# Are They Swinging Together?

## A measure of linear comovement with an application to Swedish and foreign business cycles.*

by

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## Abstract

This is a revised version of chapter II in my forthcoming thesis at Massachusetts Institute of Technology. A frequency band specific measure of the degree of linear comovement is defined. The distribution of the measure is simulated using Monte Carlo methods. The sensitivity of the distributions to the characteristics of the underlying processes is substantially reduced if low frequencies are excluded from the analysis. It is argued that "business cycle facts" established using filters that include low frequencies, for example the Hodrick-Prescott filter, may be dominated by the behavior at low frequencies. Explicit treatment of different frequency bands may then be preferable. The measure of linear comovement is applied to Swedish and foreign macro time series spanning the period 1861 to 1988. A substantial degree of comovement between Swedish consumption related variables and foreign GDP is found while Swedish GDP and manufacturing production is clearly less covariant with foreign GDP.

# 1. Introduction

This paper attempts to contribute to a descriptive research agenda, inspired by the seminal paper "Understanding Business Cycles" by Lucas (1977), but is also part of a longer tradition involving, for example, Burns and Mitchell (1946). The goal of this agenda is the establishment of empirical regularities or "stylized facts" about business cycles. One of the most problematic pitfalls in the strive to establish business cycle facts is the sensitivity of these facts to the statistical methods used to find them. A recent illustration of this is Canova (1991). Using a set of US real macroeconomic time series, Canova shows the that linear statistics typically used to establish "stylized facts" are highly sensitive to the filtering method used. One answer to this problem has been to try to find facts that are robust to variations in the statistical methods. It should be more fruitful to try to understand why the differences occur and then choose a suitable method.

Measures of comovement are typically included as a part of business cycle facts. Typically, they involve cross-correlations at some pre-specified number of leads and lags. Sometimes coherence functions are estimated. Given assumptions about exogeneity, measures of comovement may be given a causal interpretation. In this paper I will present and discuss the properties of a measure of linear comovement that, to my knowledge, not have been used before in economics. The proposed measure is easily interpreted in the time domain as a frequency decomposition of the standard regression $R^2$. In addition to this measure's own value, an examination of its properties may help us understand why different measures will behave differently when applied to macro time series.

An important and early contribution to the idea of distinguishing between linear dependence on different frequencies for economic variables is Engle (1974). He shows how to estimate linear regressions on different frequencies. In effect he is using a band pass filter and then estimates a regression on the filtered data. A problem with his method is that a process with zero spectral density over a frequency range with positive measure is linearly deterministic. Standard asymptotic inference can thus not be applied.[1]

More recent studies where frequency domain statistics are used are Englund, Persson and Svensson (1992), Hassler, Lundvik, Persson and Söderlind (1992) and Stock and Watson (1990). They all examine macro time series using statistics in both the time and the frequency domain. The preferred filters in these papers are variants of band pass filters.

Since long it has been known that the stochastic characteristics of the underlying data, particularly at zero and low frequencies, is of critical importance for the behavior of many statistics (e.g., Nelson and Kang, 1981). I will show that the distribution of the proposed measure is likely to be less sensitive than other measures to the characteristics of the underlying data generating process. This means that the consequence of inappropriate detrending is less severe.

I also show that without explicit treatment of different frequency bands, the behavior at low frequencies is likely to dominate time domain statistics of macroeconomic time series. This may also happen if the Hodrick-Prescott filter is used. King and Rebelo (1989) argue that there may be important information in the low frequency region that is "thrown out with the bath water" if the Hodrick-Prescott filter is used. My results show that the Hodrick-Prescott filter may pass enough low frequencies for them to dominate time domain statistics like cross-correlations. Explicit treatment of different frequency ranges may then be preferable. Inclusion of low frequencies can also heuristically be blamed for the spurious cross-correlations discussed in Harvey and Jaeger (1993).

After discussing the properties of the measure I propose I apply it to a long Swedish data set. I find surprisingly low degrees of comovement between Swedish GDP, manufacturing production and foreign variables. Exports, imports, consumption and unemployment has a distinctly higher degree of comovement. I also show that these

[1] See e.g. Whittle (1983) p. 26 for a formal proposition.

differences are concealed if low frequencies are included. My interpretation of the results is that business cycles in Swedish production to a large extent are home-made. Swings in long run growth, on the other hand, seem to be well correlated over the national borders.

Section 2 argues that it may be fruitful to study business cycles by separating different sets of frequencies in the series. In section 3 I define and simulate the distribution of the proposed frequency specific measure of linear comovement. Section 4 uses the proposed measure on a Swedish data set and section 5 concludes the paper.

## 2. Detrending and Definition of Business Cycles

The concept of business cycles (implicitly) assumes that data can be decomposed in one non-stationary component and one component that is without a trend in at least its first moment.[2] Let me then write

$$Y_t = Y_t^t + Y_t^s \tag{2.1}$$

where $Y_t^t$ denotes the (log of the) trending component and $Y_t^s$ is the (log of the) non-trending component.

A priori, it is not unlikely that the trending and the non-trending components are generated by somewhat different mechanisms. An example would be that the non-stationary component is generated by some mechanism producing variations in long run growth rates. An endogenous growth model could, for example, be used to model this mechanism. Monetary shocks in combination with sticky prices could, on the other hand, produce stationary fluctuations. The distinction between the two mechanisms may be useful if they have different characteristics and if the links between them are relatively simple and can be identified.

As is shown in Quah (1991), there is in general no unique decomposition of a I(1) series in a stationary and a non-stationary component. By assuming different characteristics of the non-stationary component we can make its share of the sample variance of differenced data arbitrarily small. The assumptions used to identify the components in (2.1) should thus be based on theoretical considerations.

One identifying assumption is that the two components $\Delta Y_t^t$ and $Y_t^s$ have most of their spectral power in different frequency bands. In this paper I have chosen to define business cycle frequencies as frequencies corresponding to periods between three and

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[2] We often make the additional assumption that the non-trending component is stationary. This is necessary for the application of many statistical methods including spectral analysis. Clearly this assumption may be too strong. In an evolutionary society, the higher moments of the business cycle may very well be changing over time, without business cycles looses its meaning as a concept.

eight years. Fluctuations with higher frequencies are not included on the grounds that they may represent measurement errors stemming from erroneous "shuffling" between adjacent years. With more than one observation per year higher frequency fluctuations may also come from seasonals, not typically thought of as business cycles.

Before applying spectral methods the series have to be made stationary. Below I have done this by taking first-differences of the logarithms and subtracting the sample mean. The first-difference filter will make the series stationary unless they are integrated of a higher order than one. Some evidence against integration of order higher than one for real macro variables have been presented in Jones (1992). A first difference filter also makes a spectrum with "the typical spectral shape"[3] flatter and thus easier to estimate.

The spectrum of the business cycle component is estimated as

$$\hat{h}_{Y^s}(w) = \left|(1-e^{-iw})\right|^{-2} \hat{h}_{\Delta Y}(w), \ \forall w \in \Omega, \ 0 \notin \Omega \tag{2.2}$$

where $\Omega$ is the set of frequencies corresponding to periods between 3 and 8 years. Clearly this estimate will be contaminated by the trending component unless

$$h_e(w) << h_{\Delta Y_t^s}(w), \qquad \forall w \in \Omega.^4 \tag{2.3}$$

(2.3) implies that long run growth, i.e., the non-stationary component in (2.1), is generated by a process with a fairly high degree of persistence in its growth rates – a smooth trend. It is possible to generate such a process within, for example, a model where growth is generated by the slow diffusion of inventions to new markets.[5] Consider the case when technological innovations, denoted as $z_t$, are i.i.d. white noise. New technology is adopted through the economy according to a standard S-shaped diffusion mechanism.

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[3] Granger (1966) coined this expression. Note, however, the result by Nelson and Kang (1981) that inappropriately detrended time series typically posses "the typical spectrum" and that this is an artifact of the detrending procedure rather than the underlying data generating process.

