The Dark Corners of the Labor Market

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Abstract

What can happen to unemployment after a severe disruption of the labor market? Standard models predict a reversion to a long-run steady state. By contrast, this paper shows that a large shock may set the economy on a path towards a different steady state with possibly extreme unemployment. This result follows from the empirical behavior of the U.S. job finding rate over the last 25 years. First, I estimate a reduced-form model for the labor market and show that –once allowing for nonlinearities– it implies a stable steady state around 5 percent unemployment and an unstable one around 10 percent unemployment. Second, I consider an extension of a basic Diamond-Mortensen-Pissarides (DMP) model in which multiple steady states arise due to skill losses upon unemployment, following Pissarides (1992). Based on only observed rates of job loss, this model endogenously explains most of the observed fluctuations in the job finding rate and the unemployment rate, thereby dramatically improving over a basic DMP model with a single steady state.

Key Words: Matching models, multiple steady states

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The main lesson of the crisis is that we were much closer to those dark corners than we thought—and the corners were even darker than we had thought too. – Olivier Blanchard (2014), in “Where Danger Lurks”.

1 Introduction

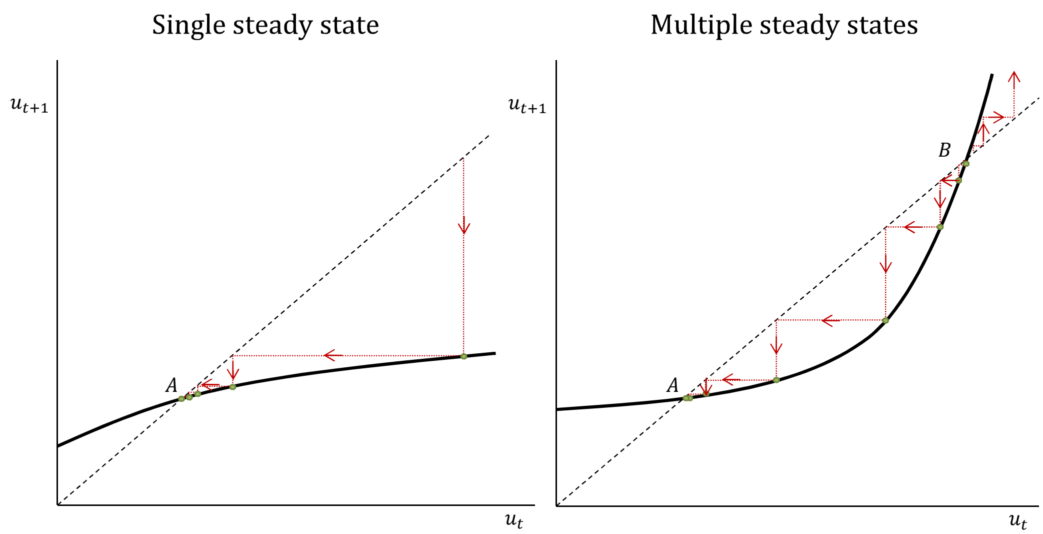
A large body of literature has developed models of cyclical swings in the labor market, often within the search and matching paradigm of Diamond, Mortensen and Pissarides (DMP). Most of these models predict that, following a one-time shock, unemployment gradually reverts back to a unique steady-state level (see Figure 1, left panel). An episode of sustained high unemployment is then the result of an unfortunate repetition of adverse shocks. By contrast, in models with multiple long-run equilibria, a large shock may set the labor market on a path towards a “dark corner”: an economic state with high unemployment and without any tendency to revert back (see Figure 1, right panel).[1]

This paper shows that models with multiple steady states –while seldom used for quantitative purposes– can provide a superior account of the dynamics of the U.S. labor market over the last 25 years. I reach this conclusion based on two complementary sets of evidence, deriving from (i) a reduced-form model estimated using data over the period 1990 until 2015 and (ii) a calibrated search and matching model, confronted with the same data.

In the reduced-form analysis, I estimate steady-state rates of unemployment based on a forecasting regression for the job finding rate, combined with a transition identity for the unemployment rate and an estimate of the long-term rate of job loss. If one assumes that the job finding rate forecast is a linear function of current labor market transition rates, the implied steady state is by construction unique. This, however, is a special case: including the unemployment rate in the linear regression is already

[1] For examples of models with multiple steady states, see Diamond (1982), Pissarides (1992), and Kaplan and Menzio (2014), as well as references mentioned in the conclusion of this paper.

Figure 1: Illustrated models for the unemployment rate (u_t).



sufficient to obtain multiple steady states.[2] Further, multiple steady states can arise when allowing for non-linearities in the forecasting equations. I show that allowing for both sources of non-linearity can substantially improve the statistical performance of the model.

The reduced-form estimates imply a stable steady state of around five percent unemployment and an unstable steady state of around ten percent unemployment. From these numbers it follows that, during the aftermath of the recent Great Recession, the U.S. economy may have narrowly escaped a transition towards an extreme level of unemployment: a dark corner. This conclusion is robust to the choice of forecasting horizon and alternative measurements of labor market flows.

The second set of evidence is based on a quantitative horse race between two calibrated search and matching models of the labor market, both allowing for exogenous shocks to the rate of job loss. The first one is a basic DMP model with a single steady

[2] A simple way to see this is to consider the transition identity $u_{t+1} = u_t\,(1 - ue_t) + (1 - u_t)\,eu_t$, where u_t is the unemployment rate in period t, ue_t is unemployment outflow rate, and eu_t is the unemployment inflow rate. If either of the two transition rates depends linearly on u_t, the right-hand side becomes quadratic in u_t, giving rise to two solutions for a steady state level $\overline{u} = u_{t+1} = u_t$.

state. The second model is an extension in which unemployment creates a loss of human capital, following Pissarides (1992). In this model, skill losses associated with higher unemployment discourage hiring, which further pushes up unemployment and, as a result, may give rise to multiple steady states. I calibrate the parameters of this model such that the implied steady states are consistent with the reduced-form evidence.

I simulate both search and matching models, feeding in observed fluctuations in the rate of job loss as the only source of aggregate uncertainty. I find that the extended model with multiple steady states can closely reproduce the job finding rate and unemployment rate observed over the sample. This is not a mechanical finding, since fluctuations in the job finding rate are more persistent than fluctuations in the rate of job loss. Instead, the result is due to a strong internal propagation mechanism, governed by the unemployment rate, which describes the data surprisingly well. By contrast and in line with previous results in the literature, the basic DMP model fails to explain the data by a wide margin. The fluctuations in the job finding rate and the unemployment rate as predicted by this model are much too small and only mildly correlated with their empirical counterparts.

Considering the aftermath of the Great Recession of 2008, the multiple-steady-state models (both the structural and reduced-form version) can account particularly well for the slow recovery of the labor market. The right panel of Figure 1 clarifies this point. Suppose the economy starts from the stable steady state steady state with low unemployment (point A in the figure, about 5 percent in the data). Next, a one-time wave of job losses brings unemployment just below the second, unstable steady state (point B in the figure, about 10 percent in the data). Ultimately, unemployment will revert back to its initial level, following the step-wise path illustrated by the red line. Initially, however, the speed of this transition is slow.[3] By contrast, in the single-steady-state model (Figure 1, left panel), the speed of transition back is the fastest in the initial periods following the shock. The latter property conflicts with the fact that

[3] The speed of the transition can be inferred from the distance of the points on the step-wise path back to the initial steady state.

the increase in unemployment during the Great Recession was not only the largest over the sample, but also the most persistent.

The reduced-form and structural models are shown to be closely linked, as the forecasting equations are the reduced-form equivalents of the core Euler equation for the vacancy posting decision of the firms in the DMP models. In the basic DMP model, the unemployment rate is not a state variable and is therefore irrelevant for the firm's vacancy posting decision. However, I find that a baseline regression without unemployment produces autocorrelation in the residuals of non-overlapping forecasts, invalidating the model. Including the unemployment rate in the forecasting regression in accordance with the skill loss model, however, absorbs this autocorrelation and improves forecast accuracy. This analysis is reminiscent of an empirical literature that sheds light on structural models by taking their Euler equations to the data. Since Hall (1978), researchers have used this approach to scrutinize a large variety of theories, including models of investment, asset pricing models, and New-Keynesian models. Somewhat surprisingly, the DMP model has not received the same kind of attention, even though its core can be conveniently summarized by a single Euler equation.[4]

The empirical analysis also relates to a literature investigating the time series properties of labor market transition rates, see e.g. Hall (2005), Shimer (2005), Elsby *et al.* (2006), Fujita and Ramey (2009). Barnichon and Nekarda (2012) develop a forecasting model for unemployment and emphasize the benefits of conditioning separately on unemployment in- and outflows. The rationale is that the two flow rates have distinctly different time series properties and therefore each contain valuable information on the state of the economy, which would be lost when conditioning on only the current level of unemployment. I follow the "flow approach", but focus on estimating steady-state rates of unemployment rather than constructing near-term forecasts. Finally, the finding that there may be multiply steady state connects this paper to an empirical literature investigating the possibility of "hysteresis" in unemployment, see Ball (2009)

[4]Instead, many researchers have focused on whether plausible model calibrations can generate sufficient volatility in unemployment and vacancies (Shimer (2005), Hagedorn and Manovskii (2008)).

for an overview.

On the theoretical side, I evaluate Pissarides' skill-loss mechanism in a more realistic environment. An important question is whether moderate skill losses can be consistent with multiple steady states.[5] I find that reproducing the steady states estimated from the data requires a skill loss that is equivalent to less than three weeks of wage income. This number is small in comparison to empirical evidence on the effect of unemployment on re-employment wages, as reported in for example Schmieder *et al.* (2014). This is reassuring, given that these empirical estimates may not purely reflect a loss of human capital, but also the effect of unemployment on a worker's bargaining position.[6]

The remainder of this paper is organized as follows. Section 2 presents the reduced-form empirical evidence. Section 3 describes the search models and presents quantitative simulation results.

2 Empirical evidence

The unemployment rate can be thought of as a stock that is determined by rates of in- and outflows of workers, which in turn may be affected by the unemployment rate. This section uses U.S. data to study the dynamic interactions between these stocks and flows and uncover the resulting steady states. This is done without specifying a full structural model, but rather using a reduced-form framework consisting of forecasting equations for the flow rates and a transition identity for the unemployment rate.

[5] Pissarides (1992) analyzes a model in which agents live for two periods and all lose their after the first period. His main point is to highlight the existence of multiple steady states, under certain parameter values. However, the stylized nature of his model makes it difficult to assess whether such parameterizations are realistic.

[6] Jarosch (2014) uses an estimated job ladder model to distinguish between the various sources of wage losses upon displacement. He finds that a loss of general human capital is the most important contributor to the overall wage loss. In his framework, a substantial part of the costs associated with the human capital loss is borne by the firms, in line with a key assumption required for the skill loss mechanism to operate. However, the possibility of multiple steady states does not arise in his framework as the vacancy posting decision is exogenous.

