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This paper provides an analytical characterization of Markov perfect equilibria in a model with repeated voting, where agents vote over distortionary income redistribution. A key result is that the future constituency for redistributive policies depends positively on current redistribution, since this affects both private investments and the future distribution of voters. The model features multiple equilibria. In some equilibria, positive redistribution persists forever. In other equilibria, even a majority of beneficiaries of redistribution vote strategically so as to induce the end of the welfare state next period. Skill-biased technical change makes the survival of the welfare state less likely. (JEL D72, E62, H11, H31, P16)

There is now a growing literature bringing politico-economic aspects into macroeconomics. Rather than treating policies as exogenous instruments in the hands of benevolent policy makers, this recent literature describes government policies as endogenous outcomes collectively determined by rational self-interested individuals. While many important issues are dynamic in nature, technical limitations have,

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however, so far prevented a thorough investigation of dynamic political choices in macroeconomics. This paper takes a step towards overcoming these difficulties.

In particular, we construct a tractable positive theory of the dynamics of income redistribution where policy is set through repeated voting by forward-looking rational agents, and current policy affects future political outcomes through changes in the distribution of voters. Until now, the literature has resorted to numerical techniques to study this link and our contribution is therefore partly methodological. Equally important, our theory provides insights on salient aspects of the debate on the determinants of redistributive policies (the "welfare state"). First, it predicts that welfare state policies and their effects on distribution are persistent: shocks to the income distribution that would have transitory effects if policies were exogenous may lead to permanent changes in the demand for redistributive policies. These, in turn, affect private investment behavior and the future dynamics of income distribution. Second, it suggests that welfare state institutions are intrinsically fragile, since even political majorities benefiting from redistribution may want to strategically vote for policies leading to the dismantlement of the welfare state. The latter prediction hinges, in our model, on rational dynamic voting and would be absent if agents voted myopically, ignoring the effect of current political decisions on future political outcomes. Thus, our theory provides an example of how

forward-looking political behavior may qualitatively change the predictions of theory.

Our model is close in spirit to the canonical politico-economic model of Allan H. Meltzer and Scott F. Richard (1981), where agents vote over redistribution financed by distortionary taxes in a static setting. Our economy is populated by two-period lived agents who are ex ante identical, but ex post heterogeneous. The overlapping generation structure is intended to capture the idea that, as life goes by, uncertainty about lifetime income is resolved. While young individuals are born identical, the old individuals have heterogeneous preferences for redistribution, since the resolution of uncertainty has turned some of them into high-income ("successful") individuals and others into low-income ("unsuccessful") individuals. A key assumption is that young individuals can affect their chances of becoming successful by making a private (human-capital) investment when young. The optimal investment is negatively affected by the extent of current and future redistribution, which is set period by period in political elections. Voters are fully rational, and take into account the effects of policies on current investments and on the future distribution of voters.

The focus of the paper is the ex post conflict over redistribution and if it can, on its own, lead to the perpetual survival of the welfare state. To this end, we assume that individuals are risk neutral, abstracting from a standard alternative motivation for the welfare state, i.e., that a government can deliver the insurance missing markets fail to provide. Our assumption of risk neutrality and the fact that redistribution is distortionary, imply that any allocation with some redistribution (after the first period) would not be Pareto optimal. Thus, the welfare state would not survive if the future path of redistribution were set by a utilitarian planner attaching any arbitrary sequence of positive weights on current and future generations. In this sense, the survival of a welfare state would constitute a "political failure," as defined by Timothy Besley and Stephen Coate (1998). Similarly, there would be no welfare state if young agents could commit to vote in a particular way in the future. However, as such commitments are not feasible in democratic systems, ex post conflicts influence political outcomes.

The theory has two main predictions. First,

the political mechanism can sustain the welfare state. In particular, if the economy starts with a pro-redistribution majority, high levels of redistribution can be sustained over time, whereas there will not be a welfare state if the economy starts with an anti-redistribution majority. Moreover, if a one-time shock creates a temporary political majority in favor of redistribution, the model predicts that the support for the welfare state can continue and regenerate a constituency for such policies. This result is due to a selfreinforcing mechanism linking private and collective choices: high current redistribution reduces investments, implying that a larger share of future voters will benefit from redistributive policies.¹

Second, there exist equilibria where an existing welfare state is irreversibly terminated by forward-looking voters, even when benefit recipients are initially politically decisive. In these equilibria, an initial pro-welfare state majority votes strategically for moderate redistribution so as to induce a future anti-welfare state majority. The expectation that the welfare state will vanish strengthens the incentives of the young to invest, thereby reducing the dependency ratio and current taxes. Furthermore, in an extension, we show that the breakdown of the welfare state becomes more likely when the pretax wage inequality is large, since such inequality strengthens the incentives for private investment and reduces, ceteris paribus, the constituency of the welfare state.

The first prediction of the theory, i.e., that redistributive programs tend to be persistent, is consistent with a number of empirical observations. For instance, a number of welfare state institutions were introduced in the aftermath of the Great Depression and after World War II, when large masses of people were impoverished, thereby creating a demand for public intervention.² The size of government pro-

² Between 1929 and 1934, government spending as a fraction of GNP doubled in the United States as well as in

¹ In related papers, Hassler et al. (1999, 2001a, 2002), we explore other examples of this mechanism in settings where unemployment interacts with the provision of unemployment insurance. Other related contributions are Assar Lindbeck (1995) and Lindbeck et al. (1999) who stress that policy persistence may arise from gradual changes in social norms vis-à-vis recipients of social assistance when a large mass of agents become dependent on such safety nets.

grams, redistributional policies and public employment did not diminish after the economies had recovered from the shocks. Instead, these policies persisted and were further expanded in the 1960's. According to our theory, this persistence stems from the fact that once they are in place, redistributive programs, government employment, etc., affect private incentives in a way generating a sustained demand for their continuation. Our theory predicts that not only policies, but also their effects on income distribution, are persistent. Such joint persistence is consistent with the dynamics of unemployment and unemployment insurance in European countries after the oil shocks. The sharp increase in unemployment after these shocks was followed by increasing unemployment benefits and larger tax wedges, which contributed to sustaining large unemployment and generating hysteresis.³ The second prediction of the theory is more difficult to assess empirically. It is, however, broadly consistent with the observation that conservative governments proposing drastic reductions in social policies were elected and reelected in the 1980's in Anglo-Saxon countries, in times associated with a significant increase in wage inequality. This political development has not been mirrored in continental European countries, where changes in wage inequality were less pronounced.

Our theory may contribute to the understanding of various aspects of the dynamics of redistribution, although several important elements are missing in our highly stylized setting. In reality, the political debate is multidimensional, with different issues being salient in different elections. Social groups have conflicting interests on different aspects of the welfare state, and governments have access to more sophisticated policies than our simple rich-to-poor transfer system. For instance, the accumulation of government debt is ruled out throughout the paper. This is as an important limitation for understanding the intergenerational conflict. Finally, we have, for simplicity, abstracted from risk aversion, while fully acknowledging that the insurance motive may be important for understanding the demand for redistribution. While these are all important limitations, we believe that our tractable framework can be enriched and further developed to account for some of these and other aspects of how distributional conflicts are resolved in a dynamic political context.⁴

There are a number of instances, both in the economic literature and the policy debate, where dynamic links between current and future political choices and constituencies are of firstorder importance. For instance, in the debate on the optimal speed of transition in postcommunist countries, a number of economists have stressed that gradualism in reforms such as restructuring and reorganizing labor markets, privatizing firms and liberalizing prices, may be preferable to a "big bang" approach. The reason is that the latter may give rise to majorities of stakeholders with an interest in blocking or reverting the path of reform at some stage in the process. In contrast, gradual reforms are argued to allow "building constituencies for further reforms" by starting with easier reforms designed to increase future support for more difficult reforms (see Mathias Dewatripont and Gerard Roland, 1995).⁵ Dynamic voting aspects have also been regarded as important in understanding the transition to democracy. Daron Acemoglu and James A. Robinson (2000, 2001) argue that the political elites extended franchise over the nineteenth century in order to commit to

the major European countries, e.g., France and Italy. A change of the same magnitude occurred in the United Kingdom in the 1930's, although most of the expansion took place after 1934.

³ Unemployment in OECD Europe rose from an average of 2.4 percent during 1969–1973 to 8.0 percent during 1985–1989, while unemployment benefit replacement ratios in OECD Europe rose from 18 percent to 30 percent for the same time periods. (Source: OECD Economic Outlook and OECD database on Benefit Entitlements and Gross Replacement Ratios.)

⁴ Hassler et al. (2003), for instance, build on the setup of this paper and consider the choice of redistribution in a more elaborate political model with probabilistic voting and risk-averse agents.

⁵ Gradual reforms are also argued to allow for divideand-rule tactics (Dewatripont and Roland, 1992a, b): if it is too expensive to compensate a sufficient constituency of workers today for the costs of massive layoffs, the government could instead compensate a minority of workers today, so that these will, together with those workers who eventually are retained, secure a constituence in favor of reform in the next period. Philippe Aghion and Olivier J. Blanchard (1994) argue that fast reforms would raise the demand for social policies to compensate losers (e.g., unemployment benefits), and the fiscal effects of these policies will slow down the entry of new firms in the reformed sectors.

MARCH 2003

sustained redistributive policies in return for less social unrest. Moreover, land reforms prior to democratization were implemented in order to reduce inequality, thereby ensuring that the poor, once in power, would limit their future demands for redistribution. Such restraints consolidated democracy by reducing the risk of the rich mounting a coup.⁶

Several earlier papers have analyzed the political economy of redistribution, but earlier models had to assume either myopic voting behavior, as in Alberto Alesina and Dani Rodrik (1994); or that current voters can commit to future policies once and for all, as in Richard W. Boadway and David E. Wildasin (1989) and Giuseppe Bertola (1993); or, finally, had to limit the attention to environments with no strategic interaction between voters at different dates, as in Roland Benabou (1996, 2000) and Torsten Persson and Guido Tabellini (1994). In contrast, our paper provides an analytical characterization of Markov perfect equilibria without commitment in a politico-economic model where rational voters face a strategic voting incentive.

To the best of our knowledge, the only previous paper that works out an analytical solution to the Markov perfect equilibria of a dynamic political economy model is Gene M. Grossman and Elhanan Helpman (1998). They analyze the political determination of intergenerational redistribution in a growth model with overlapping

⁶ Strategic voting consideration are often concealed in political debates, arguably for reasons of political correctness. In some cases, however, they have been made explicit. In the debate on the European Monetary Union, for example, Euro-sceptics have argued that a monetary union is intended as a stepping stone towards a more politically integrated federal Europe (see, for instance, Lord Skidelsky's speech in the British House of Lords, January 20, 1999; or, on the opposite side, Romano Prodi's speech to the European Parliament, April 13, 1999). Among the negative effects, critics see the consolidation of a European bureaucracy that will lobby for further centralization of power in the future.

generations, lobbies, and an AK technology. In their model, however, agents make no private economic decisions, and thus there is no feedback between public policy and individual behavior, a central mechanism in our analysis. The equilibrium of their linear model features a broad range of indeterminate political choices. Other models have incorporated repeated voting with strategic interactions, but only yielded numerical solutions (e.g., Krusell and Ríos-Rull, 1996, 1999; Krusell et al., 1996; Marco Bassetto, 1999; and Gilles Saint Paul, 2001). Saint Paul (2001), in particular, numerically solves a politico-economic model with dynamic voting, where redistribution may fall after increases in inequality if such inequality is concentrated at the lower tail of the distribution, so that the median voter becomes richer. Saint Paul's (2001) paper documents that this may be a realistic description of the political changes that occurred in Anglo-Saxon countries in the 1980's, an episode that is also consistent with our theory, as discussed above.

