Can and should a pay-as-you-go pension system mimic a funded system?*

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Abstract

This paper considers the possibility of letting a pay-go pension system mimic a fully funded pension system in the sense that both systems provide the same return on pension fees. In a stochastic setting, the condition under which a less than fully funded pension system can provide an actuarially fair return *on average* are shown to be very restrictive. A non-funded pay-go pension system can provide an actuarially fair return *on the margin*, which increases economic efficiency. But the benefits of this fall entirely on current pensioners as a windfall gain unless compensating transfers are implemented.

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1. Introduction

Obviously, a compulsory pension system violates the (short-term) intertemporal preferences of individuals if they are constrained by liquidity. Indeed, this is often a basic purpose of such systems to prevent free riding and to mitigate asserted myopic behavior of some individuals. It is also well known that such a system distorts labor supply decisions if the system is financed by taxes on labor income, for example, in the form of a proportional payroll tax. Of course this is also the case if the individual is not liquidity constrained, provided the system is not actuarially fair. Here, an *actuarially fair* system means that the expected present value of pension benefits and of fees (contributions) are equal. Usually, compulsory pay-as-you-go pension systems in the real world are not actuarially fair, even disregarding intra-generational transfers (see, for example, Feldstein 1996).

This is the background for various suggestions to make existing pay-go systems more actuarial – perhaps even fully actuarially fair. This paper discusses the possibility and desirability of doing just that. A study of this type is worthwhile because several countries plan to move in this direction, and many observers have argued that it is possible and desirable to mimic an actuarially fair funded system by providing the same return to the individual as in a funded system, without tying the pension to the return on a previously accumulated fund. This would side-step the major complication with a transition to a funded system, namely that the individuals who work during the transition needs to pay *both* the pensions of the currently old *and* the build-up of the pensions fund.

However, making a pay-go pension system more actuarial is likely to be *generically* inconsistent with balancing the budget in each period. So the question of the stability of the pension system is naturally raised. More specifically, we ask two questions with respect to this issue:

1. Under what conditions will an actuarially fair pay-go systems be stable, in the sense non-explosive?
2. Is it possible for a pay-go pension system to be actuarially fair without having a fund of the same size as in a fully funded system?

It is well known that the implicit return in a balanced-budget, pay-go pension system is determined by the growth rate of the tax base. If this growth rate is not much lower than the interest rate, a balanced-budget, non-funded pay-go pension system then provides a return that is close to that of an actuarially fair system. In this case, one might think that a small fund would be sufficient to generate the extra revenues necessary to finance an actuarial pay-go system. This apparently intuitive conjecture turns out to be false—the system must be *fully* funded to provide an actuarially fair return.

So generically, a pay-go pension system is inconsistent with actuarial fairness even if it is partially funded. But there may still be a case for mimicing a fully funded pension system only *on the margin*, that is, by providing an actuarially fair return on marginal contributions. The reason would be that most labor-market distortions depend on the degree of *marginal* actuarial fairness, that is, on the relation between marginal contributions and marginal benefits. However, this observation is not sufficient for making policy recommendations. A pay-go pension system, in contrast to a fully funded one, creates intergenerational transfers. It is therefore important to calculate how the benefits of removing the labor-market distortion are distributed between generations. This turns out to be a crucial issue. For example, consider the experiment of increasing the marginal implicit return above the average return while keeping the proportional contribution rate that finances the pensions constant. While this would reduce the labor market distortions, the gains from the improved efficiency is entirely a windfall gain to the generation that is already retired at the time of the policy change. All other generations will actually be worse off. So such a policy change cannot be described as a move in the direction of mimicking a fully funded system.

To highlight our main messages, we make several simplifying assumptions. In particular, factor prices and population growth are exogenous stochastic variables. We make this assumption partly for convenience and analytical tractability, but we believe that our qualitative results do not critically depend on this. It is well known that the introduction of a pay-as-you go pension system may have considerable negative effects on net savings of a country by diverting part of the savings of the working generation into wind-fall gains for the elderly. This may have substantial effects on capital formation and factor prices. In principle, these effects should be considered when determining the optimal size of a pay-go pension system.[1] But this issue is outside the scope of this paper, where we take the size of the pension system as given. For this reason, we believe that that an analysis, where the effects on factor prices are disregarded is worthwhile as a step toward an understanding of the difference between a fully funded and a pay-go pension system. In particular, this is true for small economies with highly open, capital markets.