2.1 Reduced-form framework

Consider a labor market in which workers flow stochastically between employment and unemployment. Time is discrete and indexed by t. Job losses occur at the very beginning of each period and the probability that a worker loses her job is denoted by $\rho_{x,t}$. Right after job losses occur, a labor market opens and a search and matching process between job searchers and firms takes place. The pool of job searchers consists of those workers who just lost their jobs and those who were already unemployed in the previous period.[7] The probability that a job searcher finds a job is denoted by $\rho_{f,t}$. Those who find a job during period t become employed within the same period. Hence, job losers may immediately find a new job without becoming unemployed. It follows that the unemployment rate, u_t, evolves according to the following transition identity:

$$u_t = \left(1 - \rho_{f,t}\right) u_{t-1} + \rho_{x,t} \left(1 - \rho_{f,t}\right) \left(1 - u_{t-1}\right), \tag{1}$$

where the first term on the right-hand side captures the number of previously unemployed workers and the second term is the number of newly unemployed workers.

The job finding rate and the job loss rate are determined as policy functions of the aggregate state of the economy. In a fully structural model, these functions would result from agents' decisions. Here, I avoid imposing such structure and use a reduced-form approach instead. Let $\rho_{f,t} = \rho_f\left(\mathcal{S}_t\right)$ and $\rho_{x,t} = \rho_x\left(\mathcal{S}_t\right)$ denote the policy functions for the two transition rates, where $\mathcal{S}_t$ is the aggregate state. Without loss of generality, one can partition the aggregate state as $\mathcal{S}_t = \left\{\mathbf{s}_{1,t}, \mathbf{s}_{2,t}\right\}$, where $\mathbf{s}_{1,t}$ is a vector of length n_1 containing various lagged values of $\rho_{f,t}$, $\rho_{x,t}$, and u_t, and $\mathbf{s}_{2,t}$ is a vector of length n_2 containing additional state variables, such as exogenous shocks.

How can this framework be used to identify steady-state rates of unemployment? One possibility is to obtain direct estimates of the policy functions $\rho_f\left(\mathcal{S}_t\right)$ and $\rho_x\left(\mathcal{S}_t\right)$ using a regression and combine these with the transition identity (1). However, this approach is fraught with problems. First, estimating the policy functions requires full knowledge of what variables are contained in the economic state vector, as they are to

[7] I abstract from flows in and out of the labor force and on-the-job search.

be included in the regression. However, any particular stance on the variables in $\mathbf{s}_{2,t}$ rests on ad hoc assumptions. Further, these state variables may not be observable to the researcher. In fact, many search and matching models in the literature rely on aggregate shocks to the productivity of workers, which are difficult to observe directly in the data. Finally, using estimated policy functions is a rather indirect way of estimating steady states, since the policy functions describe short-run dynamics, whereas the steady states summarize the joint long-run behavior of the variables. As such, the approach may be sensitive to misspecification and estimation error.

I address these problems by estimating *forecasting* equations for the transition rates, rather than policy functions. This approach uses direct information on the longer-run behavior of the transition rates and requires much weaker assumptions on the state vector. In particular, it neither forces the researcher to decide which particular variables enter $\mathbf{s}_{2,t}$, nor does it create problems when $\mathbf{s}_{2,t}$ is not entirely observable. These advantages are due to the fact that the economic state can be revealed implicitly by observed outcomes. To fix ideas, let $\mathbf{s}_{2,t}$ contain two variables ($n_2 = 2$) so that $\mathbf{s}_{2,t}$ is uniquely pinned down by the two policy functions $\rho_{f,t} = \rho_f(\{\mathbf{s}_{1,t}, \mathbf{s}_{2,t}\})$ and $\rho_{x,t} = \rho_x(\{\mathbf{s}_{1,t}, \mathbf{s}_{2,t}\})$.[8] Further, let q denote the mapping that solves for $\mathbf{s}_{2,t}$ from these two restrictions, given $\mathbf{s}_{1,t}$, $\rho_{x,t}$, and $\rho_{f,t}$. These three arguments are all observable and jointly contain the same information content as $\mathcal{S}_t = \{\mathbf{s}_{1,t}, \mathbf{s}_{2,t}\}$. Thus, if one were to make a forecast of the transition rates, $\mathbf{s}_{1,t}$, $\rho_{x,t}$, and $\rho_{f,t}$ would contain all available information. Exploiting this, one can express the k^{th}-period ahead forecasts for the two transition rates as:

$$\begin{aligned}
\mathbb{E}_t\rho_{r,t+k} &\equiv \mathbb{E}\left[\rho_r(\mathcal{S}_{t+k})\middle|\,\mathcal{S}_t\right], \\
&= \mathbb{E}\left[\rho_r(\mathcal{S}_{t+k})\middle|\,\mathbf{s}_{1,t}, q\left(\mathbf{s}_{1,t}, \rho_{f,t}, \rho_{x,t}\right)\right], \\
&= \mathbb{E}\left[\rho_r(\mathcal{S}_{t+k})\middle|\,\mathbf{s}_{1,t}, \rho_{f,t}, \rho_{x,t}\right],
\end{aligned}$$

for $r = f, x$, and where $\mathbb{E}$ is the expectations operator. It follows that the forecasts are functions of observables only, which can be estimated without reference to the nature

[8] It is straightforward to allow for $n_2 > 2$. This requires adding an additional observable to the model, as well as a forecasting equation for this variable.

of the state variables in $\mathbf{s}_{2,t}$.

Once the forecasting equations have been estimated, it is straightforward to solve for the steady state(s). Let upper bars denote steady-state values. Necessary conditions for a steady state are that $\overline{\rho}_r = \mathbb{E}\left[\rho_r\left(\mathcal{S}_{t+k}\right)\middle|\,\overline{\mathbf{s}}_1, \overline{\rho}_f, \overline{\rho}_x\right]$ for $r = f, x$. Note that these two conditions are not sufficient since $\overline{\mathbf{s}}_1$ needs to be jointly determined. To obtain the n_1 additional conditions required, one can exploit the fact that any of element in $\mathbf{s}_{1,t}$ is given by either $\rho_{f,t-l}$, $\rho_{x,t-l}$, or u_{t-l}, for some lag $l \geq 1$. The steady-state condition for these variables are, respectively, $\rho_{f,t-l} = \overline{\rho}_f$, $\rho_{x,t-l} = \overline{\rho}_x$ and $u_{t-l} = \overline{u} = \frac{\overline{\rho}_x\left(1-\overline{\rho}_f\right)}{\overline{\rho}_x\left(1-\overline{\rho}_f\right)+\overline{\rho}_f}$, where the final equality is the steady-state solution of Equation (1). Selecting the appropriate condition for each variable in $\mathbf{s}_{1,t}$ gives exactly enough restrictions to solve for the steady state.

What can be said on the possibility of multiple steady states? At this point, no restrictions have been placed on the functional forms of the forecasting equations. However, when estimating these equations parametrically, such assumptions are unavoidable. If one allows the forecasts to be non-linear functions of $\mathbf{s}_{1,t}$, $\rho_{x,t}$, and $\rho_{f,t}$, multiple steady states can arise, since non-linear systems of equations can have multiple fixed-points.[9] But even if the forecasts are restricted to be linear in the aforementioned variables, multiple steady states may arise if the state $\mathbf{s}_{1,t}$ some lagged value of the unemployment rate, u_{t-l}. In that case, the steady-state solutions $\overline{\rho}_x$ and $\overline{\rho}_f$ are linear functions of $\overline{u}$ and therefore $\overline{u} = \frac{\overline{\rho}_x\left(1-\overline{\rho}_f\right)}{\overline{\rho}_x\left(1-\overline{\rho}_f\right)+\overline{\rho}_f}$ becomes a non-linear function of itself, with multiple fixed points.

By contrast, if the two forecasting equations are assumed to be linear *and* $\mathbf{s}_t$ is assumed not to include any lagged value of u_t, multiple steady states are ruled out by construction.[10] This latter case may strike one as overly restrictive, but it warrants special attention since it corresponds to a basic DMP model, as will be shown in the next section. Finally, the stability properties of the steady state(s) can be analyzed by considering small perturbations of the variables around their steady state levels,

[9] A simple example is the forecasting equation $\mathbb{E}_t\rho_{f,t+k} = \gamma_0 + \gamma_1\rho_{f,t} + \gamma_2\rho_{f,t}^2$, where γ_0, γ_1, and γ_2 are coefficients. The corresponding steady-state condition has two solutions.

[10] In that case, all steady-state conditions are linear, allowing for only one solution.

denoted by $\overline{\rho}'_f$, $\overline{\rho}'_x$ and $\overline{\mathbf{s}}'$. Stability requires that both transition rates are expected to move closer to the steady state, i.e. $\left|\mathbb{E}\left[\rho_r\left(\mathcal{S}_{t+k}\right)\middle|\,\overline{\mathbf{s}}', \overline{\rho}'_f, \overline{\rho}'_x\right] - \overline{\rho}_r\right| > |\overline{\rho}'_r - \overline{\rho}_r|$ for $r = f, x$.

In practical applications, there are several ways to choose between different candidate specifications for the forecasting functions and to diagnose potential problems. The first is to analyze the residuals of the forecast equations over the sample. For the model to be valid, these residuals should be uncorrelated across non-overlapping observations. Correlation among the residuals can indicate that a state variable is missing from the specification. Second, one can check if results are consistent across different forecast horizons. A final check on the preferred specification is to verify that the steady-state solution is robust to allowing for stronger non-linearities.[11]

It should be emphasized that the procedure by no means guarantees that $\overline{\rho}_f$, $\overline{\rho}_x$ and $\overline{u}$ are between zero and one for any steady-steady state solution. If any of these values is outside the unit interval, then the solution is not practically relevant. Such a finding, however, could be indicative of a corner solution (i.e. a solution in which one or more variables has value zero or one). The stability properties of the various steady states determine whether this is the case. I will come back to this issue in more detail when discussing the empirical findings.

2.2 Data and estimation

2.2.1 Data

I use monthly data for the U.S. labor market, observed over the period from January 1990 until November 2014. There are three variables to be measured: the unemployment rate, u_t, the job finding rate, $\rho_{f,t}$, and the job loss rate, $\rho_{x,t}$. I measure u_t as the civilian unemployment rate from the Current Population Survey (CPS). The job finding rate is measured as the transition rate from unemployment to employment, as reported in the gross flow data from the CPS. The job loss rate is constructed to be

[11] For example, if polynomials are used to characterize the forecasting functions, additional higher-order terms may be added.

consisted with the transition equation (1) and the data on u_t and $\rho_{f,t}$.[12]

[Figure 4 here]

Figure 4 presents a plot of the three time series over the sample period. The job finding rate, plotted in the upper panel, is subject to slow-moving fluctuations. The autocorrelation of the series at a one year horizon is 0.73. Particularly striking is the slow recovery of the job finding rate after the sharp decline in during 2008. The job loss rate, plotted in the middle panel, displays much less persistence.[13] The increase in $\rho_{x,t}$ around 2008 is also less persistent than the decline in $\rho_{f,t}$.