In our model, expectations of high future redistribution leads to lower investments, which, in turn, increase future demand for redistribution. Such a feedback mechanism is present in a number of previous papers, including Gerhard Glomm and B. Ravikumar (1995), Saint Paul and Thierry Verdier (1997), and Benabou (2000).⁷ In these papers, however, the median voter has no stake in future political outcomes and there is no motive for strategic voting. In our model, in contrast, such motive is present and plays a crucial role. In particular, old voters

⁷ Saint Paul and Verdier (1997) show that multiple equilibria can arise in a politico-economic model where agents vote over capital taxation and have access to opportunities of expatriating their savings at costs varying exogenously across individuals. The expectation of the level of future taxation of domestic savings determines the extent to which young individuals exploit these opportunities. Under the assumption that the median voter has better than average access to international capital markets, multiple equilibria may arise. In Glomm and Ravikumar (1995), the endogenous determination of public expenditure creates multiple equilibria in a model where voters are identical. In their model, expectations of high (low) taxes reduce (increase) private educational investments and future income. Under the assumption that taxes are used to finance an inferior public good, high (low) income implies that the homogeneous voters prefer low (high) public good provision and taxes, leading to the possibility of multiple equilibria.

face an incentive to strategically moderate their demand for current redistribution so as to induce the expectation of a future majority against redistribution, which would lead to lower current taxes. If voters myopically ignored the effect of their political choice on the future distribution of voters, there would be a unique equilibrium where an initial majority of unsuccessful voters sustains the welfare state forever.

While in our paper redistribution has no intrinsic value, there is a large literature arguing that redistribution ameliorates capital market imperfections. In Benabou (2000), for instance, the political support for redistribution is high when the efficiency-enhancing effect dominates the purely redistributive one. This occurs when inequality is sufficiently small and thus, on the one hand, low inequality induces high redistribution. On the other hand, high redistribution sustains low inequality and hence, multiple steady states are possible. Thus, his paper is consistent with the observation that inequality and redistribution are negatively correlated across developed countries.⁸

Among other related papers, Stephen Coate and Stephen Morris (1999) construct a model of special interest groups where firms choose their location on the basis of geographical subsidies and have, ex post (though not ex ante), an incentive to bribe politicians for the subsidies to be continued. Their model features multiple steady states and policy persistence, but the political mechanism is very different from ours. In Thomas Piketty (1995), social learning about the trade-off between efficiency and incentives gives rise to multiple steady states with different levels of redistribution. Finally, a series of papers provide positive theories of social security in repeated voting models (Jose I. Conde Ruiz and Vincenzo Galasso, 1999; Thomas F. Cooley and Jorge Soares, 1999; Michele Boldrin and Aldo Rustichini, 2000). In these papers, inter-

⁸ Politico-economic models a la Meltzer and Richard (1981) typically predict that higher inequality increases the demand for redistribution. In our two-group setup, however, the median voter can be richer than the average voter, and the predictions of our theory about the sign of the correlation between inequality and redistribution is ambiguous. For instance, the equilibrium features zero redistribution if inequality is large and the median voter is richer than average, whereas it features positive redistribution if inequality is voter of the median voter is poorer than the average.

generational redistribution is sustained by trigger strategies in infinite horizon games. In contrast, our results would survive in an environment with a finite horizon.

The plan of the paper is the following. Section I describes the model. Section II characterizes the political equilibria, and Section III explores the implications of rising wage premia. Section IV concludes. All proofs are in the Appendix.

I. The Model

The model economy consists of a continuum of risk-neutral, two-period lived agents. Each generation has a unit mass. All agents are born identical, but their subsequent earnings are stochastic. "Successful" agents earn a high wage, normalized to unity, in both periods of their life, whereas "unsuccessful" agents earn a low wage, normalized to zero. At birth, each agent undertakes a costly investment, thereby increasing the probability of subsequent success. The cost of investment, which can be interpreted as the disutility of educational effort, is e^2 , where *e* is the probability of success.⁹

The dynamics of redistribution from successful to unsuccessful agents is the focal point of the paper. In each period, a transfer $b \in [0, 1]$ to each low-income agent is determined, financed by collecting a lump-sum tax τ .¹⁰ The transfer, and the associated tax rate, are determined before the young agents decide on their investment, and is assumed to be age-independent. We shall, however, maintain that the government budget balances in every period.

The expected utility of agents alive at time t is given as follows:

⁹ It is important for the analysis that agents earn income in both periods. The assumption that first and second period income are perfectly correlated is, however, not essential the qualitative results will be preserved provided that earnings in the two periods are positively correlated.

¹⁰ The assumption that $b \leq 1$ can be motivated by incentive considerations. If redistribution were larger than 100 percent, successful agents would decide not to work and to claim benefits. The assumption that $b \geq 0$ is a useful benchmark, and it can be regarded as the effect of some constitutional principle that public redistribution cannot be regressive. An important consequence of the restrictions on *b* is that optimal effort is bounded between zero and one and therefore can be interpreted as a probability.

(1)
$$\tilde{V}^{os}(b_{t}, b_{t+1}, \tau_{t}) = 1 - \tau_{t}$$

 $\tilde{V}^{ou}(b_{t}, b_{t+1}, \tau_{t}) = b_{t} - \tau_{t}$
 $\tilde{V}^{\prime}(e_{t}, b_{t}, b_{t+1}, \tau_{t}, \tau_{t+1}) = e_{t}(1 + \beta) + (1 - e_{t})$
 $\times (b_{t} + \beta b_{t+1})$
 $- e_{t}^{2} - \tau_{t} - \beta \tau_{t+1},$

where \tilde{V}^{os} , \tilde{V}^{ou} , and \tilde{V}^{y} denote the objective of old successful, old unsuccessful, and young agents, respectively. \tilde{V}^{y} is computed prior to individual success or failure and $\beta \in [0, 1]$ is the discount factor. It is straightforward to show that the solution to the optimal investment problem of the young, given b_t and b_{t+1} , is $e_t^* = e^*(b_t, b_{t+1}) = (1 + \beta - (b_t + \beta b_{t+1}))/2$.

Since agents are *ex ante* identical, agents of the same cohort choose the same investment, which implies that the proportion of old unsuccessful in period t + 1 is given by

(2)
$$u_{t+1} = 1 - e_t^* = \frac{1}{2}(1 - \beta + b_t + \beta b_{t+1}).$$

Thus, the future proportion of old unsuccessful depends on benefits in period t and t + 1. To balance the budget, tax revenues must amount to $2\tau_t = (u_t + u_{t+1})b_t$, yielding

(3)
$$\tau_t = \frac{1}{4}(1 - \beta + b_t + \beta b_{t+1} + 2u_t)b_t$$
.

By substituting for τ_i and e_i^* in equation (1), the indirect utility functions can be written as:

(4)
$$V^{vs}(b_{t}, b_{t+1}, u_{t}) = 1 - \frac{1}{4}((1 - \beta) + (b_{t} + \beta b_{t+1}) + 2u_{t})b_{t}$$
$$V^{vu}(b_{t}, b_{t+1}, u_{t}) = b_{t} - \frac{1}{4}((1 - \beta) + (b_{t} + \beta b_{t+1}) + 2u_{t})b_{t}$$
$$V^{v}(b_{t}, b_{t+1}, b_{t+2}, u_{t}) = \frac{1}{4}((1 + \beta)^{2} + ((1 - \beta) - 2u_{t})b_{t} - (b_{t+1} + \beta b_{t+2})\beta b_{t+1}).$$

Note that taxes per unit of benefits, $\tau_t/b_t = (1 - \beta + b_t + \beta b_{t+1} + 2u_t)/4$, increase in u_t (because higher u_t implies a higher dependency ratio among the old) and in b_t and b_{t+1} (because higher b_t and b_{t+1} reduce investment, implying a higher dependency ratio among the young). Since the old in period t cannot enjoy benefits in period t + 1, their utility is decreasing in b_{t+1} .

The old successful agents obviously prefer zero benefits, since redistribution implies positive taxes without providing any benefits. In contrast, the old unsuccessful agents are better off with some redistribution, even though their preferences for redistribution may be nonmonotonic, as the marginal cost of redistribution is increasing in b_t . Concerning the preferences of the young, note that positive benefits lead to positive (negative) intergenerational redistribution from the old to the young, if the number of old unsuccessful is sufficiently small (large). Holding future benefits constant, the young prefer positive redistribution if and only if $u_t < (1 - \beta)/2$.

Before proceeding to the main analysis, we note that any Pareto-efficient allocation is characterized by zero redistribution in every period except, possibly, in the first.¹¹ The reason is that redistribution distorts the effort choice of the young, but has no insurance value as agents are risk neutral.

¹¹ More formally, we define the class of Pareto-optimal sequences of benefits, $\{b_t\}_{t=1}^{\infty}$, as those which would be chosen by a social planner whose objective function is given by

$$\max_{\{b,b_{t-1}^{\infty}\}} \left\{ \lambda_{0t}(1-u_{1})V^{os}(b_{1}, b_{2}, u_{1}) + \lambda_{0u}u_{1}V^{ou}(b_{1}, b_{2}, u_{1}) + \sum_{t=1}^{\infty} \lambda_{t}V^{y}(b_{t}, b_{t+1}, b_{t+2}, u_{t}) \right\}$$

subject to $b_t \in [0, 1] \forall t$, where the planner weights λ_{0s} , λ_{0u} , and $\{\lambda_t\}_{t=0}^{\infty}$ are strictly positive. It is straightforward to show that the planner would choose zero benefits after the first period, for any arbitrary sequence of (positive) planner weights. Moreover, a utilitarian planner with equal weights on all initially living individuals would set $b_0 = 0$, for any u_0 . The proof is available upon request.

II. Political Equilibrium

The purpose of this paper is to explore the impact of the *ex post* conflict of interest between groups on the dynamics of redistribution. More specifically, can an "inefficient" welfare state survive over time? Or will dynamic voting decisions make redistribution vanish in the long run?

In answering this question, we restrict the attention to Markov perfect equilibria, where the state of the economy is summarized by the proportion of current unsuccessful old agents (u_i) . The political equilibrium is defined as follows.

