Unemployment Insurance Design: inducing moving and retraining[*]

John Hassler[†] José V. Rodríguez Mora[‡]

September 8, 2006

Abstract

Evidence suggests that unemployed individuals can sometimes affect their job prospects by undertaking a costly action like deciding to move or retrain. Realistically, such an opportunity only arises for some individuals and the identity of those may be unobservable *ex-ante*. The problem of characterizing constrained optimal unemployment insurance in this case has been neglected in previous literature. We construct a model of optimal unemployment insurance where multiple incentive constraints are easily handled. The model is used to analyze the case when an incentive constraint involving moving costs must be respected in addition to the standard constraint involving costly unobservable job-search. In particular, we derive closed-form solutions showing that when the moving/retraining incentive constraint binds, unemployment benefits should increase over the unemployment spell, with an initial period with low benefits and an increase after this period has expired.

JEL Classification: J65, J64, E24

Keywords: Unemployment benefits, search, moral hazard, adverse selection

[*]This is a substantial revision of a previous version that circulated under the title "Should UI benefits really fall over time?". We thank Christina Lönnblad for editorial assistance and Kjetil Storesletten, Jaume Ventura and Nicola Pavoni for valuable comments. JVRM acknowledges financial support from the Spanish Ministry of Education, project number SEJ2004-06877. JH thanks the Swedish Research Council for financial support.

[†] Institute for International Economic Studies, Stockholm University, S-106 91 Stockholm, Sweden, IZA, CEPR and CESIfo. Email: John@Hassler.se. Telephone: +46-8-162070.

[‡] University of Southampton (SO17 1BJ, Southampton, United Kingdom), CREA, Universitat Pompeu Fabra and CEPR. Email: sevi@soton.ac.uk, Telephone: +442380592530

1 Introduction

An important feature of the modern welfare state is the existence of an extensive unemployment insurance (UI) system. It is now well established that the design of the unemployment insurance affects the incidence of unemployment by distorting the incentives of unemployed to search for a job (see, e.g., Holmlund (1998) for a survey). This has motivated a growing literature on how the UI system should be designed to make an optimal trade-off between providing good insurance, on the one hand, and not distorting the incentives too much, on the other. The key informational friction in this literature is that search activity cannot be monitored, so sufficient search incentives must be provided.

The main contribution of this paper is that we cast the focus on another important informational friction, largely neglected in the literature. We will consider the case when individuals who become unemployed have different opportunities to find a new job. However, we assume that the insurer cannot (perfectly) observe these differences. Specifically, we assume that some, but not all, unemployed can increase the probability of being hired by undertaking a costly investment, e.g., by retraining or moving to a location with better employment prospects. Under the realistic assumption that the insurer is unable to observe who has this option, an incentive problem arises and failure to take this into account may lead to sub-optimal UI-design. One direct way of mitigating the problem would be to offer subsidies to moving or retraining. While we will discuss this case at the end of the paper, our main case is when full cost-compensation is not feasible, for example because the insurer cannot fully distinguish voluntary and involuntary job-separations.

Although an empirical investigation is outside the scope of this paper, we argue that the consequences of not providing reasonable incentives for people to move or retrain may be of substantial quantitative importance. For instance, Bartel (1979) documents that the proportion of geographical mobility in the U.S. caused by the decision to change jobs is one-half of all migration decisions for young workers and one third of all migration decisions for workers aged above 45. Furthermore, geographical mobility is substantially lower in continental Europe, and Hassler, Rodríguez Mora, Storesletten and Zilibotti (2004) document in panel-data a negative correlation between geographical mobility and UI-generosity as well as between mobility and aggregate unemployment rates. Other empirical documentations of the link between unemployment and geographical mobility are DaVanzo (1978), Pissarides and Wadsworth (1989) and McCormick (1997).

Search incentives and incentives to move are generally not independent and should therefore be jointly analyzed. The reason why moving incentives are not included in the standard analysis is that multiple incentive constraints with different characteristics are difficult to analyze. Including both search and moving/retraining incentive constraints complicates the analysis**,** since it is difficult to evaluate which of many constraints are binding, in particular when unemployment benefits are allowed to be non-constant. Suppose, for example, that the benefit schedule contains x tiers, so that the benefit level b is an element of $B \equiv \{b_1, b_2, ..b_x\}$. The incentive constraint for an individual at a particular tier then depends on benefits in all tiers that the individual could eventually end up, in general all elements of B. The methodological contribution of the paper is to show that the problem of finding the optimal benefit structure can be formulated in such a way that that all incentive constraints are linear and parallel or independent of each other. It is then immediate to check which constraints are binding and optimal benefits can easily be characterized, both graphically and analytically. Our model easily lends itself to allowing multiple incentive problems and adding, for example, a moral hazard problem in job-retention effort **as** in Wang and Williamson (1996) should be straightforward.

There is empirical evidence indicating that precautionary saving is used to self-insure against unemployment risk. Using PSID, Gruber (1997) finds that, in the absence of UI, consumption falls by 22% when an individual becomes unemployed, showing that individuals are able to smooth consumption also when there is no UI. Similarly, Engen and Gruber (2001) show that UI crowds out financial savings, indicating that households use financial markets to self-insure against unemployment risk.[1] The assumption in most of the early literature on optimal UI (e.g., the seminal papers by Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997)), namely that the insurer can perfectly control individual consumption, is thus not entirely realistic. Following the emerging tradition in a recent line of papers (e.g., Pavoni (2006), Arpad and Pavoni (2005) and Werning (2002)), we will therefore allow the individual to make her own consumption decisions, allowing access to a market for saving and borrowing.

We will provide analytical expressions for the (constrained) optimal benefit schedule and, in particular, focus on the issue of whether benefits should increase or decrease over time. Two important assumptions are key to analytical tractability. First, individuals have access to a perfect market for borrowing and lending. Second, we assume constant absolute risk-aversion. These assumptions imply search incentives to be independent of asset holdings. This allows us to focus on simple benefit schemes not contingent on the full employment history of the agent and with a limited number of benefit levels. Neither of the key assumptions is perfectly realistic and a quantitative analysis might require wealth effects, either because of non-constant absolute risk aversion and/or because of variations in the bite of liquidity constraints. Nevertheless, we hope that illustrating a mechanism not previously explored in the literature might provide guidance for future quantitative work.

The paper is structured in the following way. The model is presented in section 2, where the relevant value functions are derived in subsection 2.1. The formal optimality problem is defined and solved in section 3. In subsection 3.1, we show the methodology in the simplest case with a constant benefit level and in subsection 3.2, we allow time varying benefits. In section 4, the optimal insurance scheme is characterized under different assumptions on search and moving costs. The section ends with some extensions and section 5 concludes. Some proofs are given in the

[1] Also if access to the formal capital market is limited, alternative means of smoothing consumption may exist, see e.g., Cullen and Gruber (2000).

main text, others in the appendix and the remaining ones are available from the authors upon request.

2 The model

Consider an economy in continuous time where individuals can either be employed or unemployed. They have access to a market for safe saving and borrowing with an exogenous return r, equal to the subjective discount rate (possibly including a positive probability of dying). Unemployed individuals can affect their chances of finding a job. As noted in the introduction, we will focus on the case where some, but not necessarily all, individuals can make a costly investment increasing their chances of becoming employed. Allowing unobservable heterogeneity in this respect creates an informational problem similar to an adverse selection problem.[2] In addition, we will allow a more standard moral hazard problem where search activity entails a flow cost.

Specifically, we assume that an employed individual, who is said to be in state 1, loses her job at the exogenous rate q. A share $p \in [0,1]$ of those who lose their job can undertake a costly investment. We will interpret this as representing a cost of moving, denoted $m > 0$ (for example between geographical locations or between occupations requiring some retraining). For simplicity, we assume that if the unemployed pays this cost ("moves"), she is immediately rehired.[3] Unemployed who cannot move or decide not to move and who search for a job find one at rate h. Searching has a cost of $s \geq 0$ per unit of time. We may consider this cost as representing the opportunity cost of searching, arising from, for example, some alternative economic activity. Whether the agent actually searches or not

[2] There are few papers on UI which deal with adverse selection. One recent paper is Hagedorn, Kaul and Memmel (2003), where individuals with different hiring rates are separated by being offered different "benefit menus".

[3] This assumption reduces the number of states, since there are no unemployed movers, which makes an easy graphical representation of the results possible. However, it would be straightforward to allow a higher but finite hiring rate of movers.

and whether she has the opportunity to move are assumed to be her own private information. To make the problem interesting, we assume that it is socially optimal to induce individuals to search and move (if they have the opportunity). It is easily shown that under this assumption, agents with the option of moving should be induced to do so immediately. Therefore, in the optimal solution, no mass of agents should be unemployed while having the opportunity to move.

A key question we want to analyze is if and how UI benefits should change over the duration of the unemployment spell. To answer this question, we make two assumptions that will simplify the analysis and make graphical representations of our results possible. First, we assume the benefit schedule to be a ladder with a finite number of steps. In fact, we only allow two benefit levels, b_2 and b_3, but the extension to any a finite number of benefit levels is straightforward. Moreover, we can show that our main results would not change by allowing more than two benefit tiers – with x benefit tiers, only the first should have a unique value, all latter benefit tiers should be identical.[4] Second, we assume transition between the steps in the benefit schedule to occur with a constant hazard rate f. Individuals who lose their jobs enter state 2 and receive benefits b_2. In state 2, they face a constant hazard rate f of entering state 3 and then receiving benefits b_3.[5] Motivated by real-world practical considerations, and in contrast to, e.g., Hopenhayn and Nicolini (1997), we assume that benefit levels can only be given conditional on current unemployment status (2 or 3), not conditional on employment history or asset holdings.[6] Given the multiple incentive constraints, an extended unemployment insurance, where individuals can choose between different menus, may be better than a simple two-tier system. In subsection 4.3, we allow such a scheme, showing that our results

[4] Proof available upon request.

