

The Consequences of Uncertainty: Climate Sensitivity and Economic Sensitivity to the Climate

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Abstract

We construct an integrated assessment model with multiple energy sources—two fossil fuels and “green energy”—and use it to evaluate ranges of plausible estimates for the climate sensitivity as well as for the sensitivity of the economy to climate change. Rather than focusing on uncertainty explicitly, we look at extreme scenarios defined by the upper and lower limits given in available studies in the literature. We compare optimal policy with *laissez faire* and we point to the possible policy errors that could arise. By far the largest policy error arises when the climate policy is “overly passive”; “overly zealous” climate policy (i.e., a high carbon tax applied when climate change and its negative on the economy are very limited) does not hurt the economy much as there is considerable substitutability between fossil and non-fossil energy sources.

1 Introduction

The economy-climate nexus involves three large blocks: how the economy works, how the climate is determined, and how emitted carbon circulates between different reservoirs (the carbon cycle). These blocks interact. The key links are that the economy feeds carbon dioxide into the atmosphere and hence sets carbon circulation in motion, carbon that then constitutes a key input into the determination of the climate, which in turn affects how our economies work; hence, human welfare is affected. The description of the joint system is often referred to as *integrated assessment modeling* and in this paper we employ an integrated assessment model to address one of the key questions in this area: uncertainty. In particular, there is imperfect knowledge of the climate system, about the carbon cycle, and about the economic damages caused by climate change, as well as about how these systems interact. We focus on two of these uncertainties here: the climate system and the economic damages.

A key feature of our analysis is that, unlike the literature on this issue so far, we do not formally model uncertainty. Rather, we look at the range of estimates and focus on the extremes. The extremes are naturally defined as upper and lower bounds of intervals given in the literature. (True tails events, occurring with extremely low probability, are not considered here.) First, we look at how sensitive global temperature is to carbon dioxide in the atmosphere and select two extreme values: an upper bound and a lower bound. These are selected from IPCC's 2013 report, which states a range of values for *climate sensitivity*—which has a direct counterpart in our model—within which the outcome will “likely” land, i.e., with a probability that the IPCC considers to be higher than around 2/3. For economic damages, we rely on the recent meta-study by Nordhaus and Moffat (2017) and similarly select upper and lower bounds. We then combine these into four distinct possibilities, thus combining the bounds into four logically possible outcomes. We find this approach more easily interpretable, and arguably even more relevant, than an approach which formally looks at uncertainty, since we perceive the main issue to be a concern with extreme outcomes (both good ones and bad ones) rather than with random fluctuations within the range defined by the extreme cases. Thus, we do not think that it is the imperfect consumption smoothing that is worrisome in the climate-economy area but rather fears of a highly damaging outcome, either because insufficiently aggressive policy is undertaken when the damages of emission turn out to be large, or because of policy that is too aggressive when carbon emission by itself (through climate change) does not harm economic welfare much.¹

¹For examples of studies of risk and uncertainty, see for example Lemoine (2010), Jensen and Traeger

Our integrated assessment model is based on Golosov et al. (2014).² This framework is highly tractable and yet quantitatively specified, i.e., it is specified based on a specific (optimal neoclassical growth) structure that can be straightforwardly tied to empirical estimates of utility- as well as production-function based parameters. The model is augmented to include some richness on the side of energy supply and in order to include a carbon cycle and a climate model. One of the key features of this framework is that it captures the sensitivity of climate to atmospheric carbon dioxide concentration *jointly* with the economic damages inflicted by global warming in *one parameter*: γ . This parameter also has a concrete interpretation: the percentage loss in world GDP from a one-unit increase in the carbon dioxide concentration in the atmosphere (thus baking together how carbon creates warming, which in turns causes economic damages), where “one unit” is expressed as 1,000 GtC. Hence, we will look at four values of γ defined by the four combinations of high and low climate sensitivity and high and low economic sensitivity.

To begin with, then, one interesting issue is whether the uncertainty, as expressed by the ranges in the two studies we refer to (IPCC and Nordhaus-Moffat), generates a larger span of values for γ due to the uncertainty about climate sensitivity than that due to uncertainty in economic sensitivity, or the other way around. We find the following: low-low (climate-economic) sensitivities deliver a γ of 0.27, low-high gives a value of 1.79, high-low yields 1.44, and high-high 10.39. Thus the effects are not additive—they interact nonlinearly somewhat—but, roughly speaking, the difference between high and low climate sensitivities amount to a factor of about 5.5 in the γ , whereas the uncertainty due to economic sensitivity approximately amounts to a factor of 7 in γ . Thus, these are of the same order of magnitude. Clearly, it is hard to argue that the bounds selected from the two studies represent exactly the same amount of uncertainty, but we note at least that there is significant economically relevant uncertainty both about the climate and about the economy. Our priors were that the former would be swamped by the latter, which turned out not to be correct.

When we compute optimal taxes we obtain values that are in line with numbers in the literature and we then use these to simulate eight scenarios: for each of the four γ cases, we look both at the laissez-faire market outcome and at optimal policy. We see, in brief, that the negative welfare effects of carbon emission are sizeable, unless both the climate and economic sensitivities are low. We also see, however, that the optimal tax is quite

(2014), Cai, Judd and Lontzek (2013), Gollier (2013) and Weitzman (2011).

²It is also related to recent work where we endogenize technology: Hassler, Krusell, and Olovsson (2017) and, with more detail on energy supply, Hassler, Krusell, Olovsson, and Reiter (2017).

potent in containing climate change and its economic effects. In terms of energy supply, we see that coal use will grow significantly in all of the scenarios, the one exception being the worst outcome—with both sensitivities being high and under the corresponding optimal tax. Finally, we look at the kinds of errors that arise if one adopts a climate policy in a way that is poorly matched to the actual sensitivities. Here we see that the negative consequences of adopting a high tax—computed optimally based on the assumption that both the climate and economic sensitivities are high) are not very large, chiefly because energy substitution is quite effective: using green energy (which would be the equilibrium implication of a high tax on carbon—when coal really should be used more is not very costly for the economy as the two are rather close substitutes. On the other hand, incorrectly adopting a low tax—that is appropriate if both sensitivities are low when they are actually high—is very costly.

