Lecture Notes

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Overview:1

Purpose; Study models of the interaction between the global economy and the climate to

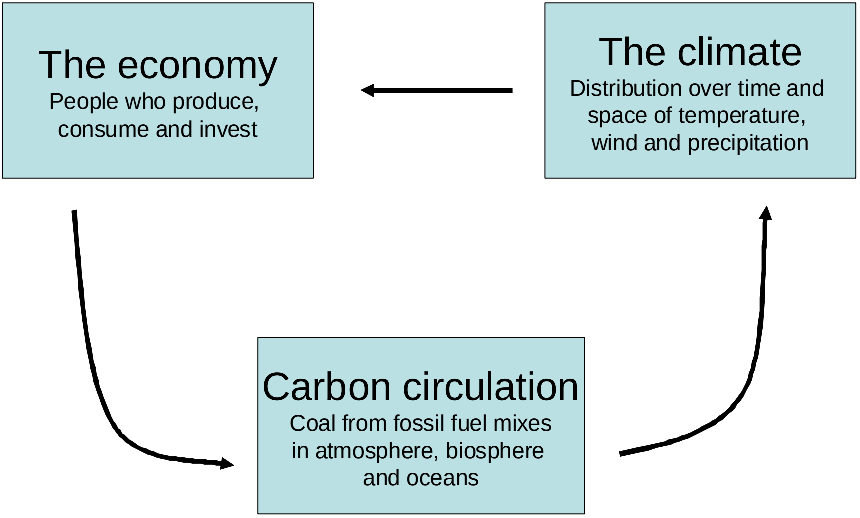
- provide understanding of important mechanisms,
- analyze optimal policy.
- Involves results from both social and natural sciences.
- But we are economists – use our comparative advantage to contribute and critically analyze the economic side but take "conventional wisdom" from the natural science as given.
- Economics is key for analyzing effects of policy.

Overview:2

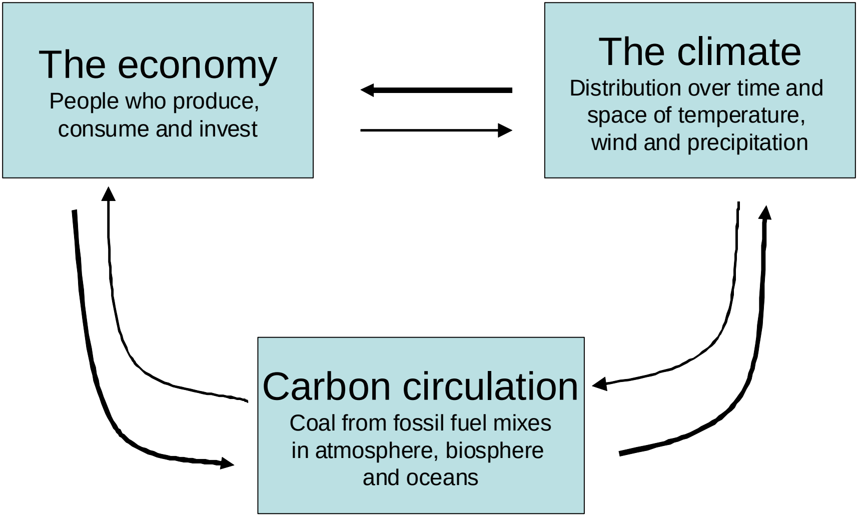
Emissions are caused by decisions taken by billions of people, firms and other agents acting on markets. Cannot be understood without economics. Economics is important for

- analyzing effects of policy,
- understanding endogenous adaptation and technical change,
- making forecasts.

A schematic IAM - interactions



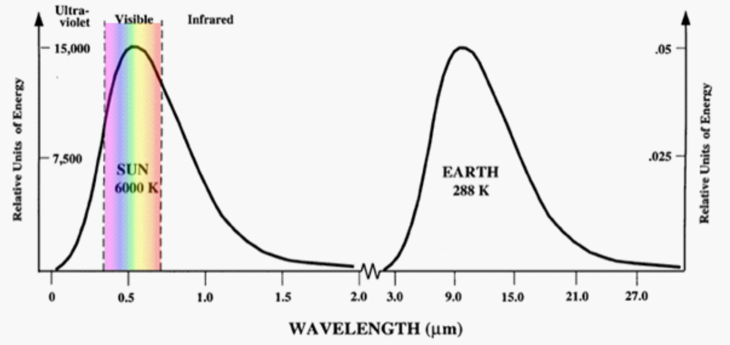
A schematic IAM - dynamic and bidirectional



Climate models - forcing and the energy budget

- Incoming *energy flow of energy* from sun (342 W/m^2 $= 2400kW$ per football field), in steady state equals;
- Inflow is largely in the form of sunlight.
- Outgoing *energy flow*, consisting of
  - direct reflection (1/3)
  - Heat radiation (2/3).
  - The latter is a function of, in particular temperature and greenhouse gases.
- Without greenhouse gases and atmosphere, ground temperature would be -19°C.