[5] This assumption implies that search incentives remain constant as long as the individual remains in state 2. An alternative would be to use discrete time and assume that short-term UI benefits are paid for one period only, as done by e.g., Cahuc and Lehmann (2000). Assuming that UI benefits change after some fixed period of time would make search incentives depend on the remaining time of current benefits and considerably complicate the analysis with little gain.

[6] In fact, under CARA utility, also this assumption is innocuous.

regarding when benefits should be increasing and when they should be decreasing remain valid in the case of menu-based insurance.

The simplest and most obvious way of interpreting the unemployment states is as an indication of the passage of time: individuals in state 3 have, on average, been unemployed longer than individuals in state 2. Therefore, we label state 2 as *short-term unemployment* and state 3 as *long-term unemployment.* Our preferred interpretation of the third state is that it is a purely administrative state and we may allow the insurance provider to choose f. In this case, it is natural to assume that search costs (s) and hiring probability (h) are the same in both states.

We may also interpret the third state as representing loss of skills during unemployment in the sense of job-finding rates and search costs developing disadvantageously over the unemployment spell. As an extension, we modify the model so that with a constant instantaneous probability f, unemployed individuals suffer a shock, and their search costs increase ($s_2 < s_3$) and/or their hiring probabilities decrease ($h_2 > h_3$). Although this interpretation raises issues about observability, we abstain from these and assume benefits to be paid contingent on whether the individual is in state 2 or 3.

Individuals maximize their intertemporal utility, given by

$$E \int_0^\infty e^{-rt} U\left(c_t\right) dt,$$

where c_t is consumption at time t and r **is** the subjective discount rate. To facilitate analytical solutions when individuals have access to markets for saving and borrowing, we choose the CARA utility function

$$U\left(c_t\right) \equiv -e^{-\gamma c_t},$$

where γ is the coefficient of absolute risk aversion. All individuals are born (enter the labor market) as employed without assets and are identical at that point.

The purpose of this paper is to discuss how an unemployment insurance system should be constructed when there are incentive problems. To this end, we want to remove other motives for unemployment benefits than providing insurance.

In particular, in this paper, we are not interested in motives for using the UI system to create non-actuarial transfers between individuals with different characteristics.[7] Therefore, we assume that individuals face an actuarially fair insurance. This means that when an individual enters the labor force, the expected present discounted value of the benefits she will receive during her life-time exactly balances the expected present discounted value of her contributions. An alternative interpretation of actuarial fairness is that in a decentralized equilibrium, where individuals can sign binding insurance contracts with competitive insurance companies when entering their first job, actuarial fairness is identical to a break-even condition for the insurance companies, which would be satisfied under perfect competition.[8]

Without loss of generality, we let individuals pay lump-sum taxes, denoted τ, implying that

$$\dot{A}_t = rA_t + y - c_t - \tau, \tag{1}$$

except at the points in time when the cost of moving is paid, and where $y \in \{w, b_1 - s, b_2 - s\}$, depending on the employment state. We define the average discounted probabilities (ADP's) of being in state 2 and 3, respectively, by

$$\Pi_2 \equiv r\int_0^\infty e^{-rt}\mu_{2,t}dt,$$
$$\Pi_3 \equiv r\int_0^\infty e^{-rt}\mu_{3,t}dt,$$

where $\mu_{2,t}$ and $\mu_{3,t}$ are the probabilities of being short-term and long-term unemployed at time t, respectively, conditional on being employed at time zero, provided that individuals who can move do so and that unemployed search for a job.[9] The actuarial fairness requirement of the UI system is now a simple linear function of the benefits

$$\tau = \Pi_2 b_2 + \Pi_3 b_3. \tag{2}$$

[7] For positive implications, the redistributive elements of unemployment insurance are, however, likely to be central. See e.g., Wright (1986).

[8] Since we use the CARA specification, individual assets do not affect preference over insurance, so that older employed agents with non-zero asset holdings would not want to renegotiate their contract.

[9] It is straightforward to calculate that

2.1 Value functions and consumption

It is well known that under constant absolute risk aversion and stationary income uncertainty, the value functions for the three states $j \in \{1, 2, 3\}$ can be separated

$$V(A_t, j) = W(A_t)\tilde{V}_j(\tau, b_2, b_3), \tag{3}$$

where

$$\begin{aligned} W(A_t) &\equiv \frac{e^{-\gamma A_t}}{r} \\ \tilde{V}_j &\equiv -e^{-\gamma c_j}, \end{aligned} \tag{4}$$

and σ_j are state-dependent consumption constants and where the state dependent consumption functions are

$$c_j(A_t) = rA_t + \sigma_j. \tag{5}$$

The consumption constants σ_j are nonlinear functions of income in all states and thus, depend on the planner choice variables τ, b_2 and b_3. The constants are found as the unique solutions to the Bellman equations for each state:[10]

$$\begin{aligned} \sigma_1 &= w - \tau - \frac{q\left(e^{\gamma\Delta_2} - 1\right)}{\gamma r} \\ \sigma_2 &= b_2 - s - \tau + \frac{h\left(1 - e^{-\gamma\Delta_2}\right)}{\gamma r} - \frac{f\left(e^{\gamma(\Delta_3 - \Delta_2)} - 1\right)}{\gamma r} \\ \sigma_3 &= b_3 - s - \tau + \frac{h\left(1 - e^{-\gamma\Delta_3}\right)}{\gamma r}, \end{aligned} \tag{6}$$

where

$$\begin{aligned} \Delta_2 &\equiv \sigma_1 - \sigma_2, \\ \Delta_3 &\equiv \sigma_1 - \sigma_3, \end{aligned} \tag{7}$$

$$\begin{aligned} \Pi_2 &\equiv \frac{q(1-p)(h+r)}{(r+h+q(1-p))(r+h+f)}, \\ \Pi_3 &\equiv \Pi_2 \frac{f}{h+r}. \end{aligned}$$

[10] See the appendix for proof.

are the consumption differences between state 1 and 2 and between state 1 and 3, respectively.

3 Optimal Insurance

Given the discussion above, the problem we set out to solve is to maximize the *ex-ante* value of unemployment insurance, that is, we want to maximize the welfare of an individual upon entering the economy. This welfare is given by $V(0,1)$, since we assume that agents enter the economy as employed with no assets.[11] Due to the separability and the fact that $W(A_t)$ is independent of the insurance system, we immediately see that this is equivalent to maximizing $\tilde{V}_1$ over $\{\tau, b_2, b_3\}$. Using the budget constraint $\tau = \Pi_2 b_2 + \Pi_3 b_3$, our objective is therefore to solve

$$\max_{b_2,b_3} \tilde{V}_1 \left(\Pi_2 b_2 + \Pi_3 b_3, b_2, b_3\right) \tag{8}$$
$$\text{s.t. IC2, IC3, and ICM,}$$

where IC2 and IC3 are the incentive constraints that unemployed individuals voluntarily search for a job and ICM is the constraint that individuals with the opportunity to move to get a job voluntarily do so.

In the direct formulation of the problem, the incentive constraints are highly non-linear functions of the choice variables b_2 and b_3. This makes it hard to find the binding constraints, which is necessary to find the solution. However, it turns out that we can formulate the problem so that the incentive constraints are linear and either parallel or orthogonal. Finding out which is binding is then trivial. Furthermore, adding more states and incentive constraints is also very simple. We regard this as the methodological contribution of the paper.

Finding the constrained optimal insurance now involves the following steps:

[11] Obviously, we could equally well have chosen any other initial condition. Note also that the separability implies that the insurance system that maximizes the *ex-ante* utility also maximizes the utility of all employed, regardless of their history.

1. Note that $\tilde{V}_1 \equiv -e^{-\gamma c_1}$ is a monotone transformation of σ_1. For convenience, we therefore use σ_1 as the objective function and express it as a function of the consumption differences, using the budget constraint (2) to replace τ.
2. Express the incentive constraints in terms of consumption differences Δ_j.
3. Maximize σ_1 over the consumption differences, subject to the incentive constraints.
4. Verify that the optimal consumption differences Δ_2 can be implemented by some combination of b_j's.

3.1 Two states

For illustrative purposes, we start with the simplest case of two states, i.e., we assume that $f = 0$ so unemployment benefits are constant forever.

The first step is now to derive an expression for σ_1 in terms of Δ_2 where the budget constraint (2) is used to replace the tax rate. For this purpose, we subtract the second line of (6) from the first and solve for b_2. Then, we use this expression in the budget constraint $\tau = \Pi_2 b_2$ and substitute for τ in the first line of (6). This yields

$$\sigma_1 = \kappa + \Pi_2 \left(\Delta_2 - \frac{he^{-\gamma\Delta_2}}{\gamma r} \right) - (1 - \Pi_2)\, q \frac{(1-p)\, e^{\gamma\Delta_2}}{\gamma r}, \tag{9}$$

where κ is a constant, independent of the choice variables. Straightforward calculus shows that (9) defines σ_1 as a concave function of Δ_2 with a unique maximum at 0. The reason for σ_1 being maximized at $\Delta_2 = 0$ is obvious – when actuarial insurance is available, full insurance maximizes utility. However, $\Delta_2 = 0$ is not incentive compatible. Neither searching nor moving will occur voluntarily under full insurance.