In Section 2 we describe the economy-climate model. Section 3 then shows how we calibrate the model and Section 4 covers the results. We offer some concluding remarks in Section 5.

2 Model

In the following, we describe our benchmark model, block by block, and then discuss the tax assumptions implemented in the market economy.

2.1 Economy

Overall, our framework is an integrated model of the economy and the climate: these two systems have a feedback between them. As such, it is a close relative of Nordhaus’s DICE and RICE models.³ However, and for focus, we consider a world economy that is highly stylized in a number of ways and, in that sense, is much simpler than some of the existing leading models. First, the world has two regions, defined by whether they are oil-consuming or oil-producing. Second, we look at three types of energy sources: oil, which is produced at zero marginal cost, coal, which is produced at a constant marginal cost, and “green”, which is also produced at a constant marginal cost. The amount of oil is finite and the amounts of coal and green are infinite (for coal, this is a simplification but not a severe one since there is a very large amount of coal and, hence, a very small associated rent). Third, the only trade between the regions is intratemporal: oil for consumption (there is a homogeneous

³Nordhaus and Boyer (2010) and later updates.

consumption good). There is no technical change of any sort—we take technology trends as given—and we only consider policy in the form of a carbon tax, which if used properly would suffice to render the world equilibrium Pareto optimal.⁴ We use a simple utility and production-function specification in order to obtain closed-form solutions as far as possible.

Both regions are inhabited by representative consumers; these all have preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \log(C_t). \quad (1)$$

From now on, we use C_t to denote consumption in the oil-consuming region and $C_{o,t}$ to be consumption in the oil-producing region.

The oil-consuming region have an aggregate production functions for the final good Y_t that is given by

$$Y_t = A_t L_t^{1-\alpha-\nu} K_t^\alpha E_t^\nu$$

where A_t is total-factor productivity, L_t is the part of overall labor used in final-good production, K_t is the total capital stock, and E_t is energy services.

The assumption that the elasticity of substitution between energy and the other inputs (capital and labor) is unity is hard to defend when a time period is short—then, a much lower elasticity is called for. However, for longer time periods—and indeed our focus here is a long-run one—the Cobb-Douglas assumption does not appear unreasonable. In fact, as demonstrated for this particular application in Hassler, Krusell, and Olovsson (2017), one can express the higher long-run substitutability between inputs in terms of endogenous technology choice. Suppose, namely, that the production function is of a CES form between energy services and a Cobb-Douglas capital-labor composite and that technology choice involves the ability to choose “input saving” in the form of two technology parameters multiplying these two inputs, subject to a constraint. Then if that constraint is specified as a log-linear relationship, the outcome is a reduced-form production function in the basic inputs that is Cobb-Douglas, regardless of the degree of short-run substitutability between these inputs.⁵

Energy services, in turn, are provided by firms that act competitively with a constant-

⁴The implications of endogenous technical change in a similar setting is explicitly analyzed in Hassler, Krusell, Olovsson, and Reiter (2017), which in turn builds on the simpler endogenous-technology in Hassler, Krusell, and Olovsson (2017).

⁵This statement holds so long as the CES function has an elasticity parameter less than or equal to one (this is the empirically reasonable case for this application). For a parameter greater than one, the result is complete specialization: the production function becomes linear in one of the inputs.

returns-to-scale production function in n distinct energy inputs:

$$E_t = \mathcal{E}(e_{1,t}, \dots, e_{n,t}) = \left(\sum_{k=1}^n \lambda_k (e_{k,t})^\rho \right)^{\frac{1}{\rho}}. \quad (2)$$

Here, $e_{1,t}$ is the import of oil in period t . The other energy sources $\{e_{2,t}, \dots, e_{n,t}\}$ are energy sources assumed to be produced and supplied entirely domestically within the oil-consuming region. The associated production technology is linear in the final good; in particular, to produce $e_{k,t}$ units of energy source $k \in \{2, \dots, n\}$, $p_{k,t}$ units of the final good is required. Thus, we allow for these marginal costs to change over time. Final goods not engaged in energy production are consumed or invested in a standard neoclassical way. In sum, the resource constraint for the final good reads

$$C_t + K_{t+1} = A_t L_t K_t^\alpha E_t^\nu - p_{1,t} e_{1,t} - \sum_{k=2}^n p_{k,t} e_{k,t} + (1 - \delta) K_t.$$

We take the world market price of oil $p_{1,t}$ to be expressed in units of the global final good.

The oil-producing region, finally, produces oil without any resource cost. Its constraints are

$$\begin{aligned} R_{t+1} &= R_t - \sum_{i=2}^n e_{1,i,t}, \\ R_t &\geq 0 \forall t, \\ C_{o,t} &= p_{1,t} (R_t - R_{t+1}), \end{aligned} \quad (3)$$

where R_t is the remaining stock of oil in ground in the beginning of period t .

2.2 The carbon cycle

The use of energy leads to carbon emission in the form of CO₂. Specifically, emissions in period t are given by

$$M_t = \sum_{k=1}^n g_k e_{k,t},$$

where g_k measures how “dirty” energy source k is.⁶ We measure fossil energy sources in terms of their carbon content, implying that for each of them $g_{k,t} = 1$. Conversely, purely

⁶It is straightforward to allow time and region dependence of g_k .

green energy sources have $g_{k,t} = 0$. We could also, but do not currently, have intermediate cases.

We use the structure in Golosov et al. (2014) so we assume that the law of motion for the atmospheric stock of carbon S_t in excess of its preindustrial level is given by

$$S_t = \sum_{s=0}^t (1 - d_{t-s}) \sum_i M_t,$$

where

$$1 - d_s = \varphi_L + (1 - \varphi_L) \varphi_0 (1 - \varphi)^s$$

captures carbon depreciation from the atmosphere: the share of emissions that remains forever in the atmosphere is φ_L , the share that leaves the atmosphere within a period is $1 - \varphi_0$ and the remainder $(1 - \varphi_L) \varphi_0$ depreciates geometrically at rate φ .