This paper is organized as follow: Section 2 presents the basic model. In section 3, we study the relation between stability and actuarialness of pension systems in a stochastic world. In section 4, we analyze the issue of marginal actuarial fairness. Section 5 concludes.

2. The model

We consider a two-period, overlapping generations' model, where individuals work in the first but not in the second period of their lives. All individuals in a generation are identical. The size of the generation born in t is N_t, which is taken to be a large number (in a sense to be made more precise later). To denote a single individual, we use the index i. But because all individuals of a given generation are identical, we can suppress this index most of the time.

We denote the ratio between generations born in t+1 and t, that is, N_{t+1}/N_t, by $1+n_{t+1}$, so n_{t+1} is the rate of population growth.[2] Let consumption in the two periods of life of an individual born in time period t be $c_{1,t}$ and $c_{2,t+1}$, labor supply l_t and the subjective rate of time preference θ. An individual born in period t is assumed to choose consumption and labor supply to maximize a time-additive, utility function of the following form

$$U_t^i \equiv u^1\left(c_{1,t}^i, -l_t^i\right) + E_t\left(1+\theta\right)^{-1} u^2\left(c_{2,t+1}^i\right), \tag{1}$$

subject to the budget constraint

$$\frac{c_{2,t+1}^i}{1+r_{t+1}} + c_{1,t}^i = w_t l_t^i (1-\tau) + \frac{B_{t+1}^i}{1+r_{t+1}}, \tag{2}$$

[1] Siandra, (1994), analyzes the optimal size of a pay-go pension system when the negative effect on capital formation is considered. Smith (1982) and and Endes & Lapan (1982) analyze optimal intergenerational risk sharing with endogeneous factor prices.

[2] Aggregate longevity risk could be incorporated in the analysis by interpreting $1+n_t$ as the ratio between the number of working individuals in period t and the number of living retirees.

where τ is a pension contribution rate that finances pension benefits denoted B_t. Individuals have access to a capital market where they may invest their savings and receive a return r_{t+1}.

At each time period t, three exogenous stochastic variables are realized: w_t the wage of the young generation in t, r_t the rate of return on the investments in the preceding period of the currently retired, and n_t the rate of growth of the number of working (young) individuals. We denote the growth rate of the aggregate wage income by g_t so that $1+g_{t+1} \equiv N_{t+1}w_{t+1}l_{t+1}/N_t w_t l_t$. We also define the (average) implicit return in the pension system as

$$r^p \equiv \frac{{}_i B_{t+1}^i}{{}_i w_t l_t^i \tau} - 1\,, \tag{3}$$

and the *marginal* implicit return for an individual as

$$r_i^{pm} = \frac{\partial B_{t+1}^i}{\partial w_t l_t^i \tau} - 1\,, \tag{4}$$

that is, the return on the fees paid on a marginal unit of working time. Note that the marginal implicit return is the marginal return for a single individual, holding the behavior of other individuals fixed.

3. Stability and actuarial fairness

This section explores necessary conditions for the stability of pay-go pension systems with fixed average implicit returns. It is well known that a pay-as-you-go pension system cannot provide an actuarially fair return unless the economy is dynamically inefficient, which in the non-stochastic case requires that the growth rate of the economy is larger than the real interest rate. We show that the requirements are stronger in a stochastic setting. We also show that actuarial fairness requires a fund of the same size as in the fully funded system, and that an actuarially fair pay-as-as-you-go pension system under certain circumstances automatically converges to a fully funded system.