Definition 1: A (Markov perfect) political equilibrium is defined as a pair of functions $\langle B, U \rangle$, where B: $[0, 1] \rightarrow [0, 1]$ is a public policy rule, $b_i = B(u_i)$, and U: $[0, 1] \rightarrow [0, 1]$ is a private decision rule, $u_{t+1} = 1 - e_i^* = U(b_i)$, such that the following functional equations hold:

- 1. $B(u_t) = \arg \max_{b_t} V(b_t, b_{t+1}, b_{t+2}, u_t)$ subject to $b_{t+1} = B(U(b_t)), b_{t+2} = B(U(B(U(b_t))))$, and $b_t \in [0, 1]$, and $V(b_t, b_{t+1}, b_{t+2}, u_t)$ is defined as the indirect utility of the current decisive voter.
- 2. $U(b_t) = (1 \beta + b_t + \beta b_{t+1})/2$, with $b_{t+1} = B(U(b_t)).$

The first equilibrium condition requires that b_t maximizes the objective function of the decisive (median) voter V, taking into account that future redistribution depends on the current policy choice via the equilibrium private decision rule and future equilibrium public policy rules. Furthermore, it requires $B(u_t)$ to be a fixed point in the functional equation in part 1 of the definition. In other words, suppose that agents believe future benefits to be set according to the function $b_{t+j} = B(u_{t+j})$. Then, we require that the same function $B(u_t)$ defines optimal benefits today.

The second equilibrium condition implies that all young individuals choose their investment optimally, given b_t and b_{t+1} , and that agents hold rational expectations about future benefits and distributions of types. In general, Umight be a function of both u_t and b_t . In our model, however, u_t has neither a direct effect on the investment choice of the young, nor, consequently, on the future distribution of voters. Thus, we choose to focus on equilibria where their equilibrium investment choice is fully determined by the current benefit level.

Finally, in order to single out the effects of dynamic rational voting, it is useful to define an alternative *myopic voting equilibrium*, where voters ignore the impact on future political decisions when deciding on current policies. A myopic equilibrium is defined as in Definition 1, but with condition 1 being replaced by

1'. $B(u_t) = \arg \max_{b_t} V(b_t, \bar{b}_{t+1}, \bar{b}_{t+2}, u_t)$ subject to $b_t \in [0, 1]$, where \bar{b}_{t+1} and \bar{b}_{t+2} are taken as parametric, subject to rational expectations.

This condition requires that b_i maximizes the objective function of the decisive (median) voter V, taking future redistribution as given. Voters correctly anticipate future redistribution, but ignore that they could affect its path through their current political choice. In the rest of the analysis, we refer to a Markov perfect equilibrium as an equilibrium, and state explicitly when referring to a myopic voting equilibrium.

A. Dictatorship

For expositional reasons it is convenient to start the analysis by describing the equilibrium under the assumption that the political power permanently rests in the hands of one of the two groups of old agents in the society. Then, we extend the analysis to the case of majority voting.

We define "plutocracy" (PL) and "dictatorship of the proletariat" (DP) as the regimes where the level of redistribution is chosen at the beginning of each period by the currently living successful and unsuccessful old agents, respectively. Formally, under DP, $V(b_t, b_{t+1}, b_{t+2}, u_t) \equiv V^{ou}(b_t, b_{t+1}, u_t)$ whereas, under PL, $V(b_t, b_{t+1}, b_{t+2}, u_t) \equiv V^{os}(b_t, b_{t+1}, u_t)$. The equilibrium under dictatorship is characterized in the following proposition.¹²

¹² The gist of the derivation of the equilibrium functions is the following. Start by assuming *B* to be linear in u_{r} , ignoring the constraint that $b \in [0, 1]$. Then, as the young

(5)
$$B^{pl}(u_t) = 0$$

 $U^{pl}(b_t) = \frac{1}{2}(1 - \beta + b_t).$

Given $u_0 \in [0, 1]$, for all $\star \geq l$, $u_l = u^{pl} \equiv \frac{1-\beta}{2}$.

The DP equilibrium, $\langle B^{dp}, U^{dp} \rangle$, is characterized as follows:

$$(6) \ B^{dp}(u_{t}) = \begin{cases} \frac{3}{2} - u_{t} \\ \text{if } u_{t} > \bar{u}(\beta); \\ b^{dp} - \frac{2}{2 - \beta} (u_{t} - u^{dp}) \\ \text{if } u_{t} \in \left[\frac{3}{2} - \frac{2}{2 + \beta}, \bar{u}(\beta)\right]; \\ 1 \\ \text{if } u_{t} \in \left[0, \frac{3}{2} - \frac{2}{2 + \beta}\right]; \end{cases}$$

$$(7) \quad U^{dp}(b_{t}) = \begin{cases} u^{dp} + \frac{2 - \beta}{4} (b_{t} - b^{dp}) \\ \text{if } b_{t} \in \left[\frac{2\beta}{2 + \beta}, 1\right]; \\ \frac{1}{2} (1 + b_{t}) \\ \text{if } b_{t} \in \left[0, \frac{2\beta}{2 + \beta}\right], \end{cases}$$

are risk neutral with quadratic effort costs, the function U, satisfying condition 2 in equilibrium Definition 1, is linear in b_i . Moreover, the indirect utility is also linear quadratic in b_i , once B and U have been substituted into (4). It turns out that, in the absence of constraints, the optimal choice of b_i is indeed linear in u_i . Imposing condition 1 in equilibrium Definition 1, it is straightforward to solve for the coefficients in B. What remains is to impose the constraints on b_i , and check that no deviations from this constrained linear rule can be optimal. Such deviations may, in some cases, be optimal, as, e.g., explained in footnote 13, in which case the policy rule B must be modified in a nontrivial way.

where $u^{dp} \equiv \frac{1}{6} \left(5 + \frac{\beta^2}{2+\beta} \right)$, $b^{dp} \equiv \frac{4}{3} \frac{1+\beta}{2+\beta}$ and $\bar{u}(\beta) = \frac{\beta+6-\beta\sqrt{4-2\beta}}{2(2+\beta)}$. The equilibrium law of motion, $u_{t+1} = U^{dp}(B^{dp}(u_t))$, is as follows:

(8)
$$u_{t+1} = \begin{cases} \frac{5}{4} - \frac{u_t}{2} \\ \text{if } u_t > \bar{u}(\beta); \\ u^{dp} - \frac{1}{2} (u_t - u^{dp}) \\ \text{if } u_t \in \left[\frac{3}{2} - \frac{2}{(2+\beta)}, \bar{u}(\beta)\right]; \\ \frac{\beta}{4} + \frac{2}{2+\beta} \\ \text{if } u_t \in \left[0, \frac{3}{2} - \frac{2}{(2+\beta)}\right]. \end{cases}$$

Given $u_0 \in [0, 1]$, the economy converges with an oscillatory pattern to a unique steady state, $u = u^{dp}$ and $b = b^{dp}$.

Figure 1 represents the equilibrium public policy rule and private decision rule for the PL and DP equilibrium, for a case in which the range $u_t > \bar{u}(\beta)$ is empty, i.e., $\bar{u}(\beta) \ge 1$.¹³ In the PL case (upper figures), the policy function is constant at $B^{pq}(u_t) = 0$, and the private decision rule is upward sloping, reflecting the expectations of the young agents that $b_{t+1} = 0$,

¹³ All figures, including Figure 1, represent a parametric case where $\beta = 0.75$. Note that $\beta < (\sqrt{17} - 1)/4 \approx 0.78$ implies that $\bar{u}(\beta) > 1$. If, in contrast, $\tilde{u}(\beta) < 1$, there is a range $u_i \in [\bar{u}, 1]$ such that the unsuccessful old in period t induce a u_{t+1} where the constraint $b_{t+1} \leq 1$ is binding. This creates a downward discontinuity of $B(u_i)$ at \bar{u} . The range $[\bar{u}, 1]$ is ephemeral, in the sense that in equilibrium, $u_t < \bar{u} \ \forall t > 0$. Therefore, if the economy starts at $u_0 < \bar{u}$, the policy rule and law of motion are qualitatively identical to those in Figure 1. Otherwise, they differ for one period only. To understand the origin of the downward discontinuity, note that b_{r+1} is negatively related to b_r when the constraint $b_{\ell+1} \leq 1$ is not binding. Thus, the marginal distortion of current benefits shifts upward at \hat{u} , as the constraint on b_{i+1} becomes binding. Therefore, $B(u_i)$ falls discontinuously at $u_1 = \overline{u}$.



FIGURE 1. PUBLIC POLICY RULE AND PRIVATE DECISION RULE UNDER PLUTOCRACY (UPPER PANELS) AND DICTATORSHIP OF PROLETARIAT (LOWER PANELS)

irrespective of the choice of b_r . In the DP case (lower figures), the equilibrium redistribution is always strictly positive, and it is 100 percent for sufficiently low u_t . Furthermore, it is downward sloping, reflecting the fact that the marginal cost of redistribution increases, as the current proportion of old successful agents falls [see equation (3)]. The private decision rule is also positively sloped, but less steep than in the PL case, since an increase in b_t negatively affects the choice of b_{t+1} , hence, the current effort choice of the young responds less to an increase in current benefits than if future redistribution were constant.

B. Majority Voting

We now assume that political decisions are taken through majority voting. Agents vote on

the single issue of redistribution. It is straightforward to show that if young agents were pivotal in voting over current benefits, then, for all t > 0, benefits would be zero in equilibrium. Intuitively, as the young are still behind the veil of ignorance, they oppose distortionary redistribution.

However, we regard it as unrealistic to assume that voters behind the veil of ignorance are pivotal.¹⁴ Thus, this paper explores the consequences of letting the political decisions be determined by the *ex post* conflict of interests between individuals who know their type, i.e.,

¹⁴ For instance, in a model where agents live for more than two periods and make their investment decision in the first period only, the "young" would constitute a small proportion of the electorate and are not likely to be decisive.

the old. Two alternative assumptions can deliver a political preponderance of the old in our model. The first is to assume that young individuals have a lower voting turnout than the old, maintaining that current benefits are set at the beginning of each period. This assumption can be defended empirically as the voting turnout increases with age. For example, Raymond E. Wolfinger and Steven J. Rosenstone (1980) document that turnout in U.S. elections is sharply increasing in age, rising from 45 percent for the 20-year-old to 75 percent for the 65vear-old.¹⁵ Alternatively, it might be assumed that elections were held at the end of each period, and agents voted over benefits in the next period, after the uncertainty about their individual success had been unraveled. In this case, by not being alive in the next period, the old would have no interests at stake and could be assumed to abstain from voting. Clearly, this is equivalent to assuming that the choice of current benefits is taken at the beginning of each period, but only the old vote. For expositional ease, we maintain in the presentation the interpretation that agents vote over current benefits and only the old vote.¹⁶

Benefits are chosen to maximize the indirect utility of the old successful (unsuccessful) if $u_t \leq \frac{1}{2} (u_t > \frac{1}{2})$. As we shall see, majority voting can generate persistence in the equilibrium choice of redistribution. If the economy starts with a pro-welfare state majority ($u_t > \frac{1}{2}$), then there exists an equilibrium where the welfare state and the political majority supporting it is sustained over time. Conversely, if $u_t \leq \frac{1}{2}$, the welfare state will never arise. The positive feedback mechanism giving rise to the persistence of policies and distributions of vot-

¹⁵ Furthermore, Casey B. Mulligan and Xavier Sala-i-Martin (1999) construct a model where the elderly have a preponderant influence on the determination of redistribution policies. In their paper, this arises due to the old having a low opportunity cost of time.

