DRAFT: Retirement policy in a frictional labor market with health heterogeneity

PRELIMINARY AND INCOMPLETE

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Abstract

I study the effect of labor market frictions on pension reform outcomes using a search and matching model with an overlapping generations structure, endogenous retirement decisions and exogenous health shocks. I consider both random and age-directed search to analyze how pension reform outcomes are affected by labor market structure. By conducting an experiment where official retirement age is increased from 62 to 67 years, I analyze the mechanisms through which retirement policy affects labor supply decisions, and how these decisions interact with firms' vacancy creation. I find that retirement policy affects not only timing of retirement but also job starts and quit decisions close to retirement. Worker decisions, in turn, affect vacancy creation through changes in the number of unemployed, composition of unemployment pool and value of hiring a worker. Quantitatively, the interaction between labor supply decisions and firms' vacancy creation is shown to have an effect on timing of retirement and old-age employment when search is age-directed, while with random search the effect is negligible.

1 Introduction

Western economies' aging populations and concerns over the sufficiency of labor supply have recently raised a lot of discussion on the timing of retirement and potential policy measures for delaying it. While existing research on retirement decisions and policies is extensive, the interaction of labor market frictions and retirement has not received much attention. Instead, retirement (policy) analysis has usually been carried out in the context of perfect labor markets. Labor market frictions and conditions might, however, have interesting implications for retirement policy analysis as shown by, for example,

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Hairault et al.[11]. The aim of this paper is, thus, to study the effect of labor market frictions on pension reform outcomes under health heterogeneity and different labor market structures.

Frictional labor market setting enables one to investigate mechanisms and questions that are absent from traditional retirement analysis such as "Does retirement policy affect unemployed and employed workers differently?" and "How do changes in workers' labor supply decisions, induced by retirement policy, interact with firms' vacancy creation decisions?". Furthermore, in a frictional labor market environment, one can address such common public concerns as "retirement age increases are not that effective, because old people have difficulties finding jobs". Frictional labor market setting, thus, provides new views to retirement analysis.

I analyze the mechanisms through which pension reforms affects unemployed and employed workers' labor supply decisions and how these decisions interact with firms' vacancy creation. Since workers differ in terms of employability, I also consider how worker heterogeneity affects these mechanisms. As a source of heterogeneity I focus on health, because it is closely linked to age and it affects labor market outcomes. Health deteriorates with age and affects workers' employability, for example, via expected work horizon through disability risk.

Shorter remaining work horizon makes hiring and training older workers (other things equal) less valuable to firms. Poor health may further reduce attractiveness of hiring older workers by decreasing expected work horizon via higher disability risk. The age and health distribution of the unemployment pool affects the expected, average hiring value and, thus, firms' incentives to create vacancies. Composition of the relevant unemployment pool and average expected hiring value depends on labor market structure - whether there is a single labor market or several age-specific labor markets. Both assumptions can be justified. On one hand, labor market discrimination by age is illegal in many countries. However, on the other hand, as brought up by Menzio et al. [14], in reality firms may be able to age-discriminate job applicants by arguing that a job applicant is rejected based on quality although the true reason is age.

To study the effect of labor market frictions and health on retirement policy reform outcomes, I build a search and matching model with endogenous retirement decisions, and overlapping generations structure where workers live for T periods. Health evolves exogenously according to an age-dependent $1st$-order Markov chain and affects labor market outcomes through disability risk. I present two versions of the model with differing labor market structures: one with random search another with age-directed search. To illustrate the mechanisms in the model and to get quantitative results, I calibrate the model to US data on labor markets and health. I analyze the effect of frictions on retirement policy implications by conducting a policy experiment where official retirement age is raised from 65 to 67 years while the early retirement age is kept intact at

62 years.

I show that the policy change increases directly the optimal retirement age for unemployed by reducing the attractiveness of retirement relative to unemployment. As for employed workers, policy change decreases the productivity threshold for which continuing to work instead of retiring is optimal and, thus, increases the expected retirement age for employed. The increase in expected retirement age for employed has a positive effect also on the retirement age for unemployed. This is because it increases the value of employment and, therefore, the return to job search investments. Moreover, I show that worker decisions affect vacancy creation through changes in labor market tightness, composition of unemployment pool and value of hiring a worker of given type.

In the quantitative experiment, I find that health has only a minor effect on workers' job start, quit and retirement decisions. Moreover, health makes a difference only if a worker is in fair health. Quantitatively, the effect of the pension reform on effective retirement age in the random search model and age-directed model are very close to each other. There is, however, differences in the mechanism through which the change takes place. When search is age-directed, the interaction between labor supply decisions and firms' vacancy creation is shown to have an effect on timing of retirement and old-age employment, while with random search the effect is negligible. This is because, with a single labor market, older workers affected by the policy change represent only a small share of the entire labor market whereas with age-specific labor markets, at older ages, everyone in the labor market is affected by the policy change. Lastly, I find that in the age-directed model, the policy experiment leads to an increase of 0.4 percentage points in the unemployment rate for 60-64 olds whereas in the random search model, the unemployment rate for 60-64 olds decreases by 0.2 percentage points. This implies that public concerns about retirement age increases being ineffective due to old people having difficulties finding jobs might be justified if labor markets are perfectly age-segmented, whereas with random search these concerns seem unwarranted.

Rest of the paper is structured as follows. In Section 2, I discuss earlier research related to this paper and in Section 3, I lay out the theoretical model. In Section 5, I present a calibrated version of the model together with quantitative results for the policy experiment. In Section 6, I go through an extension of the model with age-directed search. I conclude in Section 7.

2 Literature

This paper connects to three strands of research. First, it links to literature on life-cycle search and matching models that aim to explain all or a subset of labor market patterns over the life-cycle which include, for example, age-declining job finding [8, 14, 2] and separation probabilities [8] and hump-shaped employment rate [10, 2].[1] People use different approaches in explaining these labor market patterns, but all models share one key mechanism. This is the so-called horizon effect [3] which arises from the existence of an exogenous retirement age and refers to the impact of remaining work career on a value of a job and, thus, labor market outcomes.

Majority of the life-cycle search and matching papers - Hahn [10], Cheron et al.[4], Esteban-Pretel and Fujimoto [8] - do this using a Mortensen-Pissarides type search and matching labor market framework. Menzio et al. [14] present, however, an exception by assuming age-directed search. Hahn [10] explains the hump-shaped employment rate using a continuous time model with exogenous separations and trainings costs. Cheron et al.[4] introduce a more complex model with endogenous job destruction, persistent idiosyncratic shocks and endogenous search effort. Their model is able to qualitatively explain either the decreasing employment rate at older ages and age-declining job finding provability or the declining job separation rate, but not all three phenomena simultaneously. Esteban-Pretel and Fujimoto [8] and Menzio et al. [14] are, on the other hand, able to quantitatively replicate the hump-shaped employment rate as well as age-declining job finding and separation probabilities. Esteban-Pretel and Fujimoto's [8] model features skill and age heterogeneity as well as uncertainty about match quality, which is central for the model's ability to reproduce age-declining unemployment and job separation rates. Menzio et al. [14], in turn, have on-the-job search, human capital formation and a learning friction regarding productivity of the match. All the above listed papers, nonetheless, abstract from the participation decision.

Secondly, this paper relates to research on labor market flows in three-state labor market models, such as the work by Garibaldi and Wasmer [9] and Krusell at al. [13]. Garibaldi and Wasmer [9] use a continuous time search and matching model to study labor market flows including the participation margin. The key element of their model is valuation of non-market activities which is heterogeneous across workers and subject to idiosyncratic shocks. Krusell at al. [13], on the other hand, use a growth model with indivisible labor and an "island" structure to capture search frictions. Both papers, however, abstract from life-cycle considerations.

Thirdly, this paper builds on Hairault et al.'s [11] work on the interaction of labor market frictions and retirement decisions. Using a a continuous time, Mortensen-Pissarides type search and matching model with perfectly age-segmented search and exogenous separations, Hairault et al. [11] show that labor market frictions lead unemployed workers to retire earlier labor market tightness affects the retirement age for unemployed, but not for employed workers. Hairault et al.'s [11] paper is, however, analytical without any quantitative results.

[1] Contrary to the others, Chéron et al. [2], argue for a U-shaped separation probability, claiming that the difference arises from a different definition of the relevant pool of unemployed workers (see for [4])

3 Model

The model is a discrete time, Mortensen-Pissarides type search and matching model with an overlapping-generations structure where agents live for T periods. In the benchmark model, job search is assumed to be random.

The economy is inhabited by two types of agents: firms and workers. Firms are risk-neutral, small and identical to each other. Firms post vacancies and employ either 0 or 1 workers. There is no capital in the model, and firms produce using only labor as an input.

Workers are risk-neutral and heterogeneous with respect to age and health. Each worker lives for T periods. There is a continuum of workers of each age with the mass of workers in each age group normalized to 1. When a cohort dies and exits the model, it is replaced by a new cohort of the same size. Worker health is modeled with a discrete variable which can take three values $h \in \{excellent = 1, good = 2, fair = 3\}$. Health evolves according to an age-dependent first-order Markov chain $\pi_j(h', h)$. Health affects the probability, $P_j(h)$, with which workers experience permanent disability and, therefore, the expected work horizon.

Each worker has one indivisible unit of labor, and the key decision worker makes is regarding his labor market status. Workers can be in one of four labor market states: unemployed and searching for a job, employed, disabled or retired. Employed workers receive a wage w, and experience a disutility of working, d_j. Unemployed receive an unemployment benefit b_u and suffer a disutility of job search, c_j. Disutility of working and searching are both assumed increasing in age: the older one gets, the more straining working (searching) becomes. Retired and disabled workers both receive a benefit transfer b^r and enjoy a leisure value of retirement ν.

Unemployed workers are matched with open vacancies with probability p (which is assumed exogenous for now). Worker-firm matches differ with respect to match-specific productivity ϵ. For newly formed matches, match-specific productivity is drawn from a distribution $F(\epsilon)$. Over time, match-specific productivity evolves according to the following AR(1) process in logs: $log\,\epsilon_{j+1} = \rho log\,\epsilon_j + \xi_{j+1}, \quad \xi_{j+1} \sim N(0, \sigma_\xi^2)$. Low match specific productivity may lead to endogenous job destruction if the productivity value is too low to make the match profitable. Jobs are also destroyed exogenously due to two reasons: exogenous job destruction shocks that occur with probability δ and disability incidence.

Retirement is endogenous, but constrained by a pension system. The earliest a worker can retire is at the early retirement age T^e. Worker is, however, entitled to the full retirement benefit only if he retires at, or after the official retirement age T^{off}. Retirement and disability are both irreversible and together constitute the pool of inactive people.

Wages are set via Nash bargaining. Accordingly, wage equals a fixed proportion of the match surplus, worker's and firm's combined gain from the match versus the outside option. Wages are renegotiated every period.

Let $W_j(h, \epsilon, n)$ be the value of employment for a worker of age j, health h, match-specific productivity ϵ and type of employment relationship n. n, can take a value of 0 or 1, where $n = 0$ denotes an on-going match and $n = 1$ a newly created job. Similarly, let $U_j(h)$ be the value of unemployment, and R_j and R_j^D the values of retirement and disability. The we can define $N_j(h, \epsilon, n)$ and $O_j(h)$ as follows:

$$N_j(h, \epsilon, n) = \max\{W_j(h, \epsilon, n), O_j(h)\} \qquad O_j(h) = max\left\{U_j(h), R_j\right\} \tag{1}$$

Then the value of employment is then

$$\begin{aligned} W_j(h, \epsilon, n) =& w_j(h, \epsilon, n) - d_j + \beta(1 - P_j(h))E_{\epsilon', h'}\Big[(1-\delta)N_{j+1}(h', \epsilon', 0) \\ &+ \delta\, O_{j+1}(h') \mid h, j\Big] + P_j(h)\beta R_{j+1}^D \end{aligned} \tag{2}$$

and the value of unemployment

$$\begin{aligned} U_j(h) =& b^u - c_j + \beta(1 - P_j(h))E_{h'}\Big[p(1-\delta)\int N_{j+1}(h', \epsilon', 1)\, dF(\epsilon') \\ &+ (1 - p(1-\delta))\, O_{j+1}(h') \mid h, j\Big] + P_j(h)\beta R_{j+1}^D \end{aligned} \tag{3}$$

Since retirement and disability are absorbing states, the value of retirement R_j and disability D_j are defined in a different manner than the values for employment and unemployment. Value of retirement R_j is the value of retirement for a person retiring at the age of j, and D_j the value of disability that incurs disability at age j.

$$R_j = \begin{cases} (\alpha_j b^r + \nu)\frac{1-\beta^{T-j}}{1-\beta} & if \quad j \geq T^e \\ 0 & otherwise \end{cases} \tag{4}$$

where $0 < \alpha < 1$ for $T^{early} \leq j < T^{off}$ and $\alpha = 1$ for $j \geq T^{off}$.

$$R_j^D = \begin{cases} (\alpha_{T^e} b^r + \nu)\frac{1-\beta^{T-j}}{1-\beta} & if \quad j < T^e \\ R_j & otherwise \end{cases} \tag{5}$$

where α_j depicts the fact that workers retiring early receive reduced benefits. Starting from early retirement age, value of disability is equal to that of retirement to ensure that value of disability doesn't implicitly affect the retirement decision.