[4] A weaker requirement that may be enough is that $\int_{w\in\Omega} h_e(w) << \int_{w\in\Omega} h_{\Delta Y_t^s}(w)$

[5] For a recent study on diffusion see Lippi and Reichlin (1993).

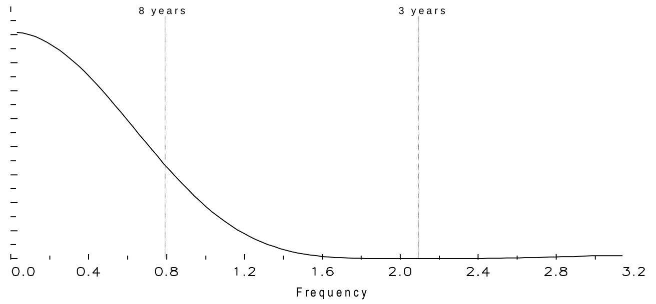
Assume for example that the maximum impact on productivity comes after two years and the diffusion process is completed after four years. We may then represent the trend as

$$y_t^t = a + y_{t-1}^t + \frac{z_t}{3} + \frac{z_{t-1}}{2} + \frac{z_{t-2}}{1} + \frac{z_{t-3}}{2} + \frac{z_{t-4}}{3}. \tag{2.4}$$

As is shown in figure 1, this trend, after differencing, has most of its spectral power outside business cycle frequencies. The same will be true also for other S-shaped diffusion patterns. For a linear time trend (2.3) is, of course, trivially true.

Assume it is true that the economy may be modeled as consisting of two mechanisms generating fluctuations in different frequency ranges. We should then use statistical methods that can distinguish between the behavior of our series at different frequencies.

**Figure 1 Spectrum of First Differences of a Diffusion Process**



# 3. Frequency Domain Comovement Measures

When studying the linear relationship between two processes we are typically assuming a model of the form

$$\begin{aligned}\mathbf{Y}_t &= \sum_{s=-\infty}^{s=+\infty} \mathbf{X}_{t+s} b_s + e_t \quad x_t \perp e_s, \forall s,t \\ &= \mathbf{B}(L)\mathbf{X}_t + e_t\end{aligned} \tag{3.1}$$

where $\mathbf{B}(L)$ denotes a potentially double-sided and infinite lag-polynomial. Both $\mathbf{Y}_t$ and $\mathbf{X}_t$ may be multidimensional although below I will consider the LHS variable to be a scalar. Without any knowledge or assumptions about the structure of $\mathbf{B}(L)$, the task of estimating the parameters of this model is formidable. Assume we are only interested in the *degree* of linear comovement between **Y** and **X**. This could be the case if want to examine to what extent one process influences another, for example how much the Swedish business cycle covaries with a foreign counterpart.[6] In this case spectral analysis offers a method of estimating this without making assumptions about $\mathbf{B}(L)$. This contrasts with the common procedure in business cycle studies where cross-correlations at pre-specified leads and lags are reported and interpreted as measures of comovement.

To simplify, consider first the case where both **Y** and **X** are univariate. Following Priestley (1986), the spectral densities of Y and X, denoted $h_y(w)$ and $h_x(w)$, are related by

$$h_y(w) = \left|\mathrm{B}(e^{-iw})\right|^2 h_x(w) + h_e(w). \tag{3.2}$$

We also have

$$\mathrm{B}(e^{-iw}) = \frac{h_{yx}(w)}{h_x(w)}, \tag{3.3}$$

where $h_{yx}(w)$ denotes the cross spectral density between Y and X, and

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[6] Given that we find significant degrees of covariation we may want to estimate the structure of $\mathbf{B}(L)$.

$$s_y^2 = \int_{-p}^{p} h_y(w)dw \qquad \forall y . \tag{3.4}$$

Using (3.1)-(3.4) we have

$$\begin{aligned} s_e^2 &= \int_{-p}^{p} h_e(w)dw \\ &= \int_{-p}^{p} \left[ h_y(w) - \left| \mathrm{B}(e^{-iw}) \right|^2 h_x(w) \right] dw \\ &= \int_{-p}^{p} \left[ 1 - \frac{\left| h_{yx}(w) \right|^2}{h_x(w) h_y(w)} \right] h_y(w)dw \\ &= \int_{-p}^{p} \left[ 1 - \left| w_{yx}(w) \right|^2 \right] h_y(w)dw \end{aligned} \tag{3.5}$$

where $\left| w_{yx}(w) \right|^2$ denotes the squared coherence between X and Y.

From (3.4) we can easily get an expression for the standard measure of linear relationship – $\mathrm{R}^2$ of regression (3.1).

$$\begin{aligned} \mathrm{R}^2 &= 1 - \frac{s_e^2}{s_y^2} \\ &= 1 - \frac{\int_{-p}^{p} \left[ 1 - \left| w_{yx}(w) \right|^2 \right] h_y(w)dw}{s_y^2} \\ &= \frac{\int_{-p}^{p} \left| w_{yx}(w) \right|^2 h_{yy}(w)dw}{s_y^2} \end{aligned} \tag{3.6}$$

Note that the interpretation of this $\mathrm{R}^2$ is the amount of the variation in Y that can be accounted for in a regression on X *given that we are using the correct model as defined in equation* (3.1). Correct means that we are including all (possibly infinitely many) positive and negative lags of X that have a true regression coefficient different from zero.

The usefulness of (3.6) is that we may use it to decompose $\mathrm{R}^2$ into frequency specific components. Assume that we are interested in the linear relationship at some frequency set Ω. Let $R^2_\Omega$ denote the degree of linear covariation at the frequency set of interest. This is then defined as

$$R^2_\Omega = \frac{\int_{w\in\Omega} |w_{yx}(w)|^2 h_y(w)dw}{\int_{w\in\Omega} h_y(w)dw}. \tag{3.7}$$

Also a multivariate regression $R^2$ may be decomposed. In this case we only replace the univariate coherence $w_{yx}(w)$ with its multivariate counterpart $w_{yx_1,\ldots,x_q}(w)$ and then use (3.6) or (3.7). The multivariate coherence is defined as

$$w^2{}_{yx_1,\ldots,x_q}(w) = \frac{\mathbf{h}_{yx}(w)\mathbf{h}_{xx}(w)^{-1}\mathbf{h}_{yx}(w)^*}{h_y(w)} \tag{3.8}$$

where * denotes the complex conjugate of the transpose. Note here the close analogy to the corresponding time domain measures. Replacing the spectral density vectors and matrices with covariance vectors and matrices we get the regression $R^2$.

An important feature of coherence is its invariance under linear transformations. The coherence at frequency $w$ measures the correlation between spectral components of frequency $w$ in the spectral representation of different processes. Since linear transformation will affect these components proportionally, it is intuitive that this correlation measure is unaffected.[7] If a linear transformation cancels a frequency, the invariance result is, however, not true for that frequency. The coherence at that frequency is analogous to the correlation between two zero variance processes and is not well defined. If for example, a first difference filter is applied to some stationary series, the zero frequency is canceled. The coherence at frequency zero is thus not invariant to the first difference filter. The coherences at all other frequencies are, however, unaffected.

