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A LEADING INDICATOR OF HOUSE-PRICE BUBBLES

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Danmarks Nationalbank

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RESUME

Forud for den finansielle krise i midten af 2000'erne steg huspriserne dramatisk, og de fleste økonomer er enige om, at en del af stigningen i huspriserne i Danmark havde karakter af en boligprisboble. Fremkomsten af en boligprisboble kan have betydelige konsekvenser for makroøkonomisk og finansiel stabilitet. En boligprisboble er som oftest et resultat af selvopfyldende forventninger, der fører til eksplosivitet i boligprisudviklingen. Dette arbejdspapir undersøger dynamikken i boligpriserne i Danmark for at kunne identificere bobler rettidigt.

Vi opstiller et boligprisindeks, som er justeret for fundamentale faktorer, og anvender testproceduren i Phillips m.fl. (2015) til at identificere boligprisbobler. De empiriske resultater indikerer, at udviklingen fra midten af 2005 var i overensstemmelse med en prisboble i Danmark. Når testet anvendes på lejlighedspriser i København, så indikerer udviklingen i den reale pris i 2015-16 også spekulativ adfærd, men det kan ikke udelukkes, at udviklingen er drevet af fundamentale økonomiske faktorer.

ABSTRACT

Prior to the financial crisis in the mid-2000, house prices increased dramatically and most economists agree that part of the increase in Danish house prices can be characterized as a house- price bubble. The emergence of a house-price bubble can have sizeable implications for macroeconomic as well as financial stability. A house-price bubble is often a result of self-exciting beliefs, leading to explosiveness of the developments in house prices. This paper investigates the dynamics of house prices in Denmark in order to identify emerging bubbles in due time.

We develop a fundamentals-adjusted house price index and apply the testing procedure of Phillips et al. (2015) to date-stamp house-price bubbles. The empirical results identify developments in line with a price bubble from mid-2005 in Denmark. When applied to flats in Copenhagen, real price developments in 2015-16 indicate speculative behaviour but it cannot be ruled out that developments are driven by fundamental economic factors.

A leading indicator of house-price bubbles*

Simon Juul Hviid†

Abstract

Prior to the financial crisis in the mid-2000s, house prices increased dramatically and most economists agree that part of the increase in Danish house prices can be characterized as a house-price bubble. The emergence of a house-price bubble can have sizeable implications for macroeconomic stability as well as financial stability. A house-price bubble is often a result of self-exciting beliefs, leading to explosiveness of the development in house prices. This paper investigates the dynamics of house prices in Denmark in order to identify emerging bubbles in due time.

We develop a fundamentals-adjusted house price index and apply the testing procedure of Phillips et al. (2015) in order to date-stamp house-price bubbles. The empirical results indicate developments in line with a price bubble from mid-2005 in Denmark. When applied to flats in Copenhagen, real price developments in 2015-16 indicate speculative behaviour but it cannot be ruled out that developments are driven by fundamental economic factors.

JEL Classification: C22, G12, R31
Keywords: House price dynamics, right-tailed unit root tests, date-stamping bubble periods.

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1 Introduction

Over the past decades house prices have moved dramatically with large price increases in the 1990s and the first half of the 2000s. In an international comparison these developments were especially pronounced in Denmark with a further acceleration in the capital, Copenhagen. House prices rose to unsustainable levels and in the mid-2000s, at the wake of the international financial crisis, a dramatic correction occurred challenging macroeconomic and financial stability. Most economist agree to characterize these developments as a house-price bubble. In recent years house prices have been on the rise again, leading to concerns that developments are unsustainable accompanying a risk that a housing related crisis may reoccur.

In order for policy makers to take timely actions, reliable leading indicators of house-price bubbles are crucial. The aim of this paper is to develop such an indicator.

Much effort has been put into theoretical modelling of price bubbles in recent decades.[1] In general the evolution of prices is driven either by changes in the underlying fundamental economic factors, by the emergence or collapse of a speculative bubble, or both.[2] Inspired by the user-cost model of Poterba (1992) and the framework in Bergman et al. (2015) and Sørensen (2013) along with Diba and Grossman (1988b), this paper develops a two-period model of housing ownership to determine how fundamentals and bubbles impact house prices. The model is used to develop an index which adjusts for evolving fundamentals such as income, housing stock, and user costs in order to isolate the effects of a rational house-price bubble. The index will be referred to as the fundamentals-adjusted house price (FAHP) index.

Investigations of price bubbles, especially in stock markets, have a long history.[3] Evolving econometric techniques have inspired a series of tests for price bubbles. West (1987) proposed a combination of two tests where only one is consistent with the presence of a bubble. A test of equality of parameters then indicates the presence of bubbles. However, the test is asymptotically inconsistent, cf. West (1985). Based on a unit root test following Bhargava (1986), it was proposed by Diba and Grossman (1988b) to test for rational bubbles by testing against the explosive alternative and not the stationary alternative. However, Evans (1991) shows that the approach has low power in detecting periodically, partially collapsing bubbles. Additionally, Diba and Grossman (1988b) propose to test for a cointegrating relation between fundamentals and prices assuming a fixed discount factor and argue that cointegration and the presence of a rational bubble are not compatible. A similar claim can be found in many

[1] Key contributions include e.g. Blanchard (1979), Blanchard and Watson (1982), Shiller (1984), Tirole (1982), Tirole (1985), Evans (1989), Evans and Honkapohja (1992), and Olivier (2000).

[2] Contributions to the literature on fundamentals include Hamilton and Schwab (1985), Richard and Nancy (1994), Himmelberg et al. (2005), Gallin (2008), Brunnermeier and Julliard (2008), Campbell et al. (2009), Plazzi et al. (2010), Cochrane (2011), Ghysels et al. (2013), Engsted and Pedersen (2015), and Gelain and Lansing (2014), where rents are typically considered the key fundamental factor.

[3] See Flood and Hodrick (1990) and Gurkaynak (2008) for a methodological and empirical review of asset price bubble detection.

studies, e.g. Phillips et al. (2011). Prices and fundamentals can, however, still share a common stochastic trend as argued by Engsted et al. (2016). However, the presence of a bubble, i.e. an explosive root in prices, in general disables estimation of the common stochastic trend. This follows from the non-stationarity of the error term when a bubble component is not explicitly allowed for as in the co-explosive framework by Engsted and Nielsen (2012).

