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Source: *The Journal of Political Economy*, Vol. 101, No. 3 (Jun., 1993), pp. 485-517
Published by: The University of Chicago Press
Stable URL: http://www.jstor.org/stable/2138773

Optimal Taxation in Models of Endogenous Growth

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We study the problem of optimal taxation in three infinite-horizon, representative-agent endogenous growth models. The first model is a convex model in which physical and human capital are perfectly symmetric. Our second model incorporates elastic labor supply through a Lucas-style technology. Analysis of these two models points out the danger of assuming that government expenditures are exogenous. In our third model, we include government expenditures as a productive input in capital formation, showing that the limiting tax rate on capital is no longer zero. In numerical simulations, we find similar effects on growth and welfare in all three models.

I. Introduction

In recent years, considerable interest has developed in the determinants of the divergent paths of development both across countries at

We would like to thank Tony Braun, Christophe Chamley, V. V. Chari, Ken Judd, Robert King, Robert Lucas, Richard Rogerson, Paul Romer, Tom Sargent, Nancy Stokey, and an anonymous referee for their comments and the National Science Foundation for financial support. Remaining errors are our own.

[*Journal of Political Economy,* 1993, vol. 101, no. 3]

the same time and within the same country at different times. A body of research has developed that traces differences in development paths to differences in government policies. This literature has emphasized simple convex models of the growth process. Examples of this line of work are Eaton (1981), Barro (1990), Jones and Manuelli (1990, 1992), King and Rebelo (1990), and Rebelo (1991).

In this paper, we continue the study of the connection between government policies and growth. Specifically, we present a quantitative assessment of the effects of making drastic changes in the structure of fiscal policies relative to the current situation. We explore the effects of the switch to this optimal tax scheme on both the growth rate and level of welfare in a representative-agent calibrated economy.

Recent examples of work on the quantitative effects of dynamic tax policies in a general equilibrium framework include Chamley (1981), Judd (1987, 1990), Auerbach and Kotlikoff (1987), Chari, Christiano, and Kehoe (1990), King and Rebelo (1990), Lucas (1990), and Yuen (1990). These studies differ greatly in both the models that they analyze and the types of fiscal experiments undertaken. Chamley (1981) explores the effects of both marginal and global effects on tax changes in a model with exogenous growth and a representative agent. Judd (1987) treats the case of the effects of marginal changes for a wide variety of different time paths for tax changes with exogenous growth. Auerbach and Kotlikoff (1987) consider global changes in taxes in an overlapping generations setting with exogenous growth. Judd (1990) and Chari et al. (1990) consider Ramsey optimal taxation problems in stochastic environments with exogenous growth. Both of these studies compare the business cycle frequency properties of optimal state-contingent tax policies with U.S. tax policies. King and Rebelo (1990) consider the effects of tax policy changes in a simple model of endogenous growth and compare them to tax effects in an exogenous growth model. Lucas (1990) examines the growth effects of Ramsey optimal taxation in a model of endogenous growth driven by a human capital externality. He uses an approximation to characterize the steady-state behavior of the optimal tax policy conditional on an exogenously specified level of steady-state debt service. Yuen (1990) analyzes a similar problem, using a linear approximation around the steady state to study optimal taxation.

We examine three separate models of the process of growth. The first is a fully convex model with no externalities in which physical and human capital are perfectly symmetric both in their usage and in their accumulation laws. The second model deviates from the first in that there is a nonconvexity at the household level in the production of "effective labor" (following Heckman [1976], Rosen [1976], and Lucas [1988]), and the human capital accumulation process depends on both market goods and nonmarket goods. In both of these models we follow the standard public finance practice of assuming that the flow of government expenditure is viewed as exogenous by the planner.

In both of these models, the growth effects of the switch to optimal tax policies cause government expenditures to shrink to a negligible fraction of output. A more realistic approach includes government expenditures as a productive input. Our third experiment makes the sequence of government expenditures endogenous to the planner's problem. This has drastic effects on the nature of optimal taxes. It is shown that, in a setting in which government spending has direct positive effects on investment, the asymptotic tax rate on capital income is strictly positive. These results contrast with those of the existing literature, which shows that the limiting tax rate on capital income is zero in the Ramsey optimal tax scheme in cases in which government spending is unproductive.

The models that we study do not admit closed-form solutions. Hence, our strategy is to compute exact solutions to finite-horizon versions of an optimal taxation problem in a deterministic setting with endogenous growth. We study the full solution to the optimal policy choice problem, which includes the determination of debt service as part of the optimal policy.

For all three models, we find large growth and welfare effects from a switch to optimal tax policies. This occurs regardless of whether the supply of labor is inelastic or elastic and whether government expenditures are taken as exogenous or endogenous. Moreover, the sizes of both the growth and welfare effects that we find are similar for the three models examined. Finally, we find a shift from a reliance on labor to consumption taxes in the second model examined (the only place in which this can be considered).

An important omission is consideration of issues of time consistency. Would new governments in power in the future choose to adopt the continuation of the policies we find as Ramsey optimal? Or in "resolving" the Ramsey problem from their point of view, would they choose to adopt different policies? (See Chari and Kehoe [1989, 1990] and Stokey [1991] for a discussion of these issues.) Throughout, we ignore the constraints these considerations impose on governments. Again, a more complete treatment of the problem including these considerations would be of considerable interest.

In all these exercises, we assume that both solutions to the planner's problems exist (in particular, the feasible set of policy plans is nonempty) and that the time paths of these solutions converge to steady-state growth paths. Neither of these assumptions is innocuous. In the case in which government spending is taken as exogenous (as in the models in Secs. II and III below), it is a simple exercise to choose time paths for spending that give rise to either nonexistence or nonstationary behavior. In addition, a recent example (Chamley 1990) shows that optimal policy can be nonstationary even if government policies are both exogenous and stationary.

The remainder of the paper is organized as follows. In Section II, we analyze a simple convex model of endogenous growth. In Section III, we modify the model to allow for a nonconvexity at the individual level in the production of effective labor from human capital and raw labor. In addition, we allow for a more general formulation of the human capital accumulation process. Section IV contains the results of the experiments when government expenditure is allowed to be endogenous. Finally, Section V offers some concluding remarks.

II. A Simple Version of the Problem: Model 1

Throughout the paper, we study variants of the following Ramsey problem: Choose tax rates to maximize the welfare of the representative agent subject to the constraints that the government's budget be balanced (in the present value sense) and that the resulting allocation is a competitive equilibrium.

In this section, we examine a simple case of this problem in which labor is inelastically supplied and physical and human capital are treated symmetrically. The representative household solves

$$\max \sum_t \beta^t u(c_t) \quad \text{subject to}$$

(i) $k_{t+1} \leq (1 - \delta_k)k_t + x_{kt}$,

(ii) $h_{t+1} \leq (1 - \delta_h)h_t + x_{ht}$,

(iii) $\sum_t p_t(c_t + x_{kt} + x_{ht}) \leq \sum_t p_t[(1 - \tau_{Kt})r_t k_t + (1 - \tau_{Ht})w_t h_t + T_t]$,

where k_t is physical capital and h_t is the stock of human capital. We interpret the household as supplying effective labor (as in Heckman [1976] and Lucas [1988]) given by $u_t h_t$, where u_t is the number of hours worked in the market sector and the household inelastically supplies the total raw labor endowment of one unit. The term T_t captures transfers from the government that are treated as lump sum by the household. The terms r_t and w_t are the rental prices of capital and labor in terms of time t consumption, and p_t is the price of time t consumption in terms of the numeraire. Finally, τ_{jt}, $j = H, K$, are the tax rates on the two factors.

The firm solves

$$\max p_t[F(k_t, z_{2t}) - w_t z_{2t} - r_t k_t],$$

where $z_{2t} = u_t h_t$ is the number of effective labor hours purchased by the firm from the market.

Simple manipulations coupled with standard no-arbitrage conditions allow us to rewrite the consumer's budget constraint as

$$\sum_t p_t(c_t - T_t) \leq W_0 \equiv k_0[(1 - \tau_{K0})r_0 + 1 - \delta_k] + h_0[(1 - \tau_{H0})w_0 + 1 - \delta_h],$$

where we have normalized $p_0 = 1$ and have assumed throughout that the solution is interior: $x_{ht} > 0$ and $x_{kt} > 0$ for all t. Since we shall want to impose this later on, we restrict the planner to choices of taxes that guarantee that this will hold in equilibrium.

The planner's problem can be phrased as choosing time paths of the variables c_t, k_t, h_t, x_{ht}, x_{kt}, τ_{Kt}, τ_{Ht}, p_t, w_t, and r_t to maximize the representative agent's welfare subject to the constraints embodied in the conditions describing competitive equilibrium. Following the approach of Lucas and Stokey (1983) and Lucas (1990), we can simplify the problem to eliminate p_t, w_t, r_t, τ_{Kt}, and τ_{Ht}.

After this is done, the planner's problem becomes

$$\max \sum_t \beta^t u(c_t) \quad \text{subject to}$$

$$\begin{aligned}
&(a)\ \sum_t \beta^t(c_t - T_t)u'(t) = W_0, \\
&(b)\ c_t + x_{ht} + x_{kt} + g_t = F(k_t, h_t), \\
&(c)\ k_{t+1} = (1 - \delta_k)k_t + x_{kt}, \\
&(d)\ h_{t+1} = (1 - \delta_h)h_t + x_{ht}, \\
&(e)\ \text{all variables nonnegative, } h_0 \text{ and } k_0 \text{ given.}
\end{aligned} \tag{P1}$$

Here

$$W_0 = [(1 - \tau_{K0})F_k(0) + 1 - \delta_k]k_0 + [(1 - \tau_{H0})F_h(0) + 1 - \delta_h]h_0,$$

and the sequences g_t and T_t are viewed as fixed.

