A wake-up call: information contagion and speculative currency attacks

Toni Ahnert† and Christoph Bertsch‡

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Abstract

A successful speculative attack against one currency is a wake-up call for speculators elsewhere. Currency speculators have an incentive to acquire costly information about exposures across countries to infer whether their monetary authority’s ability to defend its currency is weakened. Information acquisition per se increases the likelihood of speculative currency attacks via heightened strategic uncertainty among speculators. Contagion occurs even if speculators learn that there is no exposure. Our new contagion mechanism offers a compelling explanation for the 1997 Asian currency crisis and the 1998 Russian crisis, both of which spread across countries with seemingly unrelated fundamentals and limited interconnectedness.

Keywords: contagion, coordination failure, information acquisition, speculative currency attacks

JEL classification: C7, D82, F31, G01

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†London School of Economics and Political Science, Financial Markets Group and Department of Economics, Houghton Street, London WC2A 2AE, United Kingdom. Part of this research was conducted when the author was visiting the Department of Economics at New York University and the Federal Reserve Board of Governors. Email: t.ahnert@lse.ac.uk

‡Department of Economics, University College London, Gower Street, London WC1E 6BT, United Kingdom. Email: c.bertsch@ucl.ac.uk
1 Introduction

Financial contagion can happen even if countries have seemingly unrelated fundamentals and limited interconnectedness. A prominent example is Brazil that got affected by the 1998 Russian crisis although Brazil’s exposure to Russia was very limited. Our paper is motivated by this phenomenon and provides a novel contagion mechanism in coordination games that does not rely on common exposures and interconnectedness. It explains why a contagious spread of a crisis can occur even if agents learn that their country’s fundamentals are not exposed to crisis events elsewhere.

We define contagion as an increase in the likelihood of a financial crisis in one country after another country has been affected by a financial crisis. Our new contagion mechanism is developed in an incomplete information game of speculative currency attacks, based on Morris and Shin [29, 30] and following the tradition of the global games literature. The main finding of our paper is that contagion occurs even if agents get informed and learn that their country is not exposed to a crisis event elsewhere. But what is more, we find that the scenario where agents learn good news about their country’s fundamentals can be associated with a higher likelihood of financial crises relative to the scenario where agents learn no news and stay uninformed about the exposure. At first glance this second result may be surprising. However, the underlying mechanics are intuitive. Key is that learning the news of no exposure can lead to more financial fragility if the news of no exposure is not only associated with a more favourable public information, but also affects the information precision of speculators.

We demonstrate that the above described contagion effect prevails as an equilibrium phenomenon if learning is endogenous. Furthermore, endogenous information helps us to capture the idea of contagion-through-alertness. Observing a ‘trigger event’ in another country or region such as a banking crisis, a balance-of-payments crisis or a sovereign debt crisis is a wake-up call for a domestic investor and makes her alert. Taking the example of speculative currency attacks, a successful speculative currency attack against one county is a wake-up call for currency speculators elsewhere. Speculators wonder whether their country’s fundamentals are affected and, hence, the ability of their monetary authority to defend its
currency is weakened. While it is ex-ante unknown whether fundamentals are correlated across countries, there is some chance of a positive correlation. For that reason speculators expect that their monetary authority’s ability to defend its currency can be detrimentally affected. This may be due to macroeconomic factors such as common shocks or due to interconnectedness and institutional similarities of their country with the ”ground zero country”, which was attacked initially. Consequently, currency speculators wish to determine the extent of their exposure to the trigger event in the ground zero country by acquiring costly information. We call information acquisition after such a trigger event elsewhere an alertness effect and demonstrate that it can cause contagion. Interestingly, when currency speculators learn that there is no correlation, financial fragility can be higher than without information acquisition. Thus, fragility in one country can lead to fragility in a second country although fundamentals are independent – a contagion-through-alertness effect. The fragility in the second country is a direct consequence of the change in the information precision of speculators due to learning. It arises in the context of coordination problems. This is the case because additional information about the cross-country correlation of fundamentals has two effects in our incomplete information game.

Mean effect Having observed a successful currency attack due to a low realisation of fundamentals in the other country, a speculator’s posterior mean about her country’s fundamentals improves upon learning that there is no cross-country correlation. This is because the low fundamental realisation in the other country shows to be irrelevant for her country’s ability to defend its currency. The mean effect is associated with a lower likelihood of successful speculative currency attacks after learning that fundamentals are uncorrelated.  

1 In practise an exposure may arise due to trade-links, financial links or institutional similarities. Both, macroeconomic and financial similarities show to play an important role. In early empirical work Glick and Rose [19] find that ”currency crises tend to be regional” (page 603) and underline geographic proximity as an important factor. Instead Van Rijckeghem and Weder [39] [40] find that for the most recent episodes of currency crises spillovers through bank lending played a more important role. Finally, Dasgupta et. al. [14] find that institutional similarity to the ”ground zero country” is an important determinant for the direction of financial contagion.

2 A good description of the mean effect can be found in Vives [41].
**Variance effect** The information about the cross-country correlation of fundamentals also affects the information precision. In particular, the relative precision of public information is lowest if speculators learn that fundamentals are uncorrelated. We find that a lower relative precision of public information increases (decreases) the likelihood of successful speculative currency attacks if the prior belief is that fundamentals are strong (weak). In other words, the result depends on whether the equilibrium fundamental threshold is above or below the public information of speculators.\(^3\) The reason being, that a lower relative precision of public signals increases strategic uncertainty – the variance effect. This increase in strategic uncertainty is reflected in a more dispersed belief about other speculators’ posterior. Given a prior belief that fundamentals are strong (weak), the increase in strategic uncertainty makes speculators more concerned about other speculators receiving a bad (good) private signal. The shift in beliefs about other speculators’ posterior induces more (less) aggressive speculative currency attacks. As a result, the variance effect can be associated with an increase or a decrease in the likelihood of successful currency attacks when comparing the scenario where speculators learn that fundamentals are uncorrelated relative to scenario where speculators do not learn about the correlation (and, hence, expect a potentially positive cross-country correlation). The direction of the variance effect crucially depends on whether the prior belief is that fundamentals are strong or weak.

In sum, the *mean effect and the variance effect go in opposite directions* given a prior belief that fundamentals are strong (which implies a large degree of coordination failure). Having observed a successful currency attack due to a low realisation of fundamentals in one country, the news of no cross-country correlation implies more fragility in another country through heightened strategic uncertainty if the variance effect dominates the mean effect. Contagion can occur even if agents learn that they are not exposed to the crisis event elsewhere which triggered learning in the first place.

The novel contagion mechanism prevails as an equilibrium phenomenon with endogenous information acquisition about the cross-country correlation. This is because currency

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\(^3\)Similar results have been discussed in a global-games model by Metz [27], and by Rochet and Vives [36]. He and Xiong [21] provide an alternative framework in which they establish a “volatility effect”. Their volatility effect is to some degree related to the variance effect, but is not based on a change in strategic uncertainty.
speculators have an ex-ante incentive to acquire information on the cross-country correlation whenever the cost of information is sufficiently low. Intuitively, the information on the cross-country correlation of fundamentals helps a speculator to improve her forecast about her country’s fundamental as well as the behaviour of other agents. We demonstrate that a speculator can earn a higher gross expected payoff after adjusting her attack strategy because of being informed. In particular, an informed speculator obtains a higher expected payoff than an uninformed speculator by acting more (less) aggressively after receiving information that lowers (improves) her forecast for fundamentals. By doing so, an informed speculator increases her expected benefits (reduces her expected costs) from participating in a successful (unsuccessful) currency attack when fundamentals are weak (strong).

**Literature** Our paper is related to Morris and Shin [29, 30] who develop an incomplete information game of speculative currency attacks in the tradition of the global games literature pioneered by Carlsson and van Damme [8]. We differ in two main aspects. First, we consider a two-country model with potentially correlated fundamentals and address the issue of contagion across countries. Currency speculators move sequentially such that speculators in the second country decide whether to attack their country’s currency after observing the outcome in the first country (wake-up call). Second, speculators in the second country can acquire information about the cross-country correlation of fundamentals (alertness effect). Similar to Corsetti et. al. [10] speculators can be asymmetrically informed, but our focus is on contagion.

There is a large existing literature on contagion in financial economics and in international finance. With few exceptions the existing literature relies on common exposures and interconnectedness. We demonstrate with our new contagion mechanism that contagion

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4 The authors also consider different sizes of currency speculators and its effect on the likelihood and severity of a currency crisis. Instead our speculators are of equal size as our contagion-through-alertness mechanism does not require signalling or herding.

5 An excellent recent literature survey can be found in Forbes [18] and a more detailed description of the relation of our paper to the literature on contagion is given in section 4.

6 Similar to us Goldstein and Pauzner [20] do not rely on correlated fundamentals. The authors obtain contagion because of risk averse speculators who are invested in two countries. After a crisis in the first country speculators become more averse to strategic risk and have a larger incentive to withdraw their investment.
can occur in the absence of interconnectedness or common exposures. This allows us to offer an explanation for the occurrence of contagious currency or banking crises even if the fundamentals of the affected countries or financial institutions are seemingly unrelated and even if there is only limited interconnectedness. Such a situation does not only apply to the aforementioned example of Brazil, which was affected by the 1998 Russian financial crisis, but it is also relevant for the Asian balance-of-payment and banking crisis in 1997. In Asia it was at least for some of the affected countries the case that the spread of contagion is difficult to explain without leaning on models with multiple equilibria and the possibility of sudden unexplained shifts in market confidence (see Radelet and Sachs and Krugman).

The specialty of our new contagion mechanism is that contagion can occur even if speculators learn the good news that there is no cross-country correlation. What is more, the phenomenon of a higher likelihood of currency attacks after learning that there is zero correlation can be the consequence of ex-ante optimal information acquisition. In complementary subsequent work, Ahnert examines the amplification of the probability of bank runs or sovereign debt crises via endogenous acquisition of private information after learning bad news. Moreover, he investigates the strategic aspects of information acquisition choices and equilibrium multiplicity. By contrast, we examine learning about the stochastic exposure to a crisis country and demonstrate how contagion can arise even after good news.

Our new contagion mechanism is general and lends itself to several applications. It applies to coordination problems in which the payoff from acting depends on both, the underlying state of the world and the proportion of other agents acting. In the example of bank runs, the *trigger event* is that bank creditors of one bank observe a run on another bank. In the Arab spring, political activists in one country observe a revolution in a neighbouring country and decide whether or not to attempt a revolution themselves (see Edmond). Alternative applications are sovereign debt crises or foreign direct investment across emerging markets (see Dasgupta). Common to these examples is that agents are not directly

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7The spread of the 1998 Russian financial crisis to Brazil cannot be attributed to fundamental reasons or cross-country linkages on which most of the international finance literature on contagion relies. Although Brazil had a very limited exposure, it was still one of the most affected countries (see Bordo and Murshid and Pavlova and Rigobon).
affected by the trigger event but might be affected indirectly.

This paper is organised as follows. The model is described in section 2 and solved in section 3. Our main focus is on establishing the novel contagion mechanism with exogenous information. In section 3.5 we extend our model and show that our contagion mechanism can be an equilibrium phenomenon with endogenous information acquisition. A more detailed discussion of the related literature is offered in section 4. Finally, section 5 concludes. All proofs and most derivations are relegated to the appendices.

2 Model

The economy extends over two dates \( t \in \{1, 2\} \) and consists of two countries, where the first (second) country only moves at the first (second) date. Each country is inhabited by a unit continuum of risk-neutral agents interpreted as currency speculators indexed by \( i \in [0, 1] \).

A country is characterised by its fundamental \( \theta_t \) that measures the difficulty of a successful currency attack from the perspective of speculators. For example, a country’s fundamental represents the government’s strength to defend its currency with its foreign reserves (e.g. Morris and Shin [29]).

Speculators play a simultaneous-move game with binary action space \( a_{it} \in \{0, 1\} \): each speculator either attacks the currency \( (a_i = 1) \) or does not attack \( (a_i = 0) \). The success of a currency attack depends on both the fundamental \( \theta_t \) and the proportion of attacking speculators denoted by \( A_t \equiv \int_0^1 a_{it} \, di \). A speculative attack is successful if the fraction of acting speculators weakly exceeds the strength of the fundamental \( (A_t \geq \theta_t) \). The benefit of a speculator from participating in a successful currency attack is given by \( b > 0 \). Her loss from participating in an unsuccessful currency attack is given by \( l > 0 \). As in Vives [41], the payoff from not attacking is constant for simplicity and normalised to zero, i.e.

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8Currency speculators may act nationally or internationally. While speculators can be identical across countries, we only require the sequential timing of events.
\( u(a_{it} = 0, A_t, \theta_t) = 0: \)
\[
u(a_{it} = 1, A_t, \theta_t) = \begin{cases} 
    b & \text{if } A_t \geq \theta_t \\
    -l & \text{if } A_t < \theta_t 
\end{cases} \tag{1}
\]

A key feature of our model is the initial uncertainty about the correlation between fundamentals across countries denoted by \( \rho \equiv \text{corr}(\theta_1, \theta_2) \). The correlation follows a bivariate distribution and is zero with probability \( p \in (0, 1) \) and takes a positive value \( \rho_H \in (0, 1) \) with probability \( 1 - p \):
\[
\rho = \begin{cases} 
    \rho_H & \text{w.p. } 1 - p \\
    0 & \text{w.p. } p 
\end{cases} \tag{2}
\]

Fundamentals in both countries are conditionally distributed as a bivariate normal with mean \( \mu_t \equiv \mu \), precision \( \alpha_t \equiv \alpha > 0 \), and correlation \( \rho \). Speculators in country 2 observe whether a currency attack in country 1 was successful. If an attack was successful, speculators also observe the realisation of \( \theta_1 \). The bounds on the positive correlation \( \rho_H \) ensure that a speculator in region 2 who observes the realisation \( \theta_1 \) in region 1 and learns that the correlation takes the positive value has more precise information but is still imperfectly informed.

As in the global games literature pioneered by Carlson and van Damme \cite{Carlson1997}, each speculator receives a private signal \( x_{it} \) about her country’s fundamental before deciding whether to attack:
\[
x_{it} \equiv \theta_t + \epsilon_{it} \tag{3}
\]
where the idiosyncratic noise \( \epsilon_{it} \) is identically and independently normally distributed across speculators and countries with zero mean and precision \( \gamma > 0 \). All distributions are common knowledge.

