Lecture Notes

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IIES

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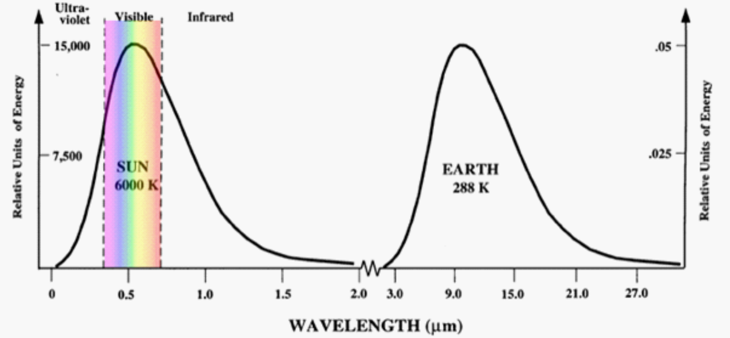
Climate models - forcing and the energy budget

- Incoming *energy flow of energy* from sun is on average across time and space 342 W/m^2 $= 2400kW$ per football field.
- Inflow is largely in the form of sunlight.
- Outgoing *energy flow*, consisting of
  - direct reflection (1/3)
  - Heat radiation (2/3).
  - The latter is a function of, in particular temperature and greenhouse gases.
- If inflow = outflow, *energy balance,* defined as difference between inflow and outflow is zero. A surplus leads to heat accumulation, temperature goes up.

Radiation

- Visible sunlight and infrared heat waves are both electromagnetic radiation, but with different frequencies (freq=Speed of light/wave length).
- Frequency of radiation emitted depends on temperature. Compare with dimmer on halogen lights,

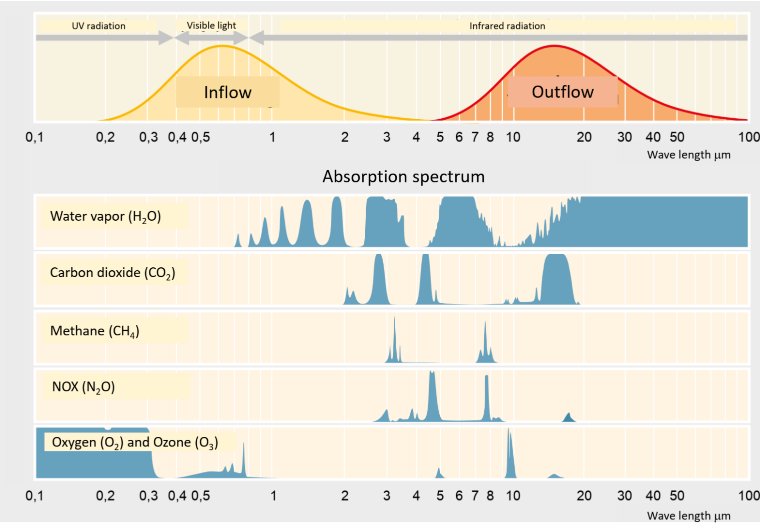


Comparison of the emission spectra of the sun and the earth. Note the huge disparity in the amount of energy emitted by the sun (left-hand scale) and the earth (right-hand scale).

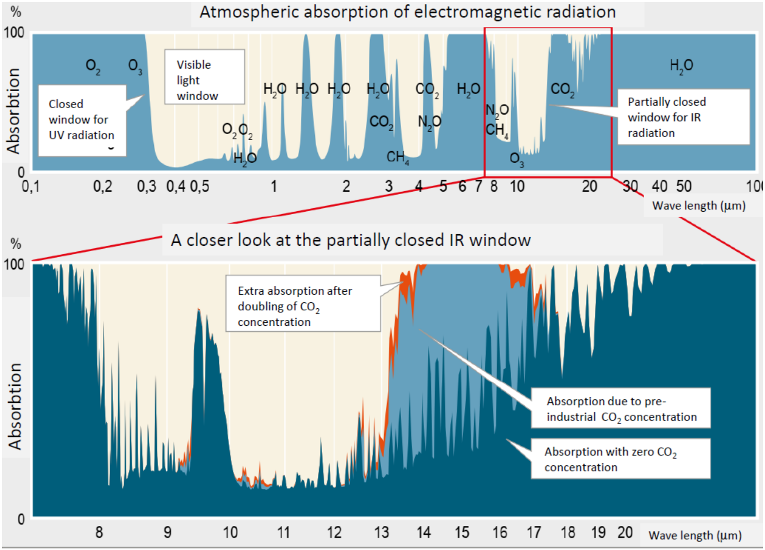
Greenhouse effect

- When electromagnetic radiation passes through gases energy can be absorbed by the radiation making the molecules vibrate.
- For this to occur, the molecules resonance frequency (like the particular frequency a guitar string vibrates) must be aligned with the frequency of the radiation.
- CO_2 (and other molecules with three or more atoms) have resonance frequencies aligned with infrared radiation. Sunlight has a higher frequency.
- Thus, CO_2 absorbs energy from heat radiation but not from sunlight.
- Gases with molecules with two atoms have resonance frequencies that do align with neither IR nor visible light. Thus, oxygen (O_2) and nitrogen (N_2), making up 99% of the atmosphere are not greenhouse gases.
- Compare to a band playing in a bar. The bass guitar can make some objects, e.g. cups and cutlery vibrate, but a high pitched tone from the guitar has no effect.

Absorption of different radiation



More on absorption

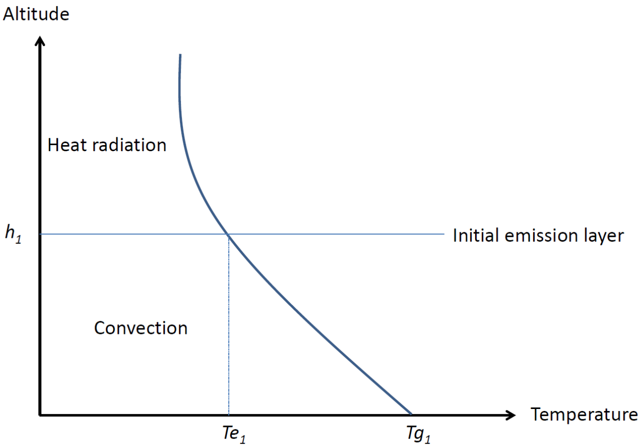


Source: Bernes, Claes, (2016), “En varmare värld”, Naturvårdsverket.

