

Political commitment and loss-aversion*

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April, 2007

Abstract

At least since the work of Kydland and Prescott (1977), it is acknowledged that the ability for policy makers to commit to future policies often is of key importance for outcomes and welfare. An example is capital income taxation, where the commitment solution typically involves zero capital income taxes after some initial periods, while no commitment may imply empirically much too high taxes. A more realistic intermediate outcome can be supported by trigger strategies where a deviation from a "good" equilibrium is punished by a possibly infinite reversion to a "bad" Nash-equilibrium. In a political economy setting, the degree of coordination between voters of different generations required to sustain such an equilibrium is large, arguably unreasonably large. We propose loss-aversion as an alternative explanation for how a commitment-like equilibrium can arise. We set up a politico-economic OLG-model where individuals make investments and dynamically form reference points for future consumption, around which they are loss-averse. Without loss-aversion, the only Markov equilibrium involves 100% taxation of any investments and trigger strategies are required to sustain lower taxes and positive investments. With loss-aversion, we find a Markov equilibrium with positive investments. In contrast to the case of trigger strategies, this equilibrium is renegotiation proof and independent of discounting, surviving also for arbitrarily high rates of discounting.

*We thank seminar participants at IIES, Hebrew University, Fudan University and at the ESSIM/CEPR conference for comments and suggestions.

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1 Introduction

A celebrated result in the literature on optimal taxation is that capital income taxes should be set high at few initial periods and then roughly to zero after that (Judd 1985),(Chamley 1986). This means that governments optimally should rely on other sources of income, like labor income taxes and consumption taxes, to finance their spendings also if these sources generate distortions. Furthermore, in stochastic settings, its shown the capital income tax should be state dependent, responding strongly to productivity shocks in the economy while being approximately zero on average. Other taxes, like labor income taxes, on the other hand, should as in the non-stochastic case be the main source of financing and also be more or less constant (Chari, Christiano and Kehoe 1994). Given these results, the fact that volatile capital income taxes with an average close to zero is not observed in most countries calls for an explanation.¹

The zero capital income tax result has proven surprisingly robust (Chari and Kehoe 1999) but it is well known that one assumption is of key importance – commitment. The optimal sequence of taxes discussed so far is generically time-inconsistent. The reason is that when a planner at some point in time, say time t , analyzes the trade-offs involved in setting some future tax rate τ_{t+s} , she takes into account the distortionary effect this tax has on investments undertaken in the periods in between t and $t + s$. However, as time passes, less investment opportunities are affected by tax τ_{t+s} and to the extent that τ_{t+s} only affects the capital income from investments made prior to time $t + s$, tax τ_{t+s} has no *ex-post* distortionary effect at all at time $t + s$ – it is lump sum.

(Klein and Ríos-Rull 2003) shows that the capital income tax rate chosen by a benevolent planner is very sensitive to the amount of commitment power of the planner. As already note, the tax rate should be approximately zero with commitment but with no commitment at all, a much too high tax rate would result. Hence, empirics seems to suggest limited commitment.

Clearly, the positive and normative importance of commitment is not confined to capital income taxes only. At least since the seminal work by (Kydland and Prescott 1977), time inconsistency and the difficulty to commit to optimal policies has been recognized as a problem inherent in many aspects of policy making. (Kydland and Prescott 1977) discuss several examples where the ability or inability to commit is of keys importance for the chosen policy and for welfare. The examples include; insurance against natural disasters, patent policy and monetary policy in addition to capital income taxation. The ideas in this seminal paper has lead to a very large literature on the importance of commitment.² A key implication of this literature is that the welfare loss of no ability to commit future policy can be high and that positive predictions are very sensitive to the degree of commitment. More specifically, Markov equilibria in models of time-inconsistencies in the vein of (Kydland and Prescott 1977) can be very different from the commitment solution, both in terms of observables and welfare.

Given the key importance of the degree of commitment in standard models , the literature on how commitment is achieved is quite limited. One line of literature assumes that power can be delegated to a person with preferences different from society or the decisive agent. The seminal paper here is (Rogoff 1985). In a sense, however, this explanation begs the question how commitment actually is achieved.

The second explanation is to consider infinite horizon games with multiple equilibria in non-Markovian strategies. By the folk theorem, the commitment solution or at least something close to it can be achieved by the threat of reverting to a bad equilibrium for an extended period of time unless discount rates are too high. The trigger strategies used to achieve the commitment solution in a game with a sequence of different voters require a substantive amount of intergenerational coordination sometimes labeled a "social contract". In particular, to prevent deviations, voters must be confident that future voters will punish

¹(Klein and Ríos-Rull 2003) report a capital income tax rate with an average of 51% and a standard deviation of 0.04% for the operiod 1947-90. However, (Krusell and Ríos-Rull 1999) argue that the capital income tax rates varies substantially between countries.

²A list may begin with (Barro and Gordon 1983),

current deviations from the social contract by coordinating on a "bad" equilibrium without attempting to re-negotiate the contract. Arguably, the amount of coordination within and between generations required to support such equilibria is unrealistically large. There is also some experimental evidence suggesting that even in the much more simple lab-environment, trigger strategies are too difficult to form a basis equilibria better than Markovian (Cabrales, Nagel and Mora 2006).

In this paper we present a new and alternative explanation to how an equilibrium resembling the commitment solution, i.e., involving moderate taxation on sunk investments, can be sustained. In contrast to alternative explanations, the equilibria in our model require no intertemporal coordination being instead Markovian in nature.

Our approach builds on the old idea that people dislike being disappointed. We build on prospect theory, with loss aversion as formalized by (Kahneman and Tversky 1991)). The key ingredient in the theory is that individuals build reference levels for payoffs and if the payoff falls short of the reference level, a utility loss is experienced. It is now well known that standard expected utility theory often fails to explain observed individual behavior and that prospect theory can provide better explanations.³ The accumulating evidence on the empirical relevance of reference dependent utility provides a strong argument for an explorative analysis of the consequence of including such utility in models of political economy. To our knowledge, however, we are the first to introduce loss aversion in dynamic political economy models. We argue that prospect theory can help understanding important issues in politics that are difficult to account for in standard theory. The focus in this paper is on commitment, although other applications, like *status quo* bias and explanations for why some issues become salient in political campaigning and others not, are quite apparent.

Given that our mechanism has not been used before in a dynamic, macro-model of taxation, we believe that it is important to use a transparent model that allows analytical characterization of equilibria. Therefore, we use the simplest dynamic politico-economic framework we could think of to illustrate our point – that loss-aversion can lead to a commitment-like outcome in a model with time-inconsistent preferences over capital income taxes also if no explicit commitment technology exists. We do think, however, that our mechanism may prove to have implications that are partially different from previous explanations allowing empirical tests. We will return to this below.

For simplicity, we assume finite lives and no intergenerational altruism. Specifically, individuals and their investment projects live only for two periods in an overlapping fashion. We assume that the policy maker cares only for currently living individuals, young and old alike. We can interpret the policy maker as a benevolent planner. Alternatively, the policy could be determined as the outcome of a game of probabilistic voting where political candidates compete to get elected in elections where all living individuals vote. For presentational consistency only, we will stick to the latter interpretation throughout the paper.

Due to the OLG structure of the economy, there is a stock of sunk investments in every period. Therefore, there is temptation to use this *ex-post* non-distortionary source of revenue. In fact, we will make sure that this temptation is strong enough so that without loss-aversion, the only Markov-equilibrium being a limit of a finite horizon game is one with 100% taxation. Since the high tax rate is foreseen by rational investors, no investment will be undertaken. Although this is stark, it reflects the finding in the literature discussed above, that no commitment may lead to unreasonably high taxes. However, we will show that loss-aversion creates a form of limited commitment – if individuals expect to get back something from their investments, it becomes costly *ex-post* for the policy maker not to give it, i.e., to disappoint

³(Bateman, Munro, Rhodes, Starmer and Sugden 1997) provides experimental evidence on individual valuation of private goods. (Bowman, Minehart and Rabin 1999) show that the behavior of aggregate consumption deviates from the predictions of standard expected utility in a way consistent with prospect theory. (Quattrone and Tversky 1988) provide evidence on systematic deviations from standard expected utility in voting-like experiments. (Kahneman and Tversky 1991) also argues that the empirical finding that the political incumbent advantage is stronger in good times is evidence in favor of voters having reference dependent utility. The argument is based on the fact that under loss-aversion, individuals are risk-loving in losses. Then, it is better to take a chance with a more risky outsider of equal expected quality if the incumbent gives something below reference for sure.

them.

For an exogenous reference level, our model becomes almost trivial. Allowing a dynamic reference point formation changes this. We will consider two polar cases for how reference points are formed, arguing that the real world features elements of both, albeit not as dichotomous as in our analysis. In the first case, we assume that the reference point is forward looking and satisfying rational expectations about future consumptions levels. A similar assumption on reference point formation is done in (Kőszegi and Rabin 2006). In the opposite case, we assume that the reference point is backward looking and, in particular, (partly) determined by past taxation. Dynamics in the two cases will differ, but interestingly, the steady states are identical.

In the forward looking case, we assume that the reference point of an individual can be anything that is consistent with rational expectations, i.e., that it is in the set of equilibrium outcomes of the political game. Of course, this implies the possibility of a large set of self-fulfilling equilibria. However, we will allow a mechanism whereby politicians can affect future reference points and we believe that we can interpret this as political promises. Governments or political candidates can make promises about their future intentions. If such a promise is incompatible with all future equilibria, it will not be believed and have no impact on the outcome. However, if a promise is consistent with a future equilibrium, we allow individual reference point formation to be coordinated on the promise, affecting future policy outcomes. Empirically, it is an understatement to say that making promises is part of political campaigning. It seems as these promises are far from meaningless but are an important part of policy making, both before elections and after. The right promise can make a candidate win but can also haunt the winning candidate afterwards if it becomes difficult or impossible to fulfill the promise. The strong effect of promises is, however, difficult to understand in standard models since they often seem not be supported by any clear commitment mechanism. Here, however, we provide a foundation for the effects of seemingly empty promises.