¹⁶ In a previous version of this paper, Hassler et al. (2001b), we also analyzed an intermediate case, when the young vote before knowing their type, albeit with a lower turnout than the old, in elections held at the beginning of each period. In particular, only a share $\varepsilon \in [0, 1]$ of the young individuals was assumed to participate in the voting process. The key insight of that extension is that our results when only the old vote ($\varepsilon = 0$) remain unchanged for $\varepsilon > 0$, provided that ε is not too large.

ers is that high (low) benefits today affect private incentives so as to induce a large (small) proportion of unsuccessful agents tomorrow, and therefore a broad (narrow) future constituency for redistribution.

An initial majority of unsuccessful individuals does not guarantee, however, the eternal survival of the welfare state. For sufficiently high discount factors, and given an initial majority of unsuccessful individuals, there exist, in addition, equilibria where any existing welfare state is dismantled in, at most, two periods. The survival of the welfare state is in this case a matter of self-fulfilling expectations.

As we shall see, expectations (beliefs) about the identity of the future median voter play a crucial role in driving such multiplicity. If the agents expect that a majority of successful agents will materialize in the next period, that majority is then expected to implement zero redistribution. Conversely, if they expect a majority of unsuccessful agents in the next period, they expect next period redistribution to be strictly positive. We assume that in the case of a tie, benefits are chosen by the successful agents. Before characterizing equilibria, it is useful to discuss some general properties of the expectations. According to Definition 1, expectations must be rational, which imposes two restrictions. First, it would be irrational to believe that the old successful will be in majority next period if, in the current period, voters set $b_t > \beta$, since the private decision rule would then imply that $u_{t+1} = (1 - \beta + b_t + b_t)$ $\beta b_{r+1}/2 > 1/2$. Second, it would be irrational to believe that a majority of unsuccessful individuals will materialize in the next period if the current majority sets $b_t = 0$, since, then, $u_{t+1} = (1 - \beta + \beta b_{t+1})/2 \le \frac{1}{2}$. We assume in addition, that expectations about the identity of the median voter are "monotonic": if agents believe that $b_t = x$ induces $u_{t+1} \le \frac{1}{2}$, then they must also believe that $b_t < x$ induces $u_{t+1} \le \frac{1}{2} \cdot \frac{17}{2}$

¹⁷ The assumption of "monotonicity" plays no role in the characterization of the equilibrium path, since, as we shall see, the only essential feature of beliefs is the highest level of b inducing a majority of successful individuals. However, the assumption simplifies the characterization of out-of-equilibrium behavior and reduces the set of observationally equivalent equilibria.

Given these assumptions, we can summarize beliefs about the identity of the future median voter as a threshold benefit level, denoted by $\theta \in [0, \beta]$, such that all agents expect that zero benefits will be provided at t + 1, if and only if current redistribution is smaller or equal to θ . Formally, if and only if $b_t \leq \theta$, the equilibrium private decision rule and policy function must feature, respectively, $U(b_t) \leq \frac{1}{2}$ and $B(U(b_t)) = 0.^{18}$ Finally, we assume that θ is constant over time and the same for all agents.

In the rest of this section, we will prove that, conditional on the existence of an initial majority of old unsuccessful agents, equilibria featuring the survival of the welfare state ("pro-welfare equilibria") are sustained if the welfare state is believed to be sufficiently robust, i.e., for sufficiently low θ . Instead, equilibria where the ruling old unsuccessful vote strategically so as to induce a future political majority of successful agents that will vote for zero redistribution ("anti-welfare equilibria") are sustained if the welfare state is believed to be sufficiently "fragile," i.e., for sufficiently high θ . In both cases, the beliefs that determine private investments and political choices are fulfilled in equilibrium.

1. Pro-Welfare Equilibria.—In this part, we consider pro-welfare equilibria. These equilibria feature multiple steady states: if the initial median voter is unsuccessful, the welfare state survives forever. If, instead, the initial median voter is successful, no welfare state will ever arise. Recall that insofar that this equilibrium determines an allocation with positive redistribution after the first period, its outcome is not Pareto optimal.

Essentially, the equilibrium functions B^{pw} and U^{pw} are found by splicing together the equivalent functions from the equilibrium under dictatorship in Proposition 1 (i.e., B^{pl} and B^{dp} , and U^{pl} and U^{dp}), and specifying beliefs (θ) such that no switch of political majority ever occurs along the equilibrium path.

PROPOSITION 2: For all $\beta \in [0, 1]$ and $\theta \leq \overline{\theta}(\beta) \in (0, \beta]$, there exists a "pro-welfare equi-

librium" (PWE), $\langle B^{pw}, U^{pw} \rangle$, featuring multiple steady states, with the following characteristics:

(9)
$$B^{pw}(u_t) = \begin{cases} B^{dp}(u_t) > 0 & \text{if } u_t \in (\frac{1}{2}, 1] \\ B^{pl}(u_t) = 0 & \text{if } u_t \in [0, \frac{1}{2}] \end{cases}$$

(10)
$$U^{pw}(b_t) = \begin{cases} U^{dp}(b_t) > \frac{1}{2} & \text{if } b_t \in (\theta, 1] \\ \\ U^{pl}(b_t) \le \frac{1}{2} & \text{if } b_t \in [0, \theta], \end{cases}$$

where the expression of $\hat{\theta}(\beta)$ is in the proof in the Appendix, and $B^{dp}(u_t)$, $B^{pl}(u_t)$, $U^{dp}(b_t)$ and $U^{pl}(b_t)$ are defined in Proposition 1.

The equilibrium law of motion, $u_{t+1} = U^{pw}$ ($B^{pw}(u_t)$), implies that there are two locally stable steady states;

- 1. if $u_0 \leq 0.5$, the economy converges in one period to a steady-state equilibrium with $\{b, u\} = \{b^{pl}, u^{pl}\}$ as defined in Proposition 1.
- 2. if $0.5 < u_0 \le 1$, the economy converges asymptotically with an oscillatory pattern to an equilibrium with $\{b, u\} = \{b^{dp}, u^{dp}\}$ as defined in Proposition 1.

Figure 2 depicts the equilibrium policy rule and private decision rule [for, again, $\beta = 0.75$, implying $\bar{u}(\beta) > 1$]. The left-hand panel shows that, when $u_t \leq \frac{1}{2}$, then $B(u_t) = 0$ in equilibrium. At $u_1 = \frac{1}{2}$, the policy function increases discontinuously, as the unsuccessful become pivotal. In fact, for an intermediate range of u_n the equilibrium policy function prescribes 100-percent redistribution, being downward sloping thereafter. The right-hand panel depicts the private decision rule. A majority of old unsuccessful materializes at t + 1 if and only if $b_t > \theta$. Since a majority of unsuccessful at t + 1 would set $b_{t+1} > 0$, whereas a majority of old successful would set $b_{t+1} = 0$, the private decision rule exhibits an upward discontinuity at θ .

This discontinuity implies a discrete increase in the current tax level at $b_r = \theta$. Thus, the utility of the old unsuccessful is discontinuous at θ . The left-hand side of Figure 3 illustrates this point, by plotting the utility of the old unsuccessful as a function of b, given u and β

¹⁸ To simplify the notation, they will not be specified as arguments of the equilibrium functions $\langle B, U \rangle$. In particular, with some abuse of notation, we will write $B(u_i)$ and $U(b_i)$ rather than $B(u_i; \theta)$ and $U(b_i; \theta)$.



Figure 2. Public Policy Rule and Private Decision Rule Under Majority Voting; Pro-Welfare Equilibrium ($\beta = 0.75$)



Figure 3. Indirect Utility of the Old Unsuccessful, $V^{ou} = V^{ou}(b_t, B(U(b_t)), u_t)$ Under PWE and AWE $(\beta = 0.75 \text{ and } u_t = u^{dp})$

(the figure depicts $\beta = 0.75$ and $u_t = u^{dp}$) and for sufficiently low θ , as in Proposition 2. Formally, the discontinuity is due to b_{t+1} entering negatively in the utility of the old in equation (4). As a result, the preferences of the median voter are not single-peaked with respect to b_t . The old unsuccessful face the temptation to vote strategically for $b_t = \theta$, so as to change the identity of the future median voter, thereby ensuring that the welfare state disappears. A PWE is sustained if, as in the left-hand figure, this strategic voting option is not globally optimal, i.e., the peak in preferences corresponding to $B^{dp}(u)$ yields higher utility than that corre-

sponding to θ . This condition must hold for all $u \in (0.5, 1]$. Note the role of the threshold θ . The lower is θ , the higher the cost in terms of forgone pretax earnings (b_t) required to induce the breakdown of the welfare state at t + 1. Intuitively, a low θ means that agents regard the welfare state as "robust," and think that only very low current redistribution can induce its termination.

To further grasp the intuition, it is useful to focus on the extreme belief, $\theta = 0$. In this case, the welfare state is expected to break down only if current redistribution is set equal to zero. But this cannot occur in equilibrium, since this would imply zero net earnings for the old unsuccessful. This explains why, for any β , there exist beliefs (i.e., a range of sufficiently low θ) sustaining the survival of the welfare state.

Proposition 2 implies that both redistributive policies (b) and their effect on voters' distribution (u) tend to be persistent. An existing welfare state can in fact regenerate its political support and survive in the long run. However, without an initial majority in favor of redistribution, the welfare state would never arise. It is possible to account for the creation (as opposed to survival) of the welfare state by introducing in the model aggregate shocks affecting the mapping from the individual educational effort choice to the aggregate proportion of unsuccessful. For instance, we have seen that a majority of successful would tend to regenerate itself, since the absence of a welfare state induces high investments. Nevertheless, a sufficiently negative shock would imply a change of majority, from successful to unsuccessful. The Great Depression can be interpreted as a once-and-for-all shock that impoverished a sufficiently large share of the voters so as to increase the political support to redistributive programs. Then, the mechanism described by Proposition 2 would explain the persistence of such programs.¹⁹

To conclude the discussion of pro-welfare equilibria, we examine the role of rational voting. If agents voted myopically, according to our definition in Section II, there would, for all β , exist an equilibrium qualitatively similar to the PWE of Figure 2. It would exhibit multiple steady states and, if $u_0 > 0.5$, b_t and u_t would converge with an oscillatory pattern to a steady

state with positive redistribution. This steady state would, however, feature less redistribution and a *lower* proportion of unsuccessful in steady state than under rational voting. The perceived increase in current taxes due to an increase in current redistribution, is smaller for rational voters than for myopic voters. This is so because rational voters understand that, by increasing current redistribution, they increase the proportion of unsuccessful agents next period. This implies a lower demand for redistribution in the next period [recall that the policy function B(u) is, in the relevant range, downward sloping, so that b_{t+1} falls when u_{t+1} increases], which lowers the number of young beneficiaries and, therefore, current taxes. Myopic voters ignore this effect and are therefore less prone to choose high redistribution. In this sense, the rational voters use current redistribution strategically so as to manipulate future redistribution.

2. Anti-Welfare Equilibria.—So far, we have analyzed equilibria where an existing welfare state survives. In this section, we show that for β sufficiently large there exist other rational expectations equilibria with beliefs such that the welfare state breaks down in finite time (i.e., in one or two periods). In such equilibria, the old unsuccessful vote strategically so as to change the identity of the future median voter, thereby ensuring that the welfare state disappears. We will also show that, as long as $\beta < 1$, no myopic voting equilibrium featuring the breakdown of an existing welfare state exists. Thus, the results of this section hinge on rational forward-looking voting.