Given the time paths for the variables c_t, x_{ht}, x_{kt}, k_t, h_t, τ_{H0}, and τ_{K0}, which solve this problem, the remainder of the variables (i.e., prices and tax rates) can be reconstructed using the conditions describing competitive equilibrium.

As is, this problem has a very simple solution: to set τ_{K0} or τ_{H0} high enough to finance the entire sequence of government expenditures and to set taxes to zero thereafter (this may involve $\tau_{K0} > 1$ for some choices of g_t and T_t). Of course, this is simply a form of lump-sum taxation since h_0 and k_0 are in fixed supply. Since there is more interest in the solution to (P1) in environments in which lump-sum taxation is not available, we shall have to put some restrictions on how the planner can set taxes. To this end, we set τ_{K0} and τ_{H0} at their historical levels and restrict the size of τ_{Kt} and τ_{Ht}. Further, the bounds on tax rates must be chosen with care. If the tax rate bounds are too high, then investment at time 0 will be zero, with the period 1 capital stocks fixed at their depreciated time 0 levels. Given this, capital taxation in period 1 takes on a lump-sum character. To avoid this problem, we choose our bounds on tax rates low enough so as to guarantee that investment will remain strictly positive in all periods.

To implement tax bounds, we use the fact that bounding tax rates above is equivalent to bounding consumption growth rates below. The bound we use in our simulations is a zero consumption growth rate.

It is worth noting that versions of the results of Judd (1985) and Chamley (1986) (which can be easily extended to the setting of endogenous growth) apply to this case. These imply that $\lim_{t\to\infty} \tau_{Kt} = 0$. Moreover, if $T_t = 0$ for all t, the tax rate bounds will be attained at the optimum for some finite number of periods after which (plus one period) $\tau_{Kt} = 0$. Because of the symmetry of the model, it follows that $\lim_{t\to\infty} \tau_{Ht} = 0$ as well. This is true in spite of the fact that labor is inelastically supplied. Roughly, although the planner would like to tax the inelastically supplied labor endowment, his only avenue to accomplish this is to jointly tax the (in the limit perfectly elastically supplied) stock of human capital as well. Because of this, the planner taxes neither labor nor capital income in the limit.

A. *Computational Methods*

In choosing a method for numerical solution of variations on the Ramsey planner's problem, we confront two major difficulties. First, we know very little about the qualitative nature of the solution path. What we do know is limited largely to the steady-state behavior of the system. For some variants of the problem, this is dependent on steady-state revenue requirements of the solution, which is, in turn, determined by the revenue raised in the initial periods. Second, the Ramsey planner's problem cannot be posed as a time-invariant dynamic program; if we follow the strategy of Chari et al. (1990) of conditioning on the budget constraint multiplier, we can write the problem as a dynamic program only from the first period forward. Even if this is done, the addition of constraints on the maximum tax rate complicates the behavior of the policy functions. Finally, this strategy requires iteration over the budget constraint multiplier and the time 0 investment choices.

To numerically solve these infinite-horizon Ramsey problems, we form truncated versions of the problems with T periods. We experimented with different values of T to obtain accurate solutions of the problem. As is standard, dropping returns after period T gives rise to "end effects" on the capital stock and consumption path due to the implicit understatement of the value (in the truncated version of the problem) of the terminal capital stocks. Even in an undistorted problem (i.e., with no taxes) with $T = 50$, this effect is felt throughout the entire time path. To compensate for this problem, we added a term to the objective function reflecting the continuation value of the terminal capital stocks. We assumed that after period T, the economy would follow the theoretically calculated steady-state growth path from then on (as in Auerbach and Kotlikoff [1987]). A similar correction was made to the last period of the constraints that are infinite horizon in nature (i.e., the Ramsey budget constraints). In some of the problems that we shall solve below, the steady-state growth behavior of the system is dependent on the solution in the first few periods. In these cases, adjustments to this procedure are used. See the discussion in Sections III and IV for details.

Once the truncated versions of the Ramsey problems are formed, they are simply nonlinear programming problems in which the value function is maximized subject to linear, nonlinear, and bounds constraints. We chose nonlinear programming methods to solve these problems. We made this choice for three reasons: (1) the modern methods that are available have known error properties, (2) experience with these techniques on a very wide variety of problems has shown them to be very robust to even extremely ill-behaved objective functions, and (3) the code for these techniques has been extensively tested and is available on a wide variety of computers from personal computers to supercomputers. The exact nonlinear programs solved for the models considered are presented in the Appendix.

We employed a sequential quadratic programming method each iteration of which uses a quadratic approximation to each problem to obtain a search direction for minimization of an augmented Lagrangian merit function (see Gill, Murray, and Wright [1981] for a discussion of this method). We used the implementation of the NPSOL algorithm in the NAG subroutine library routine E04UCF. Analytical objective function gradients and constraint Jacobians were used for all solutions. We used SUN SparcStation 330 and IBM

RS6000/520 work stations to perform all computations. Computer programs are available on request from the authors.

B. Simulations of the Simple Model

For the simulations, we use a calibrated version of the model outlined above. In particular, we assume that

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}, \quad F(k,h) = Ak^{\alpha}h^{1-\alpha}.$$

It can be shown (see Jones and Manuelli 1990) that for certain values of the parameters the tax-distorted equilibrium of this economy converges to a steady-state growth path. When δ_h and δ_k are equal, the characteristics of this path are

$$\frac{h_t}{k_t} = \frac{(1-\alpha)(1-\tau_H)}{\alpha(1-\tau_K)} \tag{1}$$

and

$$\begin{aligned}\gamma \equiv \frac{c_{t+1}}{c_t} = \frac{k_{t+1}}{k_t} = \frac{h_{t+1}}{h_t} \\ = (\beta\{A[(1-\alpha)(1-\tau_H)]^{1-\alpha}[\alpha(1-\tau_K)]^{\alpha} + 1 - \delta_k\})^{1/\sigma},\end{aligned} \tag{2}$$

where τ_K and τ_H are the limiting tax rates on human capital and physical capital income, respectively, and γ is the growth rate.

Given this, we chose parameters for the model consistent with U.S. time-series observations. Thus we set $\alpha = .36$, $\beta = .98$, $\delta_k = \delta_h = .1$, $\tau_K = .21$, $\tau_H = .31$, and $\gamma = 1.02$. This estimate of α comes from a computation of capital's share in national income, which includes durables as part of the capital stock (see Prescott 1986). Kydland and Prescott (1982) use $\delta_k = .1$ as their estimate. Data from Jorgenson and Yun (1991) suggest a smaller value, near .06. On the other hand, in a calculation using capital consumption allowances, Judd (1987) estimates $\delta_k = .12$. Heckman's (1976) estimates of δ_h range from 4 percent to 9 percent but seem very sensitive to the specification of the model. Rosen's (1976) estimates vary from 5 percent (high school graduates in 1960) to 19 percent (college graduates in 1970). Because of the wide variance in these estimates, we decided initially to treat human and physical capital symmetrically, setting $\delta_k = \delta_h = .1$. Kydland and Prescott (1982) estimate $\beta = .96$. However, empirical studies in both macroeconomics and finance find higher values (exceeding one in some cases). We chose $\beta = .98$ as an intermediate value. Given that we have fixed β in this way, different values for other parameters of preferences imply different rates of return. These are presented in table 1 below for the purpose of reference. The value of τ_H that we have selected is consistent with the estimates given in Barro and Sahasakul (1986). Given this, τ_K is given by the requirement that the government budget constraint be satisfied. Below we investigate the sensitivity of model solutions to changes in depreciation and tax rates.

Given these choices for parameter values, equation (2) then gives a joint restriction on σ and A. We experimented with various values along this frontier for comparative purposes. We chose $\sigma = 1$ (the log case), 1.5, 2.0, and 2.5. These give rise to A's of .37, .40, .43, and .46, respectively. It follows that the asymptotic growth rates of consumption in these cases are 1.072, 1.057, 1.049, and 1.044, respectively (these can be calculated using [2] with $\tau_K = \tau_H = 0$).

All calculations were done with the initial share of government spending on consumption in gross national product equal to .20 and transfers' share of output equal to .0726 with (the U.S. historical) 2 percent growth, $T = 50$, and the same initial capital stocks. Note that this figure for transfer payments is less than that reported in most sources. The difference amounts to approximately 50 percent of social security taxes. The idea behind this difference is that individuals treat at least some portion of social security as forced savings (for retirement) rather than transfer payments in the strict sense.[1]

An example of the time paths of a Ramsey solution is given in figure 1. The top half of the figure shows the paths of the real variables in the solution to the problem. The bottom half shows the corresponding paths of the tax rate, government revenues, and expenditures. As can be seen, consumption stays constant during the first few periods as the planner builds a surplus of revenue over expenditure because of our particular implementation of the tax bounds constraints. Following this initial phase, c, h, and k asymptotically ap-

[1] The figures we used for government spending are based on spending and revenue data for government at all levels. Government expenditure as a fraction of GNP has varied over the period from .266 to .349. We chose as our base case .33. Of this quantity, government consumption has been about 20 percent of GNP. Thus the figure we used in our calculations for g_t is .2 of output. The remainder of government expenditures is made up of transfer payments and interest payments. Of this quantity, approximately 1.8 percent of GNP over the period has been devoted to interest payments. Transfer payments have been growing over the period and are currently approximately 11.1 percent of GNP. These payments are made up of two parts, social security and other payments. Social security payments are complicated because they correspond in part to what is called transfer payments in the model and part savings since payments are linked (indirectly) to contributions. To handle this problem, as a first step, we attributed half of social security payments to forced savings, with the remainder being treated as true transfer payments. Since social security tax payments are approximately 12 percent of labor income, this gives forced savings through social security of approximately $.06 \times .64 = 3.84$ percent of GNP (.64 is labor's share in output). This gives transfers of 7.26 percent of GNP ($= 11.1 - 3.84$). This is the number we used for T_t in our calculations.