The game in country 2 has two stages. In stage 2 speculators play the speculative

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\footnote{If information about the fundamental is complete, then multiple equilibria arise for \( \theta_t \in (0, 1) \). Some authors therefore restrict attention to \( \mu \in (0, 1) \) to match this parameter restrictions in the incomplete information setup.}

\footnote{The motivation for this assumption is as follows. After a successful currency attack it becomes public information why the monetary authority in country 1 was too weak to defend its currency. In contrast, if the speculative currency attack in country 1 is unsuccessful, then the actual strength of country 1’s monetary authority \( \theta_1 \) remains unknown.}
attack coordination game. If information is exogenous, a proportion of speculators $n \in [0, 1]$ learn the correlation between fundamentals and $n$ is common knowledge. We derive our main result in this setup. As an extension, we consider endogenous information acquisition such that speculators play an information acquisition game in stage 1. Each speculator may simultaneously purchase a signal about the correlation of the fundamentals at a cost $c > 0$. The signal is common to all speculators and publicly available. A figurative example of such a signal is a newspaper, which takes money to buy and time to absorb. In terms of wholesale investors or currency speculators, it could be the access to Bloomberg and Datastream terminals or for the hiring of analysts to interpret the publicly available information. Information acquisition is costly in each case. The signal about the correlation of fundamentals is perfectly revealing for simplicity again.

The model is summarised in the following timeline:

**Date** $t = 1$

- $\rho$ is drawn. Then $\theta_1$ and $\theta_2$ are drawn from a bivariate normal with correlation $\rho$.
- Speculators in country 1 receive their private signals $x_{i1}$ and decide simultaneously whether to attack the currency.
- Payoffs are realised. The fundamental $\theta_1$ is publicly observed by speculators in both countries after a successful speculative currency attack.

**Date** $t = 2$

- **Stage 1:**
  - Exogenous information: a known proportion of speculators $n \in [0, 1]$ learn the realisation of the correlation.
  - Endogenous information: speculators in country 2 simultaneously decide whether to purchase a publicly available signal about the correlation of fundamentals $\rho$ at cost $c > 0$.

- **Stage 2:**
  - Speculators in country 2 receive their private signals $x_{i2}$ and decide simultaneously whether to attack the currency.
3 Equilibrium

The focus of this paper is on the equilibrium in country 2. In particular, we describe how events in country 1 influence this equilibrium, contrasting the situation of known and unknown correlation between fundamentals. It is therefore useful to revise briefly the equilibrium in country 1, which is a standard coordination game as in e.g. Vives [41].

3.1 Country 1

Observe that the differential payoff $u(a_{i1} = 1, A_1, \theta_1) - u(a_{i1} = 0, A_1, \theta_1)$ is increasing in $A_1$ and decreasing in $\theta_1$. Denote with $\phi(x)$ the pdf of a normal distribution. We can define a symmetric Bayesian equilibrium as follows.

**Definition 1** An equilibrium in country 1 is a speculative attack decision $a(x_{i1})$ and an aggregate mass of attackers $A_1 \equiv A(\theta_1)$, such that:

$$a(x_{i1}) \in \arg\max_{a_{i1} \in \{0, 1\}} E[u(a_{i1}, A_1, \theta_1)|x_{i1}]$$

$$A(\theta_1) = \int_{-\infty}^{+\infty} a(x_{i1})\sqrt{\gamma}\phi(\sqrt{\gamma}(x_{i1} - \theta_1))dx_{i1}$$

We consider monotone equilibria, i.e. equilibria where there exists an individual private signal threshold $x^*_1$ and a fundamental threshold $\theta^*_1$ such that (a) an individual speculator only attacks if $x_{i1} \leq x^*_1$ and (b) a speculative currency attack is successful if and only if $\theta_1 \leq \theta^*_1$.

In equilibrium two conditions have to be satisfied. First, the critical fraction of attacking speculators $A(\theta^*_1)$ has to equal the critical fundamental threshold above which it pays

\[11\] Notice that it is (not) a dominate strategy to attack the currency if $\theta_t \leq 0$ ($\theta_t \geq 1$). Instead fundamentals are "critical" in the intermediate range $\theta_t \in (0, 1)$. Here multiple equilibria can be sustained by self-fulfilling expectations if the realisation of $\theta_1$ is common knowledge.
to attack. Second, a speculator with the threshold signal \( x^*_1 \) has to be indifferent between attacking and not attacking the currency given \( \theta^*_1 \). These two equilibrium conditions can be combined to one equation which implicitly defines \( \theta^*_1 \):\(^{12}\)

\[
F_1(\theta^*_1) \equiv \Phi \left( \frac{\alpha}{\sqrt{\alpha + \gamma}} (\theta^*_1 - \mu) - \frac{\sqrt{\gamma}}{\sqrt{\alpha + \gamma}} \Phi^{-1}(\theta^*_1) \right) = \frac{l}{b + l}
\]

It can be shown that there exists a unique \( \theta^*_1 \) solving equation (6) if \( \frac{\alpha}{\sqrt{\gamma}} < \sqrt{2\pi} \). Hence, there exists a unique Bayesian equilibrium in threshold strategies whenever the relative precision of the private signal is sufficiently high. Following Morris and Shin \[30\] we can use an iterated dominance argument to show that there do not exit non-monotone equilibria, meaning that the above equilibrium is in fact unique.

If \( \gamma \) is sufficiently high, then \( \theta^*_1 \) is decreasing in \( \mu \) and in \( \frac{l}{b+l} \). Hence, there are two possible rankings of the equilibrium thresholds depending on the belief about the prior mean of the fundamental. First, the prior belief about fundamentals is said to be weak when \( \mu \in (0,1) \) takes on a low value, while the relative cost of an unsuccessful attack \( \frac{l}{b+l} \) is low, i.e. if:

\[
\sqrt{\gamma} \Phi^{-1}(\mu) + \sqrt{\alpha + \gamma} \Phi^{-1} \left( \frac{l}{b + l} \right) < 0
\]

A prior belief that fundamentals are weak leads to strong attacks on the currency: \( 0 < \mu < \theta^*_1 < 1 \), implying little coordination failure. Second, the prior belief about fundamentals is said to be strong when the above inequality is reversed, meaning that \( \mu \) takes on a high value, while the relative cost of an unsuccessful attack \( \frac{l}{b+l} \) is high. A strong prior on fundamentals leads to less frequent currency attacks: \( 0 < \theta^*_1 < \mu \), implying a large degree of coordination failure. These rankings are for finite private noise (\( \gamma < \infty \)); refer to Appendix A.1.2 for a detailed examination.

While the equilibrium analysis in country 1 is standard, the analysis in the next three sections contains the main contribution of our paper. For the remainder we can abstract from all the details of the equilibrium in country 1 and consider the equilibrium in country 2 as a function of the realisation of the observed fundamental \( \theta_1 \). In other words, we treat

\(^{12}\)See Appendix section A.1.1 for details.
as a public signal and focus on the equilibrium in country 2. We are interested in the case where the public signal is low, i.e. \( \theta_1 < \mu \), and associated with a successful speculative attack in country 1.\(^{13}\) The aim is to analyse how country 2 is affected after observing a country-1 fundamental realisation below average.

### 3.2 Country 2: Symmetrically informed speculators

To demonstrate the core mechanics of our novel contagion mechanism endogenous information acquisition is not necessary. Furthermore, we also do not need to allow for asymmetrically informed speculators. For that reason we abstract in this section from the first stage of the game at date \( t = 2 \) and analyse a simplified model with symmetrically informed speculators where either everybody or nobody can observe the publicly available signal on the cross-country correlation, i.e. the "polar cases" \( n = 1 \) and \( n = 0 \), respectively. In the discussion of the polar case when all speculators are informed (\( n = 1 \)) we can already uncover the mean effect and the variance effect, which are at the core of our novel contagion mechanism. Based on this analysis, we establish our novel contagion mechanism in section \( \text{3.3} \).

However, we emphasise that endogenous information acquisition is an interesting part of the contagion-through-alertness effect developed in this paper. For that reason we extend the analysis of section \( \text{3.2} \) by introducing the information acquisition stage at the beginning of date \( t = 2 \) in section \( \text{3.5} \). Here we allow for asymmetrically informed speculators (\( 0 < n < 1 \)) and demonstrate how endogenous information acquisition can be triggered by a wake-up call event that makes speculators alert. Most importantly, for a sufficiently low cost of information on \( \rho \), there exists a unique equilibrium where all speculators want to be informed and where there is a contagious spread of speculative currency attacks even if speculators learn that they are not exposed to \( \theta_1 \).

\(^{13}\)Notice that we can guarantee that any fundamental realisation \( \theta_1 < \mu \) implies that speculative currency attacks are successful whenever the relative cost of attacking in country 1 is sufficiently low. A numerical example is provided when we establish the novel contagion mechanism in section \( \text{3.3} \).
**Notation**  Let’s introduce some notation to distinguish informed and uninformed speculators. Using the subscripts $I$ and $U$, let $a_{i2I}$ ($a_{i2U}$) denote the action of informed (uninformed) speculator $i$ in country 2 and let $A_{2I}$ ($A_{2U}$) denote the aggregate proportion of attacking speculators in country 2 that are informed (uninformed).

**Informed speculators**  The belief of an informed speculator about the mean of $\theta_2$ depends on $\rho$. Let’s denote the conditional mean with $\mu'_2(\rho, \theta_1) \equiv \rho \theta_1 + (1 - \rho) \mu_2$. An informed speculator who learns that $\rho = 0$ believes that $\theta_2$ is distributed according to:

$$\theta_2 \sim N\left(\mu'_2(0, \theta_1), \frac{1}{\alpha}\right),$$

where $\mu'_2(0, \theta_1) = \mu_2 = \mu$. Instead if she learns that $\rho = \rho_H$, then she believes $\theta_2$ follows:

$$\theta_2|\theta_1 \sim N\left(\mu'_2(\rho_H, \theta_1), \frac{1 - \rho_H^2}{\alpha}\right),$$

where $\mu'_2(\rho_H, \theta_1) = \rho_H \theta_1 + (1 - \rho_H) \mu$. We can see that the mean of the belief shifts towards $\theta_1$ and the precision unambiguously increases, the larger $\rho_H$.

**Uninformed speculators**  An uninformed speculator observes $\theta_1$ and uses the prior distribution about the correlation $\rho$ to update her belief about the fundamental in country 2. As a result, the belief about the distribution of $\theta_2$ is a mixture distribution. Before receiving her private signal, the uninformed speculator believes that $\theta_2$ is drawn with probability $p$ from the normal distribution described in equation (8) and with probability $1 - p$ from the normal distribution described in equation (9).

Figure 1 depicts the beliefs about the distribution of $\theta_2$ for informed speculators given in equations (8) and (9) as brown dashed and blue dotted lines, respectively. The uninformed speculators’ belief about the distribution of $\theta_2$ is described by the red solid line and denoted with $\theta_2|U$, where $U$ stands for uninformed.\(^{14}\) Informed speculators who learn that there is no cross-country correlation, i.e. $\rho = 0$, have a belief with the highest mean and dispersion,\(^{14}\) For the chosen parameters (high $p$ and not too small $\theta_1$) the mixture distribution is not bimodal and relatively close to a normal distribution.

\(^{14}\)For the chosen parameters (high $p$ and not too small $\theta_1$) the mixture distribution is not bimodal and relatively close to a normal distribution.
while informed speculators who learn that $\rho = \rho_H$ have a belief with the lowest mean and dispersion.

Figure 1: Highest after learning that $\rho = 0$. Parameters used: $p = 0.7$, $\alpha = 0.7$, $\rho_H = 0.5$ and $\theta_1 = 0$.

We continue in section 3.2.1 by discussing the polar case $n = 1$ where all speculators are informed. In section 3.2.2 we analyse the role of public information and information precision, which lays the foundation for our novel contagion mechanism. Then we shift in section 3.2.3 focus to the analysis of the problem faced by uninformed speculators who do not learn the realisation of $\rho$. Finally, we examine the equilibrium for the polar case $n = 0$ where all speculators are uninformed in section 3.2.4.

### 3.2.1 Equilibrium for the special case $n = 1$: classical information contagion

The special case of completely informed speculators captures the classical information contagion channel, which is distinct from our novel contagion mechanism to be established in section 3.3. A low fundamental realisation in country 1 constitutes ”bad news” for the fundamentals in country 2 if speculators learn that fundamentals are positively correlated, i.e. $\rho = \rho_H$. The strength of this effect is measured by $\rho_H$.\[\text{An example of this information contagion channel is Acharya and Yorulmazer [1] who show that funding costs of one bank increase after bad news about another bank if the banks’ loan portfolio returns have a common factor.}\]
We show that there exists again a unique equilibrium in threshold strategies as in our analysis of country 1. The sufficient condition on the relative precision of private information is given by \( \frac{\alpha}{\sqrt{\gamma}} < \sqrt{2\pi} \). A successful currency attack occurs in country 2 if its fundamental realisation is below its unique equilibrium threshold, i.e. \( \theta_2 \leq \theta^*_2(\rho) \in (0, 1) \), where the subscript \( I \) stands for informed. The critical threshold for the country’s fundamental \( \theta^*_2(\rho) \) depends on \( \rho \) and is implicitly defined by:

\[
F_2(\theta^*_2(\rho), \rho) \equiv \Phi\left( \frac{\alpha}{1-\rho^2} \left( \theta^*_2(\rho) - [\rho \theta_1 + (1 - \rho) \mu] \right) - \sqrt{\gamma} \Phi^{-1}(\theta^*_2(\rho)) \right) = \frac{l}{b + l} \tag{10}
\]

where \( \rho = 0 (\rho = \rho_H) \) if speculators learn that there is (no) exposure. Again the left-hand side is monotone and decreasing in \( \theta^*_2(\rho) \) for a sufficiently high relative precision of the private signal. If \( \rho = \rho_H \), the left-hand side of equation (10) is decreasing in \( \theta_1 \) and we can conclude that \( d\theta^*_2(\rho_H)/d\theta_1 < 0 \). A lower observed fundamental in country 1 implies that the fundamental in country 2 is likely to be low as well if \( \rho = \rho_H \). Speculators expect little defence by the country-2 government against a currency attack. Consequently, it is optimal for speculators to attack the currency in country 2 more aggressively, thus raising the fundamental equilibrium threshold of country 2 below which a currency attack is successful.

### 3.2.2 The role of public information and information precision

The aim of this section is to shed light on the interplay between the *mean effect* and the *variance effect*, which crucially influences the ordering of equilibrium thresholds \( \theta^*_2(0) \) and \( \theta^*_2(\rho_H) \) in the two states of the world. This interplay between the mean effect and the variance effect will serve as a basis for the novel contagion mechanism developed in section 3.3.