The bottom panel of Figure 4 shows the evolution of the unemployment rate over the sample. The figure also plots an approximations for the unemployment rate, defined as $u_t^* \equiv \frac{\rho_{x,t}(1-\rho_{f,t})}{\rho_{x,t}(1-\rho_{f,t})+\rho_{f,t}}$, which is the unemployment rate that would prevail if the current transition rates, $\rho_{x,t}$ and $\rho_{f,t}$, would remain fixed at their current levels permanently (see Hall (2005)). The approximation is very close to the actual unemployment rate, highlighting the direct link between transition rates and the unemployment rate.

2.2.2 Estimation

The forecasting equations are estimated parametrically. For brevity, I report results for three model specifications.[14] Model (I) assumes that the forecast $\mathbb{E}_t\rho_{f,t+k}$ is a linear function of $\rho_{f,t}$ and $\rho_{x,t}$. In model (II), the forecast is a linear function of $\rho_{f,t}$, $\rho_{x,t}$ and

[12] An alternative way to measure transition rates is to use CPS data on the number of unemployed workers with durations lower than one month to construct $\rho_{x,t}$, and to use the transition identity to back out a time series for $\rho_{f,t}$ that is consistent with the observed unemployment rate. I consider this alternative data source in the appendix and show that the results are very similar to those obtained from the gross flow data.

[13] The autocorrelation of the series at a one year horizon is only 0.15. Possibly, there is downward bias in this estimate, since the series appears to be subject to noisy measurement error. Computing the same statistic based on 3-month moving averages of both series gives a one-year correlation of 0.78 for $\rho_{f,t}$ and 0.36 for $\rho_{x,t}$, still a large difference.

[14] I have considered various alternative specifications, including an AR(1). These alternatives either perform poorly in terms of model diagnostics, or generate results similar to model (III).

u_t. In model (III) the forecast is linear in $\rho_{f,t}$, $\rho_{x,t}$, u_t and u_t^2.[15] All three models also include a constant.

For the rate of job loss, $\rho_{x,t}$, various alternative specifications are considered as well. In contrast to the job finding rate, I find that this data series is well described by a simple AR(1) process. To save space, results for $\rho_{x,t}$ are reported in the Appendix.

Figure 4 suggests that measured transition rates are noisy. This is not very surprising since CPS data are based on a survey among about 60,000 respondents, out of which only a small fraction experiences a change in employment status in a given month. The presence of noise can induce coefficient bias when estimating the forecast equations. To avoid such bias, I use an Instrumental Variable estimator, implemented through a standard Two Stage Least Squares procedure. As instruments, I use lags of the three variables, $\rho_{f,t-1}$, $\rho_{x,t-1}$, and u_{t-1}.

2.3 Empirical Findings

2.3.1 Model diagnostics

To assess the three forecasting specifications for $\rho_{f,t}$, I consider two types of diagnostics statistics, presented in Figure 5. Each of these statistics is computed for a range of forecast horizons, between $k = 1$ and $k = 36$ months. The left panel plots correlations of the residuals in month t and month $t+k+1$.[16] Models (I) and (II) produce positively correlated residuals at a wide range of forecasting horizons, indicating model misspecification. The corresponding residuals of model (III), by contrast, are not positively correlated. Thus, adding a non-linear function of the unemployed rate as a regressor absorbs the residual autocorrelation, which favors model (III) over the other two

[15] Strictly speaking, the reduced-form model specificies allows the forecast to be a function of $\rho_{f,t}$, $\rho_{x,t}$ and lagged values of $\rho_{f,t}$, $\rho_{x,t}$, u_{t-1}. The three specifications are consistent with this even though models (II) and (III) include u_t. To see why, note that given $\rho_{f,t}$ and $\rho_{x,t}$, u_{t-1} and u_t are directly linked via the transition identity, so they contain the same information.

[16] Due to overlapping observations, residuals of closer time periods are generally correlated and hence not useful to diagnose misspecification.

models.[17]

Allowing for non-linearities also improves the forecast accuracy of the model. This is shown in the right panel of Figure 5, which plots the R^2 statistic for the three specifications, again for a range of forecast horizons. Especially at longer horizons, model (III) produces a much better fit than the other two specifications.

[Figure 5 here]

To help understand which particular episodes drives the above results, Figure 6 plots the job finding rate, as well as a forecasts made two years in advance. To facilitate a visual comparison, all series have been smoothed using a one-year moving average filter. Considering the period until 2008, the forecast made using model (III) produces smaller average forecast errors and shows less variability than models (I) and (II). The latter indicates that model (III) implies stronger mean reversion around the steady state with low unemployment. Turning to the period between 2008 and 2011, none of the two models predicts the initial sharp decline in $\rho_{f,t}$, although the forecast of model (III) is somewhat closer to the realization than the forecast of the other two models. The lack of forecast accuracy over this period is to be expected, given that the financial crisis was a large unexpected shock to the economy. Turning to the aftermath of the Great recession, the period from 2011 onwards, there is a stark difference between the two forecasts. Models (I) and (II) predicts a substantial recovery, which conflicts with the persistently low job finding rate. By contrast, the two year ahead forecast made with Model (III) closely tracks the realization. The non-linearity thus helps to account for the persistence of the job finding rate in the aftermath of the Great Recession.

[Figure 6 here]

The Appendix explores alternative specifications and shows that adding additional variables to the third specification has little effect on the diagnostics statistics or the

[17] At a range of longer horizons, all models produce negative correlations. This, however is less concerning, since omitted variables typically do not produce negative autocorrelation in the residuals.

implied steady states. I therefore use model (III) as the baseline for further analysis, combined with an AR(1) model for $\rho_{x,t}$.

2.3.2 Implied steady states

Having estimated the forecasting equations for the transition rates, the implied steady state(s) can be computed following the procedure described in the previous section. Figure 7 visualizes the steady states by plotting $u_t^* \equiv \frac{\rho_{x,t}(1-\rho_{f,t})}{\rho_{x,t}(1-\rho_{f,t})+\rho_{f,t}}$ against u_{t+k}^*, with $\rho_{x,t}$ and $\rho_{x,t+k}$ both set to the sample average, and for a range of values for $\rho_{f,t}$with, and with $\rho_{f,t+k}$ computed using the forecasting equations estimated previously.[18] Intersections with the 45 degree lines satisfy all steady-state requirements, being $\rho_{x,t} = \mathbb{E}_t\rho_{x,t+k}$, $\rho_{f,t} = \mathbb{E}_t\rho_{f,t+k}$ and $u_t^* = u_t = \mathbb{E}_t u_{t+k} = u_{t+k}^*$. The four panels in Figure 7 plot these curves for four different forecast horizons equal to, respectively 6, 12, 24 and 36 months.

The point estimates of the baseline model, Model (III), deliver one steady state around 5.5 percent and one around 9.5 percent, which is a robust finding across the various forecast horizons.[19] The shape of the curve implies that the steady state with low unemployment is stable, whereas the one with high unemployment is not. It follows that there must be a third steady state with even higher, possibly extreme unemployment. However, the data have little to say about the precise location of this third steady state, given that this region of the economic state is not visited over the sample period.[20]

For illustration purposes, Figure 7 also plots the corresponding curve for Model (II), even though this model was rejected in favour of model (III). This model has a single steady state, which is located between six and seven percent unemployment. The shape

[18] Recall that $\rho_{x,t}$ follows an AR(1) process. Accordingly, its steady state value is estimated as the sample average.

[19] The shaded area's denote 95 percent confidence bands, computed using Newey-West standard errors. These bands include a wider range of values in the neighborhood of the point estimates, but almost without exception imply two distinct steady states.

[20] Outside the range of values for unemployment observed in the data, confidence bands become very wide.

of the curve implies that this steady state is stable.[21]

[Figure 7 here]

3 Theoretical model

According to the estimates of the previous section, a model with multiple steady-state rates of unemployment can provide a better description of the data than a model with a single steady state. But is it possible to construct a plausible structural model that generates the steady states estimated from the data? If so, how realistic are the dynamics of such a model vis-à-vis a more standard model of unemployment with a single steady state?

To answer these questions, I quantitatively evaluate two theoretical models of the labor market. The first is a basic search and matching model, à la by Diamond, Mortensen and Pissarides (DMP), with a single steady state. The second is an extension in which unemployment generates a loss of human capital, following Pissarides (1992).[22]

The remainder of this section is organized as follows. Subsection 3.1 describes the two models. Since the model with skill losses nests the basic model, I do not present the two models separately. Subsection 3.2 explores under what parameter values multiple steady states can arise, and discusses the relation between the reduced-form and the DMP models. Finally, Subsection 3.3 conducts a quantitative horse race between the two DMP models.

[21] For this model, there is a second intersection with the 45-degree line. However, this is for a negative unemployment rate. Given that the steady state with positive unemployment is a stable attractor, there is no corner solution with zero unemployment.

[22] Esteban-Pretel and Faraglia (2008) and Laureys (2014) also simulate extensions of a DMP model with skill losses, but do not consider calibrations with multiple steady states.

3.1 Model

The economy is populated by a unit measure of risk-neutral workers who own the firms. Time is discrete.

3.1.1 Workers

The transition structure and timing of the labor market are the same as in the reduced-form model. Employed workers lose their job with a probability $\rho_{x,t}$ at the very beginning of a period. This probability is exogenous, but subject to stochastic shocks, which are revealed when job losses occur. Subsequently, a labor market opens up to firms and to workers searching for a job. The pool of job searchers consists of those workers who just lost their jobs and those who were previously unemployed. The labor market is subject to search and matching frictions and only a fraction $\rho_{f,t} \in [0,1]$ of the job searchers meets with a firm. In the equilibrium, all workers who meet a firm become employed, so $\rho_{f,t}$ is also the job finding rate. It follows that the aggregate unemployment rate, u_t, evolves as in Equation (1). After the labor market closes, production and consumption take place. Unemployed workers obtain a fixed amount of resources b from home production, whereas employed workers receive wage income. Note that some job losers immediately find a new job, whereas others become unemployed.

As in Pissarides (1992), workers who become unemployed lose some skills. In particular, the productivity of any worker who is hired from unemployment is reduced by a certain, time-invariant amount in the initial period of re-employment. After being employed for one period, a worker regains her old productivity level. One can think of the skill loss as the cost required to re-train a worker to become suitable for employment. The fraction of job searchers with reduced skills is equal to the ratio of the number of previously unemployed workers to the total number of job searchers:

$$p_t = \frac{u_{t-1}}{u_{t-1} + \rho_{x,t}\left(1 - u_{t-1}\right)}. \tag{2}$$

Wages are determined by Nash bargaining between workers and firms. It will be shown that workers who need to be re-trained are subjected to a wage dectuion upon being hired, reducing their net wage relative to the wages of other workers. Aside from this deduction, wages of all workers are the same, since wages are re-bargained in every period.

