Sluggish Mobility and the Stock and Flow of Employment Opportunity*

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Abstract

The canonical equilibrium job ladder model of on-the-job search, wage posting and firm heterogeneity is much used in labor research due to its ability to replicate empirical features relating to the wage distribution and employment dynamics. The model has a strong sense of memorylessness. Workers with a long duration of unemployment have the same job finding rate as those newly unemployed. Similarly the distribution of accepted wages is independent of status and duration conditional on the current wage. We provide empirical evidence in support of a model with history dependence. Frictions such as slow relocation between jobs, stochastic employment prospects and hiring time implies that the distribution depends on the employment status and duration. We extent the Burdett and Mortensen (1998) model to allow for stochastic employment prospects. Retaining the tractability of the original model our setup reconciles three disparities between theory and empirics. (i) Job finding rates of unemployed workers decline with the duration of unemployment. (ii) To describe the right tail of the wage distribution a less implausible tail of the firm productivity distribution is required. (iii) The min-mean ratio of wages, as described in Hornstein et al. (2011) need not be rationalized by a large cost associated with unemployment. The estimated model suggests a quantitatively important role for a foot in the door effect whereby the employed are exposed to, on average, better jobs than their counterparts in unemployment.

Keywords: On-the-job search, wage dispersion, wage posting, efficiency of search, employment prospects, relocation frictions, stock-flow matching, mismatch unemployment

1 Introduction

The Burdett and Mortensen (1998) model (henceforth BM) with a continuum of firm types, as extended in Bontemps et al. (2000) has become a workhorse equilibrium model for empirical labor market research. Because of its effectiveness in replication of labor turnover and wages many papers have used this setup to quantitatively answer a variety of questions.[1] Implicit in most of these models is the assumption that workers sample job offers from a distribution independent of their employment history. We extend the BM model to incorporate stochastic employment, which breaks the assumption of independent sapling distributions.[2] Some workers have a lot of employment opportunities whereas others have none and have to wait for the market to improve. The modeling of the stochastic employment is inspired by the intuition from stock-flow matching and miss-match unemployment. The model nests BM as a special case. Notably, despite the additional level of complexity we retain an analytical closed form solution to our model. Additionally, our model, like BM, is well identified and empirically tractable, allowing us to estimate all of the parameters using only wage and employment dynamics obtained from the Current Population Survey (CPS). The estimated model matches a number of well documented empirical regularities that the BM model fails to replicate (for further discussion see Mortensen (2003)). They are: (i) a declining job finding hazard rate; (ii) a primitive firm productivity distribution without an implausibly fat right tail; (iii) the min-mean ratio as described by Hornstein et al. (2011) need not be rationalized by a large flow cost associated with unemployment. Further, our model allows us to decompose wage premium into a retention and hiring motive in a richer framework than other models of the labor market. Our results suggest that the hiring motive is quantitatively more important, but this importance dissipates as you move up the wage distribution.

The transition rate, for a worker, from unemployed to employment job declines with their unemployment duration. There are many potential explanations for why this may be true, for example worker heterogeneity, discouragement, statistical discrimination and human capital depreciation

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[1]For recent examples see Meghir et al. (2015), Bradley et al. (2016) and Shephard (2016).

[2]This model is isomorphic to a model where after a worker and firm meet, it takes time to form a match, we label this 'relocation time'.

while in unemployment. The mechanism at play in our setup has a similar flavor to the stock-flow matching function explanation pioneered by Coles (1994) and Coles and Smith (1998). The flow entering unemployment have better prospects than the stock of unemployed workers. The unemployed with on average fewer opportunities at hand have a lower job finding rate and are consequently those in the pool of long term unemployed. While this is different from the stock having exhausted vacancy opportunities it is a fairly similar mechanism and we believe it incorporates the stock-flow approach tractably into a job ladder model. If the friction is interpreted to be that it takes time for workers to relocate to a new job then workers will collect multiple job offers concurrently but only move to the best one - the one that pays the highest wage.[3] Using transition rates to identify offer arrival rates neglects the possibility that workers can hold multiple offers simultaneously.

The estimated wage offer distribution in models with on-the-job search features a fat right tail (see Gottfries and Teulings (2016)). In order for the Burdett and Mortensen (1998) model to rationalize the right tail of the wage distribution, the underlying firm distribution requires an implausibly fat right tail. The motivation for firms to post higher wages is that they attract and retain more workers, the cost is they make less profit per worker. The increase in hiring and retention thus depends on the density of wage offers at the given wage. In the right tail of the wage distribution there are few workers employed, and few firms posting at a given wage. For the most productive firms, increasing ones wage has thus little impact on their size and in order to rationalize wages observed empirically the profit per worker needs to be very large, and hence so does the productivity of a firm. Our setup introduces another source of firm competition, similar to the mechanism in Burdett and Judd (1983). Firms not only need to attract potential employees away from their current employer but also from employers who may have offered them jobs yet to be processed. We argue through quantitative results that this additional channel of competition is sufficient to replicate a plausible distribution for firm productivity.

Typically, job-to-job transition rates are small compared with the rate at which the unemployed

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[3]Allowing agents to receive or make multiple offers or applications is not new, in a labor market context see, Albrecht et al. (2006), Kircher and Galenianos (2009), Kircher (2009), Gautier and Wolthoff (2009), Wolthoff (2014), Gautier and Holzner (2016) and Carillo-Tudela and Smith (2016).

find jobs. This suggests that the unemployed give up a lot of search option by taking a job. Hornstein et al. (2011) argue that for a large class of search models, frictional dispersion cannot explain the degree of residual wage dispersion in the labor market, without a small (and maybe negative) flow value of unemployment. The question is: why do workers accept very low wages when they can wait for much higher ones? One answer, is that in the process of rejecting a number of job, some aspects change. If the distribution of wages for job-to-job movers stochastically dominates that of the long term unemployed, then measuring job-to-job transitions is not sufficient to calculate the value of unemployment. Using the estimated model, we find that the distribution that the employed move to, stochastically dominates that of the long term unemployed. The value of search for the employed is therefore, to a large extent, hidden in the types of jobs they take. The estimated flow value of unemployment is positive compared to a large negative value in the BM model.

In addition to being consequential to wage dispersion from an empirical micro perspective, the value of unemployment has large implications for the macro labor literature as well. Shimer (2005) shows that feeding the labor productivity process into the standard calibration of the Mortensen and Pissarides (1994) model cannot generate the empirically observed variations seen in market tightness (level of vacancies divided by unemployment). A free entry conditions pins down the level of market tightness and any cyclical fluctuations must come from variation in profits. The variation in labor productivity over the business is typically small. In order for that to translate into large percentage changes in profits it has to be that the level of profits is small (Ljungqvist and Sargent (2015)). If the flow benefits of unemployment is high, then profits will be low and the model is able to generate sufficient amplification to productivity shocks (Hagedorn and Manovskii (2008)). Elsby and Michaels (2013) consider a model in which firms exhibit decreasing returns to scale. Marginal productivity is then less than average productivity and therefore movements in marginal can be greater than the movements in aggregate productivity. In this paper we find that in fact the flow value of unemployment is high compared to the marginal job.

A further contribution to the paper is a deeper assessment of a firm's motivation to pay higher

wages. The labor search literature has been very fruitful in its explanation of what governs overall wage dispersion, but embedded in these equilibrium models are some quite restrictive assumptions regarding why different firms pay different wages. Common to all, are the two fundamental motivators: for greater retention of workers; and to hire more workers. As will be shown in the paper, the BM model imposes that these two sources carry exactly equal weight in firm's wage posting decisions. This type of implicit restriction is commonplace in the literature. If wages are set after firm and workers meet (Shimer (2006) and Gautier et al. (2010)) or the hiring cost does not depend on the wage (Coles and Mortensen (2016)) then increased retention is the only reason to increase pay. In the sequential auctions framework proposed by Postel-Vinay and Robin (2002) firms are able to match outside offers. In wage negotiation between two firms the wage is set such that the worker is indifferent between the two firms, where the low productivity firm offers the full productivity. The wage is thus governed by the participation motive only. In fact only the quality of the match and not the wage, determine the retention rate. The model proposed in this paper is the first model with the both a hiring and retention motive where the relative importance of the two channels is an endogenous outcome. We find that the hiring margin in quantitatively more important, and this importance diminishes as one moves up the wage distribution.

The rest of this paper is structures as follows. In section 3 we set up the model and derive analytical solutions to the model. In sections 4 and 4.3 we present the estimation of the model and the quantitative results. Lastly section 5 concludes.

2 Empirical Motivation

The standard random search model is memoryless. The worker problem is a binary choice, accepting or rejecting job offers that arrive sequentially. This may well be a good approximation of the labor market and it proves to be analytically very convenient. That said, in this section we show a number of statistics that indicate that the memoryless property might not apply. The first salient feature of the data is that the transition rate to employment falls sharply with the duration of unemployment, see Figure 1. Data are taken from the Current Population Survey (CPS) and we

report transition rates for three different skill groups: the college educated; those with a high school diploma and those lacking formal education qualifications.

