Is There Excess Volatility of Stock Prices?

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Abstract

Previous literature has shown that price dividend ratios are volatile if dividend growth rates are positively correlated also when discount rates are constant. However, it has been argued that to produce empirically observed volatilities, an autocorrelation unreasonably close to one is required. In this note, we show that to account for the observed volatility, a much lower autocorrelation, within the range of reasonableness, is needed.

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1 Introduction

The papers by Shiller (1981) and Leroy and Porter (1981) have shown that the volatility of stock prices grossly exceeds the volatility of ex-post dividends or earnings. Although these findings were initially interpreted as a failure of the stock market model of rational expectations, criticism by Kleidon (1986) and others led to the understanding that such findings do not contradict this model if dividends or earning are serially correlated and non-stationary. In a response to some of the criticism Shiller (1983) claimed that another variable that is excessively volatile is the price to dividend, or price to earning ratio, which should be constant in the non-stationary case when log dividends follow a random walk with drift and the discount rate is constant. Since then there have been a number of studies trying to find under what conditions the price-dividend ratio should present volatility similar to that in the data. Unless the discount rate is highly volatile, a volatile price dividend ratio requires some persistence in dividend growth so that a small change in dividends today signifies large changes in future dividends. So far studies have found that the degree of persistence must be quite high. Barsky and DeLong (1993) find that if the rate of growth of dividends has a unit root, then volatility of the price dividend ratio is close to that observed in the data. In a recent paper, that also incorporates risk aversion, Bansal and Yaron (2004) find that persistence of 0.98, which is close to 1, is sufficient to get the right volatility. In this note we show that a much lower persistence is required to get a high variability of the price-dividend ratio even by using the simplest model of stock price valuation with risk neutrality and a fixed required rate of return.

2 The model

We use the simplest model of stock prices. Time is discrete and in each period stocks are traded before dividends are paid. Hence buying a stock entitles to current and future dividends. It is assumed that stock holders are risk neutral, that they face a fixed alternative rate of return r, that expectations are rational, that markets are perfectly competitive and that bubbles are ruled out. Under these assumptions stock prices should be equal to the present discounted sum of current and future dividends:

$$P_t \equiv E_t \sum_{s=0}^{\infty} (1+r)^{-s} D_{t+s}, \tag{1}$$

where P_t is the stock price and D_t are dividends in period t, and E_t is expectation based on information in time t.

We assume dividends follow a stochastic process with high persistence. More precisely, we assume that the growth rate of dividends follows an AR process described by:

$$\Delta d_t = \rho \Delta d_{t-1} + \varepsilon_t, \tag{2}$$

where $d_t \equiv \ln D_t$ is the log of the realized dividend at time t, Δ is the first-difference operator, $\rho \in [0,1]$ and ε_t is an i.i.d. shock drawn from a normal distribution with mean zero and standard deviation σ_ε, implying an unconditional standard deviation of the dividend growth rate given by $\sigma_{\Delta d} = \sqrt{\frac{\sigma_\varepsilon^2}{(1-\rho^2)}}$.

To find the price P_t given the assumed process for dividends we first rewrite (2);

$$D_t = \frac{D_{t-1}^{1+\rho}}{D_{t-2}^{\rho}} e^{\varepsilon_t}. \tag{3}$$

It follows immediately from forward substitution that

$$D_{t+s} = \frac{D_t^{\frac{1-\rho^{s+1}}{1-\rho}}}{D_{t-1}^{\rho\frac{1-\rho^s}{1-\rho}}} e^{\left(\sum_{j=1}^{s} \varepsilon_{t+j}\left(\frac{1-\rho^{s+1-j}}{1-\rho}\right)\right)}. \tag{4}$$

Let us now denote the period t price of a claim to the period $t+s$ dividend by $p_{t,s} \equiv \frac{E_t D_{t+s}}{(1+r)^s}$ noting that

$$P_t = \sum_{s=0}^{\infty} p_{t,s}. \tag{5}$$

Using the fact that for any a and under the distributional assumptions on ε, $Ee^{a\varepsilon_t} = e^{a^2\frac{\sigma^2}{2}}$, we get[1]

$$\frac{p_{t,s}}{D_t} = \frac{E_t D_{t+s}}{(1+r)^s D_t} = \frac{1}{(1+r)^s}\left(\frac{D_t}{D_{t-1}}\right)^{\rho\frac{1-\rho^s}{1-\rho}} e^{\sigma_s^2}. \tag{6}$$

where

$$\sigma_s^2 \equiv \frac{\sigma_\varepsilon^2}{2}\sum_{j=1}^{s}\left(\frac{1-\rho^{s+1-j}}{1-\rho}\right)^2.$$