Therefore, we turn to step 2 – where we find the incentive constraints. The ICM constraint implies that a person who has lost her job and has the opportunity to move must be induced to do so. We first note that if her assets upon separation were A_t, her value immediately after moving is

$$V(A_t - m, 1) = -\frac{1}{r} e^{-\gamma r (A_t - m)} e^{-\gamma \sigma_1},$$

since she has paid the moving cost, m. We compare this to the value of a one-period deviation, i.e., the value if the individual does not move during this unemployment spell. Immediately after being laid off, her assets are A_t and she is unemployed, i.e., in state 2, since she did not take the opportunity to move to get a job. Her value is therefore,

$$V(A_t, 2) = -\frac{1}{r} e^{-\gamma r A_t} e^{-\gamma \sigma_2}.$$

To induce moving, we need $V(A_t - m, 1) \geq V(A_t, 2)$. It immediately follows that this requires

$$\Delta_2 \geq rm. \tag{10}$$

We label (10) the *ICM-condition.*

Now, consider the incentive to search. Remember that for now, we assume unemployment benefits to be flat (the assumption $f = 0$ implies that b_3 is irrelevant). If the individual does not search, she therefore gets an income $b_2 - \tau$ for ever, since she will not find a new job without searching. Without uncertainty, she consumes exactly her total income $rA_t + b_2 - \tau$ (since r coincides with the subjective discount rate) and her utility is therefore

$$-\frac{1}{r} e^{-\gamma r A_t} e^{-\gamma (b_2 - \tau)}.$$

The utility if the individual instead searches is $-\frac{1}{r} e^{-\gamma r A_t} e^{-\gamma \sigma_2}$ so to induce search, we clearly need

$$\sigma_2 \geq b_2 - \tau.$$

Note that the consumption of the unemployed who search is $rA_t + \sigma_2$. Furthermore, her total income net of search costs is $rA_t + b_2 - \tau - s$. Therefore, the search condition implies consumption to be strictly higher than income. Over time, the unemployed depletes her assets and consumption therefore falls, despite the benefits being constant. The celebrated result by Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997) that consumption should optimally fall over the unemployment spell when the insurer can fully control consumption (no hidden savings) is therefore mimicked in this case, where hidden savings are allowed.

The final part of step 2 is to express the search constraint in terms of the consumption difference Δ_2. Using the second line of (6) and setting $f = 0$, the search constraint can be written

$$\Delta_2 = -\frac{\ln\left(1 - \gamma r \frac{s}{h}\right)}{\gamma}, \tag{11}$$

which we label the *IC2-condition*. As can be seen, the incentive constraints are simply constants and it is immediate to see which one is binding.

The problem is now simply depicted in Figure 1, where we note that the two constraints are parallel.

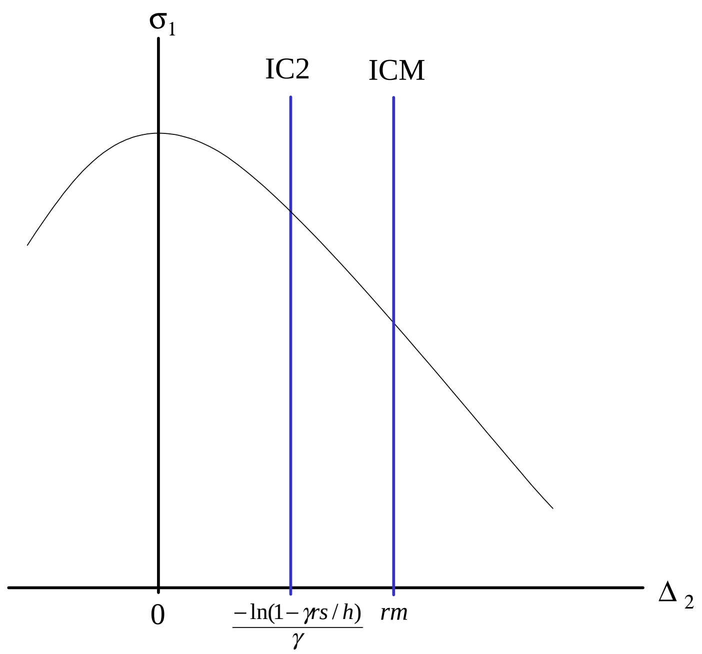

Figure 1: Objective function and constraints in **a** two-state case.

In the depicted case, it is the ICM-constraint that binds and step 3 is trivial.

Maximizing σ_1 over Δ_2 subject to the ICM constraint implies

$$\Delta_2 = rm.$$

Finally, we want to implement this. This is easily done using (6); set the difference between the first and the second line equal to rm and solve for b_2, giving

$$b_2 = w + s - rm - \frac{q\left(e^{\gamma rm} - 1\right) + h\left(1 - e^{-\gamma rm}\right)}{\gamma r}.$$

In the alternative case, where the IC2 constraint binds, we instead get

$$b_2 = w + \frac{\ln\left(1 - \gamma r \frac{s}{h}\right)}{\gamma} - \frac{sq}{h - \gamma rs}, \tag{12}$$

where both expressions are unique and easily lend themselves to comparative statics.

3.2 Three states

The procedure in the case of three states is exactly analogous to the two-state case and simply extends to any number of finite states. We use (6) and the budget constraint (2) to express σ_1 as a function of the consumption differences, now Δ_2 and $\Delta_3 \equiv \sigma_1 - \sigma_3$ (step 1). Then, we express the incentive constraints in terms of Δ_2 and Δ_3, check which are binding (step 2), maximize σ_1 over $\{\Delta_2, \Delta_3\}$ subject to the binding constraints (step 3) and find the implementing b_2, b_3 (step 4).

3.2.1 Objective and constraints

Using the equations for the consumption constants (6) and the budget constraint (2), the objective becomes

$$\begin{aligned}\sigma_1 = {} & \kappa_2 + \Pi_2\Delta_2 + \Pi_3\Delta_3 - (1 - \Pi_2 - \Pi_3)\frac{q\,(1-p)}{\gamma r}e^{\gamma\Delta_2} \\ & - \Pi_2\left(h\frac{e^{-\gamma\Delta_2}}{\gamma r} + f\frac{e^{\gamma(\Delta_3 - \Delta_2)}}{\gamma r}\right) - \Pi_3 h\frac{e^{-\gamma\Delta_3}}{\gamma r},\end{aligned} \tag{13}$$

where κ_2 is an unimportant constant. In figure 2, we make a graphical representation of the objective function by drawing indifference curves in a figure with Δ_3 on the $x-$axis and Δ_2 on the $y-$axis.[12] The bliss point is at full insurance, when

[12] The indifference curves in figure 2-6 are drawn for $\{h, f, q, r, \gamma, p\} = \{1, 1, .1, .05, .5.\}$ but the results below hold for all parameter values.

$\{\Delta_3, \Delta_2\} = \{0, 0\}$, again, for the reason that the insurance is actuarially fair. The indifference curves have elliptical shapes around the bliss point, of which we are only interested in the segment in the positive quadrant, since incentive compatibility certainly requires $\Delta_3, \Delta_2 \geq 0$. For the later analysis, we should note that the slope of an indifference curve is strictly positive if $\Delta_3 = 0$ and $\Delta_2 > 0$ and that it is downward sloping at $\Delta_2 = \Delta_3$, regardless of the parameter choice.[13]

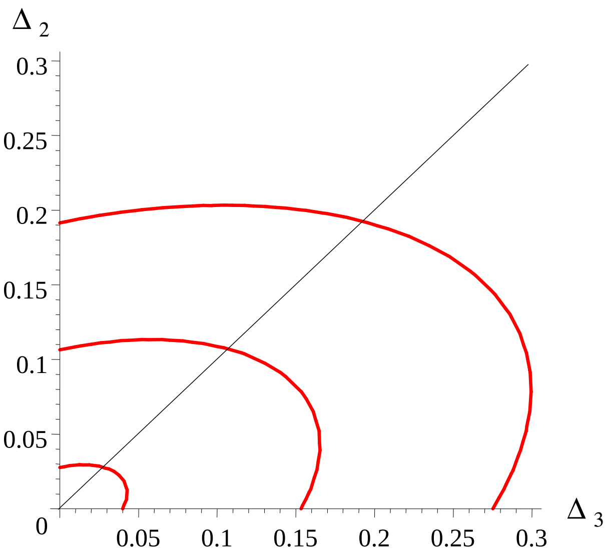

Figure 2: Indifference curves.

Regarding the three incentive constraints, it is straightforward to see that they are identical to the case of two states,[14] i.e., the ICM is $\Delta_2 \geq rm$ and the IC2 and IC3 constraints are,

$$\Delta_2, \Delta_3 \geq -\gamma^{-1} \ln \left(1 - \gamma r \frac{s}{h}\right). \tag{14}$$

The intuition for the fact that IC2 and IC3 are identical is simple. In our base line case, hiring probabilities and search costs of searching individuals are the

[13] Differentiating the objective function, we find the derivative of the indifference curve to be $\frac{fe^{-\gamma\Delta_2}}{r+(h+f)\left(1+e^{-\gamma\Delta_2}\right)} \in (0,1)$ at $\Delta_3 = 0$ and $\frac{-e^{-\gamma\Delta_2}}{1+\frac{r}{h}+\frac{(h+r)^2}{fh}+\frac{(h+r)\left(e^{-\gamma\Delta_2}\right)}{f}} \in (-1,0)$ at $\Delta_2 = \Delta_3$.

[14] See the appendix for a formal proof.

same for long- and short-term unemployed. The incentives in terms of utility and thus, in terms of consumption increases upon successful search, must therefore be the same. Allowing different search costs and/or hiring probabilities in the two states is, however, very simple by allowing s and h to be state dependent in the IC conditions; this is done in section 4.2. Therefore, we reach the key conclusion that the incentive constraints for the two states (IC2 and IC3) are identical and orthogonal in the $\{\Delta_2, \Delta_3\}$ −space. We emphasize that this does not mean that only b_2 (b_3) is of importance for search incentives of the short-term (long-term) unemployed. On the contrary, both b_2 and b_3 affect consumption and therefore incentives in all states. However, individual optimization and access to markets for saving and borrowing imply that the value function is a monotonous transformation of consumption. Thus, the wedge between consumption in the current state and during employment is a sufficient statistic to determine whether search incentives are sufficiently strong.