Following the idea of Diba and Grossman (1988b) that right-sided unit root tests can identify rational bubbles and the insight of Evans (1991), Phillips et al. (2011) proposed using the test of Dickey and Fuller (1979). This approach proved to have much better power properties than earlier tests. In Phillips et al. (2015) the approach is generalized to allow for a varying starting point of the emergence of the bubble which only increased the power of the test. In a simulation study, Homm and Breitung (2012) find that the recursive procedure of Phillips et al. (2015) is particularly well suited for real-time detection of emerging price bubbles compared to other testing procedures.[4] The test has been widely applied over the past years to detect price bubbles in stock markets and in particular to various international housing markets.[5] The test of Phillips et al. (2015) is applied to a univariate time-series and is not explicitly estimating the discount factor or the relation between the house price and fundamentals.

In this paper, we combine the insight from the economic model by creating a univariate model-based index that subtracts the effects of evolving household income, user-cost components, and housing stock. We then apply the Phillips et al. (2015) test to this index in order to identify explosive developments in house prices in excess of what can be explained by the fundamental factors. Thereby we date-stamp periods in which house-price bubbles are emerging. Glaeser and Nathanson (2014) have documented that house price changes are persistent. This allows overoptimism to develop and hence the possibility of overshooting on the fundamental price level down the road. Therefore, in the context of early warnings, it is advantageous to focus on the change rather than the level of prices.

The empirical results indicate that developments from mid-2005 in prices of single-family homes in Denmark were in line with the presence of a rational house-price bubble, i.e. self-exciting accelerations of house prices. House prices were on the rise in Denmark from 2012 to 2016, however, there is no indication of an evolving speculative house-price bubble on a nationwide basis. The market for flats in Copenhagen is found to be closer to compatible with the presence of speculative price developments, i.e. the presence of an explosive root cannot be rejected, when testing on real prices or the price-income ratio. However, when the FAHP index is used as input, the presence of a speculative bubble is rejected at any conventional level of significance.

In general the recursive testing procedure is found to be robust to choices of estimation

[4] The focus in Homm and Breitung (2012) is on comparison with a set of regime switching models.

[5] See e.g. Yiu et al. (2013) for Hong Kong, Pavlidis et al. (2014) for a several OECD member countries, and Caspi (2014) for Israeli home prices.

window size, depreciation rate, risk premium, and housing-demand elasticities.

The remainder of this paper is organized as follows: Section 2 sets up the economic model which provides the framework for understanding rational bubbles. The econometric model used to date-stamp periods of explosive behaviour is described in section 3. Our results are outlined in section 4 including some robustness analysis. Section 5 concludes the paper.

2 The economic model

Inspired by the user-cost model of Poterba (1992) and the framework in Bergman et al. (2015) and Sørensen (2013) along with Diba and Grossman (1988b), this paper develops a two-period model of housing ownership to determine how fundamentals and bubbles impact house prices. The model is used to develop an index which adjusts for evolving fundamentals in order to isolate the effects of a rational, i.e. model consistent, house-price bubble.

Denote by P_t the price of a unit of housing in period t and let R_t be the flow value of owning the housing unit measured in a monetary unit. The time t expectation of the price of the unit at the beginning of the subsequent period, $t+1$, is denoted P^e_{t+1}. Then the equilibrium imputed value of owning the housing unit is

$$R_t = \gamma_t P_t - (P^e_{t+1} - P_t), \tag{1}$$

where γ_t is the relative user costs excluding expected capital gains. The components of γ_t is described below.

The underlying framework is a two-period model where a household maximizes utility by allocating income and wealth between housing, savings, and consumption of a numeraire good. The only uncertainty is about house prices in the second period where the household sells the housing unit, and hence consumption as the household consumes all remaining assets in the second period. We denote consumption by C_t, housing by H_t, and savings in a risk-less asset by S_t. Household utility in consumption and housing is assumed to be additively separable with functional forms $u(\cdot)$ and $h(\cdot)$, respectively, both being strictly increasing and concave. In principle, the household could choose to rent housing at the rental price $\gamma_t P_t$, rather than buy, which would exclude the second period uncertainty about capital gains. The objective of the household is to maximize lifetime utility which means maximizing

$$U_t = u(C_t) + h(H_t) + E_t \beta \left[u(C_{t+1}) \right]. \tag{2}$$

As the household receives income, Y_t, and can save in either housing or the risk-less asset, the household budget constraint at time t is

$$S_t + P_t H_t = Y_t - C_t. \tag{3}$$

But the household might face a down-payment constraint for housing investments

$$\alpha P_t H_t \leq Y_t - C_t, \quad \alpha \geq 0, \tag{4}$$

which is equivalent to a limit on the loan to value ratio (LTV) of $(1 - \alpha)$. The household sells the house at the end of the period at price P_{t+1} and consumes the remaining assets. This second period price is the only source of uncertainty in the model. Over the period, the agent pays housing taxes, τ, contributions, c_t, depreciations, δ, inflation, π_t, interests, i_t,[6] and receives capital gains, g. The expected capital gain is denoted $\bar{g} = P^e_{t+1} - P_t$.

Solving the problem of the agent includes a second order approximation of the expected value of second period consumption which implies risk aversion to influence consumption choices. The household is assumed to have CRRA preferences, with relative risk aversion parameter $\rho \equiv -\frac{C_{t+1}u''(C_{t+1})}{u'(C_{t+1})}$. The solution is founded in identities related to the second period consumption and the first order conditions of the maximization problem. These conditions lead to the following relation that implies that the marginal willingness to pay for an additional unit of housing, in terms of consumption units, is equal to the marginal cost, including a risk premium and expected capital gains.

$$R_t \equiv \frac{h'(H_t)}{u'(C_{t+1})} = \left[i_t(1 - \tau^i_t) + c_t - \pi_t + \tau_t - \bar{g} + \eta\right] P_t \tag{5}$$

which is the equation that underlies equation (1). Here interest payments are tax-deductible at the rate τ^i_t, η is the real depreciation rate of the housing stock, risk premium as well as a premium for being credit constrained, which are unobservable components that are assumed to be constant over time.[7] By inserting for $\bar{g}$, equation (5) can be inverted to determine the price of the housing unit as

$$P_t = \frac{R_t + P^e_{t+1}}{1 + \gamma_t}, \qquad \gamma_t = i_t(1 - \tau^i_t) + c_t - \pi_t + \tau_t + \eta. \tag{6}$$

By recursive substitution we get

$$P_t = E_t\left[\frac{R_t}{1 + \gamma_t} + \frac{R_{t+1}}{(1 + \gamma_t)(1 + \gamma_{t+1})} + \frac{P^e_{t+2}}{(1 + \gamma_t)(1 + \gamma_{t+1})}\right] \tag{7}$$

and so on. Malinvaud (1953) has found that this equation has the following general solution

[6] Interest payments can be tax-deductible at the rate τ^i_t.