Energy transmission in atmosphere

- Even the small amount of CO_2 in the atmosphere (0.04%) makes it quite opaque for heat waves. One might think adding more does then not have any effect (Ångström - Arrhenius controversy).
- Turns out to be wrong. Heat is transferred up in the atmosphere since it is colder the higher the altitude. Eventually, the radiation can escape into space. The altitude this occurs is called *emission level.* An IR-camera in space sees only emission level.
- More CO_2 implies the emission level is moved up, where it *ceteris paribus* is colder.
- If the temperature at the emission level is colder, less energy is transmitted. This leads to a surplus – less energy escapes than comes in from the sun.
- The accumulation of energy increases the temperature in the atmosphere until the temperature at the emission level again is high enough to imply that the energy flow out in space is the same as the flow into earth.

Heat transfer and temperature gradient



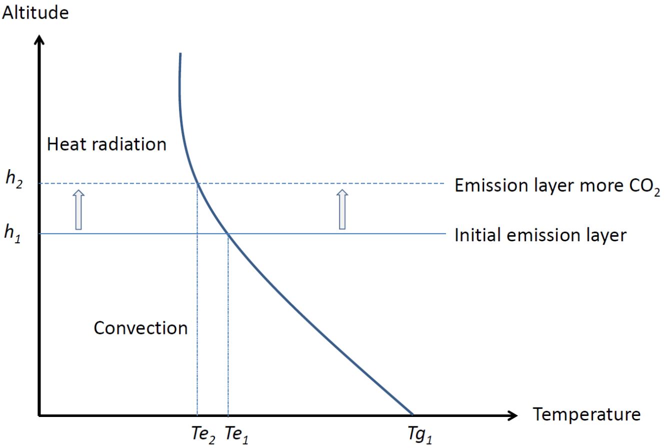


Figure: Lower temp at emission level -> less energy outflow. Surplus in energy budget.

Surplus leads to higher temperatures

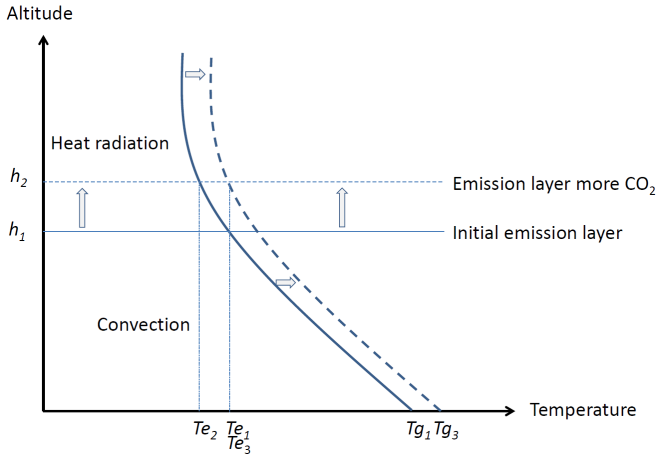


Figure: Heat accumulation gradually increases temperature. Gradient shifts rightwards until temp at h_2 has returned to Te_1 and ground temperature increased to Tg_3.

Te

- What is the temperature at the emission level?
- In at steady state, it must equal inflow minus direct reflection, 342-100=242 W/m^2.
- Use **Stefan–Boltzmann law** that says the heat radiation per m^2 is a function of temperature satisfying

$$energy\ flow = 5.67 * 10^{-8} * T^4 W/m^2.$$

  where T is temperature in $^\circ K$ (centigrades above absolute zero). Solving $242 = 5.67 * 10^{-8} * T^4$ yields $T = 256$, which is $255 - 273 = -18^\circ C$.This would be the ground temperature without greenhouse gases.
- Greenhouse gases work like a blanket, heat is transported up implying a negative temperature gradient of $0.65^\circ C$ per 100m. Emission layer at around 5000 meter. Thus, at ground level $50 * 0.65 = 32.5^\circ C$ warmer. A healthy ground temperature of $32.5 - 18 = 14.5^\circ C$.
- Without the GHG blanket, life as we know it could not have started.

The energy balance

- Consider a system (e.g., the earth) in a situation of net energy flow = incoming-outgoing flow=0.
- In such a case, the *energy balance,* is zero and no heat is accumulated or lost.
- Suppose now that the energy balance is perturbed by a positive amount f (inflow increased and/or outflow decreased). Now balance is no longer 0 but in surplus.
- Leads to an accumulation of heat in the system, temperature rises, quicker the larger is the energy balance surplus.
- Speed of temperature increase also depends on heat capacity of the system (mass and material). Compare a balloon with air and a balloon with water.
- As the temperature goes up, outgoing energy flow increases. Hotter things radiate more. Called *Planck feedback*.

Planck feedback

- Outflow is an *increasing* function of temperature, **Stefan–Boltzmann law.**
- Define the increase in the outflow over the pre-industrial steady state as $O(T_t)$, where T_t is the increase in temperature over the pre-industrial one (now it is around 1.2 °C).
- Approximate the increase in outflow as $O(T_t) \approx \kappa_{Planck} T_t$ where $\kappa_{Planck} \equiv O'(0)$.

