## Sparse Expectations: A Unified Explanation of Forecast Data

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## **Extended Abstract**

Expectations are central to macroeconomics. Whether we consider how much households consume, firms employ or how much either invests, the choices that determine macroeconomic fluctuations depend critically on household and firm expectations about the future. Yet, despite its importance to macroeconomics, the best model of expectation formation is still a question open to debate. Macroeconomists have so far considered a broad spectrum of possibilities (Coibion and Gorodnichenko, 2012): from naïve, backward-looking rules to the omniscient expectations that are the hallmark of modern full-information rational expectations. Troublesomely, however, each of these models have also yielded dramatically different results for macroeconomic dynamics and the associated optimal policy responses. This, in turn, leads to the basic question of which model best describes the process by which households and firms form their expectations; the main question that this paper seeks to address.

To do so, I propose a new model of expectation formation that is consistent with household and firm micro-forecast data, which I denote sparse expectations. Specifically, the model seeks to explain three features of survey-based forecast data that have previously been taken as evidence of irrationality or limited information processing on the part of households and firms (*cf.* Coibion and Gorodnichenko, 2015; Bordalo *et al.*, 2016 and Barberis *et al.*, 2016): that forecasts are (*a*) extrapolative, (*b*) biased and (*c*) predictable from forecast revisions. Forecast errors in both macroeconomic and financial forecast data are commonly found to exhibit serial correlation and non-zero means, in clear violation of standard tests of forecast rationality and *the Law of Iterated Expectations* (*cf.* Laster *et al.*, 1999 and Gennaioli *et al.*, 2016).

My model is based on Tibshirani's (1996) idea of *Sparsity*, recently introduced to economics by Gabaix (2014) and Gabaix (2016). The model features rational expectations and an innate desire for simplicity. But unlike the prominent *Rational Inattention* model (Sims, 2003), my model assumes that households and firms form simple, sparse, model-consistent representations

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of the economy and its dynamic stochastic properties. Unlike in Gabaix (2014, 2016), the signals that people use to form their sparse representation are here also noisy, in the tradition of Lucas (1972). I show how the combination of a sparse and a noisy representation of the economy is able to rationalize extrapolative, biased and predictable forecasts within a tractable framework. *Rational Inattention*, by contrast, is only consistent with the latter feature.<sup>1</sup>

The analysis in this paper proceeds in three steps. First, I document and review the evidence on extrapolative and predictive forecasts using a set of commonly used surveys: "the Michigan Survey of Consumers", "the Livingstone Survey", "the Survey of Professional Forecasters", and last "the Duke University Quarterly Survey". I also compare these forecasts to those from the staff of the Federal Reserve Board ("the Greenbook Forecasts"). Second, I construct a simple time-series model in which agents build imperfect, sparse representations about the level of an unobserved state variable. I use this example to show how sparse expectations create serially correlated, non-zero means forecast errors that optimally violate the Law of Iterated Expectations. Last, I embed the simple model into a benchmark micro-founded, macroeconomic model with nominal frictions and contrast the expectations that arise from it with those from the survey data. Here, I also briefly discuss how sparse expectations compare to the state-dependent expectations proposed by Mankiw and Reis (2002).

A variety of different explanations have been proposed for the extrapolative and biased nature of survey-based forecasts. These include systematic violations of *Bayes' Rule* (Bordalo *et al.*, 2016) and the reputational considerations that professional forecasters have when reporting forecasts (Laster *et al.*, 1999), among others. Unlike any of these explanations, however, sparse expectations are micro-founded and extend to, for instance, households who have no clear reputational considerations when reporting forecasts. Indeed, all that is required of sparse expectations is that households and firms, like economists, attempt to build simple representations of the economy based on noisy, incomplete information. Hendry and Mizon (2014) provide additional evidence in favor of such sparse, noisy representations of the economy.

<sup>&</sup>lt;sup>1</sup>I also demonstrate how these results relate to recent advances in statistics (Candes and Tao, 2007).

## References

- BARBERIS, N., GREENWOOD, R., JIN, L. and SHLEIFER, A. (2016). *Extrapolation and bubbles*. Tech. rep., National Bureau of Economic Research.
- BORDALO, P., GENNAIOLI, N. and SHLEIFER, A. (2016). *Diagnostic expectations and credit cycles*. Tech. rep., National Bureau of Economic Research.
- CANDES, E. and TAO, T. (2007). The dantzig selector: Statistical estimation when p is much larger than n. *The Annals of Statistics*, pp. 2313–2351.
- COIBION, O. and GORODNICHENKO, Y. (2012). What Can Survey Forecasts Tell Us about Information Rigidities? *Journal of Political Economy*, **120** (1), 116–159.
- and (2015). Information rigidity and the expectations formation process: A simple framework and new facts. *The American Economic Review*, **105** (8), 2644–2678.
- GABAIX, X. (2014). A sparsity-based model of bounded rationality. The Quarterly Journal of Economics, 129 (4), 1661–1710.
- (2016). Behavioral macroeconomics via sparse dynamic programming.
- GENNAIOLI, N., MA, Y. and SHLEIFER, A. (2016). Expectations and investment. NBER Macroeconomics Annual, **30** (1), 379–431.
- HENDRY, D. F. and MIZON, G. E. (2014). Unpredictability in economic analysis, econometric modeling and forecasting. *Journal of Econometrics*, **182** (1), 186–195.
- LASTER, D., BENNETT, P. and GEOUM, I. S. (1999). Rational bias in macroeconomic forecasts. The Quarterly Journal of Economics, 114 (1), 293–318.
- LUCAS, R. E. (1972). Expectations and the neutrality of money. *Journal of economic theory*, **4** (2), 103–124.
- MANKIW, N. G. and REIS, R. (2002). Sticky Information versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve. *The Quarterly Journal of Economics*, **117** (4), 1295–1328.
- SIMS, C. A. (2003). Implications of rational inattention. *Journal of monetary Economics*, **50** (3), 665–690.
- TIBSHIRANI, R. (1996). Regression shrinkage and selection via the lasso. Journal of the Royal Statistical Society. Series B (Methodological), pp. 267–288.