In the next subsection, we will use our model to characterize the optimal UI-scheme under different assumptions on which the constraint is binding. As in the two-state case, the analysis is greatly simplified by the incentive constraints in $\{\Delta_3, \Delta_2\}$ space being linear and parallel or orthogonal. When the optimal $\{\Delta_2, \Delta_3\}$ are found, we find the optimal benefits from the implementation mapping, which is derived by taking the difference between lines 1 and 2 and between 1 and 3 in (6) and solving for b_2 and b_3:

$$\begin{aligned} b_2 &= w + s - \Delta_2 - \frac{q\left(e^{\gamma\Delta_2} - 1\right) + h\left(1 - e^{-\gamma\Delta_2}\right) - f\left(e^{\gamma(\Delta_3 - \Delta_2)} - 1\right)}{\gamma r}, \\ b_3 &= w + s - \Delta_3 - \frac{q\left(e^{\gamma\Delta_2} - 1\right) + h\left(1 - e^{-\gamma\Delta_3}\right)}{\gamma r}. \end{aligned} \tag{15}$$

4 Characterization of optimal UI-schemes

In this section, we use our model to characterize (constrained) optimal unemployment insurance under three different scenarios. In the first, the hiring probability is the same independent of unemployment duration. In the second scenario, we look at the consequences for the optimal UI scheme of having a hiring probability that is decreasing with unemployment tenure. Finally, in the third scenario, we allow the insurer to offer a menu of possibilities to the unemployed, including the choice of a lump-sum transfer.

4.1 Constant hiring rates and search costs

4.1.1 Small search costs

We start the analysis with the assumption that search costs are sufficiently small to be ignored, later they are re-introduced. First, we analyze the problem graphically by including the ICM constraint, i.e., $\Delta_2 \geq rm$ in the indifference curve graph (Figure 3), and then we provide analytical results.

The ICM constraint is satisfied for all values of Δ_2 above the ICM-curve, which is horizontal. The optimizing choice of Δ_3 is where the ICM constraint is tangent to an indifference curve. This occurs for the solid indifference curve in figure 3. As noted above, the indifference curve is positively sloped at $\Delta_3 = 0$ and negatively sloped at $\Delta_2 = \Delta_3$ implying that the tangency must be at a point where $\Delta_3 > 0$ and $\Delta_3 < \Delta_2$. This means that state 2 should be "worse" than state 3 in the sense that, given assets, utility and consumption are higher in state 3 than in state 2. It is intuitive (and easily proved) that $\Delta_2 > \Delta_3 > 0$ implies that $b_2 - s < b_3 - s < w$. The intuition for this is that when $b_2 - s = b_3 - s$, the two unemployment states are, by construction, identical so that $\Delta_2 = \Delta_3$. Making Δ_2 larger than Δ_3 requires a reduction in benefits for short-term unemployed and/or an increase in benefits for long-term unemployed.

Result 1: *If search costs are sufficiently low, only the ICM constraint is binding and benefits should optimally increase over time.*

The economic reason for our results can be phrased in the following way. To separate individuals with the option of moving from those who have not, a positive Δ_2 is required. However, this does not call for an inefficient structure of the benefit

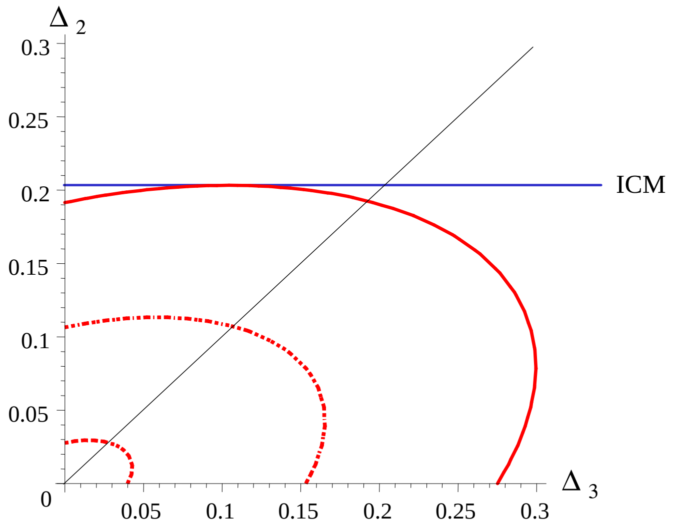

Figure 3: Indifference curves and Incentive Constraint for Moving (ICM).

schedule. Specifically, starting from a flat benefit schedule (along the 45 degree line where $\Delta_2 = \Delta_3$), the welfare in all states can be increased, while maintaining the necessary wedge $\Delta_2 = rm$, by increasing benefits for long-term unemployed and reducing benefits for short-term unemployed. The reason for this is that the expected marginal utility is higher for individuals who have been unemployed for a long time. The optimum is, however, reached before benefits to long-term unemployed are sufficiently high to make the latter indifferent between having a job and remaining unemployed. On the other hand, when $\Delta_3 = 0$ while $\Delta_2 = rm$, long-term unemployed are as well off as the employed (given assets) and their expected marginal utility is relatively low. A reallocation from long-term to short-term benefits therefore increases the value of the insurance so that the tax-cost of providing a given insurance value can be reduced.

Now, let us derive closed-form solutions to our problem. Using the binding ICM condition $\Delta_2 = rm$ to substitute for Δ_2, the objective function (13) simplifies and the problem can then be written

$$\max_{\Delta_3 \in R^+} \left\{ \Pi_3 \left(\Delta_3 - h \frac{e^{-\gamma \Delta_3}}{\gamma r} \right) - \Pi_2 f \frac{e^{\gamma(\Delta_3 - rm)}}{\gamma r} \right\}, \tag{16}$$

These terms have straightforward interpretations. The first term is due to the benefit of reducing the tax-cost of long-term benefits. This term is increasing in Δ_3, since higher Δ_3 is achieved by lower benefits for long-term unemployed, which reduce taxes in proportion to the ADP of long-term unemployment, Π_3. Note that this tax reduction comes from two sources; there is a direct effect that is proportional to Δ_3 but there is also an indirect effect, captured by the second term inside the parenthesis. Long-term unemployed find jobs at a positive rate, h. The prospect of finding a job keeps up consumption, so that it falls less than proportionally to the reduction in benefits. Conversely, given an increase in Δ_3, benefits can be reduced more than proportionally.

The second term in (16) is due to the benefit of reducing the tax cost of short-term benefits. It is decreasing in Δ_3 since less consumption for long-term unemployed has a negative impact on consumption also of the short-term unemployed, proportional to f. As Δ_3 increases, benefits to the short-term unemployed must therefore increase to keep $\Delta_2 = rm$. This has a tax-cost proportional to the ADP of short-run unemployment Π_2.

The objective function in (16) is concave in Δ_3. Thus, the unique solution to the problem is obtained by the solution to the first-order condition, given by

$$\Delta_3^* = -\frac{\ln\left(\sqrt{\left(\frac{r}{2h}\right)^2 + e^{-\gamma rm}\left(\frac{h+r}{h}\right)} - \frac{r}{2h} \right)}{\gamma} > 0.$$

Using the implementation mapping (15), we can find the optimal insurance scheme. In particular, in optimum

$$b_3^* - b_2^* = rm - \Delta_3^* + \left(f + he^{-\gamma \Delta_3^*} \right) \frac{1 - e^{-\gamma\left(rm - \Delta_3^*\right)}}{\gamma r} > 0. \tag{17}$$

Notice also that since the solution for Δ_3 is independent of f, the difference $b_3 - b_2$ should increase in f. It can be shown that the derivative of the objective function with respect to f is always positive. Low values of f is an inefficient way of inducing separation between those who can move and those who cannot, as agents expect to spend a longer stochastic time suffering the low short-run benefits. Without formally showing this, we conjecture that if lump-sum benefits were allowed, the best policy would be to punish unemployment by a lump-sum unemployment tax when an individual becomes unemployed. In reality, however, it may be politically difficult or even infeasible to implement a lump-sum punishment on those who lose their jobs. Furthermore, a lower bound on b_2, for example zero, might be imposed for political reasons, in which case this would pin down f from (17).

As is clear from the above analysis, a reduction in m reduces Δ_2 and allows **a** more generous unemployment insurance. Such a reduction could be achieved by subsidies to moving or retraining. However, full compensation is unlikely to be optimal in reality. Suppose, realistically, that individuals with a job sometimes experience a preference or productivity shock, making another job or a job in another location more attractive than the current one. Suppose also that these shocks are not sufficiently large to induce voluntary separation and moving if the individual must pay the moving cost herself. Clearly, such moves are then not socially optimal. The insurer would like to fully subsidize the moving cost of individuals who are involuntarily separated from their job, but not subsidize it for individuals who voluntary separate to claim the subsidy. However, this is is infeasible if the insurer cannot distinguish voluntary and involuntary separations. Therefore, we argue that although partial subsidies may be feasible and, in fact, optimal, full subsidization may not. More specifically, it seems clear that subsidies should be as large as possible, without inducing inefficient voluntary separation. Thus, we could interpret m as the cost of moving or retraining, net the optimal subsidy. Furthermore, a large subsidy to moving might lead unemployed individuals to claim the subsidy, which is likely to be inefficient. This issue is analyzed below.

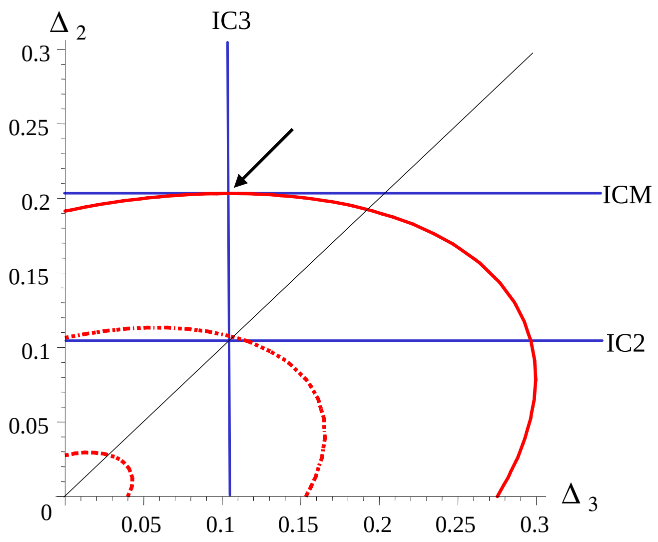

Figure 4: Low search costs

4.1.2 Larger search costs

We can now easily analyze the conditions such that IC2 and IC3 are satisfied, despite positive search costs. Graphically, the constraints are simply horizontal and vertical lines and all values of $\Delta_2(\Delta_3)$ above (to the right of) these lines imply that the respective constraints are satisfied. If search costs are sufficiently small, none of the search constraints bind, as shown in figure 4, where IC2 is slack while IC3 almost binds at the tangency between ICM and an indifference curve. This occurs at a point indicated by the arrow on the solid indifference curve.