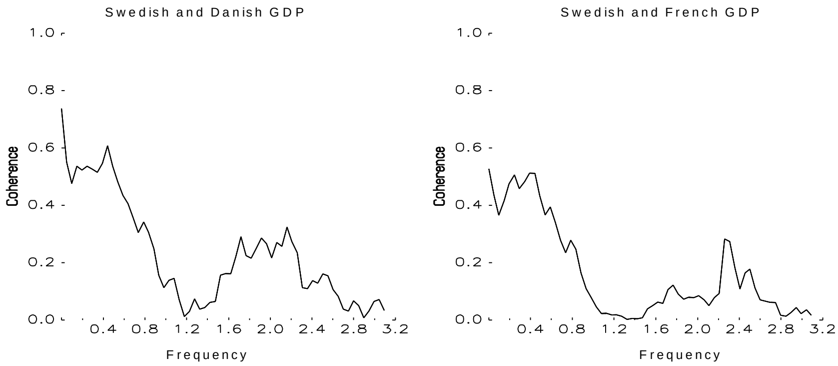
It will be useful to note from (3.7) that $R^2_\Omega$ is a weighted average of the coherences where the weights are the spectral densities $h_y(w)$. Since the latter clearly are affected by

[7] See Priestley (1986), page 661, for a proof.

linear transformations, $R^2_\Omega$ will not be invariant. The procedure of first filtering two processes with a common linear filter and then studying the degree of linear relationship between the filtered processes is equivalent to manipulating the weights in (3.7). By weighing the coherences differently we may vary the degree of linear comovement, often within wide ranges. For any $a \in [0,1]$ and $\Omega \in [-p, p]$, if the coherence in the interval $\Omega$ takes the value $\alpha$ we can produce an $R^2_\Omega$ arbitrarily close to $\alpha$ by appropriately choosing a weighing function (or equivalently; the right linear filter).[8]

The following illustrates the point that we may manipulate the degree of linear comovement. Take the time series for (the logarithm of) Swedish and French GDP from 1870 to 1988. The estimated coherence functions are depicted in Figure 2. We see that in both cases there is a low frequency region where the coherence is fairly high. In the frequency range between 1.2 and 1.6, on the other hand, the coherence is very low. We may thus get very different degrees of linear dependence if we apply linear filters that pick out the low frequencies and the frequencies in the range (1.2, 1.6), respectively.

**Figure 2 Coherence Functions**

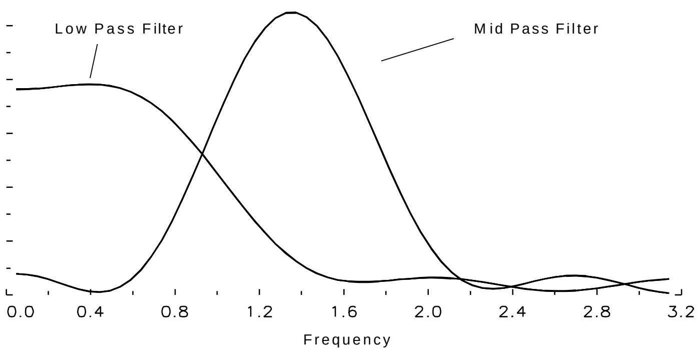



Filters with very peaked transfer functions typically have long lag structures. But also with rather short lag structures we may get sufficiently peaked transfer functions to illustrate the point. In Figure 3, I have graphed the transfer functions of two quite simple filters that are given by

[8] Here we also get an intuitive explanation for the fact that a spectrum with zero density over a range of frequencies with positive measure is equivalent to the process being deterministic. In this case we may apply a linear filter such that the process has exactly zero variance.

$$y_t^f = y_t + .9y_{t-1} + .6y_{t-2} + .2y_{t-3} - .2y_{t-5} \qquad \text{Low pass filter}$$
$$y_t^f = y_t + .3y_{t-1} - .8y_{t-2} - .6y_{t-3} + .3y_{t-4} + .6y_{t-5} \qquad \text{Mid pass filter.} \qquad (3.9)$$

**Figure 3 Transfer Functions of Two Simple Filters**




The low pass filter picks out low frequency fluctuations while the mid pass picks out frequencies in the mid range. Another filter that may tend to pick out low frequencies is the Hodrick-Prescott filter. As is shown in Cogley and Nason (1992), the spectrum of a random walk detrended with a Hodrick-Prescott filter has a pronounced peak in the low frequency region.[9] The exact location of the peak and its relative height depends on the smoothing coefficient, higher λ implies higher peaks at lower frequencies. With λ = 1600 and yearly data, the peak is at periods of 30 years.

Figure 4 shows the theoretical spectrum of a random walk detrended with a HP-filter with λ = 1600, 40 and 1 respectively.[10] Note also that if we let λ go to infinity the HP-filter becomes identical to removing a (log)linear time trend, implying (heuristically) that the latter will give us a process with an infinitely high peak in the low frequency region. Using these filters on Swedish, French and Danish GDP we would suspect that the low pass filter and the HP-filters with high λ's would pick out the high coherence ranges and thus give us high degrees of linear comovement. The mid pass filter and the HP-filters with low λ's would, on the other hand, give low degrees of comovements.

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[9] For enough persistent series that are not random walks, we also get a peak in the low frequency region, although it is less pronounced than for the random walk.

[10] Or, equivalently, the transfer function of the filter {(1-L)$^{-1}$ (HP-filter)}. The transfer functions of the HP-filter are computed as the Fast Fourier transform of the filter weights of the 64th observation in a series of 128. In the figures the spectra are normalized to the interval [0,1].

This conjecture is confirmed in Table 1. There we see that the degree of linear comovement, as measured by $R^2_\Omega$, where $\Omega$ is taken to be all frequencies, is much higher for the low pass filter than for the mid pass filter.[11] Furthermore, the high $\lambda$ filter behaves as a low pass filter while the converse is true for low $\lambda$'s. In this case the structural relations between the processes seem to be quite simple, we can thus see the same thing by looking only at the contemporaneous correlations. Attempts to establish stylized facts about the business cycle that are robust to different filters is here likely to fail. If we are interested in describing business cycles, we must use methods that describe the time series' statistical properties *at business cycle frequencies*.

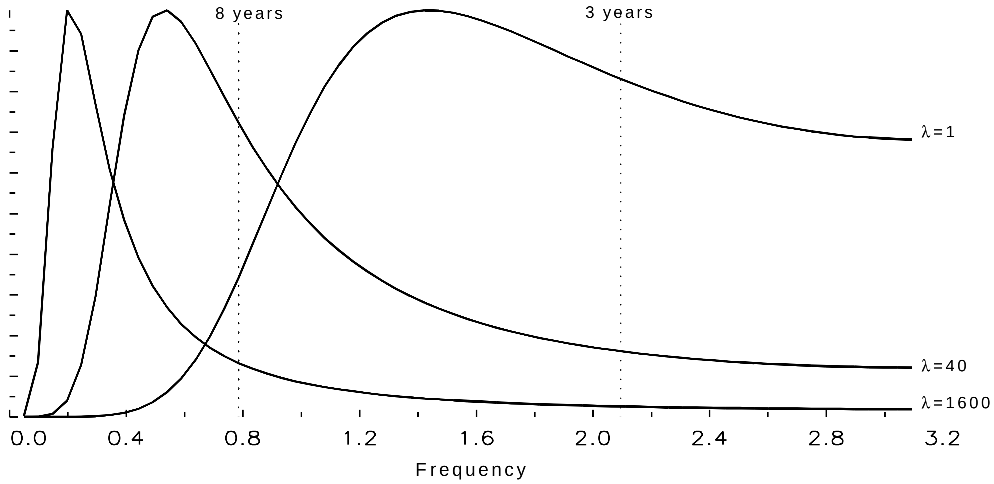
**Table 1 Manipulated Linear Comovements**

| | Sweden and Denmark | | Sweden and France | |
|---|---|---|---|---|
| | $R^2$ | Corr.† | $R^2$ | Corr.† |
| HP-Filter $\lambda$ = 1600 | 0.51 | 0.64 | 0.42 | 0.64 |
| Low Pass Filter | 0.44 | 0.59 | 0.36 | 0.54 |
| HP-Filter $\lambda$ = 40 | 0.33 | 0.48 | 0.28 | 0.43 |
| Mid Pass Filter | 0.18 | 0.30 | 0.08 | 0.14 |
| HP-Filter $\lambda$ = 1 | 0.17 | 0.31 | 0.08 | 0.03 |

† Contemporaneous Correlation

[11] The series are first differenced before the low pass and the mid pass filters are applied.