We find that the mean effect increases \( \theta^*_2(0) \) relative to \( \theta^*_2(\rho_H) \), while the variance effect tends to decrease (increase) \( \theta^*_2(0) \) relative to \( \theta^*_2(\rho_H) \) if the prior belief is that fundamentals are strong (weak). As a result, we can only have that \( \theta^*_2(0) > \theta^*_2(\rho_H) \) if the prior belief is that

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\(^{16}\)See Appendix A.3 for a detailed analysis, where the equilibrium and sufficient conditions for uniqueness are derived.
fundamentals are strong and the variance effect outweighs the mean effect. However, it is important to notice that the ordering of the equilibrium thresholds for different states of the world should not be confused with the ordering of likelihoods of successful currency attacks because $\theta_2$ is drawn from a different distribution depending on the state of the world.

The prior belief on fundamentals  Similar to before, a weak prior belief on fundamentals leads to strong attacks against the currency independent of the realisation of $\rho$, i.e. $\mu < \theta_{2L,0}^* < 1$ and $\rho_H \theta_1 + (1 - \rho_H) \mu < \theta_{2L,\rho_H}^* < 1$. Instead, a strong prior belief on fundamentals leads to weak attacks against the currency independent of the realisation of $\rho$, i.e. $0 < \theta_{2L,0}^* < \mu$ and $0 < \theta_{2L,\rho_H}^* < \rho_H \theta_1 + (1 - \rho_H)$\footnote{Recall the discussion for country 1 in section 3.1 and the derivations in Appendix section A.1.2}. For ”intermediate” values of $\mu$ the prior belief on fundamentals depends on the realisation of $\rho$. More formally:

**Definition 2** The prior belief is that fundamentals are strong independently of the realisation of $\rho$ if $\mu' \in S_1$, and that fundamentals are weak independently of $\rho$ if $\mu' \notin \{S_1, S_2\}$. Fundamentals are expected to be strong or weak depending on the realisation of $\rho$ if $\mu' \notin \{S_1, S_2\}$.

Where:

$$S_1 = \left\{ \{\mu', \theta_1, \rho_H, b, l\} : \left[ \mu' > \max\{X(0), X(\rho_H)\} \right] \right\}$$

(11)

$$S_2 = \left\{ \{\mu', \theta_1, \rho_H, b, l\} : \left[ \mu' < \min\{X(0), X(\rho_H)\} \right] \right\}$$

(12)

and:

$$X(\rho) \equiv \Phi\left( -\frac{\sqrt{1-\rho^2} + \gamma}{\sqrt{\gamma}} \Phi^{-1}\left( \frac{l}{b+l} \right) \right) - \rho(\theta_1 - \mu).$$

(13)

**Mean effect**  It is well known that more favourable public information, i.e. a higher prior mean $\mu$, is associated with a lower equilibrium fundamental threshold. In our model not only a decrease in $\theta_1$, but also an increase in $\rho_H$ are associated with a decrease in the prior mean. Given that the prior mean is higher if fundamentals are not correlated, i.e. if $\rho = 0$,
than if fundamentals are correlated, i.e. if $\rho = \rho_H$, we have that the mean effect tends to lower $\theta^*_{2I,0}$ relative to $\theta^*_{2I,\rho_H}$.

**Variance effect**  It crucially depends on the prior belief on fundamentals if the equilibrium fundamental threshold $\theta^*_{2I,0}$ increases or decreases in the precision of the private signal $\gamma$ and the public signal $\alpha$. To our knowledge this was first analysed in detail by Metz [27]. For the special case $b = l = \frac{1}{2}$, the equilibrium fundamental threshold $\theta^*_{2I,0}$ increases (decreases) in the precision of the private signal $\gamma$ when the prior belief is that fundamentals are strong (weak). This result is consistent with the findings of Rochet and Vives [36]. A related result is that the above relationship is opposite when considering a change in the precision of the public signal $\alpha$.

Notice that the precision of the public signal is lower in the state where there is no correlation ($\alpha < \frac{\alpha}{1-\rho_H}$). As a consequence, the variance effect tends to increase (decrease) $\theta^*_{2I,0}$ relative to $\theta^*_{2I,\rho_H}$ if the prior belief is that fundamentals are strong (weak) independent of the realisation of $\rho$. For a prior belief that fundamentals are strong there is a clear tension between the mean and the variance effect, which go in opposite directions. A formal derivation can be found in Appendix section A.2.1. Here we also discuss the general case for any $b, l > 0$, which requires somewhat stronger conditions on $\mu^2_2(\rho, \theta_1)$. The intuition for the results is developed in the next paragraph.

**Intuition**  Given a private signal precision $\gamma$, a speculator with a prior belief that fundamentals are strong who receives a bad signal places the more weight on her bad private signal, the more dispersed the prior (the smaller $\alpha$). Other speculators knowing this, believe that more speculators will have a low posterior that induces them to attack the currency if $\alpha$ is smaller. They optimally decide to attack the currency more aggressively. To see this consider the probability that a given informed speculator $i$ (with signal $x_{i2}$) attaches to the event that another informed speculator $j$ has a smaller posterior. Denote with $\Theta_{i2I,0}$ the posterior of a given informed speculator $i$:

$$\Theta_{i2I,0} \equiv \theta_{2} | x_{i2} \sim \mathcal{N}(\frac{\alpha \mu + \gamma x_{i2}}{\alpha + \gamma}, \frac{1}{\alpha + \gamma}).$$

(14)
She believes that another informed speculator $j$ has a smaller posterior with probability:

$$
\Pr\{\Theta_{j2t,0} < \Theta_{i2t,0} | \Theta_{i2t,0}\} = \Phi\left(\frac{\alpha(\alpha + \gamma)}{\alpha + 2\gamma} [\Theta_{i2t,0} - \mu]\right)
$$

(15)

Notice that the equilibrium posterior mean $\Theta^*_{i2t,0}$ is smaller than $\mu$ if the prior belief is that fundamentals are strong. A given speculator $i$ with a low private signal $x_{i2}$ close to the equilibrium attack threshold $x^*_i$ expects a larger fraction of other speculators receiving a signal that corresponds to a lower posterior if $\alpha$ is smaller. This induces speculator $i$ to optimally respond by attacking more aggressively. As a result, the variance effect tends to increase $\theta^*_{2t,0}$ relative to $\theta^*_{2t,\rho_H}$.

### 3.2.3 Bayesian updating by uninformed currency speculators

Uninformed speculators do not know the realisation of $\rho$. However, they use their private signal $x_{i2}$ to update their prior belief on the distribution of $\rho$. In particular, uninformed speculators use Bayes’ rule to form a belief on the probability that $\theta_2$ is not correlated to $\theta_1$. Using Bayes’ rule we can derive $\Pr\{\rho = 0|\theta_1, x_{i2}\}$ as:

$$
\Pr\{\rho = 0|\theta_1, x_{i2}\} = \frac{\Pr\{x_{i2}|\theta_1, \rho = 0\} * p}{p \Pr\{x_{i2}|\theta_1, \rho = 0\} + (1 - p) \Pr\{x_{i2}|\theta_1, \rho = \rho_H\}
$$

(16)

18 If instead the prior belief is that fundamentals are weak, then there is only a relatively small degree of coordination failure. Here the increase in strategic uncertainty caused by a smaller level of $\alpha$ has an opposite effect. Now speculators who receive a private signal that contradicts the prior, i.e. a good signal relative to the low prior mean, play a key role as they place more weight on their good private signal. Other speculators knowing this belief that more speculators will have a high posterior that induces them not to attack the currency if $\alpha$ is smaller. They optimally decide to attack the currency less aggressively. We have an increase in coordination failure. This tends to decrease $\theta^*_{2t,0}$ relative to $\theta^*_{2t,\rho_H}$. 

17
where

\[
\Pr\{x_{i2}|\theta_1, \rho = 0\} = \frac{1}{\sqrt{\text{Var}[x_{i2}|\theta_1, \rho = 0]}} \phi\left(\frac{x_{i2} - E[x_{i2}|\theta_1, \rho = 0]}{\sqrt{\text{Var}[x_{i2}|\theta_1, \rho = 0]}}\right)
\]

\[
= \left(\frac{1}{\alpha} + \frac{1}{\gamma}\right)^{-\frac{1}{2}} \phi\left(\frac{x_{i2} - \mu}{\sqrt{\frac{1}{\alpha} + \frac{1}{\gamma}}}\right)
\]

\[
\Pr\{x_{i2}|\theta_1, \rho = \rho_H\} = \frac{1}{\sqrt{\text{Var}[x_{i2}|\theta_1, \rho = \rho_H]}} \phi\left(\frac{x_{i2} - E[x_{i2}|\theta_1, \rho = \rho_H]}{\sqrt{\text{Var}[x_{i2}|\theta_1, \rho = \rho_H]}}\right)
\]

\[
= \left(\frac{1 - \rho_H^2}{\alpha} + \frac{1}{\gamma}\right)^{-\frac{1}{2}} \phi\left(\frac{x_{i2} - [\rho_H \theta_1 + (1 - \rho_H)\mu]}{\sqrt{\frac{1 - \rho_H^2}{\alpha} + \frac{1}{\gamma}}}\right)
\]

An examination of \(\Pr\{\rho = 0|\theta_1, x_{i2}\}\) reveals that:

\[
\frac{d\Pr\{\rho = 0|\theta_1, x_{i2}\}}{d\theta_1} \begin{cases} 
  \geq 0 & \text{if } x_{i2} \leq \rho_H \theta_1 + (1 - \rho_H)\mu \\
  < 0 & \text{otherwise}.
\end{cases} 
\]

We can see that for a relatively low private signal, an increase in \(\theta_1\) leads speculators to belief that a zero cross-country correlation of fundamentals is more likely.

Furthermore, we can find that

\[
\frac{d\Pr\{\rho = 0|\theta_1, x_{i2}\}}{dx_{i2}} \begin{cases} 
  > 0 & \text{if } \theta_1 < \mu \text{ and } x_{i2} \geq \rho_H \theta_1 + (1 - \rho_H)\mu \\
  \leq 0 & \text{if } \theta_1 \geq \mu \text{ and } x_{i2} \leq \rho_H \theta_1 + (1 - \rho_H)\mu \\
  = 0 & \text{otherwise}.
\end{cases} 
\]

The results are intuitive. We are interested in a scenario where speculators in country 2 observe a successful currency attack with a realisation of \(\theta_1\) smaller than \(\mu\) as described in section 3.1. Recall that the prior distribution is more dispersed if \(\rho = 0\). As a result we

\[\text{Notice that the variance terms are unconditional on } \theta_2. \text{ Hence, we have to compute the sum of } \text{Var}[\epsilon_{i2}] \text{ and the variance of } \theta_2, \text{ which is } \frac{1}{\alpha} \text{ or } \frac{1 - \rho_H^2}{\alpha}.\]

\[\text{See Appendix section A.4 for a derivation.}\]
have that an extremely high or low private signal induces uninformed speculators to believe that the state of the world is very likely to be $\rho = 0$, i.e. $\lim_{x_{i2} \to +\infty} \Pr\{\rho = 0|x_{i1}, x_{i2}\} = 1$ and $\lim_{x_{i2} \to -\infty} \Pr\{\rho = 0|x_{i1}, x_{i2}\} = 1$.

Whenever speculators in country 2 observe a relatively good signal (i.e. $x_{i2} \geq \rho_H \theta_1 + (1 - \rho_H)\mu$), while observing a successful currency attack in country 1 (i.e. $\theta_1 < \mu$ given that fundamentals are strong), an increase in their private signal leads them to belief that cross-country fundamentals are with a higher probability not correlated. Instead if speculators in country 2 observe a relatively bad signal (i.e. $x_{i2} < \rho_H \theta_1 + (1 - \rho_H)\mu$), while observing a successful currency attack in country 1, then the relationship between $\Pr\{\rho = 0|x_{i1}, x_{i2}\}$ and $x_{i2}$ is non-monotone. In case the private signal is low but not too low, we still have that $\frac{d\Pr\{\rho = 0|x_{i1}, x_{i2}\}}{dx_{i2}} > 0$. However, in case the private signal is very low we have that $\frac{d\Pr\{\rho = 0|x_{i1}, x_{i2}\}}{dx_{i2}} \leq 0$ due to the more dispersed prior distribution if $\rho = 0$.

### 3.2.4 Equilibrium for the special case $n = 0$: how Bayesian updating changes the analysis

As before we are interested in monotone equilibria. Again two conditions have to be satisfied in equilibrium. The critical mass condition and the indifference condition. A combination of both leads to\textsuperscript{21}

\[ G(\theta_{2U}^s, \theta_1) \equiv \Pr\{\rho = 0|\theta_1, x_{i2}(\theta_{2U}^s)\} \Phi_{I, \rho = 0}(\theta_{2U}^s) \]
\[ + \Pr\{\rho = \rho_H|\theta_1, x_{i2}(\theta_{2U}^s)\} \Phi_{II, \rho = \rho_H}(\theta_{2U}^s, \theta_1) = \frac{l}{b + l} \quad (21) \]

where:

\[ \Phi_{I, \rho = 0}(\theta_{2U}^s) \equiv \Phi\left( \frac{\alpha}{\sqrt{\alpha + \gamma}}(\theta_{2U}^s - \mu) - \frac{\sqrt{\gamma}}{\sqrt{\alpha + \gamma}}\Phi^{-1}(\theta_{2U}^s) \right) \]
\[ \Phi_{II, \rho = \rho_H}(\theta_{2U}^s, \theta_1) \equiv \Phi\left( \delta(\rho_H)(\theta_{2U}^s - [\rho_H \theta_1 + (1 - \rho_H)\mu]) - \frac{\sqrt{\gamma}}{\sqrt{1 - \rho_H^2} + \gamma}\Phi^{-1}(\theta_{2U}^s) \right) \]

\textsuperscript{21}See Appendix section A.5.1 for details.
and \( \delta(\rho_H) \equiv \frac{\alpha}{1-\rho_H}/\sqrt{\frac{\alpha}{1-\rho_H} + \gamma}. \) Recall that the subscript \( U \) stands for uninformed. \( G(\theta^*_2, \theta_1) \) looks like a mixture of \( F_2(\theta^*_2, \rho = 0) \) and \( F_2(\theta^*_2, \rho = \rho_H). \) But now there is only one fundamental threshold \( \theta^*_2 \) for both states of the world, as uninformed speculators use the same strategies in both states. Different to before \( G(\theta^*_2, \theta_1) \) is now harder to characterise due to the dependency of the weights on the private signal. Is our focus on monotone equilibrium still justified?

First, we can prove that \( \Pr\{\theta_2 \leq \theta^*_2 \mid \theta_1, x_{i2}\} \) is monotonically decreasing in \( x_{i2} \) using the result of Milgrom [28]. This is true although the probability weights in the indifference condition are non-monotone in \( x_{i2}. \) Refer to Appendix section A.5.2 for the derivation. The essentially same argument is used in Chen et. al. [9].