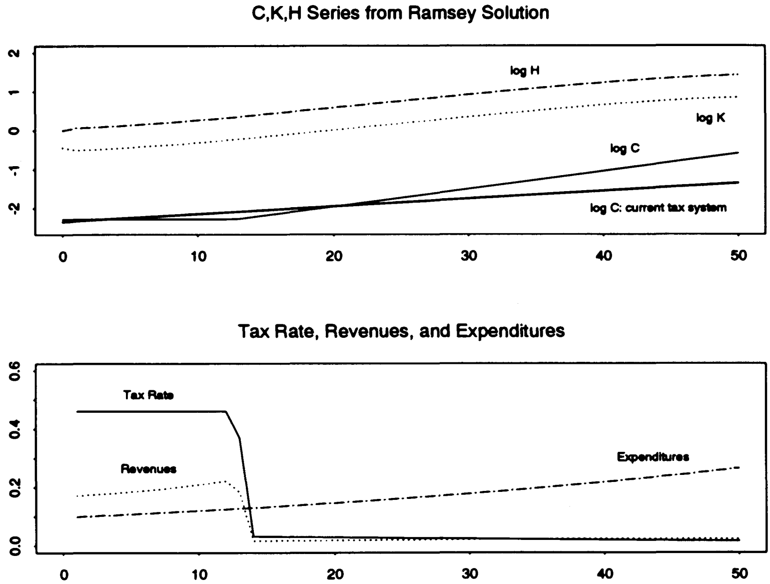

FIG. 1.—Model 1, $\sigma = 2.0$

proach the steady-state growth levels. Because of the symmetry of human and physical capital in the model, $\tau_{Kt} = \tau_{Ht}$ along the optimal path for all t. After an initial stage of high taxation, there is a one-period transition in which the tax rate declines dramatically, followed by a gradual reduction to zero.

A summary of the results obtained from different parameter settings is presented in table 1. In the table, N denotes the number of periods during which the taxes are at their theoretical upper bound, $\bar{\tau}$ denotes the tax rate upper bound that is implied by the consumption growth constraints, γ_1 is the calibrated growth rate (always 2 percent), and γ_2 is the asymptotic growth rate under optimal taxation. The term r_1 denotes the before-tax rate of return on capital along the

TABLE 1

CALCULATIONS FOR THE SIMPLE MODEL

σ	γ_1	A	r_1	γ_2	r_2	N	$\bar{\tau}$	Welfare*
1.01	1.02	.37	.05	1.072	.09	16	.38	3.92
1.50	1.02	.40	.06	1.057	.11	15	.42	1.37
2.00	1.02	.43	.08	1.049	.12	12	.46	1.15
2.50	1.02	.46	.09	1.044	.14	10	.49	1.09

* This is the factor by which the $\{c_t\}$ path of consumption must be raised in order to bring utility under the current system up to the level attained in the Ramsey resolution of tax paths.

steady-state growth path under the current fiscal system, and r_2 denotes the rate of return to capital in the steady state of the Ramsey solution. It should be emphasized that r_1 represents the rate of return paid to households for rental of capital in a world in which there is no firm-level taxation of capital income. For this reason, comparison of these rates of return with interest rates measured on existing assets in the economy should be done with caution.

There are several important qualitative features of the solutions. First, there is a substantial realignment in the ratios of physical to human capital arising from the differences between effective marginal tax rates on income derived from human and physical capital. The resulting "static" realignment gives rise to an increase in growth rates. In addition to this, the fact that the limiting tax rates are zero diminishes the intertemporal distortion present in the current system and causes faster accumulation of both types of capital.

Second, as σ is increased, the limiting growth rate (γ_2 in table 1) decreases monotonically. The nature of our model calibration induces two competing effects. As σ increases, for a given technology the growth rate falls. In contrast, as A increases for a given σ, the growth rate increases. In our simulation studies, the growth effects of changes in σ dominate those of A, resulting in a reduction in limiting growth as σ increases. As can be seen, the welfare impact of the tax reform envisaged in this exercise is substantial. Further, in our experiments, the welfare gains from tax reform are highly sensitive to and decreasing in σ.

To test the sensitivity of these results to our choice of parameters, we performed several additional computations with different choices for the depreciation rates on human and physical capital and for the tax rates on the two types of income. We performed three sets of additional computations. In the first two, we adjusted tax rates to increase capital's share of revenue; the cases considered were $\tau_K = \tau_H = .274$ and $\tau_K = .31$ with $\tau_H = .254$. For the final experiment, we reduced depreciation rates on both types of capital to $\delta_k = .07$ and $\delta_h = .05$. In all cases, σ was held fixed at two and α at .36. The results of these computations suggest that the findings reported in table 1 are fairly robust. In particular, these changes in tax rates result in almost no change in either the welfare gain or the limiting growth rate. The estimated growth rates are all close to 4.9 percent with welfare increases between 13 and 14 percent. These estimates should be compared with 4.9 percent for the growth rate and 15 percent as the welfare change in table 1. Changes in the depreciation rates also have little effect on the growth results presented in table 1. We find a limiting growth rate of 4.2 percent and a welfare gain of 20 percent for the case corresponding to a reduction in depreciation rates.

For comparative purposes, additional simulations excluding transfers were performed. We left the initial share of government spending on consumption at 20 percent of initial GNP, which was again increased by the historical average of 2 percent per year. In order that the government would be running a roughly balanced budget in the pre-Ramsey state, taxes were reduced to $\tau_K = .13$ and $\tau_H = .24$. To maintain the steady-state growth rate of 2 percent, the values of A were adjusted down accordingly. In this case, the tax rates in the optimal solution take on a "bang-bang" character in which the tax rates achieve the bound for an initial phase and after a one-period transition are set to zero. These computations give rise to similar growth and welfare effects.

In the case that excludes transfers, there is no time consistency problem because of the special form of the optimal taxes. That is, at any date t, if the problem were resolved, the resulting solution would agree with the time path from the first solution. This result holds only for the special case in which the two capital goods are perfectly symmetric and labor is inelastically supplied.

The estimates of the welfare changes that we obtain from this exercise are both large and highly sensitive to assumptions about the intertemporal elasticity of substitution in consumption. For the log case, we find a 390 percent gain with a marked decline down to 15 percent for the case in which $\sigma = 2$. To our knowledge, no directly comparable results exist in the literature. There is, however, a large related literature. King and Rebelo (1990) consider both exogenous and endogenous growth models in which the government finances a transfer policy using capital income taxation. They study the effect of using lump-sum taxes to reduce the tax rate from 30 percent to 20 percent and find very different results depending on whether the model is one with exogenous or endogenous growth. In the exogenous growth case, the gain is less than 2 percent, whereas in the endogenous growth case it exceeds 60 percent using logarithmic utility and inelastic labor supply. They find that changes in the elasticity of substitution do not have significant effects on their estimates. Chamley (1981) uses an exogenous growth model to evaluate the welfare gain in an exercise similar to that of King and Rebelo. He computes estimates of both the effects of a global tax reform from the elimination of capital income taxes and the marginal effect of small reductions in tax rates. His estimates of the welfare gain due to global changes range from 3.19 percent (when the capital tax rate is 50 percent) to less than 1 percent (when the capital tax rate is 30 percent) with logarithmic preferences. Judd (1987) estimates the marginal effect of lowering both capital and labor income tax rates. For this calculation, he obtains estimates roughly four times as large as those of Chamley for a similar exercise. Even if we use this factor to inflate Chamley's results for the global case, we obtain estimates of welfare change that are less than 13 percent for the log case.

Our own results indicate a much larger effect than the estimates given by the exogenous growth literature cited above. They are more in line with the findings of King and Rebelo (1990) for the endogenous growth case. A key difference is the greater sensitivity to σ that we find.

III. Complications of the Simple Model: Model 2

In this section, we add two features to model 1: a labor-leisure choice and a modification of the human capital accumulation process. In doing this we introduce some asymmetries between physical and human capital.

These changes have three qualitatively important effects on the model. First, they change the model from a one-sector one to a two-sector one. That this can have important impacts on the growth process is well known (see Rebelo 1991; Jones and Manuelli 1992). Second, some parts of the human capital accumulation process go untaxed. Specifically, direct labor services used in the production of human capital (i.e., a student's time spent in school) are untaxed. This is the sense in which "nonmarket" goods are introduced. Third, because of the form of the production function assumed for effective labor (see below), there is now an intrinsic source of inelasticity in the supply of human capital. Although there are two inputs into the production of effective labor, we allow the planner to use only one tax on labor income. The effect of this is to add a set of constraints not found in the existing optimal taxation literature to the Ramsey problem. These added constraints complicate the computation of the solution to the Ramsey problem.

As discussed in Section II, we assume that effective labor is supplied to both market activities and investment in human capital. Specifically, let $v_t h_t$ and $u_t h_t$ be the amount of effective labor supplied to the formation of human capital and market work, respectively. Then the problem faced by the consumer is to choose time paths for c_t, u_t, v_t, h_t, k_t, x_{kt}, and x_{ht} to maximize

$$\sum_t \beta^t u(c_t, 1 - u_t - v_t) \quad \text{subject to the constraints}$$

$$(a)\ \ k_{t+1} \leq (1 - \delta_k) k_t + x_{kt},$$

(*b*) $h_{t+1} \leq (1 - \delta_h)h_t + G(x_{ht}, v_t h_t)$,

(*c*) $\sum_t p_t[(1 + \tau_{ct})c_t + x_{kt} + x_{ht}]$

$$\leq \sum_t p_t[(1 - \tau_{Kt})r_t k_t + (1 - \tau_{Ht})w_t u_t h_t + T_t],$$

(*d*) all variables nonnegative, h_0 and k_0 fixed.