A stable pension system must have a non-explosive stock of debt. We postulate two necessary conditions for such stability. Letting D_t denote the accumulated debt in the pension system at time t divided by aggregate wage income the two conditions are:

Condition 1. A pension system is not stable unless

$$\begin{aligned}&\lim_{s\to\infty} E_t D_{t+s} < \infty, \text{ and}\\ &\lim_{s\to\infty} E_t(D_{t+1}^2) < \infty.\end{aligned} \tag{5}$$

The first condition states that if the debt share of the system approaches infinity, the system is not stable. The second condition requires that the variance is bounded. Without that, we could have a situation where the debt approaches plus or minus infinity with equal probability, satisfying the first condition. Nevertheless, one could hardly call such a system stable. Now let us consider the implications of the two stability conditions.

Using (3), we can express the pension benefit in period t for each pensioner in a pay-go pension system as $\left(1+r_t^p\right)\tau w_{t-1}$. So the per-period deficit in the pension system can be written

$$N_{t-1}(1+r_t^p)w_{t-1}l_{t-1}\tau - N_t \tau w_t . \tag{6}$$

To analyze the behavior of D_t, we express the deficit as a share of the wage bill of the currently young by dividing by $N_t w_t$.

$$\frac{N_{t-1}(1+r_t^p)w_{t-1}l_{t-1}\tau}{N_t w_t l_t} - \tau = \tau\left(\frac{1+r_t^p}{1+g_t} - 1\right) = \tau\left(\frac{r_t^p - g_t}{1+g_t}\right). \tag{7}$$

The RHS of (7) simply says that the deficit share is non-zero if the rate of return in the pension system differs from the growth rate of the wage bill. There is a deficit if r_t^p is larger than g_t and a surplus (negative deficit) otherwise. Now assume that accumulated deficits are borrowed and surpluses are invested at the market interest rate r. The time path of the debt share D_t, that is, the accumulated deficit share, is then

$$\begin{aligned}&D_{t+1} = D_t \frac{1+r_{t+1}}{1+g_{t+1}} + \tau\left(\frac{1+r_{t+1}^p}{1+g_{t+1}} - 1\right), \text{ and hence}\\ &D_{t+s} = D_t \prod_{i=1}^{s} \mu_{t+i} + \tau \sum_{j=1}^{s} \left(d_{t+j} - 1\right) \prod_{i=j+1}^{s} \mu_{t+i},\end{aligned} \tag{8}$$

where μ_t and d_t denote $(1+r_t)/(1+g_t)$ and $(1+r_t^p)/(1+g_t)$.

Equation (8) is a difference equation with stochastic parameters. Since general stability conditions for such processes are unknown, we have to restrict the attention to the case when the following assumption is satisfied.

Assumption 1. *For a given value of τ, the ratios $(1+r_t)/(1+g_t)$ and $(1+r_t^p)/(1+g_t)$ are stochastic variables that are i.i.d. over time with expected values $\bar{\mu}$ and $\bar{d}$ respectively.*[3,4]

Now consider the expected value of D_{t+1} as a function of D_t. This function has one fixed point D^* at which $E\left[D_t \middle| D_{t-1}\right] = D_{t-1}$. Under the assumption that $\bar{\mu} \neq 1$, this point is given by[5]

$$D^* = \tau \frac{\bar{d} - 1}{1 - \bar{\mu}}. \tag{9}$$

Now, if the pay-go pension system attempts to mimic the fully funded pension system by providing benefits with the same stochastic properties as the market return, $r_t^p = r_t$, and $\bar{\mu} = \bar{d}$. The pay-go pension system is then, of course, actuarially fair. In this case, the RHS of (9) is just $-\tau$. Thus, if the pay-go system is actuarially fair in this way, a debt share of $-\tau$ is a fixed point. A debt share of $-\tau$ implies that the pension system has accumulated a fund equal to $N_t w_t l_t \tau$. This fund is of the same size as the fund in a fully funded system, which by construction (in our two-period model) is each period's pension fee, that is, $N_t w_t l_t \tau$. Now let us consider the stability of the pay-go system in this setting. First consider the first part of condition 1. By iterating on (8) and using the i.i.d. assumption, it is straightforward to show that

$$E\left[D_{t+s} \middle| D_t\right] = D_t \bar{\mu}^s + \tau \sum_{i=0}^{s-1} (\bar{d} - 1)\bar{\mu}^i . \tag{10}$$