In the case of backward looking reference points, a reduction in current tax rates increases reference point consumption and will tend to restrain future taxation. However, this does not necessarily lead benevolent policy makers to reduce taxation today. In fact, if it were an equilibrium policy to restrain taxation today, such a policy must be an equilibrium also next period. Then, it would be better to tax away any sunk investment today, and wait for next period's policy makers to restrain taxation. In other words, policy makers would have an incentive to procrastinate and delay to the following period the cost of committing to a policy. However, since a cut in taxation can beneficially reduce taxation next period, 100% taxation in every period is not an equilibrium either. In fact, the possibility that future policy makers *will not* restrain taxation voluntarily, creates an incentive for policy makers to restrain it today *and vice-versa*. In a sense that will be clear below, forward looking behavior of current policy makers requires the possibility of future political myopia. This strategic substitutability implies oscillations in political strategies when these are required to be pure. Allowing, mixed strategies randomization between forward looking and myopic strategies constitutes a Markov equilibrium with limited taxation in the infinite horizon version of the model. This equilibrium critically hinges on the possibility that future tax rates might be set without consideration of the future.

As already noted, we set up our model so that 100% taxation is the only finite horizon equilibrium. Of course, a large set of other equilibria exists if more elaborate trigger strategies are allowed in the infinite horizon case. Such strategies would require a large, arguably unreasonably large, amount of coordination among voters and across generations to sustain equilibria with lower taxes. Nevertheless a natural, and very important question is whether the hypothesis of loss-aversion can be empirically distinguished from trigger strategies. We argue that is, at least in principle, is possible. To sustain "good" trigger strategy equilibria, it is key that the subjective discount rate is not too large. Under loss-aversion, the rate of discounting is irrelevant for the outcome. Furthermore, trigger strategy equilibria are sustained by threats of reversion to a worse equilibria and the "best" equilibrium is sustained by the threat of reverting to the worst equilibrium for ever. Under loss aversion, these punishment phases are non-existent. It is of course not immediate that this difference can empirically distinguish loss-aversion from trigger strategies since

punishments are out of equilibrium. However, in some stochastic games with incomplete information, punishment phases must occur with positive probability. We think that this may provide a way to test our theory.

The paper is organized as follows: In section 2, we present the model, including the economic environment and preferences. Section 3 describes the determination of taxes. Section 4 and 5 describe respectively, equilibria without and with loss aversion. Section 6 provides some concluding remarks.

2 The model

Our model economy has a two-period OLG structure and in each generation, there are two types of agents; workers and entrepreneurs. There is a unitary mass of identical, atomistic and non-altruistic agents of each type who live for two periods and, consequently, there is in each period a cohort of young and old of both types alive. Workers have a simple private life. We assume they have an exogenous wage in their second period of life, when they also consume. Young entrepreneurs have access to an investment project and individually choose an investment level i_t but are, for simplicity, assumed not to consume until the second period. The investment is costly, incurring an immediate utility cost i_t^2 . In the second period of life, the individual consumption of the representative entrepreneur is denoted c_{t+1} . We normalize the gross expected return on the investment to unity. Given a tax-rate τ_t , an entrepreneur born in period $t - 1$ solves

$$\begin{aligned} U_t &= \max_{c_{t+1}, i_t} -\frac{i_t^2}{2} + \beta u(c_{t+1}; r_{t+1}, i_t), \\ \text{s.t. } c_{t+1} &= i_t(1 - \tau_{t+1}). \end{aligned} \quad (1)$$

u is the utility derived from consumption and $\beta \in (0, 1]$ is the discount factor. The specific properties of u will be discussed below but we note that in order to allow *loss aversion*, we will let u depend not only on the level of private consumption but also on a reference point r_{t+1} and investments in the previous period.

The tax rate, τ_{t+1} is determined in the beginning of period $t+1$, when investments i_t are sunk. Taxes are used for transfers benefitting the workers. The government budget constraint is therefore

$$T_{t+1} = i_t \tau_{t+1}.$$

To simplify, we normalize the private income of workers to zero. The utility of young and old workers in period t is therefore

$$\begin{aligned} V_t^y &= \beta u(d_{t+1}; r_{t+1}, 0), \text{ s.t. } d_{t+1} = T_{t+1}, \\ V_t^o &= u(d_t; r_t, 0), \text{ s.t. } d_t = T_t, \end{aligned}$$

where d_t is the consumption of (the representative) old worker in period t and $i_t = 0$ since workers make no investments.

2.1 Loss aversion

We assume that individuals have *loss aversion*. Following (Kahneman and Tversky 1991)), we note that the key ingredients of loss aversion are that

1. Individuals care strictly more about losses relative to the reference point than about gains – their utility shows first-order risk aversion. Formally, there is an $\varepsilon > 0$ such that

$$\frac{u(r; r, i) - (r - x; r, i)}{x} - \frac{u(r + x; r, i) - u(r)}{x} \geq \varepsilon \forall x > 0.$$

2. Individuals are risk loving in losses in the sense that

$$pu(r - x; r, i) + (1 - p)u(r; r, i) > u(r - px; r, i) \forall x > 0, p \in (0, 1). \quad (2)$$

Since previous work (e.g., (Hassler, Rodríguez Mora, Storesletten and Zilibotti 2003)) has shown that (piecewise) linear utility makes it possible to analytically characterize Markov equilibria in dynamic political economy models, we also want to assume this here. However, we want to stress that loss-aversion is possible to model while restricting utility to be piece-wise linear. Specifically, we assume that

$$u(c_{t+1}; r_{t+1}, i_t) = c_{t+1} - h \cdot I(c_{t+1} < r_{t+1}) i_t \quad (3)$$

Here, $I()$ is an indicator function that is unity if the argument is true and zero otherwise. As (Köszegi and Rabin 2006), we assume that utility is not only a function of the deviation of consumption from the reference point. Specifically, the first term in (3) represents the pure utility of consumption, while the second represents loss-aversion and is a function of consumption relative to the reference level. The parameter $h \geq 0$ measures the degree of loss-aversion and for $h = 0$, utility is linear in consumption with a unitary slope.

An important implication of our preference specification should be noted; According to (3), the utility loss associated with consumption falling below the reference point (in our case, taxes being too high), depends positively on the investment the individual has done. We label this feature *fairness*. In plain words, it implies that an individual that invested little (a lot) and gets fooled in the sense of getting to keep less of the return than implied by the reference point, will feel a smaller (larger) disutility or anger. In particular, our formulation implies that in the limit, as investments and consumption go to zero, the disutility of being taxed too heavily goes to zero.⁴

A less important implication of our specification is that workers, who do not make investments, effectively do not experience any loss-aversion costs. Thus, worker utility is given by $u(d_{t+1}; r_{t+1}, 0) = u(d_{t+1}) = d_{t+1}$. However, this is not important, which we show in 5.3 where we introduce loss aversion also for workers by assuming that workers experience a fixed utility loss if consumption falls short of the reference level.

In Figure 1, we plot u against c , for $h, i > 0$ and a given value of r . We have also included a more "standard" continuous loss-averse utility function (the dotted line).

Clearly, our preference formulation induces *first-order* risk-aversion around the reference point. Second, the preferences imply *risk-loving* behavior for losses – equation (2) is satisfied. Thus, the key implications of loss-aversion are also implications of our preference formulation.⁵

An individual with the preferences specified by (3) is *loss-averse* but riskneutral. We argue that loss-aversion and risk-aversion are quite different concepts and there seems to be no conceptual difficulty in allowing risk-neutral individuals to be loss-averse. A loss averse risk neutral individual do, however, certainly care about risks. In particular, since the individual has first-order risk aversion, she cares a lot about small risks. On the other hand, a mean preserving spread does not change expected utility as long as the probability of a loss is unchanged. That a mean preserving spread necessarily reduce expected utility is a key feature of risk-aversion – thus, we prefer to label these preferences as riskneutral.

Finally, let us comment on the choice of letting loss-aversion operate through a discontinuity at r . We do this for simplicity, believing that our results do not hinge on this assumption. Specifically; fix a small

⁴We considered two alternatives to this assumption. First, one could assume that the loss is proportional to actual consumption c . But then, a tax rate of 100% removes the effects of loss aversion which is unreasonable. The second alternative we considered is to assume that the loss is a constant. As shown below, such an assumption complicates the analysis since it makes the political payoff function non-linear (but affine) in investments. However, we conjecture that such an assumption would produce similar results.

⁵In particular, assumption 1-4 in (Bowman, Minehart and Rabin 1999) are satisfied, except that we have replaced strong concavity by weak above the reference point and that we allowed loss-aversion to operate through a discontinuity at $c - r = 0$.

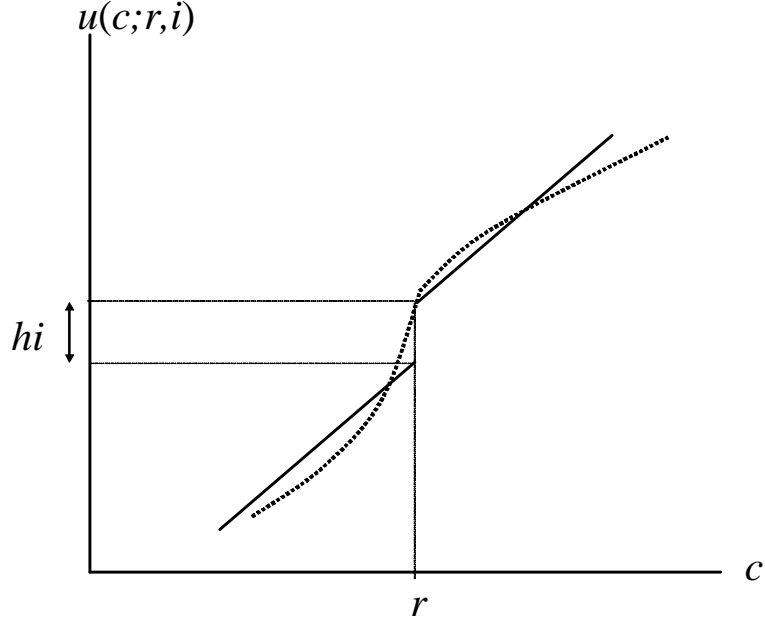


Figure 1: Figure 1. Risk-neutral, loss averse (solid) v.s. "standard" loss-averse utility (dotted).

but strictly positive ε . It then seems safe to conjecture that all our results below would go through also if preferences were piecewise linear, given by the continuous function

$$u(c_t; r_t, i_{t-1}) = \begin{cases} c_t & \text{if } c_t \geq r_t, \\ c_t - h \cdot I(c_t < r_t) i_{t-1} \frac{r_t - c_t}{\varepsilon} & \text{if } c_t - r_t \in (-\varepsilon, 0), \\ c_t - h \cdot I(c_t < r_t) i_{t-1} & \text{if } c_t - r_t \leq -\varepsilon. \end{cases} \quad (4)$$

2.2 Reference points

2.2.1 Reference consumption and investments

Our focus in this paper is on dynamic effects of loss-aversion. Unfortunately, the literature on prospect theory does not share this focus and much is therefore yet to be explored about what drives changes in the reference point over time. Bowman et al. (1999) construct a two-period model in which the first period reference point is exogenous while it in the second period is a weighted average of the first period's reference point and first period consumption. In this way, the author's can vary the degree of history dependence by changing the relative weights on the two determinants of the reference point.