2.3 The climate and the economic damages therefrom

The climate is affected by the atmospheric carbon concentration through the well-known greenhouse effect. The climate, in turn, causes economic damages, which in principle appear in a variety of forms but Golosov et al. (2014) demonstrate that, at least relative to the literature, the net effect of CO₂ concentration on productivity is quite well captured by a simple log-linear specification. Specifically, they use a specification for TFP that reads

$$A_t = e^{z_t - \gamma_t S_{t-1}}, \quad (4)$$

where z_t is exogenous technical change and γ_t captures the possibly time-varying sensitivity to atmospheric CO₂ concentration.

The effect of atmospheric carbon concentration on TFP can be thought of in two steps. In step one, there a logarithmic effect of CO₂ on the energy budget and, hence, on global warming. This effect is known since long: see Arrhenius (1896). It can be expressed, using T_t , which denotes the global mean temperature (in excess of its preindustrial level), and the stock of carbon in the atmosphere as

$$T_t = \frac{\lambda}{\ln 2} \ln \left(\frac{S_t + S_0}{S_0} \right). \quad (5)$$

Here, λ represents the “climate sensitivity” and S_0 the pre-industrial atmospheric carbon

stock. We abstract from dynamics in the relation between S_t and T_t and assume that the long-run equilibrium temperature associated with a level of carbon concentration is achieved immediately. In this sense, we exaggerate the effect of emissions on temperature but the dynamics are rather fast (it would be straightforward to include them and we leave them out for convenience mostly).

2.4 Markets and equilibrium

All agents in our model are price takers. Consider the oil-producing region first. We thus assume that there are many oil producers operating under perfect competition, with the representative oil producer choosing how much oil to store for next period, R_{t+1} , taking the world market price of oil as given. Using the last part of (3) to substitute out $C_{1,t}$ in (1), and taking the first-order condition with respect to R_{t+1} then yields

$$\frac{1}{R_t - R_{t+1}} = E_t \frac{\beta}{R_{t+1} - R_{t+2}}.$$

This second-order difference equation, which really represents an Euler equation for consumption of the oil producer, is easily solved: it delivers $R_{t+1} = \beta R_t$ implying $C_{1,t} = p_{1,t} (1 - \beta) R_t$. Note, in particular, that even if $p_{1,t}$ is stochastic, it has no effect on oil supply. The reason is simply that the income and substitution effects exactly cancel with logarithmic preferences. Conversely, note that our setup allows us to side-step the Hotelling price formula, by which the price of oil—in case its marginal production cost is zero and there is no monopoly power—would have to rise at the real rate of interest. The key behind this is that oil producers cannot invest their proceeds from an oil sale (say, in case of an oil-price hike) at a “global rate of interest”, since they do not have access, by assumption, to the global capital market. This assumption is of course unrealistic in its extreme form but the notion that there are at least some restrictions on these kinds of trade should not be controversial.

These assumptions allow us to write the behavior of energy service providers as the solution to the cost-minimization problem

$$\min_{e_{k,t}} \sum_{k=1}^n p_{k,t} e_{k,t} - \Lambda_t \left(\left(\sum_{k=1}^n \lambda_k (e_{k,t})^\rho \right)^{\frac{1}{\rho}} - E_t \right). \quad (6)$$

Here we note that by construction the Lagrange multiplier $\Lambda_t = P_t$, the price index of energy services.

The first-order condition for $e_{k,t}$ yields, for $k \in \{2, n\}$,

$$e_{k,t} = E_t \left(\frac{P_t \lambda_k}{p_{k,t}} \right)^{\frac{1}{1-\rho}} \quad (7)$$

and similarly oil consumption satisfies

$$e_{1,t} = E_t \left(\frac{P_t \lambda_1}{p_{1,t}} \right)^{\frac{1}{1-\rho}}. \quad (8)$$

Using this finding in the expenditure function, we arrive at

$$P_t = \left(p_{1,t}^{\frac{\rho}{\rho-1}} \lambda_1^{\frac{1}{1-\rho}} + \sum_{k=2}^n p_{k,t}^{\frac{\rho}{\rho-1}} \lambda_k^{\frac{1}{1-\rho}} \right)^{\frac{\rho-1}{\rho}}. \quad (9)$$

Producers of the final good maximize profits the oil price as given, so that

$$P_t = \nu \frac{A_t L_t^{1-\alpha-\nu} K_t^\alpha E_t^\nu}{E_t}.$$

This can be solved for energy-service demand:

$$E_t = \left(\nu \frac{A_t L_t^{1-\alpha-\nu} K_t^\alpha}{P_t} \right)^{\frac{1}{1-\nu}}.$$

Output net of energy expenses reads $(1 - \nu) Y_t \equiv \hat{Y}_t$. Note, however, that the shares of spending on the different energy sources are not constant unless $\rho = 0$, i.e., unless the overall production function is Cobb-Douglas in all inputs.

Households in the oil-consuming economy supply labor inelastically; we will normalize its value to unity.⁷ The households thus maximize (1) subject to the budget constraint

$$C_t + K_{t+1} = w_t L_t + r_t K_t + (1 - \delta) K_t.$$

Here $w_t = (1 - \alpha - \nu) \frac{Y_t}{L_t}$ and $r_t = \frac{\alpha Y_t}{K_t}$, so that $w_t L_t + r_t K_t = \hat{Y}_t$.

We will take one period to be long enough that we can make the assumption that $\delta = 1$. Now define the savings rate out of net output to be $s_t = \frac{\hat{Y}_t - C_t}{\hat{Y}_t}$. We can then write the Euler

⁷It would very simple to include endogenous labor supply.

equation for the households

$$\begin{aligned}\frac{C_{t+1}}{C_t} &= \beta \frac{\partial Y_{t+1}}{\partial K_{t+1}} \\ \frac{(1-s_{t+1})(1-\nu)Y_{t+1}}{(1-s_t)(1-\nu)Y_t} &= \beta \frac{\alpha Y_{t+1}}{s_t(1-\nu)Y_t}.\end{aligned}$$

By inspection we see that the savings rate must be constant across time and regions at $s = \frac{\alpha\beta}{1-\nu}$.