Radiation

- Visible sunlight and infrared heat waves are both electromagnetic radiation, but with different frequencies.
- Frequency of radiation emitted depends on temperature. Compare with dimmer on halogen lights,
- 

Comparison of the emission spectra of the sun and the earth. Note the huge disparity in the amount of energy emitted by the sun (left-hand scale) and the earth (right-hand scale).

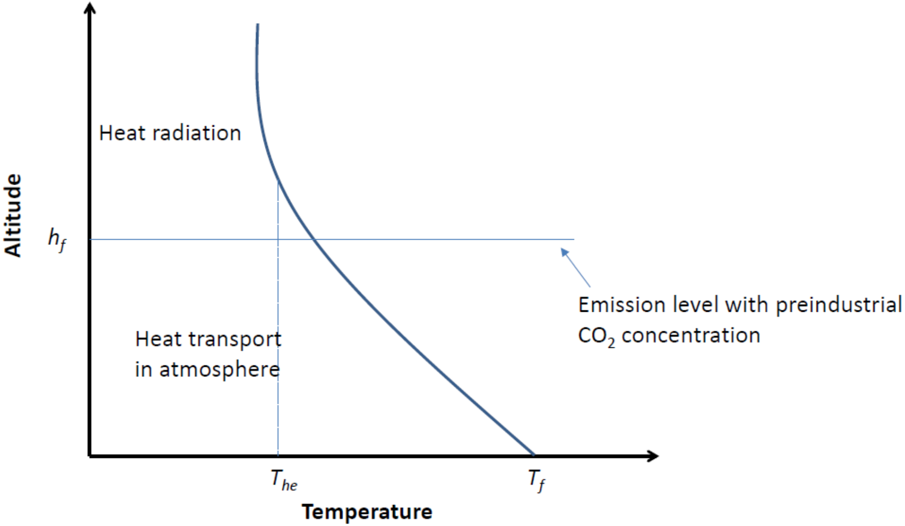
Greenhouse effect

- When electromagnetic radiation passes through gases energy can be absorbed by the radiation making the molecules vibrate.
- For this to occur, the molecules resonance frequency (like the particular frequency a guitar string vibrates) must be aligned with the frequency of the radiation.
- CO_2 (and other molecules with three or more atoms) have resonance frequencies aligned with infrared radiation. Sunlight has a frequency much higher.
- Thus, CO_2 absorbs energy from heat radiation but not from sunlight.
- Gases with molecules with two atoms have much higher resonance frequencies but not as high as the frequency of visible light. Thus, oxygen (O_2) and nitrogen (N_2), making up 99% of the atmosphere are not greenhouse gases.
- Compare to a band playing in a bar. The bass guitar can make some objects, e.g. cups and cutlery vibrate, but a high pitched tone from the guitar has no effect.

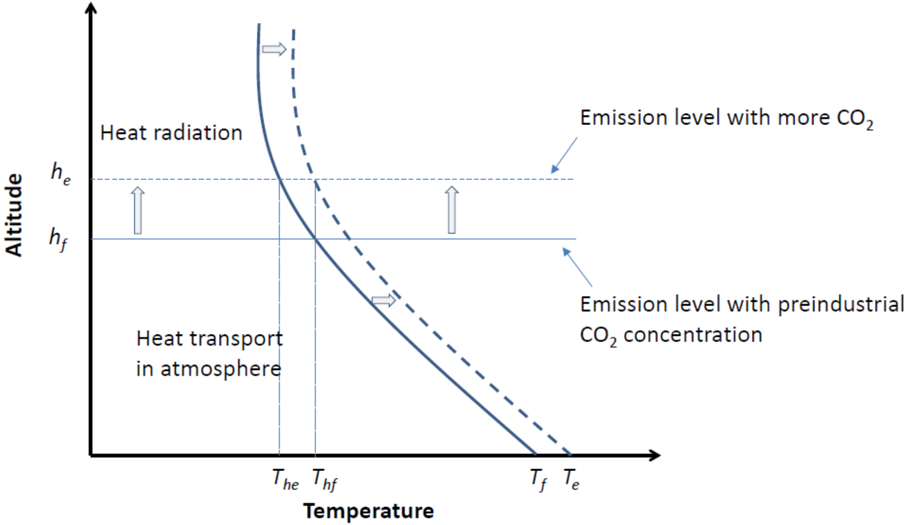
Energy transmission in atmosphere

- Even the small amount of CO_2 in the atmosphere (0.04%) makes it quite opaque for heat waves. One might think adding more does then not have any effect.
- Turns out to be wrong. Heat is transferred up in the atmosphere since it is colder the higher the altitude. Eventually, the radiation can escape into space. The altitude this occurs is called *emission level.*
- More CO_2 implies the emission level is moved up, where it *ceteris paribus* is colder.
- If the temperature at the emission level is colder, less energy is transmitted. This leads to a surplus – less energy escapes than comes in from the sun.
- The accumulation of energy increases the temperature in the atmosphere until the temperature at the emission level again is high enough to imply that the energy flow out in space is the same as the flow into earth.

