(^8\) I owe to a referee for this simple illustration. An alternative illustration is to use the formulas for $\phi^*_a$ and $\sigma^*_a$ in the Appendix.

\(^9\) The two variables may be negatively correlated in the short run (see Section 4).

\(^10\) Under time additive preferences, the size of the portfolio can be changed if there is adjustment cost (Bovenberg 1986) or if the country is large (Sibert 1990).

\(^11\) A capital income tax also raises long-run welfare in the standard infinite-horizon model if the initial tax is high and the tax change is large. Neither case is our focus.
the world interest rate, as in the case of a small open economy, a capital income tax unambiguously reduces the country's long-run welfare.\textsuperscript{12}

Now we compare with an infinite-horizon, time-additive model with adjustment cost. An example of the latter is Sen and Turnovsky (1989). First note that adjustment cost is insufficient for the long-run effects. To see insufficiency, introduce adjustment cost into our model but maintain weak separability (\( \lambda = 0 \)). The dynamic system under adjustment cost has two more dimensions, one for the capital stock and another for the shadow price of capital—Tobin's \( q \). It is not difficult to show that the new system has identical steady state to that in (3.1), with additional characterization for the steady-state shadow price of capital. The same argument as in (3.3) then shows that domestic taxes have no long-run welfare or wealth effect when \( \lambda = 0 \), despite adjustment cost. What really generates the long-run welfare effects in Sen and Turnovsky (1989) is the dependence of steady states on the initial conditions of the economy, generated by the constant rates of time preference.\textsuperscript{13} Thus the key difference between our model and Sen and Turnovsky (1989) is that the long-run results in our model do not depend on initial conditions. As remarked in the introduction, the independence is desirable for evaluating the welfare costs of taxes. The independence implies that temporary taxes have no long-run effects, a result contrary to Sen and Turnovsky.

There are additional differences between the current model and Sen and Turnovsky (1989). First, the long-run wealth and welfare effects of taxes can be either positive or negative in our model, depending on \( \lambda \), but unambiguously positive in Sen and Turnovsky (1989). Second, as will be discussed in Section 4, our model generates different dynamic movements in the current account from those in Sen and Turnovsky. In any case, the following discussion on a tax on international borrowing and lending is novel with respect to the time-additive models.

As stated in the introduction, it is difficult to examine a tax on international borrowing and lending in a small open economy with time-additive preferences and an infinite horizon. In contrast, the examination is easy in our model. A tax on international borrowing and lending, \( \tau_F \), induces a lower long-run rate of time preference and hence a negative wealth effect. This effect is represented by a downward shift of the \( \beta \) curve in Figure 1. In addition, an increase in \( \tau_F \) raises the relative return to domestic investment and hence creates a wealth effect opposite to a tax on domestic capital income. The overall wealth effect is ambiguous, depending on the nature of weak nonseparability. If consumption is more welfare stabilizing, both components of wealth effects are negative and hence long-run wealth is lowered by the tax \( \tau_F \); if leisure is more welfare stabilizing, the two wealth effects are opposite. In this case, long-run wealth and welfare can be raised by the tax if leisure is sufficiently more welfare stabilizing. In either case, there is a positive correlation between long-run wealth and long-run welfare as in the case of domestically based taxes.

\textsuperscript{12} In a small open economy, the condition (25) for a utility-increasing capital income tax in Silbert (1990) becomes \( (r - \kappa)/\kappa < 0 \), which is apparently violated when the population growth rate \( \kappa \) is zero.