Furthermore, let us do the thought experiment and analyse the best-response of a given speculator if varying the critical attack threshold used by other speculators. Letting \( \hat{\theta}_2^*_U(x_2) \) be the critical fundamental threshold when players other than \( i \) use a threshold strategy with the critical attack threshold \( \hat{x}_2, \) we can show that \( \Pr\{\theta_2 \leq \hat{\theta}_2^*_U(x_2) \mid \theta_1, x_{i2}\} \) is increasing in \( \hat{x}_2. \) The best response of a player \( i \) is to use a threshold strategy with critical attack threshold \( \tilde{x}_{i2}, \) where \( \Pr\{\theta_2 \leq \hat{\theta}_2^*_U(x_2) \mid \theta_1, \tilde{x}_{i2}\} = \frac{1}{b+1}. \) Following Vives (2005) [41], we can show that the best-response function is increasing:

\[
r' = -\frac{d\Pr\{\theta_2 \leq \hat{\theta}_2^*_U(x_2) \mid \theta_1, x_{i2}\}}{dx_{i2}} > 0 \quad (22)
\]

Hence, our interest in the existence of monotone equilibria is justified. Although the problem is now more complicated than for the polar case with \( n = 1, \) it is still possible to show that there exits a unique equilibrium in threshold strategies if the relative precision of the private information is sufficiently high. Here \( G(\theta^*_2, \theta_1) \) is monotonically decreasing in \( \theta^*_2. \) But the condition differs from the standard global games setup due to the use of mixture distributions. The proof is relegated to the Appendix and the result is summarised

\[\text{They developed a global game model with mixture distributions at the same time as we did. To our knowledge both papers are the only papers doing that. However, the focus of Chen et. al. is different to ours. They examine the role of rumours in a model of political regime change, while we consider contagion and learning about correlations.}\]
in Proposition 3.

Proposition 3  *Equilibrium existence, uniqueness and characterization*

For a finite precision of the public signal, there exists a finite value \( \gamma \) such that there exists a unique monotone equilibrium in this sub-game for all \( \gamma > \gamma \). Each uninformed speculator attacks if and only if her private signal is smaller than the threshold \( x^*_U \). A speculative currency attack is successful if and only if \( \theta^*_U \leq \theta^*_U \). The two equilibrium thresholds are implicitly defined by the solution to equations (21) and (48).

**Proof** See Appendix section A.5.3.

Finally, notice that \( \theta^*_U \) is just a weighted average of the two fundamental equilibrium thresholds from the polar case \( n = 1 \). As a result: \( \min\{\theta^*_L, 0, \theta^*_I, \rho_H\} \leq \theta^*_U \leq \max\{\theta^*_L, 0, \theta^*_I, \rho_H\} \).

### 3.3 The novel contagion mechanism

Suppose there was a successful currency attack in the first country, such that the ability of the government in country 1 to defend its currency must have been low. If fundamentals are possibly positively correlated across countries, the government’s ability to defend is likely to be also low in country 2, therefore making a successful currency attack likely to take place in country 2 as well. However, and perhaps surprisingly, the likelihood of successful currency attacks can be higher if speculators learn that fundamentals are not correlated (i.e. \( \rho = 0 \)) than if speculators do not learn about the correlation.

In particular we demonstrate in this section that the *ex-ante* likelihood of speculative attacks when all speculators are informed \( (n = 1) \) and learn that fundamentals are uncorrelated (i.e. \( \rho = 0 \)) can be higher than the *ex-ante* likelihood of attacks when all speculators are uninformed \( (n = 0) \). We call this effect *contagion-through-alertness*, as it arises following a successful currency attack in country 1. Learning good news about the strength

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23 Notice that *ex-ante* refers to the beginning of stage 2, that is before \( \theta_2 \) is realised.
of a central bank’s ability to defend its currency might have "detrimental" effects. This is because good news can lead to a higher likelihood of crises when it increases the variance of the posterior distribution relative to the case of not learning any news. The variance matters despite risk neutrality as knowing what others do is payoff-relevant information in coordination problems. This effect via the variance of the posterior distribution may lead to contagion via it’s impact on coordination failure.

The contagion effect can be present for a prior belief that fundamentals are strong and therefore a large degree of coordination failure. While our result holds more generally, the special polar cases in which either all speculators are uninformed \((n = 0)\) and all speculators are informed \((n = 1)\) help to build intuition. What is more than that, our focus on the polar cases when discussing the contagion case can be sufficient. This will be shown in section \[3.5\] We are interested in uncovering when the ex-ante likelihood of currency attacks is higher upon learning that fundamentals are uncorrelated, that is when \(\theta_{2t,0}^* > \theta_{2u}^*\). From our discussion of the role of public information and of information precision in section \[3.2.2\] we learned that there are two effects at work when varying \(\rho_H\): a mean effect and a variance effect. These two effects play a key role in what follows.

The mean effect occurs when the informed speculator has a higher posterior mean relative to the uninformed speculator, which is always true in the case of interest where the observed fundamentals of country 1 are bad, i.e. \(\theta_1 < \mu\):

\[
E[\theta_2 | x_j] < E[\theta_2 | \theta_1, x_j] = Pr\{\rho = 0 | \theta_1, x_j\} \cdot \frac{\alpha \mu + \gamma x_j}{\alpha + \gamma} + (1 - Pr\{\rho = 0 | \theta_1, x_j\}) \cdot \frac{\alpha}{1 - \rho_H} \cdot \left[\rho_H \theta_1 + (1 - \rho_H) \mu + \gamma x_j\right] + \gamma
\]

Notice that the mean effect works against us because \(\frac{d\theta_{2u}}{d\theta_1} < 0\) for \(\gamma\) sufficiently high. The mean effect vanishes if \(\theta_1 \to \mu\) or \(\rho_H \to 0\).

The variance effect refers to a larger variance of the posterior distribution for informed

\footnote{When generalising the results of section \[3.2.4\] to asymmetrically informed speculators and endogenous information acquisition in section \[3.5.2\] we will show how a symmetric equilibrium in country 2 with \(n^* = 1\) can emerge endogenously.}
speculators relative to uninformed speculators. If the prior belief is that fundamentals are strong it works in the opposite direction of the mean effect, as it tends to increase $\theta_{2I,0}^*$. The intuition established in section 3.2.2 goes through. However, the analysis is complicated because we now have to work with mixture distributions. For that reason it shows to be more attractive to directly analyse under what conditions the fundamental equilibrium thresholds satisfy: $\theta_{2I,0}^* > \theta_{2U}^*$. If the latter is the case, then it is due to the variance effect being sufficiently strong relative to the mean effect. The result is formally summarised in Proposition 4 below and is derived under the premise that the private signal is sufficiently precise. Recall that: $\delta(\rho_H) \equiv \alpha_{1-\rho_H}/\sqrt{\alpha_{1-\rho_H} + \gamma}$.

**Proposition 4 Existence of the contagion-through-alertness effect**

$\theta_{2I,0}^* > \theta_{2U}^*$ holds for the prior belief that fundamentals are strong if $\theta_1 \in [\theta_1, \mu]$, where:

$$\theta_1 \equiv \mu + \left( \frac{\rho_H}{\delta(\rho_H)} \left( (\theta_2^* - \mu) \left[ \delta(\rho_H) - \frac{\alpha}{\sqrt{\alpha + \gamma}} \right] + \Phi^{-1}(\theta_2^*) \left[ \sqrt{\frac{\gamma}{\alpha + \gamma}} - \sqrt{\frac{\gamma}{\alpha_{1-\rho_H} + \gamma}} \right] \Phi^{-1}(\theta_2^*) \right) \right)$$

and $\theta_2^*$ solves:

$$\left( (\theta_2^* - \mu) \frac{\alpha}{\sqrt{\alpha + \gamma}} - \sqrt{\frac{\gamma}{\alpha + \gamma}} \Phi^{-1}(\theta_2^*) \right) = \Phi^{-1} \left( \frac{l}{l + b} \right)$$

**Proof** See Appendix section A.6.

The desired result of $\theta_{2I,0}^* > \theta_{2U}^*$ obtains for independent and strong fundamentals if the variance effect is sufficiently strong relative to the mean effect. Intuitively, the mean effect is stronger, the lower $\theta_1$. As a consequence, the variance effect prevails only if $\theta_1$ is not too small. The contagion-through-alertness effect can only be present for a prior belief that fundamentals are strong, which implies a large degree of coordination failure. Only here it can be the case that the right-hand side of equation (23) is negative and, hence, $\theta_1 < \mu$.

**Intuition** Contagion-through-alertness can be present even after learning that there is no exposure. This happens if the higher variance of the posterior distribution ”weighs more” than the change in the mean of the posterior distribution after good news. At the core of
the novel contagion effect is that a higher posterior variance translates into more strategic uncertainty. Strategic uncertainty refers to the uncertainty about the behaviour of other speculators as perceived by a given speculator.

In figure 2 we consider a thought experiment that can help us to consolidate the intuition gained so far. We contrast graphically the posterior distributions of informed and uninformed speculators to illustrate the effect of additional variance of the posterior distribution when the contagion-trough alertness effect exists. Given $\rho = 0$, informed speculator $i$ expects a larger fraction of speculators receiving a signal that corresponds to a lower posterior when the other speculators $j$ are informed. Figure 2 sketches this effect as an increase in the area under the curve left of $\theta'$ for the more dispersed posterior distribution. Therefore, a larger share of informed speculators than uninformed speculators attack the currency despite expecting stronger defence of the currency by the government.

More strategic uncertainty only causes a higher equilibrium likelihood of attacks by informed speculators if the prior belief is that fundamentals are strong. Then, learning that fundamentals are uncorrelated reduces the posterior variance and increases strategic uncertainty. This effect outweighs the mean effect whenever $\theta_1 \in (\hat{\theta}_1, \mu]$.
Numerical example  Let us conclude this section with a numerical example. Consider the following parameters: $\alpha = \sqrt{\gamma} = 2$, $\mu = 0.8$, $p = 0.5$, $\rho_H = 0.7$, $\theta_1 = 0.5$, $l_1 = 0.2$, $b_1 = 0.6$, $l_2 = b_2 = 0.5$. Notice that the relative cost of attacking is lower in country 1. We find that $\theta_1^* \approx 0.8$. As a result speculators in country 2 observe $\theta_1$ after a successful speculative currency attack in country 1, since $\theta_1 < \theta_1^*$. Furthermore, it shows that $\theta_{2I,0}^* \approx 0.31$, $\theta_{2I,\rho_H}^* \approx 0.25$ and $\theta_{2U}^* \approx 0.29$. The likelihood of successful speculative attacks in the state when $\rho = 0$ is higher if agents get informed than if they stay uninformed. Notably the effect is stronger, the higher $\theta_1$ (i.e. the weaker the mean effect).

Finally, we have that the likelihood of a spread of the crisis is higher if the cross-country correlation is positive than if the correlation is zero: $\Pr\{\theta_2 \leq \theta_{2I,0}^* | \rho = 0\} \approx 0.24 < \Pr\{\theta_2 \leq \theta_{2I,\rho_H}^* | \rho = \rho_H\} \approx 0.25$. However, the latter result does not hold in general because $\theta_{2I,\rho_H}^*$ decreases in $\theta_1$.

3.4 Discussion & relation to empirical literature on contagion

Currency crises show to have a contagious nature. In early empirical work on contagion Eichengreen et. al. [16] find striking evidence that a crisis elsewhere increases the likelihood "of a speculative attack by an economically and statistically significant amount" (page 2). Our theoretical model is consistent with this evidence. In the model the likelihood of successful currency attacks in country 2 is higher after country 1 was successfully attacked than in the scenario where there is no crisis in country 1. What is more, this result even holds if speculators learn that fundamentals are independent, i.e. $\rho = 0$. Hence, our contagion mechanism offers a compelling explanation for the abovementioned contagious spread of the Russian crisis to Brazil, which happened although the interlinkages between the countries showed to be limited even from an ex-post perspective (compare Bordo and Murshid [6]). In fact, the likelihood of attacks can be even higher if speculators learn that $\rho = 0$ than if they

---

\[25\] When $\theta_1 > \mu$ there is no successful currency attack in country 1. Hence, speculators in country 2 do not observe $\theta_1$ and remain uncertain about its realisation. However, speculators in country 2 can infer that the realisation of $\theta_1$ must have been sufficiently high, as to prevent a successful attack in country 1. Notice that for $\theta_1 > \mu$ the mean and variance effect go in the same direction given a prior belief that fundamentals are strong. As a result, the likelihood of a successful currency attack in country 2 must be lower when country 1 was not successfully attacked.
stay uninformed. This is due to an increase in strategic uncertainty caused by the variance effect. The increased strategic uncertainty is consistent with the view by many "observers [who attribute the spread of the Russian crisis to] . . an enhanced perception of risk" (Van Rijckeghem and Weder [39], p. 294).

The surprising result that the likelihood of successful currency attacks in country 2 may be higher in the state of the world where \( \rho = 0 \) when speculators learn about the correlation instead of staying uninformed arises if \( \theta_1 \in (\theta_1, \mu] \), which implies that \( \theta_{2I,0}^* > \theta_{2U}^* > \theta_{2I,\rho H}^* \). As a consequence, it may in our model happen that the likelihood of a currency attack is lower if the cross-country correlation is positive than if the correlation is zero.\(^{26}\) At first glance this implication is at odds with the existing empirical literature. The empirical literature prescribes that the likelihood of a spread of the crisis is higher with a positive correlation, which could be interpreted as a higher institutional similarity or stronger financial and trade links (compare Dasgupta et. al. [14], Van Rijckeghem and Weder [39] or Glick and Rose [19]). However, the model implication can potentially offer an explanation why Glick and Rose found that macroeconomic variables (such as domestic credit, government budget, current account, international reserves and a devaluation of the real exchange rate) do not help to explain contagion. When we interpret the cross-country correlation of fundamentals in our model as reflecting a correlation of macroeconomic variables, then the model suggests that a positive or zero correlation has an ambiguous effect. Whenever the realisations of \( \theta_1 \) are relatively high, but still causing a crisis in the ground zero country, the likelihood of a spread of the crisis may be higher or lower if \( \rho > 0 \). Instead if \( \theta_1 \) is low, a positive correlation clearly increases the likelihood of a spread of the crisis. As a result, the empirical measurement may not find a significant effect of macroeconomic variables when not accounting for this non-linearity.

\(^{26}\)Although this is only the case if the realisation of \( \theta_1 \) is sufficiently close to \( \mu \) (see also the numerical example at the end of section 3.3).
3.5 Extension: Asymmetrically informed speculators & endogenous information acquisition

In this section we extend the previous analysis of country 2 to the general case with asymmetrically informed speculators \((0 < n < 1)\) and demonstrate how endogenous information acquisition can be triggered by a wake-up call event that makes speculators alert. The game at date \(t = 2\) has two stages and is solved backwards. First, we solve in section 3.5.1 for the equilibrium in the second stage, taking \(n\) as given. Then we solve the information acquisition game at the first stage of date \(t = 2\) in section 3.5.2.