Heat transfer through atmosphere



Effect of more carbon dioxide



The energy budget

- Consider a system (e.g., the earth) in a situation of net energy flow = incoming-outgoing flow=0.
- In such a case, the *energy budget is balanced* and no heat is accumulated or lost.
- Suppose now that the energy budget is perturbed by a positive amount F (inflow increased and/or outflow decreased)
- Now budget is now longer balanced but in surplus.
- Leads to an accumulation of heat in the system, temperature rises, quicker the larger is the energy budget surplus.
- Speed of temperature increase also depends on heat capacity of the system (mass and material). Compare a balloon with air and a balloon with water.
- As the temperature goes up, outgoing energy flow increases when temperature rises. Called *Planck feedback*.

A new balance is achieved

- Suppose there is an initial surplus in energy budget of F (forcing).
- As long as there is a surplus in the budget, temperature increases.
- Outflow is an *increasing* function of temperature $(O(T))$ (thermal radiation). So a higher temperature *reduces* surplus.
- Approximate the increased outflow as proportional to temperature increase. $O(T) \approx O(\bar{T}) + (T - \bar{T})\, O'(\bar{T})$
- Denote $\kappa \equiv O'(\bar{T})$, let $\bar{T}$ be pre-industrial temperature and redefine T_t as actual temperature in period t as actual temperature minus $\bar{T}$.
    - Energy budget is then $F - \kappa T_t$.
- Approximate rate of change in temperature is proportional (with constant σ) to surplus in budget: $\frac{dT_t}{dt} = \sigma(F - \kappa T_t)$.
- What determines σ? Will there be a new equilibrium? Yes, when $T_t = \frac{F}{\kappa}$
- If earth were a blackbody without atmosphere with a temperature of 15°C, $\kappa \approx 3.3 \frac{W/m^2}{K}$. Due to feedbacks, likely to be smaller.

Forcing

- As discussed above, more greenhouse gases pushes emission level outwards which creates a surplus in energy budget relative to preindustrial situation.
- Surplus depends on greenhouse gas concentration.
- Most important is water vapor. Second is CO_2.
- Human activities has increased concentration of CO_2 and other greenhouse gases, e.g., methane.
- Surplus (forcing) 1.7 and 1 W/m^2, respectively.

Forcing in 2011 relative to 1750

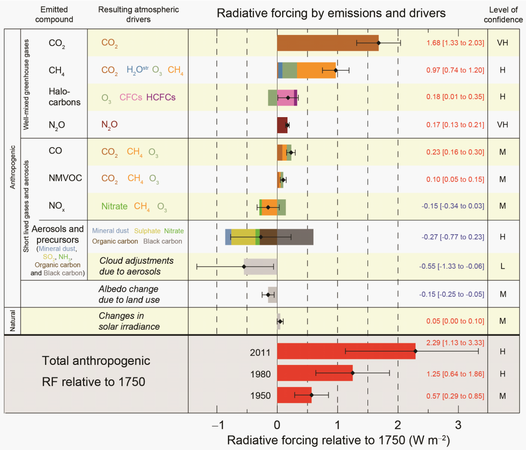
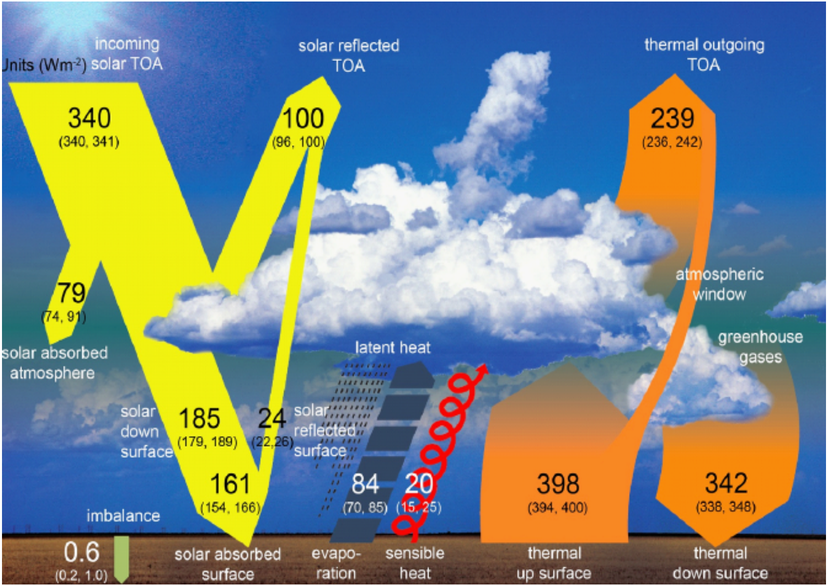


Figure: Radiative forcing estimates in 2011 relative to 1750 and aggregated uncertainties for the main drivers of climate change. Source: IPCC, Assessment report 5, Summary for policy makers fig 5.