To determine wages, I introduce firms. Let V be the value of an open vacancy (a parameter for now), and $J_j(h,\epsilon,n)$ the value of a filled job. Then $J_j(h,\epsilon,n)$, value of a vacancy filled with a worker of age j, health h, match-specific productivity ϵ and type of employment relationship n is given by

$$\begin{aligned} J_j(h,\epsilon,n) =& a_j\epsilon - w_j(h,\epsilon,n) - n\kappa \\ &+ \beta(1-P_j(h))E_{\epsilon',h'}\Big[(1-\delta)max\{J_{j+1}(h',\epsilon',0),V\} + \delta V \mid h,j\Big] + P_j(h)\beta V \end{aligned} \tag{6}$$

where a_j is the average productivity in age-group j, which captures the life-cycle profile of productivity (since the model does not explicitly take into account human capital formation), and the term $n\kappa$ represents training costs. Training costs are equal to κ for a new employee and 0 for a worker in an on-going employment relationship.

Wage is the solution to a Nash bargaining problem over the match surplus, $S_j(h,\epsilon,n) = J_j(h,\epsilon,n) - V + W_j(h,\epsilon,n) - O_j(h)$, and characterized by sharing rule

$$(1-\gamma)[W_j(h,\epsilon,n) - O_j(h)] = \gamma[J_j(h,\epsilon,n) - V] \tag{7}$$

where γ denotes bargaining power of the worker. The wage schedule takes two forms - both for on-going and new matches - depending on whether retirement or unemployment is the relevant outside option. Match surplus, and thus also wage, is lower in the case of a newly hired worker. This is due to the training costs: under Nash bargaining, worker pays a fixed share of the trainings costs in the form of a lower wage.

At the beginning of each period, exogenous disability transitions take place and matches from the previous period are realized. After this, the value of health and productivity shocks are revealed. Upon observing the health status and productivity level and (possibly) learning about their match quality, firms and workers decide whether to continue the relationship or to separate. If the worker and firm separate, the worker can either retire or become unemployed and start searching for a new job. If the match is continued, the worker and firm (re)negotiate a wage and start producing. Unemployed workers, on the other hand, decide whether to continue searching for a job or to retire. Unemployed workers search for jobs and firms with vacancies look for workers simultaneously with production taking place.

3.1 Worker decision rules

Workers' decision rules consist of the optimal retirement age from unemployment and the decision rules for when to start and end an employment relationship. When retirement is the relevant outside option, the decision rule for ending an employment relationships characterizes the retirement decision of an employed worker.

An unemployed worker retires when the value of retirement is equal to, or exceeds the value of unemployment, $U_j(h) \leq R_j$. Optimal retirement age, $T^u(h)$, depends on worker health. This is because the health-dependent disability probability gives rise to a so-called "horizon effect of health". High-disability risk shortens expected work horizon thus reducing the gains from job search and promoting earlier retirement. Therefore, if disability probability increases with bad health, a worker in poor health never retires later than a worker in good health.

When an unemployed worker is matched with an open vacancy, the match does not necessarily become productive. Instead, upon observing the match-specific productivity and his own health, the worker decides whether to start working. Similarly, a worker in a on-going employment relationship chooses whether to continue or to quit upon observing his new health and match-productivity. Job start and the quit decisions are both characterized by age- and health-dependent threshold values for the match-specific productivity ϵ. Thus, given age and health, there exists a value of match-specific productivity ϵ below which an unemployed worker never chooses to start working. Likewise, there is a value of ϵ below which a worker in an on-going match always chooses to quit. Due to Nash bargaining, these threshold values coincide with the values of ϵ at which the firm wants to start (end) an employment relationship.

The threshold value for the job start decision is given by $\epsilon_j^0(h, 1)$ that satisfies the equation $W_j(h, \epsilon, 1) = U_j(h)$, whereas the cut-off point for quitting is given by $\epsilon_j^0(h, 0)$ satisfying the equation $W_j(h, \epsilon, 0) = O_j(h)$. There are two versions of the quit threshold depending on whether the relevant threat point is unemployment or retirement. However, there is only one version of the start margin since there is no labor market flows from retirement back to the labor force in the model. When retirement is the relevant outside option, the quit margin gives the threshold productivity for which an employed worker of age j and health h chooses to quit his job and retire. The quit margin, thus, characterizes the retirement decision of an employed worker. The quit decision can also be used to find the optimal retirement age for an employed worker in health h and with match-specific productivity ϵ; optimal retirement age is the first age at which $W_j(h, \epsilon, 1) \leq U_j(h) \leq R_j$ for given h and ϵ. Because value of employment $W_j(h, \epsilon, n)$ is increasing in ϵ,the optimal retirement age from employment is also increasing in match-specific productivity. The underlying logic is that when the match is very productive, opportunity cost of retirement is high. We can also deduce that an unemployed worker in health h never retires later than an employed worker in same health. This is because an employed worker always has the option of quitting and becoming unemployed. Thus, if an employed worker chooses to retire, it must not be optimal for an unemployed worker to continue searching for a job after this age.

Using the wage equation derived in Appendix A, the job start and quit margins can be expressed as follows:

(1) Outside option: unemployment

$$\begin{aligned}\epsilon_j^0(h,n) = & \frac{\gamma}{a_j}\underbrace{(b^u + d_j + n\kappa - c_j)}_{\text{current gain from outside option}} - \frac{\gamma}{a_j}\underbrace{(V-\beta V)}_{\text{option value of vacancy}} \\ & + \frac{\gamma}{a_j}\underbrace{\beta(1-P_j(h))(1-\delta)pE_{h'}\left[\int N_{j+1}(h',\epsilon',1)dF(\epsilon') - O_{j+1}(h') \mid h,j\right]}_{\text{expected gain from job search}} \\ & - \frac{1}{a_j}\underbrace{\beta(1-P_j(h))(1-\delta)E_{\epsilon',h'}\left[N_{j+1}(h',\epsilon',0) - O_{j+1}(h') \mid h,j\right]}_{\text{expected gain from continuing in existing job}}\end{aligned} \tag{8}$$

(2) Outside option: retirement

$$\begin{aligned}\epsilon_j^0(h,0) = & \frac{\gamma}{a_j}\underbrace{(\alpha_j b^r + d_j + \nu)}_{\text{current gain from outside option}} - \frac{\gamma}{a_j}\underbrace{(V-\beta V)}_{\text{option value of vacancy}} \\ & - \frac{1}{a_j}\underbrace{\beta(1-P_j(h))(1-\delta)E_{\epsilon',h'}\left[N_{j+1}(h',\epsilon',0) - R_{j+1} \mid h,j\right]}_{\text{expected gain from continuing in existing job}} \\ & + \frac{\gamma}{a_j}\underbrace{\beta\frac{1-\beta^{T-j-1}}{1-\beta}b^r(\alpha_j - \alpha_{j+1})}_{\text{gain/loss from postponing retirement by one period}}\end{aligned} \tag{9}$$

The job start margin is always higher than the quit margin due to training costs. Both margins are increasing in the attractiveness of the outside option: the higher are b^r, b^u and ν and the lower is the disutility of job search, c_j, the higher are the job start and quit margins. Furthermore, a high expected gain from unemployment search contributes to a higher quit (start) margin. This is because when there is easy access to profitable job opportunities through unemployment search, a high match-specific productivity is needed for workers to continue in (accept) a job. High age-specific average productivity a_j and high continuation value of existing job, on the other hand, lower the quit (start) margin.

Shape of the job start (quit) margin over the life-cycle is indeterminate prior to the parametrization of the model. The shape is sensitive to the age-profile of average productivity a_j as well as the life-cycle behavior of disutility of job search c_j and disutility of searching d_j, disability probability $P_j(h)$ and health transition matrices. Furthermore, parameters that affect the attractiveness of continuing in the existing job versus unemployment search - probability of finding a match p, distribution $F(\epsilon)$ from which match-specific productivity of a new job is drawn, and process governing the evolution of match-specific productivity - have a key role in shaping the life-cycle profile. Figure 1 illustrates the decision rules for job start and quitting over the life-cycle and the optimal retirement age from unemployment for a worker in excellent health for one parametrization, discussed in more detail in Section 5.2.

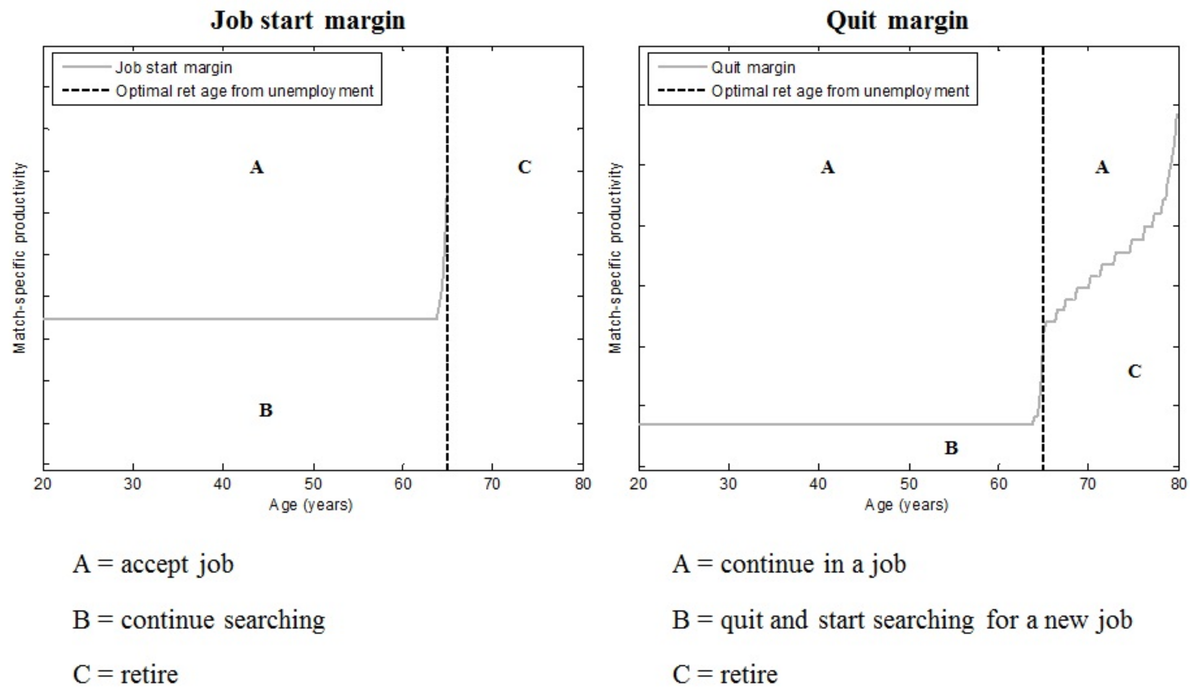


Figure 1: Job start and quit margin and retirement age from unemployment - excellent health

Similarly to the life-cycle profile of the start (quit) margin, the behavior of the job start (quit) margin as a function of health is also indeterminate. In this case too, parameters that affect the attractiveness of job search versus continuation in current job are important along the disability probabilities $P_j(h)$ and health transition matrices.

3.2 Labor demand

To endogenize p and V, I introduce a matching process and make the assumption of free entry of firms. In line with the literature, number of new matches at a given point in time is determined by a constant returns to scale matching function $M(v,u)$, where v is the number of vacancies and u the number of unemployed. Denoting labor market tightness by $\theta = \frac{v}{u}$, the probability that a firm with a vacancy finds a worker is $\frac{M(v,u)}{v} = M(1,\theta^{-1}) = q(\theta)$ whereas the probability that an unemployed worker is matched with a vacancy is $p(\theta) = \frac{M(v,u)}{u} = \theta q(\theta)$.

Let k denote firm's flow cost of maintaining a vacancy and recall that $J_j(h, \epsilon, n)$ is the value of a filled vacancy. Then the value of a vacancy to a firm is

$$V = -k + \beta q(\theta) \sum_{j=1}^{T-1} \sum_{m=1}^{H} \Bigg((1 - P_j(h^m)) \frac{u_j(h^m)}{u} E_{h'} \Bigg[(1-\delta) \int max\{J_{j+1}(h', \epsilon', 1), V\} \, dF(\epsilon') \\ + \delta V \mid h, j \Bigg] + \frac{u_j(h^m)}{u} P_j(h^m) V \Bigg) + \beta(1 - q(\theta))V \tag{10}$$

Under free entry of firms, the value of a vacancy in equilibrium is zero, $V = 0$. Using this, the job creation condition, which pins down labor demand in equilibrium, becomes

$$\frac{k}{q(\theta)} = \beta \sum_{j=1}^{T-1} \sum_{m=1}^{H} (1 - P_j(h^m)) \frac{u_j(h^m)}{u} E_{h'} \left[(1-\delta) \int max\{J_{j+1}(h', \epsilon', 1), 0\} \, dF(\epsilon') \mid h, j \right] \tag{11}$$

The job creation condition (11) states that in equilibrium the expected cost of hiring a worker equals the expected discounted gain from a filled vacancy. The expected gain from a filled vacancy depends on the composition of workers in the unemployment pool and the expected gain from hiring a worker of given age and health. Since $q(\theta)$ is decreasing in θ, a reduction in the expected gain from a filled vacancy reduces vacancy creation, in other words labor demand. This implies that less vacancies will be created if the share of workers with low hiring value increases in the pool of unemployed, and/or the expected return to hiring a worker of given age and health decreases.

3.3 Worker flows

Model has six types of workers flows: 1) from unemployment into employment, 2) from employment into unemployment, 3) from unemployment into disability, 4) from employment into disability, 5) from unemployment into retirement and 6) from employment into retirement. In addition to these worker flows within in the model, there are worker flows into and out of the model every period. Every period a new cohort enters the model unemployed whereas the cohort that turns $j = T$ exits the model. Worker flows are depicted in Figure 2.