These equilibria are labeled "anti-welfare equilibria" (AWE). Their exact characterization, including a complete set of conditions for their existence, is provided in Proposition 3, deferred to the Appendix. We summarize here the main findings. There are two types of equilibria, depending on the discount factor, β , and beliefs, θ . In one type, the welfare state is terminated in one period, whereas in the other it may be terminated in either one or two periods. We now discuss these two cases separately.

¹⁹ In an extension available upon request we introduce stochastic shocks by assuming that, with a positive probability, p, the return to investment is as in the benchmark model, implying that $u_{t+1} = 1 - e_t$, whereas, with probability 1 - p, the probability of individual success in period t [individually and identically distributed (i.i.d.) across agents] is drawn from a probability density function (p.d.f.) with support on the unit interval and mean 1/2. Thus, with probability 1 - p the investment in education has no effect on the probability of success of agents. Whether or not effort matters is revealed after benefits are set and effort is chosen. The main result of this extension is that an existing pro-welfare or anti-welfare majority regenerates itself with probability (1 + p)/2, whereas a change of majority occurs with probability (1 - p)/2. While pro- and anti-welfare majorities are occasionally reverted, the mechanism that we highlight in this section continues to apply.

²⁰ Under myopic voting, the steady-state redistribution is equal to $2(1 + \beta)/(3 + 2\beta) \le b^{dp}$. The formal characterization of the myoping voting equilibrium has been omitted and is available upon request.



FIGURE 4. PUBLIC POLICY RULE AND PRIVATE DECISION RULE UNDER MAJORITY VOTING: ANTI-WELFARE EQUILIBRIUM WITH SWITCH IN ONE PERIOD (UPPER PANELS, $\theta = 0.75$) AND WITH SWITCH IN TWO PERIODS (LOWER PANELS, $\theta = 0.58$) ($\beta = 0.75$)

The two upper panels in Figure 4 illustrate equilibria with a breakdown of the welfare state in one period. These equilibria require high discount factors and sufficiently large θ . In this case, the welfare state is terminated in one period, and the equilibrium policy rule and private decision rule are as follows (see also part 1 of Proposition 3 in the Appendix):

(11)

$$B^{aw}(u_t) = \begin{cases} \theta & \text{if } u_t > \frac{1}{2} \\ 0 & \text{if } u_t \in [0, \frac{1}{2}] \end{cases}$$

(12)

$$U^{aw}(b_i) = \begin{cases} \frac{1}{2}(1-\beta+\beta\theta+b_i) & \text{if } b_i > \theta \\ \\ U^{pl}(b_i) & \text{else,} \end{cases}$$

where $U^{pl}(b_t)$ is defined in Proposition 1. Note that, if $u_t \leq \frac{1}{2}$, the AWE policy function is as in a PWE. If $u_t > \frac{1}{2}$, however, the policy function prescribes $b_t = \theta$, i.e., benefits are set equal to the highest level that can induce $u_{t+1} \leq \frac{1}{2}$. Accordingly, the private decision rule is discontinuous at θ . As Figure 4 shows, if, at time zero, the old unsuccessful are in majority, then, given the policy function and beliefs,

they find it optimal to set $b_0 = \theta$ and induce the switch of majority. The unsuccessful in majority at time one, then, choose $b_1 = 0$ and terminate the welfare state. The sequence of equilibrium redistribution is, therefore, $b_0 = \theta$, and $b_t = 0$, $\forall t > 0$.²¹ The expectation of zero future redistribution induces the young to exert high investment, which in turn leads to a low dependency ratio, granting low taxes in the first period.

The right-hand side of Figure 3 represents the utility of the old unsuccessful as a function of b, given the same values of u and β as in the left-hand-side panel, but for different beliefs, so as to be consistent with equations (11)-(12). In particular, θ is larger, implying that agents regard the welfare state as more "fragile" and believe that a switch of majority will occur for a larger range of current redistribution policies, b. As before, the effort choice of the young falls discontinuously at $b = \theta$. But, here, the global maximum occurs at $b = \theta$. The AWE with switch in one period is sustained if $b_t = \theta$ is the global maximum for all $u_t \in [0.5, 1]$. Namely, given the private decision rule, $U^{aw}(b)$, any majority of old unsuccessful will find it optimal to induce the breakdown of the welfare state. The belief that the welfare state is "fragile" (high θ) is crucial in making it attractive for the old unsuccessful to induce the breakdown.

An interesting observation is that, given parameters, AWE with switch in one period Paretodominate PWE, at least for economies starting at the steady state of a PWE, $u_0 = u^{dp}$.²² Any

²¹ Thus, this equilibrium is Pareto efficient in the sense that, given any initial state u_0 , there exists planner weights such that the allocation would have been chosen by a social planner maximizing the objective function in footnote 11. All other equilibria considered in this paper, including the AWE with switch in two periods, are not Pareto optimal because they may imply positive benefits even after the first period.

²² Consider the AWE sustained by the belief that $\theta = \beta$. Then, for all $\beta \in [(\sqrt{5} - 1)/2, 1]$, it is easy to verify that:

$$V^{ou}(\beta, 0|u^{dp}) = \beta \frac{8 + 4\beta - \beta^2}{12(2 + \beta)}$$
$$> \frac{2(2 - \beta)(1 + \beta)^2}{(3(2 + \beta))^2}$$
$$= V^{ou}(b^{dp}, b^{dp}|u^{dp})$$

coordination device inducing the young to believe that the welfare state is sufficiently fragile would improve the welfare of all agents in the society, including the ex ante utility of all future generations. This result can be related to the debate on so-called "fiscal increasing returns." It has been argued (e.g., Blanchard and Lawrence H. Summers, 1987) that labor market reforms such as reductions in the bargaining strength of insiders, or lower replacement ratios, may lead, in high unemployment economies, to efficiency gains that are so large that even the groups that suffer a reduction in their pretax earnings would benefit from the reforms in after-tax terms. In other words, economies may get stuck in bad equilibria along the declining side of the Laffer curve. In our model, equilibria with inefficiently high taxes are the outcome of expectational traps: under pro-welfare expectations, a system of inefficient redistribution is believed to be very robust, and private and public decisions reinforce each other in sustaining a Pareto-inferior outcome. Changes in expectations (θ) may lead the society to a superior outcome.

It is important to emphasize two effects driving this result. First, current benefits have a direct impact on the government budget that includes a static distortionary effect [τ_t is increasing and convex in b_t , see equation (3)]. However, in our model, this standard effect would not be sufficient to induce the median

$$V^{os}(\beta, 0|u^{dp}) = \frac{2}{2+\beta} - \frac{\beta(4+8\beta+\beta^2)}{12(2+\beta)}$$

$$> \frac{(2-\beta)(8+7\beta+2\beta^2)}{(3(2+\beta))^2}$$

$$= V^{os}(b^{dp}, b^{dp}|u^{dp})$$

$$V^{y}(\beta, 0, 0|u^{dp}) = \frac{(6-\beta)(1+\beta)^2}{12(2+\beta)}$$

$$> \frac{(7\beta+10)(2-\beta)(1+\beta)^2}{4(3(2+\beta))^2}$$

$$= V^{y}(b^{dp}, b^{dp}, b^{dp}|u^{dp}),$$

where each left-hand-side (right-hand-side) term of the inequality represents the utility in an AWE (PWE). The result can be generalized to any $\theta \ge \underline{\theta}(\beta)$. For some $u > u^{d_p}$, however, counterexamples can be constructed.

voter (who, recall, is a net recipient of current transfers) to set b to a level low enough to induce a switch of majority. The second factor, which reinforces the previous effect, is the dependence of future redistribution on current redistribution. Rational voters understand that, by voting for lower benefits today, they can trigger a change in the identity of the future median voter, and indirectly reduce future benefits and current taxes. This indirect effect increases the gain from restraining b_r .

Rational dynamic voting are important for the existence of multiple equilibria. Formally, for any $\beta < 1$, no myopic voting equilibrium featuring the termination of an existing welfare state would exist. To construct a contradiction, consider a candidate myopic equilibrium where agents at time zero take as parametric, $\bar{b}_t = 0$ for all t > 0. The old unsuccessful would then choose b_0 so as to maximize $V^{ou}(b_0, \bar{b}_1, u_t) =$ $b_0 - ((1 - \beta) + (b_0 + \beta \bar{b}_1) + 2u_0)b_0/4$, given $\bar{b}_1 = 0$. The solution yields $b_0 = \min\{\frac{3}{2} + \frac{1}{2}\beta - u_0, 1\}$. But this level of redistribution is inconsistent with a majority of old unsuccessful at time t = 1, since

$$U(b_0|\bar{b}_1 = 0)$$

= $\frac{1 - \beta + \min[\frac{3}{2} + \frac{1}{2}\beta - u_0, 1]}{2}$
> $\frac{1}{2}$

for any $\beta < 1.^{23}$ Thus, the expectation that $b_1 = 0$ is not rational, and a myopic voting AWE does not exist. Intuitively, myopic voters do not recognize that they need to restrain their demand for current redistribution in order to terminate the welfare state. The expectation of zero future redistribution would, on the con-

trary, induce them to demand a very high level of redistribution in the current period, so that no switch of majority would materialize.

Finally, let us discuss the other type of equilibria, where the switch of majority and the end of the welfare state may occur in either one or two periods, depending on the initial state. Sufficient conditions for such equilibria to be sustained are provided in part 2 of Proposition 3 in the Appendix. In short, these equilibria are sustained for intermediate values of β and θ . An example is depicted in the two lower panels of Figure 4.24 The main change, relative to the other case, is the existence of a range of intermediate levels of initial old unsuccessful, $u \in$ $(\frac{1}{2}, \hat{u})$, where the old unsuccessful in majority choose 100-percent redistribution. In this case, the sequence of equilibrium redistribution is: $b_0 = 1$, $b_1 = \theta$, and $b_t = 0 \forall t > 1$. Formally, the equilibrium policies are given by:

(13)
$$B^{aw}(u_t) = \begin{cases} \theta & \text{if } u_t \ge \hat{u}(\beta, \theta) \\ 1 & \text{if } u_t \in (\frac{1}{2}, \hat{u}(\beta, \theta)) \\ 0 & \text{if } u_t \in [0, \frac{1}{2}] \end{cases}$$

(14)
$$U^{aw}(b_t) = \begin{cases} \frac{1}{2}(1 - \beta + \beta\theta + b_t) > \frac{1}{2} \\ \text{if } b_t > \theta; \\ U^{pl}(b_t) \le \frac{1}{2} \\ \text{if } b_t \le \theta. \end{cases}$$

The intuition for AWE with switch in two periods is the following. Since the cost of redistribution, τ_t/b_t , is increasing with u_t , setting $b_t = \theta$ is optimal, for a range of large u. This opens up a new strategic opportunity for economies starting out with an *intermediate* $u [u_0 \in (\frac{1}{2}, \hat{u}(\beta, \theta))$, see Figure 4]. Namely, the

²⁴ The necessary and sufficient conditions on the range of parameters (β) and beliefs (θ) that sustain AWE with switch in two periods can also be characterized, but are involved, and are therefore not stated here. Details are available upon request. As a more general remark, we are unfortunately unable to provide a proof that the equilibria characterized exhaust the set of equilibria consistent with Definition 1.