Proposition 1. *In each period the allocation is determined by the state variables K_t, R_t and S_{t-1} such that i) the capital savings rate is constant at $\frac{\alpha\beta}{1-\nu}$, ii) oil supply is $(1-\beta)R_t$, iii) energy price is $P_t = \left(p_{1,t}^{\frac{\rho}{\rho-1}} \lambda_1^{\frac{1}{1-\rho}} + \sum_{k=2}^n p_{k,t}^{\frac{\rho}{\rho-1}} \lambda_k^{\frac{1}{1-\rho}} \right)^{\frac{\rho-1}{\rho}}$, iv) energy service demand is $E_t = \left(\nu \frac{e^{(z_t - \gamma_t S_{t-1}) L_t^{1-\alpha-\nu} K_t^\alpha}}{P_t} \right)^{\frac{1}{1-\nu}}$, v) domestic fuel demand is $e_{k,t} = E_t \left(\frac{P_t \lambda_k}{p_{k,t}} \right)^{\frac{1}{1-\rho}}$, and vi) oil demand is $e_{1,t} = E_t \left(\frac{P_t \lambda_1}{p_{1,t}} \right)^{\frac{1}{1-\rho}}$. The price of oil is determined from equilibrium at the world oil market $e_{1,t} = (1-\beta)R_t$. The laws of motion for the state variables are $K_t = \frac{\alpha\beta}{1-\nu} \hat{Y}_t$, $R_{t+1} = \beta R_t$, and $S_t = \sum_{v=0}^t (1-d_{t-v}) M_t$.*

Two things are noteworthy here. First, the allocation is determined sequentially without any forward-looking terms; this is a result of the combination of functional forms. Second, conditional on a world market price of oil, all equilibrium conditions have closed-form solutions. Finding the equilibrium in any period t is therefore only a matter of finding the equilibrium oil price, where supply is “predetermined” (as a result of optimal oil extraction) at $(1-\beta)R_t$.

2.5 Taxation

A key goal of the present analysis is to analyze the consequences of taxing fossil fuel and, in particular, to assess the effectiveness of less than fully optimal taxation. With this aim, we allow the oil-consuming region to tax the users of fossil energy inputs. A carbon tax rate τ_t is thus imposed, implying that the total cost for the energy service provider of using energy type k becomes $(1 + \tau_t g_k) p_{k,t}$.

The immediate result of adding taxes is that the price of energy and the mix of fuels changes. These are straightforward to calculate. The prices $p_{1,t}$ and $p_{k,t}$ are simply replaced

by after tax-prices in (9), (8), and (7). The aggregate use of energy services is still given by

$$E_t = \left(\nu \frac{A_t L_t^{1-\alpha-\nu} K_t^\alpha}{P_t} \right)^{\frac{1}{1-\nu}}$$

but now using the tax-inclusive energy price.

The only complication resulting from taxes is that it matters for outcomes how the government revenues are handled. Due to the implied income effects, if the revenues are redistributed lump-sum to households, the savings rate will no longer be exactly $\alpha\beta/(1-\nu)$. However, our numerical analysis suggests that the quantitative effect of this effect is negligible, essentially because the income share of energy is small (ν is on the order of a few percent): energy taxes simply cannot generate much revenue. An alternative is to assume that the revenues from taxing fossil fuel are “wasted” or spent on goods whose consumption value do not interfere with how consumption is determined. In order to maintain tractability we opt for the former assumption along with savings rules that remain at $\alpha\beta/(1-\nu)$, hence implying that consumers do not smooth consumption fully optimally over time (conditional on their revenues). This is unlikely to lead to a sizeable bias compared to the one that would arise if the tax revenues were wasted. Given this approach, all the other features of proposition 1 remain intact.

3 Calibration

We first describe how we calibrate the benchmark model and then how the key uncertainty is captured.

3.1 Basic model parameters

We use a discount factor of 0.985¹⁰ with the understanding that a period is a decade. In the final-good production function, we set $\alpha = 0.3$ and the fuel income share ν to 0.055. We assume that labor input is constant and normalize it to 1.

The production of energy services is calibrated as follows. For the elasticity of substitution between the three sources of energy, we use a meta-study (Stern, 2012) of 47 studies of inter-fuel substitution. The unweighted mean of the oil-coal, oil-electricity, and coal-electricity elasticities is 0.95, i.e., slightly below unity. This elasticity implies $\rho = -0.058$, which we use as the main case. Note, in this context, that the meta-study is based on substitution

elasticities for different time horizons. At the same time, our arguments in Hassler, Krusell, and Olovsson (2017) discussed above suggests that a close to Cobb-Douglas elasticity is likely a reasonable outcome from endogenous input-saving technology choice.

In order to calibrate the λ 's we need prices and quantities of the three fuel types. Here we follow Golosov et al. (2014), who used a coal price of \$74/ton and a carbon content of 71.6%. The (pre-financial crises) oil price was \$70/barrel, corresponding to \$70·7.33 per ton and a carbon content of 84.6%. This implies a relative price between oil and coal in units of carbon of 5.87 (oil being worth more per carbon unit).

We then use the same source for the global ratio of oil to coal use in carbon units. With the use of equations (7) and (8) we find that $\frac{\lambda_1}{\lambda_2} = 5.348$. For green energy we use data for the sum of nuclear, hydro, wind, waste, and other renewables, also from Golosov et al. (2014), and retain their assumption of a unitary relative price between oil and renewables. This delivers $\frac{\lambda_1}{\lambda_3} = 1.527$. Along with the normalization $1 = \lambda_1 + \lambda_2 + \lambda_3$, this implies that $\lambda_1 = 0.543$, $\lambda_2 = 0.102$, and $\lambda_3 = 0.356$. We also need a value for the initial stock of conventional oil. Again following Golosov et al. (2014), it is set to 300 Gtoe.