Equation (10) defines a converging sequence if and only if $\bar{\mu}$ is smaller than unity. This is the well known result that the accumulated deficit/fund in a pay-go system is stable only if the expected value $(1+ r_t^f)/(1+g_t) < 1$. Now, consider $\lim_{s\to\infty} E\left[D_{t+s} \middle| D_t\right]$. In the case of stability, this limit equals $\tau(\bar{d} - 1)/(1 - \bar{\mu})$. Note that if $\bar{d} = \bar{\mu}$, this limit is $-\tau$, which gives the following proposition.

Proposition 1**.** *If the pay-go system is stable, and it is actuarially fair by providing a return with the same stochastic properties as the market return, its expected accumulated fund converges to that of the fully funded system.*

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[3] The assumption of independence over time can be relaxed quite easily.

[4] Note, that g_t is affected by the endogenous labor supply l_t. However, for a fixed τ, labor supply is fully determined by the exogenous variable w_t.

[5] If $\bar{\mu} = 1$, $E_{t-1}(D_t) = D_{t-1} + E_{t-1}D_t = D_{t-1} + \bar{d} - 1$. This has no fixed point unless $\bar{d} = 1$ in which case the expected deficit is zero. Then all debt levels are fixed points (the debt is a random walk).

Now turn to the second part of condition 1, that the variance should be bounded. As seen in the second equation of (8), D_{t+s} contains product chains of stochastic terms. In general, it is hard to characterize the stability of such processes. However, it turns out that there is a simple sufficient condition that we can use if we add the assumption that $(1+r_t)/(1+g_t)$ can take on only a finite number of values.

Proposition 2. *Under the assumption that* $(1+r_t^f)/(1+g_t)$ *is i.i.d. and can take only a finite number of values, the debt share of pay-go system with a stationary stochastic deficit* d_t *has a non-exploding variance if*[6]

$$\bar{\mu} < \sqrt{1 - \operatorname{var}\mu}\ . \tag{11}$$

Proof: See Appendix.

Note that this condition for stability is stronger than the condition $\bar{\mu} < 1$, which would imply the possibility of having an actuarially fair pay-go pension system with an expected surplus in each period. From the previous proposition we see that the variance of μ inserts a wedge in this condition, i.e., the larger is the variance, the higher must be the expected surplus in each period to guarantee stability. It should also be noted that the stability of the pensions system is determined by the stochastic properties of μ but not of d.

Let us now consider the special non-stochastic case when growth and interest rates are constant. In this case, actuarial fairness requires $r^p = r$, which, by (9) implies that D^* is $-\tau$. Furthermore, the condition in proposition 2 is now both sufficient and necessary for stability and the variance of μ is zero. This result can alternatively be formulated: In a dynamically efficient economy, no actuarially fair pay-go pension system can be introduced and in the dynamically inefficient case an actuarially fair pay-go pension system will converge to a fully funded one. Along the transition phase, there will be differences between the two systems. In particular, as originally shown by Samuelson (1958) and Aaron (1966), the generation that is in retirement when a pay-go system is introduced will receive a windfall gain that they would not receive if a fully funded system had been introduced from the very beginning. It is well known that this "free lunch" does not exist in a dynamically efficient economy.

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[6] Results in Warne (1996) suggest that the condition (11) is also necessary for stationarity. It is also straightforward to replace the assumption of i.i.d. by the assumption that the distribution of $(1+r_t)/(1+g_t)$ depends on a finite number of previous states of the world.

An actuarial pay-go system with assets equal in size to the fund in the fully funded pension system is in steady state identical to a fully funded pension system even if the pension payments are not formally tied to the return on the fund in the system. So we can say that any pension system that is actuarially fair and has a constant fund (or debt), expressed as a share of GDP in steady state, is a fully funded system.