\textsuperscript{13} Precise characterization of steady states in Sen and Turnovsky (1989) only if \( \beta = \kappa (1 - \tau_F) \) and \( F(t) + \epsilon k(t) = F_0 + \epsilon k_0 \) for all \( t \). With elastic labour supply, \( \xi > 1 \). Since \( d\kappa^* = (1 - \xi) d\kappa^* \), domestically based taxes reduce \( \kappa^* \), increases \( \sigma^* \) and increases long-run welfare.
4. TRANSITIONAL EFFECTS OF DISTORTIONARY TAXES

Since the short-run responses of variables are connected to the long-run responses by stability, the variables respond to the long-run welfare and wealth effects even in the short run. We examine the immediate impact of the taxes and the implied comovement among variables. To do so, replace the $\tau$'s in the dynamic system by $\tau_0 + h\tau_1$, differentiate the system with respect to $h$ and evaluate the result at $h = 0$. We then obtain a dynamic system of $(\phi_h, n_h, a_h)$. Linearizing the dynamic system, we have

\[(\phi_h, n_h, a_h)^T = J(\phi_h^*, n_h, a_h^*) \]

where $a_h(0) = 0$ and $J$ is a $3 \times 3$ matrix (see the Appendix).

The Appendix shows that system (4.1) is locally stable and the stable root is $\theta_1$, given by (A.3) in the Appendix. Furthermore, the stable path of the system is characterized by

\[a_h(t) = a_h^*(1 - e^{\theta_1 t})\]

\[(4.2)\]

\[
\begin{bmatrix}
\phi_h(t) - \phi^*_h \\
n_h(t) - n^*_h
\end{bmatrix} = \frac{\theta_1 - r}{\Omega}
\begin{bmatrix}
\left(\frac{\lambda q_e^2}{q_{ee}} + \frac{\lambda^2 q_e^{2*}}{q_{ee}}\right) - \frac{\beta \beta_e}{q_e} + \frac{\lambda^2 q_e^{2*}}{q_{ee}}
\end{bmatrix}
\]

\[
(a_h(t) - a_h^*)
\]

where

\[\Omega = -A q_e + q_e \left(1 - r - \delta\right) \frac{f_{kn}}{f_{kk}}, \quad A = \left(\lambda \theta_1 + \frac{\beta \beta_e}{q_e} + \frac{\lambda^2 q_e^{2*}}{q_{ee}}\right)\]

The immediate responses of variables to tax changes are given by (4.2) by setting $t = 0$. (Note that $a_h(0) = 0$.) These responses can be decomposed into an autonomous effect and a wealth effect. The autonomous effect is the long-run response of the variable; the wealth effect is the influence of the long-run response of wealth on that variable. Since the wealth effects of taxes depend crucially on weak nonseparability, so do the short-run effects of taxes. For example,

\[(4.3)\]

\[n_h(0) = n_h^* - \frac{(r - \theta_1) q_e}{\Omega} a_h^*.
\]

Note that $\Omega > 0$ because $A > 0$ as shown below. Labor employment immediately falls by less than in the long run if and only if long-run wealth increases with the taxes. The same result holds for domestic capital and output. Under weakly separable preferences, however, the wealth effect is absent for domestically based taxes and in this case the immediate responses of the variables are entirely determined by their long-run responses.

The comovement in variables on the transition path can be analyzed by examining the comovement of variables with wealth. An interesting question is
about the comovement between labor employment and consumption. Barro and King (1984) have shown that taxes generate opposite movements in the two variables under weakly separable preferences. They suggest that weakly nonseparable preferences may enable taxes to generate the stylized positive comovement between these two variables. Unfortunately, the negative comovement is robust to the perturbation of preferences to the present form of weakly nonseparable preferences. To see the robustness, derive the transition path of consumption from (2.11) and (4.2):

\[(4.4) \quad c_h - c^*_h = -(Aq_c q_{ee} / q_e q_{cc})(n_h - n^*_h),\]

If both consumption and leisure are normal in the sense that $\delta c_h / \delta a_h > 0$ and $\delta e_h / \delta a_h > 0$, then $\lambda > 0$ and hence consumption and labor employment are negatively correlated along the transition path.\(^{14}\)