For the carbon cycle parameters, we also follow Golosov et al. (2014) and set $\varphi_L = 0.2$, $\varphi_0 = 0.393$, and $\varphi = 0.0228$. We assume that initial global GDP is 75 trillion US\$ per year and set initial productivity and capital so that the economy is on a balanced growth path. Productivity in final-goods production, e^{z_t} , is assumed to grow at 1.5% per year and we assume that the cost of producing coal and green fuel is constant in terms of the final good. This rate of productivity increase implies an annual GDP growth rate of about 2%.

3.2 The climate and damage parameters

As discussed in the introduction, the purpose of this paper is to explore the range of economic outcomes at the endpoints of a range of plausible estimates for (i) the sensitivity of the climate to the carbon concentration and (ii) the sensitivity of the economy to the climate. For the former, we use the range given in a 2013 IPCC report, where they state that the equilibrium climate sensitivity (λ) is “likely in the range 1.5 to 4.5°C”.⁸ Since we are interested in the end points of the ranges, denoted λ_H and λ_L , we set $\lambda_H = 4.5$ and $\lambda_L = 1.5$.

To provide a similar range for the sensitivity of the economy to global warming, we build on the recent paper by Nordhaus and Moffat (2017). There the authors provide a rather comprehensive survey of studies of global damages from climate change. They also argue

⁸(IPCC, 2013a, page 81 and IPCC, 2013b, Box 12.1). The report also makes explicit that “likely” should be taken to mean a probability of 66-100%.

that the different studies should not be given equal weight in trying to distill a representative estimate of the aggregate effects of global warming. One particularly convincing argument for the unequal weights is that some studies are derivatives of earlier studies. What they do is somewhat judgemental, but they operationalize their approach by assigning a weight between zero and one, representing in reliability/originality, to each of the studies. All in all, they find 36 “usable” estimates of damages, expressed as percentages of global GDP, for different temperatures and it is based on these 36 studies that they then construct their ranges.

In Figure 1, we show the estimates reported by Nordhaus and Moffat (2017). The x axis measures the increase in the average global temperature and the y axis represents the percentage loss in world GDP; the size of bubbles indicate the attached weight.

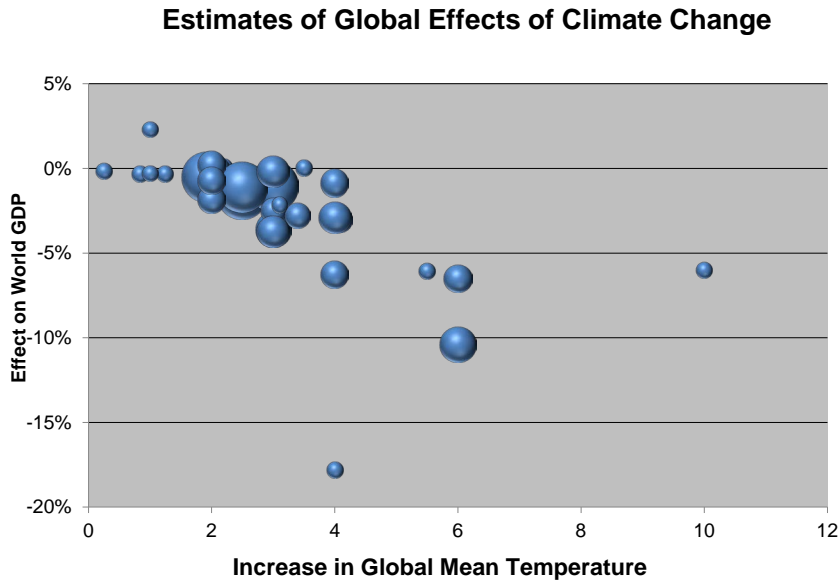


Figure 1: The Nordhaus-Moffat study

We use the Nordhaus-Moffat estimates to calibrate the likely range of economic sensitivity translated into our damage-function formulation. In our formulation, thus, we will need to derive a range for the parameter γ in (4). To accomplish this, we first observe that given a value of λ , the Arrhenius equation (5) can be inverted to yield S as a function of T :

$$S(T; \lambda) = S_0 \left(e^{\frac{T \ln 2}{\lambda}} - 1 \right).$$

By assumption, the damage associated with a given amount S of atmospheric carbon

concentration in our formulation is

$$1 - e^{-\gamma S}.$$

Thus, let $\hat{\Delta}_i(T_i)$ be a particular estimate of the effect on GDP at a temperature T_i . Then each one of the 36 estimates implies an estimate $\hat{\gamma}_i$ that satisfies

$$\hat{\gamma}_{i;\lambda} = -\frac{\ln(1 - \hat{\Delta}_i(T_i))}{S(T_i; \lambda)}.$$

For each of the two climate sensitivities under consideration, λ_H and λ_L , we thus obtain a set of damage elasticities $\hat{\gamma}_{i;\lambda}$. Within each of the sets, we define two subsets, high and low damage elasticities, denoted $\Gamma_{H,\lambda}$ and $\Gamma_{L,\lambda}$. These are constructed as follows. Let $\pi_i, i \in \{1, \dots, 36\}$ denote the weight Nordhaus and Moffat (2017) assigned to the different studies on global climate damages. Then the sets of high-damage elasticities is defined as the smallest set of the highest $\hat{\gamma}_{i;\lambda}$ such that

$$\sum_{i \in \Gamma_{H,\lambda}} \pi_i \geq 0.2 \sum_{i=1}^{36} \pi_i.$$

The set of low-damage elasticities is defined by instead collecting the lowest damage elasticities. Finally, our endpoint elasticities are defined as the weighted average value in the respective sets. Our result is that for λ_L , the endpoints, denoted γ_{L,λ_L} and γ_{H,λ_L} , are 0.27 and 1.79. For λ_H , we obtain $\gamma_{L,\lambda_H} = 1.44$ and $\gamma_{H,\lambda_H} = 10.39$, all expressed as percent (of global GDP) per 1,000 atmospheric GtC.