In any case, it seems reasonable that the reference point should be positively affected by the individual's investment level. If an individual invests a lot, we believe that *ceteris paribus*, she expects to be able to consume more and perceives a loss if she is deprived of this. Specifically, we therefore assume that

$$r_{t+1} = i_t (1 - \tau_{t+1}^r), \quad (5)$$

where τ_{t+1}^r is a period t determined reference level for the period $t+1$ tax-rate. Using the budget constraint

$$c_{t+1} = i_t (1 - \tau_{t+1}) \quad (6)$$

we see that

$$I(c_{t+1} < r_{t+1}) \iff I(\tau_{t+1} > \tau_{t+1}^r),$$

i.e., consumption falls below the reference iff taxes are higher than reference taxes. Using this, the private budget constraint (6) and the expression for the reference point (5) in the expression for private consumption utility (3), we get

$$u(c_{t+1}) = i_t (1 - \tau_{t+1} - h \cdot I(\tau_{t+1} > \tau_{t+1}^r)) \quad (7)$$

Before discussing how τ_{t+1}^r is determined, we want to stress that our assumption here makes the theory conceptually different from habit formation. In our model, the reference point is determined by the investment level. Under habit formation, if the habit and the investment level are directly related, the causal effect is rather in opposite direction, namely that the individual invests a lot because the habit for consumption is high. In contrast, (7) implies that an expectation that consumption will not reach reference consumption has a *negative* impact on the investment incentive.

2.2.2 Reference tax dynamics

In a dynamic political context there are reasons to believe the determination of τ_{t+1}^r is backward looking, but there are also reasons to consider the opposite, a forward looking reference point formation. The latter rests on the observation that political promises appear to have a large impact on outcomes and rhymes with the notion that individuals dislike being fooled in the sense of not given what they were promised. On the other hand, the evidence in (Quattrone and Tversky 1988) suggest a *status quo* bias consistent with backward looking reference point formation where precedence and tradition affects level of return an investor feels "entitled" too. We have no reason to discard any of these arguments and we will therefore consider both cases.

The first case is that the reference point is forward-looking and independent of the past. Furthermore, we assume that individuals have rational expectations implying that the reference point for $t+1$ must an equilibrium at $t+1$. The second, polar opposite case, is that the reference point is fully backward looking, namely that τ_{t+1}^r is determined by the current tax rate.

Clearly, forward-looking reference points may imply a multiplicity of equilibria. To shrink the set of equilibria, we assume that the political candidates, or the government, can make an announcement of its intentions for next period's tax rate. We denote the period t announcement of next period's tax rate τ_{t+1}^p and call it a *promise* although no exogenous commitment mechanism prevents the candidates from renegeing on their promises. If the promise is an equilibrium, it is used to form a reference point for consumption. If the promise is not an equilibrium, it has no effect on reference point formation. Since political competition will ensure that promises are made to maximize voter welfare, the economy manage to achieve the best element in the set of equilibria. This interpretation is in line with (Kőszegi and Rabin 2006) who focus on what we call forward looking reference points and defines preferred equilibrium as the one that maximize individual utility.

Our two assumptions are therefore;

Assumption F: Forward-looking reference points – $\tau_{t+1}^r = \tau_{t+1}^p$ if τ_{t+1}^p is in the set of equilibrium tax rates in period $t+1$. Otherwise, τ_{t+1}^r equals some tax-rate in the set of equilibrium tax rates for $t+1$.

Assumption B: Backward-looking reference points – $\tau_{t+1}^r = \tau_t$.

3 Equilibrium definition

As in all politico-economic environment two sets of decisions are taken, private and collective. The private decision is to choose investments. This is done in a decentralized manner – agents are atomistic and maximize private utility for given and rational expectations about future taxes and aggregate investment. Taxes, on the other hand, are chosen collectively in a centralized manner. As we will see below, the equilibrium under loss-aversion will create a link between current and future tax decisions, τ_{t+1} is in equilibrium a function of τ_t . Since future tax rates affects current welfare via private investments, there is an incentive to affect future tax rates. The link between current and future taxes is therefore exploited in the equilibrium under loss-aversion. In other words, tax decisions are taken strategically.

Let us now formally define the private and collective decisions.

Private decision – investment We require that investments are chosen privately rational by young entrepreneurs after observing current tax rates and taking the reference tax rate and the expected future tax rate as given

$$\begin{aligned} i_t &= \arg \max_{i_t} -\frac{i_t^2}{2} + \beta i_t (1 - \tau_{t+1} - h \cdot I(\tau_{t+1} > \tau_{t+1}^r)) \\ &= \max \{0, \beta (1 - \tau_{t+1} - h \cdot I(\tau_{t+1} > \tau_{t+1}^r))\} \end{aligned} \quad (8)$$

Collective decision – taxes. We assume that the political process is such that policies are chosen every period t without commitment in order to maximize a weighted sum of the utility of old and young living individuals. As noted in the introduction, there are two interpretations of our formulation of the collective decision making. It can be read as the outcome of a political process characterized by probabilistic voting, where a weighted sum of voter preferences is maximized in the Nash-equilibrium between the political candidates.⁶ Alternatively it can be read as representing the case of a benevolent planner who chooses taxes to maximize average expected utility of living individuals without commitment. To make the problem interesting, we assume that there is a political incentive to use taxes to transfer resources to the poor workers. We do this by giving an extra weight $\gamma > 0$ to workers. A natural interpretation of this is that poor workers have higher marginal utility than entrepreneurs.⁷ Using the expression for r_t from (5), and the resource constraints,

$$\begin{aligned} d_t &= \tau_t i_{t-1}, \\ d_{t+1} &= \tau_{t+1} i_t, \\ c_t &= i_{t-1} (1 - \tau_t), \\ c_{t+1} &= i_t (1 - \tau_{t+1}). \end{aligned} \quad (9)$$

we can then define the political objective function as

$$\begin{aligned} W_t &= W(\tau_t, \tau_{t+1}^p, i_t, \tau_{t+1}, \tau_{t+1}^r; i_{t-1}, \tau_t) \\ &\equiv u(i_{t-1} (1 - \tau_t); i_{t-1} (1 - \tau_t^r), i_{t-1}) \\ &\quad + (1 + \gamma) u(\tau_t i_{t-1}) - \frac{i_t^2}{2} + \beta u(i_t (1 - \tau_{t+1}); i_t (1 - \tau_{t+1}^r), i_{t-1}) + \beta (1 + \gamma) u(\tau_{t+1} i_t). \end{aligned} \quad (10)$$

⁶In probabilistic voting, two election candidates competes for power by proposing policy platforms. Voters care about policy and also over candidate-specific exogenous traits. The equilibrium election outcome is identical to the case when a planner maximizes a weighted sum of the utility of all voters where the weights depend positively on the tendency of specific voter groups to be "swing-voters". See (Persson and Tabellini 2000) for a text-book description of probabilistic voting.

⁷See (Hassler, Krusell, Storesletten and Zilibotti 2005), where it is assumed that the utility function is piecewise linear and that workers and entrepreneurs are always on a different segments for any $\tau \in [0, 1]$.

In period t , the state variable at the time when policy is chosen is i_{t-1} and τ_t^r and we denote the equilibrium policy function by $\tau_t = \tau(i_{t-1}, \tau_t^r)$. The private agents also observe the current tax rate and the tax-promise when making their investment decision, while i_{t-1}, τ_t^r are not pay-off relevant for the young entrepreneurs. Investments are therefore allowed to depend on current τ_t and τ_{t+1}^p . Our Markov is then defined as follows⁸:

Definition: A Markov equilibrium in our model is a collection of functions $\langle \tau, \tau^p, i \rangle$, such that $\tau_t = \tau(i_{t-1}, \tau_t^r)$, $\tau_{t+1}^p = \tau(i_{t-1}, \tau_t^r)$ and $i_t = i(\tau_t, \tau_{t+1}^p)$, where $\tau : R^+ \otimes [0, 1] \rightarrow [0, 1]$ and $i : [0, 1] \otimes [0, 1] \rightarrow R^+$ satisfying simultaneously

1. $\tau(i_{t-1}, \tau_t^r) = \arg \max_{\tau_t} W(\tau_t, \tau_{t+1}^p, i(\tau_t, \tau_{t+1}^p), \tau(i(\tau_t, \tau_{t+1}^p), \tau_{t+1}^r), \tau_{t+1}^r; i_{t-1}, \tau_t^r)$ s.t.
 - (a) τ_{t+1}^r satisfying **F** or **B**.
2. $\tau^p(i_{t-1}, \tau_t^r) = \arg \max_{\tau_{t+1}^p} W(\tau_t, \tau_{t+1}^p, i(\tau_t, \tau_{t+1}^p), \tau(i_t, \tau_{t+1}^r), \tau_{t+1}^r; i_{t-1}, \tau_t^r)$ s.t.
 - (a) τ_{t+1}^r satisfying **F**.
3. $i(\tau_t, \tau_{t+1}^p) = \max \{0, \beta(1 - \tau_{t+1} - h \cdot I(\tau_{t+1} > \tau_{t+1}^r))\}$, where
 - (a) $\tau_{t+1} = \tau(i_t, \tau_{t+1}^r)$ and
 - (b) τ_{t+1}^r satisfying **F** or **B**.

We should note that point 1 and 2 implies that the policy maker explicitly takes into account that the choice of τ_t and τ_{t+1}^p can affect next periods taxes since $\tau_{t+1} = \tau(i(\tau_t, \tau_{t+1}^p), \tau_{t+1}^r)$. The private choice in point 3, on the other hand, is taken for a given τ_{t+1} and 3.a requires that this value satisfies rational expectations. Of course, under assumption **B**, τ^p is not payoff relevant so $i(\tau_t, \tau_{t+1}^p)$ should be independent of τ_{t+1}^p and 3. is therefore superfluous.