As before, if $x_{kt} > 0$ for all t, considerable simplification occurs in constraint c. After simplification using the conditions defining competitive equilibrium, the planner's problem can be reformulated as

$$\max \sum_t \beta^t u(c_t, 1 - u_t v_t) \quad \text{subject to}$$

$$(a) \sum_t \beta^t \left[u_1(t)c_t - u_2(t)\frac{T_t}{h_t}\frac{G_1(t)}{G_2(t)} + u_2(t)\frac{x_{ht}}{h_t}\frac{G_1(t)}{G_2(t)} - u_2(t)u_t \right]$$

$$= W_0 \frac{u_2(0)}{h_0}\frac{G_1(0)}{G_2(0)},$$

$$(b)\ k_{t+1} = (1 - \delta_k)k_t + x_{kt}, \tag{P2}$$

$$(c)\ h_{t+1} = (1 - \delta_h)h_t + G(x_{ht}, v_t h_t),$$

$$(d)\ c_t + x_{kt} + x_{ht} + g_t = F(k_t, u_t h_t),$$

$$(e)\ \frac{u_2(t)}{u_2(t+1)} = \beta \frac{G_2(t)}{G_2(t+1)}\frac{h_t}{h_{t+1}}[1 - \delta_h + G_2(t+1)(u_{t+1} + v_{t+1})],$$

where $W_0 = [1 - \delta_k + (1 - \tau_{K0})F_k(0)]k_0$ and part e captures the constraint that the same tax rate, τ_{Ht}, must be used for both raw labor, u_t, and human capital, h_t.

Again, there is a problem with lump-sum taxation if the planner is allowed to set taxes without any restrictions. To solve this problem, we impose constraints on the maximum tax rate.

As before, one can show that under the optimal plan, $\tau_{Kt} \rightarrow 0$. However, since human and physical capital are no longer perfectly symmetric, it is no longer necessarily true that $\tau_{Ht} \rightarrow 0$. However, if transfers disappear asymptotically, then for the functional forms that we use, the labor tax rate and the consumption tax rate converge to zero. See Bull (1992) and Jones, Manuelli, and Rossi (1992) for a derivation.

For the purposes of calibration, we used the following specific functional forms:

$$u(c, 1-u-v) = \frac{[c(1-u-v)^{\eta}]^{1-\sigma}}{1-\sigma},$$

$$F(k, uh) = A_1 k^{\alpha}(uh)^{1-\alpha},$$

$$G(x_h, vh) = A_2(x_h)^{\psi}(vh)^{1-\psi}.$$

Under these assumptions, the steady-state equations of the competitive system are given by

$$\gamma^{\sigma} = \beta\left[1 - \delta_k + (1-\tau_K)\alpha A_1 u^{1-\alpha}\left(\frac{h}{k}\right)^{1-\alpha}\right], \tag{3}$$

$$\begin{aligned}\gamma^{\sigma} = \beta\Bigg[& 1 - \delta_h + A_2(1-\psi)\left(\frac{x_h}{h}\right)^{\psi} v^{1-\psi} \\ & + (1-\tau_H)A_2\psi\left(\frac{x_h}{h}\right)^{\psi-1} v^{1-\psi}A_1(1-\alpha)\left(\frac{k}{h}\right)^{\alpha} u^{1-\alpha}\Bigg],\end{aligned} \tag{4}$$

$$\frac{c}{h}\frac{\eta}{1-u-v} = \frac{1-\tau_H}{1+\tau_c}(1-\alpha)\frac{A_1}{u^{\alpha}}\left(\frac{k}{h}\right)^{\alpha}, \tag{5}$$

$$\frac{c}{h}\frac{\eta}{1-u-v} = \frac{1-\psi}{\psi}\frac{x_h}{h}\frac{1}{v}\frac{1}{1+\tau_c}, \tag{6}$$

$$\gamma = 1 - \delta_k + \frac{x_k}{k}, \tag{7}$$

$$\gamma = 1 - \delta_h + A_2\left(\frac{x_h}{h}\right)^{\psi} v^{1-\psi}, \tag{8}$$

$$\frac{c}{h} + \frac{x_h}{h} + \frac{x_k}{k}\frac{k}{h} + \frac{g}{h} = A_1\left(\frac{k}{h}\right)^{\alpha} u^{1-\alpha}. \tag{9}$$

To calibrate the model, we fix $\beta = .98$, $\delta_h = \delta_k = .1$, $A_2 = 1$, $u = .17$, $v = .12$, $\gamma = 1.02$, and $\alpha = .36$.[2] The tax and government spending variables are the same as those in model 1 with the excep-

[2] To obtain estimates of the quantities of work in the market sector and in human capital formation, we first estimated the number of hours available in total:

hours avail = (pop over 16) × 14.5 (hrs/day) × 7 (days/wk) × 52 (wks/yr)
+ (pop 5–15) × 8 (hrs/day) × 7 (days/wk) × 52 (wks/yr).

This gives an estimate of the (aggregate) time available. Note that we allotted less usable time to younger individuals. Second, we constructed series of actual hours tion that we added $\tau_c = .083$.[3] The steady-state equations are used to solve for η, ψ, and A_1 for a grid of σ values. For the parameters chosen in our calibrations, the implied intertemporal elasticity of labor supply ranges from 1.3 (for $\sigma = 1.1$ and $\eta = 7.09$) to .67 (for $\sigma = 2.5$ and $\eta = 4.38$). These values are within the upper range of the estimates reported in MaCurdy (1985) and are slightly lower than Lucas and Rapping's (1969) preferred estimate of 1.4.

Numerical solution of this problem is considerably more difficult than the inelastic labor supply case discussed in Section II. The labor supply decisions increase the number of variables twofold. In addition, the form for effective labor supply introduces nonconvexities into the problem. For this reason, we started with an initial problem with zero fiscal activity and gradually increased government expenditures to the desired level, resolving the problem at each intermediate step. Having obtained a solution for one set of parameter values, we deformed the problem by gradual shifts in the parameters. This allowed us to trace the solution over an interesting region of the parameter space.

Figures 2 and 3 show the time path of the solution to the optimal tax problem for the case of $\sigma = 2.0$ (with $\eta = 4.99$, $\psi = .44$, and $A_1 = 1.60$). The solutions are qualitatively similar for the other values of the parameters we studied. The top half of figure 2 shows the time path of the two capital variables h and k as well as the path of consumption. These paths converge very quickly to the limiting growth rate of 5.5 percent.

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worked from the *Economic Report of the President.* Our estimate of u is then (hours worked)/(hours avail). This has fluctuated from .169 to .176 over the period 1960–85. This was the basis of the calibrated value we chose of $u = .17$. To obtain an estimate of v, we used the hours avail estimate above in conjunction with estimates of the hours used in human capital formation. This last quantity was formed by

$$\text{hcap1} = (\text{pop } 5\text{–}19) \times 30\ (\text{hrs/wk}) \times 40\ (\text{wks/yr}),$$

$$\text{hcap 2} = (\text{number employed}) \times 9\ (\text{hrs/wk}) \times 52\ (\text{wks/yr}).$$

These are estimates of time spent in schooling and on-the-job training, respectively (see Juster and Stafford 1990). Our estimate of v is then given by $v =$ (hcap1 + hcap2)/(hours avail). This has varied from .104 to .123 over the period 1960–85 and forms the basis of our estimate of $v = .12$.

[3] In keeping with our discussion concerning our tax rates given in conjunction with model 1, we chose tax rates of .37 for labor income and .21 for capital income. This tax on labor income includes social security payments; thus, in keeping with our treatment of social security payments outlined above, .31 is the effective rate affecting marginal decisions and .06 is the component we are treating as forced savings through the social security system. This gives revenue of 31.24 percent of GNP. The remainder of revenue is made up of a variety of tariffs and excise, sales, and other indirect business taxes. To handle this last part, we attributed the entire quantity to general taxes on consumption. Since consumption taxes are lump sum in the model of Sec. II, we ignored this source in that section. Here, we used a tax rate of 8.3 percent on consumption to account for this extra source of revenue.

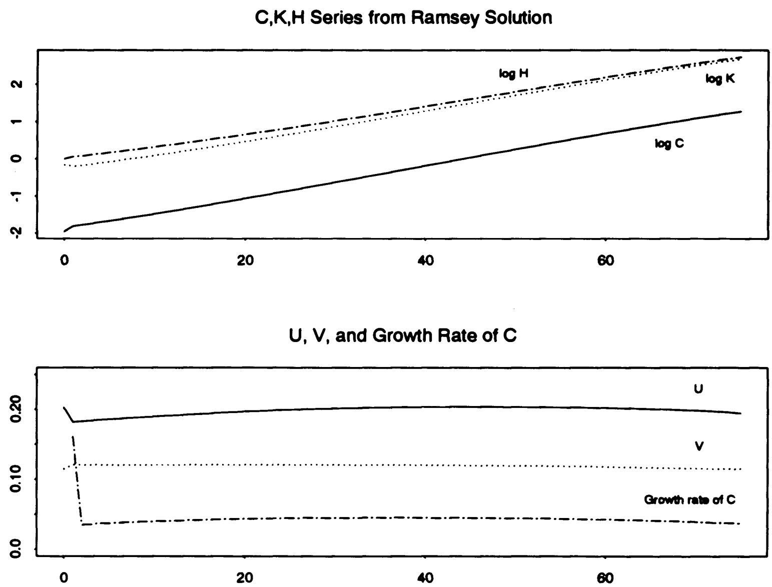

FIG. 2.—Model 2, $\sigma = 2.0$

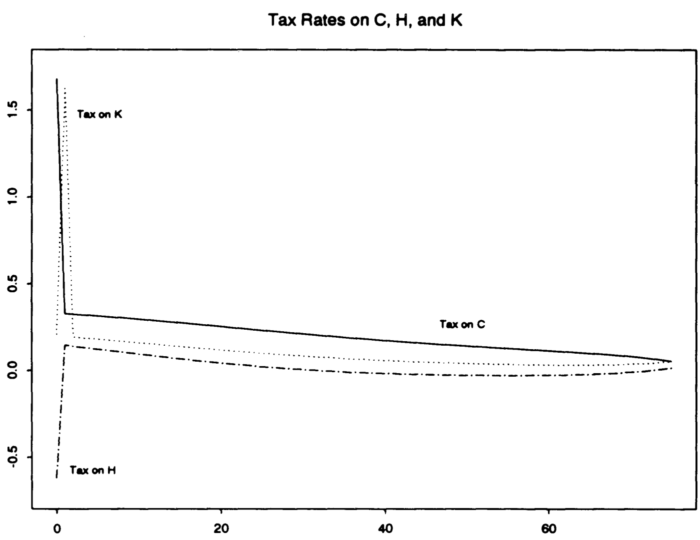

FIG. 3.—Model 2, $\sigma = 2.0$

The bottom half of figure 2 shows the time paths of some of the variables that are converging to constants along the optimal path, u, v, and γ. These are very well behaved and smoothly approach their limiting values. Also, the steady-state growth path values of u and v are considerably above those from the current calibration. It can be seen from the steady-state equations that both u and v have an effect on the limiting growth rate. This is one source of the very large change in the growth rate from 2 to 5.5 percent. Lower taxes result in a significant increase in the number of hours devoted to market work with a higher utilization of human capital.