Figure 1: Monthly transition rate to employment by duration of unemployment


High-Skill

Medium-Skill

Low-Skill

All
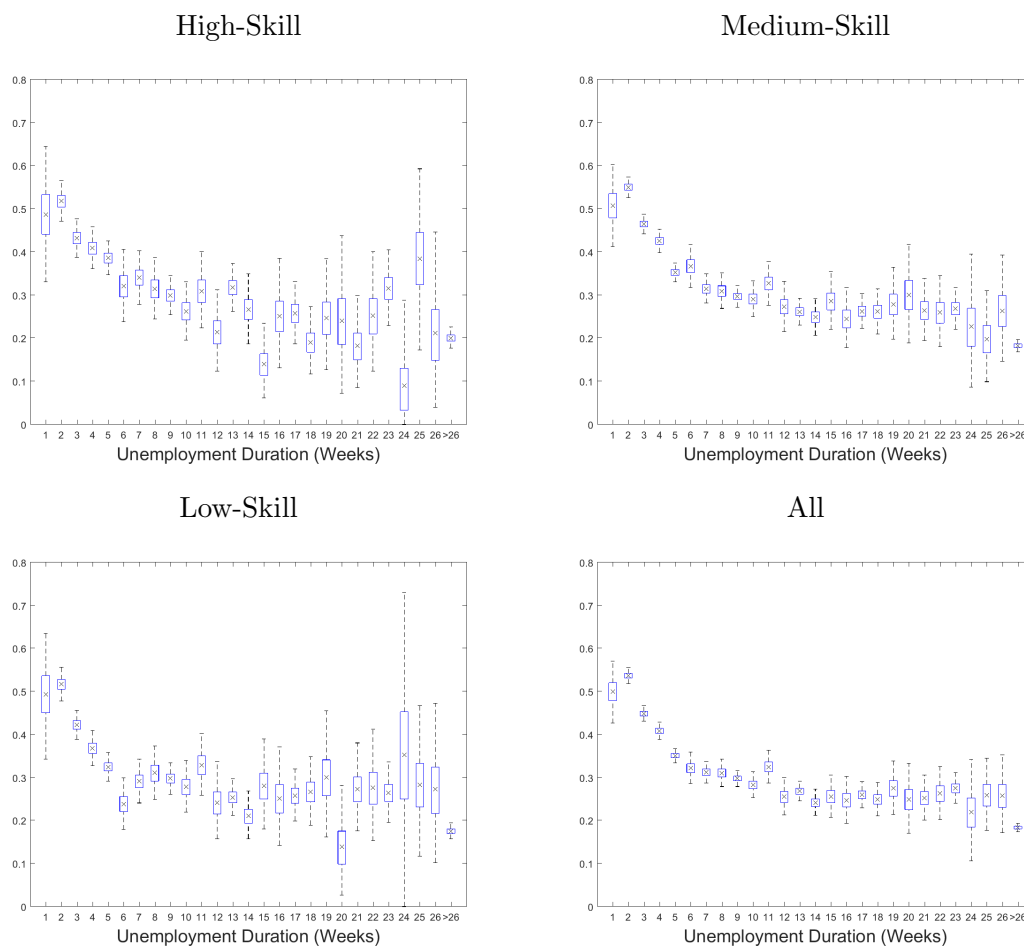
Moments are based on a bootstrap, re-sampling the data 10,000 times. The crosses represent the mean, the boxes the 25th and 75th percentile and black whiskers span the 1st and 99th percentile.

Eyeballing Figure 1 reveals that across all three skill groups the decline in job finding rate follows a similar pattern, starting at a monthly probability of around 0.5 and after two months dropping to around 0.3. This inter-skill similarity is suggestive evidence that the decline is not simply explained by a changing composition - some workers having a consistently higher job finding rate than others. This conjecture is supported by Alvarez et al. (2016). Using Austrian data with repeated claimants

Alvarez et al. (2016) reject the hypothesis of worker's having heterogeneous Poisson exit rates from unemployment.

The rest of this section uses data on unemployed job seekers in New Jersey. The data, and its construction, is detailed in Krueger and Mueller (2011) and available for public download at http://opr.princeton.edu/archive/njui/. 6,025 unemployed workers in the New Jersey area are surveyed at a weekly frequency for up to 24 weeks. The data is unique in it asks workers about the job offers they receive (not necessarily take) and their contemporaneous reservation wage. We make a broader stratification of this data - only distinguishing between those with and without a college education. We do this because we have an insufficient number of workers without high school education. In all that follows we have weighted our results by the sampling criterion and weights provided and described in Krueger and Mueller (2011).

Figure 2: CDF of Offered Wages
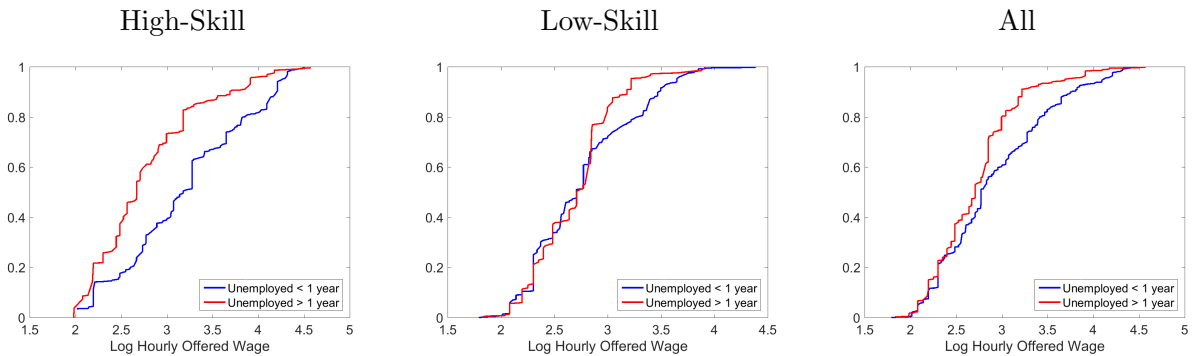
Respondents are asked the highest wage offer they received in the last week. Figure 2 plots the cumulative distribution of these offers. There appears to be first order stochastic dominance for those who have spent less time in unemployment, particularly for the college educated. This could suggest, as will be present in our model's mechanism, that the short term unemployed have on average better employment opportunities. Again, an alternative explanation would be a compositional difference in innate quality between the short and long term unemployed.

To further substantiate our claim that the recently unemployed face better job prospects we turn to information regarding reservation wages. Every week respondents are asked: "*Suppose someone*

offered you a job today. What is the lowest wage or salary you would accept (before deductions) for the type of work you are looking for?". Figure 3 plots the hourly rate reported as a CDF. Once again, there is evidence of stochastic dominance of the wages of the short term unemployed over those who have spent more time in unemployment. This could be indicative of people suspecting they have a higher employment option value when unemployed for a shorter time, a feature the model presented in the following section will contain.

Figure 3: CDF of Reported Reservation Wages

High-Skill	Low-Skill	All
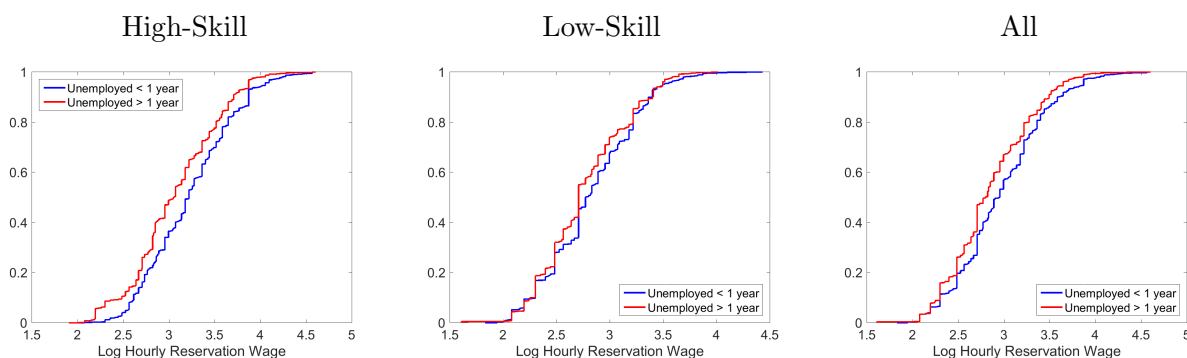
In the survey the respondents were also asked *"In the last 7 days, did you receive any job offers? If yes, how many?"*. Figure 4 shows the distribution of the number of offers, among those with at least one offer, in the data versus that implied by the Poisson distribution as well as their log ratio. A salient feature of the figure is that the number of worker with more than one offer is substantially higher than what would be implied by the Poisson distribution. This suggest that there might be a lot of heterogeneity in the employment prospects of workers.

3 Model

3.1 The Environment

The aim in our modeling approach is to stay as true to BM as possible. Time is continuous and the labor market is populated by risk-neutral workers and firms. Firms are infinitely lived and workers leave the labor force at a Poisson rate μ. Worker's are ex-ante homogeneous and ex-post vary in their employment state $s \in \{u, e\}$, their employment opportunities, and if employed, their wage w.

Figure 4: Number of Job Offers

(a) Data	(b) Poisson Process	(c) Log Ratio
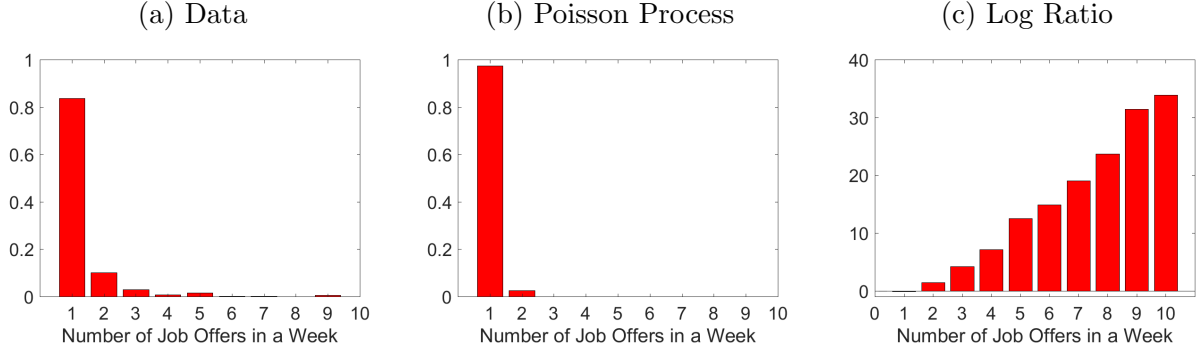
Panel (a) shows the empirical distribution of reported job offers received in a week, conditional on receiving at least one offer, weighted by sampling weights. Panel (b) shows the equivalent distribution that would be implied by a Poisson job offer process. The parameter of the Poisson distribution is inferred by the frequency at which workers receive at least one offer. Panel (c) is the log of the former divided by the log of the latter.

In unemployment workers earn a flow income of b. Firms are heterogeneous in their productivity y, the cumulative distribution of productivity amongst firms is given by $\Gamma(\cdot)$ and is a primitive of the model. All job ties including ones employer are exogenously destroyed at a rate δ. Workers leave the labor market at constant rate μ, independent of their wage and employment state and are replaced by young workers who enter unemployment without any job offers. Finally, we do not allow workers to quit other than to move to a new job.

3.2 The Frictions

The market is characterized by search frictions with an additional feature capturing aspects of stock-flow matching. In particular there is a stock of outstanding vacancies, which we refer to as the workers employment prospects or simply their stock. In the stock-flow model of Coles (1994) and Coles and Smith (1998) workers can always match with the stock, whereas we assume, this opportunity arises at a Poisson rate γ_s. If workers can always match with the stock the model has an instantaneous property. Where after being fired the worker will either immediately match or otherwise wait for the inflow of new vacancies. With a finite value of γ_s, this becomes a gradual process. The γ_s thus captures an additional search friction. The finite value of γ_s further means that all workers match with the stock. Some workers will have an empty stock and will then be

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waiting for the flow to increase the stock, this corresponds to workers waiting to match with the flow in the standard stock-flow matching. When the worker matches with the stock of vacancies, the worker chooses the most appropriate option. We assume that all rejected vacancies will no longer consider the worker. We believe that this assumption captures a realistic feature of the labor market. A vacancy enters the stock for a workers in state s at an exogenous Poisson rate λ_s.