Increasing search costs shift out IC2 and IC3 since from (14) we see that the RHS is increasing in s. Eventually (for a search cost which is sufficiently large) IC3 is no longer satisfied at the point where the ICM constraint is tangent to the indifference curve. This situation is depicted in figure 5. Here, the point where the ICM is tangent to the most outward dotted indifference curve satisfies the IC2 constraint, but not the IC3 constraint. Thus, Δ_3 must be increased but since the IC3 and the ICM constraint are orthogonal, Δ_2 need not be changed. The optimal

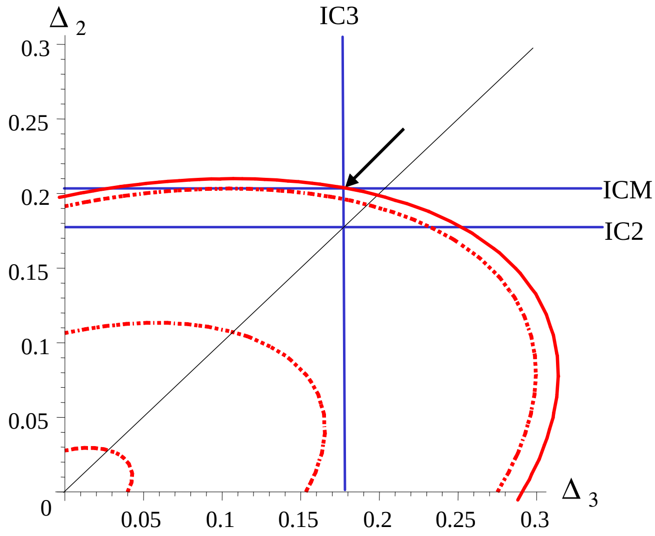

Figure 5: Moderate search costs

point is where the ICM and the IC3 constraint cross. This point is indicated by the arrow and on the solid indifference curve. Clearly, Δ_3 remains smaller than Δ_2 implying an upward sloping benefit profile, i.e., $b_2 < b_3$. Specifically, Δ_2 should be set equal to rm and Δ_3 equal to $-\gamma^{-1} \ln \left(1 - \frac{\gamma rs}{h}\right)$. This means that individuals will be indifferent in the choice of moving and that long-term unemployed are indifferent to searching, while the short-term unemployed strictly prefer to search.

Result 2: *For an intermediate range of search costs, the ICM and the IC3 constraints are binding and benefits should optimally increase over time.*

A further increase in search costs will eventually call for a situation like that in figure 6. Here, both search constraints bind, while the moving constraint is slack. Once more, the optimum is indicated by the arrow and on the solid indifference curve. Benefits are constant and given by expression (12) since $\Delta_2 = \Delta_3 = -\gamma^{-1} \ln \left(1 - \frac{\gamma rs}{h}\right)$.[15] We conclude:

[15]This is a special case of the result in Werning (2002) which shows that constant benefits are optimal under CARA utility in a general class of UI-schemes.

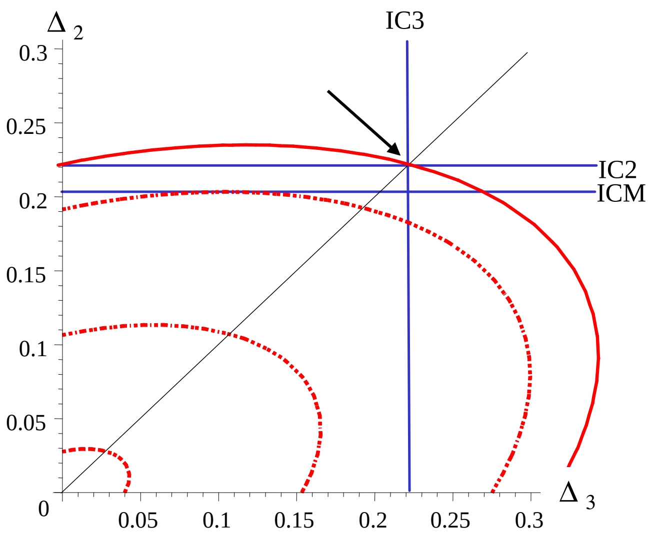

Figure 6: High search costs.

Result 3: *For sufficiently high search costs, the IC2 and the IC3 constraints are binding and benefits should optimally be constant over time.*

The conclusion so far is that when the moving cost is large relative to the search costs, then the optimal unemployment insurance scheme involves an increasing benefit profile in order to, on the one hand, generate incentives to move for those agents who can and, on the other hand, not too much limiting insurance for the possibility that an unemployment period becomes long-lasting.

If the search costs are sufficiently high relative to the moving cost, strong search incentives are needed and the moving constraint is slack. In this case, the optimal benefit profile is flat. The intuition behind this result is that, one the one hand, search incentives are strengthened by falling benefits. On the other hand, when private savings are allowed, buffer stock savings provide a good substitute for short but not for long unemployment spells, calling for an upward sloping benefit profile.[16]

[16] See Hassler and Rodríguez Mora (1999) for an analysis of the relative value of insurance against long and short unemployment spells under CARA and CRRA utility.

These two effects cancel exactly under CARA utility. With other utility functions both effects are present but will in general not cancel each other.

4.2 Loss of skills and long-term unemployment

So far, we have considered the third state as an administrative state, used as a proxy for the unemployment duration of the agent. Unemployment was assumed to have no other effect than depleting the financial assets of the agent; hiring rates and search costs remained constant. However, it is easy to relax this assumption and analyze how the path of benefits should be constructed if the unemployment duration also has real direct effects on, e.g., search costs and hiring probabilities.[17] Specifically, let s_2 and s_3 denote the search costs in states 2 and 3 and, correspondingly, h_2 and h_3 denote the state dependent hiring probabilities. The idea that the human capital of the unemployed depreciates during the unemployment spell (or that the individual "learns how to be unemployed") is captured by the assumption $h_2 > h_3$ and/or $s_2 < s_3$, implying $\frac{s_2}{h_2} < \frac{s_3}{h_3}$.

It is straightforward to show that the IC2 and IC3 constraints now become

$$\Delta_2 \geq -\gamma^{-1} \ln\left(1 - \gamma r \frac{s_2}{h_2}\right),$$
$$\Delta_3 \geq -\gamma^{-1} \ln\left(1 - \gamma r \frac{s_3}{h_3}\right),$$

respectively, where $-\gamma^{-1} \ln\left(1 - \gamma r \frac{s_2}{h_2}\right) < -\gamma^{-1} \ln\left(1 - \gamma r \frac{s_3}{h_3}\right)$ so that the IC3 constraint crosses the IC2 condition below the 45 degree line. If the binding constraints are IC2 and IC3 (small moving costs), we must then $\Delta_3 > \Delta_2$. Using the implementation equations (using the different search costs and hiring rates), we find that in this case, the optimal benefit schedule should be downward sloping ($b_2 > b_3$). If the ICM constraint binds, rather than IC2, the possibility that the optimal benefit profile should be upward sloping remains.

[17] Similarily, we could easily analyze the case when the prospective wage depends on unemployment duration.

4.3 A menu of contracts

Finally let us note that our model can also easily handle more complicated UI schemes, e.g., menus.[18] In particular, let us consider the case when the insurer allows individuals losing their job to either get a lump-sum transfer M, or a possibly non-constant UI-benefit stream.[19] Since the effective cost of moving is now $m - M$, the incentive constraint for individuals with the opportunity to move now becomes,

$$\Delta_2 = r\left(m - M\right),$$

i.e., a positive M slackens the constraint (moves it down in the figures). Increasing M to a sufficiently large extent leads to a situation like that in figure 6, where IC2 and IC3 bind. Potentially, its optimal to set $M = m$ – full subsidization. This is the case if unemployed without moving opportunities prefer UI benefits over M, so that a separation between the groups is achieved also when the moving cost is fully insured. If such separation is not achieved under full insurance but should be in optimum, M must be reduced so that unemployed individuals choose UI benefits. We note that we cannot increase the relative attractiveness of UI-benefits by raising the latter, since this would violate the IC2 and IC3 conditions, which continue to bind.

To analyze whether separation is achieved, we need to add another state to the analysis, namely to be unemployed without benefit, which makes a two-dimensional graphical analysis impractical. The analytical analysis remains simple, however. Setting the income of unemployed to zero, the consumption constant associated with being unemployed without benefits is given by

$$\sigma_u = -s - \tau + \frac{h\left(1 - e^{-\gamma(\sigma_1 - \sigma_u)}\right)}{\gamma r},$$

so that σ_u is a function of σ_1 only. The incentive constraint implying that unemployed do not choose the lump-sum transfer is then $\sigma_2 - \sigma_u \geq rM$, and it is easily

[18] Some UI schemes offer this type of menus; in particular, in the period of large unemployment (end of the 80's and beginning of the 90's) the Spanish Unemployment agency offered the option of a lump-sum transfer or standard UI payments.

[19] For simplicity, let us disregard the case of voluntary separations as discussed above.

checked if this is satisfied in the equilibrium. If not, M must be reduced. If the ICM condition is slack, benefits should be constant. However, as M is reduced, the ICM condition might eventually bind, once more calling for an upward sloping benefit schedule.

5 Conclusion

In this paper, we have argued that there are reasons to believe that an important information problem associated with unemployment insurance has been neglected in the previous literature. This problem stems from the fact that unemployed individuals sometimes have the option of making an investment that could increase their chances of finding a job. Examples of such investments are retraining and moving to another location. Since it is reasonable to assume that it is difficult or impossible to observe who has these options, the UI system should give incentives for people to take advantage of any reasonable option to increase their labor market prospects. By deriving graphical and analytical closed-form solutions for how a simple UI system should be constructed to provide sufficient incentives without excessively reducing the value of the unemployment insurance. Unless the hiring rates of long-term unemployed are very low and search costs high, this requires an initial period of relatively low benefits. The intuition here is straightforward, by setting initial benefits at a low level, individuals with good opportunities to get new jobs are induced to exploit these. On the other hand, individuals with worse opportunities value insurance against long-term unemployment more than insurance against short-term unemployment. The value of the UI system can therefore be maintained by providing more generous benefits for long-term unemployment, calling for an upward sloping benefit profile.