4 Equilibria without loss aversion

Before characterizing the Markov equilibria with loss-aversion, we will in this section provide some benchmarks. To this end, we assume away loss-aversion by setting $h = 0$. We start with the commitment case and then derive results that characterize the set of equilibria under non-Markovian (trigger) strategies.

4.1 Taxes under commitment

Let us now find the sequence of tax rates that maximize political welfare if there is commitment. First, substituting the investment choice (8) into the political objective setting $h = 0$, gives

$$W_t = i_{t-1}(1 + \tau_t \gamma) - \frac{(\beta(1 - \tau_{t+1}))^2}{2} + \beta^2(1 - \tau_{t+1})(1 + \tau_{t+1} \gamma)$$

Clearly, at the time of planning, the current tax τ_t should be set to its maximum (unity) since welfare is linear in τ_t with a slope γ . Solving the first order condition for τ_{t+1} yields the optimal next period tax under commitment,

$$\tau_{t+1} = \frac{\gamma}{1 + 2\gamma} \equiv \tau_c.$$

⁸We let the definition be general enough to encompass both assumptions **F** and **B** although, under assumption **F**, τ_t investments should not depend on current taxes while under assumption **B**, τ_{t+1}^p is irrelevant. Furthermore, although i_{t-1} clearly is a payoff relevant variable, it will turn out that it will not matter for the choice of τ_t since the payoff is linear in i_{t-1} .

If, in period, $t + 1$, we let a new planner set τ_{t+2} , she would set it to τ_c but would like to change τ_{t+1} to unity since i_t is then sunk.

4.2 Equilibrium without commitment

We will consider both the finite and the infinite horizon case. In the former, we assume that no young is born in the final period T , so when $h = 0$, the payoff in the final period is simply

$$W_T = i_{T-1}(1 - \tau_T) + (1 + \gamma)i_{T-1}\tau_T = i_{T-1}(1 + \gamma\tau_T). \quad (11)$$

Obviously, this is maximized by $\tau_T = 1$, implying that in period $i_{T-1} = 0$ and the following proposition immediately follows:

Proposition 1 *If $h = 0$, the only finite horizon equilibrium feature $i_t = 0$ and $\tau_t = 1$ for all t . Clearly, this is only the only infinite horizon Markov equilibrium that is a limit of a finite horizon equilibrium.*

4.3 Non-Markovian equilibria without loss-aversion

In this paper, we argue that loss-aversion can provide a type of limited commitment. The standard alternative explanation for why economies might avoid bad equilibria like the one in proposition 1, is that trigger strategies could be used. In order to analyze whether loss aversion and trigger strategies are observationally equivalent it is of interest to look at a broader class of equilibria in the case when $h = 0$ and when the horizon is infinite. Although the set of equilibria of course is large, we can use the method of (Abreu, Pearce and Stacchetti 1990) to characterize the *best* equilibrium, i.e., the one that maximizes the welfare of the living individuals. We do this by first finding the *worst* equilibrium. Clearly, this is the finite horizon equilibrium with zero investments. Using this equilibrium as "punishment", we can sustain better equilibria than the one with zero investments.

Suppose $\hat{\tau}$ is a steady state equilibrium with an associated investment level $i = \beta(1 - \hat{\tau})$.⁹ Using this in (10) we calculated the political payoff as

$$\beta(1 - \hat{\tau})(1 + \hat{\tau}\gamma) - \frac{(\beta(1 - \hat{\tau}))^2}{2} + \beta^2(1 - \hat{\tau})(1 + \hat{\tau}\gamma).$$

Now, the best deviation from this is to set $\tau_t = 1$, realizing that in period $t + 1$ the finite horizon equilibrium will prevail as punishment for the deviation. The deviation yields a payoff

$$\beta(1 - \hat{\tau})(1 + \gamma),$$

since investment in the current period will be zero.

The difference between the steady state payoff and the deviation, i.e., the value of not deviating, is $\beta(1 - \hat{\tau})\left((1 + \beta)(1 + \gamma\hat{\tau}) - \frac{\beta(1 - \hat{\tau})}{2} - (1 + \gamma)\right)$, which is positive for $\hat{\tau}$ sufficiently close to unity. Therefore, tax rates smaller than unity can be sustained by the threat of reverting to the finite horizon equilibrium. However, τ_c might not necessarily be achievable. Specifically, setting $\hat{\tau} = \tau_c$ yields a negative value of not deviating if $\beta < \frac{2\gamma}{1 + 2\gamma}$. If this is the case, the best equilibrium implies setting τ so that the value of deviating is zero, giving

$$\hat{\tau} = \frac{2\gamma - \beta}{\beta + 2\gamma(1 + \beta)},$$

which is decreasing in β and approach unity as $\beta \rightarrow 0$.

We conclude:

⁹We have abstained from dynamic considerations here, although we believe these are straightforward.

Proposition 2 *If $h = 0$, the best steady state infinite horizon equilibrium feature*

$$\tau_t = \tau_b = \max \left\{ \frac{2\gamma - \beta}{\beta + 2\gamma(1 + \beta)}, \frac{\gamma}{1 + 2\gamma} \right\} \forall t.$$

Corollary 3 *If $\beta < \frac{2\gamma}{1+2\gamma}$, $\tau_b < \tau_c$.*

In the next subsections, we consider equilibria under loss-aversion, first under assumption **F** and then **B**.

5 Equilibria with loss-aversion

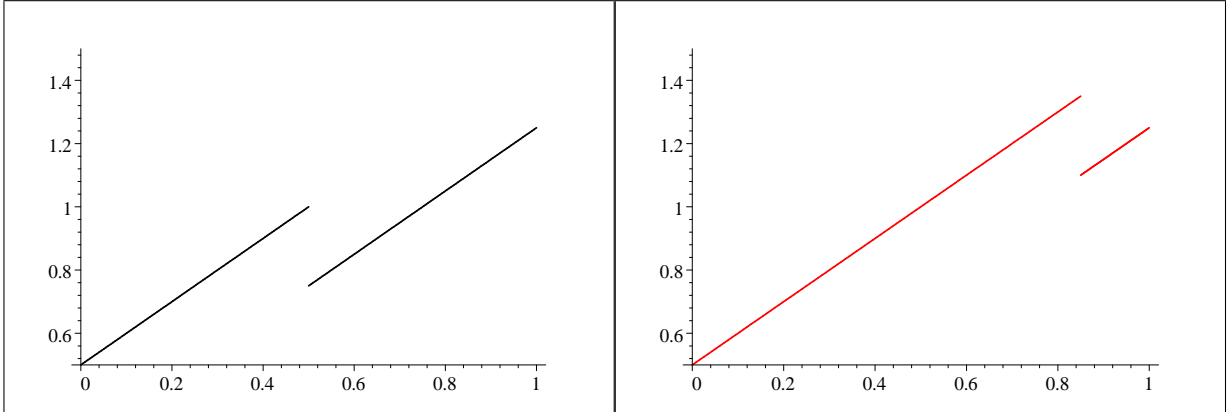
Now, we will characterize Markov equilibria under loss-aversion, i.e., when $h > 0$. To this end, we solve for the equilibrium using backward induction, assuming a finite horizon, then analyzing what happens in the limit as the time-horizon goes to infinity. As we will see, this last step is trivial since only the near future affects the equilibrium decisions.

5.1 Forward-looking references

Consider first now case **F**, i.e., when the reference point is forward-looking and independent of the past. In parallel to (11), the political objective in the final period is

$$\begin{aligned} W_T &= i_{T-1}(1 - \tau_T) + (1 + \gamma)i_{T-1}\tau_T - h \cdot I(\tau_T > \tau_T^r) i_{T-1} \\ &= i_{T-1}(1 + \gamma\tau_T - hI(\tau_T > \tau_T^r)). \end{aligned}$$

Here, we note that the political objective is increasing in τ_T , but has a downward discontinuity at $\tau_T = \tau_T^r$. In figure 2, we plot W_t for $i_{T-1} = \frac{1}{2}$ and $\gamma = 2$. In the left panel, we have set the reference tax $\tau_T^r = \frac{1}{2}$ and in the right to 0.85. Clearly, in the left panel the political objective is maximized at $\tau_T = 1$ while in the right panel, $\tau_T = \tau_T^r$ is the optimal choice.



More generally, if the reference tax is low enough, it is worth taking the loss-aversion cost of making the return to investments below the reference level, i.e., setting taxes above the reference tax. If, on the other hand, the reference tax is high, it is not worth it. We therefore define the threshold value such that the policymaker is indifferent between setting the tax to the reference level and setting it to unity. This threshold, which will turn out to be of key importance is given by

$$\tau^* \equiv 1 - \frac{h}{\gamma}.$$

Now, in the final period, we have

$$\tau_T = \arg \max_{\tau_T} W_T = \begin{cases} \tau_T^r & \text{if } \tau_T^r \geq \tau^*, \\ 1 & \text{else,} \end{cases} \quad (12)$$

independently of period $T-1$ investments. Consequently, any announced tax rate at or above τ^* is believed and will under assumption **F** form the reference point for consumption. Thus,

$$\tau_T^r = \begin{cases} \tau_T^p & \text{if } \tau_T^p \geq \tau^*, \\ \bar{\tau} & \text{else.} \end{cases} \quad (13)$$

where $\bar{\tau} \in [\tau^*, 1]$ is the (out of equilibrium) belief if $\tau_T^p < \tau^*$ and investments are given by

$$i_{T-1} = \begin{cases} \beta (1 - \tau_T^p) & \text{if } \tau_T^p \geq \tau^*, \\ \beta (1 - \bar{\tau}) & \text{else.} \end{cases} = i_{T-1}(\tau_T^p)$$

The period $T-1$ payoff is maximized by

$$\begin{aligned} \tau^p &= \max\{\tau_c, \tau^*\} \\ \tau_{T-1} &= \begin{cases} \tau_{T-1}^p & \text{if } \tau_{T-1}^p \geq \tau^*, \\ 1 & \text{else.} \end{cases} \end{aligned}$$

Note that if $\bar{\tau} > \max\{\tau_c, \tau^*\}$, the choosing the promise $\tau^p = \max\{\tau_c, \tau^*\}$ is a strictly dominating strategy for the candidates.

Continuing by backward induction we derive the following proposition.