Each employment prospect arrives with a wage that is set optimally by profit maximizing firms. Workers draw a wage from a cumulative distribution function $F(w)$ - an endogenous object of the model. Notice, the BM model is nested in our setup, our model will converge to BM as $\gamma_s \to \infty$. The friction analyzed above is the stock-flow interpretation. Alternatively, imagine that there are frictions in the relocation for workers of hiring for the firm that gives rise to the same model. There are described in section 3.2.1. The model with those types of frictions is observationally equivalent to the one with stock-flow matching. For the remaining of this paper we will set it up in terms of the stock-flow perspective.

3.2.1 Alternative interpretation of the friction

There are a number of frictions in relocation or hiring that can also result in workers not moving to the same distribution of wages independent of their employment status and job duration. Workers might not be able to immediately move to a new job. This could be due to restrictions on the end date of the current job or start date of the new job. It could also be because the new job might require geographical relocation or the old job has a mandatory notice period. These mechanisms imply that the arrival rate of job offers is not enough to determine the frequency at which workers change employers. A simple and transparent way to capture this is to introduce a relocation friction. In the baseline workers can relocate all the time whereas we introduce a Poisson process, with intensity γ_s that governs when workers can relocate. The relocation friction is therefore similar to the modeling of price frictions in New Keynesian models. When the worker is able to reallocate, the worker can choose not to move, or to move to their best offer. There is no recall of past or rejected offers. Compared to frictionless relocation there are some job moves that are not observable as job to job transitions as the worker received a better offer before being able to relocate.

3.3 Steady-state distribution of vacancy stock

We denote the number of vacancies in the stock by $j \in \mathbb{N}_+$. The probability that a worker in employment state s has j vacancies in the stock is given by $p_s(j)$. The inflow of employed workers with $j \geq 1$ offers comes from two sources. Those with $j - 1$ offers who received an additional offer and those with $j + 1$ offers who lose an offer. The inflow for employed workers with no offers $j = 0$ is from two sources: employed workers with one offer which they lose and from all workers who just searched through the stock, independent of employment state.

$$
\begin{aligned}
\text{inflow} &= \lambda_e p_e(j-1) + \delta(j+1)p_e(j+1) \quad \forall j \geq 1 \\
\text{inflow} &= \delta p_e(1) + \gamma_e + \gamma_u(1 - p_u(0))\frac{u}{1-u} \quad j = 0
\end{aligned}
$$

Similarly the outflow can be due to separation from the job, losing an offer in hand or because the worker was matched with the stock. The outflow is then given below.

$$
\text{outflow} = (\lambda_e + \gamma_e + \mu + \delta + \delta j)p_e(j)
$$

The steady-state and the distributions are defined by equalizing the outflow and inflow of a given number of job offers j. The number of outstanding offers is then

$$
\begin{aligned}
(\lambda_e + \gamma_e + \mu + \delta + \delta j)p_e(j) &= \lambda_e p_e(j-1) + \delta(j+1)p_e(j+1) \quad \forall j \geq 1 \\
(\lambda_e + \gamma_e + \mu + \delta)p_e(0) &= \delta p_e(1) + \gamma_e + \gamma_u(1 - p_u(0))\frac{u}{1-u} \quad j = 0.
\end{aligned}
$$

The inflow of unemployed workers with stock of $j \geq 1$ vacancies can either be because a worker had a stock of $j - 1$ vacancies and accrued one more, or because a worker has stock with $j + 1$ vacancies and lost one or that an employed worker with k offers is hit by a job destruction shock. The inflow for $j = 0$ is from unemployed workers who lose an offer and employed workers with no offers that are hit by a destruction shock.

$$
\begin{aligned}
\text{inflow} &= \lambda_u p_e(j-1) + \delta(j+1)p_u(j+1) + \delta\frac{u}{1-u}p_e(j) \quad \forall j \geq 1 \\
\text{inflow} &= \delta p_u(1) + \mu + \delta\frac{u}{1-u}p_e(0)
\end{aligned}
$$

For the unemployed, the outflow can be due to workers taking job offers, which when they match at a rate γ_u. In addition they also acquire new offers at a rate λ_u and lose offers at rate δ.

$$\text{outflow} = (\lambda_u + \gamma_u + \mu + \delta j)p_u(j) \quad \forall j \geq 1$$

$$\text{outflow} = \lambda_u p_u(0) \quad j = 0$$

The steady state distribution solves the equations

$$(\lambda_u + \gamma_u + \mu + \delta j)p_u(j) = \lambda_u p_e(j-1) + \delta(j+1)p_u(j+1) + \delta\frac{u}{1-u}p_e(j) \quad \forall j \geq 1$$

$$\lambda_u p_u(0) \quad j = \delta p_u(1) + \mu + \delta\frac{u}{1-u}p_e(0)$$

3.4 Steady-state distribution of match quality

In order to solve for the distribution of wages in outstanding matches we proceed in two steps. First we define the function Σ_s for each employment state s as

$$\Sigma_s(F) = \sum_{j=0}^{\infty} p_s(j)F^j.$$

The function $\Sigma_s(F)$ is a probability generating function. It evaluates the probability that when a random worker, in state s matches with the stock, they have no vacancy above rank F. The function $\Sigma_s(F)$ has the, steady-state, solution

$$\Sigma_e(F) = \frac{1}{1-F}\int_F^1 \exp\left[-\lambda_e/\delta\left(\tilde{F}-F\right)\right]\left(\frac{1-\tilde{F}}{1-F}\right)^{\frac{\gamma_e+\mu}{\delta}}\frac{\gamma_e+\mu+\delta}{\delta}d\tilde{F}$$

$$\Sigma_u(0) = \frac{\int_0^1 \exp\left[-\lambda_u/\delta\tilde{F}\right]\left(1-\tilde{F}\right)^{\frac{\gamma_u+\mu-\delta}{\delta}}\left[\frac{\delta+\mu+\gamma_u}{\delta+\mu}\frac{\mu}{\delta}+\frac{\gamma_u}{\delta+\mu}\frac{\delta\Sigma_e(\tilde{F})}{\delta}\right]d\tilde{F}}{1-\int_0^1 \exp\left[-\lambda_u/\delta\tilde{F}\right]\left(1-\tilde{F}\right)^{\frac{\gamma_u+\mu-\delta}{\delta}}\left[\frac{\gamma_u}{\delta}-\frac{\gamma_u}{\delta+\mu}\frac{\mu}{\delta}-\frac{\gamma_u}{\delta+\mu}\frac{\delta\Sigma_e(\tilde{F})}{\delta}\right]d\tilde{F}}$$

$$\Sigma_u(F) = \frac{1}{1-F}\int_F^1 \exp\left[-\lambda_u/\delta(\tilde{F}-F)\right]\left(\frac{1-\tilde{F}}{1-F}\right)^{\frac{\gamma_u+\mu-\delta}{\delta}}\left[\frac{\gamma_u\Sigma_u(0)}{\delta}+\frac{\mu/u}{\delta}+\frac{\delta(1-u)/u\Sigma_e(\tilde{F})}{\delta}\right]d\tilde{F}.$$

The derivation of this function is in Appendix A.1. The rate of inflow into unemployment from employment is given by $\delta + \mu$. Similarly the rate of outflow from unemployment is given by $\gamma_u(1-p_u(0))$. In steady-state the inflow is equal to the outflow which using the definition of Σ_u gives an expression for the unemployment rate.

$$u = \frac{(\delta+\mu)}{(\delta+\mu+\gamma_u(1-\Sigma_u(0)))} \tag{1}$$

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Using the function Σ_s we can further solve for the distribution of outstanding matches G. Note first that the inflow of matches below F is $\gamma_u \sum_{j=1}^{\infty} F^j p_u(j)$, ie the probability that an unemployed worker matches with an offer less than F. Similarly the outflow of matches below F is the exogenous separation $\delta + \mu$ plus the endogenous quit of $\gamma_e \left(1 - \sum_{j=0}^{\infty} F^j p_e(j)\right)$. In steady state the inflow has to equal the out flow which gives

$$(1-u)G\left(\delta + \mu + \gamma_e\left(1 - \sum_{j=0}^{\infty} F^j p_e(j)\right)\right) \quad = \quad u\gamma_u\left(\gamma_u \sum_{j=1}^{\infty} F^j p_u(j)\right) \tag{2}$$

$$\tag{3}$$

Using the definition of Σ_s we get

$$G \quad = \quad \frac{u\gamma_u \left(\Sigma_u(F) - \Sigma_u(0)\right)}{(1-u)\left(\delta + \mu + \gamma_e \left(1 - \Sigma_e(F)\right)\right)} \tag{4}$$

$$= \quad \frac{(\delta + \mu)}{\gamma_u(1 - \Sigma_u(0))} \frac{\gamma_u \left(\Sigma_u(F) - \Sigma_u(0)\right)}{(1-u)\left(\delta + \mu + \gamma_e \left(1 - \Sigma_e(F)\right)\right)} \tag{5}$$

The derivative of G is given in Appendix A.1.

3.5 Worker problem

An individual's utility is given by the present expected discounted value of their future income stream, this will depend on their employment status, if employed their wage, and on the stock of vacancies. The number of vacancies and the wage of a given offer will only become clear to a worker when they match with the stock. The value function for an unemployed worker with j offers in hand is given by equation (6). $F \in [0,1]$ is the rank of wages from the job offer sampling distribution.[4]

$$\mu U(j) = b + \gamma_u \int_0^1 (W(w(F), 0) - U(j)) dF^j + \lambda_u(U(j+1) - U(j)) + \delta j(U(j-1) - U(j)) \tag{6}$$

The value function of a worker, discounted by the rate they leave the market, is the sum of the flow value of unemployment b. The option value of employment which occurs after matching with the stock at rate γ_u. $W(\cdot)$ is the value of employment and defined in equation (7). Finally, while in

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[4]Notice, this corresponds perfectly with a firms rank in the productivity distribution. This is because, as will be seen, firms pay all workers they employ the same wage and wage is a monotonically increasing function in a firm's productivity type.

employment, the number of job offers they have follows a stochastic process with offers arriving at a rate λ_u and losing a given offer at a rate δ. The value function for an employed worker earning a wage w with j offers is the sum of the flow wage, the option value of the j offers to be processed, the option value of the stochastic process that governs the evolution of j and the option value of becoming unemployed which occurs with probability δ.

$$\mu W\left(w\left(F\right),j\right) = w(F) + \gamma_e \int_F^1 (W(w(\tilde{F}),0) - W(w(F),0))d\tilde{F}^j + \lambda_e(W(w(F),j+1) - W(w(F),j))$$
$$+ \delta j(W(w(F),j-1) - W(w(F),j)) + \delta(U(j) - W(w(F),j))$$
$$+ \gamma_e(W(w(F),0) - W(w(F),j)) \tag{7}$$

Like BM, a worker's decision is whether to accept or reject a given offer. One could imagine a more sophisticated set of strategies if a worker is aware of the number vacancies in the stock, their job tenure or the wages of individual vacancy in the stock. In this environment employed workers would under some conditions optimally quit to unemployment. However, this is beyond the scope of this paper. Unlike, BM however, once matching with the stock a worker has potentially more than one offer to contend with. Since all jobs are homogeneous except for wages, worker's will always prefer the highest wage job available to them, be it an offer or the job they are currently employed in. An unemployed worker accepts a wage if it yields a higher present value than continuing in unemployment. Since firms post wages optimally, assuming a negligible cost in wage posting no firm would post a wage less than this value, therefore we are solving for the infimum of the wage support, $\phi = w(0)$. This is found by solving the equality given in equation (8).

$$U(0) = W(\phi, 0) \tag{8}$$

To solve for this wage level is slightly more difficult than usual because of the second state variable - the number of job offers. Appendix A.2 explains how one can compute the value function.