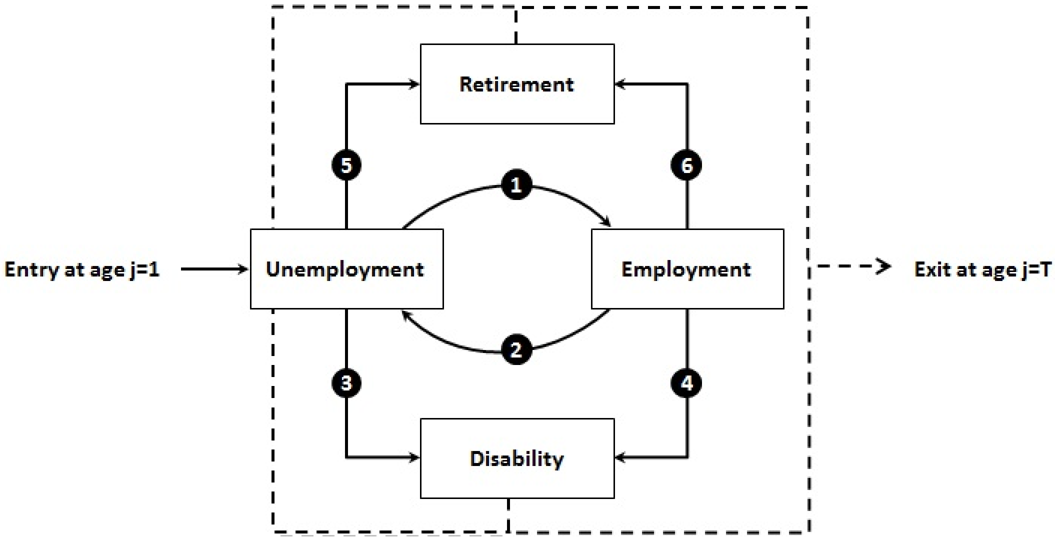


Figure 2: Workers flows

Size of each cohort is unity, so the mass of unemployed, employed and retired workers at a given age sum to one: $\sum_H \int_\epsilon e_j(h,\epsilon) + \sum_H u_j(h) + \sum_H r_j(h) + \sum_H D_j(h) = 1$, where r_j refers to the number of retired people and D_j to the number of permanently disabled. Overall unemployment in the economy is $\sum_{j=1}^{T}\sum_{h=1}^{H} u_j(h) = u$. Since new cohorts are born unemployed and all workers exit the model at the exogenous age of T, we also have that $u_1 = 1$ and $u_T = 0$. The complete age and health dynamics of employment, unemployment, retirement and disability are described in Appendix B.

3.4 General equilibrium

Equilibrium consists of the set of decision rules $T^u(h)$, $T^e(h,\epsilon,n)$, $\epsilon_j^0(h,n)$, wage schedule $w_j(h,\epsilon,n)$, labor market tightness θ and mass of workers across labor market states $e_j(h,\epsilon)$, $u_j(h)$, $r_j(h)$, $D_j(h)$ for $h \in \{1,...H\}$, $j \in \{1,2,...,T\}$, $n \in \{0,1\}$, and $\forall\epsilon$. Equilibrium is characterized by:

- the job creation equation (11)
- the sharing rule (7)
- the flow equations for unemployment, employment, disability and retirement found in Appendix B
- the equations for job start and quit margins (8), (9)
- the conditions characterizing optimal retirement age both from unemployment and employment, in other words, first age at which $U_j(h) \leq R_j$, first age at which $W_j(h,\epsilon,n) \leq U_j(h) \leq R_j$

and initial conditions: $u_1(h) = 0$ and $h_1 = 1$, in other words, all workers enter the model in excellent health and unemployed.

There is a possibility of multiple steady states in a life-cycle search and matching model with a single labor market (discussed in Hahn [10]). The possibility of multiple steady states arises from the fact that the right hand side of the job creation equation (11) is not necessarily monotonic in labor market tightness as in the basic Mortensen-Pissarides model. Hahn [10] argues numerically that multiple steady states should not be an issue for plausible parameter values. Based on numerical experiments, this seems to hold also for the model with health heterogeneity and endogenous retirement decisions presented here.

4 Policy experiment: increase in official retirement age

I consider a policy experiment where official retirement age is raised from 65 to 67 years while the early retirement age is kept unchanged at 62 years. Under the old policy, a person retiring at the early retirement age receives 80 % of the full retirement benefit while a person in the new system receives only 70%. After the early retirement age, the benefit increases so that a person retiring at the official retirement age of 65 years in the old system and 67 years in the new system is entitled to the full benefit. The old and new pension system are depicted in Figure 3.

I first analyze the effects of the policy change in partial equilibrium. This means that I treat p as a parameter and abstract from the effects of the policy reform on labor demand and labor market tightness. After this, I proceed to general equilibrium and endogenize $p(\theta)$, which allows me to consider also the labor demand and labor market tightness effects, and their interaction with worker decisions through p.

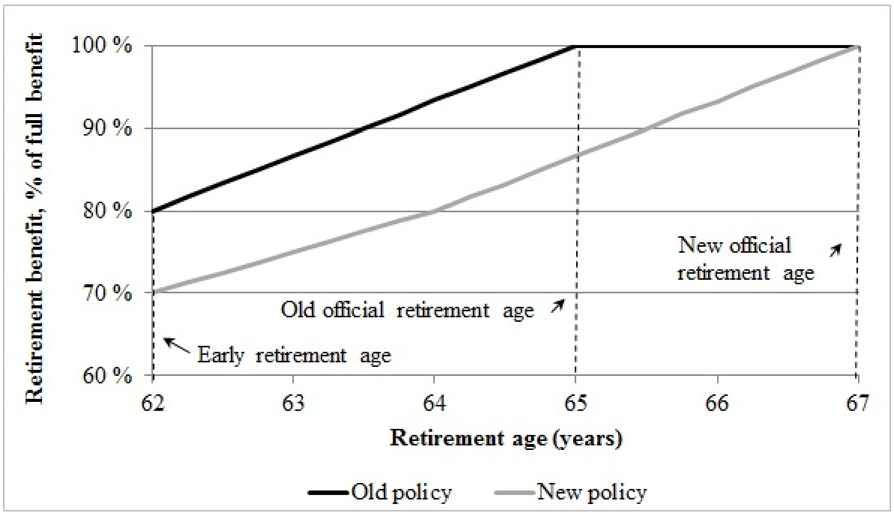


Figure 3: Pension system under old and new policy

In terms of model parameters, the policy change materializes through a decrease in α^j, for $l = \{T^{early}, ..., T^{off\_new}\}$, in other words between ages 62 and 67. The decrease in α_j, and the resulting in reduction in retirement benefits, affects worker decisions via two channels. First, it reduces directly the value of retirement and increases the attractiveness of unemployment relative to retirement. This delays the optimal retirement age for unemployed and, hence, also the threshold age at which retirement becomes the relevant outside option for employed workers. Secondly, the decrease in α_j affects job start and quit margins. The effect on job start and quit margin depends on whether unemployment or retirement is the relevant outside option.

When retirement is the relevant outside option, decrease in α_j for $j = \{T^{early}, ..., T^{off\_new}\}$ lowers the quit margin for all ages between the early retirement and official retirement age. This is due to a reduction in the current gain from outside option and an increase in the expected gain from continuing to work (as a result of the lower value of the outside option also in the future). Lower quit margin widens the range of match-specific productivities for which continuing to work instead of retiring is preferable and increases the job surplus for a given ϵ. This results in a longer expected work horizon and, consequently, higher expected retirement age for employed workers. Furthermore, longer expected work horizon makes employment more valuable which, in turn, increases the return to job search investments also contributing to later retirement for unemployed.

When unemployment is the relevant outside option, reduction in α_j for $j = \{T^{early}, ..., T^{off\_new}\}$ affects job start and quit margins only through the expected value of future surpluses. Sign of this effect is a priori indeterminate and depends on parameter values p - the probability of a worker finding a vacancy -, γ - bargaining power of worker -, κ - training costs - and distributions $G(\epsilon' \mid \epsilon)$ and $F(\epsilon')$.

Reduction in α_j decreases the value of the outside option in the future and, thus, increases expected surpluses. If continuing in existing job is a more (less) attractive strategy than unemployment search, the decrease inα lowers (increases) the job start (quit) margin. Close to retirement, remaining work horizon is short and return to search investment low, which makes continuing in existing job likely to be a better strategy than "shopping for employment opportunities". Therefore, the policy change is likely to reduce the job start and quit margins close to retirement. The lower start margin results in a larger number of profitable job opportunities for unemployed older workers and, hence, a higher probability of a match becoming productive. Lower quit margin, on the other hand, widens the range of match-specific productivities for which continuing to work instead of choosing unemployment is preferable, thus decreasing job separation probability.

The policy change also affects the demand for labor, in other words, firms' vacancy creation decisions. Job creation equation (11) implies that changes in vacancy creation are driven by expected value of hiring (the right hand side) and expected cost of hiring (the left hand side). Changes in the number of vacancies are realized through three channels.

First, changes in job start and quit margin affect the expected value of hiring. A decrease in job start margin increases the probability of a match becoming productive whereas decrease in quit margin leads to longer expected duration of employment relationship (the reverse holds for an increase). Therefore, a decrease in job start and quit margins results in a larger expected gain from a filled vacancy which incentivizes vacancy creation (the reverse holds for an increase). Secondly, changes in job start and quit margin and optimal retirement age for unemployed workers affect the number of unemployed workers in the economy and, thus, labor market tightness and the expected cost of hiring. A reduction in job start and quit margins decreases the number of unemployed workers. The rise in the optimal retirement age for unemployed, on the other hand, increases the number of unemployed workers. If the total effect is an increase in the number of unemployed, labor market tightness decreases making it easier to fill a vacancy. As a result, the expected cost of hiring decreases which has a positive impact on vacancy creation. Lastly, changes in the composition of unemployment pool affect vacancy creation through expected value of hiring. For example, if the share of unemployed older workers (with shorter expected work horizon) increases in the economy due to higher optimal retirement age for unemployed, it has a negative effect on the expected value of hiring and, thus, job creation.

Above described changes in vacancy creation have a feedback effect on worker decisions through labor market tightness. A higher labor market tightness increases the probability p at which workers are matched with open vacancies, and hence contributes to the attractiveness of unemployment search relative to retirement and employment. Increased attractiveness of unemployment relative to retirement postpones optimal retirement age for unemployed. Furthermore, when unemployment is the relevant outside option, it may increase the job start and quit margins. These changes in worker decisions again have a feedback effect on firms' vacancy creation decisions.

It is important to note that the level of unemployment among elderly in the economy prior to the policy change has a large impact on the increase in effective retirement age following the policy change. This is because unemployed and employed workers respond differently to the policy change. Furthermore, the change in the absolute number of unemployed workers has an impact on the level of job creation incentives through labor market tightness.

5 Quantitative evaluation

5.1 Labor market and health statistics

I use labor market statistics calculated based on monthly Current Population Survey data and "gross flows" codes by Robert Shimer[2] to calibrate my model. I restrict my attention to data from January 1998 to January 2005 [3] and people aged between 20 and 79 years. In line with the definition of inactivity in my model, I exclude all people that are out of the labor force for any other reason than disability or retirement. I also ignore labor market transitions that take place within a month.[4],[5]

Health transition matrices and disability probabilities are calculated from annual Panel Study of Income Dynamics (PSID) data. I restrict my attention to years 1990-1996, since after 1997 PSID has been conducted only bi-annually, and in 1997 there was a significant change in the sample weights. I use self-rated general health, which includes five health categories - *excellent, very good, good, fair, poor* -, as a measure of person's health status. To reduce the number of health states to three, I consolidate the two lowest and two highest health categories.

Health transition matrices and disability probabilities are calculated for 5-year age groups starting from 20-24 years and ending at 65-69 years. In addition, all over 69-year-olds are treated as one group. A transition probability from health state *excellent* to *good* is defined as the sample-weighted fraction of people in excellent health in a given

[2]For more detailed description of the data, please see Shimer [18] and his web-page http://sites.google.com/site/robertshimer/research/flows"

[3]I use this time period, because the official retirement age and the delayed retirement credit have been gradually increased. For example, for people turning 65 in 2002 or before, the official retirement age is 65 years, whereas for those who turn 65 in 2008-2019 it is 66 years. Furthermore, the delayed retirement credit has been increased gradually depending on the year of birth. By restricting my attention to years 1998-2004, the retirement policy is pretty much the same for all those in the "retirement window" during the time period considered.

[4]Contrary to Shimer [18], I do not adjust the transition probabilities for time aggregation bias

[5]I convert the monthly transition probabilities into quarterly first taking an average of the monthly transition probabilities and converting the corresponding "average" transition matrices into quarterly.

age group in the first year of survey that report being in *good* health next year. Similarly, a disability probability is defined as the share of people in a given age and health group who are in the labor force in the first year of the survey and permanently disabled the following year. To arrive at the values used in the parametrization, I take the median of the annual health transition and disability probabilities and convert these probabilities into quarterly.

5.2 Parameter values and functional forms

Model parameters are chosen to match the US economy. One model period corresponds to a quarter, and workers enter the model at the age of 20 and exit at the age of 80. Matching function takes a form proposed by Den Haan et al. **??**, $M(v,u) = \frac{uv}{(u^{\eta}+v^{\eta})^{1/\eta}}$. This non-standard functional forms was chosen because it ensures that matching probabilities are between 0 and 1. Discount factor β is chosen to match an annual interest rate of 4%. Bargaining power of workers is set to 0.5.

Pension system is parametrized based on the US Social Security system. Early retirement age is 62 years, whereas official retirement age is 65 years.[6] People retiring between early and normal retirement age receive reduced benefits. A table displaying the benefit schedule can be found in the Appendix. Health transition matrices and disability probabilities are parametrized based on the computations explained in more detail in Section 5.1. For simplicity, the age-profile of productivity is assumed flat in the baseline calibration. Exogenously set parameters are summarized in Table 1 (excluding the benefit schedule, health transition matrices and disability probabilities which can be found in the Appendix).