²³ An anti-welfare equilibrium with myopic voting exists in the particular case where $\beta = 1$. In this case, given the expectation that $\vec{b}_1 = 0$, the private decision rule of the myopic voting equilibrium is $U(b_0) = b_0/2$, implying that a majority of old successful materializes in period one, irrespective of the choice of b_0 . In fact, the old unsuccessful choose $b_0 = 1$ implying $u_1 = \frac{1}{2}$ and the end of the welfare state.

median voter chooses 100-percent redistribution and induces, in the next period, a majority of old unsuccessful $(u_1 \in [\hat{u}(\beta, \theta), 1])$ which will set $b = \theta$ and which, in turn, will induce the breakdown of the welfare state two periods ahead.

In summary, the previous results imply, jointly, that multiple self-fulfilling equilibria exist when $u_t > 0.5$, provided that β is not too small. In one of these equilibria, the welfare state survives, while it is terminated in the others. No AWE exists when β is sufficiently small. The reason is that strategic voting considerations become less important when agents discount the future more highly, since private investments become less sensitive to expectations about future political decisions. In fact, myopic voting and rational voting equilibria coincide for $\beta = 0$.

III. Wage Inequality and Political Support for the Welfare State

In this section, we analyze the effects of changes in pretax inequality on the political equilibrium. To this end, we extend the model by allowing the wage of the successful agents, w, to differ from unity. Note that w parameterizes the degree of technological inequality—a large w implies large inequality. The absolute wage level has no effect on the equilibrium of our linear model, which justifies maintaining the normalization that the wage of the unsuccessful agents is equal to zero. Furthermore, b_t now denotes the *benefit rate*, which implies that unsuccessful agents earn a before-tax income of b_tw , and that the constraint $b_t \in [0, 1]$ is maintained.

The optimal effort choice in this extension implies

$$u_{t+1} = 1 - e_t^*$$

= $\frac{1}{2} \max\{(2 - (1 + \beta)w + b_t w + \beta b_{t+1}w), 0\},\$

where the constraint $u_{t+1} \ge 0$ is never binding if $w \le 1$. The tax rate consistent with the balanced government budget constraint is $\tau_t = \frac{1}{2}(u_{t+1} + u_t)wb_t$. It can immediately be established that the political equilibrium necessarily features a welfare state as long as $w < 1/(1 + \beta)$. In this case, given any initial u_t , we have that $u_{t+1} > \frac{1}{2}$ irrespective of the redistribution policy chosen by the first generation, and the unsuccessful are always in majority from the second period and onwards. Thus, the equilibrium features a unique steady state characterized by positive redistribution, irrespective of initial conditions or expectations.

For a range of intermediate values of w, the equilibrium features multiple steady states, and the welfare state never arises if $u_0 \leq \frac{1}{2}$, while it survives perpetually if $u_0 > \frac{1}{2}$. An extension of the argument of Proposition 2 ensures that there does not exist an anti-welfare equilibrium for this intermediate range of wage inequality, i.e., a majority of unsuccessful agents do not strategically hand over power to the next generation of successful individuals. Next, for another range of intermediate, larger values of w, there exist both an equilibrium featuring the breakdown of the welfare state and one featuring its perpetual survival, as analyzed in Section II, subsection B, part 2 (recall that if w = 1, then there exist multiple equilibria, provided that β is sufficiently large). Finally, for sufficiently large values of w, there exists no equilibrium featuring a welfare state for more than one period.25

As suggested by this discussion, the model makes predictions about the effect of technologydriven changes in wage inequality. Assume, for

²⁵ To see why the welfare state cannot survive when w is sufficiently large, consider the case of $\beta = 0$. This is the most robust case for the survival of the welfare state, since the old unsuccessful in majority have no strategic motive to induce the breakdown of the welfare state. In this case, if $u_0 > \frac{1}{2}$, the equilibrium benefit rate is given by

$$\arg \max_{b} [b - (\max\{2 - w(1 - b), 0\} + 2u)b/4]$$

$$\geq \frac{1}{2} + \frac{1 - u}{w},$$

which is decreasing in w, so that equilibrium b_0 is bounded from below by $\frac{1}{2}$. But, then, $u_1 = \max\{1 - w(1 - b_0)/2, 0\} < \frac{1}{2}$ for sufficiently large w. Hence, irrespective of the initial value of u, the majority in the subsequent periods opposes redistribution, provided that w is sufficiently large.

instance, that there is an unexpected permanent increase in the premium to education. As a result, agents increase their investment in education and a larger proportion will, ex post, be opposed to redistributional policies. Thus, the initial impact of technological inequality is magnified by reduced support for the welfare state.²⁶ This prediction of the model is in line with important events characterizing the last quarter of the twentieth century. The skillbiased technical change that, as documented by a number of authors, started in the 1970's (see, among others, Lawrence F. Katz and Kevin M. Murphy, 1992), was followed by the electoral success of conservative governments, whose political platform included a reduction of the redistributive role of governments, especially in Anglo-Saxon countries. It is interesting to observe that not all industrialized countries went through similar political changes, though. This observation is consistent with the argument of our paper for two reasons. First, we predict that multiple self-fulfilling expectations exist. As a matter of fact, the investment in education increased more in the United States than in continental Europe, which is consistent with the expectation of less future redistribution in the United States than in Europe.²⁷ Second, if other institutions (e.g., unions) compress the wage structure and prevent the productivity differences from giving rise to large wage inequalities, the investment incentives do not change significantly, and the constituency for the welfare state does not dry up in countries where these institutions are established. This can explain why countries experiencing a lower increase in pretax inequality also reformed their welfare state institutions less radically.

Finally, in our stylized model, all agents are *ex ante* identical and there is no constraint preventing agents from making an educational investment. In reality, agents differ in both ability and the extent to which capital market imperfections restrain their educational choice. If we extend the model in this dimension, it is clear that low-skill or poor agents who cannot increase their educational effort are destined to suffer from both the increasing relative demand for skills and the induced loss of constituency for the welfare state.

IV. Conclusion

In this paper, we have analyzed the dynamics of redistribution under repeated voting, assuming agents to be fully rational and forwardlooking. Following previous research, we have restricted the attention to Markov perfect equilibria. In contrast to most previous papers, however, we have provided analytical solutions.

Our theory shows that the political support for distortionary redistribution can persist over time, even in a world where agents do not attach any ex ante value to redistribution. This arises from agents making irreversible human capital investments which shape their subsequent political preferences. Thus, a once-and-for-all shock can trigger permanent changes in the size of governments (e.g., the Great Depression). Yet, our paper also shows the possibility of reversals. This possibility is intrinsically associated with the forward-looking nature of the political process (dynamic voting). If agents believe that the breakdown of the welfare state is possible, they vote strategically so as to induce, in the future, a change in the identity of the median voter and the end of the welfare state. The model also shows when such reversals are more likely. In particular, increasing wage (productivity) inequality driven by technological factors tend to undermine rather than strengthen the welfare state. This prediction is consistent with the scaling down of redistributive policies that has been observed, especially in Anglo-Saxon countries, in the 1980's.

We believe that our approach can be fruitfully applied to a variety of areas of research. In particular, it could be integrated into the economics of transition and into positive analysis of constitutions. More in general, we hope that

²⁶ Acemoglu et al. (2001) explore a related idea, arguing that skill-biased technical change has been the cause of deunionization which, in turn, has magnified the initial increase of inequality. Their argument, however, is very different from ours, and not based on political economy mechanisms.

 $^{^{27}}$ During the period 1975–1990, the average years of secondary or higher education for the population over 25 increased by about one year in the average EU country, and by about 22 months in the United States. In the same period, the average years of higher education in the same population group increased by two months in the EU and by almost eight months in the United States. (Source: Data set of Robert J. Barro and Jong-Wha Lee, 1993.)

it can contribute to the development of dynamic macroeconomic models embedding politico-economic aspects.

In some related work in progress (Hassler et al., 2003), we introduce risk aversion, allowing for an insurance motive in redistribution programs. We also explore alternative political mechanisms, featuring more political influence for minorities. In future work, we also plan to investigate whether the political mechanism can amplify technological shocks and generate persistence at business-cycles frequencies. Finally, we plan to analyze policies that discriminate between inter- and intragenerational redistribution such as age-dependent subsidies or public debt.

APPENDIX

Statement of Proposition 3

PROPOSITION 3: Let $\underline{\beta} \simeq 0.570$ be the real solution to the equation $(1 + \sqrt{\beta})^{-1} = \beta$, where $\underline{\beta} > \overline{\theta}(\beta)$ for all $\beta \ge \beta$, and $\overline{\theta}(\beta)$ is as in Proposition 2.

Then, for all $\beta \geq \beta$ and $\theta \in [\beta, \beta]$, there exists an "anti-welfare equilibrium" (AWE), $\langle B^{aw}, U^{aw} \rangle$, with the following characteristics:

- Part 1: "Switch in one period." If $\beta \ge (\sqrt{5} 1)/2 \simeq 0.618$, then, for all $\theta \in [\underline{\theta}(\beta), \beta]$, $B^{aw}(u_t)$ and $U^{aw}(b_t)$ are given, respectively, by equations (11) and (12), where $\underline{\theta}(\beta) \equiv 1 + \beta - \sqrt{\beta(1+\beta)}$ and $\underline{\theta}(\beta) \le \beta$ for all $\beta \ge (\sqrt{5} - 1)/2$ and $U^{pl}(b_t)$ is defined in Proposition 1. Part 2: "Switch in one or two periods." If $\beta \ge \beta$, then, for all $\theta \in [\beta, \min\{\beta, \underline{\theta}(\beta)\})$, $B^{aw}(u_t)$ and
 - *Cart 2:* Switch in one or two periods. If $\beta \ge \underline{\beta}$, then, for all $\theta \in [\underline{\beta}, \min\{\beta, \underline{\theta}(\beta)\})$, $B^{arr}(u_t)$ and $U^{arr}(b_t)$ are given, respectively, by equations (13) and (14), where $\underline{\theta}(\beta) > \underline{\beta}$ for all β , and $\hat{u}(\beta, \theta) \equiv \frac{1}{2}(2 + \beta (2\beta + 3 \theta)\theta) \cdot (1 \theta)^{-1}$.

Proofs of Propositions

For notational convenience, the indirect utilities are, in this Appendix, rewritten as follows:

$$\hat{V}^{j}(b_{t}, u_{t}) \equiv V^{j}(b_{t}, B(U(b_{t})), u_{t}), \text{ for } j \in \{os, ou\}.$$

PROOF OF PROPOSITION 1:

We must show that the pair $\langle B^i, U^i \rangle$, for $i \in \{pl, dp\}$, satisfies the equilibrium conditions

1) $B^i(u_t) = \arg \max_{b_t} \hat{V}^d(b_t, u_t)$ subject to $b_t \in [0, 1]$; and 2) $U^i(b_t) = (1 - \beta + b_t + \beta B^i(U^i(b_t)))/2$, where d = os if i = pl and d = ou if i = dp.

If i = pl, it is straightforward to see that \hat{V}^{os} is maximized by setting $b_t = 0$ in every period and that, consequently, $u_t = \frac{1-\beta}{2}$ for all $t \ge 1$.