**Figure 4 Spectra of Random Walks Detrended with HP-filters**




## 3.1. Estimation

A problem arises in spectral analysis since we have to estimate the *functions* $w_{yx}(w)$ and $h_y(w)$ where the number of different ω's in general is infinite. However, since most of the processes economists are interested in have continuous spectra[12], we may approximate the integral in (3.7) with a finite sum.

Let the ω's be evenly spaced[13] on the interval Ω. We can then estimate $R^2_\Omega$ by

$$\hat{R}^2_\Omega = \frac{\sum_{w \in \Omega} \left| \hat{w}_{yx}(w) \right|^2 \hat{h}_y(w)}{\sum_{w \in \Omega} \hat{h}_y(w)} \tag{3.10}$$

where ^ indicate estimates.

The procedure of estimating the different spectral functions involves choosing a window. When estimating the coherence, the window is necessary even to get an estimate. The spectral components of a process are random variables that take some values for a

[12] For example all ARMA processes have continuos spectra. Harmonic processes (finite sums of sine waves where each frequency contributes a strictly positive amount to the variance), on the other hand, have discrete spectra.

[13] If the ω's are unevenly spaced we use a weighted average where the weights are the relative lengths between the midpoints between the ω's.

particular realization of the whole process. The coherence at frequency ω is the correlation between these components of different processes at frequency ω in *repeated* realizations. In time series analysis we only have one realization of the process and thus also only one realization of the spectral component at each frequency. It is clearly impossible to estimate a correlation from just one observation. To circumvent this problem we may use the continuity of the spectral densities in conjunction with the (asymptotic) inter-frequency independence. This allows us to approximate repeated realizations at one frequency by observations at neighboring frequencies for one realization.

The value of the spectral function at frequency ω is thus estimated as some average of neighboring "raw" values of the periodogram.[14] An unweighted average corresponds to a rectangular window, which is moved over the whole range of frequencies. Choosing a too wide window clearly leads to difficulties in detecting narrow peaks or valleys in the spectrum, while a too narrow window gives a high variance of the estimates. In this paper I have used variants of a rectangular spectral window (also called Daniell window).

Standard spectral functions are only defined for stationary processes. We are often uncertain as to the correct number of times a time series should be differenced to render it stationary. If we are studying coherences at strictly positive frequencies, over-differencing (differencing an I(k) process more than k times) should not be a serious problem. This is because the difference filter is linear and the coherence, as noted above, is invariant to differencing. Under-differencing, i.e., the application of spectral methods to non-stationary series, is, on the other hand, certainly dubious.

For independent processes $w_{yx}(w)$ is zero for all ω so $R^2_\Omega$, being a weighted average of $w_{yx}(w)$, is clearly zero for all Ω. The weights, i.e. the spectral densities, will of course be affected by the filter. We would suspect that, at least for big enough samples, the less equal the weights are, the higher is the variance of $\hat{R}^2_\Omega$ between independent

[14] This implies a somewhat disturbing similarity between the estimated coherence and $\hat{R}^2_\Omega$ – both are averages of neighboring cross spectral densities. A difference is, however, that $\hat{R}^2_\Omega$ has stochastic weights while the coherence weights are given by the window.

processes. This because the $\hat{w}_{yx}(w)$'s are asymptotically iid for independent processes. Over-differencing could thus lead to reduced power in tests of zero $\hat{R}^2_\Omega$ if the differencing makes the spectra more peaked and thus the weights less equal. If, on the other hand, we have a persistent but stationary process, differencing may make the spectra flatter. In this case differencing may potentially reduces the variance of $\hat{R}^2_\Omega$. This conjecture is supported in simulations below.

In the estimation of $\mathbf{h}_{yx}(w)$ and $\mathbf{h}_{xx}(w)$ I have used bivariate methods to estimate each element separately. In all cases I have used the Fast Fourier Transform and written the programs in GAUSS.

## 3.2. Distribution of bivariate $\hat{R}^2_\Omega$

It should be possible to calculate at least an asymptotic distribution of $\hat{R}^2_\Omega$. I have, however, chosen to simulate the distribution using Monte Carlo draws. The purpose of this paper is to propose a measure of comovement between relatively short[15] time series of macro variables. The asymptotic distributions may thus not be very helpful for inference.

The distribution of the estimates in small samples depends on the structure of the underlying processes. I have thus performed simulations using different processes. I have included processes with most of their spectral power in the low frequency region, which is typical for macro time series. I will define business cycle frequencies as those corresponding to periods between 3 and 8 years.

Using GAUSS I have constructed pairs of series of 128 independent (pseudo) random variables for which I have calculated $\hat{R}^2_\Omega$. I let the frequency band **Ω** be all frequencies, business cycle frequencies and frequencies corresponding to periods over eight years. The results are presented in Figure 1.1-6 and in Table 2. The former show relative sample frequencies in the categories $\hat{R}^2_\Omega = \{[0,0.01),[0.01,0.02), \ \ldots,[0.99,1.00]\}$ from 25 000

[15] In what appears to be a typical application Priestley gives an example with 17702 observations (Priestley, 1986, page 544).

draws of each process. In Table 2 I report the mean and the 90'th, 95'th and 99'th percentiles from 10 000 draws.

For two processes, white noise and AR(1), I have compensated for differencing when calculating $\hat{R}^2_\Omega$. Assume that the output from a process is differenced and used to estimate a spectrum. To get an estimate of the spectrum of the original, undifferenced process we can multiply the spectrum of the differenced process with the inverse of the transfer function for the first difference filter. Using obvious notation

$$\begin{aligned} h_y(w) &= \left|(1-e^{-iw})\right|^{-2} h_{\Delta y}(w) \qquad w \neq 0 \\ &= 2(1-\cos(w))^{-1} h_{\Delta y}(w). \end{aligned} \tag{3.11}$$

When I compensate for differencing I thus first estimate the spectrum. Then I multiply it with $2(1-\cos(w))^{-1}$ before using it as the weighing function in (3.10). Since $2(1-\cos(w))^{-1}$ is undefined for ω=0, the zero frequency is excluded from the calculations.

For all results reported in Figure 5.1-6 and Table 2 I have used a rectangular (Daniell) window including 15 frequencies. Figure 5 and Table 2 show that the properties of the data generating process strongly influence the distribution of $\hat{R}^2_\Omega$. We see that for all Ω the distribution of $\hat{R}^2_\Omega$ is narrowest for the uncompensated white noise (Panel 4.1) and widest for the random walk (Panel 4.3). As discussed above, it is intuitive that the more equal the weights $\hat{h}_y(w)$ in (3.10), the less wide is the distribution. The flat spectrum of a white noise process generates equal weights.[16] The finding that the white noise process generates the least dispersed distribution of $\hat{R}^2_\Omega$ is thus not surprising.

[16] The weights are stochastic but have the same mean.