However, the behaviour of the current account is very different from that suggested by conventional wisdom. It is generally conceded that capital income taxation improves the current account (e.g. Brock 1988). This is not true in the present model when leisure is more welfare stabilizing. To see this, we first examine the comovement between wealth and the country’s international credit. Note that labor employment and hence domestic capital stock moves in the opposite direction to wealth under the normality of leisure. Since $F = a - k$, the country’s international credit must move with wealth and in an opposite direction to output.\(^{15}\)

Now if leisure is more welfare stabilizing, capital income taxation reduces long-run wealth ($a^*_k < 0$). Stability requires that wealth be declining on the transition path. Since the country’s international credit is positively correlated with the level of wealth, it must also be declining on the transition path. The current account must be in deficit. The same result holds for other domestically based taxes. The conventional current account surplus can be obtained only when consumption is more welfare stabilizing. The current account behaves differently in the cases $\lambda < 0$ and $\lambda > 0$ because taxes have different wealth effects in the two cases.

To understand the occurrence of a current account deficit in the case $\lambda < 0$, we decompose the current account into savings and investment. A capital income tax causes an instantaneous portfolio shift from domestic capital to foreign assets. The magnitude of this shift differs in the cases $\lambda < 0$ and $\lambda > 0$. When $\lambda < 0$, the negative long-run wealth effect implies that labor employment instantaneously falls below its new long-run level at the time of the tax increase (see (4.3)). To maintain equal rates of return to domestic capital and foreign assets, domestic capital must also instantaneously fall below its new long-run level. As a result, domestic capital increases along the transition path, creating positive investment on the path. Together with negative savings, this produces a current account deficit on the transition path. In contrast, when $\lambda > 0$, domestic capital instantaneously falls to a

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14 The negative comovement only applies to the transition period. The instantaneous responses of the two variables can be in the same direction.

15 The negative comovement between output and the international credit is consistent with the stylized facts on business cycles in open economies (Stockman 1990).
level above the new long-run level and continues to fall along the transition path, implying negative investment and a current account surplus.

It should be noted that capital income taxation can also create a current account deficit in overlapping generations models. For example, Matsuyama (1987) illustrates such a possibility in an open economy version of Blanchard's (1985) overlapping generations model. However, such a deficit appears at the beginning of the transition path only when the steady-state aggregate capital stock is negative. With a positive capital stock, a current deficit appears only near the end of the transition.\footnote{In Matsuyama's model, a capital income tax instantaneously reduces the country's foreign assets; foreign assets begin to rise after this instantaneous fall. As a convention in continuous-time models, the current account is defined as the continuous rather than the discrete change of foreign assets.}

Also different from the case of weakly separable preferences is the comovement between wealth and welfare. The two move together if and only if
\[
\left( \frac{\theta_1 - \frac{\lambda q^2}{q_{cc}}}{q_{ee}} \right) \left( \frac{\beta \beta_e}{q_e} + \frac{\lambda^2 q^2}{q_{cc}} \right) < 0.
\]

Note that the dominator is positive if $\lambda < 0$ (see Section 1 of the Appendix). Wealth and welfare can move in opposite directions on the transition path if consumption is sufficiently more stabilizing, despite the fact that in the long run the two variables respond in the same direction to tax changes. Under weakly separable preferences, however, wealth and welfare always move together.

It is worth noting that equation (4.2) also provides a formula to calculate $\phi_k(0)$, the change in the intertemporal utility caused by tax changes. Consequently, the marginal deadweight loss of taxes can be calculated. This measure of welfare cost, adopted by Judd (1987), takes the form $\beta^* \phi_k(0)/(R q_e)$ in our case, where $R$ is the present value of tax revenue. One can simulate the model and use this measure to compare the efficiency of different taxes. Since welfare changes depend crucially on weak nonseparability, the relative efficiency of taxes should be sensitive to $\lambda$. This sensitivity casts doubt on previous simulation results which have adopted time-additive preferences. The simulation results are omitted but available from the author upon request.