[7] The outline of the derivations of the model can be found Sørensen (2013).

when the transversality condition, $\lim_{T\to\infty} E_t\left[\Pi_{j=1}^{T}\frac{1}{1+\gamma_{t+j}}\cdot P_{t+T}\right]=0$, is not invoked,

$$P_t = E_t\left[\sum_{i=0}^{\infty}\frac{R_{t+i}}{\Pi_{j=1}^{i}(1+\gamma_{t+j})}\right]+bB_t \tag{8}$$

$$\equiv F_t + bB_t. \tag{9}$$

Here F_t denotes the fundamental component of the house price, and bB_t is the rational bubble component, for which it applies that

$$B_t = E_t\left[B_{t+1}/(1+\gamma_{t+1})\right]. \tag{10}$$

A broad class of processes satisfy equation (10) as noted by Hall et al. (1999) including the periodically, partially collapsing bubbles of Evans (1991). The model implies that any rational bubble in house prices will evolve explosively. Hence, this will be the motivation for the empirical approach below. By not invoking the transversality condition, this approach differs from Bergman et al. (2015) that precludes bubbles from their model.

2.1 The Supply-demand model

To determine house prices from the model, one needs a measure of the flow value of owning a housing unit. One approach could be to use actual rents from the renting market. Renting is often, however, regulated, and it is not clear that the observed rent should reflect changes to the value of owning in such a case. Instead, we will follow the approach from Hott and Monnin (2008) in which imputed rents are determined as the result of the equilibrium on the housing market. The demand side is used to identify the imputed rent, R_t. The demand for housing depends on two factors, income and imputed rent, in a Cobb-Douglas relation with respective long run elasticities ε_Y and ε_R of housing demand, thus

$$D_t = AY_t^{\varepsilon_Y}R_t^{-\varepsilon_R}. \tag{11}$$

Here A is a constant. Assuming that supply is fixed in the short run and that supply equals demand (imputed rents adjust to satisfy this), i.e. $H_t = D_t$, then we have,

$$R_t = A^{1/\varepsilon_R}Y_t^{\varepsilon_Y/\varepsilon_R}H_t^{-1/\varepsilon_R}. \tag{12}$$

Bergman et al. (2015) have found that a reasonable coefficient of elasticities, ε_Y and ε_R, are unit-elasticity similar to the model in Hott and Monnin (2008). Inserting the equilibrium rent in equation (8) and restricting the elasticities to unity implies

$$P_t = E_t \left[\sum_{i=0}^{\infty} \frac{AY_t H_t^{-1}}{\Pi_{j=1}^{i}(1+\gamma_{t+j})} \right] + bB_t. \tag{13}$$

2.2 Implications of a rational bubble

It follows from (8) that in the presence of a rational bubble the price process will evolve explosively. Hence, in identification of bubbles in house prices it is natural to investigate the behaviour of prices. However, fundamental factors will influence the evolution and prices might be driven by such factors, e.g household income, mortgage rates, and the housing stock.

Campbell and Shiller (1987) define the spread between prices and fundamentals as a long run stationary relation, $S_t \equiv (P_t - \frac{1}{\gamma_t})R_t$, in the sense that prices and rents relative to user costs have a common stochastic $I(1)$ trend. This follows from (6) in equilibrium where $P_{t+1}^e = P_t$. In the absence of a bubble component, this implies that there is an equilibrium relation between the price of a unit of housing and the benefit of owning the unit, such that the spread is stationary, $I(0)$. However, when $B_t > 0$ and the price process additionally contains the explosive bubble component, the bubble component will be reflected in the spread and the price-rent ratio, which will likewise evolve explosively. Note that prices and rents will still share the common stochastic trend and be cointegrated in the presence of a bubble component.

Utilizing the supply-demand model for rents in the model gives a slightly different version of the spread,

$$S_t = P_t - \frac{1}{\gamma_t} \frac{AY_t}{H_t}. \tag{14}$$

In principle, we could test the cointegrating relation while allowing for a bubble component as in e.g. Nielsen (2010), Engsted and Nielsen (2012), and Engsted et al. (2016). However, the co-explosive framework is suitable for diagnosing the characteristics of the bubble and not for date-stamping price bubbles. Instead we leave the cointegrating relation unidentified, and create an index, Q_t, which handles the cointegrating properties of prices and fundamentals,[8]

$$Q_t \equiv \frac{P_t}{\frac{1}{\gamma_t} R_t} = \frac{\gamma_t P_t H_t}{Y_t}. \tag{15}$$

This index is in spirit similar to e.g. a conventional price-income index, which has been the input for a broad range of analysis of various housing markets. However, it additionally handles the interaction with the components of the user cost of housing ownership and adjustments to

[8]Craine (1993) shows that in the case of a stationary but time-varying discount factor (corresponding to the user cost in this model) the relation between prices and fundamentals will be stationary in the absence of a bubble component.

the housing stock. From here on we will refer to the index as the fundamentals-adjusted house price index. The FAHP index can be interpreted as the fraction of income, which households devote to housing. This interpretation implies that there is an equilibrium housing service ratio.

As for the spread, when a bubble is emerging, $B_t > 0$, the index will evolve explosively and it does not conflict with the presence of a common stochastic trend in prices and rents. Furthermore, as the bubble process evolves according to (10), the model does not imply that there is risk-less arbitrage opportunities, cf. Diba and Grossman (1988a), and describes a model that is in line with the efficient market hypothesis, even in the presence of a rational model, contrary to the claim in Abreu and Brunnermeier (2003).

3 The econometric model

The presence of a rational bubble implies that the spread and the proposed index will evolve explosively. Hence, the aim of this section is to outline an empirical approach that can detect explosive behaviour in a time series.

Diba and Grossman (1988a) proposed to conduct a right-tailed unit root test on the entire sample to test for the presence of an rational bubble in line with the described model. However, Evans (1991) found that the proposed test has low power in detecting periodically, partially collapsing bubbles. Based on these findings, Phillips et al. (2011) proposed an approach involving a series of right-tailed unit root tests on an expanding sample with a fixed starting date. They show that the estimated test statistics have much better power in detecting emerging bubbles. The advantage of the approach is that it additionally allows the econometrician to date-stamp periods of explosiveness in line with a rational bubble. Subsequently, Phillips et al. (2015) has generalized the approach by allowing for variation in the emergence of the bubble in addition to the collapse.