3.5.1 Hiring

Search is random and the firm posting a vacancy can either meet an employed or unemployed worker. For the worker to accept the offer it has to be that the wage is acceptable to the worker and it

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is better than any other offer the worker holds. In the BM model, workers match instantaneously which means that the offer is always the best amongst new job offers. The wage is acceptable if it is above the current wage or the reservation wage for the unemployed. In our model a workers match with the stock which means that both channels are operating. Define m as the probability that a vacancy meets a worker. The probability that a worker is hired can then be calculated using

$$
\begin{aligned}
\Pr(\text{hire}|w \geq \phi) \;=\; & m \Pr(\text{meet an unemployed worker}) \Pr(\text{unemployed worker accepts}|w) \\
& + m \Pr(\text{meet an employed worker}) \Pr(\text{employed worker accepts}|w).
\end{aligned}
$$

The probability that a vacancy meets an unemployed worker conditional on a meeting is the flow rate of meetings with unemployed workers divided by the total flow rate of all meetings. The flow rate of meetings for the unemployed workers comprises the product of of three terms: the rate at which the offers are processed; the stock of unemployed; and the expected number of offers. The expected number of offers is given by $\sum_{j=1}^{\infty} j p_s(j)$.

In order to calculate the probability that the worker accepts the offer conditional on meeting with the vacancy can be broken up into two parts, the probability that the offer is better than his current offer (1 for the unemployed and $G(w)$ for the employed) times the probability that the offer is the highest among all the offers the worker has received. The probability that the offer F is the highest offer among all offers for the worker in state s is the probability that the vacancy meets a worker with j offers $\frac{j p_s(j)}{\sum_{j=1}^{\infty} j p_s(j)}$ multiplied by the probability that the offer is higher than the $j-1$ alternative offers (F^{j-1}). This gives

$$
\frac{\sum_{j=1}^{\infty} j p_s(j) F^{j-1}}{\sum_{j=1}^{\infty} j p_s(j)}.
$$

Combining the expressions above and using the definition of Σ_s we get

$$
\begin{aligned}
\Pr(\text{hire}|w \geq \phi) \;=\; & m \frac{\gamma_u u \sum_{j=1}^{\infty} j p_u(j)}{\gamma_u u \sum_{j=1}^{\infty} j p_u(j) + \gamma_e(1-u) \sum_{j=1}^{\infty} j p_e(j)} \frac{\sum_{j=1}^{\infty} j p u(j) F^{j-1}}{\sum_{j=1}^{\infty} j p_u(j)} \\
& + m \frac{\gamma_e(1-u) \sum_{j=1}^{\infty} j p_e(j)}{\gamma_u u \sum_{j=1}^{\infty} j p_u(j) + \gamma_e(1-u) \sum_{j=1}^{\infty} j p_e(j)} (1-u) G(F) \frac{\sum_{j=1}^{\infty} j p_e(j) F^{j-1}}{\sum_{j=1}^{\infty} j p_e(j)} \\
\;=\; & m \frac{(\gamma_u u \Sigma_u'(F) + \gamma_e(1-u) G(F) \Sigma_e'(F))}{(\gamma_u u \Sigma_u'(1) + \gamma_e(1-u) \Sigma_e'(1))}
\end{aligned}
$$

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3.5.2 Duration of a job

Unlike BM, the duration of jobs are not exponentially distributed. The leaving rate is time dependent, it starts low and then increases over time as the employment prospects increase. At the beginning of a job, the worker has matched with the stock and is therefore unlikely to leave right away, as time progresses the number of offers and hence their leaving probability increases. It turns out that even though the leaving rate is not constant, the average leaving rate is a sufficient statistic for expected duration at the time of hiring. Using Little's law from queuing theory we are able to calculate the expected duration at the time of hiring. The average rate at which the working in a firm leaves the job is given by $\delta + \mu + \gamma_u(1 - \Sigma_e(F))$. The average duration in a job F is therefore just $\frac{1}{\delta + \mu + \gamma_u(1 - \Sigma_e(F))}$.

3.6 Firm problem

The firm sets a wage to optimally trade off the increased retention and hiring with the increased cost associated with a higher wage. The expected profits per vacancy for a firm with match quality rank F posting a wage w is made up by three terms: the probability that a worker is hired; the expected duration; and the markup. Combining these gives the expression for the expected profits

$$
\begin{aligned}
\pi(w, F) &= \Pr(\text{hire}) E(\text{duration})(y(F) - w) \\
&\propto \left(\gamma_e \frac{\gamma_u(1 - \Sigma_u(0))}{\delta + \gamma_u(1 - \Sigma_u(0))} G(F) \Sigma'_e(F) + \gamma_u \frac{(\delta + \mu)}{\delta + \gamma_u(1 - \Sigma_u(0))} \Sigma'_u(F) \right) E(\text{duration})(y(F) - w) \\
&\propto \left(\gamma_e \frac{(\delta + \mu)(\Sigma_u(F) - \Sigma_u(0))}{(\delta + \mu + \gamma_e(1 - \Sigma_e(F)))} \Sigma'_e(F) + \frac{(\delta + \mu)}{\delta + \mu + \gamma_u(1 - \Sigma_u(0))} \Sigma'_u(F) \right) \frac{(y(F) - w)}{\delta + \mu + \gamma_u(1 - \Sigma_e(F))}.
\end{aligned}
$$

3.6.1 Retention vs hiring

In the wage posting model firm pays more to increase retention and hiring. In particular the firm profit is given by the expression

$$
\pi(w, F) = H(w) R(w)(y(F) - w)
$$

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where $H(w)$ and $R(w)$ refers to the probability that a workers is hired and the expected duration of the match, given a wage w, respectively. The first order condition, for logarithm of profits, is given by

$$\frac{H'(F)}{H(F)} + \frac{R'(F)}{R(F)} = \frac{w'(F)}{(y(F) - w(F))}$$

The first and second terms correspond to the percentage increase in hiring and retention from a higher wage. The last term captures the increased wage cost. Defining $h(F) = \frac{H'(F)}{H(F)}$ and $r(F) = \frac{R'(F)}{R(F)}$ and solving for the wage gives

$$
\begin{aligned}
w^m(F) &= \int_0^F m(F)(y(F) - w(F))dF \quad \text{where, } m \in \{h, r\} \\
w(F) &= w^r(F) + w^h(F) + w(0).
\end{aligned}
$$

The wage $w(F)$ can therefore be decomposed into three terms, the wage increase coming from the retention $(w^r(F))$ motive and hiring motive respectively $(w^h(F))$ and the wage that satisfies the participation constraint for the worker $w(0)$. In the BM model the functions are

$$r(F) = h(F) = \frac{\lambda_e}{\delta + \mu + \lambda_e(1 - F)}.$$

The motive to pay for retention and hiring is thus the same. In our model on the other hand we have that

$$
h(F) = \frac{\left(\frac{\gamma_e}{(\delta + \mu + \gamma_e(1 - \Sigma_e(F)))} \left((\Sigma_u(F) - \Sigma_u(0))\left(\Sigma_e''(F) + \frac{\gamma_e \Sigma_e'(F)^2}{(\delta + \mu + \gamma_e(1 - \Sigma_e(F)))}\right) + \Sigma_u'(F)\Sigma_e'(F)\right) + \frac{\Sigma_u''(F)}{\delta + \mu + \gamma_u(1 - \Sigma_u(0))}\right)}{\left(\gamma_e \frac{(\Sigma_u(F) - \Sigma_u(0))}{(\delta + \mu + \gamma_e(1 - \Sigma_e(F)))}\Sigma_e'(F) + \frac{1}{\delta + \mu + \gamma_u(1 - \Sigma_u(0))}\Sigma_u'(F)\right)}
$$

$$
r(F) = \frac{\gamma_e \Sigma_e'(F)}{(\delta + \mu + \gamma_e(1 - \Sigma_e(F)))}
$$

The motives are not necessarily the same and there relative importance depends on the firm type. The relative magnitude of the two motivators is a quantitative question that depends on the specific parameterization. A question that will be explored after the model has been estimated.

3.6.2 Competition

Noting that the percentage increase in firm type, $\frac{\ell'(F)}{\ell(F)}$, is just $\frac{\ell'(F)}{\ell(F)} = \frac{G''(F)}{G'(F)} = h(F) + r(F)$ we can rewrite the first order condition as

$$\pi_w(w, F) = \frac{\ell'(F)}{\ell(F)}(y(F) - w(F)) - w'(F) = 0 \tag{9}$$

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We refer to $\frac{G''(F)}{G'(F)}$ as the degree of competition. If $\frac{G''(F)}{G'(F)}$ is low then the firm size is unresponsive to the wage and there is little reason to increase pay. The degree of competition, using our formula, can be written as

$$\frac{\ell'(F)}{\ell(F)} = \frac{\left[\Sigma_u''(F) + \frac{\gamma_e(\Sigma_u(F)-\Sigma_u(0))}{(\delta+\mu+\gamma_e(1-\Sigma_e(F)))}\Sigma_e''(F)\right]}{\Sigma_u'(F) + \frac{\gamma_e(\Sigma_u(F)-\Sigma_u(0))}{(\delta+\mu+\gamma_e(1-\Sigma_e(F)))}\Sigma_e'(F)} + 2\gamma_e\frac{\Sigma_e'(F)}{(\delta + \mu + \gamma_e(1 - \Sigma_e(F)))}$$

Whereas the competition term for the normal BM model is

$$\frac{\ell'(F)}{\ell(F)} = 2\lambda_e\frac{1}{(\delta + \mu + \lambda_e(1 - F))}$$

Note that Σ is a convex function. When the firm considers the hiring margin in the normal BM model, firms need only consider the probability that the worker is working at a lower paying firm. In our setup the firm also needs to consider the probability that the worker has a better offer in hand. This effect is captured by the term $\Sigma_s'(F)$. Secondly, the dependence of the leaving rate on F enters via the term Σ_e which is a convex function. The competition in our model therefore increases more as we move to the tail of the distribution.