5. Conclusions

This paper has examined the dynamic effects of distortionary taxes in a small open economy with perfect capital mobility. It is found that weak nonseparability between consumption and leisure is important for the evaluation of the dynamic effects of taxes. As seen from the analysis, the choice of the specific form of weak nonseparability has three benefits: (i) tractability, (ii) easy analysis of a tax on international borrowing and lending, and (iii) a clear distinction between wealth and substitution effects of taxes.

To focus on the importance of weak nonseparability, I have assumed that all tax changes are permanent. For a different timing of tax changes, such as temporary ones, the dynamic effects can be significantly different, as Judd (1987) has shown.
under time-additive preferences. For such tax changes, the analysis can be similarly conducted, except that the immediate impact of tax changes is more difficult to calculate (see Judd 1987 for a method). It is reasonable to believe that weak nonseparability will continue to play an important role.

The framework of this paper can also be used to examine the effect of other distortionary policies such as tariffs and investment tax credit. It is conceivable that these policies can have long-run wealth and welfare effects. If they exist, the long-run effects arise from weak nonseparability, not from the knife-edge property of the steady state in time-additive models such as Sen and Turnovsky (1989). Finally, the framework can be extended to a two-country model, where the Uzawa function is proved to be useful by Devereux and Shi (1991).

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**APPENDIX**

Section 1. Monotonicity and Concavity of U. We show that Assumption 1 is sufficient for local monotonicity and concavity. Denote by \( E^* = \{ (c^*, e^*) \} \) a constant path of consumption and leisure. Let \( E_\delta = \{ (c_\delta, e_\delta) \} \) be another path with \( c_\delta = c^* + \delta c \) and \( e_\delta = e^* + \delta e \). Let \( U_{\delta}(E^*) \) and \( U_{\delta\delta}(E^*) \) be the first and second derivatives of \( U(E_\delta) \) with respect to \( \delta \), evaluated along \( E^* \). If \( U_{\delta}(E^*) > 0 \) and \( U_{\delta\delta}(E^*) < 0 \), then by continuity, there exists a neighbourhood of \( E^* \) such that \( U_{\delta}(E) > 0 \) and \( U_{\delta\delta}(E) < 0 \). In this sense, \( U \) is locally monotonic and concave. Calculations reveal

\[
U_{\delta}(E^*) = \int_0^\infty (q_c c + q_e e) \exp(-\beta t) \, dt.
\]

Therefore \( U_{\delta} > 0 \) if and only if \( q_c > 0 \) and \( q_e > 0 \). To show \( U_{\delta\delta} < 0 \), compute

\[
U_{\delta\delta}(E^*) = \int_0^\infty Q(c, e, S) \exp(-\beta t) \, dt \quad \text{where}
\]

\[
Q = q_{cc} c^2 + q_{ee} e^2 - 2(v_c c + v_e e)S + vS^2, \quad S(t) = \int_0^t (\beta_c c + \beta e e) \, d\tau.
\]

Note that

\[
\int_0^\infty S^2 \exp(-\beta t) \, dt = \frac{2}{\beta} \int_0^\infty S(\beta_c c + \beta_e e) \exp(-\beta t) \, dt.
\]

Using this relation, we can obtain

\[
(A.1) \quad \int_0^\infty Q e^{-\beta t} \, dt = \int_0^\infty \left[ q_{cc} c^2 + q_{ee} e^2 - \frac{\beta q_c}{\beta} S^2 - \frac{2\lambda q_c q_e}{\beta_c} eS \right] e^{-\beta t} \, dt
\]
(A.2) \[ \int_0^\infty Q e^{-\beta t} \, dt = \int_0^\infty \left[ q_{cc} e^{2} + q_{ee} e^{2} - \frac{\beta q_{e}}{\beta e} s^{2} + \frac{2\lambda q_{ee} q_{e}}{\beta e} c s \right] e^{-\beta t} \, dt. \]