We have assumed that individuals can self-insure via unobservable savings, i.e., that individual consumption is unobservable or, for some other reason, uncontractable. If, in contrast, the insurer has control over the consumption of the individual, it is well known that there would be a tendency to provide a downward sloping path of consumption (and benefits, if the individual has no other income) to provide good search incentives. Nevertheless, the point of this paper, that a period of low initial UI benefits is an efficient way of separating individuals who can move from those who cannot, is likely to still be true.

We have also assumed constant absolute risk-aversion, even if this representation of individual preferences is not necessarily the most realistic. Under the more standard assumption of constant relative risk-aversion, the analysis is greatly complicated by the fact that search incentives would depend on asset holdings. Therefore, incentive compatibility would not in general be consistent with a finite number of benefits that are independent of individual asset holdings. However, the intuition for the results in this paper does not appear to be related to such effects. In our model, the preference for increasing benefits arises from the need to separate between the two types of workers and the fact that individual assets are depleted during unemployment, (which is true for general specifications of utility, in particular for CRRA, as shown in e.g., Hassler and Rodríguez Mora (1999)). Both mechanisms are likely to be present also under more general preference specifications. However, since search incentives in general depend on asset holdings and the duration of unemployment is likely to be correlated with the individual's asset holdings, unobservability of the latter may have consequences for optimal benefit time profiles. For example, if the search incentives are reinforced as wealth decumulates and individuals with long unemployment spells are likely to have less wealth, this might strengthen the case for increasing benefits. The analysis of optimal UI design with hidden savings when individual behavior depends on asset holdings is likely to demand numerical models. This is left to future research.

References

Arpad, A. and Pavoni, N.: 2005, The efficient allocation of consumption under moral hazard and hidden access to the credit market, *Journal of the European Economic Association* **3**, 370–81.

Bartel, A. P.: 1979, The migration decision: What role does job mobility play?, *American Economic Review* **69**(5), 775–786.

Cahuc, P. and Lehmann, E.: 2000, Should unemployment benefits decrease with the unemployment spell?, *Journal of Public Economics* **77**(1), 135–53.

Cullen, J. B. and Gruber, J.: 2000, Does unemployment insurance crowd out spousal labor supply?, *Journal of Labor Economics* **18**(3), 546–72.

DaVanzo, J.: 1978, Does unemployment affect migration?-evidence from micro data, *Review of Economics and Statistics* **6**(4), 504–14.

Engen, E. M. and Gruber, J.: 2001, Unemployment insurance and precautionary saving, *Journal of Monetary Economics* **47**(3), 545–579.

Gruber, J.: 1997, The consumption smoothing effect of unemployment insurance, *American Economic Review* **87**(1), 192–205.

Hagedorn, M., Kaul, A. and Memmel, T.: 2003, An adverse selection model of optimal unemployment insurance. Mimeo, University Pompeu Fabra.

Hassler, J. and Rodríguez Mora, J. V.: 1999, Employment turnover and the public allocation of unemployment insurance, *Journal of Public Economics* **73**(1), 55–83.

Hassler, J., Rodríguez Mora, J. V., Storesletten, K. and Zilibotti, F.: 2004, A positive theory of geographic mobility and social insurance, *International Economic Review* . forthcoming.

Holmlund, B.: 1998, Unemployment insurance in theory and practice, *Scandinavian Journal of economics* **100**(1), 113–141.

Hopenhayn, H. A. and Nicolini, J. P.: 1997, Optimal unemployment insurance, *Journal of Political Economy* **105**(2), 412–438.

McCormick, B.: 1997, Regional unemployment and labour mobility in the uk, *European Economic Review* **41**(3-5), 581–589.

Pavoni, N.: 2006, On optimal unemployment compensation, *Journal of Monetary Economics* . forthcoming.

Pissarides, C. A. and Wadsworth, J.: 1989, Unemployment and the inter-regional mobility of labour, *Economic Journal* **99**(397), 739–755.

Shavell, S. and Weiss, L.: 1979, The optimal payment of unemployment insurance benefis over time, *Journal of Political Economy* **87**(6), 1347–1362.

Wang, C. and Williamson, S. D.: 1996, Unemployment insurance with moral hazard in a dynamic economy, *Carnegie-Rochester Conference Series on Public Policy* **44**, 1–41.

Werning, I.: 2002, Optimal unemployment insurance with unobservable savings. Mimeo, University of Chicago.

Wright, R.: 1986, The redistributive roles of unemployment insurance and the dynamics of voting, *Journal of Public Economics* **31**(3), 377–399.

6 Appendix

6.1 Bellman equations and consumption constants

Guessing that the value function is $-e^{-\gamma(rA_t+\sigma_j)}$ for $j \in \{1,2,3\}$, the Bellman equation for the employed is,

$$\begin{aligned} -\frac{1}{r}e^{-\gamma(rA_t+\sigma_1)} &= \max_{\sigma} -e^{-\gamma(rA_t+\sigma)}dt \\ &\quad - (1-rdt)\left[(1-qdt)\frac{1}{r}e^{-\gamma(rA_{t+dt}+\sigma_1)} + qdt\frac{1}{r}e^{-\gamma(rA_{t+dt}+\sigma_2)}\right]. \end{aligned}$$

Using first-order linear approximations and dividing by $e^{-\gamma r A_t}$, this becomes

$$-\frac{1}{r}e^{-\gamma\sigma_1} = \max_{\sigma} -e^{-\gamma\sigma}dt$$
$$-(1-rdt)\left[(1-qdt)\frac{1}{r}e^{-\gamma\sigma_1}(1-\gamma r(w-\tau-\sigma)dt)+qdt\frac{1}{r}e^{-\gamma\sigma_2}(1-\gamma r(w-\tau-\sigma)dt)\right]$$

Adding $\frac{1}{r}e^{-\gamma\sigma_1}$ to both sides, dividing by dt and letting dt approach zero, yields

$$0 = \max_{\sigma}\left\{-re^{-\gamma(\sigma-\sigma_1)}+r+\gamma r(w-\tau-\sigma)+q\left(1-e^{-\gamma(\sigma_2-\sigma_1)}\right)\right\}. \tag{18}$$

Similarly, for the short-term and long-run unemployed, we obtain

$$0 = \max_{\sigma}\left\{-re^{-\gamma(\sigma-\sigma_2)}+r+\gamma r(b_2-s-\tau-\sigma)+h+f-he^{-\gamma(\sigma_1-\sigma_2)}-fe^{-\gamma(\sigma_3-\sigma_2)}\right\},$$
$$\tag{19}$$
$$0 = \max_{\sigma}\left\{-re^{-\gamma(\sigma-\sigma_3)}+r+\gamma r(b_3-s-\tau-\sigma)+h\left(1-e^{-\gamma(\sigma_1-\sigma_3)}\right)\right\}.$$

Equations (18) and (19) are maximized at $\sigma=\sigma_j$, implying that for the Bellman equation to be satisfied, the constants σ_j, must satisfy (6). Taking the difference between line 1 and 2 and between 1 and 3 in (6) and solving for b_2 and b_3, we **obtain** the implementation mapping (15).

6.2 The IC2 and IC3 conditions

Let us now consider the incentives for searching during unemployment. We first note if a long-term unemployed does not search, she gets an income $b_3-\tau$ forever**,** implying a utility $-\frac{1}{r}e^{-\gamma r A_t}e^{-\gamma(b_3-\tau)}$, while she gets $-\frac{1}{r}e^{-\gamma r A_t}e^{-\gamma\sigma_3}$ if she searches. Therefore, we need $\sigma_3 \geq b_3-\tau$ to induce search of the long-term unemployed. Using (6), this implies

$$\Delta_3 \geq -\gamma^{-1}\ln\left(1-\gamma r\frac{s}{h}\right), \tag{20}$$

which is the *IC3-condition.*

For the short-term unemployed, we compute the value associated with a one-period deviation, i.e., no search in the current employment state, conditional on searching in future states. This value is $-\frac{e^{-\gamma r A_t}e^{-\gamma c_{2,n}}}{r}$**,** where $\sigma_{2,n}$ satisfies

$$\sigma_{2,n} = b_2-\tau+\frac{f\left(1-e^{-\gamma(\sigma_3-\sigma_{2,n})}\right)}{\gamma r}.$$

The IC2 constraint is given by

$$\sigma_2 - \sigma_{2,n} \geq 0.$$

Furthermore,

$$\begin{aligned} \sigma_2 - \sigma_{2,n} &= \left(-s + \frac{h\left(1 - e^{-\gamma\Delta_2}\right)}{\gamma r} - \frac{f\left(e^{\gamma(\Delta_3-\Delta_2)} - e^{-\gamma(\sigma_3 - \sigma_{2,n})}\right)}{\gamma r} \right) \\ &= \left(-s + \frac{h}{\gamma r}\left(1 - e^{-\gamma\Delta_2}\right) - \frac{f}{\gamma r} e^{\gamma(\Delta_3-\Delta_2)} \left(1 - e^{-\gamma(\sigma_2 - \sigma_{2,n})}\right) \right) \\ &\equiv R\left(\sigma_2 - \sigma_{2,n}\right) \end{aligned} \tag{21}$$

Clearly, R is a monotonously decreasing function with a horizontal asymptote at $-s + \frac{h}{\gamma r}\left(1 - e^{-\gamma\Delta_2}\right) - \frac{f}{\gamma r} e^{\gamma(\Delta_3-\Delta_2)}$ (achieved as $\sigma_2 - \sigma_{2,n}$ approaches infinity), approaches infinity as $\sigma_2 - \sigma_{2,n}$ approaches minus infinity and $R(0) = -s + \frac{h}{\gamma r}\left(1 - e^{-\gamma\Delta_2}\right)$. The solution to (21) is the unique fixed-point of R. This value is non-negative if and only if $-s + \frac{h}{\gamma r}\left(1 - e^{-\gamma\Delta_2}\right) \geq 0$. So

$$\sigma_2 \geq \sigma_{2,n} \Leftrightarrow \Delta_2 \geq -\gamma^{-1} \ln\left(1 - \frac{\gamma r s}{h}\right).$$