We should note here that the price dividend ratio $\frac{p_{t,s}}{D_t}$ is strictly increasing in $\frac{D_t}{D_{t-1}}$ if $\rho > 0$.[2] The analytical expression for $\frac{p_t}{D_t}$ is obviously too involved to lend itself to interpretations. Therefore, we use a first-order Taylor approximation[3] of $p_{t,s}$ around $D_t = D_{t-1}$ to construct p_t. Hence, we use

$$\frac{p_{t,s}}{D_t} \approx \frac{1+\sigma_s^2}{(1+r)^s}\left(1 + \frac{1-\rho^{s+1}}{1-\rho}\frac{D_t - D_{t-1}}{D_{t-1}}\right). \tag{7}$$

[1] To ensure bounded prices, we need to impose the condition $\frac{\sigma_\varepsilon^2}{2(1-\rho)^2} < \ln 1+r$, implying that $\lim_{s\to\infty} E_t \frac{D_{t+s}}{(1+r)^s} = 0$.

[2] It is also straightforward to show that the standard deviation of $\ln \frac{p_{t,s}}{D_t}$, is $\left(\rho\frac{1-\rho^s}{1-\rho}\right)$ times the standard deviation of the dividend growth rate. This ratio increases in ρ and s.

[3] We have performed simulations that confirm the appropriateness of the Taylor approximation. The simulation results are available upon request.

Summing the RHS over s, we get,

$$\begin{aligned}\frac{P_t}{D_t} &\approx \sum_{s=1}^{\infty} \frac{1+\sigma_s^2}{(1+r)^s}\left(1+\frac{\rho(1-\rho^s)}{1-\rho}\left(\frac{D_t-D_{t-1}}{D_{t-1}}\right)\right) \\ &\equiv \bar{P}+\hat{P}\left(\frac{D_t-D_{t-1}}{D_{t-1}}\right),\end{aligned} \tag{8}$$

implying that the standard deviation of the price dividend ratio is $\hat{P}$ times the standard deviation of the dividend growth rate. The expression for $\hat{P}$ is quite involved, namely

$$\begin{aligned}\hat{P} \equiv\, & \frac{\rho(1+r)}{r(1+r-\rho)} \\ & +\rho\frac{\sigma_\varepsilon^2}{2}\frac{1+r}{1-\rho}\left(\frac{1}{r^2}+\frac{\rho^2}{(1+r-\rho^2)}\left(\frac{2}{(1+r-\rho)}-\frac{\rho^2}{(1+\rho)(1+r-\rho^3)}\right)\right) \\ & -\rho^2\frac{\sigma_\varepsilon^2}{2}\frac{1+r}{1-\rho}\left(\frac{r(1+\rho)+(1+r-\rho)(2+\rho)}{(1+r-\rho)^2 r(1+\rho)}\right)\end{aligned}$$

and

$$\bar{P} \equiv \frac{1}{r}+\frac{\sigma_\varepsilon^2}{2(1-\rho)^2}\left(\frac{1+r}{r^2}-\frac{\rho}{(1-\rho^2)}\left(\frac{2+\rho}{r}-\frac{2(1+\rho)\rho}{1+r-\rho}+\frac{\rho^3}{1+r-\rho^2}\right)\right).$$

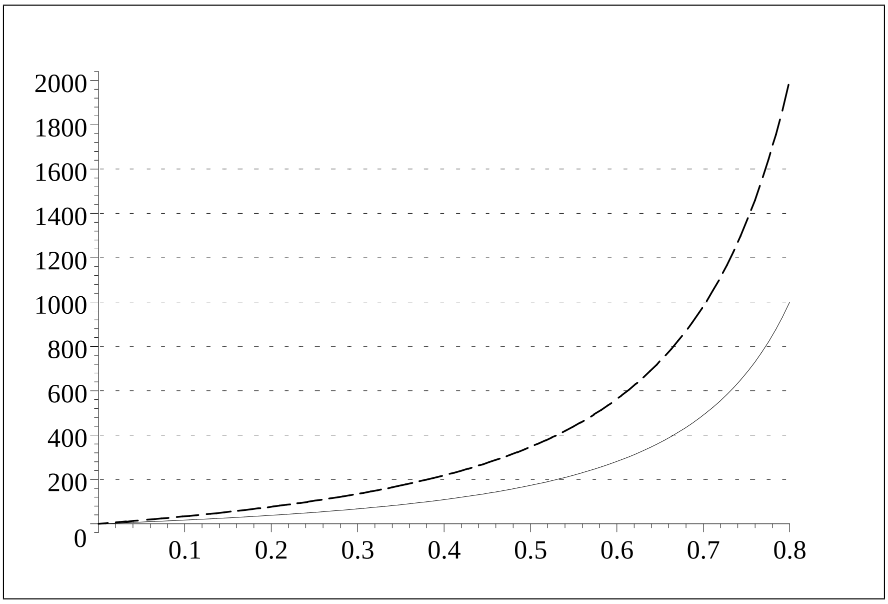
However, as we will shortly demonstrate, when r and σ_ε are of the same magnitude and $\rho<1$, a very useful approximation is