3.7 Equilibrium

An equilibrium in this economy is characterized by the function $\{W(w,j), U(j), w(y), \pi(w,y), \ell(w)\}$ such that the firm profits is given by equation 9 and the worker values is given by equation 18 in the Appendix. The wage function $W(y)$ solves the firm problem, such that that equation 9 is maximized, and the worker is indifferent between the lowest wage and unemployment, both with zero offers $W(\phi, 0) = U(0)$ with $\phi = w(0)$.

3.8 Identification

Appendix A.3 formally discusses identification. We provide a proof that the parameters of our model are identifiable using data on employment dynamics, tenure and wages. Similar data will be used when we estimate the model. Although the exact moments discussed in the Appendix are not practically implemented in estimation we use similar moments for the purpose of estimation. The aim of our estimation is to minimize a criterion defined as a distance between simulated

and empirically observable moments. While our proof does not guarantee that the estimated parameters to be presented are a global minimum of the criterion it does imply that the parameters are identifiable. However, since our model is very inexpensive computationally we can trial many different initial guesses and are confident that our estimates correspond to a global minimum.

4   Estimation - PRELIMINARY

Our estimation will focus on estimating the model presented in the previous sections, which for brevity we will henceforth refer to as BG. In addition, we will estimate the nested BM, recall BM is the limiting case of BG as γ_u and γ_e tend to infinity. Allowing both models the same opportunity to fit the data is of course the fairest way to assess their relative performance in explanatory power.

4.1   Data

The data used in this estimation are taken from the Current Population Survey (CPS). We stratify the sample according to skill level into three different stratum. They are: the college educated; those whose highest academic achievement is a high school diploma; and those who have not completed high school education. We restrict attention to male workers aged between 18 and 65 and since both models rely on steady-state assumptions, we also restrict attention to the relatively short and stable period between the years 1996 and 2000, inclusive.

Identification will rely on employment dynamics and the cross-sectional wage distribution. Table 1 reports moments on hourly earnings and hours worked per week for each stratum. These are computed by dividing the self-reported weekly earnings by self-reported hours worker per week. Inspection of Table 1 shows large systematic differences in hourly earnings across skill. These difference are the motivation for stratification. Comparing hourly earning across strata seems sensible as there is little cross strata variation in hours per week, with all subgroups working on average approximately 40 hours. Table 2 presents employment dynamics, estimated from a three state Markov process. The rows represent a worker's employment status at period t and the columns in $t+1$, all changes are conditional on a change in employer. All of the moments presented in Table

Table 1: Descriptive Statistics

	All	Low-skill	Medium-skill	High-skill
employment:				
total number employed	343,874	40,604	181,071	122,199
earnings ($/hour)				
mean earnings	16.05	9.85	13.86	21.33
standard deviation of earnings	10.59	5.98	8.42	12.28
weekly hours worked:				
mean weekly hours	42.08	39.86	41.35	43.89
standard deviation of weekly hours	9.38	9.53	9.15	9.34

Source: Data comes from the CPS, moments are based on male workers aged between 18 and 65 between 1996 and 2000, inclusive.

2 will be exactly identified in the estimation to come. Inspection of these matrices reveals the large flow from inactivity to employment and differences across strata. Particularly pronounced differences relate to job security, with a worker less likely to switch from employment to inactivity, unemployment or to another employer as they moves up the skill distribution.

4.2 Estimation Protocol

Our estimation will focus on estimating the model presented in the previous sections, which for brevity we will henceforth refer to as BG. In addition, we will estimate the nested BM, recall BM is the limiting case of BG as γ_u and γ_e tend to infinity. Allowing both models the same opportunity to fit the data is of course the fairest way to assess their relative performance in explanatory power.

Estimation of the two models is done in two parts. In a first step employment transitions are matched. The second step inverts the distribution of wages to uncover the underlying primitive firm productivity distribution. Under BM matching transition rates corresponds with exactly identifying all elements presented in Table 2. In addition to exactly identifying these moments the BG model has additional over-identifying conditions - matching the job finding rate by unemployment duration. In order to guarantee the exact identification of aggregate movements, we impose a large

Table 2: Transition Matrices

All	Inact.	Unemp.	Emp.
Inact.	—	—	0.0875
Unemp.	—	—	0.3232
Emp.	0.0172	0.0128	0.0279

Low-skill	Inact.	Unemp.	Emp.
Inact.	—	—	0.0709
Unemp.	—	—	0.3049
Emp.	0.0307	0.0265	0.0352

Medium-skill	Inact.	Unemp.	Emp.
Inact.	—	—	0.0924
Unemp.	—	—	0.3328
Emp.	0.0191	0.0142	0.0295

High-skill	Inact.	Unemp.	Emp.
Inact.	—	—	0.0996
Unemp.	—	—	0.3203
Emp.	0.0097	0.0060	0.0231

Note 1: Transition rates are (continuous time) monthly. Rows do not add up to one. The emp-emp entries represent the fractions of individuals *changing jobs* while remaining employed.

penalty on aggregate rates in our criterion function. Analytic moment conditions from both models are presented in Appendix A.4. The set of parameters to be estimated is given by the vector θ.

$$\theta = (b, \mu, \delta, \lambda_u, \lambda_e, \gamma_u, \gamma_e, \Gamma(y)) \tag{10}$$

Notice equation, (10) contains an entire distribution in $\Gamma(y)$. We make further parametric assumptions on the primitive distribution of firm type. We assume it follows a generalized extreme value distribution. This is a flexible, three parameter distributin with the cumulative distribution function given below.

$$\Gamma(y; \mu_y, \sigma_y, \xi_y) = \exp\left\{-\left[1 + \xi_y\left(\frac{y - \mu_y}{\sigma_y}\right)\right]^{-1/\xi_y}\right\}$$

The distributional parameters $\mu_y \in \mathbb{R}, \sigma_y > 0$ and $\xi_y \in \mathbb{R}$, govern the location, scale and shape of the distribution, respectively. The distributional assumption is somewhat arbitrary, we find this performs very well in matching the wage distribution in both the BM and BG setup. Further, for the time being we only consider a two state model of the labor market, ruling out inactivity, and consequently we set $\mu = 0$. Thus (10) can be reduced to the following vector of scalars. The focus

of the rest of this section is the estimation of the vector θ.

$$\theta = (b, \delta, \lambda_u, \lambda_e, \gamma_u, \gamma_e, \mu_y, \sigma_y, \xi_y)$$

4.3 Results

Running the two step estimation procedure as described yields the parameter estimates presented in Table 3. The level of the transition rate from unemployment to employment is about the same for all worker types. The transition rate from employment to unemployment on the other hand is much higher for low skilled workers. Immediately apparent from inspection of the transitional parameters, the upper cell of the table, is the two models have a very different interpretation of the functioning of the labor market. For all but the highest skilled in BM $\lambda_0 > \lambda_1$ meaning workers are exposed to a greater number of job offers in unemployment than in employment. It is this phenomenon that the BM uses to rationalize the higher rate that the unemployed find new jobs, see Table 2. In the BG setup, for all but the lowest skilled, the opposite is the case. Instead, the higher finding rate is replicated by job offers being processed more quickly for the unemployed $\gamma_0 > \gamma_1$.

4.4 Fit

Figure 5 shows the probability an unemployed agent moves to employment, by the duration of their unemployment spell. The horizontal red line represents that predicted by BM, the declining blue line is BG and the black crosses are taken from the data. Recall, both models match the aggregate transition rates exactly, however, clearly the BG model does much better by duration as the BM model is unable to give duration dependence.

Figure 6 shows the fit of the wage distribution for the two models. Both models are able to fit the wage distribution with the parametric restriction on the productivity distribution. The wage distribution is highly skewed to the right for all worker types.

Table 3: Parameter Estimates

	All		High-skill		Medium-skill		Low-skill	
	BM	BG	BM	BG	BM	BG	BM	BG
δ	0.019 ()	0.019 ()	0.009 ()	0.009 ()	0.022 ()	0.022 ()	0.04 ()	0.04 ()
λ_u	0.355 ()	1.047 ()	0.338 ()	1.026 ()	0.366 ()	0.998 ()	0.351 ()	1.179 ()
λ_e	0.16 ()	0.303 ()	0.308 ()	0.684 ()	0.16 ()	0.282 ()	0.127 ()	0.287 ()
γ_u	—	0.267 ()	—	0.235 ()	—	0.265 ()	—	0.296 ()
γ_e	—	0.522 ()	—	1.3 ()	—	0.577 ()	—	0.34 ()
b	-10.33 ()	8.95 ()	3.06 ()	20.96 ()	-8.25 ()	8.25 ()	-7.75 ()	1.82 ()
ξ_y	0.76 ()	0.75 ()	0.63 ()	0.65 ()	0.71 ()	0.68 ()	1.02 ()	0.92 ()
σ_y	1.7 ()	0.8 ()	1.28 ()	0.52 ()	1.58 ()	0.77 ()	0.92 ()	0.57 ()
μ_y	7.26 ()	5.76 ()	6.85 ()	5.6 ()	7.38 ()	5.77 ()	7.66 ()	6.31 ()

4.4.1 Flow Value in Unemployment

The results show that the implied b needed to justify the observed wage distribution is much higher under the BG model. The estimation for the BM model results in an implausible large negative cost associated with unemployment. For example taking the estimation for the full sample we find that for each hour spent in unemployment the worker would be willing to pay $3 dollar to work without pay. Whereas the estimation of the BG model suggest that the unemployed value their time at 7$ /hour. Furthermore, the difference between the productivity and the flow value received in unemployment is much closer in our model compared to the standard BM estimation. The difference between the flow benefits and the level of productivity is very important for the cyclical fluctuations in the unemployment rate (see Hagedorn and Manovskii (2008) and Ljungqvist and Sargent (2015)). The estimation of our model suggest that the difference is much smaller than in the normal estimation. The productivity distribution exhibits a right skewness suggesting that there are a number of marginal jobs that become unproductive in the recession. The shape and