3.1.2 Firms

On the supply side of the economy, there is a unit measure of identical firms who maximize the expected present value of net profits, operating a constant returns-to-scale technology to which labor is the only input. In order to hire new workers, a firms post a number of vacancies, denoted v_t, which come at a cost $\kappa > 0$ per unit. Firms' search for workers is random. When choosing the optimal number of vacancies, firms take as given the stochastically fluctuating rate of job separations, $\rho_{x,t}$, the fraction of new hires with reduced skills, p_t, and the rate at which vacancies are filled, denoted q_t. Let the total cost of retraining a worker is denoted by χ and let the deduction be denoted d_t. The value of a firm, V, can be expressed recursively as:

$$V(n_{t-1}, \mathcal{S}_t) = \max_{h_t, n_t, v_t} \overline{A} n_t - w_t n_t - (\chi - d_t)\, p_t h_t - \frac{\kappa}{q_t} h_t + \beta \mathbb{E}_t V(n_t, \mathcal{S}_{t+1}),$$

subject to

$$n_t = \left(1 - \rho_{x,t}\right) n_{t-1} + h_t,$$

$$h_t = q_t v_t,$$

$$h_t \geq 0,$$

where n_t denotes the number of workers in the firm, $\mathcal{S}_t$ is the state of the aggregate economy, h_t is the number of new hires and w_t is the wage of a worker, excluding a possible deductions related to re-training. The output of the firm is given by $\overline{A} n_t$, where $\overline{A}$ is a productivity parameter. The costs faced by the firms consist of three components. First, $w_t n_t$ is the baseline wage bill (again excluding deductions). Second, $(\chi - d_t)\, p_t h_t$ is the amount spent on re-training workers, net of the wage deductions. Third, $\frac{\kappa}{q_t} h_t$ are the costs of posting vacancies.

The first constraint in the firms' decision problem is the transition equation for the number of workers in the firm. The second constraint relates the number of vacancies to the number of new hires. The third constraints states that the number of new hires cannot be negative, preventing the firms from generating revenues by firing workers. In line with the empirical results, the rate of job loss follows an AR(1) process:

$$\rho_{x,t} = (1 - \lambda_x)\,\overline{\rho}_x + \lambda_x \rho_{x,t-1} + \varepsilon_{x,t},$$

where bars denote steady-state levels, $\lambda_x \in [0, 1)$ is a persistence parameters and $\varepsilon_{x,t}$ is a normally distributed shock innovation with mean zero and standard deviation σ_x.

3.1.3 Matching technology and wage determination

Let the number of job searchers at the beginning of period t be denoted by $s_t \equiv u_{t-1} + \rho_{x,t}\,(1 - u_{t-1})$. Job searchers and vacancies are matched according to a Cobb-Douglas matching function, $m_t = s_t^{\alpha} v_t^{1-\alpha}$, where m_t is the number of new matches and $\alpha \in (0, 1)$ is the elasticity of the matching function with respect to the number of searchers. From the matching function it follows that the vacancy yield, $q_t = \frac{m_t}{v_t}$, and the job finding rate, $\rho_{f,t} = \frac{m_t}{s_t}$, are related as:

$$q_t = \rho_{f,t}^{\frac{\alpha}{\alpha-1}}. \tag{3}$$

The evolution of the aggregate employment rate is identical to the evolution of firm-level employment due to symmetry across firms.

Wages are set according to Nash bargaining, as mentioned previously. I assume that if bargaining were to fail, the worker and the firm have to wait for the next period in order to search again. This implies that the worker would become a reduced-skill worker. Let ϕ be the bargaining power of the worker. The Appendix shows that the wage w_t and the deduction d_t are given by:

$$\begin{aligned} w_t \;=\;& (1-\phi)\left(b - \beta \mathbb{E}_t \phi \chi \rho_{f,t+1}\right) & (4)\\ & +\phi\left(\overline{A} + \beta \mathbb{E}_t \rho_{f,t+1}\left(1 - \rho_{x,t+1}\right)\left(\chi\,(1-\phi)\,p_{t+1} + \kappa \rho_{f,t+1}^{\frac{\alpha}{1-\alpha}} - \xi_{t+1}\right)\right), & \\ & d_t = \phi \chi. & (5) \end{aligned}$$

Note that the deduction d_t is equal to a fraction ϕ of the total training cost. A version of the model with a fully rigid wage ($w_t = b$) is obtained by setting ϕ equal to zero.[23]

[23] Across different parametrizations for ϕ, the home production parameter b can be adjusted to obtain

3.1.4 Equilibrium

The Appendix shows that the first-order optimality conditions for the firms' hiring decision problem can be combined to obtain the following Euler Equation for vacancies:

$$\chi\left(1-\phi\right)p_t-\xi_t+\kappa\rho_{f,t}^{\frac{\alpha}{1-\alpha}}=\overline{A}-w_t+\beta\mathbb{E}_t\left(1-\rho_{x,t+1}\right)\left(\ \chi\left(1-\phi\right)p_{t+1}+\kappa\rho_{f,t+1}^{\frac{\alpha}{1-\alpha}}-\xi_{t+1}\ \right), \tag{6}$$

where ξ_t is the Lagrange multiplier on the constrained restricting hiring to be non-negative. The left-hand side of he Euler equation captures the expected marginal costs of hiring an additional worker. Due to the presence of skill losses, these costs are increasing in p_t, which in turn is nonlinearly increasing in u_{t-1}, see Equation (2). Thus, higher unemployment discourages vacancy posting, as it introduces additional costs. The benefits of hiring a worker are captured by the right hand side and consist of a current-period profit flow, $\overline{A}-w_t$, plus savings on future hiring costs, captured by the last term on the right-hand side. The Euler equation is useful to compactly characterize the equilibrium:

Definition. *A recursive equilibrium is characterized by policy functions for the job finding rate, $\rho_f\left(\mathcal{S}_t\right)$, for the unemployment rate $u\left(\mathcal{S}_t\right)$, for the wage of new hires $w\left(\mathcal{S}_t\right)$, and for the fraction of reduced-skill hires, $p\left(\mathcal{S}_t\right)$, satisfying the unemployment transition equation (1), the equation for the fraction or reduced-skill hires (2), the wage equation (4), the vacancy Euler equation (6), as well as the exogenous laws of motion for $\rho_{x,t}$. The state of the aggregate economy can be summarized as $\mathcal{S}_t=\left\{\rho_{x,t},u_{t-1}\right\}$.*

Note that u_{t-1} is a state variable only because it enters the the definition of p_t, Equation (2), which in turn enters the vacancy posting condition (6). In the absence of skill losses ($\chi=0$), Equation (2) can be dropped from the model, eliminating p_t as a variable. Then, the equilibrium policy function for $\rho_{f,t}$ can be solved from a dynamic system containing only Equation (6) and the wage equation (4). In this system, $\rho_{x,t}$ is the only state variable. Given a simulation for $\rho_{f,t}$ and $\rho_{x,t}$, and an initial level of

the same steady-state wage.

unemployment, the path of the unemployment rate can be computed separately using Equation (1).

3.2 Steady-state properties and relation to the reduced-form model

3.2.1 Steady state properties

I now investigate under what parameter values multiple steady states can arise, closely following Pissarides (1992). Consider the Euler equation and the wage setting equation. In a deterministic steady state with positive hiring, these reduce to:[24]

$$\underbrace{\kappa\overline{\rho}_f^{\frac{\alpha}{1-\alpha}}}_{\text{vacancy cost}} = \underbrace{\frac{\overline{A}-\overline{w}}{1-(1-\overline{\rho}_x)\beta}}_{\text{NPV gross profit}} - \underbrace{\left(1-\overline{\rho}_f\right)\chi\left(1-\phi\right)}_{\text{re-training cost}}, \tag{7}$$

$$\overline{w} = (1-\phi)\left(b-\beta\phi\chi\overline{\rho}_f\right) + \phi\overline{A} + \phi\beta\left(1-\overline{\rho}_x\right)\left(\chi\left(1-\phi\right)\overline{\rho}_f\left(1-\overline{\rho}_f\right) + \kappa\overline{\rho}_f^{\frac{1}{1-\alpha}}\right) \tag{8}$$

where upper bars denote steady-state values. Here, I have used that $\overline{q} = \overline{\rho}_f^{\frac{\alpha}{\alpha-1}}$ and that $\overline{p} = 1-\overline{\rho}_f$. Since $\overline{\rho}_f$ and $\overline{w}$ are the only endogenous variables that enter these two equations, the steady-state solution can be found from this system.

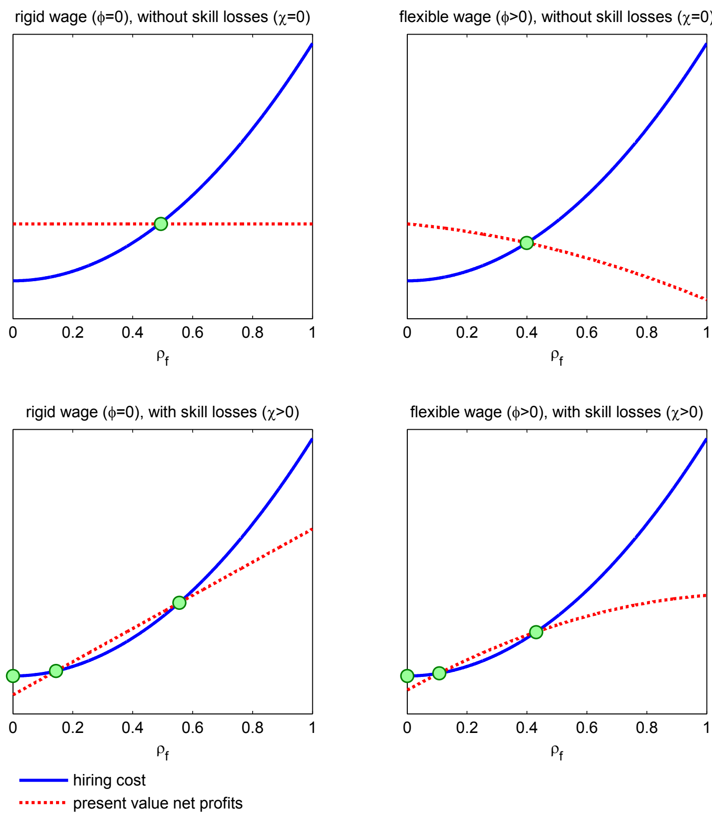
It is insightful to distinguish between versions of model with a sticky wage ($\phi = 0$) versus a flexible wage ($\phi > 0$), as well as between a model without skill losses ($\chi = 0$) versus a model with skill losses ($\chi > 0$). For each of the four cases, Figure 2 illustrates the left- and right-hand side of Equation (7) as a function of $\overline{\rho}_f$, after substituting out $\overline{w}$ using Equation (8). The solid blue lines represent the left- side of the equation, which captures the (expected) vacancy cost of hiring a worker, and does not depend on either ϕ or χ. This cost is illustrated under the assumption that $\alpha > \frac{1}{2}$, which implies that it is convexly increasing function in $\overline{\rho}_f$. The importance of this assumption is discussed below. The red dashed lines in Figure 2 illustrates the right-hand side of Equation (7), which is the expected Net Present Value (NPV) of gross profits generated by a marginal worker net of expected re-training costs, and depends on both ϕ and χ.