Figure 3 shows the time paths of the tax rate variables in the calculated Ramsey solution. All tax rates converge to zero in the limit after an initial phase of high taxation. As in model 1, there is a very high time period 1 capital tax rate followed by a smooth decline to zero. The tax on consumption follows a similar path with an even higher initial tax rate. In contrast, the initial labor tax rate is negative immediately followed by positive and smoothly declining tax rates. The time 0 labor subsidy accounts for the initial high values of u and v, which are immediately adjusted downward. The high time 0 consumption tax delays consumption, resulting in an abnormally high time 0 consumption growth rate.

The results of experiments involving changes in σ are contained in tables 2–4. As in the experiments conducted with model 1, we have adjusted the values of η, ψ, and A_1 so as to maintain a growth rate of 2 percent under the current tax system. Alternatively, given the nature of the experiment, this exercise can be viewed as adjusting η and altering σ, ψ, and A_1 accordingly.

Table 2 contains limiting growth rates calculated from the 75-period approximate solution. For all values of σ, there are large growth and welfare effects of a change to the optimal tax system. As in model 1, there is a general reduction in the level of limiting taxation. This by itself results in a substantial increase in the growth rate through its effect on investment decisions. There are both dynamic

TABLE 2

GROWTH AND WELFARE: ELASTIC LABOR SUPPLY

σ	η	A_1	ψ	γ_1	γ_2	Welfare*
1.1	7.09	1.29	.51	1.02	1.103	3.52
1.5	5.91	1.42	.48	1.02	1.055	1.46
2.0	4.99	1.60	.44	1.02	1.040	1.20
2.5	4.38	1.80	.41	1.02	1.034	1.13

* This is the factor by which the $\{c_t\}$ path of consumption must be raised in order to bring utility under the current system up to the level attained in the Ramsey resolution of tax paths.

TABLE 3

STEADY-STATE VARIABLES: ELASTIC LABOR SUPPLY

σ	γ_2	u_1	u_2	v_1	v_2	$(k/h)_1$	$(k/h)_2$	$(c/y)_1$	$(c/y)_2$
1.1	1.103	.17	.25	.12	.18	.74	.71	.24	.21
1.5	1.055	.17	.20	.12	.13	.79	.84	.29	.35
2.0	1.040	.17	.19	.12	.12	.85	.94	.34	.44
2.5	1.034	.17	.19	.12	.12	.93	1.04	.39	.51

TABLE 4

REVENUE SHARES: ELASTIC LABOR SUPPLY

σ	γ_2	rev_1^c	rev_2^c	rev_1^k	rev_2^k	rev_1^h	rev_2^h
1.1	1.103	.07	.31	.26	.47	.68	.22
1.5	1.055	.08	.49	.25	.41	.67	.11
2.0	1.040	.09	.56	.25	.39	.66	.05
2.5	1.034	.11	.62	.25	.39	.65	−.01

and static components to these investment decision effects. In addition to these investment effects, there is an additional effect of the switch to optimal taxes. This is the labor supply response to the reduction in distortion on the labor-leisure margin. In fact, for a given σ, the welfare effects seen here are larger than those calculated in model 1 (see table 1).

In tables 3 and 4, a subscript 1 denotes values corresponding to the current tax system, and a subscript 2 refers to limiting values in the optimal tax system.

The reduction in labor and consumption taxes results in an increase in $u + v$, as can be seen in table 3. For all σ, u is larger in the optimal tax system by at least 11 percent over the current value. This increase in u enhances the effectiveness of h in production, resulting in higher growth as can be seen from (3). This labor intensity effect on growth is a direct by-product of our choice of the technology for effective labor. (Any production function of the form $z = \phi(u)h$ will have this type of effect.) A similar argument could be made for v.

Reductions in any of the tax rates have complicated effects on all the steady-state variables of the system. For example, the tax reductions induce an increase in u but not always in v, which have opposite growth effects. This can be seen by comparing the results for $\sigma = 1.1$ and $\sigma = 2.5$. In the first case, the change in tax systems increases u and v and decreases c/y (a reflection of the increase in the fraction of output devoted to investment), all of which are growth enhancing.

When $\sigma = 2.5$, we still see an increase in u. However, v is slightly reduced (in the third digit) and c/y increases. These last two changes serve to temper the growth effect of the increase in u.

Table 4 shows one of the most dramatic effects of the switch to the optimal tax system. This is the change in reliance on differing sources of revenue relative to the current system. The terms rev^c, rev^k, and rev^h are, respectively, the fraction of government revenue raised through taxation of consumption, capital income, and labor income. These are calculated as present values of revenue streams in time 0 consumption units. Most striking is the switch from a labor tax–based system to one that relies heavily on consumption taxes.[4]

As is the case with model 1, we obtain large growth and welfare effects from a switch to the Ramsey optimal solution. It is interesting to note that we obtain similar magnitudes in both the inelastic and elastic labor supply cases. Our results for the elastic labor supply case differ from the findings of Lucas (1990), in which a related model is studied. He estimates the maximal gain in welfare from this experiment to be 2.7 percent using the value $\sigma = 2$. However, his policy experiment is slightly different in that he reduces the tax on capital income to zero while increasing both the labor tax rate (to keep the level of debt roughly constant) and the level of government spending (to account for the growth effect of tax changes). The requirement in Lucas's exercise that the budget be balanced in the steady state is probably the main source of the differences in growth and welfare effects.

IV. Endogenous Government Expenditure

The reader will note that in the experiments we have performed to this point, we have taken the Ramsey program very literally. Specifically, we have held a sequence of government expenditures on consumption and transfers as fixed exogenously (in real terms) and asked the question, Is there an alternative tax system that would finance this expenditure sequence more efficiently?

In particular, because of the way in which the Ramsey scheme differs from the current one, it follows that following the Ramsey scheme would give rise to long-run growth rates of output different from those we are currently experiencing. Because of this fact, it follows that the sequence of fixed g's tends to a negligible fraction of output. There is no obvious reason to think that g is an inferior good.

[4] From a policy perspective, switching to a consumption-based tax has been advocated in Bradford (1984). The results concerning welfare comparisons among alternative tax bases in Auerbach and Kotlikoff (1987) are also relevant.

This suggests that g should be adjusted at the same time that taxes are realigned. In this respect, there are both qualitative and quantitative issues that deserve attention. On the qualitative side, it is of interest to explore how different technological specifications involving productive public goods affect both the path and the asymptotic properties of optimal tax policies. From a quantitative perspective, it is important to identify those aspects of technology that are crucial in determining the effects of tax reform. This is the focus of this section.

Alternative methods for endogenizing g include explicitly introducing g into either the utility function or the production function (or both). For these versions of the Ramsey problem, the planner chooses both the tax rates and the g_t sequence. Any of the several different alternatives for introducing productive government spending would be controversial. As a start, we treat government spending as though it had a direct impact on the effectiveness of investment. Examples of this would include the provisions of roads or dams and expenditures on education and health.[5]

In order to isolate the effects of this change on the form of optimal taxes in infinite-horizon growth models, we simplify the models considered above. To do this, we consider the case of a simple one-sector model of capital accumulation and delete labor from the model entirely. The notion of equilibrium used given a time path of government expenditures is identical to that introduced in Section II. Consumers treat the sequence g_t as given when making their investment decisions.

This gives rise to the following problem for the consumer to solve:

$$\max \sum_t \beta^t u(c_t) \quad \text{subject to}$$

$$(a) \sum_t p_t(c_t + x_t) \leq \sum_t p_t(1 - \tau_{Kt}) r_t k_t,$$

$$(b)\ k_{t+1} \leq (1 - \delta_k) k_t + G(x_t, g_t),$$

where we assume that G is homogeneous of degree one in x and g jointly, concave, and smooth.

As formulated, the problem assumes that government spending is a publicly provided private good. This is not a necessary assumption. Alternatively, one could view g as a common input that is shared by each of n productive units. The qualitative character of the solution

[5] For empirical evidence concerning the effect of government spending on productivity, see Aschauer (1989) and Munnell (1990).

of the Ramsey problem presented below would not be affected by this change.