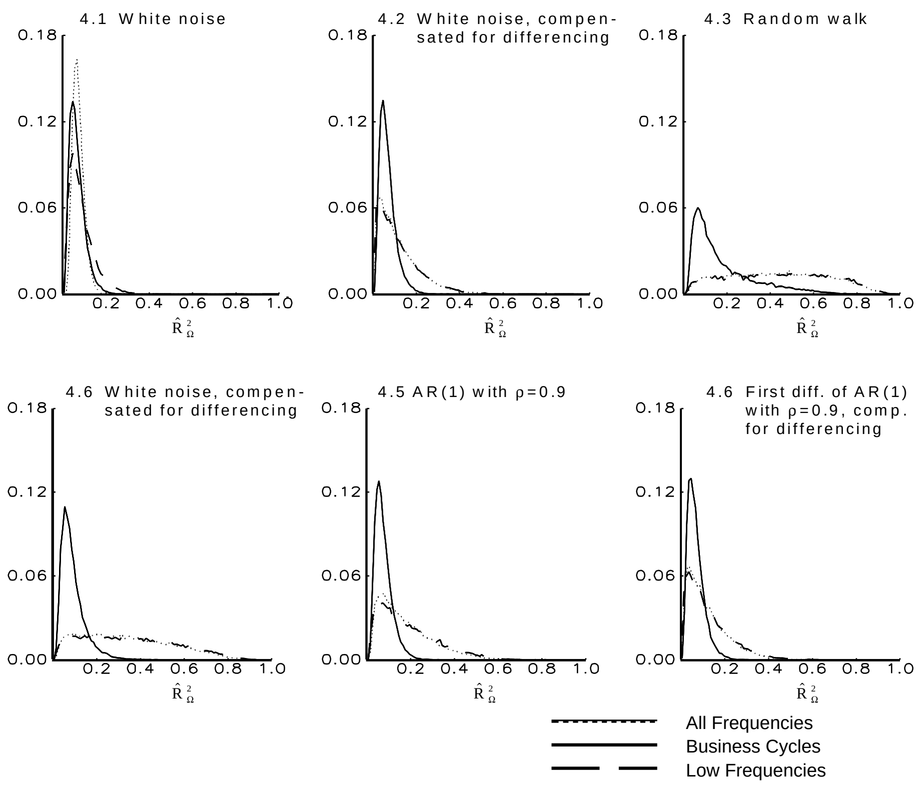
## Table 2 Empirical Distribution of Bivariate

| 128 observations | All frequencies | | | | Business Cycles | | | | Long Periods | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Percentiles | | | | Percentiles | | | | Percentiles | | |
| | mean | 90% | 95% | 99% | mean | 90% | 95% | 99% | mean | 90% | 95% | 99% |
| White noise | 0.071 | 0.106 | 0.120 | 0.147 | 0.066 | 0.114 | 0.134 | 0.180 | 0.086 | 0.162 | 0.197 | 0.263 |
| White noise* | 0.120 | 0.256 | 0.317 | 0.431 | 0.067 | 0.117 | 0.140 | 0.188 | 0.122 | 0.265 | 0.329 | 0.446 |
| Random walk | 0.433 | 0.734 | 0.793 | 0.873 | 0.181 | 0.404 | 0.501 | 0.637 | 0.445 | 0.752 | 0.809 | 0.885 |
| Random walk** | 0.338 | 0.626 | 0.693 | 0.804 | 0.091 | 0.160 | 0.199 | 0.283 | 0.351 | 0.649 | 0.715 | 0.826 |
| AR(1) with correlation 0.9 | 0.173 | 0.343 | 0.405 | 0.526 | 0.075 | 0.126 | 0.147 | 0.197 | 0.186 | 0.376 | 0.441 | 0.572 |
| First difference of AR(1)* | 0.123 | 0.254 | 0.310 | 0.425 | 0.067 | 0.116 | 0.137 | 0.185 | 0.127 | 0.266 | 0.327 | 0.448 |

*Compensated for differencing

**Detrended with a linear time trend

## Figure 5 Simulated Distributions of $\hat{R}^2_\Omega$ for 128 Observations



Over-differencing does not seem to cause any problems in the estimation. The mean is lower and the distribution seems slightly more peaked in the case where we take first differences of an AR(1) process and compensate for the differencing. This supports the conjecture above. The distribution of $\hat{R}^2_\Omega$ for the random walk is extremely wide. We thus clearly want to use the first difference filter if we are unsure of whether the process is I(1) or I(0) with a high degree of persistence.

We also see that the problems with a wide distribution seem to be most severe if $\Omega$ contains low frequencies. For $\Omega$ being business cycle frequencies the distribution of $\hat{R}^2_\Omega$ is very similar for all the processes except the random walk. For low frequencies the distribution, on the other hand, changes a lot when we change the properties of the data generating process. For the random walk and the random walk minus a linear trend the distribution is not far from uniform both for all frequencies and low frequencies. This is a serious problem since we often cannot reject first order integration in macro time series. It is important to note that the influence from the low frequencies clearly dominates that from business cycle frequencies. In Figure 5 the distribution of $\hat{R}^2_\Omega$ for $\Omega$ being all frequencies is indistinguishable from that for $\Omega$ being only low frequencies for all processes except uncompensated white noise.

These results show that it is dangerous not to filter out low frequency fluctuations if we want to study business cycles. Statistical inference may be very inaccurate if low frequencies are included and we miss-specify the characteristics of the underlying processes. A reasonable conjecture is that also other methods of estimating the linear relationship between processes will show the same strong sensitivity to the properties of the underlying processes unless we filter out the low frequencies. Unless we have a strong a priori reason to include low frequencies when studying business cycles, it seems to be safer to study business cycle frequencies and lower frequencies separately. The HP-filter is susceptible to this critique since it may, not only pass low frequencies, but also generate filtered series dominated by low frequencies.

### 3.2.1. Modification of Daniell Window

The Daniell window includes frequencies outside of Ω in the estimation of spectral measures for $\omega \in \Omega$ if ω is close to the endpoints of Ω. In finite samples his may cause a bias in the estimation of $R^2_\Omega$ due to the influence from frequencies outside of Ω. This bias could be reduced if we use a window that only "average" over $\omega \in \Omega$ in the estimation. Unless Ω is too small, this should be feasible. If a priori we believe, or at least don't exclude, that the linear relationship between two processes is qualitatively different at different frequencies, this procedure should reduce the risk of the inference for $\omega \in \Omega$ being contaminated by relationships at other frequencies. In Table 3 I show the result of the same simulation as in Table 2 using a modified Daniell window modified in accordance with

$$\hat{h}(w_s) = \frac{\sum_k l_{s,k} I(w_k)}{\sum_k l_{s,k}}, \qquad \forall w_s \in \Omega$$

$$l_{s,k} = \begin{cases} 1, & \text{if } 2|s-k| \le M \text{ and } w_k \in \Omega \\ 0, & \text{otherwise} \end{cases} \tag{3.12}$$

where $I(w_k)$ denotes the periodogram ordinate for frequency $w_k$. I have set M to 7 so I take an average over the 15 frequencies surrounding that which is estimated, but exclude those of these that are outside of Ω.

**Table 3 Empirical Distribution of Bivariate $\hat{R}^2_\Omega$**

| Modified Daniell Window, 128 observations | Business Cycles Percentiles | | | |
|---|---|---|---|---|
| | mean | 90% | 95% | 99% |
| White noise | 0.080 | 0.140 | 0.165 | 0.225 |
| White noise, compensated for differencing | 0.080 | 0.143 | 0.172 | 0.231 |
| Random walk | 0.191 | 0.426 | 0.518 | 0.662 |
| Random walk detrended with a linear time trend | 0.100 | 0.183 | 0.225 | 0.326 |
| AR(1) with correlation 0.9 | 0.085 | 0.150 | 0.180 | 0.246 |
| First difference of AR(1) compensated for differencing | 0.080 | 0.140 | 0.166 | 0.222 |

One effect of using the modified window is that it reduces the power in tests of zero $\hat{R}^2_\Omega$. This is seen in Table 3 and is explained by the fact that the modification of the Daniell window leads to a reduction in the number of periodogram ordinates involved in the estimation of $\hat{R}^2_\Omega$. On the other hand, if we have positive coherence in the complement of (and close to) $\Omega$, nominal significance levels in tests of zero $\hat{R}^2_\Omega$ based on simulations like that in Table 2 will be overstated in all small samples. This since the positive coherence outside $\Omega$ causes an upward bias in $\hat{R}^2_\Omega$ if $R^2_\Omega = 0$.