Figure 5: Job finding rate by duration of unemployment
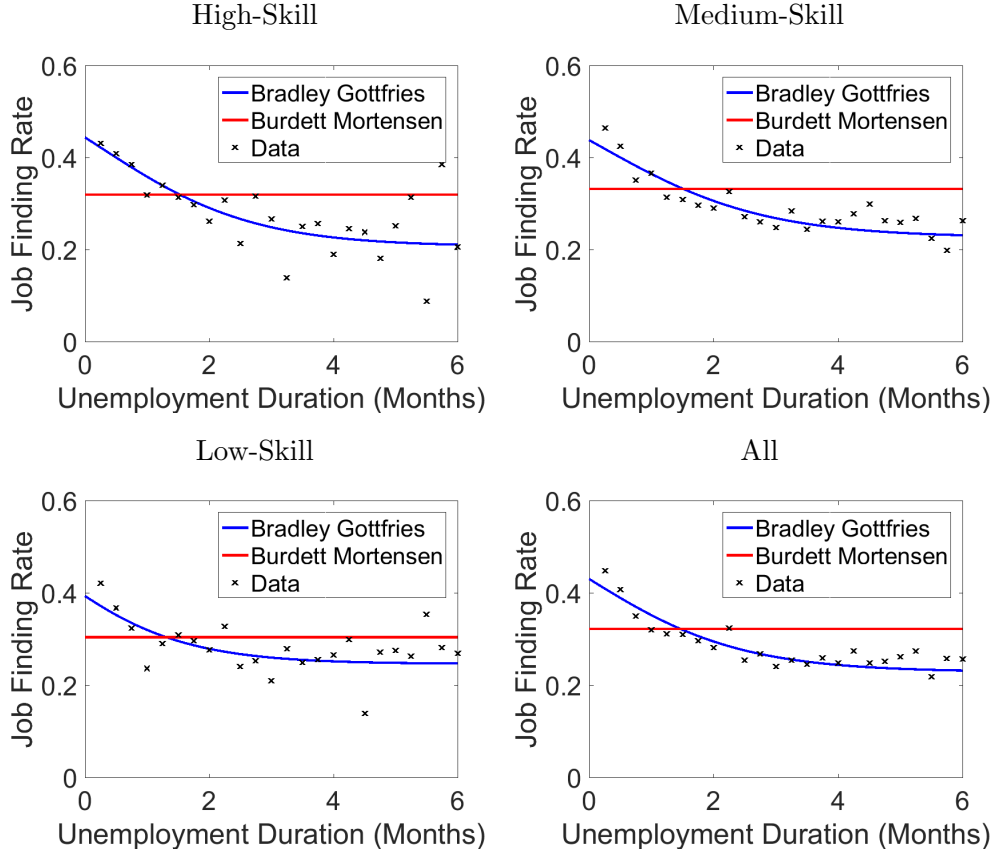
scale parameter of the productivity distributions is given in Table 3. The BG model requires, for all skill groups, a less dispersed productivity distribution as the effective competition is much greater in the right tail.

4.4.2 Firm Competition

We can use expression 9 to calculate the relative competition for different firm types for the BM and BG model. Figure 7 reveals, that for all skill groups, the low productive firms exhibit stronger wage competition in BM compared to the BG model. The intuition for this is that since some workers have many offers it is unlikely that the highest of them is in the lower end of the distribution. Similarly for firms at the upper support of the distribution it is relatively more likely that the highest offer is in the upper part of the distribution. The model therefore shifts competition from

Figure 6: Fit of the Wage Distributions
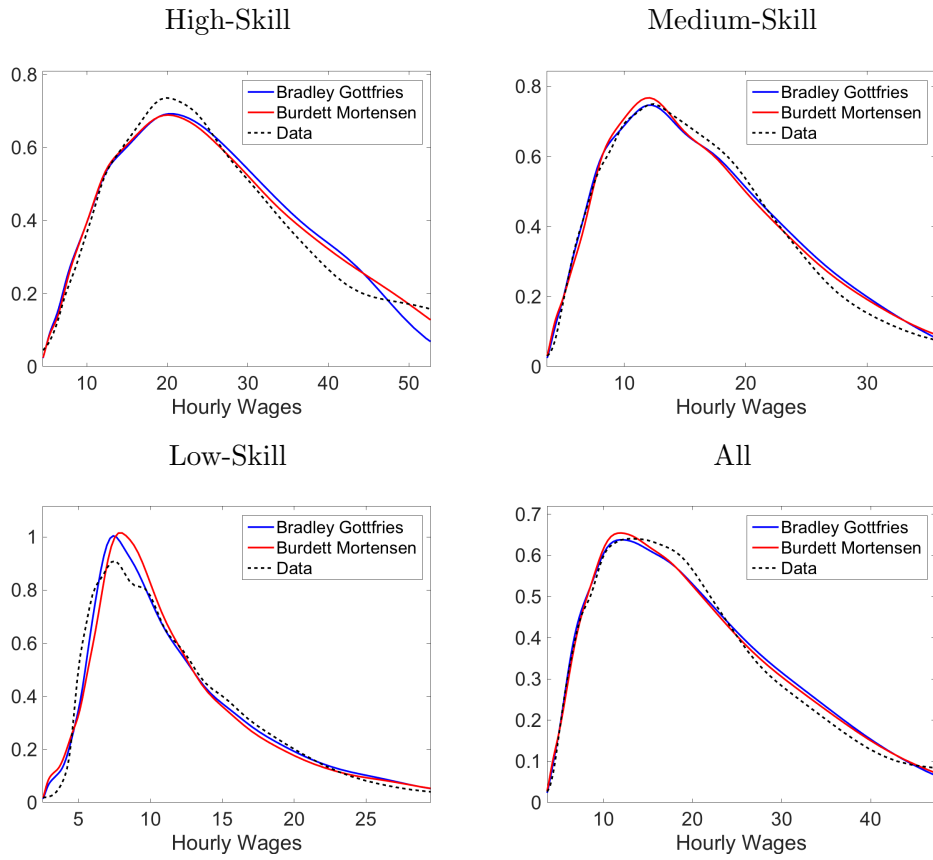
the lower to the upper part of the distribution. Thus increasing the dispersion of wages for a given primitive firm productivity.

4.4.3 Wage Posting Motivation

In the BM model firms incentive to pay for retention is equal to the incentive to pay for hiring whereas in the BG model the incentives differ. In Figure 8 we show the fraction of the wage that is paid for retention. The results suggest that the hiring motive is quantitatively more important, but relatively less so, for wages in the upper support of the wage distribution.

Figure 7: Wage Competition in BM & BG

Figure 8: Proportion of Wages Driven by the Retention Motive

High-Skill

Medium-Skill

Low-Skill

All

5   Conclusion

This paper extends the standard BM model to account for the time it takes for workers to relocate between jobs. We solve the model analytically and show that the extension results in less competition at the lower end of the wage distribution but much more competition at the right tail of the distribution. The increased competition in the tail means that a less dispersed productivity distribution is needed to match the wage distribution compared to the standard BM model. Furthermore the model delivers declining job finding rates by the duration of unemployment as observed in the data. Further, the flow value associated with unemployment required to match the wage distribution does not need to be large and negative. Our findings, in the order of 7 \$/hour, we believe are more plausible. Finally, our model has a more sophisticated mechanism of firm's wage posting motivation allowing differential importance between retention and attraction. We find that attracting new workers is quantitatively more important, but this disparity lessens the higher the wage.

References

Albrecht, J., P. Gautier and S. Vroman, "Equilibrium Directed Search with Multiple Applications," *Review of Economic Studies* 73 (2006), 869–891.

Alvarez, F., K. Borovickova and R. Shimer, "The Proportional Hazard Model: Estimation and Testing using Price Change and Labor Market Data," *mimeo* (2016).

Bontemps, C., J.-M. Robin and G. V. den Berg, "Equilibrium Search with Continuous Productivity Dispersion: Theory and Nonparametric Estimation," *International Economic Review* 41 (2000), pp. 305–358.

Bradley, J., F. Postel-Vinay and H. Turon, "Public Sector Wage Policy and Labor Market Equilibrium: A Structural Model," *forthcoming in The Journal of the European Economic Association* (2016).

Burdett, K. and K. Judd, "Equilibrium Price Dispersion," *Econometrica* 51 (1983), 955–969.

Burdett, K. and D. T. Mortensen, "Wage differentials, employer size, and unemployment," *International Economic Review* (1998), 257–273.

Carillo-Tudela, C. and E. Smith, "Search Capital," *forthcoming in Review of Economic Dynamics* (2016).

Coles, M., "Understanding the Matching Function: The Role of Newspapers and Job Agencies," *CEPR Discussion Paper 939* (1994).

Coles, M. and D. Mortensen, "Equilibrium Labor Turnover, Firm Growth, and Unemployment," *Econometrica* 84 (2016), 247–363.

Coles, M. and E. Smith, "Cross-Section Estimation of the Matching Function: Evidence from England and Wales," *Economica* 63 (1998), 589–598.

Elsby, M. W. L. and R. Michaels, "Marginal Jobs, Heterogeneous Firms, and Unemployment Flows," *American Economic Journal: Macroeconomics* 5 (January 2013), 1–48.

GAUTIER, P. AND C. HOLZNER, "Simultaneous Search and Efficiency of Entry and Search Intensity," *forthcoming in American Economic Journal: Microeconomics* (2016).

GAUTIER, P. AND R. WOLTHOFF, "Simultaneous Search with Heterogeneous Firms and Ex Post Competition," *Labour Economics* 16 (2009), 311–319.

GAUTIER, P. A., C. N. TEULINGS AND A. VAN VUUREN, "On-the-Job Search, Mismatch and Efficiency," *The Review of Economic Studies* 77 (2010), 245–272.

GOTTFRIES, A. AND C. N. TEULINGS, "Returns to On-the-Job Search and the Dispersion of Wages," Tinbergen Institute Discussion Papers 16-080/VI, Tinbergen Institute, 2016.

HAGEDORN, M. AND I. MANOVSKII, "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited," *The American Economic Review* 98 (2008), 1692–1706.

HORNSTEIN, A., P. KRUSELL AND G. VIOLANTE, "Frictional Wage Dispersion in Search Models: A Quantitative Assessment," *American Economic Review* 101 (2011), 2873–2898.

KIRCHER, P., "Efficiency of Simultaneous Search," *Journal of Political Economy* 117 (2009), 861–913.

KIRCHER, P. AND M. GALENIANOS, "Directed Search with Multiple Job Applications," *Journal of Economic Theory* 114 (2009), 445–471.