Parameter	**Description**	**Value**
T^{off}	Official retirement age	65
T^{early}	Early retirement age	62
T	Exit age from model	80
β	Discount factor	0.99
γ	Bargaining power of the worker	0.5

Table 1: Exogenous parameters (excl. α, π, P)

Rest of the parameters are determined by minimizing the distance between a set of steady state targets and model moments. [7] In the text, each parameter is associated

[6]The early retirement age has stayed the same, but the normal retirement age has been gradually increased in US. For people born prior to Jan 2, 1938 the normal retirement age is 65. From that onwards the normal retirement age is gradually increased so that for people born on Jan 2, 1962 or later the normal retirement age is 67. Since my labor market data is for the period January 1998 to January 2005, I use the normal retirement age of 65 years.

[7]The method for solving the calibration problem is described in more detail in Appendix E

with one target that is thought to best identify it, but in practice the parameters are jointly determined by all targets. All endogenously determined parameters and associated steady state targets are summarized in Table 2.

Unemployment benefit b_u is calibrated to 50% of the average wage following Esteban-Pretel and Fujimoto [8]. Retirement benefit is set equal to the unemployment benefit. This assumption is based on the OECD [**?**] estimate that, under the rules and policy parameters of 2002, retirement benefit net replacement rate for an average earner in the United States was 51%. Both disutility of work and disutility of job search are assumed linearly increasing in age. Disutility of working and disutility of searching for the youngest person are calibrated to match employment rate and unemployment rate in the youngest 5-year age group, respectively. Following the same logic, disutility of working and disutility of searching for the oldest person are set by targeting employment and unemployment rate in the oldest 5-year age group. Value of leisure while retired, ν, is chosen to match the employment rate for 65-69-year-olds.

Parameter η is calibrated to match quarterly unemployment-to- employment (UE) transition probability of 68%. Following Shimer [17] and Esteban-Pretel and Fujimoto [8], labor market tightness is normalized to 1. Vacancy cost k is then adjusted to satisfy the free entry condition (11). Training cost κ is calibrated to 32% of the average wage along the lines of Mortensen [15].
Training cost κ is calibrated to match the employment rate for 60-64 olds.

I discretize the AR(1) process for the logged match-specific productivity using the method of Tauchen [19]. I use a grid of 60 points, equally distributed between $-3\frac{\sigma_\xi}{\sqrt{1-\rho^2}}$ and $3\frac{\sigma_\xi}{\sqrt{1-\rho^2}}$. Persistence of the AR(1) process, ρ , and standard deviation of the innovation, σ_ξ, are chosen to match in the economy unemployment rate among 60-64-year-olds and the overall unemployment rate of 4.5%. $F(\epsilon)$, the distribution from which the match-specific productivity of new matches is drawn from, is assumed to follow uniform distribution. The support for $F(\epsilon)$ is derived from the discretized AR(1) process, in other words, $F(\epsilon)$, $\epsilon \in [exp(-3\frac{\sigma_\xi}{\sqrt{1-\rho^2}}), exp(3\frac{\sigma_\xi}{\sqrt{1-\rho^2}})]$.

Target	Target value	Parameter	Parameter value
Replacement rate, % of average wage	0.5	b^u	0.47
b^u/b^r	1	b^r	0.47
Employment rate, 20-24 years	0.89	d(1)	0.031
Unemployment rate, 20-24 years	0.089	c(1)	0.031
Employment rate, 75-79 years	0.081	d(240)	0.40
Unemployment rate, 75-79 years	0.032	c(240)	0.41
Quarterly UE transition probability	0.68	η	4.35
Labor market tightness, θ	1	k	0.52
Employment rate, 65-69 years	0.26	ν	0.23
Unemployment rate, 60-64 years	0.033	ρ	0.68
Unemployment rate	0.045	σ_ξ	0.17
Quarterly EU transition probability	0.023	δ	0.016
Employment rate, 60-64 years	0.51	κ	0.35

Table 2: Calibration targets and endogenously determined parameter values

5.3 Calibrated model with random search

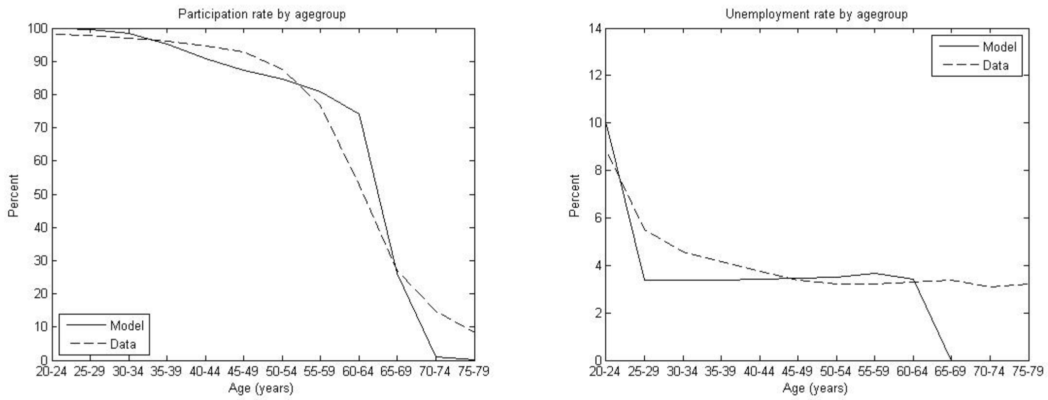
Table **??** shows the calibration targets and model generated values. Model is able to match the calibration targets quite well. However, it struggles to match the targets in one area: it predicts virtually all 70-79 olds to be out of the labor force although we do not observe this in the data. One explanation for the too low participation rate among 70-79 year-olds might be that the model features a zero-one labor supply decision without an opportunity for part-time work.

Target	Target value	Model
Replacement rate as a share of average wage	0.5	0.47
Employment rate, 70-74 years	0.14	0.013
Employment rate, 20-24 years	0.89	0.90
Unemployment rate, 20-24 years	0.089	0.098
Employment rate, 75-79 years	0.081	0.0
Unemployment rate, 75-79 years	0.032	0.0
Quarterly UE transition probability	0.68	0.71
Employment rate, 65-69 years	0.26	0.26
Unemployment rate, 60-64 years	0.034	0.035
Unemployment rate	0.045	0.040
Quarterly EU transition probability	0.023	0.024
Employment rate, 60-64 years	0.51	0.73

Table 3: Comparison of calibration targets and model generated values

Figure 4 illustrates the life-cycle profiles of participation and unemployment in the data and in the model. The model is able to replicate, at least qualitatively, the life-cycle profiles despite the fact that only part of the profiles were targeted in the calibration. The two main discrepancies are the overestimation of the participation rate for 60-64-olds and the underestimation of the participation and unemployment rate for 70-79 year olds (already discussed above). Model's pension system and the absence of savings decisions might explain at least partly the overestimation of the employment rate for 60-64 olds. In the model, the earliest a person can retire is at the age of 62 years (which is the early retirement age in the Social Security system) whereas in reality people can retire earlier if they have enough personal savings. The underestimation of the participation rate and unemployment rate at oldest ages might be due to the absence of savings decisions and the fact that the model does no have enough heterogeneity to induce more gradual transitions into retirement.

The life-cycle profiles of unemployment- to-employment and employment-to-unemployment transition probabilities were not targeted in the calibration. Due to the lack of learning frictions and human capital accumulation, the model is unable to replicate the strictly age-decreasing job finding probability combined with a employment-to-unemployment transition probability that first drops and then plateaus. Due to this tension in the model, I choose a calibration which generates a employment-to-unemployment transition probability that is flat until 60-64 years and a job finding rate that starts to decline after 59 years, because the focus of the paper is on old-age labor market dynamics .



(a) Participation rate (b) Unemployment rate

Figure 4: Data versus model - employment and participation rate

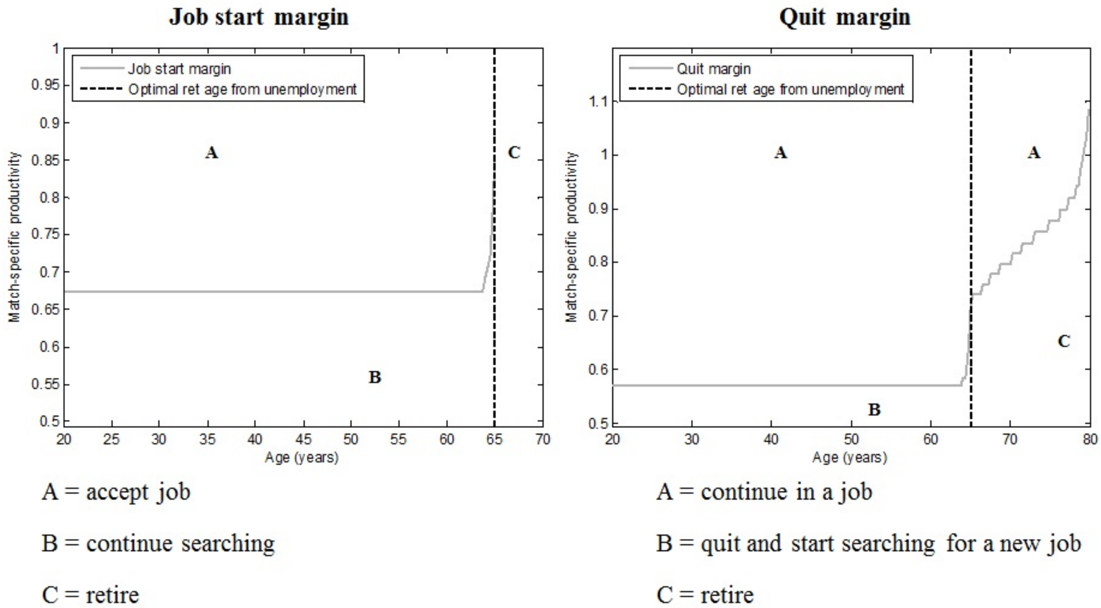
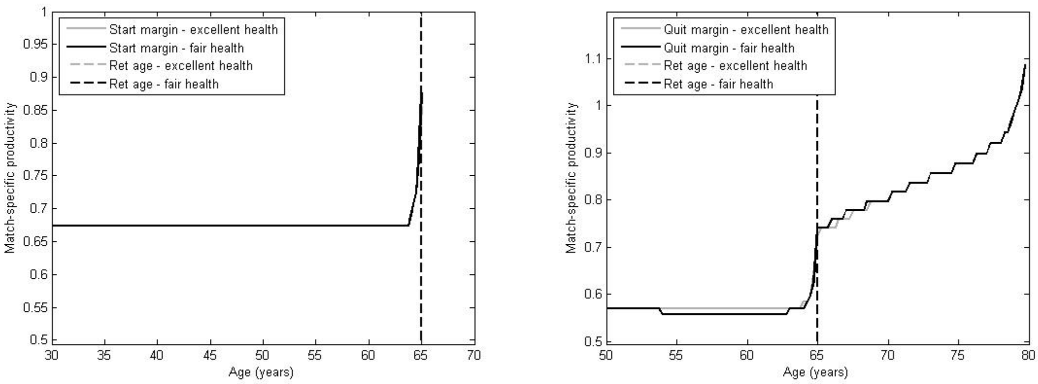


Figure 5: Job start and quit margin and retirement age from unemployment - excellent health

The life-cycle profiles of the job start and quit margin are depicted in Figure 5. Under the benchmark calibration, both margins are increasing in age, but start to rise only close to retirement. The increase in job start and quit margin at older ages reflects the horizon effect (phenomenon discussed by Hairault et al. [3]): when work horizon is short, jobs become less valuable to the worker and firm due to low continuation values. Due to the horizon effect, hiring and training an old worker is profitable only for high match-specific productivities. Furthermore, with short remaining work horizon, current productivity increases in importance, and it is difficult to compensate low current productivity with favorable expected productivity development in the future.

Figure 6 shows the job start and quit margins and optimal retirement age for an unemployed worker by health status. There is no difference in optimal retirement age for unemployed or the job start margin due to health. However, when unemployment is the relevant outside option, the quit margin is slightly higher for a worker in excellent health versus in fair health. This implies that as a result of the lower disability risk and, thus, longer expected work horizon, unemployment search is more profitable for a worker in excellent health compared to a worker in fair health. Therefore, a higher match-specific productivity is required for a worker in excellent health to stay on the job instead of quitting and searching for better employment opportunities. When retirement becomes the relevant outside option, the quit margin is lower the better one's health implying a higher expected retirement age for an employed worker in excellent health. The higher retirement age for a person in excellent health arises from "horizon effect of health":

longer expected work horizon due to lower disability risk increases value of employment contributing to later retirement.