### 3.2.2. Introducing Dependence

If we want to quantify the degree of linear comovements it will be of interest to test hypothesis that $\hat{R}^2_\Omega \geq k > 0$. It is likely that the small sample distribution of $\hat{R}^2_\Omega$ in the case where $R^2_\Omega > 0$ will depend on $\mathbf{B}(L)$. Unfortunately this implies that it is necessary to make assumptions about $\mathbf{B}(L)$ in order to make inferences about $R^2_\Omega$. To get some idea about the distribution of $\hat{R}^2_\Omega$ I have anyhow done simulations using the simplest possible linear dependence. In Table 4 I report the result from simulations where the underlying processes are constructed so that

$$\begin{cases} Y_t & = b\,X_t + e_t,\ X_t \perp e_t,\ \forall t \\ h_y(w) & = h_x(w),\ \forall w \end{cases} \tag{3.13}$$

implying that $R_\Omega^2 = b^2$. In the table we see that the power in tests of zero linear comovement is fairly strong.

**Table 4 Empirical Distribution of Bivariate $\hat{R}_\Omega^2$**

| White noise, compensated for differencing, 128 observations | | Business Cycles | | | | | |
|---|---|---|---|---|---|---|---|
| | | Percentiles | | | Prob. of rejection in tests of $R_\Omega^2$= 0 | | |
| Degree of Dependence, $R_\Omega^2$ | mean | 90% | 95% | 99% | 90% | 95% | 99% |
| 0.00 | 0.080 | 0.143 | 0.172 | 0.231 | - | - | - |
| 0.15 | 0.214 | 0.341 | 0.380 | 0.455 | 0.758 | 0.643 | 0.398 |
| 0.30 | 0.348 | 0.487 | 0.522 | 0.595 | 0.978 | 0.957 | 0.863 |
| 0.45 | 0.485 | 0.609 | 0.639 | 0.692 | 1.000 | 1.000 | 0.992 |
| 0.60 | 0.624 | 0.729 | 0.754 | 0.795 | 1.000 | 1.000 | 1.000 |
| 0.75 | 0.763 | 0.837 | 0.852 | 0.875 | 1.000 | 1.000 | 1.000 |

## 3.3. Distribution of Multivariate $\hat{R}_\Omega^2$

As was shown above the bivariate $\hat{R}_\Omega^2$ has a multivariate analog. In Table 5 I show the result from a simulation with three RHS variables. Table 6 and Figure 6 shows the result of multivariate simulations with different degrees of dependence. As in Table 3 and 4, I use a modified Daniell window with 15 included frequencies. For computational reasons the number of draws is, however, in all multivariate simulations reduced to 1000 per series.

Qualitatively, the simulation results are similar to those in the bivariate case. Not surprisingly is the upward bias higher and the distribution slightly less peaked in the multivariate case.

**Figure 6 Simulated Distributions of $\hat{R}^2_\Omega$, White Noise compensated for differencing, 3 RHS Variables**

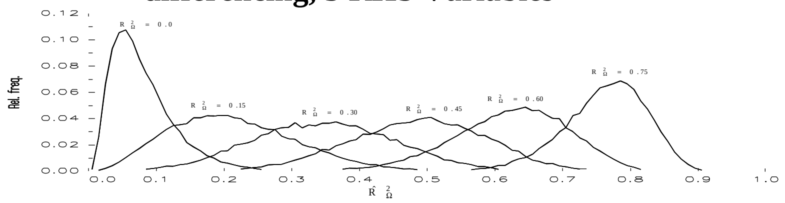



**Table 5 Empirical Distribution of Multivariate $\hat{R}^2_\Omega$, 3 RHS Variables**

| Modified Daniell Window, 128 observations | mean | Percentiles 90% | 95% | 99% |
|---|---|---|---|---|
| White noise | 0.235 | 0.323 | 0.351 | 0.410 |
| White noise, compensated for differencing | 0.237 | 0.330 | 0.364 | 0.424 |
| Random walk | 0.414 | 0.645 | 0.718 | 0.784 |
| Random walk detrended with a linear time trend | 0.319 | 0.464 | 0.520 | 0.623 |
| AR(1) with correlation 0.9 | 0.251 | 0.351 | 0.382 | 0.450 |
| First difference of AR(1) with correlation 0.9 | 0.235 | 0.321 | 0.352 | 0.404 |
| First difference of AR(1), compensated for differencing | 0.244 | 0.342 | 0.367 | 0.437 |

**Table 6 Empirical Distribution of Multivariate $\hat{R}^2_\Omega$**

| White noise, compensated for differencing, 128 observations | 2 RHS variables | | | | 3 RHS variables | | | |
|---|---|---|---|---|---|---|---|---|
| | | Percentiles | | | | Percentiles | | |
| Degree of Dependence, $R^2$ | mean | 90% | 95% | 99% | mean | 90% | 95% | 99% |
| 0 | 0.165 | 0.251 | 0.286 | 0.342 | 0.237 | 0.330 | 0.364 | 0.424 |
| 0.15 | 0.285 | 0.410 | 0.462 | 0.517 | 0.349 | 0.477 | 0.511 | 0.570 |
| 0.30 | 0.403 | 0.536 | 0.569 | 0.618 | 0.458 | 0.576 | 0.620 | 0.672 |
| 0.45 | 0.534 | 0.649 | 0.683 | 0.744 | 0.575 | 0.690 | 0.714 | 0.753 |
| 0.60 | 0.658 | 0.760 | 0.780 | 0.821 | 0.691 | 0.782 | 0.805 | 0.836 |
| 0.75 | 0.787 | 0.855 | 0.871 | 0.896 | 0.802 | 0.865 | 0.880 | 0.901 |

## 3.4. Summary of Section 3

The following points summarize the results and ideas in this section;

- linear comovements may be different at different frequencies, both in terms of degree and structure.
- for typical macro series, statistics of comovement that include all frequencies can be expected to be dominated by the comovement at low frequencies.
- we may define easily interpreted measures of comovement for a specific range of frequencies. These may be thought of as decomposed regression $R^2$.
- the sensitivity of the distribution of such measures to the characteristics of the studied process is substantially reduced if we exclude the low frequencies.

- the establishment of "business cycle facts" should not (at least not without an explicit reason) be done with methods that tend to give statistics dominated by the behavior at low frequencies.

# 4. Application to Swedish Data

## 4.1. Data Set

The time series used in this paper are taken from a data base put together at The Institute for International Economic Studies at Stockholm University. The Swedish series span the period from 1861 to 1988, except for unemployment that start at 1911. The foreign series go back to 1870. Definitions, a full description of the sources and a discussion of the sources and their quality can be found in Hassler et al (1992).

## 4.2. Results

### 4.2.1. Bivariate relations

In Table 7 I report the bivariate business cycle $\hat{R}_{\Omega}^{2}$ between Swedish and foreign macro variables.[17] Black cells represent strong significance (at 99%) and shaded weak (at 90%). I have used the simulated distribution for the case when the underlying processes are random walks that are differenced and the spectral functions compensated for the differencing. In Table 3 we find that the critical levels for these significances are 0.143 and 0.231. We also see that black cells are significant at more than 99% if the process is the autoregressive that I simulated.

The overall impression is one of low degrees of stable, linear relationships between Swedish production related variables and foreign business cycles. The $\hat{R}_{\Omega}^{2}$'s between Swedish GDP and most foreign GDP series, as well as with the index of foreign demand, are low. Only with Norwegian GDP do we get a strongly significant $\hat{R}_{\Omega}^{2}$. This is also the only estimate of comovement between Swedish GDP and foreign variables that is higher than mean simulation value for a true $R_{\Omega}^{2}$ of 15% (as given by Table 4). None of the $\hat{R}_{\Omega}^{2}$

[17] A modified Daniel window with a width of 15 frequencies is used in all what follows.

between manufacturing production and the foreign variables is significantly different from zero.