7 Proofs not intended for publication

7.1 Proof that results extend to n unemployment states

Suppose we have n states, then the consumption constants are

$$\sigma_1 = w - \tau - q\frac{pe^{\gamma r m} + (1-p)\,e^{\gamma\Delta_2} - 1}{\gamma r}, \tag{22}$$

$$\sigma_2 = b_2 - s - \tau + h\frac{1 - e^{-\gamma\Delta_2}}{\gamma r} - f\frac{e^{\gamma(\Delta_3-\Delta_2)} - 1}{\gamma r},$$

$$\sigma_3 = b_3 - s - \tau + h\frac{1 - e^{-\gamma\Delta_3}}{\gamma r} - f_3\frac{e^{\gamma(\Delta_4-\Delta_3)} - 1}{\gamma r} \tag{23}$$

$$\ldots \tag{24}$$

$$\sigma_{n-1} = b_{n-1} - s - \tau + h\frac{1 - e^{-\gamma\Delta_{n-1}}}{\gamma r} - f_{n-1}\frac{e^{\gamma(\Delta_n-\Delta_{n-1})} - 1}{\gamma r} \tag{25}$$

$$\sigma_n = b_n - s - \tau + h\frac{1 - e^{-\gamma\Delta_n}}{\gamma r}.$$

Now, $\tau = \sum_{s=2}^{n} b_s \Pi_s$, and assume the ICM constraint to be binding, so $\Delta_2 = rm$, implying that we should minimize taxes. Using the above, and $\Delta_2 = rm$ we have

$$
\begin{aligned}
\Delta_2 &= w - b_2 + s - q\frac{e^{\gamma rm}-1}{\gamma r} - h\frac{1-e^{-\gamma rm}}{\gamma r} + f\frac{e^{\gamma(\Delta_3-\Delta_2)}-1}{\gamma r} \\
\Delta_3 &= w - b_3 + s - q\frac{e^{\gamma rm}-1}{\gamma r} - h\frac{1-e^{-\gamma \Delta_3}}{\gamma r} + f_3\frac{e^{\gamma(\Delta_4-\Delta_3)}-1}{\gamma r} \\
&\dots \\
\Delta_{n-1} &= w - b_{n-1} + s - q\frac{e^{\gamma rm}-1}{\gamma r} - h\frac{1-e^{-\gamma \Delta_{n-1}}}{\gamma r} + f_{n-1}\frac{e^{\gamma(\Delta_n-\Delta_{n-1})}-1}{\gamma r} \\
\Delta_n &= w - b_n + s - q\frac{e^{\gamma rm}-1}{\gamma r} - h\frac{1-e^{-\gamma(\Delta_n)}}{\gamma r}
\end{aligned}
$$

or

$$
\begin{aligned}
b_2 &= w - \Delta_2 + s - q\frac{e^{\gamma rm}-1}{\gamma r} - h\frac{1-e^{-\gamma rm}}{\gamma r} + f_2\frac{e^{\gamma(\Delta_3-\Delta_2)}-1}{\gamma r} \\
b_3 &= w - \Delta_3 + s - q\frac{e^{\gamma rm}-1}{\gamma r} - h\frac{1-e^{-\gamma \Delta_3}}{\gamma r} + f_3\frac{e^{\gamma(\Delta_4-\Delta_3)}-1}{\gamma r} \\
&\dots \\
b_{n-1} &= w - \Delta_{n-1} + s - q\frac{e^{\gamma rm}-1}{\gamma r} - h\frac{1-e^{-\gamma \Delta_{n-1}}}{\gamma r} + f_{n-1}\frac{e^{\gamma(\Delta_n-\Delta_{n-1})}-1}{\gamma r} \\
b_n &= w - \Delta_n + s - q\frac{e^{\gamma rm}-1}{\gamma r} - h\frac{1-e^{-\gamma(\Delta_n)}}{\gamma r}
\end{aligned}
$$

$$
\begin{aligned}
\tau = {} & \Pi_2\left(w - rm + s - q\frac{e^{\gamma rm}-1}{\gamma r} - h\frac{1-e^{-\gamma rm}}{\gamma r} + f\frac{e^{\gamma(\Delta_3-rm)}-1}{\gamma r}\right) \\
& + \sum_{i=3}^{n-1}\Pi_3\left(w - \Delta_i + s - q\frac{e^{\gamma rm}-1}{\gamma r} - h\frac{1-e^{-\gamma\Delta_i}}{\gamma r} + f_s\frac{e^{\gamma(\Delta_{i+1}-\Delta_i)}-1}{\gamma r}\right) \\
& + \Pi_n\left(w - \Delta_n + s - q\frac{e^{\gamma rm}-1}{\gamma r} - h\frac{1-e^{-\gamma\Delta_n}}{\gamma r}\right)
\end{aligned}
$$

Removing constants,

$$
\begin{aligned}
\tau = {} & \text{constant} + \Pi_2\left(f\frac{e^{\gamma(\Delta_3-rm)}}{\gamma r}\right) \\
& + \sum_{i=3}^{n-1}\Pi_i\left(-\Delta_i + h\frac{e^{-\gamma\Delta_i}}{\gamma r} + f_s\frac{e^{\gamma(\Delta_{i+1}-\Delta_i)}}{\gamma r}\right) \\
& + \Pi_n\left(-\Delta_n + h\frac{e^{-\gamma\Delta_n}}{\gamma r}\right)
\end{aligned}
$$

First-order conditions are

$$\Delta_{i\in\{3,n-1\}}; \Pi_{i-1}\frac{f_{i-1}}{r}e^{\gamma(\Delta_i-\Delta_{i-1})} - \Pi_i\left(1+\frac{h}{r}e^{-\gamma\Delta_{i-1}}+\frac{f_i}{r}e^{\gamma(\Delta_i-\Delta_{i-1})}\right) = 0$$

$$\Delta_n; \Pi_{n-1}\frac{f_{n-1}}{r}e^{\gamma(\Delta_n-\Delta_{n-1})} - \Pi_n\left(1+\frac{h}{r}e^{-\gamma\Delta_n}\right) = 0,$$

where $\Delta_2 = rm$.

Suppose that this is satisfied for $\Delta_3 = \Delta_4 = ...\Delta_n = \Delta$.Then,

$$\begin{aligned}
e^{\gamma(\Delta-rm)} &= \frac{r\Pi_3}{f_2\Pi_2}\left(1+\frac{h}{r}e^{-\gamma\Delta}+\frac{f_3}{r}\right)\\
\frac{f_{i-1}}{r} &= \frac{\Pi_i}{\Pi_{i-1}}\left(1+\frac{h}{r}e^{-\gamma\Delta}+\frac{f_i}{r}\right)\\
\frac{f_{n-1}}{r} &= \frac{\Pi_n}{\Pi_{n-1}}\left(1+\frac{h}{r}e^{-\gamma\Delta}\right)
\end{aligned}$$

or

$$\begin{aligned}
e^{\gamma(\Delta-rm)} &= \frac{r\Pi_3}{f_2\Pi_2}\left(1+\frac{h}{r}e^{-\gamma\Delta}+\frac{\Pi_4}{\Pi_3}\left(1+\frac{h}{r}e^{-\gamma\Delta}+\frac{f_4}{r}\right)\right)\\
&= \frac{r\Pi_3}{f_2\Pi_2}\left(1+\frac{h}{r}e^{-\gamma\Delta}+\frac{\Pi_4}{\Pi_3}\left(1+\frac{h}{r}e^{-\gamma\Delta}+\frac{\Pi_5}{\Pi_4}\left(1+\frac{h}{r}e^{-\gamma\Delta}+\frac{f_5}{r}\right)\right)\right)\\
&= \frac{r}{f_2}\left(1+\frac{h}{r}e^{-\gamma\Delta}\right)\left(\left(\frac{\Pi_3}{\Pi_2}+\frac{\Pi_4}{\Pi_3}+...+\frac{\Pi_{n-1}}{\Pi_{n-2}}\right)+\frac{\Pi_n}{\Pi_2}\right)\\
&= \frac{r}{f_2}\left(1+\frac{h}{r}e^{-\gamma\Delta}\right)\left(\sum_{i=3}^{n-1}\frac{\Pi_i}{\Pi_{i-1}}+\frac{\Pi_n}{\Pi_2}\right)
\end{aligned}$$

Clearly, there exists a Δ^* such that this is satisfied, consequently $\Delta_i = \Delta^* \forall i \in \{3, 4, ..., n\}$ satisfies all first-order conditions. This allocation is then implemented by a $\tilde{b}_2^*$ and a constant benefit sequence $\tilde{b}_3^* = \tilde{b}_4^* = ...\tilde{b}_n^*$. Finally, we note that since individuals face identical conditions in states $3, ...n$, the allocation would not change if the number of states **were** reduced as long as $n > 3$. Thus, the optimal value of b_2 is independent of n if $n > 3$. Consequently, the optimal benefit schedule is to have $b_2 = b_2^*$ and a constant benefit level $b_3 = b_3^*$ thereafter.