Energy Flows



Feedback effects

- Gross flows as very large relative to direct greenhouse effect.
- Creates feedback effects. Example, more CO_2 increase forcing, leads to
  - higher concentration of water vapor, increase greenhouse effect.
  - melting of icecaps, decrease direct surface reflection (albedo).
  - changed cloud formation, change back radiation and reflection.
- Feedback mechanisms are very important and have been so historically.
- Direct effect of CO_2 emission, quite certain. Not the case for feedback.

Feedbacks in the energy budget

- Let us formalize feedback as follows: Increased temperature increases *effective forcing,* adding a term xT_t to the energy budget, becoming:

$$\frac{dT_t}{dt} = \sigma \left(F + xT_t - \kappa T_t \right) = \sigma \left(F - \left(\kappa - x \right) T_t \right).$$

- The steady state for a given forcing F now becomes

$$T\left(F\right) = \frac{1}{\kappa - x} F$$

- A realistic value of $\frac{1}{\kappa - x}$ is around 0.8, but with large uncertainty.

Greenhouse effect and climate sensitivity

- Higher concentration of CO_2in atmosphere reduces outgoing energy flow (long-wave (heat) radiation).
- Well approximated by a logarithmic function (Arrhenius greenhouse law, 1896). For a given concentration S of CO_2 in the atmosphere and the pre-industrial level S_0, forcing is

$$F(S) = \frac{\eta}{\ln 2} \ln\left(\frac{S}{S_0}\right)$$

- An often used approximation of η is 3.7.
- Combine with $T(F) = \frac{F}{\kappa - x}$ gives

$$T(F(S)) = \frac{\eta}{\kappa - x} \frac{1}{\ln 2} \ln\left(\frac{S}{S_0}\right).$$

- The ratio $\eta/(\kappa - x)$ has a very important interpretation and is often labelled the *Equilibrium Climate Sensitivity (ECS).*
- IPCC AR5: ECS is "likely in the range 1.5 to 4.5°C", "extremely unlikely less than 1°C", and "very unlikely greater than 6°C".

Heating of oceans

- Equation $\frac{dT_t}{dt} = \sigma\left(F - (\kappa - x)\, T_t\right)$ does not take into account heating of oceans/atmosphere separately.
- Two other terms in energy budget for atmosphere, capturing energy flow from atmosphere to ocean and *vice versa.*
- These new terms *do not balance* if temperature is different (in an average sense).
- New law-of-motion for atmosphere

$$\frac{dT_t}{dt} = \sigma_1\left(F_t - (\kappa - x)\, T_t - \sigma_2\left(T_t - T_t^L\right)\right)$$

  where T_t and T_t^L, respectively, denote the atmospheric and ocean temperature in period t.
- Complete by setting

$$\frac{dT_t^L}{dt} = \sigma_3\left(T_t - T_t^L\right)$$

- Implies a drag on heating, but no difference with respect to long-run effect of forcing.

Simulation

- Make a discrete time approximation. Yields a system of difference equations;

$$
\begin{aligned}
T_t &= T_{t-1} + \sigma_1 \left(F_{t-1} - (\kappa - x)\, T_{t-1} - \sigma_2 \left(T_{t-1} - T_{t-1}^L \right) \right) \\
T_t^L &= T_{t-1}^L + \sigma_3 \left(T_{t-1} - T_{t-1}^L \right)
\end{aligned}
$$

instead of

$$
\begin{aligned}
\frac{dT_t}{dt} &= \sigma_1 \left(F_t - (\kappa - x)\, T_t - \sigma_2 \left(T_t - T_t^L \right) \right) \\
\frac{dT_t^L}{dt} &= \sigma_3 \left(T_t - T_t^L \right)
\end{aligned}
$$

- Can easily be simulated in a spread-sheet program.

Simulation

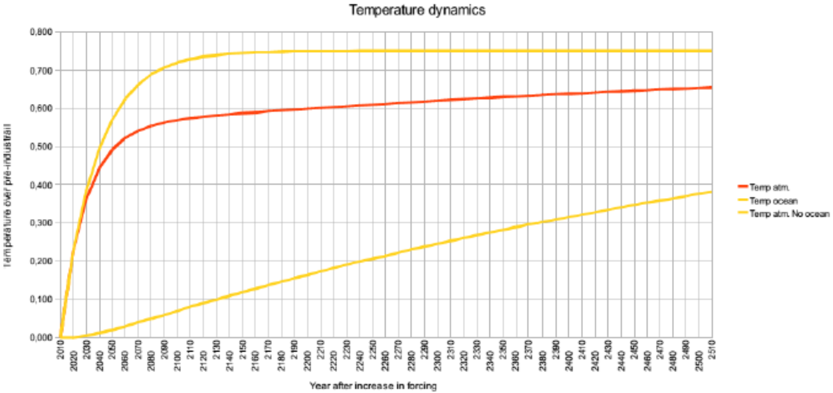
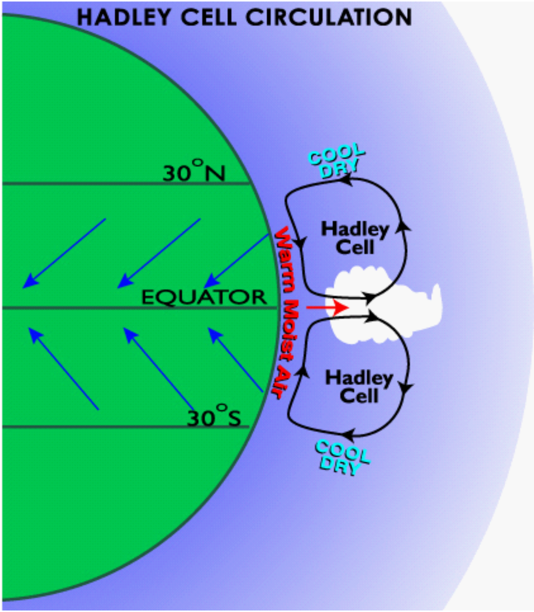