Temperature dynamics

- The energy balance surplus $f - \kappa_{Planck} T_t$ affects temperature dynamics. A surplus in the balance implies increasing temperature and vice versa.
- Assume a linear relation between $\frac{dT_t}{dt}$ and energy balance

$$\frac{dT_t}{dt} = \sigma \left(f - \kappa_{Planck} T_t \right)$$

- What determines σ? Will there be a new equilibrium? Yes, when $T_t = \frac{f}{\kappa_{Planck}}$
- Using **Stefan–Boltzmann law** and temperature at the emission level of -18$^\circ C$, κ_{Planck} would be $\approx 3.8 \frac{W/m^2}{^\circ C}$. In reality, things are more complicated. A typical number for κ_{Planck} is $3.2 \frac{W/m^2}{^\circ C}$.

Forcing

- As discussed above, more greenhouse gases pushes emission level outwards which creates a surplus in energy balance relative to preindustrial situation.
- Surplus depends on greenhouse gas concentration.
- Most important is water vapor. Second is CO_2.
- Human activities has increased concentration of CO_2 and other greenhouse gases, e.g., methane. We also emit particles and aerosols that have a direct negative effect on reflection and a quite uncertain negative effect on the energy balance via changed cloud formation.
- Best guess (IPCC) is that these in total give a forcing of 2.7 W/m^2.
- Disregarding the feedbacks, only considering the Planck feedback as derived above, we can calculated the long run effect of that on Earth's temperature as

$$\frac{2.7}{3.8} \approx 0.7^{\circ} C.$$

Forcing in 2019 relative to 1750

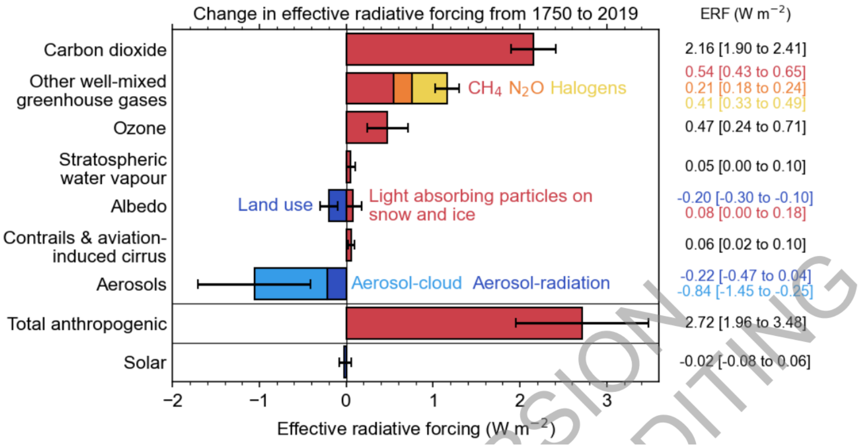
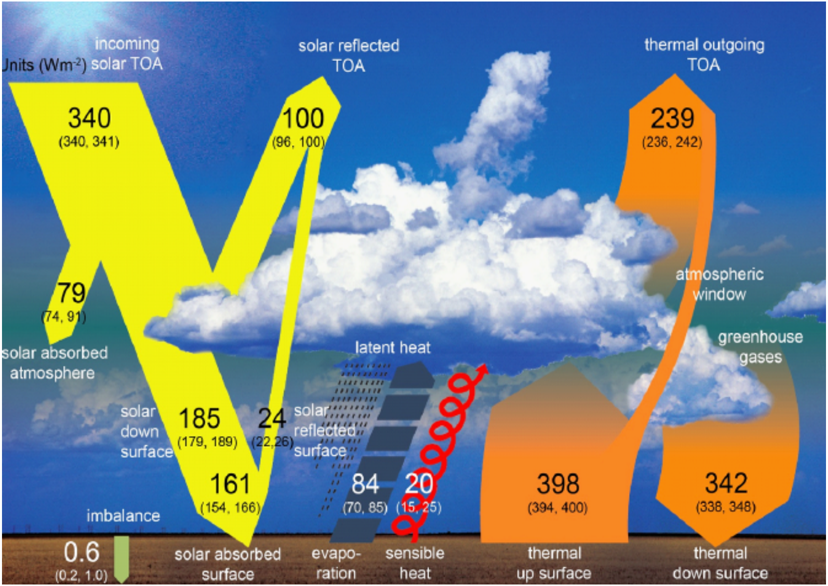


Figure: Fig 7.6 IPCC 6th report page 7-182.

Energy Flows



Orders of magnitude

- Area of Earth's surface is 510 million km^2. This is 510*$10^6 * 1000^2 = 5.1 \times 10^{14} m^2$. Thus, the inflow net of reflection is 240*$5.1 \times 10^{14} = 1.22 \times 10^{17} W$.
- A nuclear power plant is around 1000 MW, i.e., $10^9 W$. Thus, the inflow of solar energy is equivalent to $1.22 \times 10^8 = 122$ million nuclear power plants (NPP). We currently have around 440 in operation.
- The human induced forcing of 2.7W/m^2 is equivalent to $2.7 * 5.1 \times 10^{14}/10^9 = 1.4$ million NPP.
- Global yearly energy use is around 600 million TJ, i.e., $6 * 10^{2+6+12} = 6 * 10^{20} J$. Dividing by the number of seconds per year, we get the average power use. 6*$10^{20}/(365 * 24 * 3600) \approx 1.9 \times 10^{13} W$ or 19000 NPP.
- Thus, solar inflow is $\frac{1.22 \times 10^{17}}{1.9 \times 10^{13}} \approx 6400$ times global energy use. If we could harness 0.1%, it would allow 6 times current energy use.

Feedback effects

- Gross flows as very large relative to direct greenhouse effect.
- Changed climate affects outflow indirectly. Example, more CO_2, leads to
  - higher concentration of water vapor, increases greenhouse effect.
  - changed cloud formation, affects outflow.
- We approximate these as reduction in outflow being linear in temp deviation, i.e., $\kappa_{other} T_t$.
- Additionally, the reflection of incoming sunlight changes
  - changes in ice-cover (albedo) and (again) changed cloud formation.
- Approximate also these as reductions in inflow being linear, $\kappa_{refl} T_t$.