Following the same strategy as in previous sections and using the arbitrage condition that $p_t/G_1(t) = p_{t+1}\{(1 - \tau_{Kt+1})r_{t+1} + [(1 - \delta_k)/G_1(t+1)]\}$, we can write the Ramsey problem for this economy as

$$\max \sum_t \beta^t u(c_t) \quad \text{subject to}$$

$$(a) \sum_t \beta^t \left[u_1(t)c_t - u_1(t)g_t \frac{G_2(t)}{G_1(t)} \right] = \frac{u_1(0)}{G_1(0)} \tag{P3}$$

$$\times \left\{ [1 - \delta + G_1(0)(1 - \tau_{K0})F_k(0)]k_0 + \sum_t \beta^t u_1(t) \frac{G_1(0)}{u_1(0)} F_L(t) \right\},$$

$$(b)\ c_t + x_t + g_t = F(k_t, 1),$$

$$(c)\ k_{t+1} \leq (1 - \delta_k)k_t + G(x_t, g_t),$$

where the planner is choosing time paths for c, k, x, and g.

It is straightforward to show that the first-best allocation in this environment involves only lump-sum taxes. It follows from this fact that the Ramsey equilibrium will not be first-best in most cases of interest. Exceptions include situations in which τ_{K0} is viewed as variable and unlimited (so that lump-sum taxation is available to the planner) or situations in which g is sufficiently unproductive (i.e., $G_2(\ ) = 0$). (In this regard, the recent work of Barro and Sala-i-Martin [1990] on the relative benefits of lump-sum and income taxation in an unconstrained environment should be noted.)

As it turns out, this change has significant impacts on the limiting nature of taxes. In contrast to the results with models 1 and 2 above, it can be shown that in certain cases, the limiting tax on capital income is strictly positive. To study this issue and ensure that stationary growth paths can be equilibria, we restrict attention to cases in which $u(c) = c^{1-\sigma}/(1 - \sigma)$, $F(k, 1) = bk$. Any other production function consistent with long-run growth will give rise to the same limiting behavior. In Jones et al. (1991), the following proposition is proved.

PROPOSITION. The form of the solution to (P3) is determined by the following inequality:

$$\sum_t \beta^t u_1^*(t) g_t^* > k_0[\tau_{K0} u_1^*(0) F_k^*], \tag{10}$$

where asterisks indicate that variables are evaluated at their first-best quantities.

i) If (10) is satisfied, then the limiting tax on capital income is strictly positive.
ii) If (10) is not satisfied, then the Ramsey allocation is first-best and involves only lump-sum taxation.

Expression (10) has a very simple interpretation: that the planned government spending in the first-best solution exceeds the government's ability to raise taxes in the first period by taxing time 0 capital income.

It is possible to show (see Jones et al. 1992) that the reason why asymptotic tax rates on capital income are positive is that pure profits result in our formulation. Both the limiting g (as a fraction of output) and the limiting x affect the size of these profits. The planner has full control over g but can influence the choice of x only indirectly through the choice of capital tax rates. This is the reason why limiting tax rates are positive.

Alternative formulations of productive public goods (see Judd 1991; Zhu 1991) have been suggested. Under these specifications, profits do not result from the inclusion of public goods, and because of this, the limiting tax rate on capital income is zero. These are only a few of many possible specifications for the incorporation of productive public goods. A comprehensive investigation of optimal tax and spending policies across these formulations would be an interesting direction for future research.

A. *Simulations*

In this subsection, we describe the results of some simple simulation experiments based on the model outlined above. To do this, we shall simplify the model by choosing the following functional forms:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma},$$

$$f(k) = bk,$$

$$G(x, g_x) = A[\alpha x^{-\rho} + (1-\alpha)g_x^{-\rho}]^{-1/\rho}.$$

The constant elasticity of substitution functional form was chosen to enable us to study the relationship between the elasticity of substitution between public and private investment ($\epsilon = 1/[1+\rho]$) and both the limiting tax rate on capital income and the limiting share in output of government investment expenditure.

Given these choices, the steady-state growth equations for the model are

$$\gamma^{\sigma} = \beta[(1-\delta) + (1-\tau_k)bG_1], \tag{11}$$

$$\gamma = 1 - \delta + G_1 \frac{x}{k} + G_2 \frac{g_x}{k}, \tag{12}$$

$$\frac{c}{k} + \frac{x}{k} + \frac{g_c}{k} + \frac{g_x}{k} = b. \tag{13}$$

To calibrate the model, we chose $\beta = .98$, $\delta = .1$, $\gamma = 1.02$, $\tau_k = .2$, $g_c/k = .08b$, $g_x/k = .12b$, $A = 1$, $\sigma = 2$, and $\alpha = .6$. Given these choices, the steady-state equations given above determine b, c/k, and x/k as a function of ρ.

To avoid lump-sum taxation, it is necessary to bound taxes. This bound is referred to as $\tau_{\max}$ below.

In contrast to models 1 and 2, the limiting behavior of the system is dependent on the path of the solution in the first few periods. The reason is that the long-run rate of growth depends on steady-state revenue requirements. This (and hence the limiting tax rate) depends on the ability of the planner to raise revenue in the beginning of the problem. To handle this problem, we followed a procedure in which the limiting government choice variables (τ_k and g_x/k) are chosen, and then the problem is solved to give calculated values for τ_k and g_x/k from the end of the solution. We iterate on these values until a fixed point is found.

Figure 4 shows the time path of capital taxes and the share in output of government investment spending for the base case: $\sigma = 2$, $\delta = .1$, $A = 1$, $\beta = .98$, $\rho = -.5$ ($\epsilon = 2$), $\tau_{\max} = .65$, $b = .416$, and

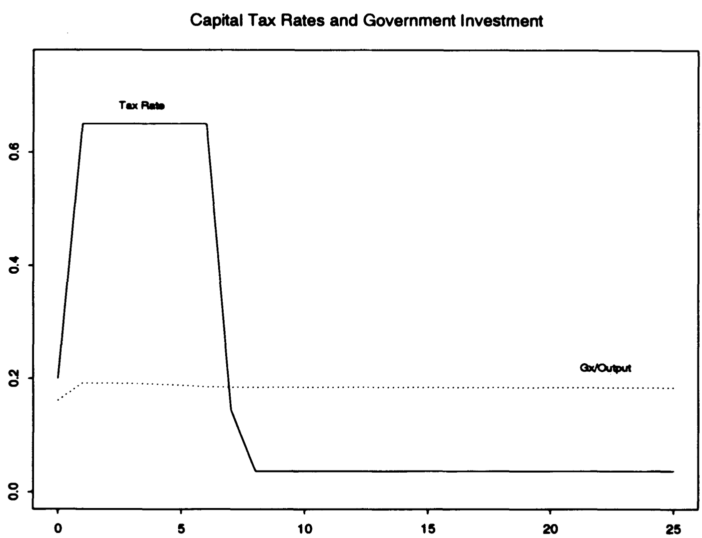

FIG. 4.—Model 3, $\sigma = 2.0$, elasticity of substitution $= 2$

$\alpha = .6$. There is a substantial increase in the limiting growth rate of output due to two separate effects. First, because of our choice of $\alpha = .6$, there is a substantial realignment of the ratio of public to private spending on investment. In addition to this, there is a change in the limiting tax rate on capital income that also increases the growth rate.

The dynamic behavior of taxes mimics that of the model considered in Section II. An initial period of high taxation is followed by a reduction of taxes to their steady-state levels. In contrast to the results in Section II, however, this limiting tax rate is strictly positive. This is in keeping with the proposition above.

In order to test the sensitivity of our numerical results to our choice of parameter values, experiments were done adjusting σ and ϵ independently. These experiments involved one-dimensional parametric adjustments around a base case in which $\sigma = 2$, $\delta = .1$, $A = 1$, $\beta = .98$, $\rho = -.5$ ($\epsilon = 2$), $\tau_{max} = .65$, $b = .416$, and $\alpha = .6$. The results of two of these experiments are included in tables 5 and 6 below.

In the first experiment, we changed σ while adjusting b to maintain a 2 percent growth rate in the steady state under the current tax system. The results of this experiment are summarized in table 5. As in the experiments in the previous sections, we see that the change to the Ramsey optimal tax system gives rise to substantial increases in both welfare and the growth rate. In contrast to the results in the previous section, however, there are now two independent reasons for these changes. First is the standard increase due to realignment across time of the tax burden. This effect is present in the models of Sections II and III as well. In this case, there is an additional effect due to the misalignment of the relative sizes of government and private investment spending in the current system as discussed above. As σ increases, the limiting rate of growth decreases for the standard intertemporal elasticity of substitution reasons. (The subtlety here is that as σ is increased, b is increased as well. These changes have opposing effects on growth, with the net effect of a decrease in

TABLE 5

CHANGING σ

σ	b	γ_1	γ_2	c/y	x/y	g_x/y	N	τ_∞	Welfare*
1.01	.37	1.02	1.042	.26	.58	.17	9	.098	2.24
1.50	.39	1.02	1.042	.31	.51	.18	7	.065	1.57
2.00†	.42	1.02	1.041	.35	.47	.19	6	.037	1.37
2.50	.44	1.02	1.038	.39	.43	.18	4	.022	1.27

NOTE.—For all cases, $\delta = .1$, $A = 1$, $\beta = .98$, $\rho = -.5$ ($\epsilon = 2$), $\tau_{max} = .65$, and $\alpha = .60$.

* This is the factor by which the $\{c_t\}$ path of consumption must be raised in order to bring utility under the current system up to the level attained in the Ramsey resolution of tax paths.

† Signifies results for the base case.

TABLE 6

CHANGING ϵ

ϵ	b	γ_1	γ_2	c/y	x/y	g_x/y	N	τ_∞	Welfare*
.66	.62	1.02	1.082	.42	.31	.27	4	.134	1.84
.99	.53	1.02	1.060	.39	.34	.26	5	.115	1.54
2.00†	.42	1.02	1.041	.35	.47	.19	6	.037	1.37
4.00	.37	1.02	1.038	.31	.59	.10	3	.002	1.34

NOTE.—For all cases, σ = .1, δ = .1, A = 1, β = .98, α = .6, and τ_{max} = .65.