Proposition 4 *Under assumption **F**, there is a unique equilibrium in the finite horizon case. This equilibrium is also a Markov equilibrium in the infinite horizon and features*

$$\begin{aligned} \tau_t &= \tau(i_{t-1}, \tau_t^r) = \begin{cases} \tau_t^r & \text{if } \tau_t^r \geq \tau^*, \\ 1 & \text{else,} \end{cases} \quad \forall i_{t-1}, \\ \tau_{t+1}^p &= \tau^p(i_{t-1}, \tau_t^r) = \max\{\tau_c, \tau^*\} \vee \{i_{t-1}, \tau_t^r\}, \\ i_t &= i(\tau_t, \tau_{t+1}^p) = \begin{cases} \beta (1 - \tau_{t+1}^p) & \text{if } \tau_{t+1}^p \geq \tau^*, \\ \beta (1 - \bar{\tau}) & \text{else} \end{cases} \quad \forall \tau_t, \end{aligned}$$

where $\bar{\tau} \in [\tau^*, 1]$ is the out-of-equilibrium belief. Starting from any i_0, τ_1^p , the equilibrium tax-rate is $\max\{\tau_c, \tau^*\} \forall t > 1$.

We should note here that the equilibrium under loss-aversion is independent of β , while this is not the case when there is no loss-aversion. In particular, the tax-rate in the best equilibrium is weakly negative in β and strictly negative for $\beta < \frac{2\gamma}{1+2\gamma}$. This implies that with some loss-aversion and a low enough discount factor, the economy can reach a better equilibrium even if is restricted to Markov strategies than without loss aversion but with no restriction on the strategy space. We believe that this provides a way of empirically distinguishing loss-aversion from reputation.

5.2 Backward-looking references

Case **B**, when the reference point is backward looking is slightly more complicated to analyze since it, for obvious reasons, tends to generate more dynamics. As we will see, however, the equilibrium is tractable for at least an interesting subset of the parameter range.

We first, we analyze the finite horizon equilibrium. Recall that in this case, reference taxes are backward-looking ($\tau_t^r = \tau_{t-1}$) and therefore, the choice of τ_t will generally depend non-trivially on τ_{t-1} .

In the final period T , the payoff is also in this case given by (12), but now $\tau_T^r = \tau_{T-1}$, rather than given by (13). Knowing this, individuals in period $T - 1$ choose

$$i_{T-1} = \begin{cases} \beta(1 - \tau_{T-1}) & \text{if } \tau_{T-1} \geq \tau^*, \\ 0 & \text{else,} \end{cases}$$

where we note that i_{T-1} is a strictly negative function of τ_{T-1} in the range $\tau_{T-1} \in [\tau^*, 1]$.

Consider now period $T - 1$. Clearly, a motive to restrain current taxation has now arisen since by reducing τ_{T-1} from unity, current investments increase from 0 as τ_{T-1} is reduced from zero. Specifically, while a tax-rate of unity would maximize the value of consumption of public and private goods in the current period, the welfare of the young able could be enhanced to the extent that restraining current taxes constrains next periods taxes. The important difference here compared to the analysis under forward-looking reference points is that now, there is an up-front cost of restricting next periods taxes since it means that the current stock of inelastic capital cannot be fully exploited.

In the somewhat uninteresting case, $\frac{h}{\gamma} > 1$, i.e., when full commitment can be achieved, either τ_{T-1} satisfies an interior first-order condition or is set equal to τ_{T-2} (see appendix for details, TBW). We will focus on a more interesting case, when loss aversion is not strong enough to completely "bind the hands" of the period T political decision maker.

In contrast the forward-looking case, commitment is now costly; In order to restrict next periods tax rate, the current tax-rate also has to be reduced since the constraint on next periods taxes is that it is costly to set them higher than the current tax rate. Because of this, a natural comparison in the current case is the tax rate would be chosen if it would apply today and forever thereafter. It is straightforward to show that this is

$$\tau_f \equiv 1 - \frac{\beta(1 + \gamma)}{\beta(1 + \gamma) + \gamma(1 + \beta)}.$$

$\tau_f > \tau_c$ since there is a current cost in terms of reducing transfers to the poor by restricting taxes.

In what follows, we will restrict attention to the case when loss-aversion provides limited commitment in the sense that $\tau^* \geq \tau_f$. In other words, loss-aversion at the most makes it possible to achieve τ_f .

Assumption h $\tau^* \geq \tau_f$.

As in the case of forward looking reference points we proceed by backward induction. In the appendix we show that ;

Lemma 5 *Under assumptions h and B, $T_{T-1} = \tau^* \forall \tau_{T-2}, i_{T-2}$*

The intuition for this result can be presented as follows. First, under assumption **h**, welfare falls in τ_{T-1} in the whole range $\tau_{T-1} \in [\tau^*, 1]$, provided i_{T-2} is done rationally, i.e., where a reduction in the current tax rate thus increases investment. In other words, h is not large enough to make it possible for the period $T - 1$ government to achieve its most preferred tax, given that it could set the same tax in the current and next period. Thus, the equilibrium tax rate in period $T - 1$ cannot be larger than τ^* .

Second, under criterion assumption **h**, h is small enough for the government in period $T - 1$ always to prefer to increase the tax rate to τ^* , if $\tau_{T-2} < \tau^*$, recognizing that this entails a cost due to loss-aversion and that if $\tau_{T-1} < \tau^*$, $I_{T-1} = 0$. Note that the fact that $i_{T-1}(\tau_{T-1})$ is discontinuous at $\tau_{T-1} = \tau^*$, is the reason for why it is optimal to increase taxes to τ^* also if τ_{T-2} is arbitrarily close to τ^* . If τ_{T-1} were to be set strictly below τ^* , individuals would know that in period T , the temptation to set $\tau_T = 1$, would not be resisted and therefore, $i_{T-1} = 0$, for all $\tau_{T-1} < \tau^*$. For this reason, the equilibrium policy in period $T - 1$ cannot be to set $\tau_{T-1} < \tau^*$.

Since we have established that first equilibrium $\tau_{T-1} \leq \tau^*$ and second, equilibrium τ_{T-1} cannot be smaller than τ^* , the equilibrium policy in $T - 1$ is clearly pinned down to $\tau_{T-1} = \tau^*$, independently of τ_{T-2} . In other words, the government in period $T - 2$ cannot affect the period $T - 1$ tax-rate. Therefore, the the problem of the period $T - 2$ government is simply to maximize *current pay-off*, i.e., they face an

identical problem as the period T government. Consequently, $T_T(\tau_{T-1})$ is optimal also in period $T - 2$. By continuing this induction, we establish:

Proposition 6 *Under assumption h , the only finite horizon equilibrium features*

$$\tau_{T-s} = \begin{cases} \tau_E(\tau_{T-s-1}) \equiv \begin{cases} 1 & \text{if } \tau_{T-s-1} < \tau^*, \\ \tau_{T-s-1} & \text{if } \tau_{T-s-1} \geq \tau^*, \end{cases} & \text{and } s \text{ is even.} \\ \tau_O \equiv \tau^* & \text{if } s \text{ is odd.} \end{cases}$$

This equilibrium involves an oscillation between forward-looking strategic behavior (the odd strategy τ_O) and complete "myopic" behavior, constrained by the previous tax rate (the even strategy τ_E). It is clear that these oscillations are key to the existence of the equilibrium. To see this, note that if a government (in period t) expects next government to behave strategically, by limiting τ_{t+1} in order to constrain later taxes, there is no need to be strategic already in period t . On the contrary, it is in this case superior to *procastinate* and make the myopically optimal decision today – the expectation of future strategic behavior, eliminates the need to be strategic today. Correspondingly, the expectation about future governments to behave myopically, creates an incentive to act strategically already in the current period. As we will discuss more below, we believe that this interaction between myopic and strategic behavior is necessary whenever commitment entails a short run-cost.

We should also note that although the tax policies must oscillate in equilibrium, the actual tax-rate does not. In fact, the tax-rate is constant at τ^* after the first period.

5.3 Loss aversion for workers

Before turning to the case of mixed strategies, let us briefly analyze the case when there is loss aversion also among workers. We focus on the case of forward looking references points and assume that the utility of old workers is

$$u(d_t; r_t) = d_t - w \cdot I(d_{t+1} < r_t^w)$$

where r_t^w is the reference level for worker consumption. As for entrepreneurial consumption, we assume the government in period $t - 1$ can make a "promise" for worker consumption, denoted d_t^p which becomes the reference for d_t if the promise is an equilibrium, otherwise it is zero.

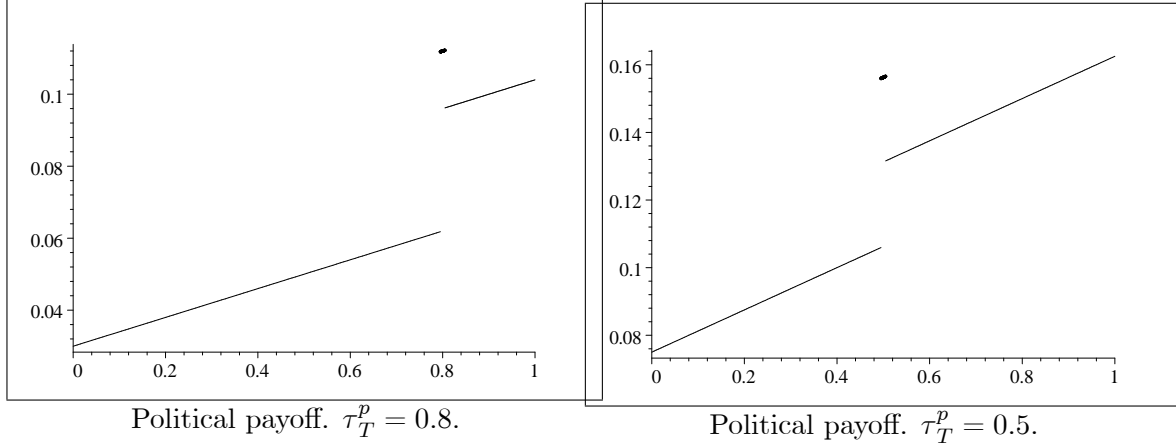
The final period political payoff is now

$$\begin{aligned} W_T &= i_{T-1}(1 - \tau_T) + (1 + \gamma)i_{T-1}\tau_T - h \cdot I(\tau_T > \tau_T^r) i_{T-1} - w \cdot I(d_T < r_T^w) \\ &= i_{T-1}(1 + \gamma\tau_T - h \cdot I(\tau_T > \tau_T^r)) - w \cdot I(i_{T-1}\tau_T - d_T^p) \end{aligned}$$

Except at the points of discontinuity, the payoff is increasing in τ_T . Furthermore, at the new point of discontinuity, $\tau_T = \frac{d_T^p}{i_{T-1}}$, the payoff jumps *upwards*. Therefore, the only possible equilibrium tax rates are $\tau_T = \tau_T^r$ and $\tau_T = 1$, exactly as in the case of loss-aversion only in private consumption. Furthermore, as the government budget constraint implies that $d_t = i_{T-1}\tau_T$, promises not equal equilibrium investments times equilibrium taxes rates will not be believed. Clearly, if the equilibrium tax rate is unity, investments will be zero and no positive promise on d_T will be believed.