Next, consider the DP equilibrium.

First, we note that

(A1)
$$B^{dp}(U^{dp}(b_t)) = \begin{cases} \frac{3b^{dp} - b_t}{2} & \text{if } b_t \ge \frac{2\beta}{2 + \beta} \\ 1 & \text{else,} \end{cases}$$

where we used the fact that for any $b_t \in [0, 1], U^{dp}(b_t) < \bar{u}$. Then, \hat{V}_t^{ou} can be expressed as:

(A2)
$$\hat{V}_{t}^{ou}(b_{t}, u_{t}) = \begin{cases} b_{t} - \frac{1}{4} \left(1 - \beta + b_{t} + \beta \frac{3b^{dp} - b_{t}}{2} + 2u_{t} \right) b_{t} & \text{if } b_{t} \ge \frac{2\beta}{2 + \beta} \\ b_{t} - \frac{1}{4} \left(1 + b_{t} + 2u_{t} \right) b_{t} & \text{else.} \end{cases}$$

Maximizing \hat{V}_t^{ou} over b_t yields;

$$b_{t} = \begin{cases} \frac{3}{2} - u_{t} & \text{if } u_{t} > \overline{u}(\beta) \\ b^{dp} - \frac{2}{2 - \beta} (u_{t} - u^{dp}) & \text{if } u_{t} \in \left[\frac{3}{2} - \frac{2}{2 + \beta}, \overline{u}(\beta)\right] \\ 1 & \text{if } u_{t} \in \left[0, \frac{3}{2} - \frac{2}{2 + \beta}\right] \\ = B^{dp}(u_{t}). \end{cases}$$

This proves that equilibrium condition 1 is satisfied.

To prove that the second condition is satisfied, we use (A1) to substitute for b_{t+1} in the optimal investment expression, giving,

$$(1 - \beta + b_t + \beta B^{dp}(U^{dp}(b_t)))/2 = \begin{cases} u^{dp} + \frac{2 - \beta}{4} (b_t - b^{dp}) & \text{if } b_t \in \left[\frac{2\beta}{2 + \beta}, 1\right] \\ \frac{1 + b_t}{2} & \text{else} \end{cases}$$
$$= U^{dp}(b_t).$$

To see the steps of the first part of the proof in more detail, define $\hat{V}^a(u_t)$ and $\hat{V}^b(u_t)$ as follows:

$$\begin{split} \hat{V}^{a}(u_{t}) &= \max_{b_{t} \in \{0, (2\beta/(2+\beta))\}} \hat{V}_{t}^{ou}(b_{t}, u_{t}) \\ &= \begin{cases} \frac{9}{16} - \frac{3}{4} u_{t} + \frac{1}{4} u_{t}^{2} \equiv \hat{V}^{a, int}(u_{t}) & \text{if } u_{t} > \frac{6-\beta}{2(2+\beta)} \\ \beta \frac{6+\beta-2u_{t}(2+\beta)}{2(2+\beta)^{2}} \equiv \hat{V}^{a, cor}(u_{t}) & \text{else} \end{cases} \\ \hat{V}^{b}(u_{t}) &= \max_{b_{t} \in [(2\beta/(2+\beta)), 1]} \hat{V}_{t}^{ou}(b_{t}, u_{t}) \\ &= \begin{cases} \frac{1}{8} \frac{(\beta^{2}-3\beta+2u_{t}(2+\beta)-6)^{2}}{(2-\beta)(2+\beta)^{2}} \equiv \hat{V}^{b, int}(u_{t}) & \text{if } u_{t} \geq \frac{3}{2} - \frac{2}{(2+\beta)} \\ \frac{1}{8} \frac{8+\beta(6-\beta)}{2+\beta} - \frac{u_{t}}{2} \equiv \hat{V}^{b, cor}(u_{t}) & \text{else}, \end{cases} \end{split}$$

where $\hat{V}^{a,cor}(u_t)$ and $\hat{V}^{b,cor}(u_t)$ result from corner solutions in the respective ranges (the corners being $b_t = \frac{2\beta}{2+\beta}$ and $b_t = 1$, respectively) while $\hat{V}^{a,int}(u_t)$ and $\hat{V}^{b,int}(u_t)$ result from the interior solutions $b_t = \frac{3}{2} - u_t$ and $b_t = b^{dp} - \frac{2}{2-\beta}(u_t - u^{dp})$, respectively.

First, standard algebra establishes that $\hat{V}^{b,int}(u_t) - \hat{V}^{u,cor}(u_t) = \frac{1}{8} \frac{(\beta^2 - 2\beta u_t - \beta + 6 - 4u_t)^2}{(2 - \beta)(2 + \beta)^2} > 0$ and that, in the range where $u_t \leq \frac{3}{2} - \frac{2}{(2+\beta)}$, $\hat{V}^{b,cor}(u_t) - \hat{V}^{a,cor}(u_t) > \frac{1}{8}(2-\beta)\frac{4(1-\beta)+\beta^2}{(2+\beta)^2} > 0$. Thus, whenever $\hat{V}^a(u_t) = \hat{V}^{a,cor}(u_t)$, then $\hat{V}^b(u_t) > \hat{V}^a(u_t)$.

Second, if $\beta < \frac{2}{3}$, then $\frac{6-\beta}{2(2+\beta)} > 1$ and $\hat{V}^a(u_t) = \hat{V}^{a,cor}(u_t)$ for all u_t . Thus, $\hat{V}^b(u_t) > \hat{V}^a(u_t)$ if $\beta < \frac{2}{3}$.

Third, note that if $\beta \ge \frac{2}{3}$, then there exists a range of u_t , where $\hat{V}^a(u_t) = \hat{V}^{a,int}(u_t)$. In this range, standard algebra establishes that $\hat{V}^{b,int}(u_t) > \hat{V}^{a,int}(u_t)$ for all u_t provided that $\beta < (\sqrt{17} - 1)/4$. Thus, $\beta < (\sqrt{17} - 1)/4$ implies that $\hat{V}^b(u_t) > \hat{V}^a(u_t)$ for all $u_t \in [0, 1]$.

Finally, consider the range of parameters such that $\beta \ge (\sqrt{17} - 1)/4$, implying $\bar{u} \le 1$. In this case, for all $1 \ge u_t > \bar{u}(\beta)$, $\hat{V}^b(u_t) < \hat{V}^a(u_t) = \hat{V}^{a,int}(u_t)$.

PROOF OF PROPOSITION 2:

We must show that for all t and u_t , $\langle B^{pw}, U^{pw} \rangle$ satisfies the two equilibrium conditions

- 1) $B^{pw}(u_t) = \arg \max_{b_t} \{ \hat{V}^{pw}(b_t, u_t) \}$ subject to $b_t \in [0, 1]$; and 2) $U^{pw}(b_t) = (1 \beta + b_t + \beta B^{pw}(U^{pw}(b_t)))/2$, where $\hat{V}^{pw}(b_t, u_t) = \hat{V}^{os}(b_t, u_t)$ if $u_t \le 1/2$ and $\hat{V}^{pw}(b_t, u_t) = \hat{V}^{ou}(b_t, u_t)$ otherwise.

We start from condition 1. Consider first $u_t \leq \frac{1}{2}$. Then, $V_t^{pw}(b_t, u_t) = V_t^{os}(b_t, u_t)$, which is maximized by setting $b_t = 0$.

Then, consider $u_t > \frac{1}{2}$. First, we define

$$\overline{\theta}(\beta) \equiv \min\{\beta, \ \widetilde{\theta}(\beta)\},\$$

where

(A3)
$$\tilde{\theta}(\beta) = \begin{cases} \frac{1+\beta}{2} - \frac{\sqrt{\beta(2+\beta)}}{2} & \text{if } \beta > \frac{\sqrt{17}-4}{4} \\ \frac{1+\beta}{2} - \frac{\sqrt{\beta(6+\beta)}}{2} \frac{1+\beta}{2+\beta} & \text{else.} \end{cases}$$

Now, note that $\theta \leq \overline{\theta}(\beta) \leq \beta$, implying that $U^{pw}(b_t) \leq (>) \frac{1}{2}$ if $b_t \leq (>) \theta$. Thus,

(A4)
$$B^{pw}(U^{pw}(b_t)) = \begin{cases} \frac{3b^{dp} - b_t}{2} & \text{if } b_t \ge \frac{2\beta}{2+\beta} \\ 1 & b_t \in \left(\theta, \frac{2\beta}{2+\beta}\right) \\ 0 & b_t \le \theta, \end{cases}$$

and

108

$$\hat{V}_{t}^{ou}(b_{t}u_{t}) = \begin{cases} b_{t} - \frac{1}{4} \left(1 - \beta + b_{t} + \beta \frac{3b^{dp} - b_{t}}{2} + 2u_{t} \right) b_{t} & \text{if } b_{t} \ge \frac{2\beta}{2 + \beta} \\ b_{t} - \frac{1}{4} (1 + b_{t} + 2u_{t}) b_{t} & b_{t} \in \left(\theta, \frac{2\beta}{2 + \beta} \right) \\ b_{t} - \frac{1}{4} (1 - \beta + b_{t} + 2u_{t}) b_{t} & b_{t} \le \theta. \end{cases}$$

Comparing this to (A2), we see that utility in the PWE is identical to utility under DP for $b_t > \theta$. Thus, a sufficient condition for equilibrium condition 1 to be satisfied for $u_t > \frac{1}{2}$ is that in this range, $\max_{b_t \geq \theta} \hat{V}_t^{ou}(b_t, u_t) \geq \max_{b_t \geq \theta} \hat{V}_t^{ou}(b_t, u_t)$. It is straightforward to verify that this is the case for sufficiently low θ , in particular when $\theta \leq \tilde{\theta}(\beta)$, which is implied by $\theta \leq \tilde{\theta}(\beta)$.

To prove that the second condition is satisfied, we use (A4) to substitute for b_{t+1} in the optimal investment expression, giving,

$$(1 - \beta + b_t + \beta B^{pw}(U^{pw}(b_t)))/2 = \begin{cases} \left(1 - \beta + b_t + \beta \frac{3b^{dp} - b_t}{2}\right)/2 & \text{if } b_t \ge \frac{2\beta}{2 + \beta} \\ (1 + b_t)/2, & b_t \in \left(\theta, \frac{2\beta}{2 + \beta}\right) \\ \frac{1 - \beta + b_t}{2} & b_t \le \theta \end{cases}$$
$$= U^{pw}(b_t),$$

where we used $\theta \leq \overline{\theta}(\beta) \leq \beta$.