Figure: Increase in atmospheric and ocean temperature after a permanent forcing of $1W/m^2$.

Climate models:1

- Circulation models.
- Energy is not evenly radiated to the earth. Highest around equator.
- Creates systematic flows of air and water.
- Used to forecast weather – but also climate.

Climate models:Circulation cells

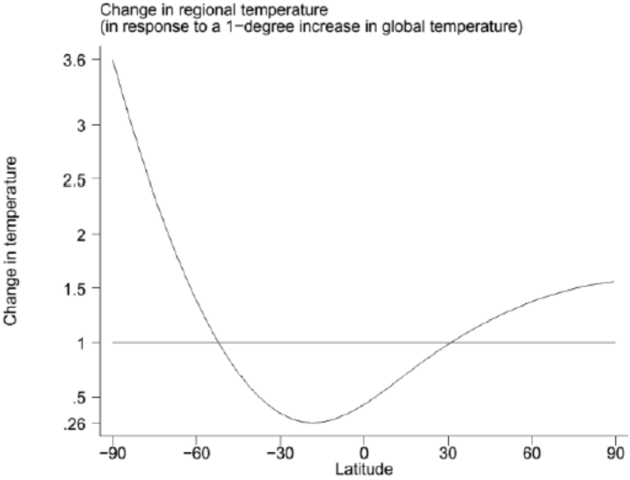


Climate models: key points

- Ocean currents also transport heat from equator towards poles.
- More accurate descriptions need to model landmasses and mountains.
- Climate models build on deterministic laws of physics but are chaotic in nature. This implies:
  - A "butterfly effect" – small variation in initial state e.g., distribution of energy, leads to unsystematic large differences in weather a few weeks later.
  - Unconditional distribution stable, e.g., mean and variance of temperature and wind speeds.
  - Best forecast is unconditional distribution for forecasts beyond a few weeks.
- State-of-the art climate models build on same principles.

Downscaling

- Circulation models (very) large and (very) time consuming to run.
- Simplification: use a statistical representation of how a change in global mean temperature affects different locations.
- Simplest case – use latitude. Estimate a different sensitivity β_i for each latitude.
- $T_{i,t} = \bar{T}_i + \beta_i * T_t + z_{i,t}$



Change in regional temperature
(in response to a 1−degree increase in global temperature)
Change in temperature
3.6
3
2.5
2
1.5
1
.5
.26
−90
−60
−30
0
30
60
90
Latitude

Historic climate

- Use various proxy data, tree rings, corals, plankton and pollen...
- Also data on greenhouse gas concentrations. Positive correlation suggests positive feedback.

Ice ages

- Small change in solar influx or variation in earth's orbit gets amplified by feed-back.
- A key mechanism may be ice-albedo feedback (Arrhenius).
- A small negative F leads to buildup of the icecap.
- Increase albedo of earth, amplifies the initial effect.
- Additional effects may come from greenhouse gases.
- See https://youtu.be/gGOzHVUQCw0

Non-linearities

- Recall that the equilibrium climate sensitivity is affected by feedbacks

$$T(F) = \frac{\eta}{\kappa - x} \frac{1}{\ln 2} \ln\left(\frac{S}{\bar{S}}\right).$$

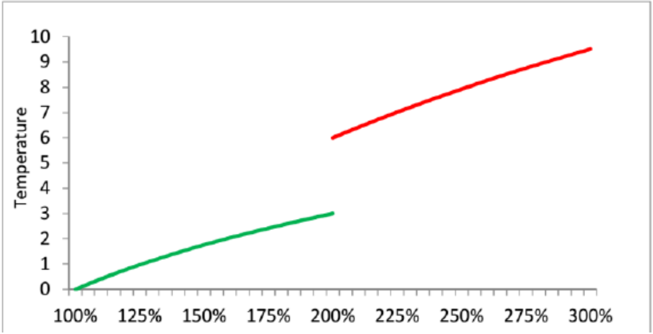
- We are quite uncertain about the value of x. One think that could happen is that it suddenly increases at some temperature. For example, suppose

$$x = \begin{cases} 2.1 \text{ if } T < 3^o C \\ 2.72 \text{ else} \end{cases}$$

- This produces a jump in the relation between CO_2 and long-run temperature.