**Definition 5** A pure strategy Perfect Bayesian Nash Equilibrium in country 2 is an information acquisition choice \(d_i^* \in \{I, U\}\) for each speculator \(i \in [0, 1]\) in stage 1, an aggregate fraction of informed speculators \(n^*\) and a decision rule \(a_{2|t}^*(\theta_1, x_{i2}, n)\) in stage 2 such that:

1. All speculators optimally choose \(d_i\) in stage 1 given \(n^*\).
2. The proportion \(n^*\) is consistent with the optimal choices implied by (1.): \(n^* = \int_0^1 1 \{d_i^* = I\} di\).
3. The speculative attack decisions for uninformed speculators in stage 2 are given by:

\[
a_{i2|U}(\theta_1, x_{i2}, n^*) = \arg \max_{a_{i2|U} \in \{0, 1\}} E[u(a_{i2|U}, A_2, \theta_2, \theta_1, n^*)|x_{i2}] \tag{25}
\]

and for a given realisation of \(\rho\) the speculative attack decisions for informed speculators in stage 2 are given by:

\[
a_{i2|I, \rho}(\theta_1, x_{i2}, n^*) = \arg \max_{a_{i2|I} \in \{0, 1\}} E[u(a_{i2|I}, A_2, \theta_2, \theta_1, \rho, n^*)|x_{i2}] \tag{26}
\]

4. For a given realisation of \(\rho\) the aggregate mass of speculative attackers \(A_2 \equiv A(\theta_2, n^*, \rho)\) in stage 2 is given by:

\[
A(\theta_2, n^*, \rho) = n^* \int_{-\infty}^{+\infty} a_{i2|I, \rho}^*(\theta_1, x_{i2}, n^*) \sqrt{\gamma} \phi(\sqrt{\gamma}(x_{i2} - \theta_2)) dx_{i2} + (1 - n^*) \int_{-\infty}^{+\infty} a_{i2|U}^*(\theta_1, x_{i2}, n^*) \sqrt{\gamma} \phi(\sqrt{\gamma}(x_{i2} - \theta_2)) dx_{i2} \tag{27}
\]
5. $A(\theta_2, n^*, \rho)$ is consistent with the optimal speculative attack decision implied by (3.).

### 3.5.1 Stage 2: The general case $0 < n < 1$

Different to before, we now allow for asymmetrically informed speculators. A fraction $n$ of speculators learns the realisation of the cross-country correlation $\rho$ (informed speculators), while a fraction $1 - n$ of speculators does not learn the realisation of the correlation (uninformed). As before speculators use threshold strategies, where uninformed speculators attack if their posterior mean is below a threshold. However, differently attack thresholds now depend on $n$ and for the informed speculators also on the observed correlation. For this reason we now have three attack thresholds. One critical attack threshold for uninformed speculators: $x_{2U}^*(n)$. And two critical attack thresholds for informed speculators: $x_{2I,\rho}^*(n)$ for the two states $\rho = 0$ and $\rho = \rho_H$. Also fundamental thresholds are now functions of $n$ and we have two of them depending on the realisation of $\rho$. We denote them with $\theta_{2,\rho}^*(n)$ for the states $\rho = 0$ and $\rho = \rho_H$.

Details on the equilibrium analysis can be found in Appendix section $\text{A.7}$ The equilibrium can be described by two equations in two unknowns $\theta_{2,0}^*(n)$ and $\theta_{2,\rho_H}^*(n)$:

\begin{align*}
M^1(\theta_{2,0}^*, \theta_{2,\rho_H}^*; n) &= 0 \quad (28) \\
M^2(\theta_{2,0}^*, \theta_{2,\rho_H}^*; n) &= 0 \quad (29)
\end{align*}

where $n$ is taken as given. We have that:

\begin{align*}
\frac{\partial M^1(\theta_{2,0}^*, \theta_{2,\rho_H}^*; n)}{\partial \theta_{2,0}^*} &> 0 \quad (30) \\
\frac{\partial M^1(\theta_{2,0}^*, \theta_{2,\rho_H}^*; n)}{\partial \theta_{2,\rho_H}^*} &< 0 \quad (31)
\end{align*}

From $M^1(\theta_{2,0}^*, \theta_{2,\rho_H}^*; n)$ together with equations (30) and (31) we can conclude that $\frac{d\theta_{2,0}^*}{d\theta_{2,\rho_H}^*} > 0$ for a given $n$. Furthermore, it shows that $\frac{\partial M^2(\theta_{2,0}^*, \theta_{2,\rho_H}^*; n)}{\partial \theta_{2,0}^*}$ and $\frac{\partial M^2(\theta_{2,0}^*, \theta_{2,\rho_H}^*; n)}{\partial \theta_{2,\rho_H}^*}$ are negative for a sufficiently high precision of the private signal $\gamma$. Consequently, we can again prove that there exists a unique equilibrium in threshold strategies for a sufficiently high precision of
the private signal. This can be seen by a similar argumentation as in the proof of Proposition 3, using the result that \( \frac{d\theta_{2,0}^*}{d\theta_{2,\rho_H}^*} > 0 \) for a given \( n \). The intuition is the same as in the polar case \( n = 0 \) and the result is formally stated in the Proposition 6.

**Proposition 6  Equilibrium existence and uniqueness**

For a finite precision of the public signal, there exists a finite value \( \gamma \) such that there exists a unique monotone equilibrium in this sub-game for all \( \gamma > \gamma \). Each uninformed speculator attacks if and only if her private signal is smaller than the threshold \( x_{2U}(n) \). Each informed speculator attacks if and only if her private signal is smaller than the threshold \( x_{2I,0}(n^*) \) when learning \( \rho = 0 \) and smaller than the threshold \( x_{2I,\rho_H}(n^*) \) when learning \( \rho = \rho_H \). A speculative currency attack is successful if and only if \( \theta_2 \leq \theta_{2,0}^*(n) \) \( (\theta_2 \leq \theta_{2,\rho_H}^*(n)) \) when \( \rho = 0 \) \( (\rho = \rho_H) \).

**Proof**  See Appendix section A.8.

The more interesting question is how a variation in \( n \) affects the equilibrium thresholds. Analytically it is not possible to characterise the equilibrium by using comparative static methods based on the implicit function theorem for simultaneous equations. In a numerical analysis we find however very intuitive patterns. Figure 3 shows a numerical example where parameters are chosen such that the above described contagion mechanism kicks in, i.e. \( \theta_{2I,0}^* > \theta_{2I}(n = 0) \). Here the likelihood of successful speculative currency attacks shows to be higher when the actual correlation is \( \rho = 0 \) (‘good news’) and informed speculators learn about it, than when speculators remain uninformed. While uninformed speculators use the same critical attack threshold no matter whether there is a correlation or not, the informed speculators adjust their critical attack thresholds depending on the observed correlation. Interestingly, the equilibrium fundamental thresholds for the state of the world when the actual correlation is \( \rho = 0 \) and the state of the world when the actual correlation is \( \rho = \rho_H \) are diverging when \( n \) increases. This relations are intuitive. Given that informed speculators attack more aggressively after learning that \( \rho = 0 \) compared to uninformed speculators, a larger population fraction of informed speculators causes the equilibrium fundamental threshold \( \theta_{2,0}^*(n) \) to be higher (see orange dot-dashed line). The opposite is true.
for the state of the world, where informed speculators learn that $\rho = \rho_H$. Here they attack less aggressively when compared to uninformed speculators. As a result, the equilibrium fundamental threshold $\theta^*_2,\rho_H(n)$ decreases in $n$ (see green dashed line).

![Figure 3: The critical fundamental thresholds as a function of the fraction of uninformed speculators $n$. (Parameters: $\mu = 0.9$, $\alpha = \gamma = 1$, $b = l = 0.5$, $P = 0.7$, $\rho_H = 0.5$ and $p = 0.8$.)](image)

Analytically, it is difficult to show when the very intuitive first-order effects described above outweigh potential second-order effects that may arise due to an equilibrium adjustment of the critical attack threshold for uninformed $x^*_U(n)$ when $n$ changes\(^{27}\).

### 3.5.2 Stage 1: Information acquisition

In the previous section we derived the equilibrium in stage 2 of date 1 for the general case $0 < n < 1$. While the amount of information was taken as given – a fraction $n \in [0,1]$ was informed, we allow for endogenous information acquisition in this section and thereby generalise our result. We argue that there exists an equilibrium in which each speculator acquires information if the cost of doing so is sufficiently small. The contagion-through-alertness

\(^{27}\)An attempt to derive comparative statics results that hold for at least restricted parameter parameter ranges using alternative methods is left for future work.
effect is present in this equilibrium: there can be more speculative currency attacks after speculators learn that fundamentals are uncorrelated than without having learned anything.

After observing country 1’s fundamental \( \theta_1 \), speculators in country 2 decide whether to acquire costly information on the cross-country correlation \( \rho \). Recall that the purchased information is a perfect signal about the realisation of \( \rho \) and that the additional signal is \emph{publicly available} to all speculators at a cost. As before, we maintain our focus on the case in which speculators in country 2 observe a crisis in country 1, that is \( \theta_1 < \theta_1^* < \mu \) for strong fundamentals.

The speculator’s problem To determine the equilibrium of the game, we consider the problem of an individual speculator. Each speculator \( i \) takes the population proportion of speculators \( n \) who purchase information as given and compares the expected payoffs from purchasing the publicly available signal (becoming informed \( s = I \)) and not purchasing the signal (remaining informed \( s = U \)). The expected utility of an informed speculator \( EU_I \) is:

\[
EU_I \equiv E[u(d = I, \alpha, \gamma, \mu, \rho_H, \theta_1, n)]
\]

\[
= p \left( \int_{\theta_1}^{\theta_2} (-l) \int_{x_{12} \leq x_{21,0}(n)} g(x_{12} | \theta_2) dx_{12} f(\theta_2) d\theta_2 + \int_{-\infty}^{\theta_1} b \int_{x_{12} \leq x_{20,0}(n)} g(x_{12} | \theta_2) dx_{12} f(\theta_2) d\theta_2 \right)
\]

\[
+ (1 - p) \left( \int_{\theta_1}^{\infty} (-l) \int_{x_{12} \leq x_{21,0}(\rho_H, \theta_1, n)} g(x_{12} | \theta_2) dx_{12} f(\theta_2) d\theta_2 + \int_{-\infty}^{\theta_1} b \int_{x_{12} \leq x_{20,0}(\rho_H, \theta_1, n)} g(x_{12} | \theta_2) dx_{12} f(\theta_2) d\theta_2 \right) - c
\]

In contrast the expected utility of an uninformed \( EU_U = E[u(d = U, \alpha, \gamma, \mu, \rho_H, \theta_1, n)] \) has the only difference that the cost \( c \) of information is not subtracted and that uninformed speculators use the same critical attack threshold \( x_{21}^*(n) \) for both states of the world. The distributions of the fundamental in country 2 for both states of the world and the distribution
of signals are given as follows:

\[ f(\theta_2) = \sqrt{\frac{\alpha}{2\pi}} \exp\left\{-\frac{\alpha}{2}(\theta_2-\mu)^2\right\} \quad (33) \]
\[ f(\theta_2|\theta_1,\rho_H) = \sqrt{\frac{\alpha}{2\pi(1-\rho_H^2)}} \exp\left\{-\frac{\alpha}{2(1-\rho_H^2)}(\theta_2-(\rho_H\theta_1+(1-\rho_H)\mu))^2\right\} \quad (34) \]
\[ g(x|\theta_2) = \sqrt{\frac{\gamma}{2\pi}} \exp\left\{-\frac{\gamma}{2}(x-\theta_2)^2\right\} \quad (35) \]

**Intuition** Before the fundamental \( \theta_2 \) is realised, speculators know the conditional distribution of \( \theta_2 \), which depends on the state of the world. For informed speculators receiving news that fundamentals are uncorrelated (correlated), the pdf is given by \( f(\theta_2) \) ( \( f(\theta_2|\theta_1,\rho_H) \) ). The difference in the expected payoffs of informed and uninformed speculators results from the informed being able to select different critical attack threshold for the two events. For each realisation of \( \theta_2 \), speculators can compute how many (un-) informed speculators decide to attack and how likely it is that they themselves receive a private signal below their critical threshold which induces them to attack. Both, informed and uninformed speculators know that the two events "no correlation" and "positive correlation" occur with probability \( p \) and \( 1-p \), respectively. For each event speculators integrate over the corresponding distribution of \( \theta_2 \).

**Benefits from and costs of attacking** To gain a better understanding consider the benefits and costs from attacking for the polar case when \( n = 0 \). Here \( \theta^*_{2,0}(n = 0) = \theta^*_{2,rho_H}(n = 0) = \theta^*_{2U} \). Taking derivatives leads to:

\[
\frac{dEU}{dx^*_{2l,0}(0)} = p\left( -l \int_{\theta^*_{2l,0}}^{+\infty} g(x^*_{2l,0}(0)|\theta_2)f(\theta_2)d\theta_2 + b \int_{-\infty}^{\theta^*_{2l,0}} g(x^*_{2l,0}(0)|\theta_2)f(\theta_2)d\theta_2 \right)
\]

The first summand is negative and represents the cost of increasing the attack threshold due to a higher likelihood to participate in unsuccessful speculative attacks. The second summand is positive and represents the benefit from a higher likelihood to participate in successful currency attacks. In equilibrium the marginal cost and the marginal benefit have to be equalised.
Strategic complementarity in information acquisition choices  A given speculator finds it optimal to purchase the publicly available signal if the expected differential payoff is positive. If the dependency of equilibrium fundamental thresholds can be characterised as in figure 3, then we have a strategic complementarity in information acquisition choices. Here we have that incentives to get informed are increasing in $n$.