It is easy to see that all promises $\tau_T^p \geq \tau^*$ with $d_T^p = \tau_T^p \beta (1 - \tau_T^p)$ are believed and will be self-enforced. The political objective in this case is depicted in the left panel of Figure 4.¹⁰ At the promised tax rate (0.8 in the figure), there is a spike. The upward discontinuity is due to that d_T is lower than the promise to for $\tau_T < \tau_T^p$ and the downward discontinuity is due to the loss associated with taxes above the reference point.

¹⁰Parameters are $\beta = 0.5$, $\gamma = 0.5$, $h = 0.2$, $g = 0.05$.



If, instead $\tau_T^p < \tau^*$ and $d_T^p = \tau_T^p \beta (1 - \tau_T^p)$, the political payoff looks like in the right panel of figure 4. Clearly, the temptation to raise taxes above the promise is now too large, precluding that $\tau_T^p < \tau^*$ with $d_T^p = \tau_T^p \beta (1 - \tau_T^p)$ is an equilibrium. We therefore conclude that the final period equilibrium is independent of w . It is then immediate that also the equilibria in preceding periods is independent of w .

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5.4 Markov equilibrium under an infinite horizon

First, we note extending the horizon backwards to infinity, the equilibrium described in proposition 6 does obviously not converge to a Markov-equilibrium in pure strategies. However, the logic behind the finite-horizon equilibrium – that expectation of future myopia breeds strategic behavior and *vice versa* – suggests the existence of a Markov equilibrium in mixed strategies in an infinite horizon game *without a fixed endpoint*.. We may thus conjecture that if next periods government mixes between a myopic and a strategic policy with the right probabilities, we could make the current government indifferent between the same two policies. It turns out that this conjecture is correct and we can establish the following proposition.¹²

Proposition 7 *Under assumption h, a Markov equilibrium exists with the following characteristics.*

$$\tau_t = \tau(i_{t-1}, \tau_t^r) = \begin{cases} \tau_e(\tau_t^r) & \text{with probability } 1 - p(\tau_t^r), \\ \tau_o & \text{with probability } p(\tau_t^r), \end{cases} \quad \text{for any } i_{t-1},$$

$$\tau_t^r = \tau_{t-1},$$

$$i(\tau_t, \tau_{t+1}^p) = \begin{cases} 0 & \text{if } \tau_t < \tau^* \\ \beta(1 - \tau_t + p(\tau_{t-1})(\tau_t - \tau^*)) & \text{if } \tau_t \geq \tau^* \end{cases} \quad \text{for any } \tau_{t+1}^p$$

with $i'(\tau_t) < 0 \forall \tau_t > \tau^*$ and where

$$p(\tau_t^r) = \begin{cases} 1 & \text{for } \tau_t^r = 1, \\ \frac{p_1 + \frac{1}{2} \sqrt{((2p_1)^2 - 4p_2(2(p_1 + \gamma h) - p_2))}}{p_2} & \text{for } 1 > \tau_t^r > \tau^* \\ < \gamma & \text{for } \tau_t^r < \tau^* \end{cases},$$

¹¹THE ANALYSIS FOR THE BACKWARD LOOKING CASE REMAINS.

¹²We extend the equilibrium definition in a straightforward way to allow for mixed strategies. Since marginal utilities are constant almost everywhere, optimal choices are also immediate to derive.

for

$$\begin{aligned} p_1 &= \gamma(\gamma(1+\beta)(1-\tau) - \beta\tau(1+\gamma) - h) \text{ and} \\ p_2 &= \beta(\gamma(1-\tau) - h)(1+2\gamma). \end{aligned}$$

Starting from any $\tau_0 \in [0, 1]$ and $i_0 = i(\tau_0)$, the equilibrium tax-rate converges with probability 1 to τ^* .

Sketch of Proof: We begin by showing that if $\tau_{t-1} = \tau^*$, it is optimal to set $\tau_t = 1 - \frac{h}{\gamma}$, which is easy since both pure strategies prescribe this. Then, we note that if $\tau_{t-1} < \tau^*$, $i_{t-1} = 0$ so for the government to want to set $\tau_t = 1$, it has to be that this does not affect the future negatively, implying $p(1) = 1$. Finally, we solve for the function $p(\tau)$ such that for any $\tau_{t-1} \in (\tau^*, 1)$, the period t government is indifferent between the two pure strategies. Details in the appendix.

As an illustration, let us draw the pay-off to a period t government as a function of its choice of τ_t , given $\tau_{t-1} = 0.6$, when we set $h = .1, \gamma = .2, \beta = .75$. We show the value of the political objective function $W(\tau_t; \tau_{t-1}, i_{t-1})$ for all $\tau \in [0, 1]$ in Figure 2.

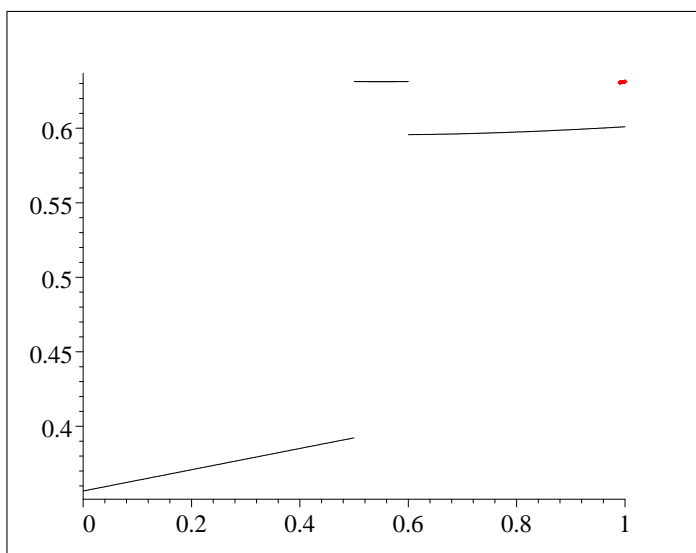


Figure 2. Political payoff against the choice of the current tax rate τ_t for $\tau_{t-1} = 0.6$.

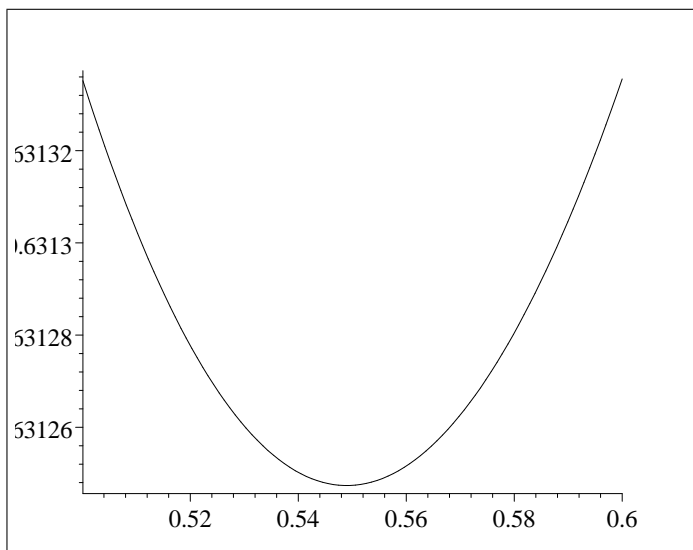


Figure 3. Political payoff against the choice of the current tax rate τ_t for $\tau_{t-1} = 0.6$.

While the function looks almost flat for the segment $\tau \in [\tau^*, \tau_{t-1}]$, a closer inspection, shown in Figure 3, illustrates that this is not the case. The government is indifferent between τ^* and τ_{t-1} and strictly prefers these values to any other $\tau \in [0, 1]$. The $p(\tau)$ function is plotted in Figure 4.

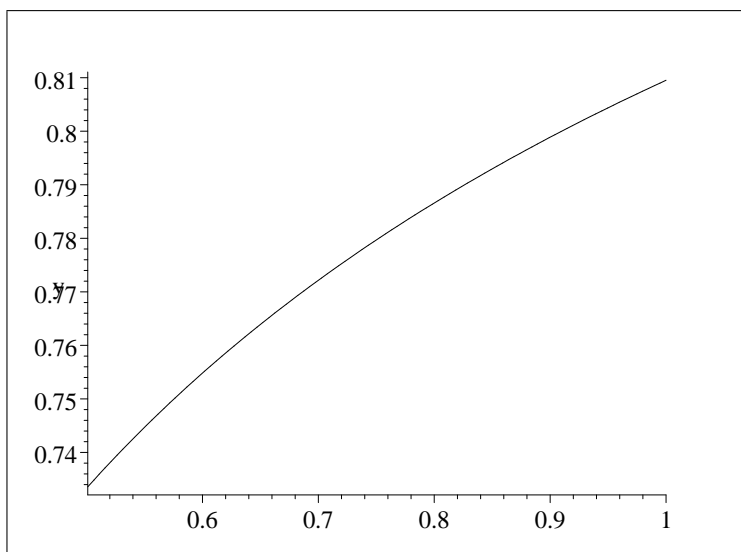


Figure 4. Mixing probability on τ^* .