There is, on the other hand, evidence for substantial linear comovements between many of the Swedish macro variables and business cycle fluctuations in Denmark and Norway, especially Norway. All the Swedish variables I used, except manufacturing production, have a $\hat{R}_{\Omega}^{2}$ with Norwegian GDP that is at least weakly significant. The consumption related measures, private consumption and imports and also exports, have $\hat{R}_{\Omega}^{2}$ significantly larger than 15%. For Denmark six of the variables are significantly comoving, although only one (private consumption) at a degree significantly larger than 15%.

**Table 7 Business Cycle $\hat{R}^2_\Omega$**

| | Y | C | G | I | X | M | MAN | H | U* | FY | UKY | DKY | NOY | GEY | FRY | USY |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Y** | 1.000 | 0.449 | 0.213 | 0.339 | 0.333 | 0.150 | 0.263 | 0.252 | 0.180 | 0.095 | 0.122 | 0.168 | 0.296 | 0.113 | 0.058 | 0.153 |
| **C** | | 1.000 | 0.262 | 0.278 | 0.466 | 0.574 | 0.126 | 0.399 | 0.496 | 0.111 | 0.083 | 0.364 | 0.537 | 0.211 | 0.106 | 0.204 |
| **G** | | | 1.000 | 0.070 | 0.174 | 0.100 | 0.043 | 0.323 | 0.458 | 0.087 | 0.120 | 0.086 | 0.205 | 0.145 | 0.277 | 0.180 |
| **I** | | | | 1.000 | 0.075 | 0.225 | 0.046 | 0.303 | 0.245 | 0.132 | 0.147 | 0.167 | 0.220 | 0.081 | 0.069 | 0.104 |
| **X** | | | | | 1.000 | 0.478 | 0.139 | 0.163 | 0.325 | 0.079 | 0.113 | 0.186 | 0.471 | 0.355 | 0.238 | 0.049 |
| **M** | | | | | | 1.000 | 0.053 | 0.175 | 0.374 | 0.151 | 0.053 | 0.268 | 0.431 | 0.464 | 0.266 | 0.093 |
| **MAN** | | | | | | | 1.000 | 0.176 | 0.122 | 0.085 | 0.061 | 0.042 | 0.134 | 0.046 | 0.044 | 0.132 |
| **H** | | | | | | | | 1.000 | 0.760 | 0.301 | 0.357 | 0.088 | 0.170 | 0.075 | 0.071 | 0.250 |
| **U*** | | | | | | | | | 1.000 | 0.187 | 0.369 | 0.173 | 0.268 | 0.101 | 0.127 | 0.253 |
| **FY** | | | | | | | | | | 1.000 | 0.582 | 0.205 | 0.036 | 0.513 | 0.056 | 0.477 |
| **UKY** | | | | | | | | | | | 1.000 | 0.055 | 0.034 | 0.105 | 0.045 | 0.406 |
| **DKY** | | | | | | | | | | | | 1.000 | 0.185 | 0.246 | 0.255 | 0.126 |
| **NOY** | | | | | | | | | | | | | 1.000 | 0.252 | 0.160 | 0.123 |
| **GEY** | | | | | | | | | | | | | | 1.000 | 0.322 | 0.188 |
| **FRY** | | | | | | | | | | | | | | | 1.000 | 0.103 |
| **USY** | | | | | | | | | | | | | | | | 1.000 |

All variables are in real terms.
Black cells represent significance at 99% and gray at 90%.

| | | |
|---|---|---|
| **Y**= GDP | **C**=Private consumption | **G**=Public consumption |
| **I**=Gross capital formation | **X**=Exports | **I**=Imports |
| **MAN**=Value added in manufacturing and mining | **H**=Hours worked in MAN | **U**=Unemployment |
| **FY**=Index for foreign demand | **UKY**=GDP in Great Britain | **DKY**=GDP in Denmark |
| **NOY**=GDP in Norway | **GEY**=GDP in Germany | **FRY**=GDP in France |

**USY**=GDP in USA

* Only 78 observations

The high degree of comovement that hours worked and unemployment show with GDP in US and Great Britain and also with foreign demand s is puzzling. This since Swedish GDP, as well as manufacturing production, shows very low degrees of comovement with these foreign variables. Hours worked in manufacturing and mining is also more covariant with foreign demand than with manufacturing and mining production itself.

Exports and imports show the highest degrees of comovement with the foreign variables. They are both at least weakly significant for Denmark, Norway, Germany and France. The degrees of comovement with Great Britain, U.S. and foreign demand are on the other hand low for both exports and imports.

### 4.2.2. Multivariate relations

The degree of comovement with a univariate index of foreign demand may be a poor substitute for a broader, multivariate analysis. In Table 8 I report the multivariate $\hat{R}^2_\Omega$ using the same Swedish macro variables as in Table 7 as LHS variables but using two sets of multivariate variables as RHS (Scandinavian and non-Scandinavian countries). Black and gray cells have the same interpretation as in Table 7.

One of the motivations for the separation of linear comovement at different frequencies is, of course, that it may be different at different frequencies. To see whether this is the case I have in Table 8 included columns for $\hat{R}^2_\Omega$ where I let $\Omega$ be all frequencies below the business cycle band, i.e., periods longer than 8 years. I have not compensated for the differencing. This since it is assumed that we are interested in the degree of linear comovement between the innovations to the non-stationary component. To illustrate the behavior of the HP-filter I have also calculated the degree of linear comovement between HP-filtered data, using three different values for $\lambda$. After filtering the series, the degree of linear comovement is calculated, in the case of the HP-filter including all frequencies. As was shown in Section 2 above, the distribution of $\hat{R}^2_\Omega$ is much more sensitive to the characteristics of the underlying process when low frequencies are included. I have thus not indicated significance levels for low frequencies and for the HP-filtered data in Table 8.

Also in the multivariate analysis we find a low degree of comovement at business cycle frequencies for the production related variables. For manufacturing production $\hat{R}^2_\Omega$ is approximately equal to the mean value in the simulations, *given no linear comovement at all.* For GDP, the comovement is significant but not impressingly large. For the second set of foreign variables, it is equal to the average simulation value given a dependence of 15%. The degree of comovement with the Nordic countries appears to be somewhat higher, but not much so. The covariation between investments and Danish and Norwegian GDP is weakly significant.

Exports and imports are clearly covariant with the foreign business cycle. The degree of comovement is high and strongly significant $\hat{R}^2_\Omega$ for exports and the Nordic countries. It is higher than the 99'th percentile given 15% linear dependence and also close to the mean simulation value for 45% dependence. For imports the degree of comovement is even higher. It is significantly higher than 30% and also higher than the mean simulation value for 45%. The same pattern is observed when we use the second set of countries as independent variables.

**Table 8 Multivariate $\hat{R}^2_\Omega$**

| | Low Frequencies[1] | | HP-filtered data[1] | | | | | | Business Cycles[2] | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | λ=1600 | | λ=40 | | λ=1 | | | |
| | Denm. Norway | Germ. Britain US | Denm. Norway | Germ. Britain US | Denm. Norway | Germ. Britain US | Denm. Norway | Germ. Britain US | Denm. Norway | Germ. Britain US |
| GDP | 0.595 | 0.641 | 0.557 | 0.565 | 0.436 | 0.446 | 0.350 | 0.297 | 0.353 | 0.342 |
| Consumption | 0.601 | 0.476 | 0.581 | 0.444 | 0.612 | 0.438 | 0.608 | 0.456 | 0.655 | 0.474 |
| Gov. Cons. | 0.268 | 0.583 | 0.232 | 0.585 | 0.226 | 0.593 | 0.290 | 0.455 | 0.285 | 0.499 |
| Investments | 0.244 | 0.193 | 0.242 | 0.196 | 0.285 | 0.224 | 0.258 | 0.258 | 0.312 | 0.258 |
| Exports | 0.639 | 0.688 | 0.642 | 0.673 | 0.576 | 0.578 | 0.497 | 0.505 | 0.519 | 0.535 |
| Imports | 0.771 | 0.616 | 0.672 | 0.648 | 0.612 | 0.571 | 0.500 | 0.580 | 0.565 | 0.619 |
| Manuf. prod. | 0.306 | 0.561 | 0.275 | 0.476 | 0.207 | 0.415 | 0.158 | 0.235 | 0.158 | 0.258 |
| Hours worked | 0.228 | 0.541 | 0.180 | 0.580 | 0.254 | 0.590 | 0.296 | 0.438 | 0.211 | 0.524 |
| Unemployment | 0.483 | 0.716 | 0.440 | 0.704 | 0.318 | 0.662 | 0.410 | 0.620 | 0.312 | 0.670 |

1. No significance levels indicated.
2. Black cells represent significance at 99% and gray at 90%.

Unemployment and hours worked are also strongly covariant with business cycle fluctuations of GDP in the non-Nordic countries. This is surprising given that hours worked refers to hours worked in manufacturing and mining. On the other hand we see in Table 7 that the bivariate degree of comovement on business cycle frequencies between hours worked and manufacturing production is low. Business cycle fluctuations in manufacturing and mining will thus to a large extent be reflected as changes in measured labor productivity.