When is it optimal to purchase information?  If the differential expected payoff $E U_I - E U_U \equiv \Delta[\alpha, \gamma, \mu, \rho_H, \theta_1, n]$ is positive, which can be written as:

$$p \left( \int_{\theta_{2,0}^* (n)}^{+\infty} (-l) \int_{x_{2,0}^* (n)}^{x_{2,I}^* (n)} g(x_{i2} | \theta_2) dx_{i2} f(\theta_2) d\theta_2 - \int_{-\infty}^{\theta_{2,0}^* (n)} b \int_{x_{2,U}^* (n)}^{x_{2,0}^* (n)} g(x_{i2} | \theta_2) dx_{i2} f(\theta_2) d\theta_2 \right) - (1 - p) \left( \int_{\theta_{2,H}^* (n)}^{+\infty} (-l) \int_{x_{2,I}^* (n)}^{x_{2,U}^* (n)} g(x_{i2} | \theta_2) dx_{i2} f(\theta_2 | \theta_1, \rho_H) d\theta_2 + \int_{-\infty}^{\theta_{2,H}^* (n)} b \int_{x_{2,U}^* (n)}^{x_{2,H}^* (n)} g(x_{i2} | \theta_2) dx_{i2} f(\theta_2 | \theta_1, \rho_H) d\theta_2 \right) - c \geq 0 \quad (36)$$

Suppose we are in the scenario where the novel contagion effect occurs, i.e. $\theta_{2,0}^* (n) \geq \theta_{2,H}^* (n)$. Given that an increase in $n$ is associated with an increase in $\theta_{2,0}^* (n)$ and a decrease in $\theta_{2,H}^* (n)$. An increase in $n$ leads to a relative increase of the benefit component in the first summand and a relative decrease of the loss component in the second summand, holding everything else equal. For any admissible combination of equilibrium attack thresholds this implies a strict increase in the differential payoff of being informed. The reason being that informed speculators can take ”full” advantage of the change in equilibrium fundamental thresholds when $n$ changes, while uninformed speculators have always to ”balance” the marginal benefit from increasing $x_{2,U}^*$ in case there is no exposure (with probability $p$) with the marginal loss of increasing $x_{2,U}^*$ in case there is an exposure (with probability $1 - p$).

A consequence of the above argument is that if the cost of information is sufficiently low as to give an incentive for an individual speculator to acquire information given that all other speculators are uninformed (i.e. $n = 0$), then it is also optional to acquire information for an individual speculator no matter how many other speculators are informed (i.e. for all

\footnote{That is if $\theta_{2,0}^* (n) > \theta_{2,U}^* > \theta_{2,H}^* (n)$ and if $\theta_{2,0}^* (n)$ is monotonically increasing in $n$, while $\theta_{2,H}^* (n)$ is monotonically decreasing in $n$. Notice that the former implies that $x_{2,U}^* (n) \in (x_{2,I,H}^* (n), x_{2,I,0}^* (n))$ for all $n \in (0, 1)$.}
\( n \in (0, 1] \). The result is summarised below.

**Result. Equilibrium of the information acquisition game**

Suppose that speculators have a prior belief that fundamentals are strong, and that private signals are sufficiently precise such that there exists a unique monotone equilibrium of the sub-game at stage 2 for a given \( n \). Then there exists a unique equilibrium of the information acquisition game at stage 1 in which all speculators acquire the publicly available signal, i.e. \( n^* = 1 \), after observing \( \theta_1 < \mu \) whenever:

1. \( \Delta[\alpha, \gamma, \mu, \rho_H, \theta_1, n = 0] > 0 \)

2. there is a strategic complementarity in information acquisition choices.

The strategic complementarity in information acquisition choices is guaranteed if parameters are such that \( \theta^*_{2,0}(n) > \theta^*_{2U} > \theta^*_{2,\rho_H}(n) \) and that \( \theta^*_{2,0}(n) \) is monotonically increasing in \( n \), while \( \theta^*_{2,\rho_H}(n) \) is monotonically decreasing in \( n \).

### 3.5.3 Discussion

In this section we demonstrate that the **contagion-through-alertness effect** described earlier can be an equilibrium phenomenon in the more general setup with endogenous information acquisition whenever the cost of information is sufficiently low. Furthermore, we found that we can have a strategic complementarity in information acquisition choices. The strategic complementarity in information acquisition choices arises quite naturally in global games models with endogenous information acquisition.\(^{29}\)

The numerical example underlying figure 3 provides a situation when the above result applies. Here, we have that \( \theta^*_{2,0}(n) > \theta^*_{2U} > \theta^*_{2,\rho_H}(n) \) and that \( \theta^*_{2,0}(n) \) is monotonically increasing in \( n \), while \( \theta^*_{2,\rho_H}(n) \) is monotonically decreasing in \( n \). Although this characterisation suggest to hold generally in our numerical analysis, it is not possible to do an analytical comparative statics analysis relying on the simultaneous equations version of the implicit

\(^{29}\)Szkup and Trevino show numerically in a model with continuous information acquisition choice over the precision of private signals and convex costs that strategic complementarity may under some parameters not be guaranteed. However, in our model with discrete information acquisition choice and publicly available signals their result should be less or not at all relevant.
function theorem. The problem is left for future research.

**Policy implications**  A *wake-up call* triggers endogenous information acquisition whenever the cost of information is sufficiently low. The benefit from being informed shows to be positively related to the difference between $\theta_{2,0}^*(n)$ and $\theta_{2,\rho_H}^*(n)$. As a result, the incentives to get informed are the higher, the stronger the contagion mechanism.

If $\theta_1 \in (\underline{\theta}_1, \mu]$, then the *contagion-through-alertness effect* prevails and we have a higher likelihood of successful currency attacks after speculators learn that there is no correlation compared with the case where they stay uninformed. If instead $\theta_1 < \underline{\theta}_1$, then the we have a higher likelihood of successful currency attacks after speculators learn that there is a positive correlation compared with the case where they stay uninformed. In both scenarios, an informed policy maker could reduce the likelihood of successful currency attacks by making information more costly, such that individual speculators optimally decide not to acquire information in the first place.

The opposite is true if $\theta_1 \in (\underline{\theta}_1, \mu]$ and informed speculators learn that there is exposure or if $\theta_1 < \underline{\theta}_1$ and informed speculators learn that there is no exposure. Here, an informed policy maker could reduce the likelihood of successful currency attacks by making information less costly, such that individual speculators optimally decide to acquire information.

4 Related literature

The literature on currency crisis is large and we do not attempt to provide a detailed review but focus on the incomplete information game introduced by Morris and Shin [29, 30]. Following the seminal contribution of Carlsson and van Damme [8], a perturbation of the information structure yields a unique equilibrium. This overcomes the multiplicity of equilibria present in many previous models of currency crisis, such as the Krugman-Flood-Garber [24, 17] first-generation currency crisis model, the second-generation currency crisis model by Obstfeld [32], and many third-generation currency crisis models.

An important ingredient of our contagion-through-alertness mechanism is the exac-
erbation of the coordination problem when the precision of the agents’ prior information changes. This element is present in earlier work on bank runs by Rochet and Vives \[36\], for example. Our contagion mechanism sheds new light on results on the role of information precision and of public information that have been established in the global games literature. The novelty of this paper is to combination the mean effect and the variance effect in a setting where both can go in opposite directions.

Furthermore, our paper is also related to the literature analysing the role of information precision. Information acquisition can have a detrimental effect in our model. This result connects to papers that stress the possible benefits of coarse information\[33\] For instance, the papers of Dang, Gorton and Holmström \[11\] as well as Pagano and Volpin \[33\] emphasise the benefits of coarse information in supporting market liquidity.

Endogenous information acquisition is considered by Hellwig and Veldkamp \[22\] who discuss the similarity in the strategic motives between choosing an action and deciding on how much information to acquire in a beauty-contest model. In their words, investors “who want to do what others do, want to know what others know” (p. 223). They show that adding a public information choice may lead to a multiplicity in equilibria. By contrast, uniqueness is always guaranteed under the usual mild condition of sufficiently precise private signals in our global games model.

**Contagion in financial economics**

While there exists a large literature on financial contagion, typically either interconnect-edness or common exposures is required to generate contagion or systemic fragility more generally. First, systemic fragility because of common exposures (correlated fundamentals) are considered in Acharya and Yorulmazer \[1\], who show that banks can have an ex-ante incentive to correlate their investment decision to avoid information contagion, and Allen, Babus and Carletti \[3\], who analyze systemic risk resulting from the interaction of common exposures and funding maturity through an information channel. Manz \[26\] explores the role of common exposures in a global-games framework. Second, financial contagion can

\[30\]See Morris and Shin \[31\].
arise from interconnectedness. Allen and Gale [4] provide a model of financial contagion as an equilibrium outcome through interbank linkages. Dasgupta [12] shows that financial contagion arises with positive probability in a global-game version of Allen and Gale [4]. In Goldstein and Pauzner [20] contagion results from a wealth effect of investors who become more averse to strategic risk after a crisis in one country. There is also a large literature on contagion through a pecuniary “fire-sale” externality related to the ideas of Shleifer and Vishny [37].

The distinct feature of the proposed contagion-through-alertness mechanism is the endogenous information acquisition such that contagion can occur in the absence of interconnectedness and common exposures. Observing an adverse event in another region is a wake-up call to investors that induces them to acquire costly information about their exposure to that event. This alertness effect can result in a higher likelihood of an adverse event in their region. Such fragility can even be present if investors learn that their investments are completely uncorrelated with the adverse event. In sum, it is sufficient that fundamentals are potentially correlated to generate the alertness effect. Once speculators are alert, the incidence of speculative attacks is increased even after speculators learn that the regional fundamentals are uncorrelated.

Contagion in international finance

The international finance literature mainly considers a terms-of-trade channel and a common-discount-factor channel to explain an international co-movement in asset prices during crisis periods. (Co-movement in asset prices is considered as contagious when “excessive”). However, these channels cannot account for the observed co-movements in the 1997/1998 emerging market crisis period. Pavlova and Rigobon [34] argue that neither channel explains the co-movements in asset prices of countries with limited trade links. They construct an open-economy dynamic stochastic general equilibrium model and show that portfolio constraints can cause a substantial amplification and help to explain the observed co-movements in asset prices in crisis periods. An alternative amplification mechanism is provided by Kodres and Pritsker [23] who establish the “cross-market portfolio rebalancing channel”, which is based
on the common discount factor channel.

Calvo and Mendoza [7] also offer a contagion mechanism that does not rely on correlated macroeconomic fundamentals, where the authors relate contagion to information acquisition. In this sense their paper is closer to our model than the existing mechanisms in the financial economics literature. In their paper a lower degree of information acquisition, as a consequence of globalisation, gives rise to contagion because market participants prefer to imitate arbitrary market portfolios instead of gathering information which can lead to a detrimental herding behaviour. By contrast, contagion is a consequence of a higher, not a lower, degree of information acquisition in our model, where fragility can arise because of heightened strategic uncertainty in coordination problems.

5 Conclusion

This paper proposes a novel contagion mechanism based on an alertness effect. Upon observing a successful currency attack elsewhere – a wake-up call – speculators wish to determine to what extent their investment position is affected by that crisis. This alertness effect per se can lead to a larger likelihood of successful currency attacks through an increase in strategic uncertainty. The contagion-through-alertness effect prevails whenever speculators in country 2 observe a successful currency attack in country 1 that results from weak but not too weak fundamentals. We consider this scenario as relevant. First, a very low fundamental realisation of \( \theta_1 \) is a low probability event. Second, the situation in practise is most of the time not so obvious and fragile countries or banks tend to be somewhat weak but not destined to fail with certainty. While we present an application to speculative currency attacks, the contagion-through-alertness mechanism occurs in general coordination problems and is applicable to bank runs, political regime change, and sovereign debt crises.
Appendix

A.1 Country 1

A.1.1 Equilibrium analysis

The first equilibrium condition is given by:

\[ A(\theta_1) = \Pr\{x_{i1} \leq x_1^*|\theta_1^*\} = \Phi(\sqrt{\gamma}(x_1^* - \theta_1)) = \theta_1^* \]
\[ x_1^* = \theta_1^* + \frac{1}{\sqrt{\gamma}} \Phi^{-1}(\theta_1^*) \]  (37)

It demands that in equilibrium the critical fraction of attacking speculators has to be equal to the critical fundamental threshold above which it pays to act.

The second equilibrium condition is an indifference condition. It implicitly defines the equilibrium fundamental threshold. Given \( \theta_1^* \), the payoff of an attacking speculator is given by:

\[ b\Pr\{\theta_1 \leq \theta_1^*|x_{i1}\} - l\Pr\{\theta_1 > \theta_1^*|x_{i1}\} = 0 \]  (38)

where:

\[ \Pr\{\theta_1 \leq \theta_1^*|x_{i1}\} = \Phi\left(\frac{\theta_1^* - \mathbb{E}[\theta_1|x_{i1}]}{\sqrt{\text{Var}[\theta_1|x_{i1}]}\gamma}\right) = \Phi\left(\sqrt{\gamma}(\theta_1^* - \frac{\alpha\mu + \gamma x_{i1}}{\alpha + \gamma})\right) \]

which is decreasing in \( x_1^* \). A speculator attacks if and only if \( x_{i1} \leq x_1^* \). At the critical equilibrium attack threshold \( x_1^* \) speculators have to be just indifferent whether to attack or not.

Combining the two equilibrium conditions leads to equation (6). The right-hand side is a constant and the left-hand side is decreasing in \( \theta_1 \) if the relative precision of the private signal is sufficiently high:

\[ \frac{dF_1(\theta_1)}{d\theta_1} = \Phi'(\theta_1^*) \frac{\alpha - \sqrt{\gamma}\Phi^{-1}(\theta_1)}{\sqrt{\alpha + \gamma}} < 0 \quad \text{if} \quad \frac{\alpha}{\sqrt{\gamma}} < \sqrt{2\pi} \]  (39)
A.1.2 Equilibrium characterization

It is useful to distinguish between a prior belief that fundamentals are strong and a prior belief that fundamentals are weak.

Consider the equilibrium condition:

\[ \Phi \left( \frac{\alpha}{\alpha + \gamma} (\theta_1^* - \mu) - \sqrt{\frac{\gamma}{\alpha + \gamma}} \Phi^{-1}(\theta_1^*) \right) = \frac{l}{b + l} < 1 \tag{40} \]

Reformulate it to:

\[ \alpha (\theta_1^* - \mu) = \sqrt{\gamma} \Phi^{-1}(\theta_1^*) + \sqrt{\alpha + \gamma} \Phi^{-1}(\frac{l}{b + l}) \tag{41} \]

and notice that, given \( \mu \in (0, 1) \), \( \theta_1^* = \mu \) if and only if:

\[ \sqrt{\gamma} \Phi^{-1}(\mu) + \sqrt{\alpha + \gamma} \Phi^{-1}(\frac{l}{b + l}) = 0 \tag{42} \]

\( \theta_1^* > \mu \) if and only if:

\[ \sqrt{\gamma} \Phi^{-1}(\mu) + \sqrt{\alpha + \gamma} \Phi^{-1}(\frac{l}{b + l}) < 0 \tag{43} \]

and \( \theta_1^* < \mu \) otherwise.