4. Marginal actuarial fairness

In the previous section, we showed that a pay-go pension system that operates in a dynamically efficient economy cannot mimic a fully funded pension system in the sense of providing an implicit return equal to the market return. But there is still the possibility of having the *marginal* implicit return, as defined in (4), equal to the market return, while keeping budget balance by setting the *average* return equal to the growth rate of the economy. This possibility is discussed in, for example, Auerbach and Kotlikoff (1987). Intuition suggests that efficiency is improved if the marginal return on pension is set such that the individual on the margin is indifferent between paying pension fees and investing on the capital market. It is straightforward to derive conditions under which this is true.[7] However, as noted in the introduction, it is also important to analyze the consequences of such reforms for the intergenerational distribution of income. This is the purpose of this section.

We consider only balanced budget pay-go pension systems with a linear (affine) relation between fees and benefits. More specifically, we consider systems where B_{t+1}^{i} in the individual budget constraint (2) is given by

$$B_{t+1}^{i} = \tau w_t l_t^i \alpha (1 + g_{t+1}) + T_{t+1}, \tag{12}$$

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[7] See, for example, Hassler and Lindbeck (1998). Note also that in some cases, the average return is important for efficiency. The choice of whether to participate in the official labor market is determined by the average, rather than the marginal, return. This is a main point in Lorz (1998), where individuals have an option of working on the black market.

where T_{t+1} is a lump sum positive or negative transfer to pensioners in t+1, constant over all *i.* The coefficient α expresses the marginal return on pension fees in excess of the return generated by the growth rate *g*.

As we already know, budget balance in the pay-go pension system implies that the average implicit return in the pension system is equal to *g*. Thus,

$$B_{t+1} = \tau w_t l_t \; (1 + g_{t+1}) \,, \tag{13}$$

where *B* and *l* denote averages over all individuals. From (12) and (13) we see that if α>1 and g_{t+1} is constant, T_{t+1} has to fall as aggregate labor supply in period *t* increases in order to insure that the aggregate implicit return remains at g_{t+1} as required for budget balance. However, we assume that each agent behaves atomistically and disregard the (very small) effect on *T* induced by his choice of labor supply. The parameter α thus affects the *marginal* implicit return for the individual, which becomes

$$r_{t+1}^{pm} = \alpha(1 + g_{t+1}) \,. \tag{14}$$

Of course the assumption of atomistic behavior is highly realistic. The effect of an individual on the aggregate budget constraint of the pension system is negligible. The first-order conditions for the individual now becomes:

$$\begin{aligned} u_{c_{1,t}} &= (1+\theta)^{-1} E(1 + r_{t+1}) u_{c_2,t+1}, \\ u_{-l_t} &= \left(w_t (1-\tau) u_{c_{1,t}} + w_t \tau\alpha E_t u_{c_2,t+1} \frac{1+g_{t+1}}{1+\theta} \right), \end{aligned} \tag{15}$$

where subscripts on *u* denote partials with respect to the relevant period utility function (u^1 or u^2).

Now, us define the following welfare function

$$W(\alpha,\tau) = E \sum_{t=1}^{\infty} \delta^t \left(u^1\left(c_{1,t}, -l_t\right) + \left(1+\theta\right)^{-1} u^2\left(c_{2,t+1}\right) \right), \tag{16}$$

where δ^t denote the welfare weight given to the representative individual in a generation born in period *t*. These weights may reflect both different sizes of generations and social time preferences. Since the utility of the current old generation is not included in (16), a pareto efficiency requires that $W(\alpha,\tau)$ is maximized over α while the utility of the currently old is held constant. This can be achieved by adjusting τ to keep $\tau w_1 l_1$ constant.