Feedbacks in the energy balance

- Let us include feedbacks in energy balance:

$$\frac{dT_t}{dt} = \sigma \left(f - \kappa_{Planck} T_t + \kappa_{other} T_t + \kappa_{refl} T\right)$$
$$= \sigma \left(f - \left(\kappa_{Planck} - \kappa_{other} - \kappa_{refl}\right) T_t\right).$$

- The steady state for a given forcing f now becomes

$$T(f) = \frac{f}{\kappa_{Planck} - \kappa_{other} - \kappa_{refl}}$$

- A realistic value of $\kappa_{Planck} - \kappa_{other} - \kappa_{refl}$ is around 1.2 while $\kappa_{Planck} = 3.2$, but with large uncertainty. If it is 1.2, the current f of 2.7 yields an increase in temperature of $2.7/1.2 = 2.25^{\circ}C$.
- Direct effect of CO_2 emission on f, (as well as of κ_{Planck}) fairly certain. Not the case for feedbacks.

Current Feedbacks

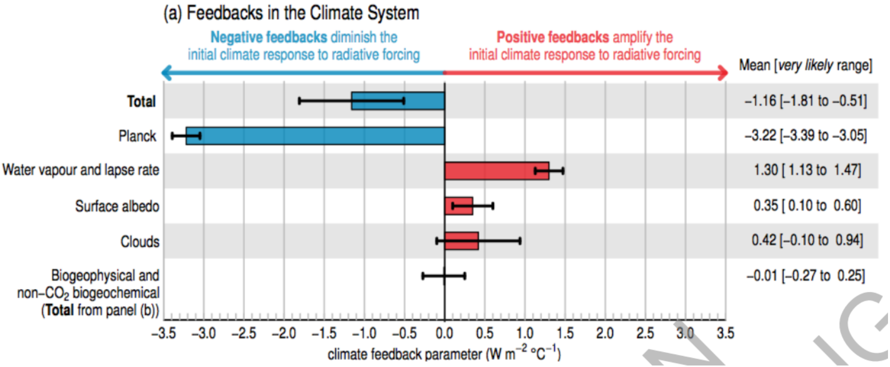


Figure: Figure TS.17 IPCC 6th report.

Quantifying greenhouse effect on energy balance

- Higher concentration of CO_2in atmosphere reduces outgoing (infra red) energy flow. Well approximated by a logarithmic function (Arrhenius greenhouse law, 1896). A concentration S of CO_2 in the atmosphere and the pre-industrial level S_0, yields

$$f_{CO_2}(S) = \frac{\eta}{\ln 2} \ln\left(\frac{S}{S_0}\right)$$

- An often used approximation of η is 3.7. Combine with $T(f) = \frac{f}{\kappa_{Planck} - \kappa_{other} - \kappa_{refl}}$ gives

$$T(f(S)) = \frac{\eta}{\kappa_{Planck} - \kappa_{other} - \kappa_{refl}} \frac{1}{\ln 2} \ln\left(\frac{S}{S_0}\right).$$

- $\frac{\eta}{\kappa_{Planck} - \kappa_{other} - \kappa_{refl}}$ is labelled the *Equilibrium Climate Sensitivity (ECS)*, measures long-run temperature impact of CO_2 doubling.
- IPCC 6th report: ECS is “likely" 2.5 to 4°C, with a "best estimate" of 3. Narrower than the 5th report's 1.5 to 4.5. "Likely" means a 2/3 confidence interval. A 90% interval is 2-5°C.

Heating of oceans

- Equation $\frac{dT_t}{dt} = \sigma \left(f - \left(\kappa_{Planck} - \kappa_{other} - \kappa_{refl}\right) T_t\right)$ does not take into account heating of oceans/atmosphere separately.
- Two other terms in energy balance for atmosphere, capturing energy flow from atmosphere to ocean and *vice versa.*
- These new terms *do not balance* if temperature is different (in an average sense).
- Let us also allow f to vary over time. Then the law-of-motion for atmosphere is

$$\frac{dT_t}{dt} = \sigma_1 \left(f_t - \left(\kappa_{Planck} - \kappa_{other} - \kappa_{refl}\right) T_t - \sigma_2 \left(T_t - T_t^L\right)\right)$$

where T_t and T_t^L, respectively, denote the atmospheric and ocean temperature in period t.

- Complete by setting

$$\frac{dT_t^L}{dt} = \sigma_3 \left(T_t - T_t^L\right)$$

- Drag on heating, but same relation btw steady state and forcing.

Simulation

- Make a discrete time approximation. Yields a system of difference equations;

$$T_t - T_{t-1} = \sigma_1 \left(f_{t-1} - \left(\kappa_{Planck} - \kappa_{other} - \kappa_{refl} \right) T_t - \sigma_2 \left(T_{t-1} - T \right.\right.$$

$$T_t^L - T_{t-1}^L = +\sigma_3 \left(T_{t-1} - T_{t-1}^L \right)$$

instead of

$$\frac{dT_t}{dt} = \sigma_1 \left(f_t - \left(\kappa_{Planck} - \kappa_{other} - \kappa_{refl} \right) T_t - \sigma_2 \left(T_t - T_t^L \right) \right)$$

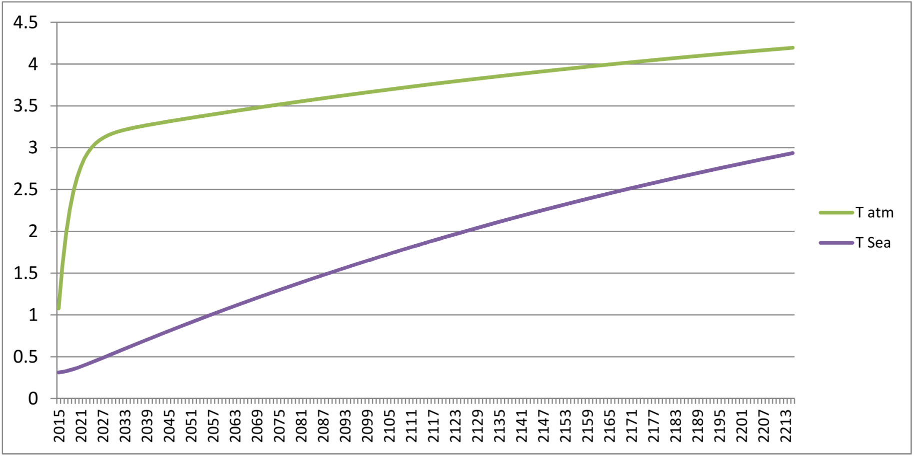
$$\frac{dT_t^L}{dt} = \sigma_3 \left(T_t - T_t^L \right)$$

- Simple to solve or simulate.