It suffices to show that the integrand in either (A.1) or (A.2) is negative definite. Since

\[ q_{e}/q_{c} = (\beta_{e} + \lambda q_{e})/\beta_{c} \text{ and } q_{e}/q_{c} = (\beta_{e} - \lambda q_{e})/\beta_{e}, \]

monotonicity requires \( \lambda \in (-\beta_{e}/q_{e}, \beta_{c}/q_{e}) \). Also \( \lambda > 0 \iff q_{e}/q_{c} > \beta_{e}/\beta_{c} \). If \( \lambda > 0 \), then (use condition (iii) of Assumption 1)

\[ \lambda^{2} < \lambda(\beta_{e}/q_{e}) < (\beta_{c}/q_{e})(-\beta_{ee}/q_{e}^{2}) = -\beta_{e}q_{ee}/q_{e}^{2}. \]

The condition \( \lambda^{2} < -\beta_{e}q_{ee}/(q_{e}q_{c}^{2}) \) is necessary and sufficient for the quadratic integrand in (A.1) to be negative definite. Similarly, we have \( \lambda^{2} < -\beta_{e}q_{ee}/(q_{e}q_{c}^{2}) \) when \( \lambda < 0 \), which is necessary and sufficient for the quadratic integrand in (A.2) to be negative definite. Therefore \( U_{\delta_{f}}(E^{*}) < 0 \) under Assumption 1.

Section 2. Derivations for Sections 2 and 3. First, we derive (2.12). Define an auxiliary state variable \( B \) by

\[ B(t) = \int_0^t \beta(c(\tau), e(\tau)) \, d\tau. \]

The Hamiltonian for problem (P) is

\[ H = \sigma(c, e) e^{-B} + \alpha_{1}(\lambda - \delta k) + \alpha_{2} \beta + \alpha_{3}[r(1 - \tau F)F + r_{d}(1 - \tau k)k + w(1 - \tau w)n - (1 + \tau e)c - I] \]

where \( \alpha_{1}, \alpha_{2} \) and \( \alpha_{3} \) are the shadow prices of respectively \( k, B \) and \( F \). Deriving and manipulating the optimization conditions from \( H \), we have

\[ q_{cc} \dot{c} = [\beta - r(1 - \tau F)]q_{c} + \beta_{c} \dot{c}. \]

(2.12) can be obtained from (2.2), (2.11) and the above equation.

Second, the long-run responses, \( \phi_{h}^{*}, n_{h}^{*} \) and \( a_{h}^{*} \), can be calculated by differentiating (2.11) and (3.1) with respect to \( h \),

\[ -|J| \phi_{h}^{*} = r^{2} \left( \frac{q_{e}^{2}}{q_{ee}} + \frac{q_{e}^{2}}{q_{cc}} \right) \tau_{F1} - \frac{r_{e} q_{ee}^{2} q_{e} f_{hk}}{q_{cc} e_{h} f_{n} f_{k}} \right) \tau_{h} \]

\[ -|J| n_{h}^{*} = -\frac{r_{e} q_{e}}{q_{ee}} \left( \beta - \frac{\lambda q_{ee}^{2}}{q_{cc}} \right) \tau_{F1} + \frac{r \beta_{e} q_{ee} q_{e} f_{hk}}{q_{cc} e_{h} f_{n} f_{k}} \right) \tau_{h} + \frac{r \beta_{e} q_{ee} q_{e} f_{hk}}{q_{cc} e_{h} f_{n} f_{k}} \right) \tau_{h}, \]