Tipping points

- Suppose $\eta = 3.7$ and $\kappa = 3.3$. and $x = 2.1$ if $T < 3^o C$ and 2.72 else. Then, the relation between CO_2 concentration and long-run temperature looks like follows



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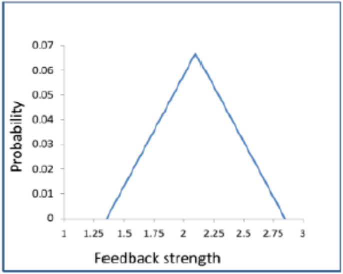
Tipping points:2

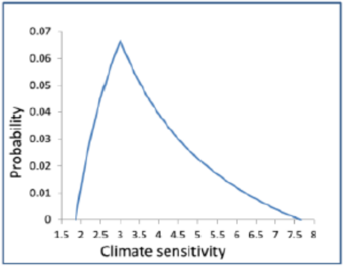
- Tipping points like then one described are possibilities and many of them are known to exist on local and regional scales.
- If they exist on a global scale and if so at which temperatures is much more debated.

Feedback uncertainty

- Uncertainty in the feedback produces a skewed distribution of the climate sensitivity.
- Since $\lambda \equiv \frac{\eta}{\kappa - x}$ is a non-linear transformation of x, uncertainty about λ becomes very skewed with possibilities of very large values.
- Suppose the uncertainty about x by a symmetric triangular density function with mode 2.1 and endpoints at 1.35 and 2.85. The mean, and most likely, value of x translates into a climate sensitivity of 3.

Feedback uncertainty





Carbon circulation - block 3 in IAM

- Externality is created from carbon emission.
- For policy analysis as well as for forecasts, we need to now the dynamic mapping from path of emissions to path of CO_2 concentrations.
- We will look at two approaches:
  1. stock-flow approach. Idea; different reservoirs of carbon. A continuos flow between these. Stable system always tending towards a steady state.
  2. Non-structural (reduced form) – define a depreciation function that specifies how much of deviation or of an emitted unit remains in atmosphere over time.

Stocks and flows

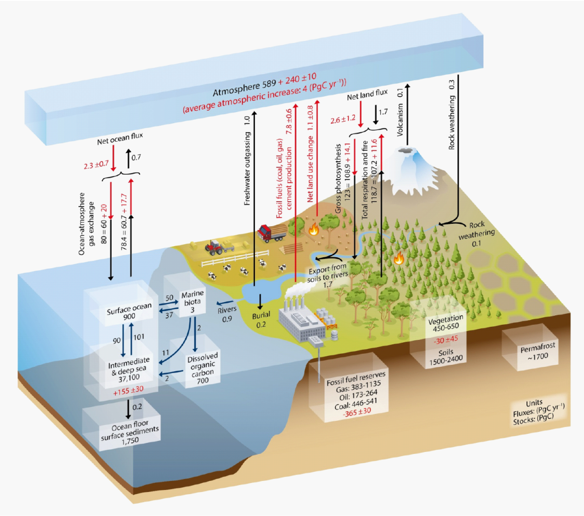


Figure: Global carbon cycle. Stocks in GtC (PgC) and flows GtC/year. Source: IPCC (2013), Figure 6.1

Easiest stock-flow case in continuous time

- Assume 2 reservoirs S_t, S_T^L. S_t represents the atmosphere in period t and S_t^L represents the deep oceans.
- Flow from S_t to S_t^L proportional to S_t, with proportionality factor ϕ_1.
- Flow from S_t^L to S_t proportional to S^L, with proportionality factor ϕ_2.
- Inflow to S_t also from emissions E_t.
- Change in stocks equal to net flows (in minus out), gives

$$\begin{aligned} \frac{dS_t}{dt} &= -\phi_1 S_t + \phi_2 S_t^L + E_t \\ \frac{dS_t^L}{dt} &= -\phi_2 S_t^L + \phi_1 S_t \end{aligned}$$

with $E_t = 0$, steady state satisfies

$$\begin{aligned} 0 &= -\phi_1 S + \phi_2 S^L \\ 0 &= \phi_1 S - \phi_2 S^L \end{aligned}$$

which cannot be uniquely solved, all solutions satisfy $S = \frac{\phi_2}{\phi_1} S^L$. Why?