* This is the factor by which the $\{c_t\}$ path of consumption must be raised in order to bring utility under the current system up to the level attained in the Ramsey resolution of tax paths.

† Signifies results for the base case.

growth.) In addition to this, both the limiting tax rate and the limiting share of government in output are decreasing in σ. As both the limiting rate of growth of output falls and the share of government spending in output falls, a higher percentage of the necessary government budget can be raised through the initial phase of high taxation, giving rise to lower steady-state revenue requirements and hence lower steady-state tax levels.

The second experiment involves adjusting the elasticity of substitution between private and public expenditures on investment. These results are contained in table 6. As ϵ is increased, public and private investment become better substitutes. Thus the higher ϵ is, the less important is any misalignment between these two sources of investment expenditure. In line with this intuition, the sizes of the welfare and growth effects are lower when ϵ is higher. For the same reason, the higher ϵ is, the lower is the limiting tax rate on income and the fraction of output devoted to public investment.

In addition, we conducted experiments in which α and τ_{max} were varied while σ and ϵ were held constant. Varying α changes the size of the realignment between x and g_x in the optimal regime. Because of this, an increase in α results in lower levels of the limiting growth rate and welfare change. The tax bound was varied between .5 and .8. As the constraint is relaxed, a higher level of the required revenue is raised at the beginning of the problem, giving rise to lower limiting tax rates and higher limiting growth rates. In addition, as is to be expected, increasing this bound increases the size of the welfare effect. While the range of tax bounds considered is fairly substantial, the differences in the welfare and growth effects across the cases are quite small, changing the limiting growth rate by less than 0.2 percent over the entire range.

In summary, the key parameters for determining the size of the growth and welfare effects of a switch to the Ramsey optimal policy are those of the production function for investment and the intertemporal elasticity of substitution. The fact that the production function parameters are so crucial in determining the size of the effects is unfortunate since our knowledge about these is particularly sparse. This suggests a high payoff to further research on the identification and estimation of these parameters.

V. Conclusions

Our goal in this paper has been to provide a quantitative assessment of the size of the effects on welfare and growth rates of radical tax reform in the Ramsey spirit in a calibrated model of the U.S. economy. In all experiments, we found large growth and welfare effects. This holds for all the cases we have studied with inelastic or elastic labor supply and with exogenous or endogenous government spending.

However, these improvements in welfare have been attained by qualitatively different tax policies. Thus the structure of optimal tax policy is highly dependent on the formulation of the model. What the models considered in Section IV suggest is that the program of strict separation between the spending and revenue sides of the government's problem envisaged in the Ramsey approach should be viewed with some skepticism. If the solution to the Ramsey problem causes a large enough realignment of private resources to necessitate a reconsideration of the planned expenditure pattern, the resulting tax policy may be seriously misleading. In more concrete terms, in the calibrated models of Sections II and III, there is a substantial increase in investment in both human and physical capital. If accomplishing this requires an increase in publicly financed activities (e.g., schooling or roads), models of the sort explored in Section IV are of considerable interest. However, in models with endogenous government spending, the limiting capital tax rate depends critically on the specification of the production technology (cf. Judd 1991; Zhu 1991; Jones et al. 1992).

Our findings contrast markedly with many of the results in the dynamic taxation literature. Chamley (1981), Judd (1987), and King and Rebelo (1990) find much smaller welfare effects from various tax reform experiments in exogenous growth settings. Our results are most similar in magnitude to those of the endogenous growth case studied in King and Rebelo. This suggests that endogenous growth is an important contributing factor to the quantitative character of the effects of optimal tax experiments.

The nature of the solutions to the Ramsey problems that we study raises several concerns about the practicality of their implementation. One consideration that arises because of the dynamic path of the taxes is time consistency. This is clearly a problem with the solutions presented in connection with the models we have analyzed (the solutions in Sec. II without transfers are time consistent). In addition to this, in a tax system with significant differences in marginal tax rates across either time or factors, tax arbitrage schemes might arise, limiting government revenue.

Finally, the solutions to all the Ramsey problems that we study in this paper are characterized by an initial phase of relatively high taxation followed by much lower (often zero) asymptotic taxes. If this feature is endemic to all dynamic Ramsey problems, the relevance of the Ramsey experiment might be called into doubt. However, further work is required on alternative technological specifications and more complicated sets of constraints suggested by implementation concerns before definitive methodological conclusions can be drawn.

Appendix

In this Appendix, we outline the nonlinear programming problems actually solved in performing the simulations.

Simple Symmetric Model with Inelastic Labor Supply (Sec. II)

Maximize over $\{k_1, \ldots, k_{T+1}; h_1, \ldots, h_{T+1}\}$

$$\sum_{t=0}^{T} \beta^t u(f(k_t, h_t) - k_{t+1} + (1 - \delta_k)k_t - h_{t+1} + (1 - \delta_h)h_t - g_t) + f^*(k_{T+1})$$

subject to

$$k_t - (1 - \delta_k)k_{t-1} \geq 0, \quad t = 2, \ldots, T,$$
$$h_t - (1 - \delta_h)h_{t-1} \geq 0, \quad t = 2, \ldots, T,$$
$$k_1 \geq (1 - \delta_k)k_0,$$
$$h_1 \geq (1 - \delta_h)h_0,$$
$$\frac{h_{T+1}}{k_{T+1}} = hk^*,$$

$$f(k_t, h_t) - k_{t+1} + (1 - \delta_k)k_t - h_{t+1} + (1 - \delta_h)h_t - g_t$$
$$\geq f(k_{t-1}, h_{t-1}) - k_t + (1 - \delta_k)k_{t-1} - h_t + (1 - \delta_h)h_{t-1} - g_{t-1}, \quad t = 1, \ldots, T$$

(this constraint ensures positive consumption growth),

$$\sum_{t=0}^{T} \beta^t[f(k_t, h_t) - k_{t+1} + (1 - \delta_k)k_t - h_{t+1}$$
$$+ (1 - \delta_h)h_t - g_t - T_t]u'(t) + g^*(k_{T+1}) = W_0,$$

where

$$W_0 = [(1 - \delta_k) + (1 - \tau^k)f_1(k_0, h_0)]k_0 + [(1 - \delta_h) + (1 - \tau^h)f_2(k_0, h_0)]h_0$$

(budget constraint).

End corrections are made to the objective function and the budget constraint by assuming that after period T consumption and the capital stocks grow at the steady-state rate. In the steady state, both capital taxes are zero, h and k are in a fixed ratio, g has dropped to a negligible fraction of output, and consumption is linear in k:

$$f^*(k_{T+1}) = \sum_{t=T+1}^{\infty} \beta^t u(\gamma^{t-T-1} ck^* k_{T+1}),$$

$$g^*(k_{T+1}) = \sum_{t=T+1}^{\infty} \beta^t (\gamma^{t-T-1} ck^* k_{T+1}) u'(\gamma^{t-T-1} ck^* k_{T+1}),$$

where hk^* is the steady-state ratio of h to k and ck^* is the steady-state ratio of c to k when all taxes are zero (see eqq. [1] and [2] above).

Elastic Labor Supply Model (Sec. III)

In this model, we think of the planner as solving a constrained infinite-horizon problem under the restriction that after period T consumption and the capital stocks must grow at a constant rate and labor supply is constant. To implement this, we pose a planner problem in which the steady-state growth rate, γ, labor supplies (u_{T+1} and v_{T+1}), and terminal values of consumption (c_{T+1}) and investment (x_{hT+1}) are explicit choice variables. Additional constraints are introduced to ensure that the terminal variable choices are feasible and satisfy the dynamic constraints. The problem can be stated as

$$\text{maximize over } \{k_1, \ldots, k_{T+1}; h_1, \ldots, h_{T+1}; u_0, \ldots, u_{T+1}; v_0, \ldots, v_{T+1}; x_{h0}, \ldots, x_{hT+1}; c_0, \ldots, c_{T+1}; \gamma\}$$

$$\sum_{t=0}^{T} \beta^t u(c_t, 1 - u_t - v_t) + f^*(c_{T+1}, u_{T+1}, v_{T+1}, \gamma)$$

subject to

$$k_t \geq (1 - \delta_k) k_{t-1}, \quad t = 1, \ldots, T,$$

$$c_t \geq c_{t-1}, \quad t = 1, \ldots, T,$$

$$(1 - \delta_h) h_t + G(x_{ht}, v_t h_t) - h_{t+1} = 0, \quad t = 0, \ldots, T,$$

$$F(k_t, u_t h_t) - k_{t+1} + (1 - \delta_k) k_t - x_{ht} - g_t - c_t \geq 0, \quad t = 0, \ldots, T,$$

$$\frac{u_2(c_t, 1 - u_t - v_t)}{u_2(c_{t+1}, 1 - u_{t+1} - v_{t+1})} - \beta \frac{G_2(x_{ht}, v_t h_t)}{G_2(x_{ht+1}, v_{t+1} h_{t+1})} \frac{h_t}{h_{t+1}} \times [1 - \delta_h + G_2(x_{ht+1}, v_{t+1} h_{t+1})(u_{t+1} + v_{t+1})] = 0,$$

$$t = 0, \ldots, T \text{ } (h_t \text{ Euler constraint}),$$

$$\sum_{t=0}^{T} \beta^t \left[u_1(t) c_t - u_2(t) \frac{T_t}{h_t} \frac{G_1(t)}{G_2(t)} + u_2(t) \frac{x_{ht}}{h_t} \frac{G_1(t)}{G_2(t)} - u_2(t) u_t \right] + g^*(c_{T+1}, u_{T+1}, v_{T+1}, \gamma) = W_0 \frac{u_2(0)}{h_0} \frac{G_1(0)}{G_2(0)},$$

where $W_0 = [(1 - \delta_k) + (1 - \tau^k) F_1] k_0$ (budget constraint).