The simplest of the four cases is one with a sticky wage and no skill losses (Figure 2, upper left panel). In this case, the net discounted profit does not depend on $\overline{\rho}_f$. Given

[24] Below I will discuss corner solutions with zero hiring.

that the hiring cost is increasing in $\overline{\rho}_f$, there is at most one steady state. The same is true in a model with flexible wages and no skill losses (upper right panel), since net profits become strictly decreasing in $\overline{\rho}_f$.[25] Thus, multiple steady states do not arise in the model without skill losses. It is straightforward to verify that this is also true when $\alpha \leq \frac{1}{2}$.

Figure 2: Illustration of the steady-sate properties of various model versions.



Multiple steady states can arise when unemployment when there are skill losses ($\chi > 0$). Under sticky wages (Figure 2, bottom left panel), the wage $\overline{w}$ does not depend on $\overline{\rho}_f$. However, net profits are increasing in $\overline{\rho}_f$ since the expected training cost, $\left(1 - \overline{\rho}_f\right) \chi \left(1 - \phi\right)$, depends negatively on $\overline{\rho}_f$. Intuitively, a lower job finding

[25] When $\chi = 0$, the steady-state wage is increasing in $\overline{\rho}_f$ and steady-state net profits are decreasing in $\overline{\rho}_f$.

rate increases the fraction of job searchers with reduced skills, and hence increases the total amount of training costs that firms need to pay. In turn, higher expected costs discourage firms to hire, further reducing the job finding rate. As a result, three steady states can arise. The steady state with the highest job finding rate, and thus the lowest unemployment rate, is stable, whereas the middle steady state is unstable. Further, there is a corner solution with 100 percent unemployment, which is a stable steady state.[26] Finally, consider a model with skill losses and flexible wages (Figure 2, bottom right panel). In this case, multiple steady states can arise as well. However, the net present value of profits depends in a more ambiguous way on the job finding rate, due to the term $\left(1-\overline{\rho}_f\right)\overline{\rho}_f$ in the wage equation.

Finally, consider the role of the matching function parameter α. As shown above, a model with skill losses and $\alpha > \frac{1}{2}$ can generate the type of steady states observed in the data. In a model with $\alpha < \frac{1}{2}$, the hiring cost becomes a concavely increasing function of $\overline{\rho}_f$ and it is straightforward to verify that multiple steady states can arise as well, but their stability properties are unlikely to be in line with the reduced-form estimates.[27] When α is exactly to $\frac{1}{2}$, the hiring cost becomes linear in $\overline{\rho}_f$, allowing for at most one interior steady state under rigid wages.

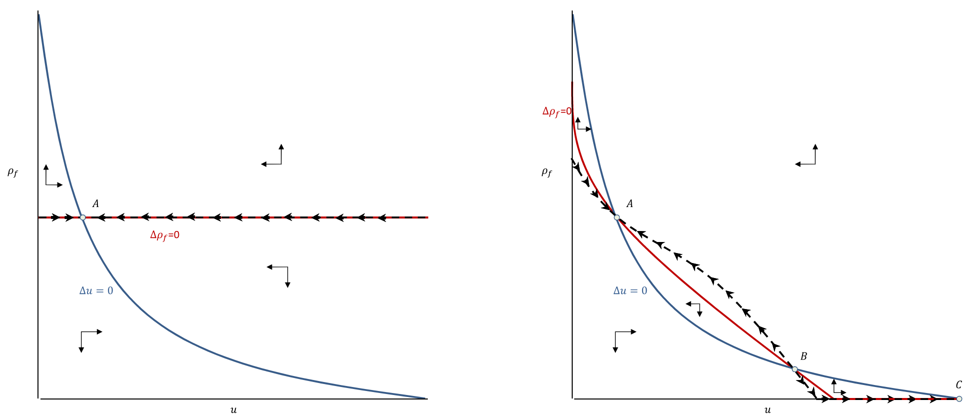
How plausible is the assumption that the elasticity of the matching function with respect to unemployment is larger than one half? Petrongolo and Pissarides (2001) survey the literature, which reports a wide range of estimates. They conclude that the matching function elasticity with respect to unemployment is likely to be between 0.5 and 0.7.

[26] In this corner steady state $h = \rho_f = 0$ and $u = p = 1$. The Euler equation implies then becomes $\xi = \chi(1-\phi) - \frac{\overline{A}-b}{1-\beta(1-\overline{\rho}_x)}$, with $\phi = 0$ in the sticky wage case.

[27] For example, in a model with skill losses and a rigid wage three steady states would again arise. However, now the corner steady state is one with zero rather than hundred percent unemployment, and is stable. The steady state with the next lowest unemployment level then becomes unstable, whereas the interior steady state with the highest unemployment level is stable. This outcome is not in line with the empirical results of the previous section, in which the interior steady state with low unemployment was found to be the stable one of the two interior solutions.

3.2.2 Dynamic properties

Figure 3: Illustrated phase diagrams for the DMP model without skill losses (left panel) and with skill losses (right panel).



The fact that there are multiple steady states does not imply that there are multiple dynamic equilibria. The dynamic properties of the model can be understood by constructing a phase diagram for a deterministic version of the model. The appendix explains how the phase diagrams can be constructed numerically for a specific calibrated model.

The left panel of Figure 3 illustrates the phase diagram for the basic DMP model without skill losses ($\chi = 0$). The blue and red line illustrate the null-curves of the model. Specifically, the blue line depicts combinations of u and ρ_f for which Equation (1), implies that u remains constant. Similarly, the red line depicts combinations for which ρ_f remains constant according to Equation (6). The two null-curves divide the space of possible paths into four segments with different directions of motion for ρ_f and u. There is a unique intersection of the two null-lines at point A, representing the steady state. It can be checked that this steady state is locally saddle-point stable. The dashed black line with arrows illustrates the saddle path that leads into the steady state. Without skill losses, the equilibrium job finding rate does not depend on the unemployment rate, and hence the equilibrium path lies on top of the null-line for ρ_f. It can be verified that the saddle path is a unique equilibrium.

The phase diagram for the DMP model with skill losses ($\chi > 0$) is illustrated in the right panel of Figure 3. There are three steady states, indicated by points A, B and C. Steady states A and C are locally saddle-path stable, whereas steady state B is unstable. The saddle path leads into A from the right and into C, and reverses direction exactly at steady state B. As in the model without skill losses, the saddle path is a unique dynamic equilibrium.[28] There is thus no scope for equilibria driven by self-fulfilling changes in expectations. To understand why the skill-loss mechanism does not invite this type of equilibrium, note that firms would be encouraged to hire more today when unemployment is expected to increase, since average re-training costs per hire would be expected to rise in the future.

3.2.3 Relation to the reduced-form model

There is a close connection between the DMP models and the reduced-form models of the previous section, due to the fact that the central Euler equation of the DMP model, Equation (6), is essentially a one-period ahead forecasting equation for a nonlinear transformation of the job finding rate, $\rho_{f,t+1}^{\frac{1}{1-\alpha}}$. The unemployment rate enters this equation non-linearly through p_t and p_{t+1}, but drops out when skill losses are removed from the model and unemployment is no longer a state variable.

3.3 Quantitative horse race

Extending the DMP model with skill losses helps to reproduce the steady states estimated from the data, but does it also improve the model's ability to explain fluctuations? Further, can the model reproduce the estimated steady states under realistic

[28] To see why, note that: (i) There cannot be alternative equilibrium paths that lead into steady states A or C since these steady states are saddle-path stable and we have already considered the saddle path. Further, there cannot be equilibrium paths that lead into steady state B, since this steady state is unstable. (ii) There cannot be alternative equilibrium paths that cycle/spiral around one or more steady states, as such paths would cross the saddle path which is not possible (in the continuous-time limit). (iii) Any other candidate path not considered would violate the condition that $\rho_{f,t} \leq 1$ and hence cannot be an equilibrium.

degrees of skill losses? I address these questions by calibrating a model with and without skill losses and confront both models with the data.

3.3.1 Calibration

The parametrization of the models is based on a period length of one month, corresponding to the frequency at which the data is observed. For both models, I calibrate real wages to be rigid ($\phi = 0$). Table 1 shows the remaining parameter values, which are set equal for both models, with the exception of χ and b.

The subjective discount factor, β, is set to imply an annual real interest rate of 4 percent. The matching function elasticity, α, is set to 0.6, which is just in the middle of the 0.5-0.7 range recommended by Petrongolo and Pissarides (2001). The vacancy cost, κ, is calibrated to imply that in the (low-unemployment) steady state, the average cost of hiring is 4.5 percent of a worker's quarterly output, see Silva and Toledo (2009). The productivity of a fully skilled worker, $\overline{A}$, is normalized to one.

The parameters of the exogenous process for the job loss rate, $\rho_{x,t}$, are obtained by estimating an AR(1) process based on its observed counterpart in the data. As mentioned above, the series is rather noisy and I therefore smooth it using a 3 month moving average filter. To take out any slow moving trend, the series is put through the HP filter with a smoothing coefficient equal to $81 \cdot 10^5$. This value corresponds to one used by Shimer (2005) for quarterly data, but is converted to the appropriate monthly value using the adjustment factor recommended by Ravn and Uhlig (2002). The estimated persistence parameter is computed based on the autocorrelation of the filtered series at a one-year horizon. Given this parameter value, the shock innovations are backed out using the AR(1) equation. I set $\overline{\sigma}_x$ equal to the standard deviation of these shock innovations over the sample.

The remaining parameters, χ and b, are set differently for both models. The flow value of an unemployed worker, b, is used to target a the steady-state unemployment rate to 5.5 percent in the basic model, and the low-unemployment steady state to 5.5 percent in the extension with skill losses. The re-training cost, χ, is naturally set to zero in the model without skill losses. In the model with skill losses, this parameter is chosen to match the second steady state to 9.5 percent unemployment, in line with the reduced-form evidence presented in the previous section. The implied training cost is equal to 0.688, which corresponds to only 2.7 weeks of output.

3.3.2 Quantitative results

The empirical performance of the two models is evaluated by conducting a simulation based on shocks that exactly reproduce the (filtered) job loss rate in the data. The job loss series is plotted in the upper panel of Figure 8. The bottom two panels plot the job finding rate and the unemployment rate as predicted by the two models, as well as the actual realization in the data. The model is solved using a global projection method, which is explained in the appendix.

The job finding rate series predicted by the model with skill losses is strikingly similar to its counterpart in the data. The correlation between the two series is 0.88. During the late 2000's, there is a discrepancy in the levels but even during this period the dynamics are close. For the years after 2010, the model with skill losses matches almost perfectly the job finding rate in the data. The low job finding rate over this period is ultimately driven by a spike in the job loss rate during 2008 and 2009. By 2011, however, the rate of job loss has returned to its pre-crisis level. The fact that the job finding rate remains persistently low highlights the strong endogenous propagation mechanism of the model. The essence of this propagation mechanism is that skill losses associated with high unemployment discourage hiring, preventing a swift recovery.