To see the steps in the proof of the first condition in more detail, we first note that $\theta \leq \beta$, implies that $\max_{b_{t\leq\theta}} \hat{V}_t^{ou}(b_t, u_t)$ is a corner solution with $b_t = \theta$, and utility equal to $\frac{3+\beta-\theta-2u_t}{4}\theta$. It is easily verified that the difference $\max_{b_t>\theta} \hat{V}_t^{ou}(b_t, u_t) - \max_{b_{t\leq\theta}} \hat{V}_t^{ou}(b_t, u_t)$ is minimized at $u_t =$ 1, where it is given by

$$\begin{cases} \frac{1}{16} - \frac{1}{4} \theta(3 + \beta - (2 + \theta)) & \text{if } \beta > \frac{\sqrt{17} - 4}{4} \\ \frac{1}{8} (1 + \beta) \frac{2 + \beta(1 - \beta)}{(2 + \beta)^2} - \frac{1}{4} \theta(3 + \beta - (2 + \theta)) & \text{if } \beta \le \frac{\sqrt{17} - 4}{4} \end{cases}$$

Solving $\max_{b_i > \theta} \hat{V}_t^{ou}(b_t, u_t) - \max_{b_{t \le \theta}} \hat{V}_t^{ou}(b_t, u_t) = 0$ for θ , yields (A3). Finally, it is trivial to verify that $u^{dp} = U^{pw}(B^{pw}(u^{dp}))$ and $u^{pl} = U^{pw}(B^{pw}(u^{pl}))$ and that the root of $u_{t+1} = U^{pw}(B^{pw}(u_t))$ is $\frac{1}{2}$, which establishes points 1-2 of the proposition.

PROOF OF PROPOSITION 3:

As in the Proof of Proposition 1, we must show that, for all t and u_t , $\langle B^{aw}, U^{aw} \rangle$ satisfies

- 1) $B^{aw}(u_t) = \arg \max_{b_t} \{ \hat{V}^{aw}(b_t, u_t) \}$, subject to $u_{t+1} = U^{aw}(b_t), b_t \in [0, 1]$ and $b_{t+1} = B^{aw}(u_{t+1})$; and
- $B^{aw}(u_{t+1}); \text{ and}$ 2) $U^{aw}(b_t) = (1 \beta + b_t + \beta B^{aw}(U^{aw}_{(\epsilon)}(b_t)))/2.$
- 1. Consider first the case when $\beta \ge \frac{\sqrt{5}-1}{2}$ and $\theta \in [\underline{\theta}(\beta), \beta]$. Note first that in the range $\beta \ge \frac{\sqrt{5}-1}{2}, \underline{\theta}(\beta) \le \beta$, ensuring that the set of beliefs under consideration is nonempty. To prove part (1), consider, first, the range where $u_t > \frac{1}{2}$.

$$\hat{V}^{aw}(b_t, u_t) = \hat{V}^{ou}(b_t, u_t)$$

$$= \begin{cases} b_t - \frac{1}{4}(1 - \beta + b_t + \beta\theta + 2u_t)b_t & \text{if } b_t > \theta \\ \\ b_t - \frac{1}{4}(1 - \beta + b_t + 2u_t)b_t & \text{if } b_t \le \theta. \end{cases}$$

Standard differentiation shows that $\hat{V}^{aw}(b_t, u_t)$ is increasing in b_t for all $b_t \leq \theta$ (since $\theta \leq \beta$) and that, as long as $\theta \geq \underline{\theta}(\beta)$,

$$\hat{V}^{aw}(\theta, u_t) = \theta - \frac{1}{4}(1 - \beta + \theta + 2u_t)\theta$$

> $b_t - \frac{1}{4}(1 - \beta + b_t + \beta\theta + 2u_t)b_t = \hat{V}^{aw}(b_t > \theta, u_t),$

for all $b_t > \theta$ and $u_t > \frac{1}{2}$. Thus, $B^{aw}(u_t) = \theta$ for $u_t > \frac{1}{2}$. If $u_t \le \frac{1}{2}$, $\hat{V}^{aw}(b_t, u_t) = \hat{V}^{os}(b_t, u_t)$, which is decreasing in b_t . Hence, $B^{aw}(u_t) = 0$, for $u_t \le \frac{1}{2}$.

To prove part (2), observe that

$$(1 - \beta + b_t + \beta B^{aw}(U^{aw}(b_t)))/2 = \begin{cases} (1 - \beta + b_t + \beta \theta)/2 & \text{if } b_t > \theta \\ (1 - \beta + b_t)/2 & \text{if } b_t \le \theta \end{cases} = U^{aw}(b_t).$$

where the equality follows from the facts that, for all $b_t \le \theta \le \beta$, $(1 - \beta + b_t)/2 \le \frac{1}{2}$, and, for all $b_t > \theta \ge \frac{\theta}{2}(\beta)$, $\frac{1}{2}(1 - \beta + \beta\theta + b_t) > \frac{1}{2}$. The latter inequality can be checked by inserting the definition of $\frac{\theta}{2}(\beta)$ in the left-hand side of the inequality.

2. Consider now the case when $\beta \ge \beta$ and $\theta \in [\beta, \min\{\beta, \underline{\theta}(\beta)\})$. Note first that $\beta < \underline{\theta}(\beta)$ for all $\beta \ge \beta$, implying that the set of beliefs under consideration is nonempty.

As to part (1), consider, first, the range where $u_t > \frac{1}{2}$.

$$\hat{V}^{aw}(b_t, u_t) = \hat{V}^{ou}(b_t, u_t)$$

$$= \begin{cases} b_t - \frac{1}{4}(1 - \beta + b_t + \beta\theta + 2u_t)b_t & \text{if } b_t > \theta \\ b_t - \frac{1}{4}(1 - \beta + b_t + 2u_t)b_t & \text{if } b_t \le \theta. \end{cases}$$

It is immediate to see that the value function has a discontinuous fall at $b_t = \theta$. Moreover, standard differentiation shows that $\hat{V}^{aw}(b_t, u_t)$ is increasing in b_t , throughout in the region $b_t \le \theta$, and provided that $u_t \ge \hat{u}(\beta, \theta)$ in the region $b_t > \theta$. Furthermore,

is a decreasing function of u_t , strictly positive for $u_t \in (0.5, \hat{u}(\beta, \theta))$, equal to zero when $u_t = \hat{u}(\beta, \theta)$, and strictly positive for $u_t \in (\hat{u}(\beta, \theta), 1]$. Thus, in the range $u_t \in (0.5, \hat{u}(\beta, \theta)]$, $\hat{V}^{aw}(1, u_t) \ge \hat{V}^{aw}(\theta, u_t)$, with equality holding if and only if $u_t = \hat{u}(\beta, \theta)$. This shows that setting $b_t = 1$ is optimal for the old unsuccessful in the range $u_t \in (0.5, \hat{u}(\beta, \theta)]$ and, hence, $B^{aw}(u_t) = 1$ in that range.

Finally, we need to show that setting $b_t = \theta$ is optimal for the old unsuccessful in the range $u_t \in (\hat{u}(\beta, \theta), 1]$. Since, as already noted, $\hat{V}^{aw}(b_t, u_t)$ is increasing in b_t for all $b_t \leq \theta$, it remains to be shown that $\hat{V}^{aw}(\theta, u_t) > \max_{b \in (\theta, 1]} \{ \hat{V}^{aw}(b, u_t) \}$ when $u_t \in (\hat{u}(\beta, \theta), 1]$. This can be shown as follows.

(a) First, note that if $u_t \in \left(\hat{u}(\beta, \theta), \frac{1}{2} + \frac{\beta(1-\theta)}{2}\right]$, then, $\arg \max_{b \in (\theta,1]} \{\hat{v}^{aw}(b, u_t)\} = 1$. In this case, as pointed out above [see equation (A5) and the following discussion], $\hat{v}^{aw}(\theta, u_t) > \hat{v}^{aw}(1, u_t)$, establishing the claim.

(b) Next, if
$$u_t \in \left\lfloor \frac{1}{2} + \frac{\beta(1-\theta)}{2}, 1 \right\rfloor$$
, then $b^*(\theta, u_t) \equiv \arg \max_{b \in (\theta,1]} \{\hat{V}^{aw}(b, u_t)\} = \frac{3}{2} + \frac{\beta(1-\theta)}{2} - u_t < 1$. Define, then,

$$\Delta \hat{V}(u_t, \beta, \theta) \equiv \hat{V}^{aw}(\theta, u_t) - \hat{V}^{aw}(b^*(\theta, u_t), u_t) = \left(-\frac{1}{2}\theta + \frac{1}{4}\beta + \frac{3}{4} - \frac{1}{4}\beta\theta\right)u_t$$
$$-\frac{1}{4}u_t^2 + \frac{3}{4}\theta + \frac{5}{8}\beta\theta - \frac{1}{4}\theta^2 - \frac{1}{16}\beta^2 - \frac{9}{16} - \frac{3}{8}\beta - \frac{1}{16}\beta^2\theta^2 + \frac{1}{8}\beta^2\theta$$

Since $\Delta \hat{V}(u_t, \beta, \theta)$ is hump-shaped in u_t , it must attain its minimum in the range $u_t \in \left[\frac{1}{2} + \frac{\beta(1-\theta)}{2}, 1\right]$ at either $u_t = \frac{1}{2} + \frac{\beta(1-\theta)}{2}$ or $u_t = 1$. It turns out that $\Delta \hat{V}\left(\frac{1}{2} + \frac{\beta(1-\theta)}{2}, \beta, \theta\right) = \frac{1}{4}\left(-1 + 2\theta - \theta^2(1-\beta)\right)$ $\geq \frac{1}{4}\left(-1 + 2\beta - \beta^2(1-\beta)\right) \geq 0$ $\Delta \hat{V}(1, \beta, \theta) = \frac{1}{4}\left(-(1 + \frac{1}{4}\beta^2)\theta^2 + \frac{1}{2}(2+\beta)(1+\beta)\theta - \frac{1}{4}(1+\beta)^2\right)$ $\geq \frac{1}{4}\left(-(1 + \frac{1}{4}\beta^2)\beta^2 + \frac{1}{2}(2+\beta)(1+\beta)\beta - \frac{1}{4}(1+\beta)^2\right) \geq 0.$

Thus, $B^{aw}(u_t) = \theta$ for $u_t > \hat{u}(\beta, \theta)$.

The proof for the range $u_t \le 1/2$ is identical to the first case and thus, part (1) is proved. To prove part (2), observe that

$$(1-\beta+b_t+\beta B^{aw}(U^{aw}(b_t)))/2 = \begin{cases} (1-\beta+b_t+\beta\theta)/2 & \text{if } b_t > \theta\\ (1-\beta+b_t)/2 & \text{else} \end{cases} = U^{aw}(b_t),$$

where the equality follows from the fact that $(1 - \beta + b_t)/2 < \frac{1}{2}$ for all $b_t < \theta$, and that $(1 - \beta + b_t)/2 < \frac{1}{2}$ for all $b_t < \theta$.

MARCH 2003

 $\beta + b_t + \beta \theta / 2 \ge \hat{u}(\beta, \theta)$ for all $b_t \in (\theta, 1]$. The latter can be checked as follows. Recall that, in the range under consideration, $\beta \ge \beta$ and $\theta \ge \beta$. Then:

$$(1 - \beta + b_t + \beta\theta)/2 - \hat{u}(\beta, \theta) > (1 - \beta + \theta + \beta\theta)/2 - \hat{u}(\beta, \theta)$$

$$\geq (-1 - 2\beta + (3 + 4\beta)\beta - (2 + \beta)\beta^2) \frac{1}{2(1 - \theta)}$$

$$\geq (-1 - 2\beta + (3 + 4\beta)\beta - (2 + \beta)\beta^2) \frac{1}{2(1 - \theta)} > 0.$$

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