Easiest case - discrete time approximation

- $$\begin{aligned} S_t - S_{t-1} &= -\phi_1 S_{t-1} + \phi_2 S^L_{t-1} + E_{t-1}. \\ S^L_t - S^L_{t-1} &= \phi_1 S_{t-1} - \phi_2 S^L_{t-1} \end{aligned}$$
- Same steady state and approximately the same dynamics.
- Such linear systems (in discrete or continuos time) can be solved analytically.
- Suppose emissions stop at t, then deviation from steady state $S_t = \frac{\phi_{21}}{\phi_{12}} S^L$ vanish over time as determined by the factor

$$\left(1 - \phi_1 - \phi_2\right)^{t+s}$$

- Specifically the law-of-motion for the stocks follow for $s \geq 0$ is given by;

$$\begin{aligned} S_{t+s} &= \tfrac{\phi_2}{\phi_1+\phi_2}\left(S_t + S^L_t\right) - \tfrac{\phi_2 S^L_t - \phi_1 S_t}{\phi_1+\phi_2}\left(1 - \phi_1 - \phi_2\right)^s \\ S^L_{t+s} &= \tfrac{\phi_1}{\phi_1+\phi_2}\left(S_t + S^L_t\right) + \tfrac{\phi_2 S^L_t - \phi_1 S_t}{\phi_1+\phi_2}\left(1 - \phi_1 - \phi_2\right)^s . \end{aligned}$$

A three reservoir system

- S_t represents the atmosphere in period t, S_t^U is the surface ocean, and finally S_t^L, which represents the deep oceans.
- Flows still assumed to be proportional to stocks and change is a reservoir is equal to net flow.
- We then have

$$
\begin{aligned}
S_t - S_{t-1} &= -\phi_{12} S_{t-1} + \phi_{21} S_{t-1}^U + E_{t-1} \\
S_t^U - S_{t-1}^U &= \phi_{12} S_{t-1} - (\phi_{21} + \phi_{23}) S_{t-1}^U + \phi_{32} S_{t-1}^L \\
S_t^L - S_{t-1}^L &= \phi_{23} S_{t-1}^U - \phi_{32} S_{t-1}^L.
\end{aligned}
$$

Calibration

- Two ways;
  - Try to choose the parameters to make model dynamics match as close as possible dynamics of more complicated models.
  - Take linear model seriously and use measured flows.
- Let's use the pre-industrial flows and stocks for the calibration.
  - Before industrialization we had 589 GtC in atmosphere and a flow to surface ocean of 60 GtC, implies $\phi_{12} = \frac{60}{589} \approx 0.102$.
  - The flow from the surface ocean to the atmospere gives $\phi_{21} = \frac{60.7}{900} \approx 0.067$
  - Use flow to deep ocean, giving $\phi_{23} = \frac{90}{900} = 0.100$.
  - Finally, the flow from the deep ocean to the surface ocean is set to the same value, giving $\phi_{32} = \frac{90}{37100} \approx 0.00243$.

Properties of steady state

- If emissions stop, this system also asymptotically approach a steady state. Solve

$$
\begin{aligned}
0 &= -\phi_{12} S + \phi_{21} S^U \\
0 &= \phi_{12} S - (\phi_{21} + \phi_{23}) S^U + \phi_{32} S^L \\
0 &= \phi_{23} S^U - \phi_{32} S^L
\end{aligned}
$$

again no unique solution, but all solutions satisfy

$$
\begin{aligned}
S &= \frac{\phi_{21}}{\phi_{12}} \frac{\phi_{32}}{\phi_{23}} S^L \\
S^U &= \frac{\phi_{32}}{\phi_{23}} S^L
\end{aligned}
$$

i.e., proportions between stocks are always restored.

Excel implementation

Table 2. Three stock carbon circulation

Year	S_t	S_t^U	S_t^L	M_t
=2011	=589+240	=900+550	=37100+155	=7.8+1.1
=1+A2	=B2-0.102*B2+0.0667*C2	=C2+0.102*B2-(0.0667+0.100)*C2	=D2+0.100*C2	=7.8+1.1
	+E2	+0.00243*D2	-0.00243*D2	
=1+A2	=B3-0.102*B3+0.0667*C3	=C3+0.102*B3-(0.0667+0.100)*C3	=D2+0.100*C3	=7.8+1.1
	+E3	+0.00243*D3	-0.00243*D3	

Non-structural carbon circulation models

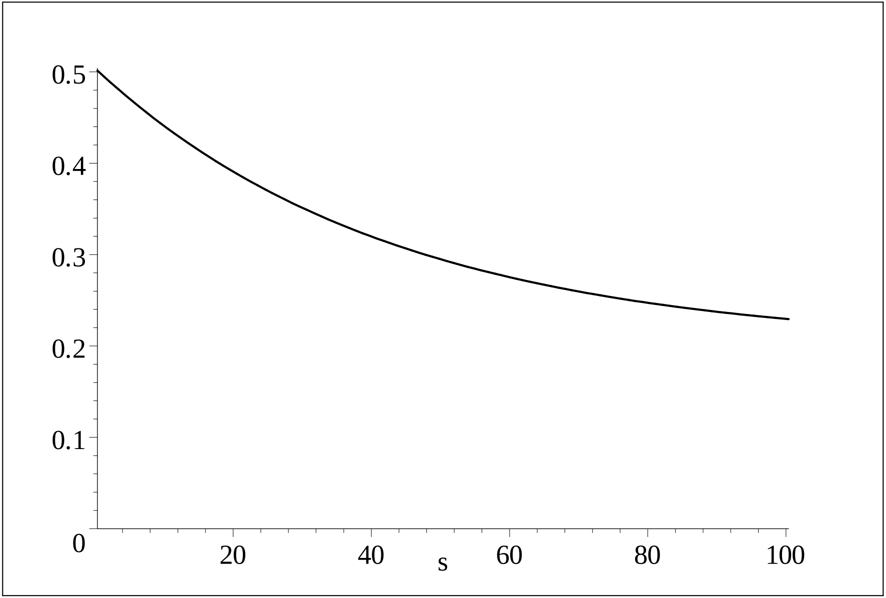
- Structural model may anyway be to simplified. Misses non-linearities, and other relevant variables.
- Could then instead try to match key characteristics directly; (IPCC and Archer 2005).
    - a share (ca 50%) is removed quite quickly (a few years to a few decades)
    - another share (ca 20-25%) stays very long (thousands of years) until CO_2 acidification has been buffered
    - remainder decays with a half-life of a few centuries.
- These features can be modeled directly by a depreciation function (rather remainder function), $d(s)$ that says how much remains of an emitted unit after s period.