By contrast, the basic DMP model without skill losses produces hardly any fluctuations in the job finding rate. As a result, this model fails to account for the large and persistent increase in unemployment following the Great Recession. The mild increase in unemployment that the model does produce, is largely a direct effect of the job loss shock. To illustrate this point, Figure 8 also plots a simulation in which $\rho_{f,t}$ is kept entirely constant over the sample. The unemployment rate series produced by this simulation is very similar to the one predicted by the basic DMP model. The simulation also highlights that most of the fluctuations in unemployment derive from fluctuations in the job finding rate, in particular the persistent increase in unemployment following the Great Recession.

The success of the model with skill losses is remarkable for several reasons. First, the shocks to the rate of job loss are relatively minor. Over the sample, these shocks account for only 23 percent of the fluctuations in unemployment. In the literature, shocks to the rate of job loss have often been dismissed as a plausible source of overall fluctuations in unemployment. Nonetheless, the model with skill losses reproduces the amount of unemployment volatility in the data based on only job loss shocks, which requires a strong internal amplification mechanism. Such a mechanism is clearly absent in the basic DMP model. Second, the model produces a mild correlation between the rate of job loss and the job finding rate, which requires a strong internal propagation mechanism.[29] Indeed, the basic DMP predicts a perfect correlation of minus one, since this model lacks any internal propagation mechanism. Finally, the test to which the DMP models are put in this paper is stringent, compared to many similar model evaluation exercises in the literature. Often, such exercises compare moments within the model to their counterparts in the data, rather than confronting a model simulation directly with observed time series.[30]

4 Conclusions

The main message of this paper is that models with multiple steady state may provide a much more satsifactory description of observed labor market dynamics than standard models with a single steady state. This follows from a reduced-form model estimated on U.S. data, as well from a simple DMP model in which skill losses give rise to multiple steady states. An additional point is that the degree of skill losses required to

[29] In the data, the correlation between the two series is -0.26, whereas in the model the correlation is -0.11.

[30] An exception is Mitman and Rabinovich (2014), who feed observed productivity into a model with countercyclical unemployment benefit extensions.

generate multiple steady states is small, adding credibility to the mechanism proposed by Pissarides (1992).

In the literature, there are almost no attempts to bring models with multiple steady states directly to the data. An exception is Kaplan and Menzio (2014) who argue that a model with shopping externalities and expectations shocks can account for the depth of the Great Recession. In a more qualitative setting, Saint-Paul (1995) studies the role of firing costs in a model with idiosyncratic production risk and on-the-job search. Blanchard and Summers (1987) consider the endogenous effect of the power of labour unions. Blanchard and Summers (1986), Rochetau (1999) and Den Haan (2007) study the role of taxes required to finance social security payments. Finally, Ellison *et al.* (2014) consider a DMP with non-constant returns to scale in the matching function. Since the models in all of these references can – at least in principle– generate multiple steady states, they provide a starting point for a further quantitative exploration and evaluation versus the model analyzed in this paper. Distinguishing between various candidate models is important, given that government policies can have dramatic impacts if they can prevent the economy from slipping into a steady state with high unemployment.

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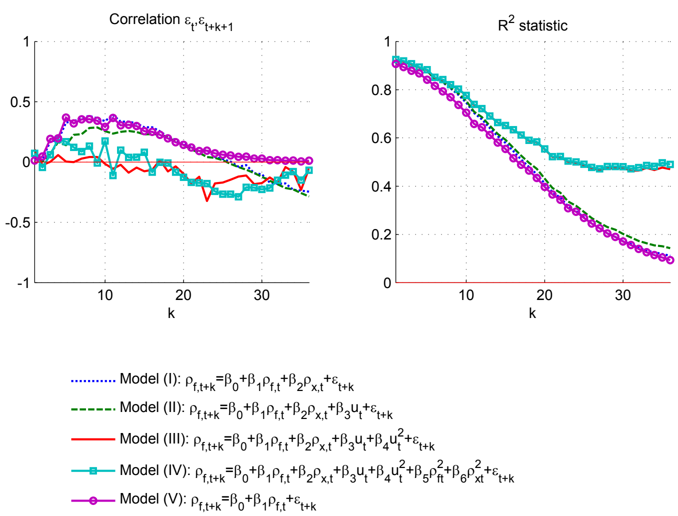
Appendix

A1. Additional empirical results

Below, I present several robustness checks on the results presented in the main text.

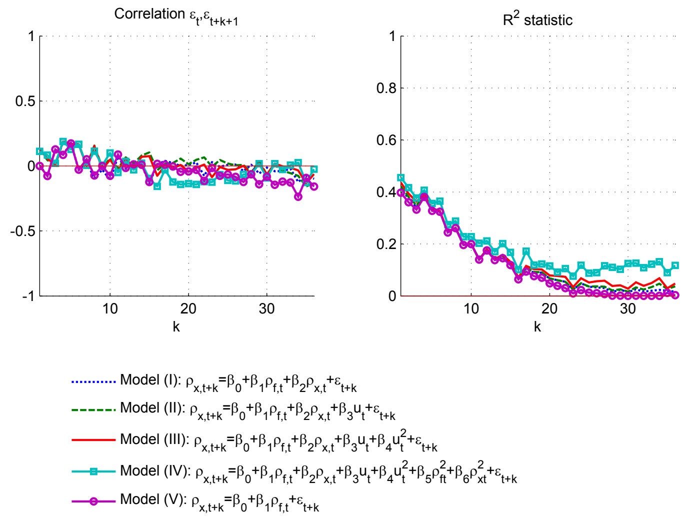
Model Diagnostics. I consider two additional specifications. In Model (IV), $\mathbb{E}_t \rho_{f,t+k}$ is a linear function of $\rho_{f,t}$, $\rho_{x,t}$, u_t, u_t^2,$\rho_{f,t}^2$, $\rho_{x,t}^2$ and a constant. In model (V), the forecast is a linear function of only a constant and $\rho_{f,t}$, which corresponds to a simple AR(1) process. Figure A1 presents the diagnostics statistics for all five specifications. Results for model (III) and (IV) are very similar, whereas results for model (V) are close to those of models (I) and (II).

Figure A1: Additional models for $\rho_{f,t+k}$.



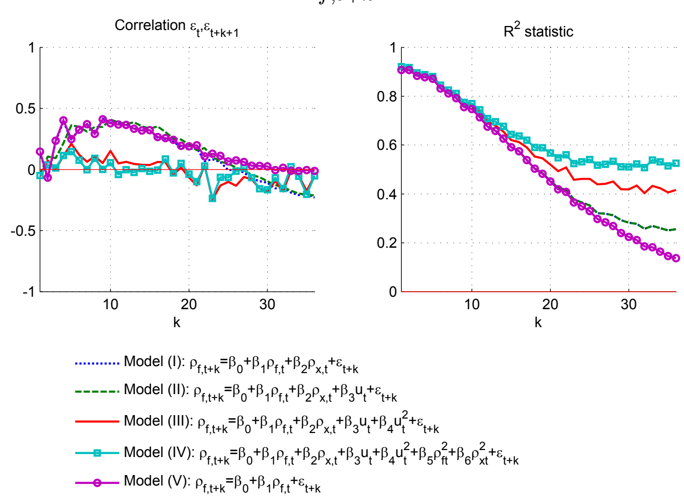
Next, consider forecasting equations for the rate of job loss, i.e. $\rho_{x,t+k}$. The specifications are the same as for the forecasting equations for $\rho_{f,t+k}$, with the exception of model (V) in which the regressor is now $\rho_{x,t}$ rather than $\rho_{f,t}$, so that the specification corresponds to an AR(1) for $\rho_{x,t}$. Figure A2 shows that, In contrast to the results for the job finding rate, model diagnostics for all five models are similar. I conclude from this that the rate of job loss is well described by a simple AR(1) process.

Figure A2: Models for $\rho_{x,t+k}$.



Finally, consider again the forecasting equations of the job finding rate, but now estimated on durations-based data from the CPS rather than the gross flow data. Figure A3 shows that the model diagnostics are very similar to those obtained from the gross flow data. In particular, model (III) is favored over models (I), (II) and (V).

Figure A3: Models for $\rho_{f,t+k}$: duration-based CPS data.



Implied steady states. While models (III) and (IV) produce very similar diagnostics statistics, model (III) was chosen as the baseline for the sake of parsimony. Figure A4 shows that the steady states implied by model (IV) are very similar to those of model (III). Finally, I compute the steady states based on model (III), but estimated using the duration-based rather than the gross-flow data. Figure A5 shows that using these alternative data produces similar steady states.

Figure A4: Steady states based on Model (IV).

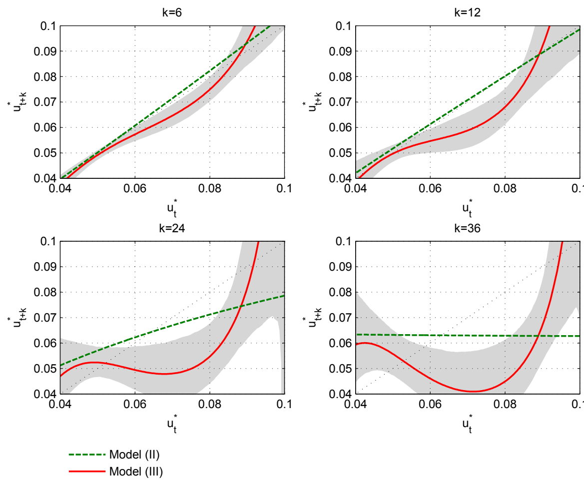
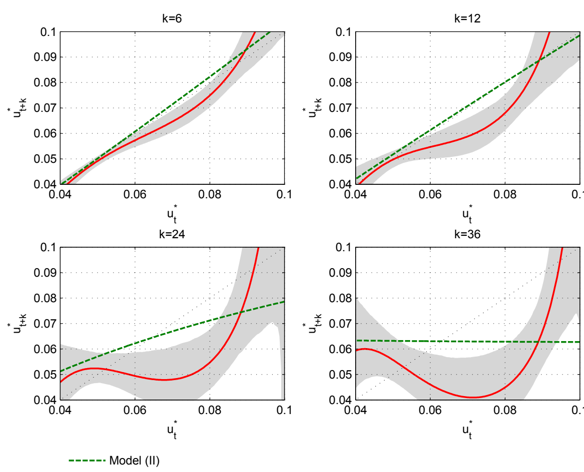


Figure A5: Steady states based on Model (III) estimated on duration-based CPS data.