(a) Start margin

(b) Quit margin

Figure 6: Job start and quit margin and retirement age for unemployed by health

5.4 Policy experiment

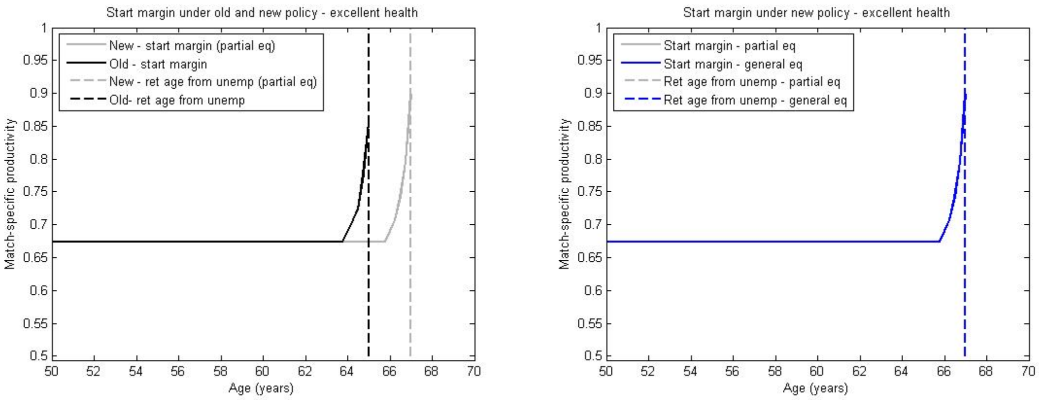
From now on, I will use the term partial equilibrium effect to refer to the implications of the policy experiment when labor market tightness and match probabilities are treated as parameters. General equilibrium effects, in turn, refer to the implications of the policy experiment when labor market tightness is endogenous and allowed to adjust to the new policy.

In the quantitative policy experiment, I keep all parameters as in the benchmark calibration except for the retirement benefit schedule and labor market tightness. In general equilibrium, when also firm responses through vacancy creation are taken into account, θ adjusts so that the free entry condition is satisfied under the new retirement policy. However, in evaluating the partial equilibrium effects, θ is the same as in the benchmark calibration, because labor market tightness is a parameter in the partial equilibrium analysis.

Figure 7 depicts the change in job start margin and optimal retirement age for unemployed following the policy change for a worker in excellent health whereas Figure 8 shows a corresponding graph for a worker in fair health. Figures 9 and 10 illustrate equivalent graphs for the quit margin. Both figures show separately the effects of the policy change in partial equilibrium and in general equilibrium (where also firms' vacancy creation response is taken into account).

[RETIREMENT AGE FOR UNEMPLOYED]
Prior to the policy reform, optimal retirement age for unemployed in all health states

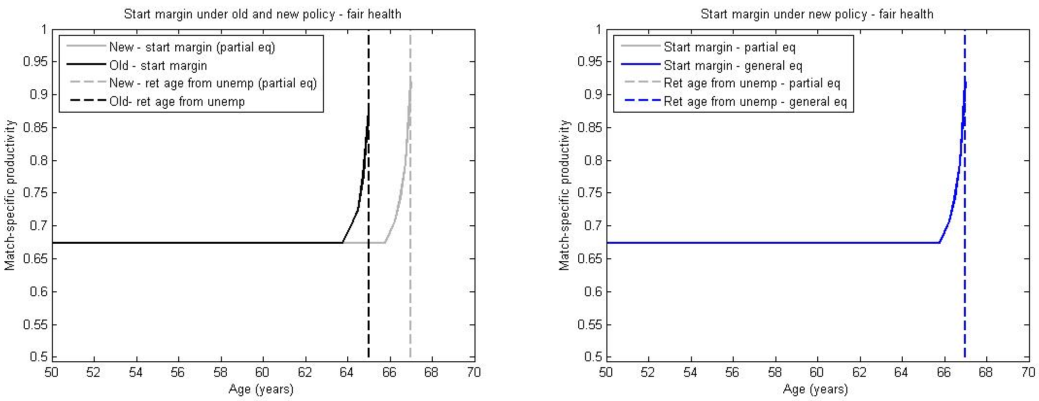
coincides with the official retirement of 65 years. Because of the lack of savings decisions, there is no inflow into early retirement except for disability. The partial equilibrium effect of the policy reform is a 2-year rise in the optimal retirement age for unemployed to the new official retirement age of 67 years (mechanisms described in section 4).[8] The inclusion of general equilibrium effects has no effect on the optimal retirement age for unemployed workers.



(a) Partial equilibrium

(b) General equilibrium

Figure 7: Job start margin and retirement age for unemployed under new and old policy - excellent health

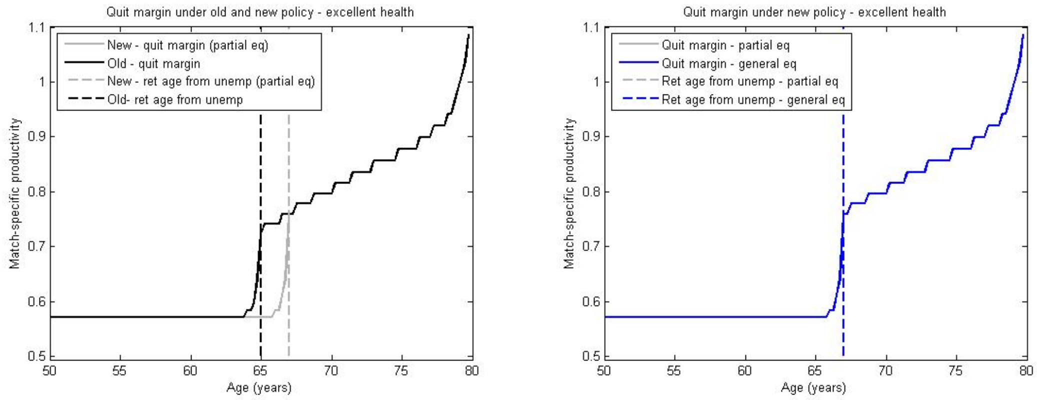


(a) Partial equilibrium

(b) General equilibrium

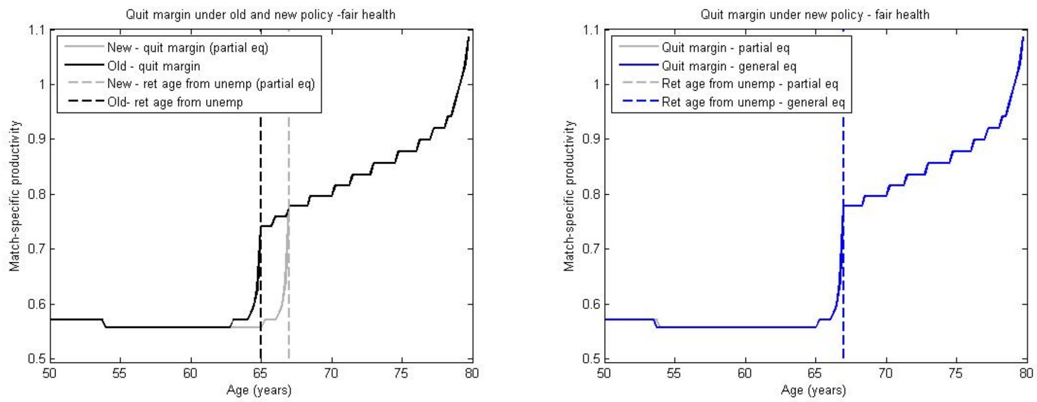
Figure 8: Job start margin and retirement age for unemployed under new and old policy - fair health

[8]The optimal retirement age is different from the effective retirement age. The effective retirement age takes into account that some workers retire early due to disability shocks



(a) Partial equilibrium (b) General equilibrium

Figure 9: Quit margin and retirement age for unemployed under new and old policy - excellent health



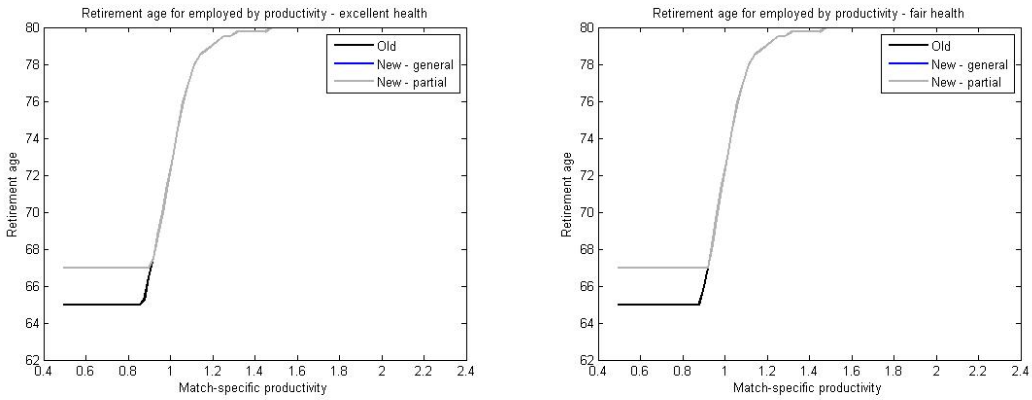
(a) Partial equilibrium (b) General equilibrium

Figure 10: Quit margin and retirement age for unemployed under new and old policy - fair health

The partial equilibrium effect is a decrease in both job start and quit margins close to retirement. As discussed in Section 4, this is due to the horizon effect: the longer expected work horizon makes employment more valuable both to the older worker and the firm employing the older worker. Thus, firms are willing to hire older workers for lower productivity values. Higher continuation value of employment due to the longer work horizon, in turn, induces workers to continue in (accept) a job for a lower current productivity value. The inclusion of general equilibrium effects has virtually no effect on the job start and quit margin. This implies that the change in labor market tightness and, thus, matching probabilities and attractiveness of unemployment are not significant following the pension reform.

The reduction in quit margin when retirement is the relevant outside option means that a lower match-specific productivity is needed for workers to continue working instead of retiring. This implies a higher optimal retirement age for employed workers. Figure 11 shows that, following the policy change, optimal retirement age rises for lower values of match-specific productivity. This, in turn, translates into higher expected retirement age for employed workers.

The inclusion of general equilibrium effects does not change the optimal retirement decision for employed workers. This is not surprising because labor market tightness does not directly affect the retirement decision for employed. It may only have a minor indirect effect in case there are workers who retire at, or very close to the age at which workers are indifferent between the two outside options. In this case, a higher $p(\theta)$ increases value of unemployment which, in turn, may postpone the age at which retirement becomes the relevant outside option.[9]



(a) Excellent health

(b) Fair health

Figure 11: Retirement age for employed under new and old policy - person in excellent health

Above, I have discussed the effect of a retirement policy change on workers' and firms' decision making. However, what is still lacking is the final effect of the policy change on the aggregate labor market outcomes such as effective retirement age and life-cycle-employment and unemployment rates.

In the new steady state, the effective retirement age is approximately 1.2 years higher than prior to the policy reform (see Table 6) which corresponds to 59% of the increase in official retirement age. The ratio of the increase in effective retirement age to the increase in official retirement age is higher than usually in the retirement literature.

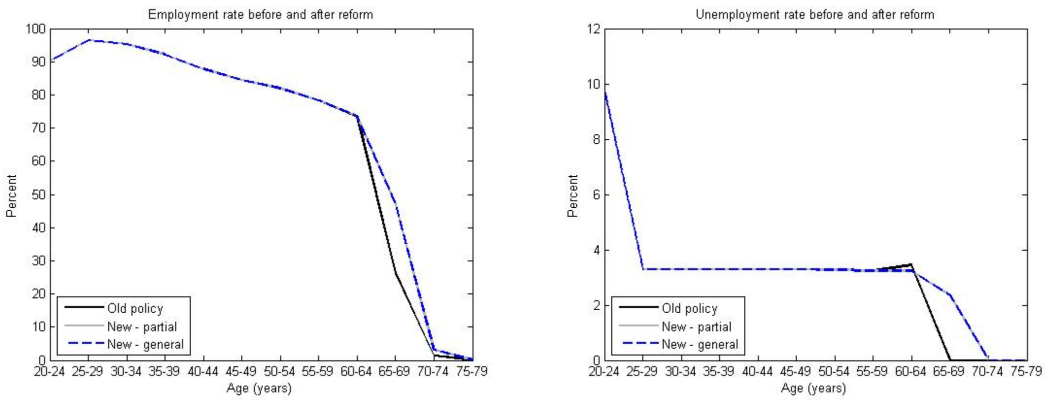
[9] In the model, it is assumed that people that are indifferent between retiring and employment/unemployment choose to retire

This is because there is no inflow into early retirement except for disability which is a result of the simplicity of the model and especially the lack of savings decisions. There is virtually no difference in effective retirement age when only partial equilibrium effects versus also general equilibrium effects are considered (Table 6).

	Old policy	**New policy - partial**	**New policy - general**
Effective retirement age	62.0	63.2	63.2

Table 4: Effective retirement age (years) under old and new policy

Life-cycle unemployment and employment rate before and after the policy change is illustrated in Figure 12. Also here, the inclusion of general equilibrium effects makes no difference compared to the partial equilibrium case. The employment rate for 65 to 69 olds rises by approximately 20 percentage points whereas for other ages, employment rate remains largely unchanged. As for the unemployment rate, there are two changes. First, a rise in the unemployment rate for 65-69 olds purely due to a participation effect: prior to the policy change, no unemployed workers aged 65 or older participated in the labor market. Secondly, there is a 0.2 percentage point decrease in the unemployment rate for 60-64 olds. This suggests that public concerns about retirement age increases being ineffective due to old people being ”unemployable”, are unwarranted if search is random.



(a) Employment rate

(b) Unemployment rate

Figure 12: Life-cycle employment and unemployment rate before and after policy change

Above, the inclusion of general equilibrium effects made a negligible quantitative difference in retirement ages and old-age employment. This is because, with a single labor market, workers primarily affected by the policy change - older workers - represent only a small share of the entire labor market. As a result, changes in retirement policy have only a small effect on the expected value of hiring, pool of unemployed workers and, thus, vacancy creation and aggregate labor market tightness. In the parametrized model, the policy change leads to a 2.2% reduction in labor market tightness and 1.1% decrease in the probability of a worker finding an open vacancy. Accordingly, also the feedback effect between vacancy creation and labor supply decisions is quantitatively small.