4 Results

We now describe and discuss our results, beginning with optimal-tax calculations and then looking at outcomes (for the economy and the climate) under different scenarios.

4.0.1 Optimal taxes

The methodology starts with an optimal-tax calculation (or, equivalently, the calculation an optimal marginal damage externality, which will equal the optimal tax in a standard Pigou manner). Thus, given any value of γ , we can use the formula from Golosov et al. (2014) for

an optimal tax—the setting here is a special case of that described there. This formula reads

$$\tau_t = \gamma Y_t \left(\frac{\varphi_L}{1 - \beta} + \frac{(1 - \varphi_L)\varphi_0}{1 - (1 - \varphi)\beta} \right), \quad (10)$$

where we note that all parameters are expressed for a period length of a decade. Note that the optimal tax is proportional to global GDP with only three kinds of parameters, representing discounting (β), carbon depreciation (the φ s), and damages (γ).

We will maintain the carbon depreciation parameters throughout and mainly focus on damages, but we will also comment on, and do robustness with respect to, discounting. For the four values of γ , the associated optimal tax rates are given in Table 1.

	Table 1	Base line	
	γ	Tax US\$/ton C	Tax US cents/gallon gasoline ⁹
	$\gamma_{L,\lambda_L} = 0.27$	6.9	1.6
	$\gamma_{H,\lambda_L} = 1.79$	45.5	10.9
	$\gamma_{L,\lambda_H} = 1.44$	36.6	8.8
	$\gamma_{H,\lambda_H} = 10.39$	264.4	63.4

Less stern discounting We can also show the optimal tax rates assuming a lower subjective discount rate. Specifically, we select an alternative discount rate to be that suggested in the Stern Review (Stern, 2006), namely 0.1% per year. The optimal tax rates for this discount rate are presented in Table 2.

	Table 2	Low discount rate	
	γ	Tax US\$/ton C	Tax US cents/gallon gasoline ¹⁰
	$\gamma_{L,\lambda_L} = 0.27$	60.3	14.5
	$\gamma_{H,\lambda_L} = 1.79$	399.5	95.9
	$\gamma_{L,\lambda_H} = 1.44$	321.4	77.1
	$\gamma_{H,\lambda_H} = 10.39$	2319	556

Clearly, the tax values are much higher here. In the high-high sensitivity case, the tax per gallon of gas would exceed \$5 and thus near ten times that with higher discounting.

⁹ We use a carbon content of 2.4 Kg/gallon.

¹⁰ We use a carbon content of 2.4 Kg/gallon.

Quasi-geometric discounting Iverson and Karp (2017) show that if discounting is quasi-geometric, we can find a Markov-perfect Nash equilibrium in a tax-setting game using the current model setting. In particular, they can extend the closed-form solutions studied here to such cases. Applying their formula to a case when the discount rate is 1.5% per year during the first decade and thereafter 0.1%, we obtain optimal taxes as in Table 3.

γ	Tax US\$/ton C	Tax US cents/gallon gasoline
$\gamma_{L,\lambda_L} = 0.27$	55.8	13.4
$\gamma_{H,\lambda_L} = 1.79$	369.6	88.7
$\gamma_{L,\lambda_H} = 1.44$	297.4	71.4
$\gamma_{H,\lambda_H} = 10.39$	2146	515

We see that the implied numbers are similar to those coming from Stern-like discounting.

4.0.2 Scenarios

Let us now use the model to compare the different scenarios. We therefore solve the model for the four different combinations of parameters, representing the four combinations of high and climate sensitivity and high and low economic sensitivity. Moreover, for each of the four cases, we solve the model without taxes and with taxes. We set the tax to the optimal level in the first period and then let it increase by 2% per year (22% per decade) which is approximately equal to the balanced-growth path for GDP.¹¹ Throughout, we use the high discount rate, i.e., a level of 1.5% per year.

In Figure 2, we show the path of global mean temperature. We graph increases in the temperature over the initial period.

When inspecting the graph, note that the model’s prediction for the global mean temperature in the initial period depends on the climate sensitivity (again, the two extreme values are 0.8 and 2.4 degrees Celsius), because we assume immediate adjustment to equilibrium in the climate system. We take the current stock of atmospheric carbon (800 GtC) as an initial condition and given this amount there are consequently two implied initial temperature levels (with a higher temperature for the higher sensitivity value). The current global mean temperature is approximately 1 degree above the average over the period 1951–1960. Using

¹¹Recall that the optimal tax should be indexed to GDP, as shown in equation (10). Thus, solving for a fully optimal equilibrium path implemented by taxes involves a fixed-point problem: at all points in time, the tax level depends on optimal GDP but optimal GDP depends on the tax. The simplification we adopt here circumvents this fixed-point problem by having slightly suboptimal taxes.

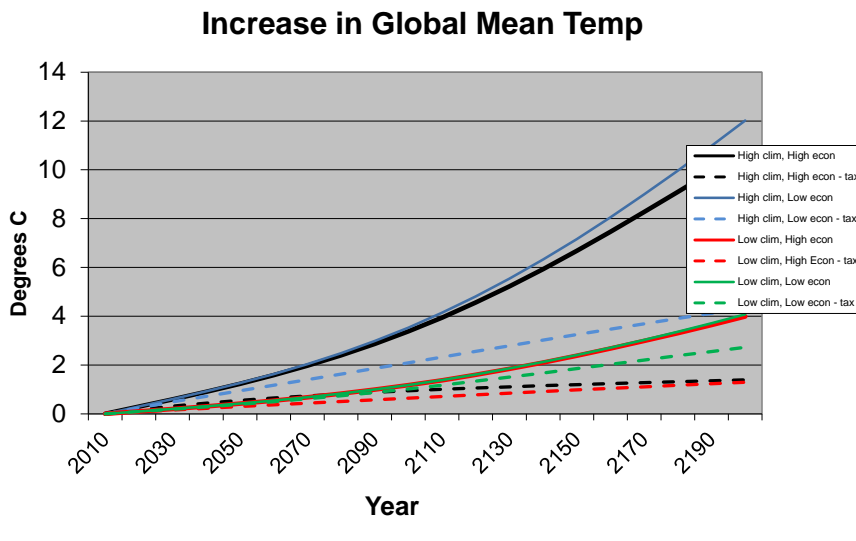


Figure 2: Climate outcomes in all scenarios

this as calibration target would yield a moderate climate sensitivity of around 2, interior to our range of uncertainty. There is no scientific consensus about whether the fairly low temperature increase is a sign of a low climate sensitivity or due to other temporary factors, such as inertia or dimming due to airborne particles.