Now let us consider the effect on the current young and all future generations by varying α while keeping τ fixed

$$\frac{\partial W(\alpha,\tau)}{\partial\alpha} = E \sum_{t=1}^{\infty} \delta^t \left(u_{c_{1,t}} \frac{\partial c_{1,t}}{\partial\alpha} - u_{l_t} \frac{\partial l_t}{\partial\alpha} + \frac{u_{c_{2,t+1}}}{1+\theta} \frac{\partial c_{2,t+1}}{\partial\alpha} \right). \tag{17}$$

Budget balance in the pension system implies that $B_{t+1} = \tau(1+n_{t+1})w_{t+1}l_{t+1} = \tau(1+g_{t+1})w_t l_t$. This means that the aggregate budget restriction is

$$\begin{aligned} c_{2,t+1} &= \left(w_t(1-\tau)l_t - c_{1,t}\right)(1+r_{t+1}) + B_{t+1} \\ &= \left(w_t(1-\tau)l_t - c_{1,t}\right)(1+r_{t+1}) + (1+g_{t+1})w_t \tau l_t. \end{aligned} \tag{18}$$

Now, let let us define $l_t(\alpha)$ as the labor supply as a function of α when, B_{t+1} is adjusted in order to keep (13) satisfied. This implies that the average return on pension fees is kept constant while the marginal is varied. Clearly, this implies that the budget restriction of the pension system is respected and that there is no income effect of varying α. Since the substitution effect is positive, $\partial l_t/\partial\alpha > 0$.

In general, $\partial l_t/\partial\alpha$ may change over time as wages and other stochastic variables are realized. To simplify the analysis, we make the following assumption.

Assumption 2. *The marginal effect of variations in α on labor supply is constant over time, i.e., $\partial l_t/\partial\alpha = \partial l/\partial\alpha$.*

By the previous assumption we can drop time subscripts on l, so we have

$$dc_{2,t+1} = \left(w_t(1-\tau)dl - dc_{1,t}\right)(1+r_{t+1}) + (1+g_{t+1})w_t \tau dl\,. \tag{19}$$

Using this in (17) yields

$$\begin{aligned} \frac{\partial W(\alpha,\tau)}{\partial\alpha} = E \sum_{t=1}^{\infty} \delta^t \Bigg(& u_{c_{1,t}} \frac{\partial c_{1,t}}{\partial\alpha} - u_{l_t} \frac{\partial l}{\partial\alpha} \\ & + \left(w_t(1-\tau)\frac{\partial l}{\partial\alpha} - \frac{\partial c_{1,t}}{\partial\alpha} \right) \frac{u_{c_{2,t+1}}(1+r_{t+1})}{1+\theta} + w_t \tau \frac{\partial l}{\partial\alpha} \frac{u_{c_{2,t+1}}(1+g_{t+1})}{1+\theta} \Bigg). \end{aligned} \tag{20}$$

Now we can use the first-order conditions from (15). Doing this and using the law of iterated expectations gives[8]

$$\frac{\partial W(\alpha,\tau)}{\partial\alpha} = E \sum_{t=1}^{\infty} \delta^t \left((1-\alpha) w_t \tau \frac{\partial l}{\partial\alpha} \frac{u_{c_{2,t+1}}(1+g_{t+1})}{1+\theta} \right). \tag{21}$$

[8] See the appendix for the steps in the derivation.

Setting (21) to zero yields the first-order condition for an optimal α, *when* τ*, rather than* τwl*, is held constant.* Since $\partial l_t/\partial\alpha>0$, also the second order condition for a maximum is satisfied.

Proposition 3. *If the contribution rate* τ *is held constant, setting* $\alpha = 1$ *maximizes the welfare of the current working and all future generations*[9].

If τ is held constant and each generation of workers work an extra hour, every individual in the current young generation get an extra income of $w(1-\tau)$ while young and $w\tau(1+g)$ while retired, regardless of α. The same applies to all future generations. It is the value of this extra income that should be set equal to the marginal utility of leisure in a welfare optimum, where the windfall gain in welfare of the initially retired generation is disregarded. Setting α different from unity distorts the labor-leisure choice by creating an externality, because variations in labor supply affect the size of the lump-sum component of pensions. This would reduce welfare. In a sense, we can consider $\alpha = 1$ as the *constrained first best* when the welfare of the current retired generation is disregarded and the contribution rate is constrained to τ.