Also private and government consumption have significant degrees of comovement with the foreign variables. Especially private consumption and GDP in the Nordic countries are strongly covariant. From Table 7 we find that $\hat{R}^2_\Omega$ between GDP and private consumption is 0.449. This is significantly different from zero. Nevertheless it is low enough to allow for a clearly higher degree of comovement between private consumption and the foreign variables than what we see for GDP. This finding is in accordance with Lundvik (1992) who, within a real business cycle model, also finds a higher degree of foreign influence on consumption than on production.

In the preceding paragraphs we noted that the production related measures are less linearly covariant with the foreign variables than the consumption related are. This is, however, only true for business cycle frequencies. In contrast to the business cycle component, there is a high degree of comovement between the non-stationary (low frequency) component of GDP and the foreign variables. For low frequencies the degree of comovement of GDP with the foreign variables is of the same order of magnitude as that of private consumption and the trade variables.[18]

As discussed above and in Cogley and Nason (1992) the application of the Hodrick-Prescott filter with a sufficiently high λ to difference stationary series produce filtered series possessing spectra with a peak in the low frequency region. The use of such filter on difference stationary (and probably also on highly persistent) series would thus produce measures of linear comovement dominated by the coherence at low frequencies. In Table 8 we clearly see that the results for HP-filtered data are very close to those for the low frequencies when a λ of 1600 is used. In particular the lower degree of comovement for production than for consumption related variables with foreign GDP is not found in this case.

For lower values of λ the results come closer to those for business cycle. For λ = 1 most of the low frequencies are removed, while high frequencies are amplified relative to

[18] This difference is not an artifact of that only the business cycle component is compensated for differencing. The difference remains if the business cycle component is not compensated for differencing. Since the compensating function $\left|(1-e^{-i\omega})\right|^{-}$ is relatively flat in the business cycle frequency band, we wouldn't expect the results to be very dependent on the compensation.

business cycle frequencies. In this case the results do not substantially change the measures of comovement. This, however, could only be checked for by actually removing their influence, using $\hat{R}^2_\Omega$.

**Table 9 Multivariate $\hat{R}^2_\Omega$**

| Last 30 observations censured | Business Cycles[1] | |
|---|---|---|
| | Denm. Norway | Germ. Britain US |
| GDP | 0.470 | 0.454 |
| Consumption | 0.764 | 0.597 |
| Gov. Cons. | 0.388 | 0.544 |
| Investments | 0.327 | 0.307 |
| Exports | 0.588 | 0.635 |
| Imports | 0.593 | 0.698 |
| Manuf. prod. | 0.179 | 0.299 |
| Hours worked | 0.305 | 0.551 |

1. No significance levels indicated.

The possibility of measuring the degree of linear comovement in subsamples is limited by the small sample size. I have anyway performed one test to check that the results at least not are driven by the last decades when Sweden, in several respects, became more open than previously. This is to calculate $\hat{R}^2_\Omega$ for the period from 1861 to 1958, i.e., censuring the last 30 observations. The results are given in Table 9. There we see that the qualitative results are not changed much, when the last three decades are deleted. All $\hat{R}^2_\Omega$ are somewhat higher but their approximate ranking remains, including the higher degrees of comovement for consumption related measures than for production.

# 5. Some Interpretations and Remarks

It is not a controversial statement that the establishment of stylized facts is an important task for the students of business cycles . More debatable are the questions; what characteristics should be measured and which statistical methods should be used to use describe them? The aim of this paper has been to convince the reader that $\hat{R}^2_{\Omega}$ should be included as a standard measure of comovement, taken for granted that comovement is an important characteristic to measure. Lastly I will make some evaluations and interpretations of the results from the application of $\hat{R}^2_{\Omega}$ in the previous section.

An important finding is that business cycles in the production related measures, GDP, value added in manufacturing and mining and investments, have had fairly low degrees of comovement with the GDP of important trading partners of Sweden. Only Norwegian and to some extent Danish GDP seems to swing together with Swedish GDP. However, because of the small size of these countries, it is not unlikely that this comovement reflects dependence where causality runs in the opposite direction, so that Norway and Denmark are dependent on Sweden rather than the opposite. Another possible explanation is that the Nordic countries have been hit by shocks common to them but not to (or as important for) other countries.

It seems likely that the mechanism generating the foreign influence on Swedish business cycles involves (or even mainly goes via) Swedish trade. We find a substantial degree of comovement between both exports and imports and the foreign variables. This gives evidence for an alternative interpretation of the data, namely that the foreign influence on Swedish trade is strong but that the link between Swedish trade and GDP is rather weak. In the data only a third of the Swedish business cycle can be linearly attributed to fluctuations in exports. Other domestic factors seem to be responsible for the lions share of business cycle fluctuations in Swedish GDP.

The difference in the degree of comovement with the foreign variables that we find between production and consumption related variables is also interesting. Consumption seems to be more dependent on the foreign variables than production. A possible explanation comes from the finding in Table 8 that the degree of comovement between Swedish and foreign GDP is highest at frequencies below the business cycle band. Assume that the non-stationary component of GDP has a high degree of international comovement, while the business cycle component is less covariant. Changes in life-time income of the agents will be driven mostly by innovations in the non-stationary component. Assume also that the consumers can (imperfectly) distinguish the non-stationary and the business cycle components. If the non-stationary component has non-negligible spectral density also at business cycle frequencies we may then expect consumption to be more covariant than production at business cycle frequencies.

Hours worked in manufacturing and mining appears to have a substantial degree of comovement with business cycles in the non-Nordic countries. This variable thus shows the same characteristics as the consumption related rather than the production related variables. In fact, business cycles in hours worked in manufacturing and mining have a clearly higher degree of comovement with private consumption than with value added in manufacturing and mining. This finding supports the hypothesis that shocks to labor supply are more important than shocks to labor demand as a driving force behind fluctuations in hours worked. This interpretation is also in line with the finding in Hassler et al (1992) of a negative contemporaneous correlation between hours worked and real wages.

The idea that it may be fruitful to model the economy as consisting of two distinct but interrelated components, one generating long run, non-stationary growth and one generating stationary business cycles, gets some preliminary support by the finding that the relations with foreign variables are different at different frequencies. This difference also warrants caution when using statistics that are some kind of weighted average of all frequencies. This may be important when the Hodrick-Prescott filter is used since it may produce statistics dominated by the behavior at low frequencies. Given an intelligent choice of $\lambda$ and that our studied processes do not have certain high frequency behavior, linear comovement business cycle statistics on HP-filtered data may come quite close to

$\hat{R}^2_\Omega$. On the other hand, this may often not be the case. If we want to establish business cycle stylized facts about linear relations between macro time series, the use of $\hat{R}^2_\Omega$, including only those frequencies we are interested in, thus seems preferable.

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