\[ -|J| a_{h}^{*} = \frac{r q_{e}}{q_{ee}} \left( \frac{q_{e} q_{ee} \lambda + \beta q_{ee}}{q_{ee}} \right) \left( f_{hk} \frac{k - r - \delta}{k} \right) \beta - \frac{\lambda q_{ee}^{2}}{q_{cc}} \right) \tau_{F1} \]
\[\begin{align*}
&+ \left[ \frac{\lambda q_e^2}{q_{ee}} \left( \beta + \frac{\lambda q_e^2}{q_{ee}} \right) + \frac{\beta \beta_e}{q_e} \left( \frac{q_e^2}{q_{ee}} + \frac{q_e^2}{q_{cc}} \right) \right] \frac{f_e - r - \delta}{f_{ek}} \\
&+ \frac{\beta q_e q_e f_{kn}}{q_{cc} q_{ee} f_{n} f_{kk}} \left[ \beta_e + \frac{\beta_e}{q_{ee}} \left( \frac{f_e - r - \delta}{f_{kk}} \right) \right] \varepsilon_{kn} \\
&+ \frac{\beta q_e^2 f_{e}}{q_{ee} q_{cc}} \left[ \beta_e + \frac{\beta_e}{q_{ee}} \left( \frac{f_e - r - \delta}{f_{kk}} \right) \right] \varepsilon_{eh},
\end{align*}\]

where $|J|$ is the determinant of $J$ specified below and

\[\varepsilon_{1h} = \frac{\varepsilon_{10} \tau_{k1} - r \tau_{F1}}{1 - \tau_{k0}}, \quad \varepsilon_{2h} = \frac{\tau_{w1} + \varepsilon_{20} \tau_{e1}}{1 + \tau_{e0}},\]

\[|J| = r \left[ \frac{\beta \beta_e}{q_e} \left( \frac{q_e^2}{q_{ee}} + \frac{q_e^2}{q_{cc}} \right) + \frac{\lambda q_e^2}{q_{cc}} \left( \beta + \frac{\lambda q_e^2}{q_{ee}} \right) \right].\]

It can be shown that $|J| < 0$ from the stability condition (A.4) below.

**Section 3. The Dynamic System.** The matrix in (4.1) is

\[J = \begin{bmatrix}
\beta - \frac{\lambda q_e^2}{q_{cc}} & q_e + \frac{q_e q_{ee}}{q_{cc}} & 0 \\
-q_e & \frac{\beta e}{q_e} \frac{\lambda}{q_{cc}} & 0 \\
-q_{cc} & \frac{q_{cc} q_{ee}}{q_{cc}} & \frac{(f - r - \delta)}{f_{kk}} \frac{f_{kn}}{f_{kk}} & r
\end{bmatrix}.
\]

To study stability, let $\theta_i (i = 1, 2, 3)$ be the eigenvalues of $J$. Then $r$ is one of these eigenvalues. The other two eigenvalues are

\[(A.3) \quad \theta_{1,2} = \frac{1}{2} \left[ \beta \pm \frac{4 \beta_e}{q_e} \left( \frac{q_e^2}{q_{ee}} + \frac{q_e^2}{q_{cc}} \right) - \frac{4 \lambda q_e^2}{q_{cc}} \left( \beta + \frac{\lambda q_e^2}{q_{ee}} \right) \right].\]

Since there is only one predetermined variable in the system (4.1), the system is saddle-path stable if and only if $\theta_1 < 0$ and $\theta_2 > 0$, or equivalently if and only if

\[(A.4) \quad \frac{\beta \beta_e}{q_e} \left( \frac{q_e^2}{q_{ee}} + \frac{q_e^2}{q_{cc}} \right) + \frac{\lambda q_e^2}{q_{cc}} \left( \beta + \frac{\lambda q_e^2}{q_{ee}} \right) < 0.\]

This condition can be shown to hold upon noting that either $\lambda^2 < -\beta \beta_e q_{ee} / (q_e q_{ce})$ or $\lambda^2 < -\beta \beta_e q_{cc} / (q_e q_{ce}^2)$ (see the first part of the Appendix).

The stable path can be obtained as follows. Let $(y_1, y_2, 1)^T$ be the eigenvector of $J$ associated with the eigenvalue $\theta_1$. Then the stable path is given by
\[(\phi_h - \phi^{*}_h, n_h - n^{*}_h, a_h - a^{*}_h)^T = (y_1, y_2, 1)^T s_0 e^{\theta_{iz}},\]

where \(s_0 = -a^{*}_h\) because \(a_h(0) = 0\). (4.2) is obtained by solving \(y_1\) and \(y_2\).

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