So far, we have neglected the utility of the initially retired generation. Setting α to unity results in an allocation that is not Pareto efficient when the initially retired generation is included in the analysis. Increasing α from unity has only second-order negative effects on the current and future young generations. Current pensioners, by contrast, enjoy a positive first-order effect from a marginal increase in α, because labor supply and thus pension benefits increase in α. So increasing α from unity tends to increase economic efficiency, which provides an opportunity for a Pareto improvement. However, without a compensating increase in transfers to the current working and all future generations, for example by reducing τ, the benefit due to the increased efficiency falls fully as a windfall gain just to the currently retired generation. The current working and all future generations would actually be worse off.

Our results in this section, applying to a dynamically efficient economy, can be summarized as follows.

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[9] We derived this result for the case of borrowing constrained individuals in Hassler & Lindbeck (1997). As we have shown, the result is more general.

1. For a given contribution rate, the marginal and average return on pension fees should coincide in order to maximize welfare of the current young and all future generations.
2. For a given contribution rate, the marginal return on pension fees should be higher than the average, and set equal the capital market return in order to maximize aggregate welfare *including* the current old generation. Such a change increases the welfare of the current old and reduces the welfare of the current young and all future generations.
3. When the contribution rate is allowed to vary, a Pareto efficient allocation can be reached by increasing the marginal return on pension fees so it equal the capital market return and reducing the contribution rate so as to hold pensions of the current old constant. This allocation maximizes the welfare of the current young and all future generations while keeping the welfare of the current old generation constant.[10]

5. Concluding remarks

We analyzed various methods of making a pay-go pension system mimic a fully funded system. But several intuitively plausible methods turned out not achieve this. Simply disregarding budget balance and paying an actuarially fair average return would make the pension debt explode if the economy is dynamically efficient. As long as the growth rate of the wage bill is lower than the capital market return, only a fully funded system can then provide an implicit return equal to the capital market return. However, if the growth rate is close to the capital market return, the efficiency gain in the labor market by moving to a fully funded system will be small. In the dynamically inefficient economy, an actuarially fair pay-go pension system generates surpluses that *automatically* accumulate into a fund of equal size as in a fully funded system.

We have also argued that it is important to analyze how the benefits of increasing the marginal return on pension fees are distributed between generations. Unless the contribution rate is lowered, or other compensating transfers are used, the benefits of the increased

[10] See, Hassler and Lindbeck (1998) for a formal proof.

efficiency goes entirely to current pensioners as a windfall gain. The current working generation and all future generations actually lose. Such a change cannot be called a move in the direction of mimicking the fully funded pension system, because it would strengthen the intergenerational transfers created by the introduction of a pay-go pension system.

We finally want to emphasize some limitations of our analysis. First, we have not dealt with intra-generational redistribution, which may be one of the purposes of real world pension systems. Such redistributional concerns may limit the possibility of lump sum elements of the pension system, as we have discussed in the paper.

Second, we have abstracted from general equilibrium effects and hence changes in factor prices. This may be somewhat more justifiable in a small open economy than in a large. Another justification is that the clarification of partial effects is an important first step in a full analysis of pension systems. It should also be noted that our conclusion regarding the possibilities of mimicing a funded system is limited to some particular aspects – the marginal return on pension fees, the stability of the system and the absence of marginal distortions on the labor market.

A fully funded pension system may also have other features than *cannot* be mimicked by a reformed pay-go system. Among those are the effects on aggregate savings and the capital stock with implications for the development of financial market. The consequences of shocks to wage rates, demography and capital market return will also necessarily be differently distributed between generations in the two types of systems. For example, demographic shocks do not influence pension in the fully funded system. It is often also argued that a fully funded system provides better protection against political interventions in promised pension benefits. However, the pay-go pension system provides a potential for sharing the consequences of shocks to wages and capital market returns that do not exist in the fully funded system.

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Appendix

Proof of Proposition 2

Assume that for all t

$$\begin{aligned} &\frac{1+r_t}{1+g_t} \in [\mu_1,\ldots,\mu_n], \text{ with} \\ &prob\left(\frac{1+r_t}{1+g_t} = \mu_i\right) = p[i], i = 1,\ldots,n, \end{aligned} \tag{A.1}$$

where $p[i]$ is an element of a vector of probabilities that sum to unity.