6 Age-directed search

Until now, I have assumed that job search is random and that there is only one labor market. Motivation for this assumption is that labor market discrimination by age is illegal in many countries. However, as brought up by Menzio et al. [14], in reality firms may be able to age-discriminate job applicants by arguing that a job applicant is rejected based on quality although the true reason is age. To examine this possibility, I release the assumption of random search, and instead, assume that search is age-directed.

Unemployed workers are no longer randomly matched with open vacancies regardless of worker age, but instead, labor markets are perfectly age-segmented (similarly to Hairault et al.[11] and Fujimoto [12]). More specifically, there is a separate sub-market for workers of each age. The underlying assumption is, as explained in Hairault et al. [11], that a job targeted for age j-workers requires attributes that only age-j workers possess. Accordingly, denoting the attributes required in an age-j position by a_j and skills of an age-j worker by s_j, match output is positive for matches of type $\{a_j, s_j\}$ and zero for matches of type $\{a'_j, s_j\}$, $j' \neq j$.

Cost of posting a vacancy, k, is the same across sub-markets, and firms are free to post vacancies in any sub-market. Each sub-market has its own matching function $M(u_j, v_j)$, where v_j is the number of open vacancies and u_j number of unemployed workers in the labor market for age j workers. Denoting labor market tightness in age-j market by $\theta_j = \frac{v_j}{u_j}$, the probability that a firm with an open vacancy finds a worker is $q_j = q(\theta_j) = \frac{M(v_j,u_j)}{v_j}$ whereas the probability that a worker finds an open vacancy is $p_j = p(\theta_j) = \frac{M(v_j,u_j)}{u_j} = \theta_j q(\theta_j)$.

Equations characterizing the value of employment, disability and retirement are the same as in the benchmark model. However, equations for value of a filled vacancy, value of an open vacancy and value of unemployment (and therefore also job start and quit margins when unemployment is the relevant outside option) are slightly different.[10] Difference in the value of unemployment arises from match finding probability p_j now being age-dependent as opposed to a constant. Moreover, the value of an open vacancy, V, is now defined as $V = max\, V_j$, $j = \{1, ..., T-1\}$, because firms are free to choose in which

[10] New versions of these equations can be found in Appendix

sub-market to post vacancies. Now, vacancy creation depends on the size and characteristics of the unemployment pool for a given age, and not the unemployed population as a whole. Although the labor market is perfectly age-segmented, workers and open vacancies in a given sub-market are randomly matched regardless of worker health.

Model is parametrized similarly to the random search model. Same values are used for the exogenously set parameters and endogenously set parameters are determined by matching the same targets as in the benchmark calibration. There are, however, two exceptions: labor market tightness θ_j for $j \in \{1, ..., T-1\}$ and cost of posting a vacancy k. I calibrate the cost of posting a vacancy to match the ratio of vacancy cost to average wage in the benchmark model[11]. Labor market tightness is then adjusted so that the free entry condition is satisfied in each age-specific sub-market.

Parameter	**Parameter value**
b^u	0.47
b^r	0.47
η	4.17
d(1)	0.035
c(1)	0.032
d(240)	0.41
c(240)	0.35
k	0.52
ν	0.22
ρ	0.68
σ_ξ	0.17
δ	0.015
κ	0.35

Table 5: Endogenously determined parameter values in the model with age-directed search

Depending on parameter values, there can be multiple steady states. This possibility arises because, contrary to the random search model, labor market tightness θ_j may affect whether or not a labor market close to retirement is at all operational. In the labor markets close to retirement, a decrease in labor market tightness benefits firms up to a point by increasing the probability of finding a worker. However, if labor market tightness becomes sufficiently low, it may decrease value of unemployment to the extent that it induces unemployed workers to retire earlier so that the whole labor market-j "disappears". This gives rise to the possibility of multiple steady states. With these parameter values, this is however, not an issue.

[11]The targeted ratio of cost of posting a vacancy to average wage is $\frac{k}{w} = 0.51$

The above calibration produces a life-cycle profile of labor market tightness (see Figure 13) that is not monotonic. Instead, the labor market tightness is age-decreasing until just before the optimal retirement age from unemployment when it increases sharply. Labor market tightness first decreases with age because firms prefer to post vacancies in sub-markets where expected value of hiring is high; horizon effect reduces the expected hiring value of older workers and, thus, results in less vacancies towards end of the life-cycle. Just before the official retirement age, firms are no longer willing to hire any workers and there exists no labor markets for workers of these ages (64.50 and older). Consequently, in the labor markets preceding these ages, value of the outside option, in other words unemployment search, is low due to the weakness of the labor markets and workers workers are willing to accept jobs with very low productivities (and wages). This increases the expected value of hiring for firms which incentivizes vacancy creation. As a result, labor market tightness increases near the official retirement age. Furthermore, the same mechanisms leads job start and quit margin (see Figures 15 and **??**) to drop prior to retirement which is opposite of what happens in the random search model.

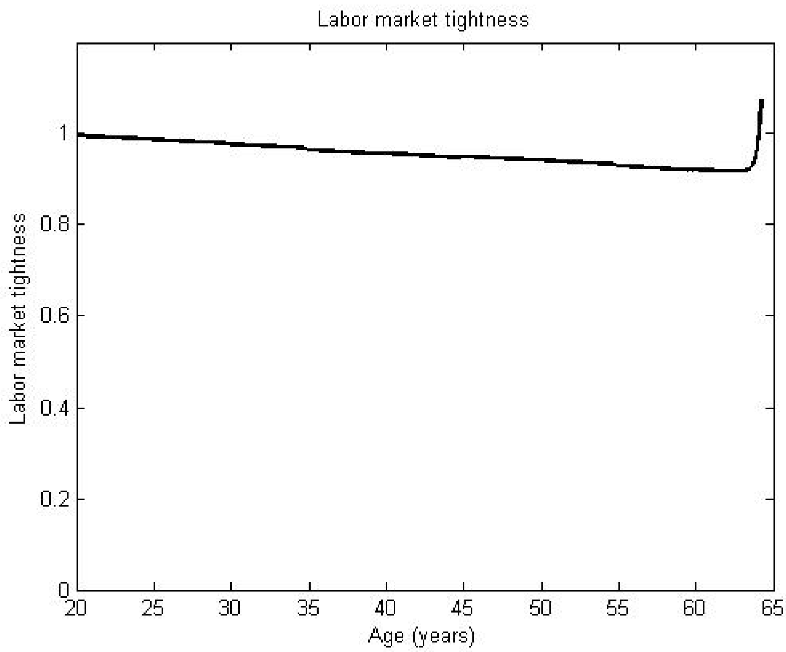
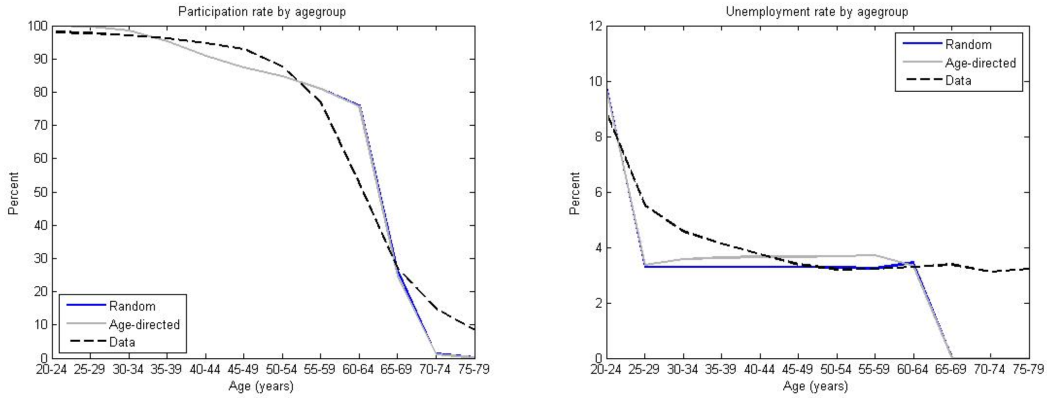


Figure 13: Labor market tightness over the life-cycle

Figure 14 depicts life-cycle participation and unemployment profiles generated by the calibrated age-directed model. These are very close to the life-cycle profiles generated by the benchmark model with random search. The key difference is that unemployment rate in the age-directed model is higher than in the benchmark model until the age of 60.



(a) Employment rate

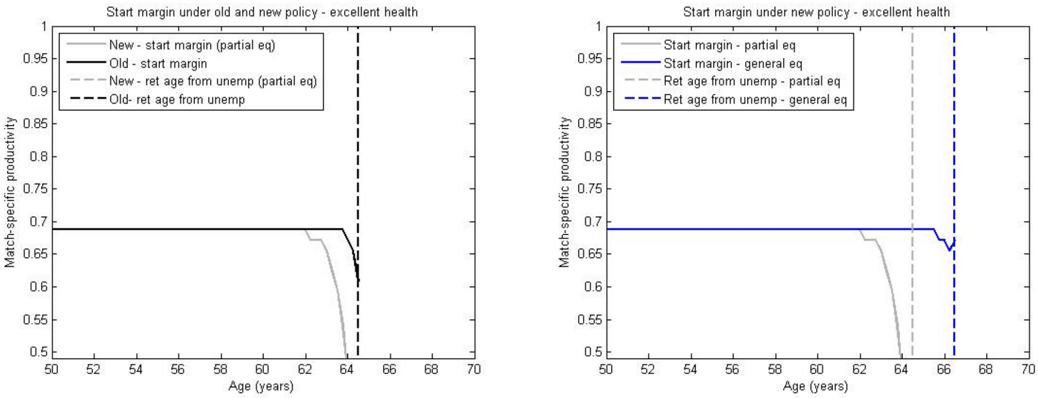
(b) Participation rate

Figure 14: Random and age-directed search - participation and unemployment rate

Figures 15 and 16 illustrate the changes in job start margin and optimal retirement age for unemployed, and Figures 17 and 18 the changes in quit margin following the policy change. Optimal retirement age for unemployed remains unaffected when only partial equilibrium effects of the policy change are considered (see Table **??**). This is because there are no labor markets and, thus, open vacancies available for workers aged 64.5 and older. When also general equilibrium effects are taken into account, optimal retirement age for unemployed workers in all health states rises by 2 years from 64.5 to 66.5 years. This results from new labor markets, that were not operational prior to the policy change, being opened for older workers.

For both job start and quit margin, the effect of the policy change in partial equilibrium (which abstracts from the effect of the policy change on firms' vacancy creation and labor market tightness) differs from that in the random search model. Here, policy change induces a sharp drop in the job start and the quit margins (when retirement is the relevant outside option) near the retirement age, with workers around 65 years willing to accept (continue in) a job for any productivity value. This is because labor markets for workers aged 64.5 years and over are not operative: weak labor market situation pushes down the value of unemployment search and, thus, decreases the job start (quit) margin. When also general equilibrium effects are taken into account, the job start margin and the q it margin when unemployment is the relevant outside option increase compared to the partial equilibrium case. This is due to labor markets that were not previously operative becoming active. The raise in official retirement age increases the expected retirement age for employed, thus, increasing the expected value of hiring an older worker (due to the horizon effect). The resulting increase in expected surplus and the increase in the number of old job seekers due to postponed retirement from unemployment makes it optimal for firms to post vacancies in labor markets for older workers that were not operational prior to the policy change. The opening of new labor markets renders unemployment more attractive for older workers, thus, increasing the job start margin.

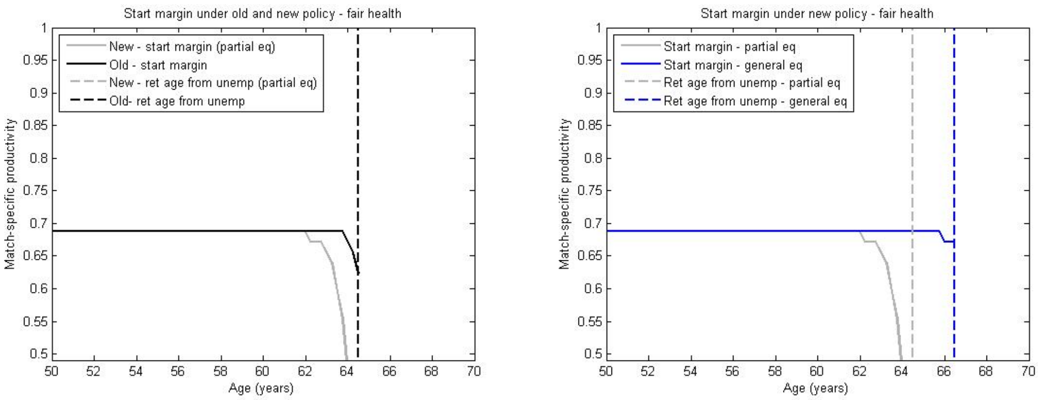
In partial equilibrium, the pension reform induces a drop also in the quit margin when retirement is the relevant outside option. This is because the pension reform makes retirement between ages 62 and 67 less attractive than before. The inclusion of the general equilibrium effect does not shift the quit margin when retirement is the relevant outside option since labor market tightness does not directly affect the retirement decision from employment.