A2. Model derivations

First, we derive explicitly the firms' Euler equation for vacancies. The firms' problem can be written as:

$$V(n_{t-1}, \mathcal{S}_t) = \max_{n_t, h_t} \left(\overline{A} - w_t\right) n_t - (\chi - d_t)\, p_t h_t - \kappa \frac{h_t}{q_t} + \beta \mathbb{E}_t V(n_t, \mathcal{S}_{t+1})$$

subject to

$$n_t = \left(1 - \rho_{x,t}\right) n_{t-1} + h_t,$$

$$h_t \geq 0.$$

The first-order conditions for n_t and h_t are:

$$\begin{aligned}
\overline{A} - w_t - \mu_t + \beta \mathbb{E}\left(1 - \rho_{x,t+1}\right) \mu_{t+1} &= 0 \\
(d_t - \chi)\, p_t - \frac{\kappa}{q_t} + \mu_t + \xi_t &= 0 \\
\xi_t h_t &= 0
\end{aligned}$$

where μ_t and ξ_t are, respectively, the Lagrange multiplier on the employment transition equation and the non-negativity constraint on hires. The third condition is the complementary slackness condition. The first two equations can be combined to obtain:

$$(\chi - d_t)\, p_t + \frac{\kappa}{q_t} - \xi_t = \overline{A} - w_t + \beta \mathbb{E}_t \left(1 - \rho_{x,t+1}\right) \left((\chi - d_{t+1})\, p_{t+1} + \frac{\kappa}{q_{t+1}} - \xi_{t+1} \right).$$

Next, we derive the wage equation under Nash bargaining. The value of an additional fully-skilled worker to a firm is given by the Lagrange multiplier μ_t. Let W_t be the value to a household of being a fully-skilled employed worker and let U_t be the value of being an unemployed worker. These two variables satisfy:

$$\begin{aligned}
W_t &= w_t + \beta \mathbb{E}_t \left(1 - \rho_{x,t+1} + \rho_{x,t+1}\rho_{f,t+1}\right) W_{t+1} + \beta \mathbb{E}_t \rho_{x,t+1} \left(1 - \rho_{f,t+1}\right) U_{t+1}, \\
U_t &= b + \beta \mathbb{E}_t \rho_{f,t+1} \left(W_{t+1} - d_{t+1}\right) + \beta \mathbb{E}_t \left(1 - \rho_{f,t+1}\right) U_{t+1}.
\end{aligned}$$

If a match were to break up endogenously, the worker would spend at least one period in unemployment and hence lose skills. Define X_t as the surplus of full-skilled employed worker, relative to being unemployed:

$$\begin{aligned}
X_t &\equiv W_t - U_t \\
&= w_t - b + \beta \mathbb{E}_t \rho_{f,t+1} d_{t+1} + \beta \mathbb{E}_t \left(1 - \rho_{x,t+1}\right) \left(1 - \rho_{f,t+1}\right) X_{t+1}
\end{aligned}$$

The total surplus of a match between a fully-skilled worker and a firm, denoted S_t, is given by:

$$S_t = \mu_t + X_t.$$

The solution to the Nash solution to the bargaining problem between a fully-skilled worker and a firm can be expressed as:

$$\begin{aligned} \phi S_t &= X_t \\ &= w_t - b + \beta \mathbb{E}_t \rho_{f,t+1} d_{t+1} + \beta \mathbb{E}_t \left(1 - \rho_{x,t+1}\right)\left(1 - \rho_{f,t+1}\right) \phi S_{t+1}, \end{aligned}$$

where $\phi \in (0,1)$ is the bargaining power of the worker. The surplus also satisfies:

$$\begin{aligned} & S_t = \mu_t + X_t \\ = \; & \overline{A} - w_t + \beta \mathbb{E}\left(1 - \rho_{x,t+1}\right)\mu_{t+1} + w_t - b + \beta \mathbb{E}_t \rho_{f,t+1} d_{t+1} + \beta \mathbb{E}_t \left(1 - \rho_{x,t+1}\right)\left(1 - \rho_{f,t+1}\right) X_{t+1} \\ = \; & \overline{A} - b + \beta \mathbb{E}_t \rho_{f,t+1} d_{t+1} + \beta \mathbb{E}\left(1 - \rho_{x,t+1}\right) \rho_{f,t+1} \mu_{t+1} + \beta \mathbb{E}_t \left(1 - \rho_{x,t+1}\right)\left(1 - \rho_{f,t+1}\right)\left(\mu_{t+1} + X_{t+1}\right) \\ = \; & \overline{A} - b + \beta \mathbb{E}_t \rho_{f,t+1}\left(d_{t+1} + \left(1 - \rho_{x,t+1}\right)\mu_{t+1}\right) + \beta \mathbb{E}_t \left(1 - \rho_{x,t+1}\right)\left(1 - \rho_{f,t+1}\right) S_{t+1}, \end{aligned}$$

It follows from the Nash Bargaining solution that:

$$\begin{aligned} w_t &= b - \beta \mathbb{E}_t \rho_{f,t+1} d_{t+1} + \phi S_t - \beta \mathbb{E}_t \left(1 - \rho_{x,t}\right)\left(1 - \rho_{f,t+1}\right) \phi S_{t+1} \\ &= b - \beta \mathbb{E}_t \rho_{f,t+1} d_{t+1} + \phi \left(\overline{A} - b + \beta \mathbb{E}_t \rho_{f,t+1}\left(d_{t+1} + \left(1 - \rho_{x,t+1}\right)\mu_{t+1}\right)\right) \\ &= (1 - \phi)\left(b - \beta \mathbb{E}_t \rho_{f,t+1} d_{t+1}\right) + \phi \left(\overline{A} + \beta \mathbb{E}_t \rho_{f,t+1}\left(1 - \rho_{x,t+1}\right)\mu_{t+1}\right). \end{aligned}$$

Next, consider the bargaining problem between a new hire with reduced skills and the firm. Let X_t^{rs} denote the surplus of a reduced-skill employed worker and let S_t^{rs} denote the total surplus of a match between a reduced-skilled worker and a firm. The Nash Bargaining solution for a reduced-skill worker and a firm is $\phi S_t^{sr} = X_t^{rs}$. Note also that $S_t - S_t^{rs} = \chi$, since the only way in which a match with a reduced-skill worker is different from a match with a fully-skilled worker is that production in the current period is lowered by an amount χ. It follows that

$$\begin{aligned} d_t &= X_t - X_t^{rs}, \\ &= \phi\left(S_t - S_t^{rs}\right), \\ &= \phi \chi. \end{aligned}$$

Thus, the worker pays for a fraction ϕ of the training cost. Given this result, the first-order condition for vacancies becomes:

$$\chi(1-\phi)p_t + \frac{\kappa}{q_t} - \xi_t = \overline{A} - w_t + \beta\mathbb{E}_t\left(1-\rho_{x,t+1}\right)\left(\chi(1-\phi)p_{t+1} + \frac{\kappa}{q_{t+1}} - \xi_{t+1}\right).$$

Further, the wage equation becomes:

$$w_t = (1-\phi)\left(b - \beta\mathbb{E}_t\phi\chi\rho_{f,t+1}\right) + \phi\left(\overline{A} + \beta\mathbb{E}_t\rho_{f,t+1}\left(1-\rho_{x,t+1}\right)\left(\chi(1-\phi)p_{t+1} + \kappa\rho_{f,t+1}^{\frac{\alpha}{1-\alpha}} - \xi_{t+1}\right)\right),$$

where I have used that $\mu_{t+1} = \chi(1-\phi)p_{t+1} + \kappa\rho_{f,t+1}^{\frac{\alpha}{1-\alpha}} - \xi_{t+1}$, which follows from the first-order condition for h_t. Note that $w_t = b$ when ϕ equals zero.

A3. Constructing phase diagrams

This appendix discusses how to construct the phase diagram for a specific calibrated model, under the assumption that $\rho_{x,t} = \overline{\rho}_x$ at all times, so there is perfect foresight. Construct null-curves of u_{t-1} versus $\rho_{f,t}$ is done as follows. The transition identity for unemployment can be used to construct a relation between u_{t-1} and $\rho_{f,t}$ given that $\Delta u \equiv u_t - u_{t-1} = 0$:

$$u_t = \frac{\overline{\rho}_x\left(1-\rho_{f,t}\right)}{\overline{\rho}_x\left(1-\rho_{f,t}\right) + \rho_{f,t}}.$$

The Euler equation for vacancies can be used to construct a relation between u_{t-1} and $\rho_{f,t}$ given that $\Delta\rho_f \equiv \rho_{f,t+1} - \rho_{f,t} = 0$:

$$\beta\left(1-\overline{\rho}_x\right)\left(\chi\frac{u^*\left(\rho_{f,t}\right)}{u^*\left(\rho_{f,t}\right) + \overline{\rho}_x\left(1-u^*\left(\rho_{f,t}\right)\right)} + \kappa\rho_{f,t+1}^{\frac{\alpha}{1-\alpha}} + \xi_{t+1}\right) = \chi\frac{u_{t-1}}{u_{t-1} + \overline{\rho}_x\left(1-u_{t-1}\right)} + \kappa\rho_{f,t}^{\frac{\alpha}{1-\alpha}} + \xi_t - \overline{A} + b$$

where $u^*\left(\rho_{f,t}\right) \equiv \left(1-\rho_{f,t}\right)u_{t-1} + \overline{\rho}_x\left(1-\rho_{f,t}\right)\left(1-u_{t-1}\right) = u_t$. Given a value of u_{t-1} one can solve this equation to find the corresponding value of $\rho_{f,t}$, setting $\xi_t = \xi_{t+1} = 0$. If it turns out that $\rho_{f,t} < 0$, then set $\rho_{f,t} = 0$.

The second step is to inspect the local stability properties of the steady states, which can be done using a standard first-order perturbation method. Next, one can numerically trace out the saddle path. To do so, use the perturbation solution to find two points arbitrarily close to each of the two stable steady states.[31] Next, one can

[31] For the low-unemployment steady state, find one point with higher unemployment, and one with lower unemployment.

numerically iterate backwards on the Euler equation and the transition identity for unemployment to trace out the saddle path leading into the stable steady states. From the directions that the saddle path takes one can infer the off-equilibrium directions in each of the segments divided by the null-lines.

A4. Global solution method

The model is solved numerically using a global projection algorithm on a grid for the state variables. The exogenous process for the rate of job loss is discretized using the method of Rouwenhorst (1995) with 31 grid points. For the unemployment rate, I use 151 grid points. The total number of grid points is equal to $N = 31 \cdot 151 = 1581$. The algorithm is based on an approximation of the policy function for $\rho_{f,t}$ on this grid. The steps are as follows:

1. Create a grid for u_{t-1} and $\rho_{x,t-1}$ as described above. At each grid point, compute the associated value of p_t using Equation (3).

2. Guess an initial value $\rho_{f,i}$ at each of the grid points, indexed by $i = 1, ..., N$.

3. At each grid point i, evaluate the left-hand side of the Equation (6) using the guess from the previous step. Given the left-hand side, solve for the value $\widetilde{\rho}_{f,i}$ that sets the right-hand side equal to the left-hand side.

4. Update the guess as $\rho_{f,i} = \widetilde{\rho}_{f,i}$ at each grid point i and repeat step 3 until $\frac{1}{N}\sum_{i=1}^{N} \left|\rho_{f,i} - \widetilde{\rho}_{f,i}\right| < \epsilon$, where I set $\epsilon = 10^{-11}$.

To find appropriate initial values to be used in step 2, I first solve a deterministic version of the model in which $\rho_{x,t} = \overline{\rho}_x$ at all times.