Calibration

- Folini et al. (2021), show that the climate model above and the linear carbon circulation model later to be presented closely replicates the mean behavior of the most advanced Earth System Models (CMIP5), if parameters are chosen appropriately.
- They choose, $\sigma_1 = 0.137$, $\sigma_2 = 0.73$, $\sigma_3 = 0.00689, \eta = 3.45$ and $\kappa = 1.06$ implying an ECS of 3.25. Note that $\sigma' s$ depend on time interval in discete approximation.
- Folini et al. choose initial temperatures $T_{2015} = 1.2778$, and $T^L_{2015} = 0.3132$ based on what the model predicts given historic emissions. Alternative: average temperature 2010-2019 over the average for the period 1880-1920 which is 1.078.
- Note that σ_1 is much larger than σ_3. Atmospheres energy balance settles to a temporary steady state of 0 quickly.

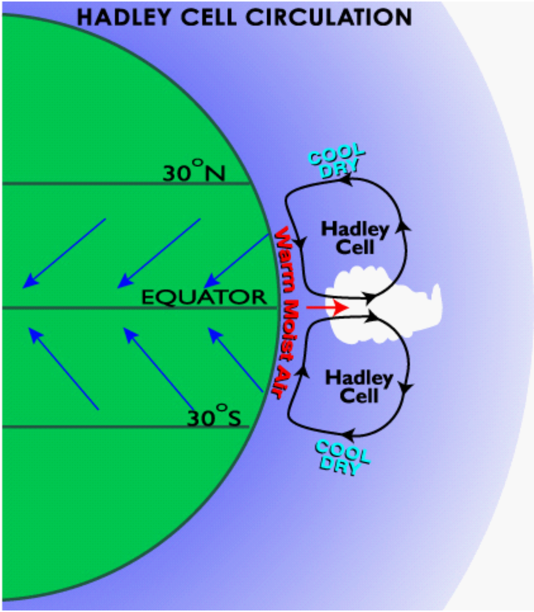
Simulation of a doubling of current forcing



Climate models:1

- Circulation models.
- Energy is not evenly radiated to the earth. Highest around equator.
- Creates systematic flows of air and water.
- Used to forecast weather – but also climate.

Climate models:Circulation cells

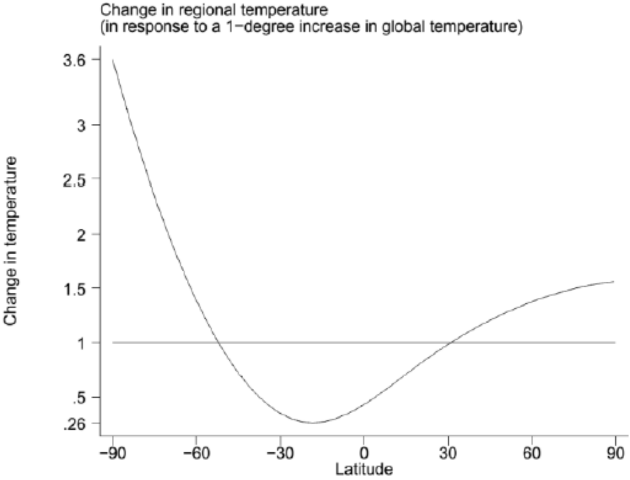


Climate models: key points

- Ocean currents also transport heat from equator towards poles.
- More accurate descriptions need to model landmasses and mountains.
- Climate models build on deterministic laws of physics but are chaotic in nature. This implies:
  - A "butterfly effect" – small variation in initial state e.g., distribution of energy, leads to unsystematic large differences in weather a few weeks later.
  - Unconditional distribution stable, e.g., mean and variance of temperature and wind speeds.
  - Best forecast is unconditional distribution for forecasts beyond a few weeks.
- State-of-the art climate models build on same principles.

Downscaling

- Circulation models (very) large and (very) time consuming to run.
- Simplification: use a statistical representation of how a change in global mean temperature affects different locations.
- Turns out that global mean temperature is a quite good summary statistic for other aspects of climate - an approximate sufficient statistic.
- Relation between GMT and other aspects can be estimated using output from advanced Earth System Models.
- Simplest case – relation between GMT and temp at differend latitudes. Estimate a different sensitivity β_i for each latitude.
- $T_{i,t} = \bar{T}_i + \beta_i * T_t + z_{i,t}$


Change in regional temperature
(in response to a 1−degree increase in global temperature)
Change in temperature
3.6
3
2.5
2
1.5
1
.5
.26
−90
−60
−30
0
30
60
90
Latitude

Historic climate

- Use various proxy data, tree rings, corals, plankton and pollen...
- Also data on greenhouse gas concentrations. Positive correlation suggests positive feedback.