For all four combinations of parameters, solid curves represent the laissez-faire allocation. The figure shows that in laissez faire, the level of economic sensitivity is not important for the climate. Instead, the speed of climate change is largely determined by the climate sensitivity. In the case of high climate sensitivity, the temperature increases very fast: it will have risen by 3.4 degrees Celsius by the end of the century and will continue to accelerate thereafter. In the opposite case, with low climate sensitivity, the increase in the global mean temperature relative to today is one degree Celsius by the end of the current century.

Figure 2 also shows that taxes are highly effective in bringing down global warming. When the climate sensitivity and the economic sensitivity both are high, the introduction of the optimal tax implies that global warming is slowed down sharply. Until 2100 the temperature increase over the current level is less than one degree Celsius and 100 years later it has increased by only an additional 0.4 degrees Celsius. This is substantially smaller than in the case of low climate sensitivity and no taxes.

Another important point shown in the figure is that with optimal taxes, there is a strong link between climate change and the economic sensitivity. In the case of a high climate sensitivity, climate change is, as just noted, almost halted. However, if the economic sensitivity

is low, substantially more climate change should be allowed—2.1 degrees Celsius relative to the initial level by 2105 and 4.3 towards the end of the simulation period.

Finally, we see that if the climate sensitivity is low, climate change is obviously slower, but it is still affected rather substantially by the tax. This is particularly so in the case of high economic sensitivity, in which case no more than a 1.3 degrees Celsius increase should be allowed over the two-century horizon. In fact, this number is close to the corresponding number when the climate sensitivity is high. Thus, although the optimal tax rates are very different in the cases with low and high climate sensitivity, the targets for the temperature increase are similar when the economic sensitivity is high. Of course, the lower tax in the case of low climate sensitivity would imply more carbon emissions than in the high climate sensitivity case, but the resulting temperature increase would be almost the same.

Moving to economic effects, Figure 3 shows the damages caused by climate change.

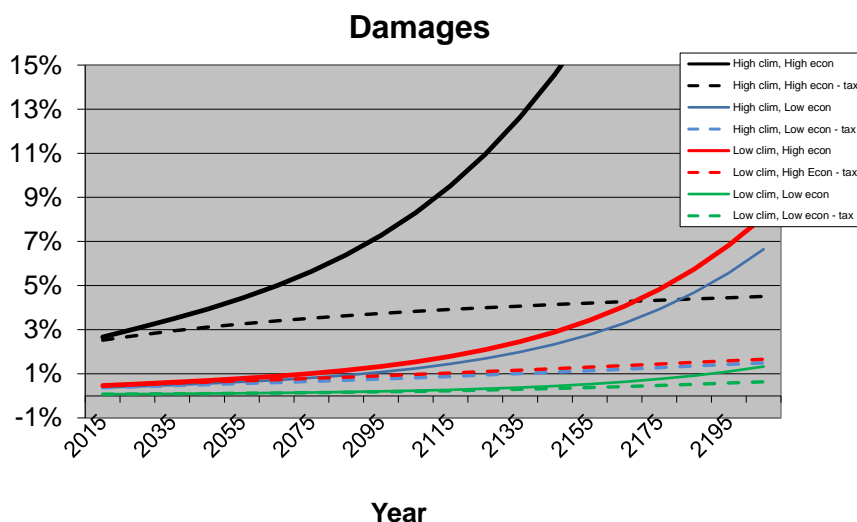


Figure 3: Economic externalities

Reflecting the finding for climate change, we see that taxes are effective in mitigating climate damages in all cases, thus keeping them on a fairly flat trajectory. Of course, the damage estimates for very high levels of climate change are especially uncertain here, but the purpose in the present paper is not to speculate on costs beyond what is reported in the Nordhaus-Moffat meta-study.

Figure 4 shows global consumption, measured *relative* to the most benign scenario: that with low climate and economic sensitivities (with optimal taxes imposed). Consumption can be viewed as a flow measure of welfare.

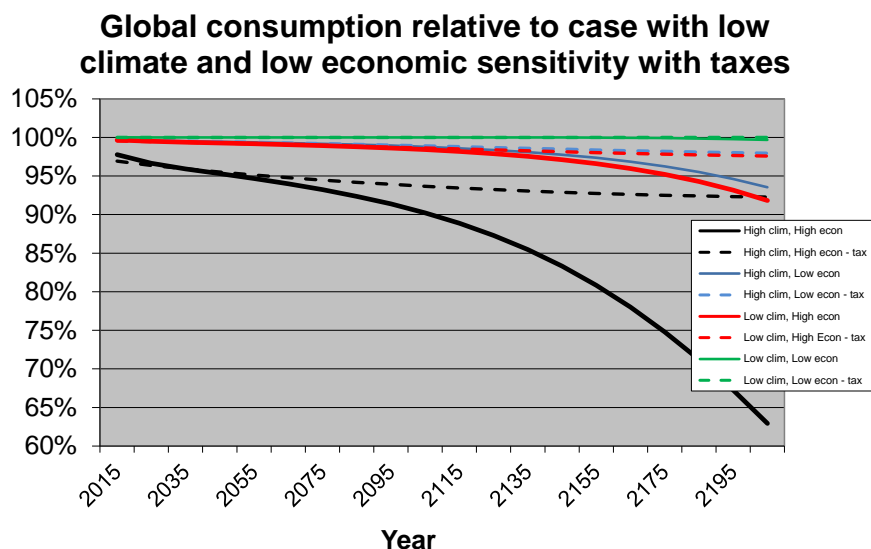


Figure 4: Relative consumption effects

The figure reveals that the stakes are very high when climate and economic sensitivities are high. Without a climate policy, consumption is significantly lower. A climate policy cannot remove all negative consequences of climate change in this case, but it can remove a very significant part. In all the other scenarios, the stakes are substantially smaller.