Equation (43) refers to the case of a prior belief that fundamentals are ”weak”. Weak fundamentals are associated with a low \( \mu \) and a relatively low cost of an unsuccessful currency attack. Here the critical equilibrium fundamental threshold is strictly larger than \( \mu \). The opposite is true if the prior belief is that fundamentals are ”strong”, meaning that \( \mu \) is high and the relative cost of an unsuccessful attack is high. Of special interest is the case when \( \frac{l}{b + l} = \frac{1}{2} \) for which the analysis simplifies. Here a prior belief that fundamentals are weak (strong) is defined as \( 0 < \mu < \frac{1}{2} (\frac{1}{2} < \mu < 1) \). For a prior belief that fundamentals are weak (strong) we can find that: \( 0 < \mu < \frac{1}{2} \) and \( \theta_1^* < x_1^* < 1 \) (\( 0 < x_1^* < \theta_1^* < \frac{1}{2} < \mu < 1 \)).
A.2 Country 2: Stage 2

A.2.1 Higher precision of the public signal ($\alpha$) and the private signal ($\gamma$)

The subsequent discussion draws in parts from Bannier and Heinemann [5]. When analysing the equilibrium condition we find that:

$$\frac{d\theta^*_{21,\rho}}{d\alpha} \begin{cases} < 0 & \text{if } \theta^*_{21,\rho} < \mu'_2(\rho, \theta_1) + \frac{1}{2\sqrt{1-\rho^2+\gamma}} \Phi^{-1} \left( \frac{l}{b+t} \right) \\ \geq 0 & \text{otherwise.} \end{cases}$$

and:

$$\frac{d\theta^*_{21,\rho}}{d\gamma} \begin{cases} > 0 & \text{if } \theta^*_{21,\rho} < \mu'_2(\rho, \theta_1) + \frac{1}{\sqrt{1-\rho^2+\gamma}} \Phi^{-1} \left( \frac{l}{b+t} \right) \\ \leq 0 & \text{otherwise.} \end{cases}$$

If $$\frac{l}{b+t} \geq \frac{1}{2},$$ then a prior belief that fundamentals are strong (i.e. $$\theta^*_{21,\rho} < \mu'_2(\rho, \theta_1)$$ for $$\rho = 0$$ and $$\rho = \rho_H$$) implies that $$\frac{d\theta^*_{21,\rho}}{d\alpha} < 0$$ and $$\frac{d\theta^*_{21,\rho}}{d\gamma} > 0.$$ And if $$\frac{l}{b+t} < \frac{1}{2},$$ then a prior belief that fundamentals are weak (i.e. $$\theta^*_{21,\rho} > \mu'_2(\rho, \theta_1)$$ for $$\rho = 0$$ and $$\rho = \rho_H$$) implies that $$\frac{d\theta^*_{21,\rho}}{d\alpha} > 0$$ and $$\frac{d\theta^*_{21,\rho}}{d\gamma} < 0.$$

Moreover, if $$\frac{l}{b+t} < \frac{1}{2},$$ then the prior belief that fundamentals are strong does not necessarily imply that $$\frac{d\theta^*_{21,\rho}}{d\alpha} < 0$$ and $$\frac{d\theta^*_{21,\rho}}{d\gamma} > 0.$$ This is only true if $$\mu'_2(\rho, \theta_1)$$ is sufficiently high. The critical values of $$\mu'_2(\rho, \theta_1)$$ can be derived from the equilibrium condition after plugging in the above inequalities. For instance we find that if $$\frac{l}{b+t} < \frac{1}{2},$$ then $$\frac{d\theta^*_{21,\rho}}{d\alpha} < 0$$ when $$\mu'_2(\rho, \theta_1) \geq [\rho \theta_1 + (1-\rho)\bar{\mu}]$$ where:

$$\bar{\mu} \equiv \Phi \left( \left( \frac{\alpha}{\sqrt{\gamma} 2\sqrt{\frac{\alpha}{1-\rho^2} + \gamma}} - \frac{\sqrt{\frac{\alpha}{1-\rho^2} + \gamma}}{\sqrt{\gamma}} \right) \Phi^{-1} \left( \frac{l}{l+b} \right) \right) - \frac{1}{2\sqrt{\frac{\alpha}{1-\rho^2} + \gamma}} \Phi^{-1} \left( \frac{l}{l+b} \right). \quad (44)$$

Similarly, if $$\frac{l}{b+t} \geq \frac{1}{2},$$ then the prior belief that fundamentals are weak does not necessarily imply that $$\frac{d\theta^*_{21,\rho}}{d\alpha} > 0$$ and $$\frac{d\theta^*_{21,\rho}}{d\gamma} < 0.$$ For instance we only have that $$\frac{d\theta^*_{21,\rho}}{d\alpha} > 0,$$ if $$\mu'_2(\rho, \theta_1)$$ is sufficiently low, i.e. if $$\mu'_2(\rho, \theta_1) \leq [\rho \theta_1 + (1-\rho)\bar{\mu}].$$
A.3 The special case $n = 1$

In this paragraph we provide details for the equilibrium analysis in section 3.2.1. The first equilibrium condition is identical to equation (38) with the only difference that $\theta^*_1$ needs to be substituted by $\theta^*_2$. Instead the second equilibrium condition, which is an indifference condition, can be computed as:

$$b \Pr\{\theta_2 \leq \theta^*_2 | x^*_2\} - l \Pr\{\theta_2 > \theta^*_2 | x^*_2\} = 0$$  (45)

where:

$$\Pr\{\theta_2 \leq \theta^*_2 | x^*_2\} = \Phi\left(\sqrt{\frac{\alpha}{1-\rho^2}} + \gamma (\theta^*_2 - \frac{\alpha}{1-\rho^2} + \gamma [\rho \theta_1 + (1-\rho)\mu] - \frac{\gamma}{1-\rho^2} + \gamma x^*_2)\right)$$

Equation (10) is constructed by combining both equilibrium conditions. Using the same argument as before, it can be shown that equation (10) has a unique solution if the relative precision of the private signal is sufficiently strong, i.e. if $\frac{\alpha}{\sqrt{\gamma}} < \sqrt{2\pi}$.

A.4 Bayesian updating

In this section we analyse how the posterior probability of facing the state $\rho = 0$ varies with the private signal. Differentiating equation (16) with respect to $x_{i2}$ leads to:

$$\frac{d \Pr\{\rho = 0 | \theta_1, x_{i2}\}}{dx_{i2}} = p(1-p) \left( \left( \frac{1}{\alpha} + \frac{1}{\gamma} \right)^{-1} \left( \sqrt{\frac{1-\rho^2_H}{\alpha} + \frac{1}{\gamma}} \right)^{-1} \phi\left( \frac{x_{i2}-\mu}{\sqrt{\frac{1}{\alpha} + \frac{1}{\gamma}}} \right) \right) \left( \phi\left( \frac{x_{i2}-[\rho H \theta_1 + (1-\rho H)\mu]}{\sqrt{\frac{1-\rho^2_H}{\alpha} + \frac{1}{\gamma}}} \right) \right)$$

$$= \left[ p \left( \frac{1}{\alpha} + \frac{1}{\gamma} \right)^{-1} \phi\left( \frac{x_{i2}-\mu}{\sqrt{\frac{1}{\alpha} + \frac{1}{\gamma}}} \right) + (1-p) \left( \sqrt{\frac{1-\rho^2_H}{\alpha} + \frac{1}{\gamma}} \right)^{-1} \phi\left( \frac{x_{i2}-[\rho H \theta_1 + (1-\rho H)\mu]}{\sqrt{\frac{1-\rho^2_H}{\alpha} + \frac{1}{\gamma}}} \right) \right]^2$$
To determine the sign, we have to inspect the nominator of equation (46). After several manipulations, we find that the nominator is weakly positive if:

\[
\left(\frac{\alpha + \gamma(1 - \rho_H^2)}{\alpha + \gamma}\right)(\mu - x_{i2}) > \left[\rho_H \theta_1 + (1 - \rho_H)\mu\right] - x_{i2}
\]

and negative otherwise. Hence, we arrive at the result summarised in equation (20) and the discussion thereafter.

A.5 Derivations related to the equilibrium analysis for the special case \(n = 0\)

A.5.1 The equilibrium condition

In this paragraph we provide details for the equilibrium analysis in section 3.2.4. Again two equilibrium conditions have to be satisfied. First, the critical fraction of attacking speculators \(A(\theta^*_{2U})\) has to equal the critical fundamental threshold above which it pays to attack. This leads to the first equilibrium condition:

\[
x^*_{2U} = \theta^*_{2U} + \sqrt{\frac{1}{\gamma} \Phi^{-1}(\theta^*_{2U})}
\]

where the subscript \(U\) stands for uninformed. Second, a speculator with the threshold signal \(x^*_{2U}\) has to be indifferent whether to attack the currency or not given \(\theta^*_{2U}\):

\[
b \Pr\{\theta_2 \leq \theta^*_{2U}|\theta_1, x^*_{2U}\} - l \Pr\{\theta_2 > \theta^*_{2U}|\theta_1, x^*_{2U}\} = 0
\]

where:

\[
\Pr\{\theta_2 \leq \theta^*_{2U}|\theta_1, x^*_{2U}\} = \Pr\{s = NE|\theta_1, x^*_{2U}\}\Phi\left(\frac{\theta^*_{2U} - \frac{\alpha \mu + \gamma x^*_{2U}}{\alpha + \gamma}}{\sqrt{\frac{1}{\alpha + \gamma}}}ight)
\]

\[
+ \Pr\{s = E|\theta_1, x^*_{2U}\}\Phi\left(\frac{\theta^*_{2U} - \frac{\alpha \rho_H \theta_1 + (1 - \rho_H)\mu + \gamma x^*_{2U}}{\frac{\alpha}{1 - \rho_H} + \gamma}}{\sqrt{\frac{1}{\frac{\alpha}{1 - \rho_H} + \gamma}}}ight)
\]
The indifference condition shows to be a mixture between the indifference conditions for the two cases \( n = 1, \rho = 0 \) and \( n = 1, \rho = \rho_H \). A combination of equations (48) and (49) leads to the equilibrium condition stated in equation (21).

### A.5.2 Monotonicity

The conditional density function \( f(x|\theta) \) is normal with mean \( \theta \) and satisfies the monotone likelihood ratio property (MLRP). For all \( x_i > x_j \) and \( \theta' > \theta \), we have:

\[
\frac{f(x_i|\theta')}{f(x_i|\theta)} \geq \frac{f(x_j|\theta')}{f(x_j|\theta)}
\]

or:

\[
\frac{\phi\left(\sqrt{\gamma}(x_i - \theta')\right)}{\phi\left(\sqrt{\gamma}(x_i - \theta)\right)} \geq \frac{\phi\left(\sqrt{\gamma}(x_j - \theta')\right)}{\phi\left(\sqrt{\gamma}(x_j - \theta)\right)}.
\]

As a consequence, we can make use of the result in Proposition 1 of Milgrom [28] and conclude that \( \Pr\{\theta_2 \leq \theta^*_2 U|\theta_1, x_i, x_j\} \) is monotonically decreasing in \( x_i \).

Furthermore, notice that \( \frac{d\Pr\{\theta_2 \leq \theta^*_2 U|\theta_1, x_i, x_j\}}{d\theta_2} > 0 \) and from equation (48) we can derive that:

\[
0 \leq \frac{d\hat{\theta}_2(x^*_2)}{dx^*_2} \leq \frac{1}{1 + \sqrt{\frac{2\pi}{\gamma}}}.
\]  

### A.5.3 Proof of Proposition 3

The result in Proposition 3 can be proven by showing that \( \frac{dG(\theta_2 U, \theta_1)}{d\theta_2} < 0 \) for some sufficiently high value of \( \gamma \). We have that:

\[
\frac{dG(\theta_2 U, \theta_1)}{d\theta_2} = \Pr\{\rho = 0|\theta_1, x_2U(\theta_2)\} \frac{d\Phi_{I,\rho=0}}{d\theta_2} + (1 - \Pr\{\rho = 0|\theta_1, x_2U(\theta_2)\}) \frac{d\Phi_{II,\rho=\rho_H}}{d\theta_2} + \frac{d\Pr\{\rho = 0|\theta_1, x_2U(\theta_2)\}}{dx_2U} \frac{d\theta_2}{d\theta_2} [\Phi_{I,\rho=0} - \Phi_{II,\rho=0}]
\]  

The proof proceeds by inspecting the individual terms of equation (51). From our earlier analysis we know that \( \frac{d\Phi_{I,\rho=0}}{d\theta_2} < 0 \) if \( \frac{\rho}{\sqrt{\gamma}} < \sqrt{2\pi} \) and that \( \frac{d\Phi_{II,\rho=\rho_H}}{d\theta_2} < 0 \) if \( \frac{\rho_H}{\sqrt{\gamma}} < \sqrt{2\pi} \). Notice that \( \lim_{\gamma \to \infty} \frac{d\Phi_{I,\rho=0}}{d\theta_2} = \lim_{\gamma \to \infty} \frac{d\Phi_{II,\rho=\rho_H}}{d\theta_2} = -1 \).
The sign of the last summand in equation (51) is ambiguous. We have that \( \Phi_{I,\rho=0} - \Phi_{II,\rho=\rho_H} \leq 0 \) whenever \( \theta^*_{2I,0} \leq \theta^*_{2I,\rho_H} \) and \( \Phi_{I,\rho=0} - \Phi_{II,\rho=\rho_H} > 0 \) otherwise. Furthermore, it shows that \( \lim_{\gamma \to \infty} [\Phi_{I,\rho=0} - \Phi_{II,\rho=\rho_H}] = 0 \). The last term to consider is \( \Phi_{I,\rho=0} - \Phi_{II,\rho=\rho_H} = \rho_H \leq 0 \) whenever \( \theta^*_{2I,0} \leq \theta^*_{2I,\rho} \) and \( \Phi_{I,\rho=0} - \Phi_{II,\rho=\rho_H} > 0 \) otherwise. Furthermore, it shows that \( \lim_{\gamma \to \infty} [\Phi_{I,\rho=0} - \Phi_{II,\rho=\rho_H}] = 0 \). The last term to consider is \( dPr\{\rho=0|\theta_1, x_2\} dx_2 dx_2 U(\theta_2) d\theta_2 \).

Given the previous sufficient conditions on the relative precision of the private signal we have that:

\[
0 < dx_2 U d\theta_2 = 1 + \frac{1}{\sqrt{\gamma}} \phi(\Phi^{-1}(\theta_2)) < 1 + \frac{\sqrt{2\pi}}{\alpha}
\]

Finally, recall \( \frac{dPr\{\rho=0|\theta_1, x_2\}}{dx_2} \) from section 3.2.3. Taking the limit \( \gamma \to \infty \) shows that \( \frac{dPr\{\rho=0|\theta_1, x_2\}}{dx_2} \) is finite as long as \( \alpha \) is finite. Hence, we can conclude that:

\[
\lim_{\gamma \to \infty} \frac{dPr\{\rho=0|\theta_1, x_2\}}{dx_2} \frac{dx_2 U(\theta_2)}{d\theta_2} [\Phi_I - \Phi_{II}] = 0
\]

As a result, there must exist a finite level of precision \( \gamma \) such that \( \frac{dG(\theta_2, \theta_1)}{d\theta_2} < 0 \) for all \( \gamma > \gamma \), as long as \( \alpha \) takes on a finite value. This concludes the proof of Proposition 3.