Now we use a result in Karlsen (1990).[11] A sufficient condition for stability of a first-order autoregressive model with state dependent AR coefficients denoted μ_i and with a state transition matrix denoted Π is that the largest eigenvalue of

$$\begin{bmatrix} \mu_1^2\Pi_{1,1} & \cdots & \mu_1^2\Pi_{n,1} \\ \vdots & & \vdots \\ \mu_n^2\Pi_{1,n} & \cdots & \mu_n^2\Pi_{n,n} \end{bmatrix} \tag{A.2}$$

is smaller than unity. In (A.2) $\Pi_{i,j}$ is the probability of moving from state i to j. In the case of proposition 2, the Π is particularly simple since the probabilities of different states are independent of previous states. This implies that

$$\begin{bmatrix} \mu_1^2\Pi_{1,1} & \cdots & \mu_1^2\Pi_{n,1} \\ \vdots & & \vdots \\ \mu_n^2\Pi_{1,n} & \cdots & \mu_n^2\Pi_{n,n} \end{bmatrix} = \begin{bmatrix} \mu_1^2 p_1 & \cdots & \mu_1^2 p_1 \\ \vdots & & \vdots \\ \mu_n^2 p_n & \cdots & \mu_n^2 p_n \end{bmatrix}. \tag{A.3}$$

Using a result in Magnus & Neudecker (1988), it can be shown that the only non-zero eigenvalue of the matrix in (A.3) is given by[12]

$$\left[\mu_1^2, \cdots, \mu_n^2\right] \begin{bmatrix} p_1 \\ \vdots \\ p_n \end{bmatrix} \tag{A.4}$$

which is the expected value of the square of $(1+r_t)/(1+g_t)$. This, in turn is equal to the square of the expected value of $(1+r_t)/(1+g_t)$ plus the variance of $(1+r_t)/(1+g_t)$, as stated in the proposition.

Derivation of equation (21)

The steps are the following

[11] We are grateful to Anders Warne for showing us Karlsen's proof.

[12] See Warne (1996).

$$
\begin{aligned}
&= E \sum_{t=1}^{\infty} \delta^t \Bigg(u_{c_{1,t}} \frac{\partial c_{1,t}}{\partial \alpha} - \left(w_t (1-\tau) u_{c_{1,t}} + w_t \tau \alpha \frac{u_{c_{2,t+1}} (1+g_{t+1})}{1+\theta} \right) \frac{\partial l}{\partial \alpha} \\
&\quad + \left(w_t (1-\tau) \frac{\partial l}{\partial \alpha} - \frac{\partial c_{1,t}}{\partial \alpha} \right) \frac{u_{c_{2,t+1}} (1+r_{t+1})}{1+\theta} + w_t \tau \frac{\partial l}{\partial \alpha} \frac{u_{c_{2,t+1}} (1+g_{t+1})}{1+\theta} \Bigg) \\
&= E \sum_{t=1}^{\infty} \delta^t \Bigg(u_{c_{1,t}} \frac{\partial c_{1,t}}{\partial \alpha} - \left(w_t (1-\tau) u_{c_{1,t}} + w_t \tau \alpha \frac{u_{c_{2,t+1}} (1+g_{t+1})}{1+\theta} \right) \frac{\partial l}{\partial \alpha} \qquad (5) \\
&\quad + \left(w_t (1-\tau) \frac{\partial l}{\partial \alpha} - \frac{\partial c_{1,t}}{\partial \alpha} \right) u_{c_{1,t}} + w_t \tau \frac{\partial l}{\partial \alpha} \frac{u_{c_{2,t+1}} (1+g_{t+1})}{1+\theta} \Bigg) \\
&= E \sum_{t=1}^{\infty} \delta^t \left((1-\alpha) w_t \tau \frac{\partial l}{\partial \alpha} \frac{u_{c_{2,t+1}} (1+g_{t+1})}{1+\theta} \right).
\end{aligned}
$$