7.2 Derivation of (9)

The consumption difference is

$$\begin{aligned}
\Delta_2 &= w - \tau - q\frac{pe^{\gamma rm} + (1-p)\, e^{\gamma(\Delta_2)} - 1}{\gamma r} - \left(b_2 - s - \tau + h\frac{1 - e^{-\gamma(\Delta_2)}}{\gamma r}\right) \\
&= w - b_2 + s - q\frac{pe^{\gamma rm} + (1-p)\, e^{\gamma(\Delta_2)} - 1}{\gamma r} - \left(h\frac{1 - e^{-\gamma(\Delta_2)}}{\gamma r}\right),
\end{aligned}$$

giving

$$b_2 = w + s - q\frac{pe^{\gamma rm} - 1}{\gamma r} - \frac{h}{\gamma r} - \Delta_2 - q\frac{(1-p)\, e^{\gamma\Delta_2}}{\gamma r} + \frac{he^{-\gamma\Delta_2}}{\gamma r}.$$

Collecting constants we get

$$\begin{aligned}
\sigma_1 &= w - \Pi_2\left(w + s - q\frac{pe^{\gamma rm} - 1}{\gamma r} - \frac{h}{\gamma r} - \Delta_2 - q\frac{(1-p)\, e^{\gamma\Delta_2}}{\gamma r} + \frac{he^{-\gamma\Delta_2}}{\gamma r}\right) \\
&\quad - q\frac{pe^{\gamma rm} - 1}{\gamma r} - q\frac{(1-p)\, e^{\gamma\Delta_2}}{\gamma r} \\
&= w - \Pi_2\left(w + s - q\frac{pe^{\gamma rm} - 1}{\gamma r} - \frac{h}{\gamma r}\right) - q\frac{pe^{\gamma rm} - 1}{\gamma r} \\
&\quad + \Pi_2\left(\Delta_2 + q\frac{(1-p)\, e^{\gamma\Delta_2}}{\gamma r} - \frac{he^{-\gamma\Delta_2}}{\gamma r}\right) - q\frac{(1-p)\, e^{\gamma\Delta_2}}{\gamma r} \\
&= \kappa + \Pi_2\left(\Delta_2 + q\frac{(1-p)\, e^{\gamma\Delta_2}}{\gamma r} - \frac{he^{-\gamma\Delta_2}}{\gamma r}\right) - q\frac{(1-p)\, e^{\gamma\Delta_2}}{\gamma r} \\
&= \kappa + \Pi_2\left(\Delta_2 - \frac{he^{-\gamma\Delta_2}}{\gamma r}\right) - (1 - \Pi_2)\, q\frac{(1-p)\, e^{\gamma\Delta_2}}{\gamma r}.
\end{aligned}$$

7.3 Derivation of 13

Doing the substitution in the text and collecting endogenous terms, we have

$$\begin{aligned}
\sigma_1 &= w - \Pi_2\left(w + s - q\frac{pe^{\gamma rm} - 1}{\gamma r} - (h + f)\,\frac{1}{\gamma r}\right) - \Pi_3\left(w + s - q\frac{pe^{\gamma rm} - 1}{\gamma r} - \frac{h}{\gamma r}\right) - q\frac{pe^{\gamma rm} - 1}{\gamma r} \\
&\quad - \Pi_2\left(-\Delta_2 - q\frac{(1-p)\, e^{\gamma\Delta_2}}{\gamma r} + h\frac{e^{-\gamma\Delta_2}}{\gamma r} + f\frac{e^{\gamma(\Delta_3 - \Delta_2)}}{\gamma r}\right) \\
&\quad - \Pi_3\left(-\Delta_3 - q\frac{(1-p)\, e^{\gamma\Delta_2}}{\gamma r} + h\frac{e^{-\gamma\Delta_3}}{\gamma r}\right) - q\frac{(1-p)\, e^{\gamma\Delta_2}}{\gamma r} \\
&= \kappa_2 + \Pi_2\Delta_2 + \Pi_3\Delta_3 - (1 - \Pi_2 - \Pi_3)\, q\frac{(1-p)\, e^{\gamma\Delta_2}}{\gamma r} \\
&\quad - \Pi_2\left(h\frac{e^{-\gamma\Delta_2}}{\gamma r} + f\frac{e^{\gamma(\Delta_3 - \Delta_2)}}{\gamma r}\right) - \Pi_3 h\frac{e^{-\gamma\Delta_3}}{\gamma r}.
\end{aligned}$$

7.4 Indifference curves

The objective function is

$$\sigma_1 = \kappa_2 + \Pi_2\Delta_2 + \Pi_3\Delta_3 - (1-\Pi_2-\Pi_3)\frac{q(1-p)}{\gamma r}e^{\gamma\Delta_2} - \Pi_2\left(h\frac{e^{-\gamma\Delta_2}}{\gamma r} + f\frac{e^{\gamma(\Delta_3-\Delta_2)}}{\gamma r}\right) - \Pi_3 h\frac{e^{-\gamma\Delta_3}}{\gamma r}. \tag{26}$$

Differentiation gives

$$\left(\Pi_2 - (1-\Pi_2-\Pi_3)\frac{q(1-p)}{r}e^{\gamma\Delta_2} + \Pi_2 h\frac{e^{-\gamma\Delta_2}}{r} + \Pi_2 f\frac{e^{\gamma(\Delta_3-\Delta_2)}}{r}\right)d\Delta_2$$
$$= -\left(\Pi_3 - \Pi_2 f\frac{e^{\gamma(\Delta_3-\Delta_2)}}{r} + \Pi_3 h\frac{e^{-\gamma\Delta_3}}{r}\right)d\Delta_3$$

$$\frac{d\Delta_2}{d\Delta_3}|_{\sigma_1\text{constant}} = -\frac{\left(\Pi_3 - \Pi_2 f\frac{e^{\gamma(\Delta_3-\Delta_2)}}{r} + \Pi_3 h\frac{e^{-\gamma\Delta_3}}{r}\right)}{\left(\Pi_2 - (1-\Pi_2-\Pi_3)\frac{q(1-p)}{r}e^{\gamma\Delta_2} + \Pi_2 h\frac{e^{-\gamma\Delta_2}}{r} + \Pi_2 f\frac{e^{\gamma(\Delta_3-\Delta_2)}}{r}\right)}$$
$$= -\frac{\frac{f}{h+r} - \frac{1}{r}\left(fe^{\gamma(\Delta_3-\Delta_2)} - \frac{fh}{h+r}e^{-\gamma\Delta_3}\right)}{1 - \frac{1}{r}\left((r+h+f)e^{\gamma\Delta_2} - he^{-\gamma\Delta_2} - fe^{\gamma(\Delta_3-\Delta_2)}\right)}$$

7.5 Different search and hiring probabilities

Here, we **formally analyze** the case when s and h are state dependent. We first have that

$$\sigma_1 = w - \tau - q\frac{pe^{\gamma rm} + (1-p)e^{\gamma(\Delta_2)} - 1}{\gamma r}, \tag{27}$$
$$\sigma_2 = b_2 - s_2 - \tau + h_2\frac{1-e^{-\gamma(\Delta_2)}}{\gamma r} - f\frac{e^{\gamma(\Delta_3-\Delta_2)} - 1}{\gamma r},$$
$$\sigma_3 = b_3 - s_3 - \tau + h_3\frac{1-e^{-\gamma(\Delta_3)}}{\gamma r} \tag{28}$$

The IC2 and IC3 conditions are

$$\Delta_2 \geq -\gamma^{-1}\ln\left(1-\gamma r\frac{s_2}{h_2}\right)$$
$$\Delta_3 \geq -\gamma^{-1}\ln\left(1-\gamma r\frac{s_3}{h_3}\right)$$

and the implementation equations

$$
\begin{aligned}
b_2 &= w + s_2 - \Delta_2 - \frac{q\left(e^{\gamma\Delta_2} - 1\right) + h_2\left(1 - e^{-\gamma\Delta_2}\right) - f\left(e^{\gamma(\Delta_3-\Delta_2)} - 1\right)}{\gamma r} \\
b_3 &= w + s_3 - \Delta_3 - \frac{q\left(e^{\gamma\Delta_2} - 1\right) + h_3\left(1 - e^{-\gamma\Delta_3}\right)}{\gamma r}.
\end{aligned} \tag{29}
$$

Fixing Δ_2 and assuming that s_3 increases while respecting $\Delta_3 = -\gamma^{-1}\ln\left(1 - \gamma r \frac{s_3}{h_3}\right)$, we see that

$$
\begin{aligned}
\frac{db_2}{ds_3} &= \frac{\partial b_2}{\partial \Delta_3}\frac{\partial \Delta_3}{\partial s_3} = \frac{fe^{\gamma(\Delta_3-\Delta_2)}}{r}\frac{r}{h_3 - \gamma r s_3} = \frac{fe^{\gamma(\Delta_3-\Delta_2)}}{h_3\left(1 - \gamma r \frac{s_3}{h_3}\right)} > 0, \\
\frac{db_3}{ds_3} &= 1 - \frac{\partial \Delta_3}{\partial s_3}\left(1 + \frac{h_3 e^{-\gamma\Delta_3}}{r}\right) \\
&= 1 - \frac{r}{h_3 - \gamma r s_3}\frac{r + h_3 e^{-\gamma\Delta_3}}{r} \\
&= 1 - \frac{r}{h_3 - \gamma r s_3}\frac{r + h_3\frac{h_3 - \gamma r s_3}{h_3}}{r} \\
&= -\frac{r}{h_3\left(1 - \gamma r \frac{s_3}{h_3}\right)} < 0.
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
\frac{db_2}{dh_3} &= \frac{\partial b_2}{\partial \Delta_3}\frac{\partial \Delta_3}{\partial h_3} = \frac{fe^{\gamma(\Delta_3-\Delta_2)}}{r}\frac{-rs_3}{h_3^2\left(1 - \gamma r \frac{s_3}{h_3}\right)} < 0. \\
\frac{db_3}{dh_3} &= -\frac{\partial \Delta_3}{\partial h_3}\left(1 + \frac{h_3 e^{-\gamma\Delta_3}}{r}\right) - \frac{\left(1 - e^{-\gamma\Delta_3}\right)}{\gamma r} \\
&= \frac{rs_3}{h_3\left(h_3 - \gamma r s_3\right)}\frac{r + h_3 e^{\ln\left(1 - \gamma r \frac{s_3}{h_3}\right)}}{r} - \frac{\left(1 - e^{\ln\left(1 - \gamma r \frac{s_3}{h_3}\right)}\right)}{\gamma r} \\
&= \frac{rs_3}{h_3^2\left(1 - \gamma r \frac{s_3}{h_3}\right)} > 0.
\end{aligned}
$$