For a given time series, y_t, that potentially contains a bubble component, the null hypothesis is that it follows a random walk with a drift that becomes negligible as the sample size, T, goes to infinity,

$$y_t = dT^{-\eta} + \rho y_{t-1} + \varepsilon_t, \quad \varepsilon_t \overset{iid}{\sim} N\left(0, \sigma_y^2\right), \quad \rho = 1. \tag{16}$$

Here d is a constant and the coefficient $\eta > 1/2$ ensures that the drift becomes negligible asymptotically. Phillips et al. (2011) effectively exclude the drift by setting $\eta = \infty$ implying a null hypothesis of a random walk without drift, whereas Phillips et al. (2015) set $d = \eta = 1$. Date-stamping involves estimations of the model for a range of subsets of the total sample. Let r_w denote the smallest window size on which estimation is conducted. Obviously this window should satisfy $r_w \leq T$. Furthermore, let r_1 and r_2 denote the first and last observation in a given sub-sample respectively, leading to a sample size of $T_{r_1,r_2} = r_2 - r_1 + 1$.

Under the null hypothesis, the time series contains a unit root, hence Phillips et al. (2015) runs an auxiliary regression for a given sub-sample

$$\Delta y_t = \mu_{r_1,r_2} + \alpha_{r_1,r_2}\Delta y_{t-1} + \sum_{j=1}^{k}\phi_{j,r_1,r_2}\Delta y_{t-j} + \epsilon_t, \quad \epsilon_t \overset{iid}{\sim} N\left(0, \sigma^2_{r_1,r_2}\right). \tag{17}$$

The estimated model can be used to test whether $\alpha_{r_1,r_2} = 0$ (corresponding to the null hypothesis that $\rho = 1$), i.e. that the sub-sample contains a unit root, against the right-sided alternative that the sub-sample evolves explosively, $\hat{\alpha}_{r_1,r_2} > 0$. Let $ADF_{r_1}^{r_2}$ denote the corresponding ADF test statistic.

The backward ADF (BADF) test statistic, first proposed by Phillips et al. (2011), is at time r_2 simply the ADF test with starting point at the beginning of the sample and ending point at r_2,

$$BADF_{r_2} = ADF_1^{r_2}. \tag{18}$$

Collecting test statistics for all sub samples with $r_2 \geq r_w$ provides a time series of test statistics. This approach identifies bubbles that have been present from the beginning of the sample period. However, bubbles can be short-lived, and as an alternative we propose a rolling window augmented Dickey-Fuller (RWADF) test specified as follows

$$RWADF_{r_2} = ADF_{r_2-r_w}^{r_2}, \tag{19}$$

which provides a sequence of test statistics that puts more weight on recent developments in the series of interest. Similarly, the backward supremum ADF (BSADF) test from Phillips et al. (2015) collects a time series of test statistics where the test statistic at r_2 is the supremum of all test statistic estimated on sub-samples ending at time r_2.

$$BSADF_{r_2}(r_w) = \sup_{r_1 \in \{1,\dots,r_2-r_w\}} \left\{ADF_{r_1}^{r_2}\right\}. \tag{20}$$

Note that the BSADF test statistic is a function of the smallest window size. Phillips et al. (2015) use these sequences to define the emergence and collapse of explosive periods denoted by $\widehat{r}_e$ and $\widehat{r}_c$ respectively,

$$\widehat{r}_e = \inf_{r_2 \in \{r_w,\dots,T\}} \left\{r_2 : BSADF_{r_2}(r_w) > cv_{r_2}^{\alpha_T}\right\}, \tag{21}$$

$$\widehat{r}_f = \inf_{r_2 \in \{\widehat{r}_e,\dots,T\}} \left\{r_2 : BSADF_{r_2}(r_w) < cv_{r_2}^{\alpha_T}\right\}. \tag{22}$$

Here $cv_{r_2}^{\alpha_T}$ denotes the critical value at $100\,(1-\alpha_T)\,\%$ level of the BSADF test statistic given r_2 observations. The identification scheme implies that only a single period of explosiveness will be identified, not allowing for periodically, partially collapsing bubbles as the bubble will emerge at the first observation of a BSADF test statistic above the critical value and subsequently evaporate with the first subsequent observation of a BSADF test statistic below the critical level. Here, we roll the identification scheme forward and identify re-emerging explosiveness by

$$\widehat{r}_e = \inf_{r_2 \in \{\hat{r}_f,\ldots,T\}} \left\{ r_2 : BSADF_{r_2}(r_w) > cv_{r_2}^{\alpha_T} \right\}. \tag{23}$$

In theory a bubble can only be present if it has been so in the infinite past, cf. Diba and Grossman (1988a), which is evident from (10). The date-stamping scheme outlined here identifies explosive behaviour when the explosive component becomes of a statistically significant order relative to the developments in the fundamental process and noise in the series.

4 Empirical results

4.1 The data

Most of the data stems from Statistics Denmark. At the national level we use an index on prices of single-family homes in Denmark and flat prices in the City of Copenhagen.[9] The price index is quality-adjusted.[10] The income measure is household disposable income from the National Accounts. For Copenhagen such a measure is not available, instead we use disposable income from the Income Statistics. The housing stock can be improved either by quality or quantity adjustments. Here we use the accumulated net investments in housing as the measure of the housing stock comprising both dimensions. The measure is unattainable for Copenhagen, instead we use the number of dwellings and adjust these by the ratio between the accumulated net investments and the number of dwellings at the national level, whereby it is assumed that the degree of repairs is constant across the country. The user cost of housing is composed of the mortgage rate on a 30 year fixed-interest mortgage, administration fees are from Realkredit Danmark, the effective housing taxation rate (property and land taxes aggregated) calculated on data from Statistics Denmark, and inflation expectations that have been calculated with some degree of persistence according to $\pi_t^e = \gamma\pi_{t-1}^e + (1-\gamma)\log(cp_t/cp_{t-4})$, where $\gamma \in [0,1]$ controls the degree of persistence and cp is consumer prices from Statistics Denmark. Data has been seasonally adjusted. An overview of data and sources is found in table 1.

Figure 1 shows the four main series that will be investigated below. That is the real price series of single-family homes in Denmark and flat prices in the City of Copenhagen as well as the proposed FAHP index for the two geographical areas. Visual inspection of the series indicate that prices evolved somewhat explosively up to the financial crisis in the mid-2000s. Furthermore, recent developments, especially in Copenhagen, give cause for concern as even the

[9]The City of Copenhagen covers the four municipalities: Copenhagen, Frederiksberg, Tårnby and Dragør.