$$\hat{P} \approx \tilde{P} \equiv \frac{\rho}{r(1-\rho)},$$

implying that we can approximate the standard deviation of the price dividend by

$$\hat{P}\sigma_{\Delta d} \approx \frac{\rho\left(1-\rho^2\right)^{-\frac{1}{2}}}{r(1-\rho)}\sigma_\varepsilon.$$

In Figure 1, we plot the coefficient $\frac{\rho(1-\rho^2)^{-\frac{1}{2}}}{r(1-\rho)}$. As we see, it becomes very high already for moderate values of ρ.



Ratio between monthly standard deviation of the price dividend ratio and of the dividend innovation for $r = 8\%$ per year (solid curve) and $r = 4\%$ per year (dashed curve).

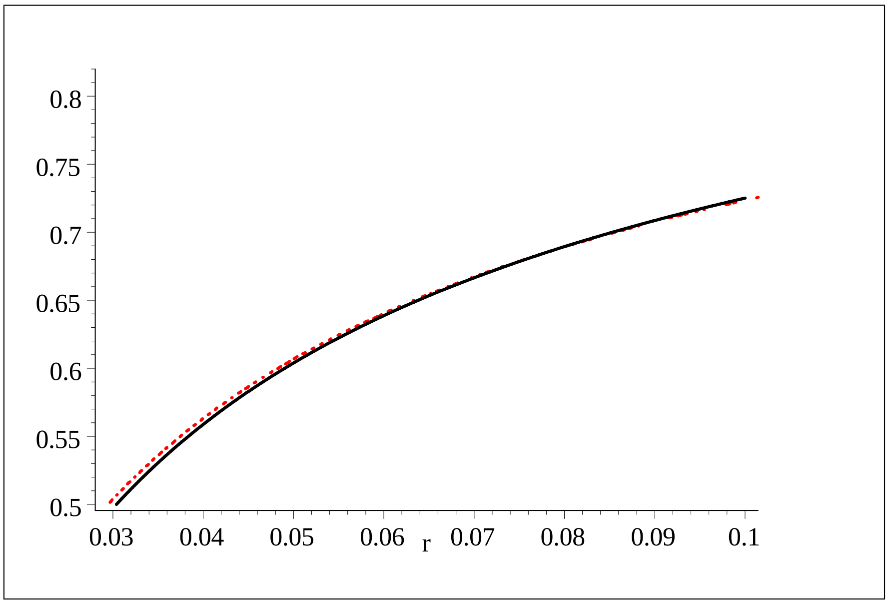
3 Results and conclusion

To calibrate our model, we use monthly data on stock prices and dividends for the period 1871 to 1995[4], published by Robert J. Shiller at http://www.irrationalexuberance.com/index.htm. The autocorrelation of monthly log dividend growth in that sample is 0.5214 and the corresponding value of the σ_ε is 1.301% per month. We use the same yearly interest rate as Shiller (1981), namely $\frac{1}{12}\ln(1.04.82)/12$ per month. Using these numbers, we find that $\hat{P} = 282.51$ and $\tilde{P} = 277.71$, implying a standard deviation of the price dividend ratio of $\hat{P} * \left(1 - \rho^2\right)^{-\frac{1}{2}} \sigma_\varepsilon = 4.307$ per month. This is high, but lower than the value of 6.21, found in Shiller's data. From the approximation $\tilde{P}$ and the expression for $\hat{P}\sigma_{\Delta d}$, we see that higher values of ρ increases the volatility of the price dividend ratio and higher r reduces it. It is therefore interesting to analyze how large the value of ρ has to be to yield the observed standard deviation of 6.21.