(a) Partial equilibrium

(b) General equilibrium

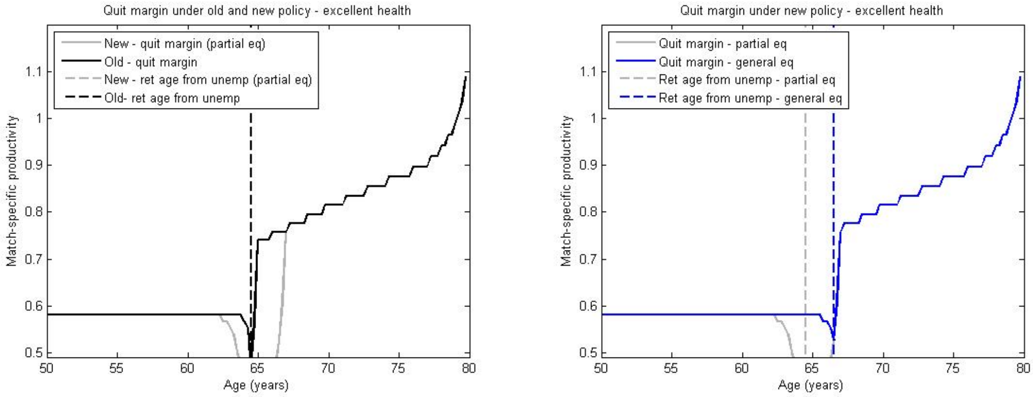
Figure 15: Job start margin and retirement age for unemployed under new and old policy - excellent health



(a) Partial equilibrium

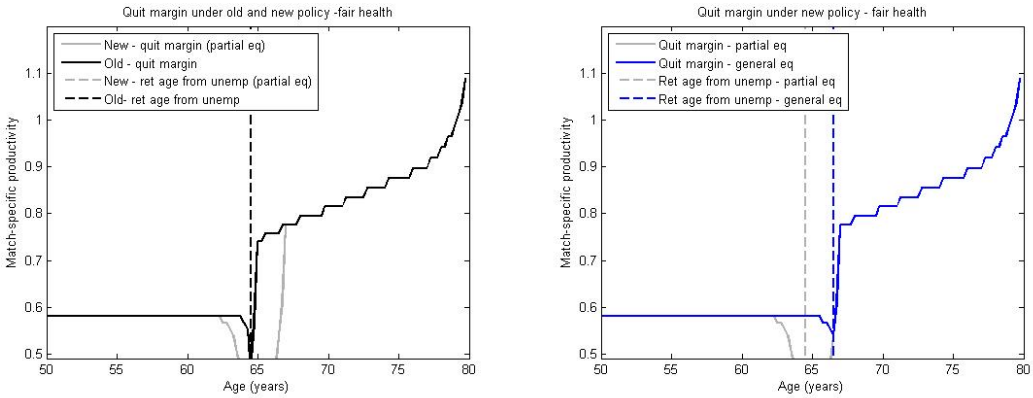
(b) General equilibrium

Figure 16: Job start margin and retirement age for unemployed under new and old policy - fair health



(a) Partial equilibrium (b) General equilibrium

Figure 17: Quit margin and retirement age for unemployed under new and old policy - excellent health



(a) Partial equilibrium (b) General equilibrium

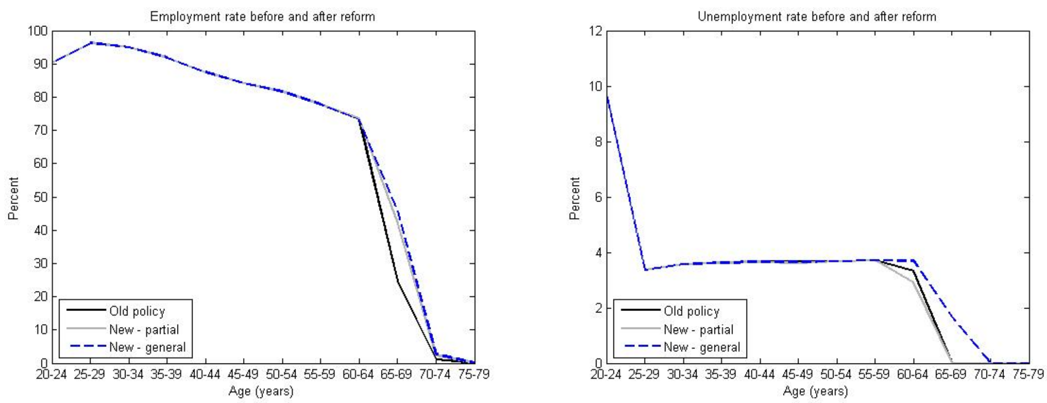
Figure 18: Quit margin and retirement age for unemployed under new and old policy - fair health

Table **??** shows the effective retirement ages prior to and after the policy change. Average effective retirement age increases by 1.2 years following the policy change. This increase corresponds to 59% of the increase in official retirement age. The change in effective retirement age is quite close to that int he random search model. However, the difference between the models arises from the mechanisms through which the increase in effective retirement age is realized. While in the random search model the labor market tightness response had no effect on the effective retirement age, it does make a difference in the age-directed model. In the age-directed model, the inclusion of general equilibrium effects raises the effective retirement age by additional 2.4 months.

	Benchmark	New policy - partial	New policy - general
Effective retirement age	61.9	62.9	63.1

Table 6: Effective retirement age (years) under old and new policy - age-directed model

Figure 19 shows that, similarly to the random search model, employment rate increases at older ages following the policy change and remains unchanged earlier in the life-cycle. However, general equilibrium effects are stronger in the age-directed model compared to the random search model. Furthermore, the key aspect in which the two models differ is employment rate for 60-64 olds. In the random search model, pension reform leads to a 0.2 percentage point decrease in unemployment rate for 60-64 olds while in the age-directed model the unemployment rate for 60-64 olds increases by 0.4%. The increase in unemployment rate for 60-64 olds in the age-directed model arises from the behavior of the labor market tightness and wage close to retirement. Near the official retirement age, old unemployed workers are willing to accept jobs with very low productivities (and wages) due to weakness of labor markets close to retirement age. When official retirement age is increased, the labor market situation for ages 64.50 and up improves significantly. This increases the value of unemployment search and makes unemployed workers more picky about the job offers they accept. Consequently, workers no longer accept the low wage and low productivity jobs that they used to accept prior to the policy change, but instead, keep on searching for better employment opportunities. This reduces the expected hiring value in these markets, thus, incentivizing firms to cut down on number of vacancies. The final general equilibrium effect is then an increase in unemployment rate for 60-64 olds.



(a) Employment rate

(b) Unemployment rate

Figure 19: Life-cycle employment and unemployment rate before and after policy change: age-directed model

Contrary to the random search mode, the general equilibrium effects of the pensions reform on effective retirement age and old-age (un)employment are notable. In the random search model, only a small share of the workers in the labor market were affected by the policy change. However, in the-age directed model, all workers in the labor markets for older workers are affected by the policy change. As a result, the effect on the expected value of hiring, number of unemployed job seekers and, thus, labor market tightness is stronger. This implies that the strength of firm response to pension reforms and its feedback effect on labor supply depends on whether one believes that there is a single labor market or a multitude of age-specific labor markets.

7 Conclusions

In this paper, I studied the effect of labor market frictions and health on retirement policy implications. I did this using a search and matching model with endogenous retirement decisions, exogenous health process and overlapping-generations structure where workers live for T periods. I introduced two versions of the model - one with random search and another with age-specific labor markets - to study how the assumed labor market structure affects results. The effect of retirement policy on unemployed and employed workers' labor supply decisions, and the interaction of these decisions with firms' vacancy creation, was studied by conducting a policy experiment where official retirement age was raised from 65 to 67 years while early retirement age was kept intact at 62 years.

I found that retirement policy affects the decision making of unemployed and employed workers through different mechanisms. Furthermore, results showed that retirement policy does not only affect retirement decisions, but also workers' job start and quit decisions. I also found that quantitatively the interaction between labor supply decisions and firms' vacancy creation has a larger effect on timing of retirement and old-age employment when one assumes that there are multitude of age-specific labor markets instead of a single labor market. Furthermore, results showed that labor market structure affects whether or not higher old-age unemployment following a retirement age increase is a matter of concern.

The model presented in this paper featured linear utility and abstracted from the savings decision. Savings are known to affect retirement decisions, and the inclusion of concave utility and savings decision would render the model more realistic. However, this extension is left for future research.

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A Wage equation

Let's first consider the case when unemployment is the relevant outside option, in other words, $O_j(h) = U_j(h)$. We can reformulate the sharing rule to

$$-(1-\gamma)U_j(h) = \gamma(J_j(h,\epsilon,0) + W_j(h,\epsilon,0) - V) - W_j(h,\epsilon,0) \tag{12}$$

$$\begin{aligned}
&\gamma\left(J_j(h,\epsilon,0) + W_j(h,\epsilon,0) - V\right) - W_j(h,\epsilon,0) = \\
&\gamma a_j\epsilon - w_j(h,\epsilon,0) + (1-\gamma)d_j - (1-\gamma)\beta(1-P_j(h))E_{\epsilon',h'}[(1-\delta)N_{j+1}(h,'\epsilon,0) + \delta O_{j+1}(h') \mid h,j] \\
&+ \gamma\beta(1-P_j(h))E_{\epsilon',h'}[max\{J_{j+1}(h',\epsilon',0),V\} + \delta V \mid h,j] \\
&- (1-\gamma)P_j(h)\beta R^D_{j+1} + (1-P_j(h))\beta\gamma V
\end{aligned} \tag{13}$$

Setting this equal to $-(1-\gamma)U_j(h)$ yields:

$$\begin{aligned}
w_j(h,\epsilon,0) =& \gamma a_j\epsilon + (1-\gamma)(b^u + d_j - c_j) \\
&- (1-\gamma)\beta(1-P_j(h))(1-\delta)E_{\epsilon',h'}[N_{j+1}(h,'\epsilon,0) \mid h,j] \\
&+ \gamma\beta(1-P_j(h))E_{\epsilon',h'}[max\{J_{j+1}(h',\epsilon',0),0\} + \delta V \mid h,j] \\
&+ (1-\gamma)\beta(1-P_j(h))(1-\delta)(1-p)E_{h'}[O_{j+1}(h') \mid h,j] \\
&+ (1-\gamma)\beta(1-P_j(h))(1-\delta)pE_{h'}\left[\int N_{j+1}(h',\epsilon',1)dF(\epsilon') \mid h,j\right] \\
&+ (1-\beta P_j(h))\gamma V
\end{aligned} \tag{14}$$

Using

$$\begin{aligned}
N_{j+1}(h',\epsilon,0) &= max\{W_{j+1}(h',\epsilon',0), O_{j+1}(h',\epsilon',0)\} \\
&= max\{W_{j+1}(h',\epsilon',0) - O_{j+1}(h',\epsilon',0), 0\} + O_{j+1}(h')
\end{aligned} \tag{15}$$

$$max\{J_{j+1}(h',\epsilon',0),V\} = max\{J_{j+1}(h',\epsilon',0) - V, 0\} + V \tag{16}$$

and the sharing rule, we have

$$N_{j+1}(h',\epsilon',0) = \frac{\gamma}{1-\gamma}max\{J_{j+1}(h',\epsilon',0)\} + O_{j+1}(h') \tag{17}$$

Plugging these in:

$$\begin{aligned}
w_j(h,\epsilon,0) =& \gamma a_j\epsilon + (1-\gamma)(b^u + d_j - c_j) \\
&+ (1-\gamma)\beta(1-P_j(h))(1-\delta)pE_{h'}\left[\int N_{j+1}(h',\epsilon',1)dF(\epsilon') - O_{j+1}(h') \mid h,j\right] \\
&- \gamma(V - \beta V)
\end{aligned} \tag{18}$$

Following the similar procedure, the wage for a new hire ($n = 1$) is

$$w_j(h, \epsilon, 1) = w_j(h, \epsilon, 0) - \gamma\kappa \tag{19}$$

The wage schedule for the case when retirement is the relevant outside option, in other words, $O_j(h) = R_j$ can be derived in the similar manner as above:

$$\begin{aligned} w_j(h, \epsilon, 0) =& \gamma a_j \epsilon + (1-\gamma)(\alpha_j b^r + d_j + \nu) + (1-\gamma)\beta b^r \frac{1-\beta^{T-j-1}}{1-\beta}(\alpha_j - \alpha_{j+1}) \\ & - \gamma(V - \beta V) \end{aligned} \tag{20}$$

where the second to last term is the gain/loss from postponing retirement by one period.

For a newly hired worker, $w_j(h, \epsilon, 1) = w_j(h, \epsilon, 0) - \gamma\kappa$.