[10]The quality adjustment is based on the SPAR (Sale Price Appraisal Ratio) method, in which compositional effects is mitigated over time.

Table 1: Data sources

Variable	Source	Comments
House price	Statistics Denmark	Price index for single-family homes.
Flat price, CPH	Statistics Denmark	Price index for flats in the City of Copenhagen.
Disposable income	Statistics Denmark	National level, also for Copenhagen.
Housing stock	MONA data bank	Accumulated net investments. For Copenhagen used investment-adjusted number of dwellings.
Housing taxes	Statistics Denmark	From 2004: Calculated on micro data. Before 2004: Revenue relative to housing wealth.
Consumer prices	Statistics Denmark	
Mortgage rate	Realkredit Danmark	Based on a 30 year fixed interest mortgage.
Administration margins	Realkredit Danmark	

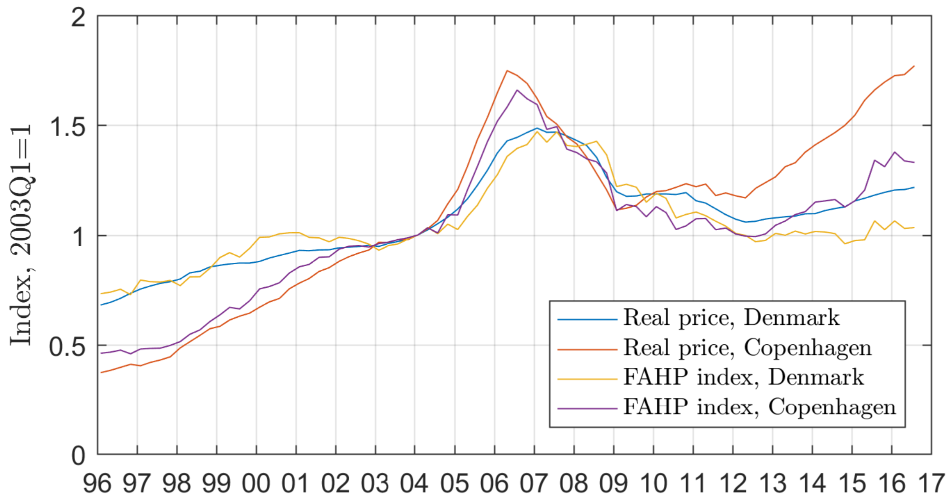

Figure 1: Real prices and the FAHP indices for single-family homes in Denmark and flats in Copenhagen

FAHP index is yet again on the rise. However, the question is whether this can be characterized as the emergence of a new house-price bubble.

4.2 Right-tailed unit root tests and date-stamping

The finite sample critical values of the ADF statistic do not follow a standard distribution under the null hypothesis. Therefore, standard errors are simulated for the relevant lag length and sample size. Specifically, we estimate (16) setting $d = 1$ and $\eta = 1$ as in Phillips et al. (2015) and $\sigma_y^2 = 1$. 5,000 simulations have been performed for each sample size of which the 90$^{\text{th}}$, 95$^{\text{th}}$,

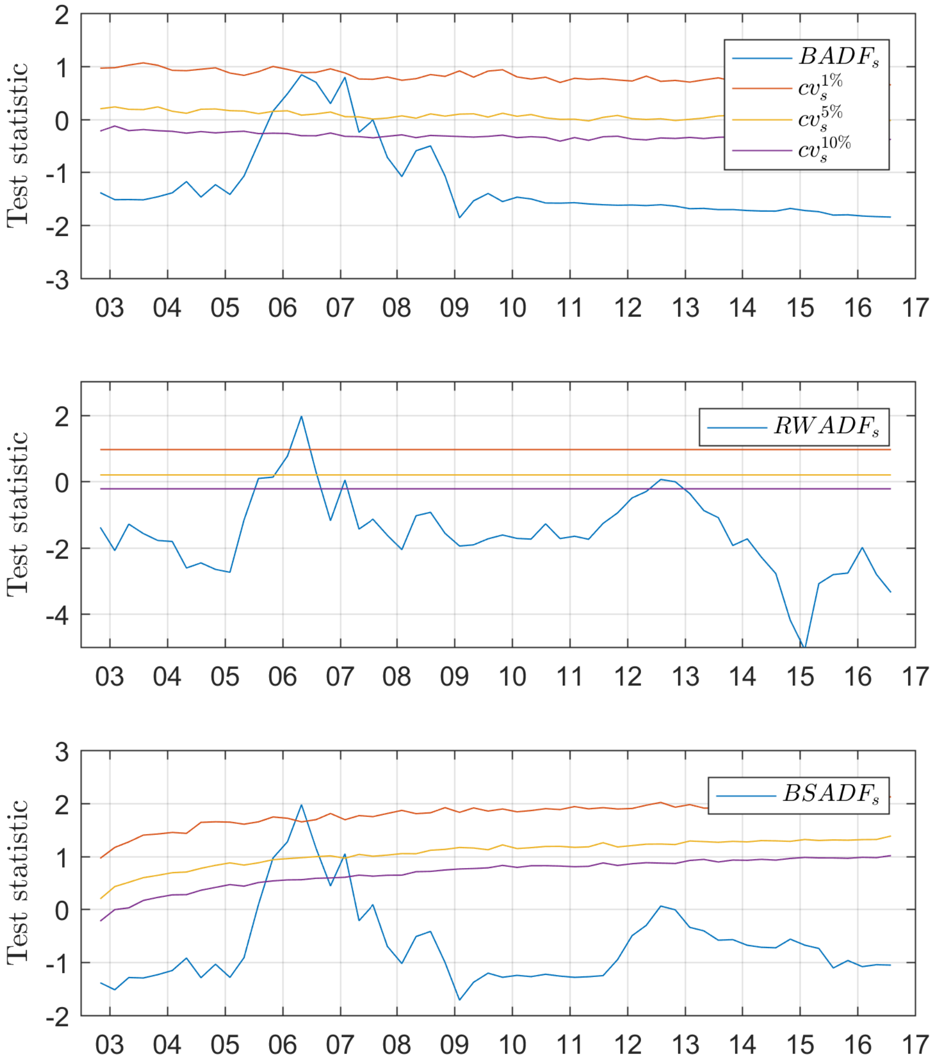

Figure 2: The BADF (top), RWADF (middle), and BSADF (bottom) tests applied to the FAHP index of single-family homes in Denmark with a (minimum) window size of 28 quarters and lag length $k = 2$. Finite sample critical values are estimated in 5,000 simulations under the null hypothesis.