Ice ages

- Small change in solar influx or variation in earth's orbit gets amplified by feed-back.
- A key mechanism may be ice-albedo feedback (Arrhenius).
- A small negative F leads to buildup of the icecap.
- Increase albedo of earth, amplifies the initial effect.
- Additional effects may come from greenhouse gases.
- See https://youtu.be/gGOzHVUQCw0

Non-linearities

- Recall that the equilibrium climate sensitivity is affected by feedbacks

$$T(f) = \frac{\eta}{(\kappa_{Planck} - \kappa_{other} - \kappa_{refl})} \frac{1}{\ln 2} \ln\left(\frac{S}{\bar{S}}\right).$$

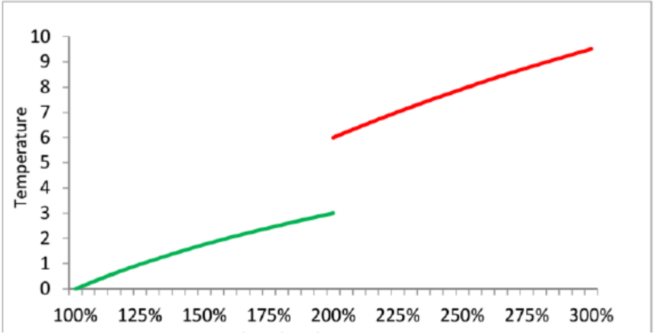
- We are quite uncertain about the value of $\kappa_{other} + \kappa_{refl}$. One think that could happen is that it suddenly increases at some temperature. For example, suppose

$$\kappa_{other} + \kappa_{refl} = \begin{cases} 2.1 \text{ if } T < 3^o C \\ 2.72 \text{ else} \end{cases}$$

- This produces a jump in the relation between CO_2 and long-run temperature.
- Also simple to make this irreversibe,
- $\kappa_{other} + \kappa_{refl} = \begin{cases} 2.1 \text{ if } T \text{ ever was larger than } 3^o C \\ 2.72 \text{ else} \end{cases}$

Tipping points

- The relation between CO_2 concentration and long-run temperature now looks like follows



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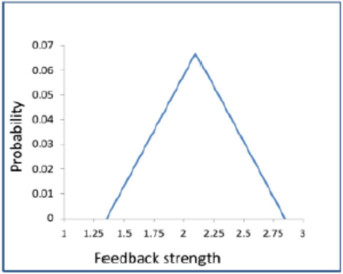
Tipping points:2

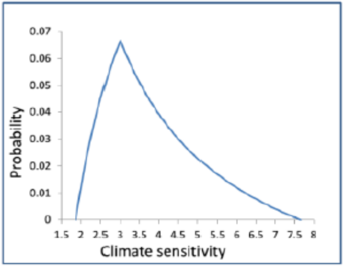
- Tipping points like the one described are possibilities. "At the regional scale, abrupt responses, tipping points and even reversals in the direction of change cannot be excluded (high confidence)." IPCC AR6 WG1 Box TS 9.
- If they exist on a global scale and if so at which temperatures is debated but not likely unless global warmning goes much further than projected for the coming century also in quite pessimistic scenarios.
- IPCC 6th report claims "there is no evidence of such non-linear responses at the global scale in climate projections for the next century, which indicate a near-linear dependence of global temperature on cumulative GHG emissions." (IPCC AR6 WG1, chap. 1 p. 202).

Feedback uncertainty

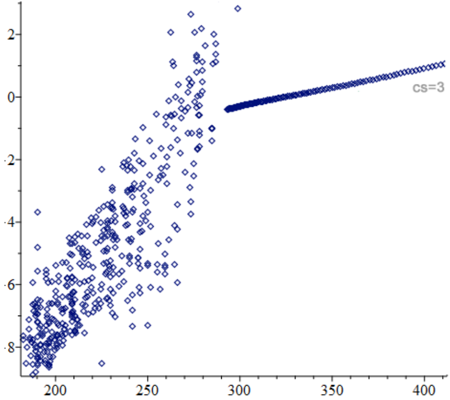
- Uncertainty in the feedback produces a skewed distribution of the climate sensitivity.
- Since $\lambda \equiv \frac{\eta}{\kappa_{Planck} - \kappa_{other} - \kappa_{refl}}$ is a non-linear transformation of of κ_{other} and κ_{refl}, uncertainty about λ becomes very skewed with possibilities of very large values.
- Suppose the uncertainty about $\kappa_{other} + \kappa_{refl}$ by a symmetric triangular density function with mode 2.1 and endpoints at 1.35 and 2.85. The mean, and most likely, value of $\kappa_{other} + \kappa_{refl}$ translates into a climate sensitivity of 3.

Feedback uncertainty





Historic relation between CO2 and temp



Carbon circulation - block 3 in IAM

- Externality is created from carbon emission.
- For policy analysis as well as for forecasts, we need to now the dynamic mapping from path of emissions to path of CO_2 concentrations.
- We will look at two approaches:
  1. stock-flow approach. Idea; different reservoirs of carbon. A continuos flow between these. Stable system always tending towards a steady state.
  2. Non-structural (reduced form) – define a depreciation function that specifies how much of deviation or of an emitted unit remains in atmosphere over time.
- Note difference between measuring emissions in CO_2 and C. A mole of carbon atoms weighs 12 grams and a mole of oxygen weighs 16. Then a kg of carbon produces $\frac{2*16+12}{12} \approx 3.67$ kg CO_2.

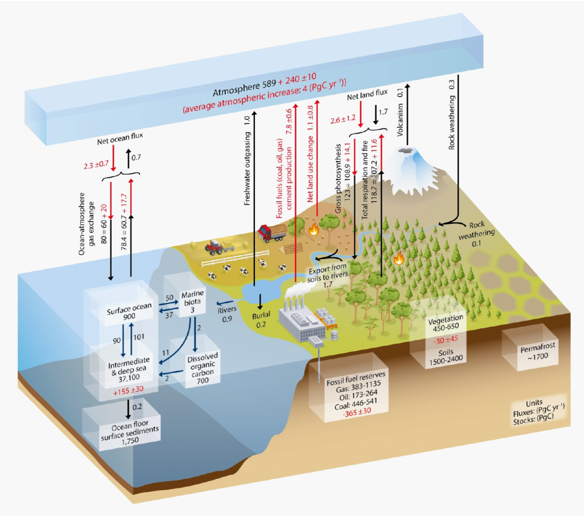


Figure: Global carbon cycle. Stocks in GtC (PgC) and flows GtC/year. Source: IPCC (2013), Figure 6.1