Figure 5 depicts coal use.

We see that in all the scenarios without taxes, coal use grows approximately exponentially.¹² In the case of high climate and high economic sensitivities, coal use is approximately flat under the optimal tax while it does increase, albeit not exponentially, as long as either of the sensitivities is low.

Let us finally consider the consequences of policy mistakes. Specifically, suppose the true state of the world is that both the climate sensitivity and the economic sensitivity are high while a “overly passive climate policy” is pursued, as represented by a tax that is optimal in the state of low sensitivities. Conversely, also consider the situation where the true state of the world is benign, with both sensitivities at their low values, but where the highest tax (which is optimal in the high-sensitivities world) is adopted: “overly zealous climate policy”. The results in terms of global consumption of these two kinds of policy errors are presented

¹²Whether this implies that we will run out of coal within the simulation period is an unsettled issue. On the one hand, standard references like BP (2017) estimates global proved coal reserves to 816 Gt which would not allow a trajectory like the higher ones in figure 5. On the other hand, other estimates of the stock of all hydrocarbon sources that potentially could be used could actually allow such trajectories (Rogner, 1997).

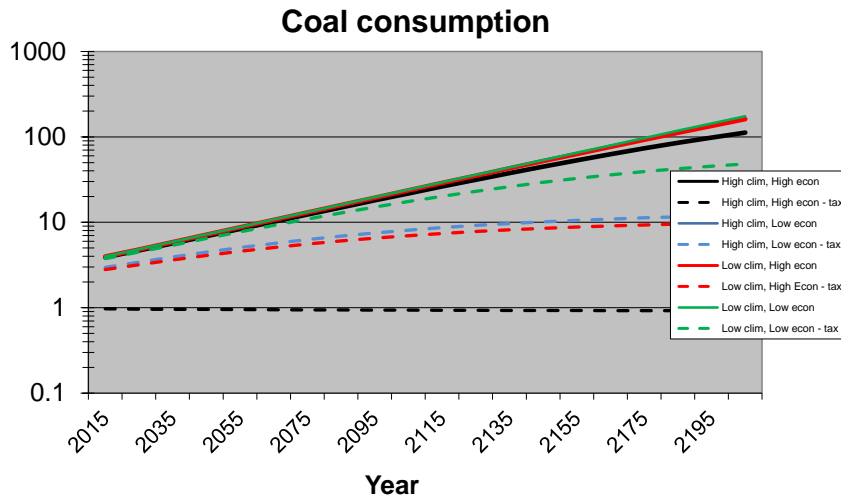


Figure 5: Coal use

in Figure 6.¹³

As we see from the graph, there is a stark difference between the two types of errors. Failing to introduce a high tax when it is necessary has dramatic consequences for consumption and welfare while unnecessarily imposing a (much) too high tax has very moderate consequences. The intuition for this result is that it is relatively cheap to replace coal-based energy production with greener sources. Thus, doing this in vain is not a great loss. On the other hand, not having replaced coal-based production with green energy if both the climate and economic sensitivities are high will inflict serious damage to welfare.

5 Concluding remarks

We have looked at two kinds of uncertainty here. There are others. For example, one could straightforwardly extend the present analysis to cover uncertainty about the carbon cycle. One could also consider uncertainty about mitigation costs, which could be accomplished by looking at a range of elasticities of substitution between green and fossil fuels in energy provision. One could also discuss uncertainty about the assumptions we have entertained here about technological change, both in its general form and in how technologies for energy production may develop. Yet another line of inquiry regards the possible irreversibilities

¹³In both cases, we let the wrong tax be in place for all of the simulation period. Obviously, such a persistent error is unlikely given that we would likely learn the true state and (hopefully) introduce the correspondingly appropriate tax.

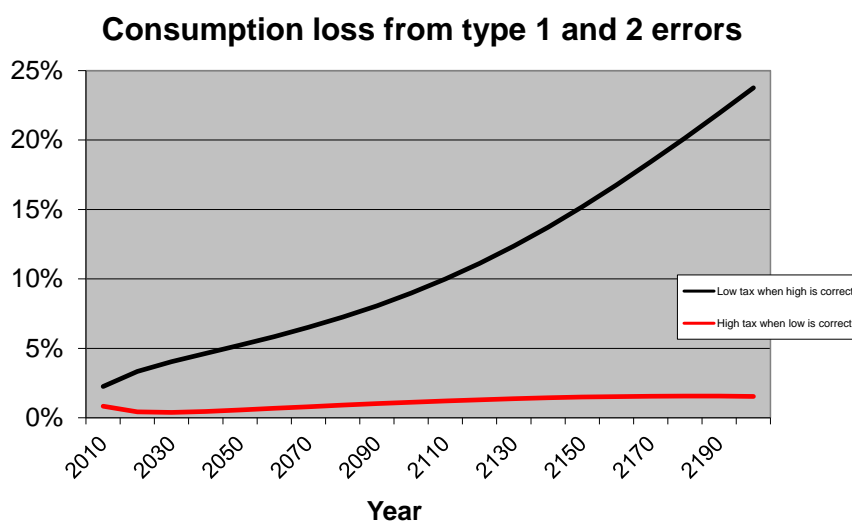


Figure 6: Overly passive and overly zealous policy

involved by incorrectly scrapping fossil-based capital and infrastructure, thus influencing the discussion of the two kinds of policy errors. There are also basic model parameters that could be altered. We assume, for example, logarithmic utility curvature, which allows for greater tractability, but limits the range of welfare consequences somewhat. Similarly, more curvature could be introduced on the damage side. We leave all these extensions for future work.

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