B Worker flows

Let $Pr(\epsilon_{j+1} \geq \epsilon' \mid \epsilon_j = \epsilon) = G(\epsilon' \mid \epsilon)$ and define

$$\mathrm{I}_{j+1}(h', \epsilon', n) = \begin{cases} 1 & if\, h' = H'\, and\, E' \geq \epsilon' > \epsilon^0_{j+1}(h', n) \\ 0 & otherwise \end{cases} \tag{21}$$

$$\mathrm{I}^H_{j+1}(h') = \begin{cases} 1 & if\, h' = H' \\ 0 & otherwise \end{cases} \tag{22}$$

Then the age, health and match-specific productivity dynamics of worker flows are given by

$$\begin{aligned} e_{j+1}(H', E') =& (1-\delta) \sum_h \int_{\epsilon'} \int_{\epsilon} \mathrm{I}_{j+1}(h', \epsilon', 0) \pi_j(h', h)(1 - P_j(h)) e_j(h, \epsilon)\, dG(\epsilon' \mid \epsilon) d\epsilon \\ & + (1-\delta) p(\theta) \sum_h \int_{\epsilon'} \mathrm{I}_{j+1}(h', \epsilon', 1) \pi_j(h', h)(1 - P_j(h)) u_j(h)\, dF(\epsilon') \end{aligned} \tag{23}$$

$$
\begin{aligned}
u_{j+1}(H') =& (1-(1-\delta)p(\theta))\sum_h \mathrm{I}^H_{j+1}(h')\pi_j(h',h)(1-P_j(h))u_j(h) \\
&+ p(\theta)(1-\delta)\sum_h \int_{\epsilon'} (1-\mathrm{I}_{j+1}(h',\epsilon',1))\pi_j(h',h)(1-P_j(h))u_j(h)dF(\epsilon') \\
&+ \delta \sum_h \int_{\epsilon} \mathrm{I}^H_{j+1}(h')\pi_j(h',h)(1-P_j(h))e_j(h,\epsilon)d\epsilon \\
&+ (1-\delta)\sum_h \int_{\epsilon'}\int_{\epsilon}(1-\mathrm{I}_{j+1}(h',\epsilon',0))\pi_j(h',h)(1-P_j(h))e_j(h,\epsilon)dG(\epsilon' \mid \epsilon)d\epsilon
\end{aligned}
\tag{24}
$$

$$
D_{j+1}(H') = \sum_h \mathrm{I}^H_{j+1}(h')\pi_j(h',h)P_j(h)(e_j(h)+u_j(h)) + \sum_h \mathrm{I}^H_{j+1}(h')\pi_j(h',h)D_j(h) \tag{25}
$$

where $e_j(h) = \int_\epsilon e_j(h,\epsilon)d\epsilon$.

$$
\begin{aligned}
r_{j+1}(H') =& \sum_h \mathrm{I}^H_{j+1}(h')\pi_j(h',h)R_j(h) + \\
& \mathrm{I}^U_{j+1}\Big[(1-(1-\delta)p(\theta))\sum_h \mathrm{I}^H_{j+1}(h')\pi_j(h',h)(1-P_j(h))u_j(h) \\
&+ p(\theta)(1-\delta)\sum_h \int_{\epsilon'} (1-\mathrm{I}_{j+1}(h',\epsilon',1))\pi_j(h',h)(1-P_j(h))u_j(h)dF(\epsilon') \\
&+ \delta \sum_h \int_{\epsilon} \mathrm{I}^H_{j+1}(h')\pi_j(h',h)(1-P_j(h))e_j(h,\epsilon)d\epsilon \\
&+ (1-\delta)\sum_h \int_{\epsilon'}\int_{\epsilon}(1-\mathrm{I}_{j+1}(h',\epsilon',0))\pi_j(h',h)(1-P_j(h))e_j(h,\epsilon)dG(\epsilon' \mid \epsilon)d\epsilon\Big]
\end{aligned}
\tag{26}
$$

where I^U_{j+1} is an indicator function that takes value 1 if $j+1 \geq T^u(h')$ and zero otherwise.

C Solving the model numerically

Solving the model includes finding:

- value functions $W_j(h,\epsilon,n)$, $J_j(h,\epsilon,n)$, $U_j(h)$, R_j and R^D_j
- reservation productivities $\epsilon^0_j(h,n)$
- wages $w_j(h,\epsilon,n)$

- labor market tightness θ
- stationary distributions, $u_j(h)$, $e_j(h)$ and $D_j(h)$ and $r_j(h)$, of people in each labor market state for each combination of h and j

D Solving the model numerically

Solving the model includes finding:

- value functions $W_j(h,\epsilon,n)$, $J_j(h,\epsilon,n)$, $U_j(h)$, R_j and R_j^D
- reservation productivities $\epsilon_j^0(h,n)$
- wages $w_j(h,\epsilon,n)$
- labor market tightness θ
- stationary distributions, $u_j(h)$, $e_j(h)$ and $D_j(h)$ and $r_j(h)$, of people in each labor market state for each combination of h and j

I follow a solution method that builds on the algorithm used in Bils et al. [1].

1. Make an initial guess for the wage $w^0(h,\epsilon,0;\theta^0)$ and $w^0(h,\epsilon,1;\theta^0)$ and labor market tightness θ^0.
2. Calculate value functions for filled vacancies and workers in different labor market states using these guesses. Use backward iteration to solve for the value functions, since the model has a finite horizon.
3. Solve for an updated wage guess from the value function for a filled vacancy (separately for those in a new and those in an on-going on employment relationship). In other words, for an on-going relationship, from

$$\begin{aligned}&w_j^1(h,\epsilon,0;\theta^0) = a_j\epsilon - J_j(h,\epsilon,0;w^0(h,\epsilon;\theta^0))\\&+\beta(1-P_j(h))E\Big[max\left\{J_{j+1}(h',\epsilon',V;w^0(h,\epsilon,0)),0\right\} \mid h,j\Big] + \beta P_j(h)V\end{aligned}$$

   where $J_j(h,\epsilon,0;w^0(h,\epsilon;\theta^0))$ is computed using the first order condition for the Nash bargaining problem
4. Compare $w^1(h,\epsilon,0;\theta^0)$ and $w^1(h,\epsilon,1;\theta^0)$ to $w^0(h,\epsilon,0;\theta^0)$ and $w^0(h,\epsilon,1;\theta^0)$, respectively. If the difference is smaller than 10^-4, move on to step 5. Otherwise, update guess $w^0(h,\epsilon,0;\theta^0) = (1-\alpha)w^0(h,\epsilon,0;\theta^0) + \alpha w^1(h,\epsilon,0;\theta^0)$ and $w^0(h,\epsilon,1;\theta^0) = (1-\alpha)w^0(h,\epsilon,1;\theta^0) + \alpha w^1(h,\epsilon,1;\theta^0)$,and start again from step 2
5. Compute the quit and job start margins from rules $W_j(h,\epsilon,0) - O_j(h) = 0$ and $W_j(h,\epsilon,1) - O_j(h) = 0$, given the converged wage schedules $w^0(h,\epsilon,0;\theta^0)$ and $w^0(h,\epsilon,1;\theta^0)$
6. Compute the optimal retirement age from unemployment and employment for each health state given the converged wage schedules $w^0(h,\epsilon,0;\theta^0)$ and $w^0(h,\epsilon,1;\theta^0)$

7. Calculate the stationary distribution of unemployed, employed, disabled and retired for each age and health state using the value functions, the reservation productivities and laws of motion in section 3.3

8. Compute labor market tightness θ^1 that satisfies the job creation condition. Compare this to θ^0. If the difference is smaller than 10^{-4}, this is the steady state. Otherwise, update guess $\theta^0_{new} = (1-\alpha)\theta^0 + \alpha\theta^1$ and go back to step 2.

E Method for solving the calibration problem

As described in section 5.2, some of the parameters are set exogenously based on outside information whereas remaining model parameters are determined endogenously by targeting selected steady state targets. The endogenously set parameters are jointly determined by minimizing the distance between the targets and model generated moments in equilibrium.

The parameter values are found by minimizing the square of percentage errors between the targets and model generated values. Denoting the vector of endogenously set parameters by $\boldsymbol{\theta}$, number of targets by T and model generated values by $model_i$, I solve the following problem using Matlab's *Patternsearch* solver with Latin hypercube search and MADS Positive Basis Np1 polling algorithm:

$$\min_{\boldsymbol{\theta}} \quad \sum_{i=1}^{T} \left(\frac{target_i(\boldsymbol{\theta}) - model_i(\boldsymbol{\theta})}{target_i(\boldsymbol{\theta})} \right)^2$$

F Reduced retirement benefit schedule

Retirement benefit, % of full benefit		
Age	**Old**	**New**
62	80,0 %	55,0 %
62.25	81.7%	56.4%
62.5	83.3%	57.8%
62.75	85,0 %	59.2%
63	86.7%	60.6%
63.25	88.3%	62,0 %
63.5	90,0 %	63.4%
63.75	91.7%	64.8%
64	93.3%	66.3%
64.25	95,0 %	67.7%
64.5	96.7%	69.1%
64.75	98.3%	70.5%
65	100.0 %	71.9%
65.25	100.0 %	73.3%
65.5	100.0 %	74.7%
65.75	100.0 %	76.1%
66	100.0 %	77.5%
66.25	100.0 %	78.9%
66.5	100.0 %	80.3%
66.75	100.0 %	81.7%
67	100.0 %	83.1%
67.25	100.0 %	84.5%
67.5	100.0 %	85.9%
67.75	100.0 %	87.3%
68	100.0 %	88.8%
68.25	100.0 %	90.2%
68.5	100.0 %	91.6%
68.75	100.0 %	93,0 %
69	100.0 %	94.4%
69.25	100.0 %	95.8%
69.5	100.0 %	97.2%
69.75	100.0 %	98.6%
70	100.0 %	100.0 %

Table 7: Reduced retirement benefit schedule

G Disability probabilities

	Health		
Age	**Excellent**	**Good**	**Fair**
20-24	0.0001	0.0000	0.0000
25-29	0.0003	0.0002	0.0003
30-34	0.0006	0.0002	0.0071
35-39	0.0008	0.005	0.009
40-44	0.0004	0.0027	0.0125
45-49	0.0002	0.0004	0.011
50-54	0.0001	0.0003	0.0095
55-59	0.0007	0.0006	0.0128
60-64	0.0000	0.0032	0.008
65-69	0.0000	0.0000	0.0088
Over 69	0.0000	0.0000	0.0000

Table 8: Quarterly disability transition probabilities by age-group

H Health transition probabilities

	Health transition								
Age	**EE**	**EG**	**EF**	**GE**	**GG**	**GF**	**FE**	**FG**	**FF**
20-24	0.94	0.05	0.009	0.168	0.801	0.031	0.046	0.189	0.765
25-29	0.952	0.045	0.003	0.142	0.826	0.032	0.051	0.163	0.785
30-34	0.952	0.045	0.003	0.118	0.85	0.032	0.027	0.121	0.852
35-39	0.953	0.046	0.001	0.109	0.848	0.043	0.014	0.12	0.867
40-44	0.945	0.051	0.004	0.098	0.862	0.041	0.007	0.096	0.897
45-49	0.942	0.057	0.001	0.109	0.852	0.039	0.003	0.107	0.889
50-54	0.939	0.058	0.003	0.08	0.867	0.053	0.008	0.079	0.913
55-59	0.94	0.059	0.001	0.068	0.884	0.048	0.005	0.061	0.934
60-64	0.922	0.076	0.002	0.082	0.869	0.049	0.005	0.066	0.929
65-69	0.898	0.102	0.001	0.079	0.865	0.056	0.002	0.062	0.937
Over 69	0.878	0.104	0.019	0.082	0.835	0.083	0.013	0.052	0.935

Table 9: Quarterly health transition probabilities by age-group

I Age-directed search

Value of unemployment is

$$\begin{aligned} U_j(h) =& b^u - c_j + \beta(1 - P_j(h))E_{h'}\Big[p_j(1-\delta)\int N_{j+1}(h', \epsilon', 1)\, dF(\epsilon') \\ & + (1-p_j)(1-\delta))\, O_{j+1}(h') \mid h, j\Big] + P_j(h)\beta R^D_{j+1} \end{aligned} \tag{27}$$

This also changes the job start and quit margins when unemployment is the relevant outside option (see Equation (28)). If the match finding probability for a given age is lower than the aggregate match finding probability in the benchmark model, $p(\theta_j) < p$, job start (quit) margin will be lower in the age-directed model. Furthermore, age-dependent job finding probability has an effect on the age-profile of job start and quit margins through attractiveness of unemployment search relative to continuing in existing job.

$$\begin{aligned} \epsilon^0_j(h, n) =& \frac{\gamma}{a_j} \underbrace{(b^u + d_j + n\kappa - c_j)}_{\text{current gain from outside option}} - \frac{\gamma}{a_j} \underbrace{(V - \beta V)}_{\text{option value of vacancy}} \\ & + \frac{\gamma}{a_j} \underbrace{\beta(1 - P_j(h))(1-\delta)p(\theta_j)E_{h'}\left[\int N_{j+1}(h', \epsilon', 1)dF(\epsilon') - O_{j+1}(h') \mid h, j\right]}_{\text{expected gain from job search}} \\ & - \frac{1}{a_j} \underbrace{\beta(1 - P_j(h))(1-\delta)E_{\epsilon', h'}\left[N_{j+1}(h', \epsilon', 0) - O_{j+1}(h') \mid h, j\right]}_{\text{expected gain from continuing in existing job}} \end{aligned} \tag{28}$$

The value of a vacancy filled with a worker of age j, health h, match-specific productivity ϵ and type of employment relationship n is now expressed as:

$$\begin{aligned} J_j(h, \epsilon, n) =& a_j\epsilon - w_j(h, \epsilon, n) - n\kappa \\ & + \beta(1 - P_j(h))E_{\epsilon', h'}\Big[(1-\delta)\, max\{J_{j+1}(h', \epsilon', 0), V\} + \delta V \mid h, j\Big] + P_j(h)\beta V \end{aligned} \tag{29}$$

Differently to the benchmark model, the value of an open vacancy V is now defined as $V = max\, V_j$, $j = \{1, ..., T-1\}$, because firms are free to choose in which sub-market to post vacancies. Value of open vacancy posted in sub-market j is, in turn, given by:

$$V_j = - k + \beta q(\theta_j) \sum_{m=1}^{H} \Bigg((1 - P_j(h^m)) \frac{u_j(h^m)}{u_j} E_{h'} \Bigg[(1-\delta) \int max\{J_{j+1}(h', \epsilon', 1), V\} \, dF(\epsilon') + \delta V \mid h, j \Bigg] + \frac{u_j(h^m)}{u_j} P_j(h^m) V \Bigg) + \beta(1 - q_j(\theta))V \tag{30}$$

In equilibrium, there is free entry in all sub-markets and, thus, $V_j = 0$ for all j. The job creation equation in sub-market j then becomes:

$$\frac{k}{q(\theta_j)} = \beta \sum_{m=1}^{H} (1 - P_j(h^m)) \frac{u_j(h^m)}{u_j} E_{h'} \Bigg[(1-\delta) \int max\{J_{j+1}(h', \epsilon', 1), 0\} \, dF(\epsilon') \mid h, j \Bigg] \tag{31}$$