and 99$^{\text{th}}$ percentile are used as the relevant critical values in the right-tailed ADF tests.[11]

Figure 2 plots the empirical results of the three versions of the test for explosiveness applied to the FAHP index of Danish single-family homes. In the estimation procedure, lag lengths of the ADF test statistics has been chosen to limit residual autocorrelation, thereby increasing the

[11]The null-hypothesis includes a constant. The constant will corresponds to a linear trend and subtract some of the explosiveness form the model, hence critical values can be negative.

power of the test statistics. The lag length, however, is fixed across sub-samples. A constant is included in the model. The three different testing procedures have individual characteristics, which are evident from the simulated finite sample critical values. The BADF test expands the sample size in the recursive estimation, increasing the precision of the point estimates in time and converge to the asymptotic Dickey-Fuller distribution. Hence, the critical values are slightly but monotonically decreasing. The variation stems from simulation uncertainty. The RWADF has a fixed estimation sample size and critical values are constant over time. Here the critical values are larger than the critical values of the BADF test. Lastly, the BSADF test is the supremum on a set of estimates where the size of the set is increasing along the time dimension. The expected value of the extreme test statistic is increasing and hence, critical values increase monotonically as the recursive estimation procedure expands. Therefore the critical values of the BSADF test is larger than those of the BADF and the RWADF tests.

The three testing procedures all identify the nationwide housing bubble of single-family homes from mid-2005 approximately. In the RWADF test there is some indication of explosive behaviour in 2012 which can be explained by an interesting feature of the testing procedure. Basically, the test includes a constant in the estimation in first differences, which in levels corresponds to a linear trend. When a price bubble is bursting and prices are adjusting to a lower level, as was the case for the index from peak to trough, the test can in some cases fit a negative linear time trend along with an explosive root. At the national level, there is no indication of a house-price bubble in recent years.

It should be noted that the tests are suitable for date-stamping the emergence and not the collapse of a speculative house-price bubble. Therefore, the time at which the tests do no longer reject the null hypothesis merely reflects that the series does no longer exhibit explosiveness, which is most likely around the top of a price bubble from which point it bursts. In the case of the BADF test, one should additionally note that the test is unlikely to identify the build-up of a new house-price bubble, as even in the presence of a bubble, developments should force an explosive trend through the dip in prices in the 2000s.

Overall, the BSADF test is preferred over the two other alternatives as it is more robust to variations over the business cycle.

Previous applications of the Phillips et al. (2015) test have typically used real price, price-income, or price-rent measures which will all evolve explosively in the presence of a self-fulfilling bubble component. However, adjusting for the developments in key fundamental factors should be focal in empirical bubble detection, ensuring that favourable developments in fundamentals are not mistaken for a price bubble. Using the BSADF testing procedure to alternative indices commonly applied, provides some insight to the index and the importance of adjusting for fundamentals. Figure 3 shows the results of an application of the BSADF to three different price indices, i.e. the real price, price-income, and the FAHP indices. All indices agree in identifying explosiveness in the mid-2000s. At first, price developments could have been driven

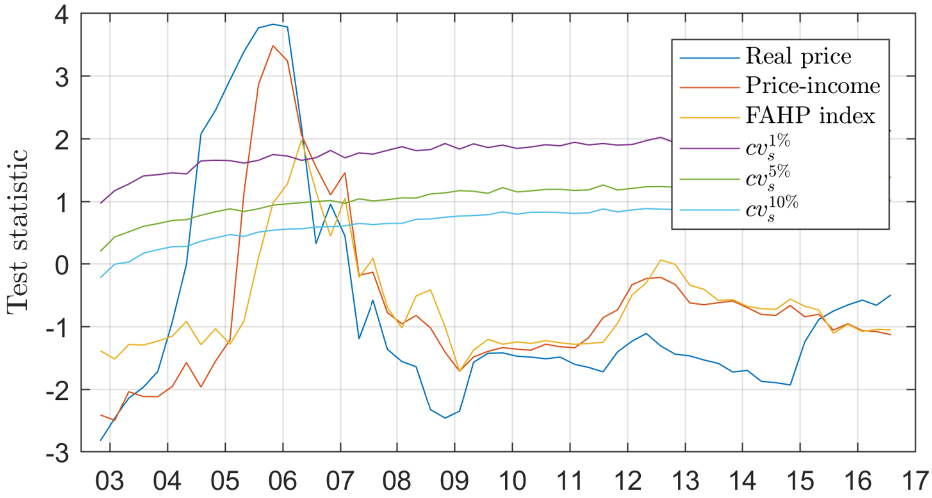

Figure 3: The BSADF test applied to the real price, the price-income ratio, and the FAHP index of single-family homes in Denmark with a minimum window size of 28 quarters and lag length $k = 2$. Finite sample critical values are estimated in 5,000 simulations under the null hypothesis.

by income developments and later user-cost components can have explained the inflation of house prices, as interest rates remained relatively low for some time into the overheating of the housing market. However, overoptimism arose and a house-price bubble emerges in the wake of these favourable developments. This suggest that the price-income (and the real price) measure is too simple to pin down periods in which house prices develop unambiguously as a house-price bubble. However, conducting the test on real prices and price-income ratios does seem to provide some early warning of developments that might fuel speculative behaviour down the road.

4.3 Flats in Copenhagen

House-price bubbles can be a regional phenomena just as well as it can be nationwide. Regional indicators of house-price bubbles can be an important indicator of national house-price bubbles, as overoptimism in one area can be transmitted to surrounding regions via the ripple effect described in e.g. Meen (1999). In itself, a regional house-price bubble might not pose systemic risks to a large and robust financial system. Approximately 45 percent of household housing wealth is located in the Capital Region and covers about 30 percent of the housing units which is used for 35 percent of the collateral of household mortgages in Denmark. Therefore an emerging house-price bubble in the Capital Region would be of great concern. In this application, we narrow the focus even further to only consider flats in the City of Copenhagen. The results from an application of the BSADF test to real price, price-income, and the FAHP index can be found in figure 4. Copenhagen seems to be a good leading indicator of the nationwide house-price bubble in the mid-2000s, when considering the FAHP index and price-income ratio. When