6 Concluding remarks

We have in this paper provided a theory for how prospect theory can be used in political economy. In particular we want to emphasize the interpretation that people dislike being disappointed and that this can help mitigate commitment problems that otherwise could have severe implications for society. Our model is simple and stylized in order to achieve analytical tractability. However, when a new economic mechanism is analyzed, analytical transparency appears valuable. In future work, we plan to develop

the model, including, in particular, stochastic elements. We believe that the stochastic properties of the model can help distinguish this theory from alternative explanations for how commitment problems are overcome. With stochastics, promises will sometimes be broken and utility losses incurred. Under trigger strategies, such events may in some circumstances of asymmetric information lead to a switch to a more worse equilibrium. In our model, such a switch should not occur and history might not matter much at all. Therefore, the strong history dependence might be a way to distinguish the mechanisms, at least under forward looking reference point formation in which case the history is irrelevant.

Another issue is that we would like to analyze political competition involving an agency problem between voters and political executives. We have assumed away this by assuming probabilistic voting, which essentially means that tax rates are determined at the election date. In practice, political promises are, of course, most often seen in cases where a political candidate makes promises about what to do *after* being elected. We think similar mechanisms as the ones analyzed in this paper, may help politicians to make such promises credible.

Furthermore, we believe that prospect theory can be used to analyze how different issues become salient in political campaigns. Suppose that preferences are such that reference points can arise for different variables, specific types of income or transfers or specific goods. Clearly, this is in line with the experimental evidence for loss aversion since what matters there is certainly not the individuals aggregate income or consumption. Political parties may then have different incentives in affecting reference points in different political dimensions. Suppose for example one party has an advantage in providing a particular public good. That party should then have an incentive to make the provision of this good the salient issue in the elections. Establishing a high reference levels for that public good can be a way to achieve this. Other parties should have an incentive to prevent this and instead establish reference levels in other dimensions. We leave these issues for future work.

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7 Appendix

7.1 Proof of Lemma 5

The payoff in period $T - 1$ is

$$W_{T-1} = i_{T-2}(1 + \gamma\tau_{T-1} - h(\tau_{T-1} > \tau_{T-2})) \\ + \beta(i_{T-1}(\tau_{T-1})(1 + \gamma T_T(\tau_{T-1}) - h(T_T(\tau_{T-1}) > \tau_{T-1}))) - \frac{i_{T-1}(\tau_{T-1})^2}{2}.$$

This is

$$i_{T-2}(1 + \gamma\tau_{T-1} - h(\tau_{T-1} > \tau_{T-2})) \\ + \begin{cases} \beta(\beta(1 - \tau_{T-1})(1 + \gamma\tau_{T-1})) - \frac{(\beta(1 - \tau_{T-1}))^2}{2} & \text{if } \tau_{T-1} \geq 1 - \frac{h}{\gamma} \\ 0 & \text{else} \end{cases} \\ = i_{T-2}(1 + \gamma\tau_{T-1} - h(\tau_{T-1} > \tau_{T-2})) \\ + \begin{cases} \frac{\beta^2}{2}(1 - \tau_{T-1})(1 + (1 + 2\gamma)\tau_{T-1}) & \text{if } \tau_{T-1} \geq 1 - \frac{h}{\gamma} \\ 0 & \text{else} \end{cases}$$

We will first show that under assumption **h**, there is no interior solution in the range $\tau_{T-1} \in \left[1 - \frac{h}{\gamma}, 1\right]$ to the problem of maximizing W_{T-1} that can be an equilibrium under rational expectations. To show this, suppose on the contrary, that such an interior solution exists. This it satisfies the first order condition

$$\frac{d\left(i_{T-2}(1 + \gamma\tau_{T-1}) + \frac{\beta^2}{2}(1 - \tau_{T-1})(1 + (1 + 2\gamma)\tau_{T-1})\right)}{d\tau_{T-1}} = 0 \\ \Rightarrow \tau_{T-1} = \gamma \frac{i_{T-2} + \beta^2}{\beta^2(1 + 2\gamma)}.$$

For this to be an equilibrium, we also need that investments in period $T - 2$ are rational, i.e., that

$$i_{T-2} = \begin{cases} \beta(1 - \tau_{T-1}) & \text{if } \tau_{T-2} \geq \tau_{T-1}, \\ \max\{\beta(1 - \tau_{T-1} - h), 0\} & \text{else.} \end{cases}$$

implying

$$\tau_{T-1} = \gamma \frac{\beta(1 - \tau_{T-1}) + \beta^2}{\beta^2(1 + 2\gamma)} \\ \Rightarrow \tau_{T-1} = 1 - \frac{\beta(1 + \gamma)}{\beta(1 + \gamma) + \gamma(1 + \beta)}.$$

However, under assumption **h**, $1 - \frac{h}{\gamma} > 1 - \frac{\beta(1 + \gamma)}{\beta(1 + \gamma) + \gamma(1 + \beta)}$, so this interior equilibrium is not possible.

Alternatively, if $\tau_{T-2} < \frac{\gamma(1 + \beta)}{\beta(1 + \gamma) + \gamma(1 + \beta)}$ and $\beta(1 - \tau_{T-1} - h) \geq 0$, we have

$$\tau_{T-1} = \gamma \frac{\beta(1 - \tau_{T-1} - h) + \beta^2}{\beta^2(1 + 2\gamma)} \\ \Rightarrow \tau_{T-1} = 1 - \frac{\beta(1 + \gamma) + \gamma h}{\beta + 2\beta\gamma + \gamma},$$

which also is below $1 - \frac{h}{\gamma}$ under assumption **h**. Clearly, this is also the case if investments would have been zero in which case no temptation to set taxes above the first best $\frac{\gamma}{1+2\gamma} < 1 - \frac{h}{\gamma}$, exists.

We therefore conclude that there cannot be a rational expectations equilibrium in period $T - 1$, where τ_{T-1} satisfies an interior first-order condition in the range $\tau_{T-1} \in \left[1 - \frac{h}{\gamma}, 1\right]$.

Remaining possibilities, except our proposed equilibrium, where $\tau_{T-1} = 1 - \frac{h}{\gamma}$ for all τ_{T-2} , is that for some value of $\tau_{T-2} < 1 - \frac{h}{\gamma}$, it is not worth to take the cost due to loss aversion.

To see that this is not the case, we first note know that for τ_{T-2} in the range $[0, 1 - \frac{h}{\gamma})$, the period T payoff is 0, if τ_{T-1} is set equal to τ_{T-2} so payoff is $i_{T-2}(1 + \gamma\tau_{T-1})$ which is increasing in τ_{T-1} . So the potential deviation from our equilibrium must be to set $\tau_{T-1} = \tau_{T-2}$. The payoff to this is $i_{T-2}(1 + \gamma\tau_{T-2})$. Under the proposed equilibrium policy, $\tau_{T-1} = 1 - \frac{h}{\gamma} > \tau_{T-2}$, so $i_{T-2} = \max\left\{\beta\left(\frac{h}{\gamma} - h\right), 0\right\}$, giving a deviation policy value no larger than $\beta\left(\frac{h}{\gamma} - h\right)(1 + \gamma\tau_{T-2})$, which, of course, is increasing in τ_{T-1} so the supremum over all deviation policies is reached as τ_{T-2} approach $1 - \frac{h}{\gamma}$, implying that the deviation payoff is bounded from above by $\beta\left(\frac{h}{\gamma} - h\right)\left(1 + \gamma\left(1 - \frac{h}{\gamma}\right)\right) \equiv W_{dev}$. We finally require that this is smaller than the payoff from the equilibrium policy of setting $\tau_{T-1} = 1 - \frac{h}{\gamma}$ for all τ_{T-2} . The payoff from this (when $\tau_{T-2} < 1 - \frac{h}{\gamma}$) is

$$i_{T-2}(1 + \gamma\tau_{T-1} - h) + \frac{\beta^2}{2}(1 - \tau_{T-1})(1 + (1 + 2\gamma)\tau_{T-1})$$

for $i_{T-2} = \beta\left(\frac{h}{\gamma} - h\right), \tau_{T-1} = 1 - \frac{h}{\gamma}$

giving

$$\begin{aligned} & \beta\left(\frac{h}{\gamma} - h\right)\left(1 + \gamma\left(1 - \frac{h}{\gamma}\right) - h\right) \\ & + \frac{\beta^2}{2}\frac{h}{\gamma}\left(1 + (1 + 2\gamma)\left(1 - \frac{h}{\gamma}\right)\right) \\ \equiv & W_* \end{aligned}$$

and the condition for our proposed policy to be an equilibrium is thus

$$W_* - W_{dev} = -h\beta\left(\frac{h}{\gamma} - h\right) + \frac{\beta^2}{2}\frac{h}{\gamma}\left(1 + (1 + 2\gamma)\left(1 - \frac{h}{\gamma}\right)\right) \geq 0.$$

Setting the last LHS expression equal to zero gives a quadratic equation in h , with roots $h = 0$ and $h = \frac{2\beta\gamma(1+\gamma)}{2\gamma(1-\gamma)+\beta(1+2\gamma)} \equiv h_m > 0$. By differentiating $W_* - W_{dev}$ with respect to h at $h = 0$, we see that $W_* - W_{dev}$ is positive in the range $h \in [0, h_m]$. Finally, we need to establish that assumption **h** implies that $h \leq h_m$. To see this, we note that assumption **h** implies $h < \frac{\gamma\beta(1+\gamma)}{\beta(1+\gamma)+\gamma(1+\beta)} \equiv h_h$, and we finally need to show that $h_m - h_h \geq 0$. At last,

$$h_m - h_h = \frac{\beta\gamma(1+\gamma)(\beta(1+2\gamma) + 2\gamma^2)}{(2\gamma(1-\gamma) + \beta(1+2\gamma))(\beta(1+\gamma) + \gamma(1+\beta))}$$

For $\beta \in [0, 1]$ the two real roots to $h_m - h_h = 0$ are $\gamma = 0$ and -1 and

$$\left[\frac{d(h_m - h_h)}{d\gamma}\right]_{\gamma=0} = 1.$$

Thus, $h_m - h_h > 0 \forall \gamma > 0$. QED.