KRUEGER, A. B. AND A. MUELLER, "Job Search, Emotional Well-Being and Job Finding in a Period of Mass Unemployment: Evidence from High-Frequency Longitudinal Data," *Brookings Papers on Economic Activity* 42 (2011), 1–81.

LJUNGQVIST, L. AND T. J. SARGENT, "The Fundamental Surplus in Matching Models," CEPR Discussion Papers 10489, C.E.P.R. Discussion Papers, March 2015.

MEGHIR, C., R. NARITA AND J.-M. ROBIN, "Wages and Informality in Developing Countries," *American Economic Review* 105 (2015), 1509–1546.

MORTENSEN, D., *Wage Dispersion: Why Are Similar Workers Paid Differently?* (MIT Press, 2003).

MORTENSEN, D. T. AND C. A. PISSARIDES, "Job Creation and Job Destruction in the Theory of Unemployment," *Review of Economic Studies* 61 (1994), 397–415.

POSTEL-VINAY, F. AND J. M. ROBIN, "Wage Dispersion with Worker and Employer Heterogeneity," *Econometrica* 70 (2002), 2295–2350.

SHEPHARD, A., "Equilibrium Search and Tax Credit Reform," *forthcoming in International Economic Review* (2016).

SHIMER, R., "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," *American Economic Review* 95 (March 2005), 25–49.

———, "On-the-job search and strategic bargaining," *European Economic Review* 50 (May 2006), 811–830.

WOLTHOFF, R., "It's About Time: Implications of the Period Length in an Equilibrium Search Model," *International Economic Review* 55 (2014), 839–867.

A   Appendix

A.1   Derivation of Σ

A.1.1   Employed Σ_e

Define the probability generating function (pgf) of the stationary distribution as

$$\Sigma_e(F) = \sum_{j=0}^{\infty} F^j p_e(j) \tag{11}$$

Using the Markov process for the number of offers for the steady state using $\frac{\partial \Sigma_e(F,j)}{\partial t} = 0$.

$$\frac{\partial \Sigma_e(F)}{\partial F} = -\frac{\gamma_e + \mu + \delta}{\delta(1-F)} + \frac{(\lambda_e(1-F) + \gamma_e + \mu + \delta)}{\delta(1-F)} \Sigma_e(F)$$

$$\frac{\partial^2 \Sigma_e(F)}{\partial F^2} = \frac{(\lambda_e(1-F) + \gamma_e + \mu + 2\delta)}{\delta(1-F)} \frac{\partial \Sigma_e(F)}{\partial F} - \frac{\lambda_e}{\delta(1-F)} \Sigma_e(F)$$

Solving the differential equation gives

$$\Sigma_e(F) = \frac{1}{1-F} \int_F^1 \exp\left[-\lambda_e/\delta\left(\tilde{F} - F\right)\right] \left(\frac{1-\tilde{F}}{1-F}\right)^{\frac{\gamma_e + \mu}{\delta}} \frac{\gamma_e + \mu + \delta}{\delta} d\tilde{F}$$

A.1.2   Unemployed Σ_{uu}

Define the probability generating function (pgf) of the stationary distribution as

$$\Sigma_{uu}(F) = \sum_{j=0}^{\infty} F^j p_{uu}(j) \tag{12}$$

Using the Markov process for the number of offers for the steady state using $\frac{\partial \Sigma_{uu}(F,j)}{\partial t} = 0$.

$$\frac{\partial \Sigma_{uu}(F)}{\partial F} = -\frac{\gamma_u + \mu}{\delta(1-F)} + \frac{(\lambda_e(1-F) + \gamma_u + \mu)}{\delta(1-F)} \Sigma_{uu}(F)$$

Solving the differential equation gives

$$\Sigma_{uu}(F) = \frac{1}{1-F} \int_F^1 \exp\left[-\lambda_u/\delta\left(\tilde{F} - F\right)\right] \left(\frac{1-\tilde{F}}{1-F}\right)^{\frac{\gamma_u + \mu - \delta}{\delta}} \frac{\gamma_u + \mu}{\delta} d\tilde{F}$$

A.1.3 Unemployed Σ_u

Define the pgf as

$$\Sigma_u(F,t) = \sum_{j=0}^{\infty} F^j p_u(j,t) \tag{13}$$

The steady state gives the differential equation

$$\Sigma_u'(F) = -\frac{\delta(1-u)/u\Sigma_e(F)}{\delta(1-F)} - \frac{\gamma_u p_u(0)}{\delta(1-F)} + \frac{\lambda_u(1-F) + \gamma_u + \mu}{\delta(1-F)}\Sigma_u(F) - \frac{\mu}{\delta(1-F)} - \frac{\mu(1-u)/u}{\delta(1-F)}$$

$$\frac{\partial^2\Sigma_u(F)}{\partial F^2} = -\frac{\delta(1-u)/u}{\delta(1-F)}\frac{\partial\Sigma_e(F)}{\partial F} + \frac{(\lambda_u(1-F) + \gamma_u + \mu)}{\delta(1-F)}\frac{\partial\Sigma_u(F)}{\partial F} - \frac{\lambda_u}{\delta(1-F)}\Sigma_u(F) + \frac{\delta}{\delta(1-F)}\frac{\partial\Sigma_u(F)}{\partial F}$$

Solving the differential equation and using $\Sigma_u(1) = 1$ gives

$$\Sigma_u(F) = \frac{1}{1-F}\int_F^1 \exp\left[-\lambda_u/\delta(\tilde{F} - F)\right]\left(\frac{1-\tilde{F}}{1-F}\right)^{\frac{\gamma_u+\mu-\delta}{\delta}}\left[\frac{\gamma_u p_u(0)}{\delta} + \frac{\mu/u}{\delta} + \frac{\delta(1-u)/u\Sigma_e(\tilde{F})}{\delta}\right]d\tilde{F}$$

where $u = \frac{\delta+\mu}{\delta+\mu+\gamma_u(1-\Sigma_u(0))}$

$$\Sigma_u(0) = \int_0^1 \exp\left[-\lambda_u/\delta\tilde{F}\right]\left(1-\tilde{F}\right)^{\frac{\gamma_u+\mu-\delta}{\delta}}\left[\frac{\gamma_u\Sigma_u(0)}{\delta} + \frac{\delta+\mu+\gamma_u(1-\Sigma_u(0))}{\delta+\mu}\frac{\mu}{\delta} + \frac{\gamma_u(1-\Sigma_u(0))}{\delta+\mu}\frac{\delta\Sigma_e(\tilde{F})}{\delta}\right]d\tilde{F}$$

gives

$$\Sigma_u(0) = \frac{\int_0^1 \exp\left[-\lambda_u/\delta\tilde{F}\right]\left(1-\tilde{F}\right)^{\frac{\gamma_u+\mu-\delta}{\delta}}\left[\frac{\delta+\mu+\gamma_u}{\delta+\mu}\frac{\mu}{\delta} + \frac{\gamma_u}{\delta+\mu}\frac{\delta\Sigma_e(\tilde{F})}{\delta}\right]d\tilde{F}}{1 - \int_0^1 \exp\left[-\lambda_u/\delta\tilde{F}\right]\left(1-\tilde{F}\right)^{\frac{\gamma_u+\mu-\delta}{\delta}}\left[\frac{\gamma_u}{\delta} - \frac{\gamma_u}{\delta+\mu}\frac{\mu}{\delta} - \frac{\gamma_u}{\delta+\mu}\frac{\delta\Sigma_e(\tilde{F})}{\delta}\right]d\tilde{F}}$$

A.1.4 Distribution of outstanding matches G

The first and second derivative of $G(\cdot)$ is given by

$$u = \frac{(\delta + \mu)}{(\delta + \mu + \gamma_u(1 - \Sigma_u(0)))} \tag{14}$$

$$G(F) = \frac{(\delta + \mu)\left(\Sigma_u(F) - \Sigma_u(0)\right)}{(1 - \Sigma_u(0))\left(\delta + \mu + \gamma_e(1 - \Sigma_e(F))\right)} \tag{15}$$

$$G'(F) = \frac{(\delta + \mu)\Sigma_u'(F)}{(1 - \Sigma_u(0))\left(\delta + \mu + \gamma_e(1 - \Sigma_e(F))\right)} + \frac{\gamma_e(\delta + \mu)\left(\Sigma_u(F) - \Sigma_u(0)\right)\Sigma_e'(F)}{(1 - \Sigma_u(0))\left(\delta + \mu + \gamma_e(1 - \Sigma_e(F))\right)^2} \tag{16}$$

$$
\begin{aligned}
G''(F) =\ & \frac{(\delta + \mu)\Sigma_u''(F)}{(1 - \Sigma_u(0))\left(\delta + \mu + \gamma_e(1 - \Sigma_e(F))\right)} + \frac{\gamma_e(\delta + \mu)\left(\Sigma_u(F) - \Sigma_u(0)\right)\Sigma_e''(F)}{(1 - \Sigma_u(0))\left(\delta + \mu + \gamma_e(1 - \Sigma_e(F))\right)^2} \\
& + 2\gamma_e\Sigma_e'(F)\frac{(\delta + \mu)\Sigma_u'(F)}{(1 - \Sigma_u(0))\left(\delta + \mu + \gamma_e(1 - \Sigma_e(F))\right)^2} \\
& + 2\frac{\gamma_e^2(\delta + \mu)\left(\Sigma_u(F) - \Sigma_u(0)\right)\Sigma_e'(F)^2}{(1 - \Sigma_u(0))\left(\delta + \mu + \gamma_e(1 - \Sigma_e(F))\right)^3}
\end{aligned} \tag{17}
$$

A.2 Value Functions

The value function can be calculated using the (expected) average flow value and the (expected) average duration using the formula

$$(\text{Avg. Duration})W(w(F), 0) = \text{Avg. Flow benefit}$$

The average duration in a job with quality F is $\delta + \mu + \gamma_e(1 - \Sigma_e(F))$. The average flow benefits are given by the wage $w(F)$ plus the search option $\gamma_e \int_F^1 W(w(\tilde{F}), 0)d\Sigma_e(\tilde{F})$ and separation value δU_{eu}. Defining $W_F(w(F), j) = W_w(w(F), j)w'(F)$ and combining gives[5]

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[5]Note that differentiating equation (7) with respect to F gives

$$(\gamma_u(1 - F^j) + \mu + \lambda_e + \delta + j\delta)W_F(w(F), j) = w'(F) + \lambda_e W_F(F, j + 1) + \delta j W_F(F, j - 1) + \gamma_u(1 - F^j)W_F(F, 0).$$

One can write the above expression in the form below, where the row and column of the matrix corresponds to the number of job offers, starting with zero.