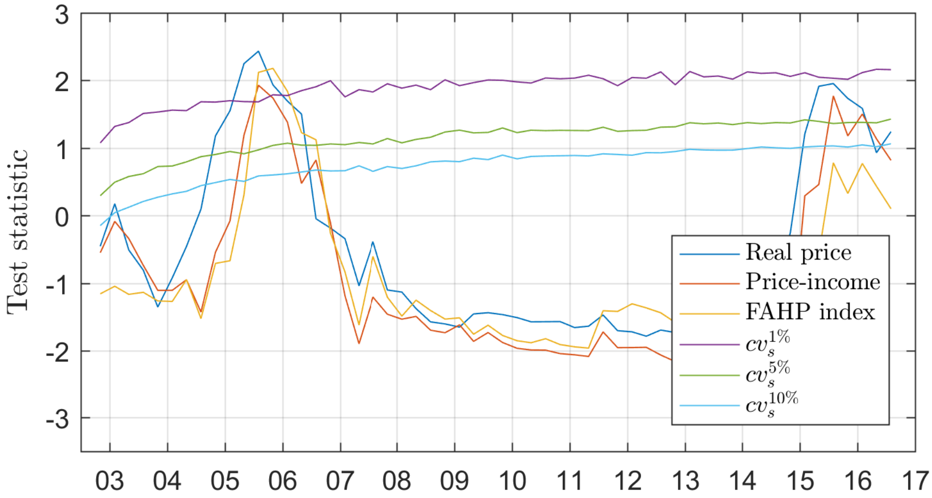

Figure 4: The BSADF test applied to the real price, the price-income ratio, and the FAHP index of flats in Copenhagen with a minimum window size of 28 quarters and lag length $k = 3$. Finite sample critical values are estimated in 5,000 simulations under the null hypothesis.

comparing to the national level, interest rates did not rise until the end of 2005 which sustained price developments relative to what was captured in the price-income ratio. In Copenhagen, there was a boom in the housing stock from 2005 which counteracted the effect from relatively low interest rates. This explains why tests of the price-income ratio and the fundamentals-adjusted index cannot reject an explosive root from roughly the same quarter in 2005. In 2015 and the beginning of 2016, real house prices in Copenhagen seem to be in line with an evolving speculative price bubble, even if developments in disposable income are taken into consideration. On the contrary, the FAHP index does not indicate a house-price bubble at any conventional level of significance, as price increases to some degree can be explained by the low interest environment that has provided households with historically low mortgage rates.

4.4 Sensitivity

The economic model and the estimation of the empirical model include certain choices. The choice of window size, r_w, pertains to econometric application, whereas choices of housing-demand elasticities, risk premium, and depreciation rates are of a theoretical nature.

Window size

When the model is estimated, the choice of minimum window size affects the period over which the bubble can evolve and ceteris paribus smaller window sizes will give larger expected BSADF test statistics. For that reason, the simulated critical values will increase and hence, the impact

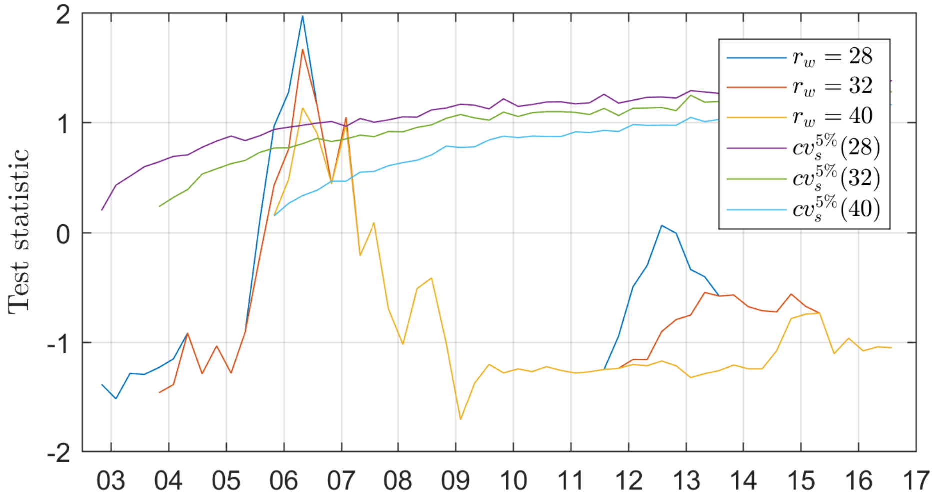

Figure 5: The BSADF test applied to the FAHP index of single-family homes in Denmark for three different choices of minimum estimation window, $r_w \in \{28, 32, 40\}$ and lag length $k = 2$. Finite sample critical values are estimated in 5,000 simulations under the null hypothesis.

on identification of house-price bubbles is ambiguous. Figure 5 shows BSADF test statistics for three choices of $r_w \in \{28, 32, 40\}$ along with corresponding 95% simulated finite-sample critical values. From the figure it is evident that $BSADF_{28} \geq BSADF_{32} \geq BSADF_{40}$. Similarly, we see that $cv_{28}^{0.05} > cv_{32}^{0.05} > cv_{40}^{0.05}$, which is a result of a larger set of ADF test statistics on which the BSADF test is determined and secondly, that the test statistic is more volatile, when the window size decreases. If the largest test statistic of e.g. the $BSADF_{28}$ stems from a sample of 33 quarters, then we will observe that $BSADF_{28} = BSADF_{32} > BSADF_{40}$.

The simulations of the critical values ensure that in theory the power of the test is constant across choices of window size. However, the bubble process in the model implicitly states that the presence of a bubble component today implies that the bubble has been present in the infinite past. Though, assuming that a bubble could be jump-started and short-lived, then choosing shorter window sizes will do better at detecting these bubbles. In particular this is relevant in the aftermath of a collapsed bubble such as the house-price bubble in the mid-2000s. As a rule of thumb, we suggest that choices of window size should as a minimum be in the ballpark of the business cycle.