In Figure 2, we plot the value of ρ that is consistent with a price dividend ratio of 6.21, as a function of the yearly interest rate (the solid line). We also show the results using the approximation $\tilde{P}$ (dotted line). As we see, we do not require very high values of ρ and/or low values of r to make the model consistent with data. Specifically, at Shiller's interest rate of 4.82% per year, the required ρ is 0.597. Of course, this is higher than in the data. However, it is substantially lower than the autocorrelation of earnings, which is 0.720. We believe that a reasonable discount rate should be should definitely be in

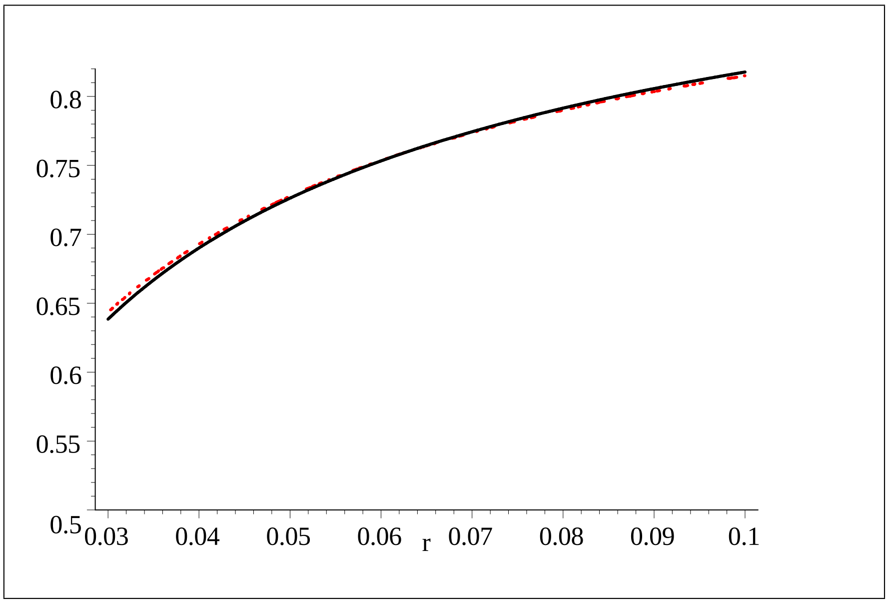
[4]The arguably abnormal period between from 1995 is excluded. Below, we report the results if it is included.

the range, 3 to 10% per year. As we see, the required ρ is in the range given by the autocorrelation of dividend growth and earnings growth for any of these discount rates.



The value of ρ that yields the empirical standard deviation of the price dividend ratio as a function of yearly discount rates (solid). Approximation $\hat{P} \approx \frac{\rho}{r(1-\rho)}$ (dotted line).

Let us now, report the results for the full Shiller sample. Due to the sharp rise and decline of the price dividend ratio in recent year, the standard deviation of is now 12.13. The value of σ_ε is 0.01267, and the autocorrelations of dividend and earnings growth are 0.522 and 0.735 respectively. The required value of ρ, calculated as for Figure 1, is depicted in Figure 2.



The value of ρ that yields the empirical standard deviation of the price dividend ratio as a function of yearly discount rates (solid) for the sample including the recen stock market boom and crash. Approximation $\hat{P} \approx \frac{\rho}{r(1-\rho)}$ (dotted line).

Finally, we have also analyzed yearly data from Shiller's data base. Here, the predictions of our model is less in line with data. Specifically, over the period until and including 1994, the autocorrelation of dividend growth rates is only 0.105, producing a $\hat{P}$ of 2.83. In the data, the ratio of the standard deviation of the dividend ratio and the standard deviation of dividend growth rates is 44.3, i.e., an order of magnitude larger. Given the observed $\sigma_\varepsilon = 0.12512$ on annual data and a yearly interest rate of 4.82%, the autocorrelation of dividends would have to be 0.668, to justify the high volatility of the price-dividend ratio.

We offer no explanation of this result. However, one should bear in mind, that $\hat{P}$ is a very convex function of ρ. Taking averages of the autocorrelation of dividends over long periods of time might then introduce a downward bias of average $\hat{P}$ in the case the autocorrelation of dividend growth varies over time.[5] It is, of course, possible that variations in the discount rate are necessary to explain low frequency movements in the price-dividend ratio. In any case, our results indicate that such variations are not necessary for higher frequency movements.

[5] Of course, a calculation of $\hat{P}$ when also ρ follows a stochastic process is substantially more involved than our calculations above.