W_F

$$= \begin{bmatrix} (\rho + \lambda_e + \delta + \mu) & -\lambda_e & 0 & 0 & \dots \\ -\delta - \gamma_u(1 - F) & (\gamma_u(1 - F) + \mu + \lambda_e + 2\delta) & -\lambda_e & 0 & \dots \\ -\gamma_u(1 - F^2) & -2\delta & (\gamma_u(1 - F^2) + \mu + \lambda_e + 3\delta) & -\lambda_e & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}^{-1} w'(F)$$

Inverting the matrix and using the first element we get the following

$$W_F(w(F), 0) = \frac{w'(F)}{\delta + \mu + \gamma_e(1 - \Sigma_e(F))},$$

$$W(w(F),0) = \frac{w(F) + \gamma_e \int_F^1 W(w(\tilde{F}),0)d\Sigma_e(\tilde{F}) + \delta U_{eu}}{\delta + \mu + \gamma_e(1 - \Sigma_e(F))}, \quad (18)$$

$$W_F(w(F),0) = \frac{w'(F)}{\delta + \mu + \gamma_e(1 - \Sigma_e(F))}, \quad (19)$$

$$W(w(F),0) - W(0,0) = \int_u^F \frac{w'(\tilde{F})}{\delta + \mu + \gamma_e(1 - \Sigma_e(\tilde{F}))} d\tilde{F}. \quad (20)$$

Similarly for the unemployed we have

$$U(0) = \frac{b + \gamma_u \int_0^1 W(w(\tilde{F}),0)d\Sigma_{uu}(\tilde{F})}{\mu + \gamma_u(1 - \Sigma_{uu}(0))}$$

$$U_{ue} = \frac{b + \gamma_u \int_0^1 W(w(\tilde{F}),0)d\Sigma_u(\tilde{F})}{\mu + \gamma_u(1 - \Sigma_u(0))}$$

$$U_{ue} - U(0) = \frac{b + \gamma_u \int_0^1 W(w(\tilde{F}),0)d\Sigma_u(\tilde{F})}{\mu + \gamma_u(1 - \Sigma_u(0))} - \frac{b + \gamma_u \int_0^1 W(w(\tilde{F}),0)d\Sigma_{uu}(\tilde{F})}{\mu + \gamma_u(1 - \Sigma_{uu}(0))}$$

Evaluating at $F = 0$ and using $W(0,0) = U(0)$ gives

$$b = w(0) + \gamma_e \int_0^1 \frac{w'(\tilde{F})(1 - \Sigma_e(\tilde{F}))}{\delta + \mu + \gamma_e(1 - \Sigma_e(\tilde{F}))} d\tilde{F} - \gamma_u \int_0^1 \frac{w'(\tilde{F})(1 - \Sigma_{uu}(\tilde{F}))}{\delta + \mu + \gamma_e(1 - \Sigma_e(\tilde{F}))} d\tilde{F}$$

$$+ \delta \left(\frac{\gamma_u \int_0^1 \frac{w'(\tilde{F})}{\delta + \mu + \gamma_e(1 - \Sigma_e(\tilde{F}))} \left(\Sigma_{uu}(\tilde{F}) - \Sigma_u(\tilde{F}) \right) d(\tilde{F})}{(\mu + \gamma_u(1 - \Sigma_u(0)))} \right),$$

A.3 Proof of Identification

Take the lowest worker $w = \phi$ is identified from the data. The transition rates μ and δ can be estimated using the rate at which the employed workers leave employment for unemployment and out of the labor force respectively. From the data we can estimate the job finding rate of someone employed in the lowest job $F = 0$ as a function of tenure. The job finding rate for the lowest match quality we are going to denote by $\gamma_e(1 - \Sigma_{e0}(0,t))$ where $\frac{(1 - \Sigma_{e0}(F,t))}{(1 - \Sigma_{e0}(0,t))}$ is the fraction of job offer that are better than F. The differential equation for $\Sigma_{e0}(F,t)$ is

$$\frac{\partial \Sigma_{e0}(F,t)}{\partial t} = \delta(1 - F)\frac{\partial \Sigma_{e0}(F,t)}{\partial F} - (\lambda_e(1 - F) + \gamma_e)\Sigma_{e0}(F,t) + \gamma_e(1 - \Sigma_{e0}(0,t))\Sigma_{e0}(F,t) + \gamma_e\Sigma_{e0}(0, \quad (21)$$

$$= \delta(1 - F)\frac{\partial \Sigma_{e0}(F,t)}{\partial F} - \lambda_e(1 - F)\Sigma_{e0}(F,t) + \gamma_e\Sigma_{e0}(0,t)(1 - \Sigma_{e0}(F,t)) \quad (22)$$

This gives the differential equation for $\gamma_e(1 - \Sigma_{e0}(F,t))$:

$$\frac{\partial \gamma_e(1 - \Sigma_{e0}(F,t))}{\partial t} = -\gamma_e \left(\delta(1-F)\frac{\partial \Sigma_{e0}(F,t)}{\partial F} - \lambda_e(1-F)\Sigma_{e0}(F,t) + \gamma_e \Sigma_{e0}(0,t)(1 - \Sigma_{e0}(F,t)) \right).$$

(23)

Replacing $\gamma_e(1 - \Sigma_{e0}(F,t))$ by $H_0(t,F)$ and $\gamma_e \frac{\partial \Sigma_{e0}(F,t)}{\partial F}$ by $-\frac{\partial H_0(t,F)}{\partial F}$, $\Sigma_{e0}(F,t) = 1 - \frac{H_0(t,F)}{\gamma_e}$ gives

$$\frac{\partial H(t,F)}{\partial t} = \frac{\partial H(t,F)}{\partial F}\delta(1-F) - (\gamma_e - H(t,0))\,H(t,F) - \lambda_e(1-F)\,(\gamma_e - H(t,F)) \qquad (24)$$

Evaluating this expression at at $t = 0$ we have $H(0,F) = \gamma_e(1 - \Sigma_e(F,0)) = 0$ and $\frac{\partial H(0,F)}{\partial F} = 0$ which implies

$$\lambda_e = \frac{-\frac{\partial H(t,F)}{\partial t}|_{t=0}}{\gamma_e(1-F)} \qquad (25)$$

using this in the "original" expression we get

$$\frac{\partial H(t,F)}{\partial t} = \frac{\partial H(t,F)}{\partial F}\delta(1-F) - (\gamma_e - H(t,0))\,H(t,F) + \frac{1}{\gamma_e}\frac{\partial H(t,F)}{\partial t}|_{t=0}\,(\gamma_e - H(t,F)). \qquad (26)$$

This gives the quadratic equation above in γ_e with the solution

$$\gamma_e = \frac{-\left[-\frac{\partial H(t,F)}{\partial t} + \frac{\partial H(t,F)}{\partial F}\delta(1-F) + H(t,0)H(t,F) + \frac{\partial H(t,F)}{\partial t}|_{t=0} \right]}{-2H(t,F)}$$

$$\frac{\pm\sqrt{\left[-\frac{\partial H(t,F)}{\partial t} + \frac{\partial H(t,F)}{\partial F}\delta(1-F) + H(t,0)H(t,F) + \frac{\partial H(t,F)}{\partial t}|_{t=0} \right]^2 - 4H(t,F)\frac{\partial H(t,F)}{\partial t}|_{t=0}H(t,F)}}{-2H(t,F)}$$

Noting that $H(t,F) \geq 0$ and $\frac{\partial H(t,F)}{\partial t}|_{t=0} < 0$ implies there is a unique positive solution. The equation therefore solves for γ_e. Using the previous equation we get λ_e. Having identified λ_e, γ_e it is straightforward to identify γ_u. Note that we can calculate $\Sigma_e(F)$ the instantaneous job finding rate following separation is

$$\gamma_u(1 - \Sigma_e(F)) \qquad (27)$$

this gives γ_u. We can then identify λ_u by the job finding rate of the long term unemployed.

A.4 Moment Conditions

We define the transition moments in model $M \in \{BM, BG\}$ between employment states $s, s' \in \{u, e\}$ as $\pi_{ss'}^M$. For BM they are,

$$\pi_{eu}^{BM} = \frac{\delta}{\delta + \mu} \left(1 - \exp\left(-\delta - \mu\right)\right)$$

$$\pi_{ue}^{BM} = \frac{\lambda_u}{\lambda_u + \mu} \left(1 - \exp\left(-\lambda_u - \mu\right)\right)$$

$$\pi_{ee}^{BM} = \int_0^1 \frac{\lambda_e(1 - F)}{\delta + \mu + \lambda_e(1 - F)} \left(1 - \exp\left(-\delta - \mu - \lambda_e(1 - F)\right)\right) dG(F)$$

and for BG,

$$\pi_{eu}^{BG} = \frac{\delta}{\delta + \mu} \left(1 - \exp\left(-\delta - \mu\right)\right)$$

$$\pi_{ue}^{BG} = \frac{\gamma_u(1 - \Sigma_u(0))}{\gamma_u(1 - \Sigma_u(0)) + \mu} \left(1 - \exp\left(-\gamma_u(1 - \Sigma_u(0)) - \mu\right)\right)$$

$$\pi_{ee}^{BG} = \int_0^1 \frac{\gamma_e(1 - \Sigma_e(F))}{\delta + \mu + \gamma_e(1 - \Sigma_e(F))} \left(1 - \exp\left(-\delta - \mu - \gamma_e(1 - \Sigma_e(F))\right)\right) dG(F).$$

Define t_0 as the instant an individual enters unemployment, t_i is the ith week of unemployment and τ_i the time interval between t_i and t_{i-1}. For BG, we match the monthly transition rates for each of the first 26 weeks of unemployment. For the ith week the theoretical counterpart is

$$\pi_{ue}^{BG}(\tau_i) = \int_{t_{i-1}}^{t_i} \frac{\gamma_u(1 - \Sigma_u(0, t))}{\gamma_u(1 - \Sigma_u(0, t)) + \mu} \left(1 - \exp\left(-\gamma_u(1 - \Sigma_u(0, t)) - \mu\right)\right) dt$$

When estimating the first step, in the case of BG, we make sure that $(\pi_{eu}^{BG}, \pi_{ue}^{BG}, \pi_{ee}^{BG})$ is tightly fitted, we do this by imposing a large penalty parameter that punishes the criterion if they are not fitted well in comparison with the vector of $\pi_{ue}^{BG}(\tau_i)$s.

The second step is identical for BM and BG. In both models the cross-sectional distribution of wages is a direct empirical counterpart to G. To match the distribution of wages well we compute 99 percentiles from 1 to 99, and match with the wage level at the corresponding percentile of G. We impose equal weight to each quantile of the distribution.