Unobservable user-cost components

The user-cost includes unobservable and, in this application, constant components, i.e. the risk premium, the rate of depreciation of housing, and the shadow price of borrowing constraints are prefixed. Figure 6 plots the test results from an application of the BSADF test on the FAHP index for three different choices of $\eta \in \{0.07, 0.10, 0.15\}$. In general, the BSADF test procedure

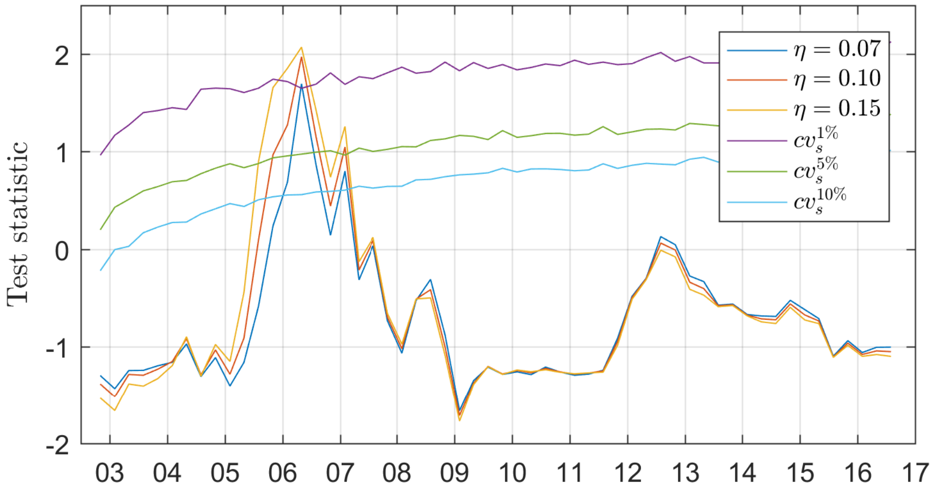

Figure 6: The BSADF test applied to the FAHP index of single-family homes in Denmark for three different choices of constant risk premium, depreciation, and liquidity premium with a minimum window size of 28 quarters and lag length $k = 2$. Finite sample critical values are estimated in 5,000 simulations under the null hypothesis.

is fairly robust to the choice of constant, even though some non-negligible differences are found in 2005. From a practical point of view, increasing the constant term puts relatively more weight on the developments in real prices and less on the time varying components of the FAHP index. The choice has implications for the ability of the test to identify house-price bubbles when there are large movements in fundamentals, but in general the test sequence seems robust to choices of the constant. In all tests presented in this paper, the constant has been fixed to 0.1.

Housing-demand elasticities

The supply-demand model is utilized for a given choice of income and price elasticity of demand, ε_Y and ε_R. In all application so far, the elasticities have been prefixed at unity, which given Bergman et al. (2015) is reasonable, though sensitivity of their results towards the choice is limited. According to Englund (2011), international empirical evidence suggests that the income elasticity of housing demand is close to unity. Englund (2011) and Girouard et al. (2006) survey the empirical literature and find that the (numerical) price elasticity of housing demand in most cases is less than one. As a robustness check to the choice of demand elasticities, figure 7 shows the results from applications of the BSADF test for the four combinations of $\varepsilon_Y \in \{0.5, 1\}$ and $\varepsilon_R \in \{0.5, 1\}$. The general representation of the FAHP index is

$$Q_t \equiv \frac{P_t}{\frac{1}{\gamma_t} R_t} = \gamma_t P_t H_t^{\frac{1}{\varepsilon_R}} Y_t^{\frac{-\varepsilon_Y}{\varepsilon_R}}. \tag{24}$$

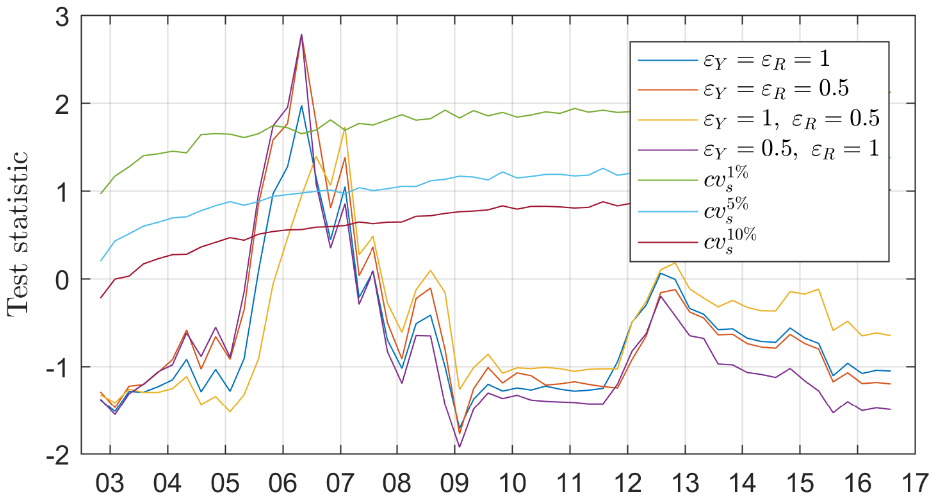

Figure 7: The BSADF test applied to the FAHP index of single-family homes in Denmark for combinations of housing-demand elasticities $\varepsilon_Y \in \{0.5, 1\}$ and $\varepsilon_R \in \{0.5, 1\}$ with a minimum window size of 28 quarters and lag length $k = 2$. Finite sample critical values are estimated in 5,000 simulations under the null hypothesis.

Note that $\varepsilon_Y = \varepsilon_R$ implies unit income elasticity of house prices and that $\varepsilon_R < 1$ corresponds to a numerical price elasticity below one. We see that the specification is fairly robust to choices of demand elasticities with the exception of $(\varepsilon_Y = 1, \varepsilon_R = 0.5)$ in which case the income elasticity of housing demand is 2. Under this assumption, the income developments in the mid-2005 are able to explain a large part of the house-price increases and the test does not indicate a house-price bubble before mid-2006.

5 Conclusion

In this paper we investigate the distinction between house price fundamentals and bubbles with the aim of providing a real-time indicator of house-price bubbles.

Our starting point is the theoretical framework of Bergman et al. (2015) which is extended to allow for a self-exciting bubble component to be present. From the theoretical model we develop the fundamentals-adjusted price index which handles co-movements in house-price fundamentals including household income, the housing stock, and various components of the user costs in order to isolate the effects of a rational house-price bubble. We argue that the index is more suitable for house-price bubble detection than conventional measures, such as the price-income ratio.

Secondly, we use the index as an input to the BSADF testing procedure developed by Phillips et al. (2015) which is suitable for identifying explosive behaviour, in line with the implications of a rational price bubble, in excess of what can be explained by economic fundamentals. Thereby we date-stamp episodes of emerging bubble behaviour.

The empirical application focuses on developments in single-family homes in Denmark and flats in the City of Copenhagen. Here we identify developments from mid-2005 in prices of single-family homes in Denmark that cannot be rejected as in line with the presence of a rational house-price bubble. In Copenhagen flat prices have been rising from 2012 to 2016 and there are indications of an evolving speculative bubble in real prices. However, when controlling for fundamentals, the presence of a speculative bubble is rejected.

Lastly, we perform some robustness checks with respect to theoretical and empirical choices. The testing procedure seems reasonably robust.

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