### A.6 Proof of Proposition 4

The result in Proposition 4 is proven by analysing the equilibrium condition for \( n = 0 \). First, recall that \( \theta^*_{2U} \) solves equation (21). Both \( \Phi_{I,\rho=0}(\theta^*_{2U}) \) and \( \Phi_{I,\rho=\rho_H}(\theta^*_{2U}, \theta_1) \) are decreasing in \( \theta^*_{2U} \) if \( \frac{\alpha}{\sqrt{\gamma}} < \sqrt{2\pi} \). Second, consider the equilibrium condition for the polar case \( n = 1 \) and observe that it can only be true that \( \theta^*_{2I,0} > \theta^*_{2U} \) if \( \Phi_{I,\rho=0}(\theta^*_2) > \Phi_{I,\rho=\rho_H}(\theta^*_{2}, \theta_1) \). The condition in equation (23) follows immediately after few manipulations. Since we are interested in a condition such that \( \theta^*_{2I,0} > \theta^*_{2U} \), equation (23) has to be evaluated at \( \theta^*_{2I,0} \). This explains equation (24) and completes the proof.

### A.7 Derivations related to the equilibrium analysis for the general case \( 0 < n < 1 \)

The equilibrium conditions can again be derived in two steps. First, in equilibrium the fraction of attacking speculators \( A_2(\theta^*_{2,\rho}) \) has to be equal to the fundamental threshold \( \theta^*_{2,\rho}(n) \).
above which it pays to act. This leads to two equilibrium conditions:

\[ \theta_{2,0}^*(n) = n \Phi \left( \frac{x_{2I,0}^*(n) - \theta_{2,0}^*(n)}{\sqrt{\gamma}} \right) + (1 - n) \Phi \left( \frac{x_{2U}^*(n) - \theta_{2,0}^*(n)}{\sqrt{\gamma}} \right) \]  

(52)

\[ \theta_{2,H}^*(n) = n \Phi \left( \frac{x_{2I,H}^*(n) - \theta_{2,H}^*(n)}{\sqrt{\gamma}} \right) + (1 - n) \Phi \left( \frac{x_{2U}^*(n) - \theta_{2,H}^*(n)}{\sqrt{\gamma}} \right) \]  

(53)

Second, in equilibrium the speculator receiving a private signal equal to the equilibrium threshold has to be indifferent whether to attack or not. This has to hold for both, uninformed speculators and informed speculators who learn that \( \rho = 0 \) or \( \rho = \rho_H \). We arrive at three equilibrium conditions. One equilibrium condition for uninformed speculators:

\[ J(\theta_{2,0}^*, \theta_{2,H}^*, x_{2U}^*; n) \equiv \Pr\{\rho = 0|\theta_1, x_{2U}^*(n)\} \Phi_{J,0}(\theta_{2,0}^*(n), x_{2U}^*(n)) + (1 - \Pr\{\rho = 0|\theta_1, x_{2U}^*(n)\}) \Phi_{J,H}(\theta_{2,H}^*(n), x_{2U}^*(n)) = \frac{l}{b + l} \]  

(54)

where:

\[ \Phi_{J,0}(\theta_{2,0}^*(n), x_{2U}^*(n)) \equiv \Phi \left( \frac{\theta_{2,0}^*(n) - \frac{\alpha \mu + \gamma x_{2U}^*(n)}{\alpha + \gamma}}{\sqrt{\frac{1}{\alpha + \gamma}}} \right) \]  

(55)

\[ \Phi_{J,H}(\theta_{2,H}^*(n), x_{2U}^*(n)) \equiv \Phi \left( \frac{\theta_{2,H}^*(n) - \frac{\alpha \mu \theta_1 + (1 - \rho_H)\mu + \gamma x_{2U}^*(n)}{1 - \rho_H + \gamma}}{\sqrt{\frac{1}{1 - \rho_H + \gamma}}} \right) \]  

(56)

And two equilibrium conditions for informed speculators:

\[ \Phi \left( \frac{\theta_{2,0}^*(n) - \frac{\alpha \mu + \gamma x_{2I,0}^*(n)}{\alpha + \gamma}}{\sqrt{\frac{1}{\alpha + \gamma}}} \right) = \frac{l}{b + l} \]  

(57)
and:

\[
\Phi \left( \frac{\theta_{2,\rho H}^* (n) - \frac{\alpha}{1 - \rho_H} [\rho_H \theta_1 + (1 - \rho_H) \mu] + \gamma x_{2L,\rho H}^* (n)}{\sqrt{\frac{1}{1 - \rho_H} + \gamma}} \right) = \frac{l}{b + l}
\] (58)

We are left with five equations in five unknowns. First, we can use equation (52) to obtain \(x_{2U}^* (n)\) as a function of \(\theta_{2,0}^*(n)\), and \(x_{2L,0}^*(n)\). Second, we can use equation (57) to obtain \(x_{2L,0}^*(n)\) as a function of \(\theta_{2,0}^*(n)\).

Plugging the second function into the first function leads to:

\[
x_{2U}^*(\theta_{2,0}^*(n); n) = \theta_{2,0}^* + \sqrt{\frac{1}{\gamma}} \Phi^{-1} \left( \frac{\theta_{2,0}^* - n \Phi \left( \frac{\alpha (\theta_{2,0}^* - \mu) - \sqrt{\alpha + \gamma} \Phi^{-1} \left( \frac{l}{b + l} \right)}{\sqrt{\gamma}} \right)}{1 - n} \right)
\] (59)

Notice that:

\[
\frac{\partial x_{2U}^*(\theta_{2,0}^*(n); n)}{\partial \theta_{2,0}^*} = \frac{1}{\gamma} \Phi^{-1} \left( \frac{\theta_{2,0}^* - n \Phi \left( \frac{\alpha (\theta_{2,0}^* - \mu) - \sqrt{\alpha + \gamma} \Phi^{-1} \left( \frac{l}{b + l} \right)}{\sqrt{\gamma}} \right)}{1 - n} \right) > 0
\] (60)

when assuming that the sufficient condition for uniqueness from the polar case \(n = 1\) holds, i.e. \(\frac{\alpha}{\sqrt{\gamma}} < \sqrt{2\pi}\). In equilibrium there can only be one critical threshold for uninformed speculators, \(x_{2U}^*(\theta_{2,0}^*(n); n) = x_{2U}^*(n)\). Hence, equation (59) can in turn be plugged into \(\Phi_{I,0}(\theta_{2,0}^*(n), x_{2U}^*(n))\), which gives us \(\Phi_{I,0}\) as a function of \(\theta_{2,0}^*(n)\) only.

Similarly, we can use equation (53) to obtain \(x_{2L}^* (n)\) as a function of \(\theta_{2,\rho H}^* (n)\) and \(x_{2L,\rho H}^* (n)\). Then we can use equation (58) to obtain \(x_{2L,\rho H}^* (n)\) as a function of \(\theta_{2,\rho H}^* (n)\).
Again plugging the second into the first function leads to:

\[ x_{2U}(\theta^{*}_{2,\rho_H}; n) = \theta^{*}_{2,\rho_H} \]

\[ + \sqrt{\frac{1}{\gamma}} \Phi^{-1}(\frac{\theta^{*}_{2,\rho_H} - n\Phi(\frac{\alpha}{1-\rho_{H}^{2}}(\theta^{*}_{2,\rho_H} - \rho_{H}\theta_{1} + (1-\rho_{H})\mu) - \sqrt{\frac{1}{1-\rho_{H}^{2}} - 1\gamma \Phi^{-1}(\frac{1}{\sqrt{\pi}})})}{1-n}) \]

(61)

Analog to before we have that:

\[ \frac{\partial x_{2U}(\theta^{*}_{2,\rho_H}; n)}{\partial \theta^{*}_{2,\rho_H}} = 1 + \sqrt{\frac{1}{\gamma}} \frac{1-n\Phi(\frac{\alpha}{1-\rho_{H}^{2}}(\theta^{*}_{2,\rho_H} - \rho_{H}\theta_{1} + (1-\rho_{H})\mu) - \sqrt{\frac{1}{1-\rho_{H}^{2}} - 1\gamma \Phi^{-1}(\frac{1}{\sqrt{\pi}})})}{\Phi^{-1}(\frac{\theta^{*}_{2,\rho_H} - n\Phi(\frac{\alpha}{1-\rho_{H}^{2}}(\theta^{*}_{2,\rho_H} - \rho_{H}\theta_{1} + (1-\rho_{H})\mu) - \sqrt{\frac{1}{1-\rho_{H}^{2}} - 1\gamma \Phi^{-1}(\frac{1}{\sqrt{\pi}})})}{1-n})} > 0 \]

given the sufficient condition for uniqueness from the polar case \( n = 1 \) holds, i.e. \( \frac{1-\rho_{H}^{2}}{\sqrt{\gamma}} = \sqrt{2\pi} \). Again we can use the argument that in equilibrium there can only be one critical threshold for uninformed speculators, \( x_{2U}(\theta^{*}_{2,\rho_H}; n) = x_{2U}(n) \). Hence, plugging equation (61) into \( \Phi_{J,\rho_{H}}(\theta^{*}_{2,\rho_H}(n), x_{2U}(n)) \) gives us \( \Phi_{J,\rho_{H}} \) as a function of \( \theta^{*}_{2,\rho_H}(n) \) only.

Equalising equations (59) and (61) gives an implicit relation between \( \theta^{*}_{2,0}(n) \) and \( \theta^{*}_{2,\rho_{H}}(n) \):

\[ M^{1}(\theta^{*}_{2,0}, \theta^{*}_{2,\rho_{H}}; n) \equiv x_{2U}(\theta^{*}_{2,0}; n) - x_{2U}(\theta^{*}_{2,\rho_{H}}; n) = 0 \]

(62)

Finally, consider \( J(\theta^{*}_{2,0}(n), \theta^{*}_{2,\rho_{H}}(n), x_{2U}(n)) \) and plug in for \( x_{2U}(n) \) from equation (59). Let us define:

\[ M^{2}(\theta^{*}_{2,\rho_{0}=0}, \theta^{*}_{2,\rho_{H}}; n) \equiv \frac{J(\theta^{*}_{2,0}, \theta^{*}_{2,\rho_{H}}; n) - \frac{l}{b + l}}{b} = 0 \]

(63)

where \( n \) is taken as given.
We can derive:

\[
\frac{\partial M^1(\theta^*_2, \theta^*_2, \rho_H; n)}{\partial n} = \begin{cases} < 0 & \text{if } \theta^*_2 > \theta^*_{2l, \rho_H} \\ \geq 0 & \text{if } \theta^*_2 \leq \theta^*_{2l, \rho_H} \\ \leq 0 & \text{otherwise.} \end{cases}
\]

(64)

\[
\frac{\partial M^2(\theta^*_2, \theta^*_2, \rho_H; n)}{\partial \theta^*_2} = \frac{d \Pr\{\rho = 0|\theta_1, x^*_{2l}(\theta^*_2, \rho_H; n)\}}{d \theta^*_2} \left[ \Phi^{-1}(\theta^*_2 - \sqrt{\gamma}) - \Phi^{-1}(\frac{\theta^*_2}{\sqrt{\gamma}}) \right] + \frac{d \Phi^{-1}(\theta^*_2; n)}{d \theta^*_2} (1 - \Pr\{\rho = 0|\theta_1, x^*_{2l}(\theta^*_2, \rho_H; n)\})
\]

(65)

\[
\frac{\partial M^2(\theta^*_2, \theta^*_2, \rho_H; n)}{\partial \theta^*_2, \rho_H} = (1 - \Pr\{\rho = 0|\theta_1, x^*_{2l}(\theta^*_2, \rho_H; n)\}) \frac{d \Phi_{1,\rho_H}(\theta^*_2; n)}{d\theta^*_2, \rho_H}
\]

(66)

A.8 Proof of Proposition 6

The proof of Proposition 6 is similar to the proof of Proposition 3. We consider equations (66) and (65) in turn.

First, observe that \(\frac{dM^2(\theta^*_2, \theta^*_2, \rho_H; n)}{d\theta^*_2, \rho_H} < 0\) is satisfied if:

\[
\frac{\alpha}{1-\rho^2_H} < \frac{1-n\Phi^{-1}\left(\sqrt{\frac{\alpha^2}{1-\rho^2_H} - \frac{\sqrt{\frac{\alpha^2}{1-\rho^2_H} - 1}}{\sqrt{\gamma}}\frac{\gamma^{-1}(\frac{\alpha^2}{1-\rho^2_H})}{\gamma^{-1}(\frac{\alpha^2}{1-\rho^2_H})}\right)}{\phi\left(\Phi^{-1}\left(\frac{\theta^*_2 - \sqrt{\gamma}}{1-n}\sqrt{\frac{\alpha^2}{1-\rho^2_H} - \frac{\sqrt{\frac{\alpha^2}{1-\rho^2_H} - 1}}{\sqrt{\gamma}}\frac{\gamma^{-1}(\frac{\alpha^2}{1-\rho^2_H})}{\gamma^{-1}(\frac{\alpha^2}{1-\rho^2_H})}\right)}\right)}
\]

Notice that the standard normal pdf cannot take values above \(\frac{1}{\sqrt{2\pi}}\). As a result the above equation holds given the sufficient condition used for the polar case \(n = 1\), i.e. \(\frac{\alpha}{\sqrt{\gamma}} < \sqrt{2\pi}\).

Second, observe that \(\frac{dM^2(\theta^*_2, \theta^*_2, \rho_H; n)}{d\theta^*_2, \rho_H} < 0\) is satisfied for a sufficiently high but finite \(\gamma\) (given a finite \(\alpha\)). This can be seen by applying the same argument as in the proof of Proposition 3.

Finally, recall that \(\frac{d\theta^*_2}{d\theta^*_2, \rho_H} > 0\) for a given \(n\). This connects the two results above and lets us conclude that the left-hand side of \(M^2(\theta^*_2, \theta^*_2, \rho_H; n)\) is strictly decreasing in \(\theta^*_2\) whenever \(\gamma\) is sufficiently high (given a finite \(\alpha\)). As the right-hand side is constant, this concludes our
proof that there must exist a $\gamma$ such there does exist a unique monotone equilibrium for all $\gamma > \gamma$ (given a finite $\alpha$).

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