A three reservoir system

- Assume 3 reservoirs (sinks). S_t represents the atmosphere in period t, S_t^U is the surface ocean (and biosphere), and finally S_t^L, which represents the deep oceans.
- Flows assumed to be proportional to stocks and change is a reservoir is equal to net flow.
- We then have

$$\begin{aligned} S_t - S_{t-1} &= -\phi_{12} S_{t-1} + \phi_{21} S_{t-1}^U + E_{t-1} \\ S_t^U - S_{t-1}^U &= \phi_{12} S_{t-1} - (\phi_{21} + \phi_{23}) S_{t-1}^U + \phi_{32} S_{t-1}^L \\ S_t^L - S_{t-1}^L &= \phi_{23} S_{t-1}^U - \phi_{32} S_{t-1}^L. \end{aligned}$$

Calibration

- Two ways;
  - Try to choose the parameters to make model dynamics match as close as possible dynamics of more complicated models, e.g., from CMIP5 (Coupled Model Intercomparison Projects).
  - Take linear model seriously and use measured flows.
- Let's use the pre-industrial flows and stocks for the calibration.
  - Before industrialization we had 589 GtC in atmosphere and a flow to surface ocean of 60 GtC, implies $\phi_{12} = \frac{60}{589} \approx 0.102$.
  - The flow from the surface ocean to the atmosphere gives $\phi_{21} = \frac{60.7}{900} \approx 0.067$
  - Use flow to deep ocean, giving $\phi_{23} = \frac{90}{900} = 0.100$.
  - Finally, the flow from the deep ocean to the surface ocean is set to the same value, giving $\phi_{32} = \frac{90}{37100} \approx 0.00243$.

Calibration to advanced Earth System Models

- Folini et al. (2021), show that the carbon cycle model above closely replicates the mean behavior of the most advanced Earth System Models (CMIP5), if parameters are chosen appropriately.
- They choose $\phi_{12} = 0.053, \phi_{21} = 0.0536$, $\phi_{23} = 0.0042$ and $\phi_{32} = 0.001422$ when the time step is a year. The initial values of the stocks are $S_{2015} = 850$, $S^U_{2015} = 765$ and $S^L_{2015} = 1799$. Note that in particular the deep oceans is much smaller than in reality. To model it that small makes the dynamics of the model more in line with the (much) more advanced models.

Properties of steady state

- If emissions stop, this system also asymptotically approach a steady state. Solve

$$
\begin{aligned}
0 &= -\phi_{12} S + \phi_{21} S^U \\
0 &= \phi_{12} S - (\phi_{21} + \phi_{23}) S^U + \phi_{32} S^L \\
0 &= \phi_{23} S^U - \phi_{32} S^L
\end{aligned}
$$

again no unique solution, but all solutions satisfy

$$
\begin{aligned}
S &= \frac{\phi_{21}}{\phi_{12}} \frac{\phi_{32}}{\phi_{23}} S^L \\
S^U &= \frac{\phi_{32}}{\phi_{23}} S^L
\end{aligned}
$$

i.e., proportions between stocks are always restored.

Non-structural carbon circulation models

- Structural model may anyway be to simplified. Misses non-linearities, and other relevant variables.
- Could then instead try to match key characteristics directly; (IPCC and Archer 2005).
  - a share (ca. 50%) is removed quite quickly (a few years to a few decades)
  - another share (ca. 20-25%) stays very long (thousands of years) until CO_2 acidification has been buffered
  - remainder decays with a half-life of a few centuries.
- These features can be modeled directly by a depreciation function $d(s)$ such that $1 - d(s)$ is what remains in atmosphere s periods after it was emitted.

$$1 - d(s) = \varphi_L + (1 - \varphi_L)\varphi_0 (1 - \varphi)^s$$

with $\varphi_L = 0.2$, $\varphi_0 = 0.38$ and $\varphi = 0.023$ (using a decadal time scale).

Generalization

- Previous model assumed three components with different depreciation rates, 0, infinite and 2.3% per decade. Can be generalized. IPCC AR5 WG1 table 2.14 uses 4 components (here in continuous time)

$$1 - d(s) = \alpha_0 + \sum_{i=1}^{3} \alpha_i e^{-\frac{s}{\tau_i}}$$

with $\alpha_0 = 0.217, \alpha_1 = 0.259, \alpha_2 = 0.338, \alpha_3 = 0.186,$
$\tau_1 = 172.9, \tau_2 = 18.51, \tau_3 = 1.186.$

- With this parametrization, 50% has left atmoshpere after 30 years, 75% after 356 years and 21.7% stays for ever.

Non-linearities

- The parameters in the models we have presented are likely to be affected by the emission scenario.
- For example, more emissions reduce the capacity of oceans to store carbon (temperature and chemistry).
- Implies that more than 20-25% stays in atmosphere for thousands of years if cumulated emissions are large.
- With 10 times current cumulated emissions a twice as big share is likely to remain, i.e., φ_L or α_0 are twice as large.

CCR – Carbon Climate Response

- Climate system and carbon circulation are dynamic and non-linear..
- Surprisingly, these non-linearities seem to cancel each other in most advanced climate models. The global mean temperature is linear in cumulative emissions. $T_t = \sigma_{CCR} \sum_{s=0}^{t} M_s$
- According to the latest (6th) IPCC report, σ_{CCR} is "likely" (which should be interreted as a 2/3 confidence interval) between 1.0 and 2.3 degrees Celsius per 1000 GtC (corresponding to 0.27-0.63°/ $Tt$$CO_2$). This constant is called CCR (some time CRE or TCRE).