$$d\left(s\right) = \varphi_L + \left(1 - \varphi_L\right) \varphi_0 \left(1 - \varphi\right)^s$$

Calibration

- Use decades Let's set $\varphi_L = 0.2$.
- $d(1) = 0.5$
- and $(1-\varphi)^{30} = \frac{1}{2}$.
- Gives $\ln(1-\varphi) \approx -\varphi = \frac{\ln\frac{1}{2}}{30} = -0.023$.
- $d(1) = 0.5 = 0.2 + (1-0.2)\varphi_0 (1-0.023)^1, \Rightarrow \varphi_0 = 0.38$

$$\left[\varphi_L + (1 - \varphi_L)\,\varphi_0\,(1 - \varphi)^s\right]_{\varphi_L=0.2,\varphi=0.023,\varphi_0=0.38}$$



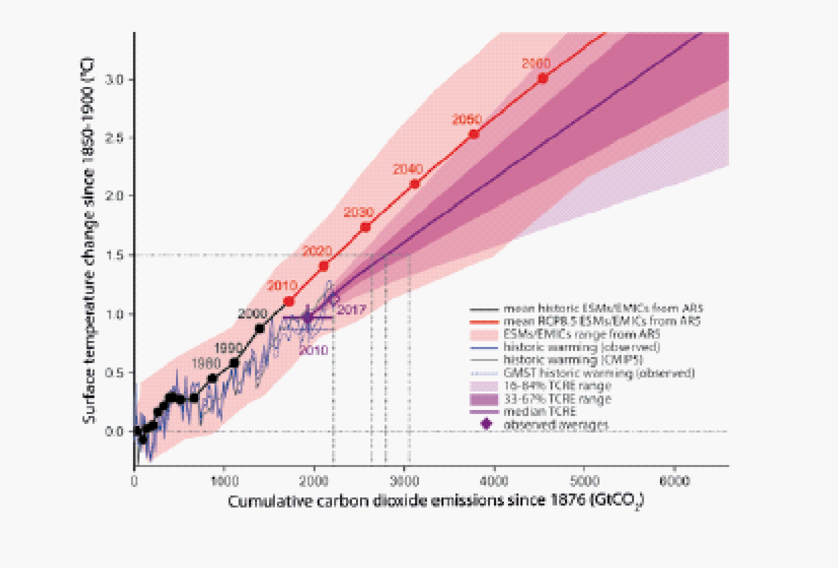
Non-linearities

- The parameters in the models we have presented are likely to be affected by the emission scenario.
- For example, more emissions reduce the capacity of oceans to store carbon (temperature and chemistry).
- Implies that more than 20-25% stays in atmosphere for thousands of years if cumulated emissions are large.
- With 10 times current cumulated emissions a twice as big share is likely to remain.

CCR – Carbon Climate Response

- Climate system and carbon circulation are dynamic and non-linear.
- An increase in forcing has a delayed impact (increasing over time) on temperature and is concave (logarithmic).
- Emission of carbon has a decaying impact (decreasing over time) on atmospheric CO_2 concentration and the relation is convex since other sinks storage capacity decreases when emissions have been large.
- Surprisingly, these non-linearities seem to cancel each other in most advanced climate models. The global mean temperature is linear in cumulative emissions.
- According to IPCC proportionality is between 0.8 and 2.5 degrees Celsius per 1000 GtC. This constant is called CCR (some time CRE or TCRE).

Linear relation between emissions and temperature



Carbon budgets

- Given a linear relation between ackumulated emissions and temperature, a remaining carbon budget can be calculated.
- The large uncertainty about the CCR coefficient, makes this problematic.
- We have now emitted around 600 GtC. If CCR is 0.8, we have committed 0.6*0.8=0.48$^{\circ}C$ and can emitt another 1250 GtC before reaching 1.5$^{\circ}C$.
- This would take more than 100 years with current emission rates.
- BUT, if CCR is 2.5, we have already passed the 1.5 heating.
- This is genuine uncertainty. Probabilities are informed guesses.