7.2 Proof of proposition 7.

Noting that under the equilibrium policy,

$$E_t \tau_{t+1} = \begin{cases} \tau_t - p(\tau_t)(\tau_t - \tau^*) & \text{if } \tau_t \geq \tau^* \\ 1 - p(\tau_t)(1 - \tau^*) & \text{else} \end{cases}$$

Now, since $(p(1 - \tau^*) - h) < 0$, iff $p < \gamma$ which is the case under the equilibrium policy, equilibrium investments are

$$i_t(\tau_t) = \begin{cases} \beta(1 - \tau_t + p(\tau_t)(\tau_t - \tau^*)) & \text{if } \tau_t \geq \tau^* \\ 0 & \text{else.} \end{cases}$$

We define the political payoff by choosing τ_t , given τ_{t-1} and the equilibrium strategy is played in the future as

$$W(\tau_{t-1}, \tau_t) \equiv i(\tau_{t-1})(1 + \gamma\tau_t - h(\tau_t > \tau_{t-1})) - \frac{i(\tau_t)^2}{2} + \beta(i(\tau_t)(1 + \gamma E_t T(\tau_t) - h E_t(T(\tau_t) > \tau_t)))$$

Let us now go over the value function in the different regions of τ_{t-1} . The proof will proceed by verifying that the equilibrium policy is optimal for all τ_{t-1} . Suppose first that $\tau_{t-1} < \tau^*$, then $i_{t-1} = 0$, and the equilibrium policy prescribes mixing between $\tau_t = \tau^*$ and $\tau_t = 1$. Clearly these choices are both optimal provided they both lead to $\tau_{t+1} = \tau^*$ which they do under the equilibrium policy.

Consider then the range $\tau_{t-1} \geq \tau^*$. Here, the equilibrium prescribes mixing between $\tau_t = \tau_{t-1}$ and $\tau_t = \tau^*$. Therefore, we need

$$W(\tau_{t-1}, \tau_{t-1}) = W(\tau_{t-1}, \tau^*)$$

Using $i(\tau) = \beta(1 - \tau + p(\tau)(\tau - \tau^*))$ and the definition of τ^* and to simplify notation using $\tau = \tau_{t-1}$ this yields the following second degree equation in p ;

$$0 = p^2 + \frac{2}{1 + 2\gamma} \left(\frac{\gamma(1 - \tau) - \tau(1 + \gamma)}{\tau - \tau_s} - \frac{\gamma}{\beta} \right) p - \frac{1}{1 + 2\gamma} \left(\frac{2(\gamma(1 - \tau) - \beta(\tau(1 + \gamma) - h))}{(\tau - \tau_s)\beta} + 1 \right). \quad (14)$$

The relevant root is given by

$$p = p(\tau),$$

as defined in the proof.

In the range $\tau_{t-1} \geq \tau^*$, it now remains to be shown;

1. That $p(\tau) \in [0, 1]$,
 2. that no choice of τ_t below τ^* is optimal,
 3. that no choice of τ_t above τ_{t-1} is optimal and
 4. that no choice of τ_t in the range (τ^*, τ_{t-1}) is optimal.
1. To show that $p(\tau) \in [0, 1]$ for all $\tau > \tau^*$ (remember that $p(\tau^*)$ is not defined and $p(\tau)$ for $\tau < \tau^*$

is only required to be smaller than γ), we first note that

$$\frac{p_1}{p_2} - \frac{1}{2} \sqrt{\left(2 \frac{p_1}{p_2}\right)^2 - 4 \left(2 \frac{p_1 + \gamma h}{p_2} - 1\right)} > \frac{p_1}{p_2} - \frac{1}{2} \sqrt{\left(2 \frac{p_1}{p_2}\right)^2} = 0,$$

since $\frac{d\left(2 \frac{p_1 + \gamma h}{p_2} - 1\right)}{dh} < 0 \forall \tau_{t-1} > \tau^*$ and using assumption **h**, we have

$$\left[2 \frac{p_1 + \gamma h}{p_2} - 1\right]_{h=\frac{\gamma \beta (1+\gamma)}{\beta(1+\gamma)+\gamma(1+\beta)}} = 1 + \frac{2\gamma}{(1+2\gamma)\beta} > 0. \text{ Thus, } p(\tau) > 0 \text{ for all } \tau > \tau^*.$$

Second, $p(\tau)$ is smaller than unity if $\frac{p_1}{p_2} < 1$, i.e. if $p_2 - p_1 > 0$. Now, since $p_2 - p_1$ is increasing in τ and decreasing in h in the relevant range ($\tau > \tau^*$ and $h < \frac{\gamma \beta (1+\gamma)}{\beta(1+\gamma)+\gamma(1+\beta)}$) we have

$$p_2 - p_1 > [p_2 - p_1]_{\tau=1-\frac{h}{\gamma}, h=\frac{\gamma \beta (1+\gamma)}{\beta(1+\gamma)+\gamma(1+\beta)}} = \frac{\gamma^2 \beta (1+\gamma)}{\beta(1+2\gamma) + \gamma} > 0.$$

2. This is immediate. Setting $\tau_t < \tau^*$ yields zero investment and lower current payoff $i_{t-1}(1 + \gamma\tau_t)$ than τ^* .

Before going to part 3 and 4, we establish the following lemma.

Lemma 2. *Under assumption **h**, $i'(\tau) < 0 \forall \tau \in (\tau^*, 1)$*

Proof below.

We can now continue to point 3. Since we consider $\tau_{t-1} \geq \tau^*$, current payoff is $i_{t-1}(1 + \gamma\tau_t - h(\tau_t > \tau_{t-1}))$ not higher by setting $1 > \tau_t > \tau_{t-1}$. Furthermore, any $\tau_t \in (\tau_{t-1}, 1)$ yields lower continuation payoff and must be suboptimal under lemma 1. Only setting $\tau_t = 1$, remains. Define the continuation payoff including current investments if future tax-rates are τ^* as

$$V^* \equiv -\frac{\left(\beta \frac{h}{\gamma}\right)^2}{2} + \beta \left(\beta \frac{h}{\gamma} (1 + \gamma\tau^*)\right).$$

By setting $\tau_t = 1$, we get $i(\tau_{t-1})(1 + \gamma - h) + V^*$. By following the equilibrium policy, e.g., by setting $\tau_t = \tau^*$, the payoff is $i(\tau_{t-1})(1 + \gamma\tau^*) + V^*$ and by the definition of τ^* these payoffs are identical, so there is no gain to be made to deviate from the equilibrium by setting $\tau_t = 1$.

4. We have chosen p so that

$$\begin{aligned} i(\tau_{t-1})\gamma(\tau - \tau^*) &= -\frac{i(\tau^*)^2}{2} + \beta(i(\tau^*)(1 + \gamma\tau^*)) \\ &\quad - \left(-\frac{i(\tau)^2}{2} + \beta(i(\tau)(1 + \gamma(p(\tau)\tau^* + (1 - p(\tau))\tau)))\right) \end{aligned}$$

if $\tau = \tau_{t-1}$. I.e., the short run temptation to set high taxes ($\tau_t = \tau_{t-1}$) is balanced by the long run gain of setting $\tau_t = \tau^*$. Now, given τ_{t-1} , could there be another $\tau \in (\tau^*, \tau_{t-1})$ that satisfies this? Suppose there is such a solution, and call it $\hat{\tau}$, then

$$\begin{aligned} i(\tau_{t-1})\gamma(\hat{\tau} - \tau^*) &= -\frac{i(\tau^*)^2}{2} + \beta(i(\tau^*)(1 + \gamma\tau^*)) \\ &\quad - \left(-\frac{i(\hat{\tau})^2}{2} + \beta(i(\hat{\tau})(1 + \gamma(p(\hat{\tau})\tau^* + (1 - p(\hat{\tau}))\hat{\tau}))\right) \end{aligned}$$

From the construction of p we know that

$$\begin{aligned} i(\hat{\tau})\gamma(\hat{\tau} - \tau^*) &= -\frac{i(\tau^*)^2}{2} + \beta(i(\tau^*)(1 + \gamma\tau^*)) \\ &\quad - \left(-\frac{i(\hat{\tau})^2}{2} + \beta(i(\hat{\tau})(1 + \gamma(p(\hat{\tau})\tau^* + (1 - p(\hat{\tau}))\hat{\tau}))) \right) \end{aligned}$$

thus, we must have

$$i(\tau_{t-1}) = i(\hat{\tau})$$

which is a contradiction since $i'(\tau) < 0$ in the relevant range under lemma 1.

7.3 Proof of lemma 2.

Totally differentiating (14) yields

$$\frac{dp}{d\tau} = \frac{(1-p)(\tau_s + \gamma - 2h) - \frac{h}{\beta}}{(\tau - \tau_s)^2(1 + 2\gamma) \left(p + \frac{1}{1+2\gamma} \left(\frac{\gamma(1-\tau) - \tau(1+\gamma)}{\tau - \tau_s} - \frac{\gamma}{\beta} \right) \right)}.$$

Therefore,

$$\begin{aligned} \frac{i'(\tau)}{\beta} &= -(1-p) + (\tau - \tau_s) \frac{dp}{d\tau} \\ &= -(1-p) \left(1 - \frac{(\tau_s + \gamma - 2h) - \frac{h}{\beta}}{(\tau - \tau_s)(1 + 2\gamma) \left(p + \frac{1}{1+2\gamma} \left(\frac{\gamma(1-\tau) - \tau(1+\gamma)}{\tau - \tau_s} - \frac{\gamma}{\beta} \right) \right)} \right) \\ &\equiv -(1-p)X \end{aligned}$$

Now, since X is increasing in p , we have

$$\begin{aligned} X &> \left(1 - \frac{\tau_s + \gamma - 2h - \frac{h}{\beta}}{(\tau - \tau_s)(1 + 2\gamma) \left(\frac{1}{1+2\gamma} \left(\frac{\gamma(1-\tau) - \tau(1+\gamma)}{\tau - \tau_s} - \frac{\gamma}{\beta} \right) \right)} \right) \\ &= \left(1 - \frac{\tau_s + \gamma - 2h - \frac{h}{\beta}}{\gamma - 2\gamma\tau - \tau - (\tau - \tau_s)\frac{\gamma}{\beta}} \right). \end{aligned}$$

The final expression is decreasing in τ , so

$$X > 1 - \frac{\tau_s + \gamma - 2h - \frac{h}{\beta}}{\gamma - 2\gamma - 1 - (1 - \tau_s)\frac{\gamma}{\beta}},$$

where the RHS is decreasing in h . Therefore,

$$X > \left[1 - \frac{\tau_s + \gamma - 2h - \frac{h}{\beta}}{\gamma - 2\gamma - 1 - (1 - \tau_s)\frac{\gamma}{\beta}} \right]_{h=\frac{\gamma\beta(1+\gamma)}{\beta(1+\gamma)+\gamma(1+\beta)}} = 1.$$

Consequently, $i'(\tau) < -\beta(1-p) < 0$.