Linear relation between emissions and temperature

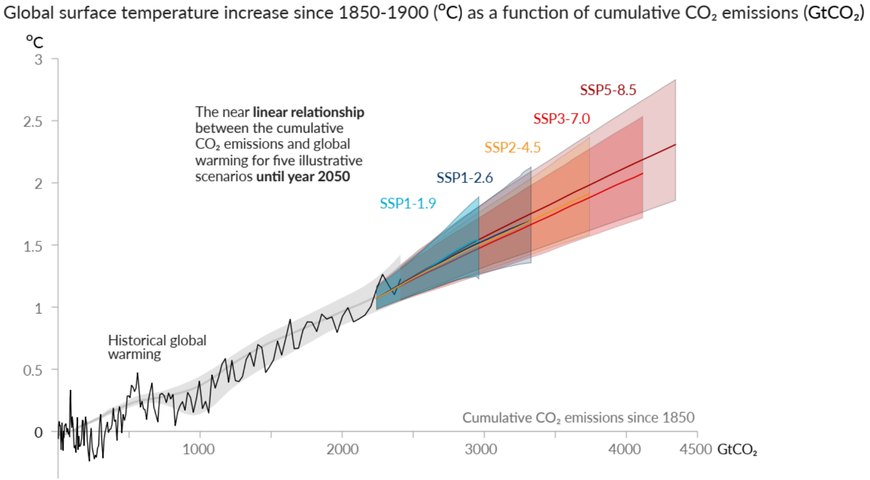


Figure: Figure SPM.10 in IPCC's 6th report.

Why does temperature stop rising when emissions stop?

- The CCR proportionality implies that global warmning continues as long emissions continue. It also implies that temperture stays constant after emissions have stopped. Why?
- Consider the dynamic system
  $\frac{dT_t}{dt} = \sigma_1 \left(\frac{\eta}{\ln 2} \ln \left(\frac{S_t}{S_0} \right) - \kappa T_t - \sigma_2 \left(T_t - T_t^L \right) \right)$ and
  $\frac{dT_t^L}{dt} = \sigma_3 \left(T_t - T_t^L \right).$
- Here $\sigma_1 >> \sigma_3, \frac{\sigma_1}{\sigma_3} \approx 20$. Thus, dynamics are much faster in the first than in the second equation. If emissions stop, the first equation therefore quickly reach a temporary steady state, by T_t adjusting.
- The first and third term instead are much slower moving, the first falling as CO_2 slowly leaves the atmosphere to the deep oceans. The third also diminsh since T_t^L slowly increase (ocean heat up). It turns out (by coincidence) that the changes of these two terms are about equally fast. Since they enter with opposite signs, the energy budget remains in balance without any changes in T_t.

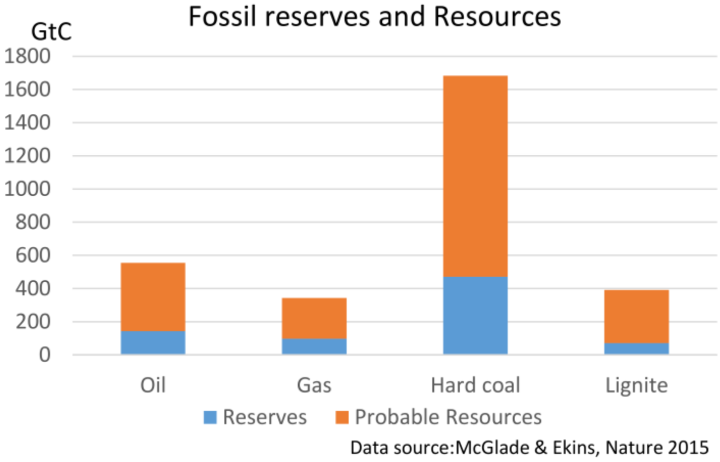
Carbon budgets

- Given a linear relation between accumulated emissions and temperature, a remaining carbon budget can be calculated.
- The large uncertainty about the CCR coefficient, makes this problematic.
- We have now burnt around 650 GtC. If CCR is 1, we have committed 0.6*1=0.65$^\circ C$ and can emit another 850 GtC before reaching 1.5$^\circ C$.
- This would take around 85 years with current emission rates.
- BUT, if CCR is 2.3, we have already passed 1.5 heating.
- This is genuine uncertainty. Probabilities are informed guesses.

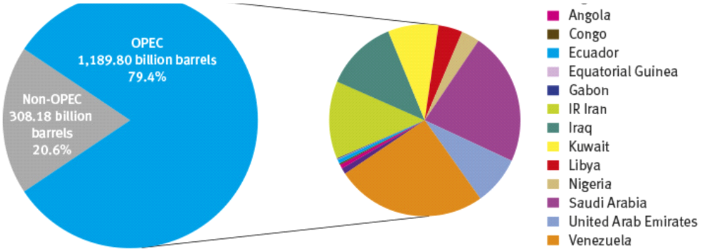
How much carbon is there?

- Fossil fuels exists in many forms.
- Different costs of recovery.
- One classification is
  1. Reserves (recoverable under current economic and technological conditions)
  2. Resources (recoverable under possible future economic and technological conditions).
- Technological developments are and have been fast. Leading to continuos reclassifications.

How much carbon is there?



OPEC's own estimates



OPEC proven crude oil reserves, at end 2018 (billion barrels, OPEC share)

Venezuela	302.81	25.5%	Kuwait	101.50	8.5%	Algeria	12.20	1.0%	Gabon	2.00	0.2%
Saudi Arabia	267.03	22.4%	UAE	97.80	8.2%	Ecuador	8.27	0.7%	Equatorial Guinea	1.10	0.1%
IR Iran	155.60	13.1%	Libya	48.36	4.1%	Angola	8.16	0.7%			
Iraq	145.02	12.2%	Nigeria	36.97	3.1%	Congo	2.98	0.3%			

Source: OPEC Annual Statistical Bulletin 2019.

- Is 1190 billion brls a lot? A barrel is 1/7.33 tons and oil contains 85% carbon. So this is 138GtC. Likely gives 0.14-0.32$^{\circ}C$ warming using IPCC's likely CCR coefficient.