Table and Figures

Table 1. Parameter values.

parameter	**description**	**no skill losses**	**skill losses**
β	discount factor	$1.04^{-\frac{1}{12}}$	$1.04^{-\frac{1}{12}}$
α	matching function elast.	0.6	0.6
κ	vacancy cost	0.989	0.989
$\overline{A}$	worker productivity	1	1
$\overline{\rho}_x$	s.s. job loss rate	0.021	0.021
λ_x	persistence job loss rate shocks	0.896	0.896
$\overline{\sigma}_x$	s.t. deviation job loss shocks	$7.91e^{-4}$	$7.91e^{-4}$
χ	re-training cost	0	0.688
b	flow from unemployment	0.997	0.985

Figure 4: Raw data.

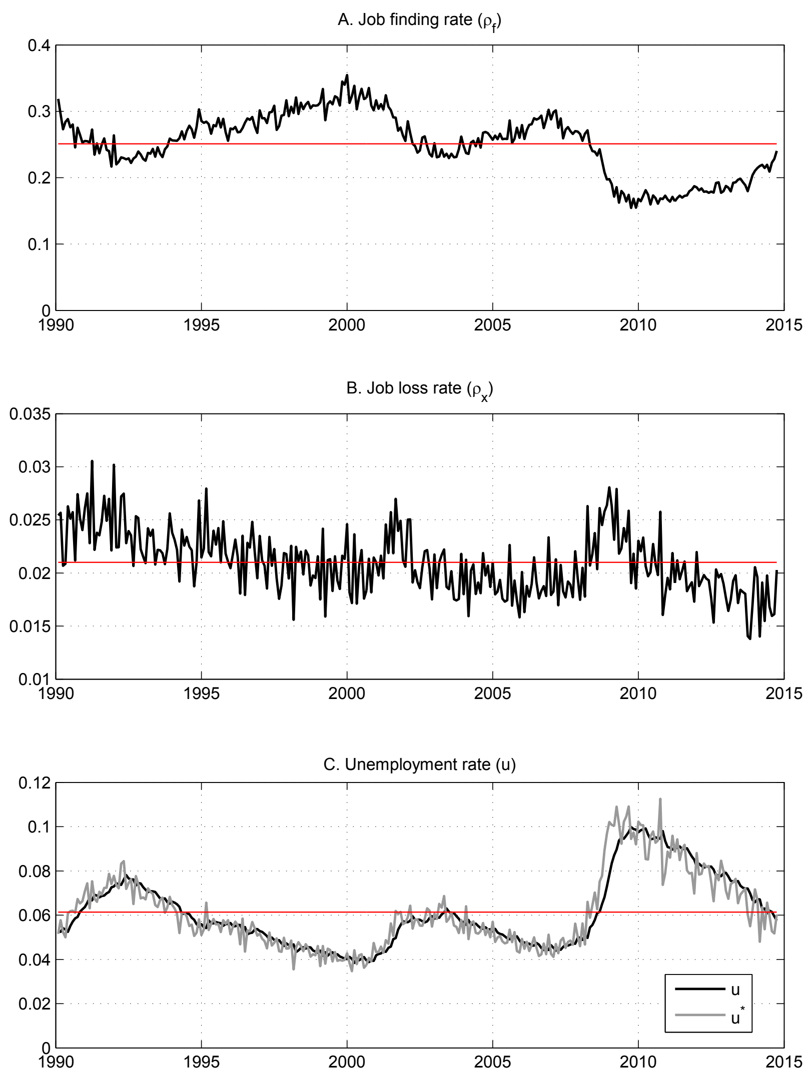


Figure 5: Diagnostic statistics.

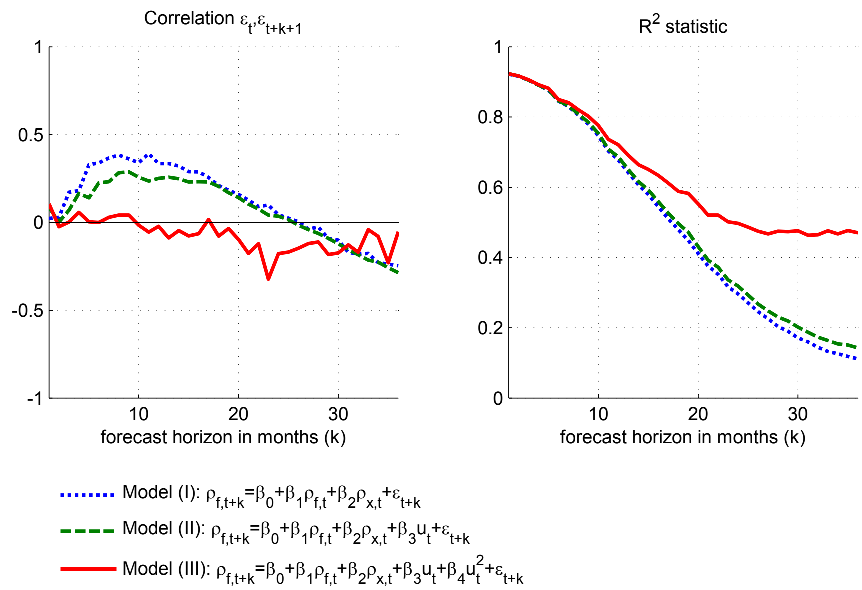


Figure 6: Job finding rate and forecasts made two years in advance.

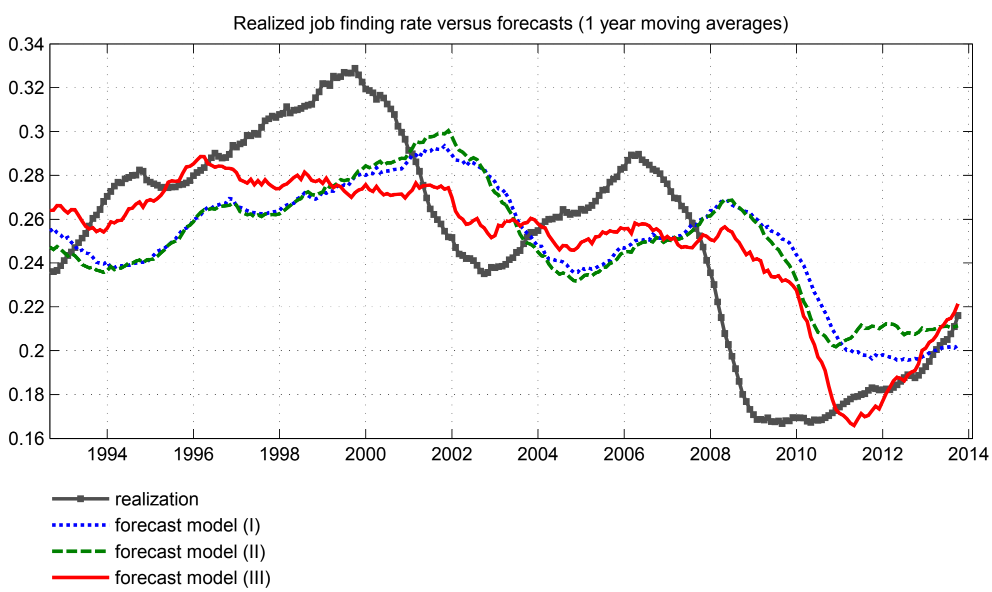
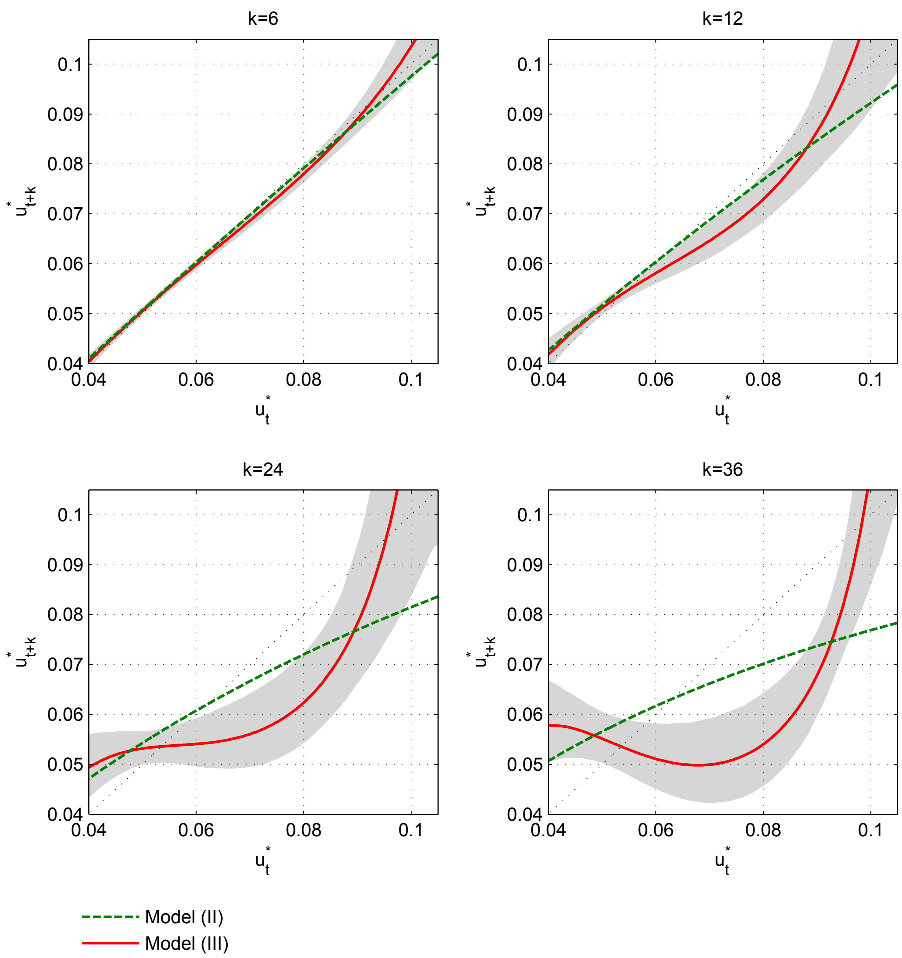


Figure 7: Estimation results of the reduced-form models.



Notes: The figure plots $u_t^* \equiv \frac{\rho_{x,t}(1-\rho_{f,t})}{\rho_{x,t}(1-\rho_{f,t})+\rho_{f,t}}$ against u_{t+k}^*, where intersections with the 45-degree line indicate steady states. Here, a range for u_t^*, is constructed for a range of values for $\rho_{f,t}$ and setting $\rho_{x,t}$ to its sample average $\overline{\rho}_x$. Further, u_{t+k}^* is computed given $\rho_{x,t+k} = \overline{\rho}_x$ and using the forecasting model to evaluate $\rho_{f,t+k}$, given $\overline{\rho}_x$ and the range of values for u_t^* and $\rho_{f,t}$. The shaded area plots a 90 percent confidence band for the model (III). These bands are uniform and have been computed based on a bootstrap method. The forecast horizon, denoted by k, is in months.

Figure 8: Model simulations versus data