To ensure that the choices of terminal variables (γ, c_{T+1}, u_{T+1}, v_{T+1}, and x_{hT+1}) satisfy feasibility and the h_t Euler constraints from period $T + 1$ on, we impose three additional constraints:

$$\gamma^\sigma v_{T+1} - \beta\{(1 - \delta_h) + (1 - \psi)[\gamma - (1 - \delta_h)](u_{T+1} + v_{T+1})\} = 0$$

(h_t Euler from period $T + 1$ on),

$$F(k_{T+1}, u_{T+1}h_{T+1}) - [\gamma - (1 - \delta_k)]k_{T+1} - x_{hT+1} - c_{T+1} \geq 0$$

(consumption feasibility), and

$$(1 - \delta_h)h_{T+1} + G(x_{hT+1}, v_{T+1}h_{T+1}) - \gamma h_{T+1} \geq 0$$

(feasibility of x_{hT+1}).

Given the choice of steady-state growth and labor supply, we can compute the continuation value of the objective and budget constraints:

$$f^*(c_{T+1}, u_{T+1}, v_{T+1}, \gamma) = \sum_{t=T+1}^{\infty} \beta^t u(c_{T+1}\gamma^{t-T-1}, 1 - u_{T+1} - v_{T+1}),$$

$$g^*(c_{T+1}, u_{T+1}, v_{T+1}, \gamma) = \frac{\beta^{T+1}(c_{T+1})^{1-\sigma}}{1 - \beta\gamma^{1-\sigma}} \times \left[1 + \frac{\eta\psi v_{T+1}}{(1 - \psi)(1 - u_{T+1} - v_{T+1})} - \frac{\eta u_{T+1}}{1 - u_{T+1} - v_{T+1}}\right].$$

Endogenous Government Spending Model (Sec. IV)

Maximize over $\{c_0, \ldots, c_T; k_1, \ldots, k_{T+1}; x_0, \ldots, x_T; g_0, \ldots, g_T\}$ given k_0

$$\sum_{t=0}^{T} \beta^t u(c_t) + f^*(k_{T+1})$$

subject to

$$f(k_t) - c_t - x_t - g_t - g_{ct} \geq 0, \quad t = 0, \ldots, T,$$

$$k_t = (1 - \delta_k)k_{t-1} + G(x_{t-1}, g_{t-1}), \quad t = 1, \ldots, T + 1,$$

$$\sum_{t=0}^{T} \beta^t \left[u_1(t)c_t - u_1(t)g_t\frac{G_2(t)}{G_1(t)}\right] + g^*(k_{T+1}) = \frac{u_1(0)}{G_1(0)}[(1 - \delta_k) + G_1(0)(1 - \tau^k)F_k(0)]k_0$$

(budget constraint), and

$$1 - \frac{1}{F_k(t)}\left[\frac{u_1(t)}{G_1(t)\beta u_1(t + 1)} - \frac{1 - \delta_k}{G_1(t + 1)}\right] \leq \tau_{\max}, \quad t = 1, \ldots, T$$

(tax bounds).

For this model, we face an additional problem. We know that the steady-state taxes are nonzero; however, the steady-state revenue requirements are determined by the initial portion of the solution path. We adopt a different strategy here: (1) we fix values of the steady-state tax rate and the size of government investment relative to output; (2) we calculate the steady-state values of growth, consumption relative to output, and private investment relative to output conditional on these estimates of steady-state taxes and government investment; (3) we make end corrections assuming these values of steady-state variables; and (4) we iterate this procedure until the growth rate and tax rate from the end of the solution agree with the assumed steady-state values. The continuation corrections to the objective and the budget constraint are

$$f^*(k_{T+1}) = \sum_{t=T+1}^{\infty} \beta^t u(ck^* k_{T+1} \gamma^{t-T-1}),$$

$$g^*(k_{T+1}) = \beta^{T+1} \frac{(ck^* k_{T+1})^{1-\sigma}}{1 - \beta\gamma^{1-\sigma}} \left[1 - \frac{gk^*}{ck^*} \frac{1-\alpha}{\alpha} \left(\frac{xk^*}{ck^*}\right)^{\rho+1}\right],$$

where gk^* is the asymptotic ratio of government investment spending to capital and ck^* is the asymptotic ratio of consumption to the capital stock given by solving steady-state equations (11)–(13) conditional on an assumed tax rate and g/k ratio.

References

Aschauer, David A. "Is Public Expenditure Productive?" *J. Monetary Econ.* 23 (March 1989): 177–200.

Auerbach, Alan J., and Kotlikoff, Laurence J. *Dynamic Fiscal Policy.* New York: Cambridge Univ. Press, 1987.

Barro, Robert J. "Government Spending in a Simple Model of Endogenous Growth." *J.P.E.* 98, no. 5, pt. 2 (October 1990): S103–S125.

Barro, Robert J., and Sahasakul, Chaipat. "Average Marginal Tax Rates from Social Security and the Individual Income Tax." *J. Bus.* 59, no. 4, pt. 1 (October 1986): 555–66.

Barro, Robert J., and Sala-i-Martin, Xavier. "Public Finance in Models of Economic Growth." Working Paper no. 3362. Cambridge, Mass.: NBER, May 1990.

Bradford, David F. *Blueprints for Basic Tax Reform.* 2d ed. Arlington, Va.: Tax Analysts, 1984.

Bull, N. "When the Optimal Dynamic Tax Rate Is None." Manuscript. Minneapolis: Fed. Reserve Bank, 1992.

Chamley, Christophe. "The Welfare Cost of Capital Income Taxation in a Growing Economy." *J.P.E.* 89 (June 1981): 468–96.

———. "Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives." *Econometrica* 54 (May 1986): 607–22.

———. "The Last Shall Be First: Foreign Borrowing and Growth with Human Capital in an Open Economy." Manuscript. Boston: Boston Univ., 1990.

Chari, V. V.; Christiano, Lawrence; and Kehoe, Patrick J. "Dynamic Ramsey Taxation in a Stochastic Growth Model." Manuscript. Minneapolis: Fed. Reserve Bank, 1990.

Chari, V. V., and Kehoe, Patrick J. "Sustainable Plans with Debt." Manuscript. Minneapolis: Fed. Reserve Bank, 1989.

———. "Sustainable Plans." *J.P.E.* 98 (August 1990): 783–802.
Eaton, Jonathan. "Fiscal Policy, Inflation and the Accumulation of Risky Capital." *Rev. Econ. Studies* 48 (July 1981): 435–45.
Gill, Philip E.; Murray, Walter; and Wright, Margaret H. *Practical Optimization.* New York: Academic Press, 1981.
Heckman, James J. "A Life-Cycle Model of Earnings, Learning, and Consumption." *J.P.E.* 84, no. 2, pt. 2 (August 1976): S11–S44.
Jones, Larry E., and Manuelli, Rodolfo. "A Convex Model of Equilibrium Growth: Theory and Policy Implications." *J.P.E.* 98, no. 5, pt. 1 (October 1990): 1008–38.
———. "Finite Lifetimes and Growth." *J. Econ. Theory* 58 (December 1992).
Jones, Larry E.; Manuelli, Rodolfo; and Rossi, Peter E. "Optimal Taxation in Models of Endogenous Growth." Working paper no. 91-108. Chicago: Univ. Chicago, 1991.
———. "On the Optimal Taxation of Capital Income." Manuscript. Chicago: Univ. Chicago, 1992.
Jorgenson, Dale W., and Yun, Kun-Young. *Tax Reform and the Cost of Capital.* Oxford: Clarendon, 1991.
Judd, Kenneth L. "Redistributive Taxation in a Simple Perfect Foresight Model." *J. Public Econ.* 28 (October 1985): 59–83.
———. "The Welfare Cost of Factor Taxation in a Perfect-Foresight Model." *J.P.E.* 95 (August 1987): 695–709.
———. "Optimal Taxation in Dynamic Stochastic Economies: Theory and Evidence." Manuscript. Stanford, Calif.: Hoover Inst., 1990.
———. "Optimal Taxation with Government Investment." Manuscript. Stanford, Calif.: Hoover Inst., 1991.
Juster, F. Thomas, and Stafford, Frank P. "The Allocation of Time: Empirical Findings, Behavioral Models and Problems of Measurement." Manuscript. Ann Arbor: Univ. Michigan, Survey Res. Center, Inst. Soc. Res., 1990.
King, Robert G., and Rebelo, Sergio. "Public Policy and Economic Growth: Developing Neoclassical Implications." *J.P.E.* 98, no. 5, pt. 2 (October 1990): S126–S150.
Kydland, Finn E., and Prescott, Edward C. "Time to Build and Aggregate Fluctuations." *Econometrica* 50 (November 1982): 1345–70.
Lucas, Robert E., Jr. "On the Mechanics of Economic Development." *J. Monetary Econ.* 22 (July 1988): 3–42.
———. "Supply-Side Economics: An Analytical Review." *Oxford Econ. Papers* 42 (April 1990): 293–316.
Lucas, Robert E., Jr., and Rapping, Leonard A. "Real Wages, Employment, and Inflation." *J.P.E.* 77 (September/October 1969): 721–54.
Lucas, Robert E., Jr., and Stokey, Nancy L. "Optimal Fiscal and Monetary Policy in an Economy without Capital." *J. Monetary Econ.* 12 (July 1983): 55–93.
MaCurdy, Thomas E. "Interpreting Empirical Models of Labor Supply in an Intertemporal Framework with Uncertainty." In *Longitudinal Analysis of Labor Market Data,* edited by James J. Heckman and Burton Singer. Cambridge: Cambridge Univ. Press, 1985.
Munnell, Alicia H., ed. *Is There a Shortfall in Public Capital Spending?* Conference Series no. 34. Boston: Fed. Reserve Bank, 1990.
Prescott, Edward C. "Theory Ahead of Business Cycle Measurement." *Fed. Reserve Bank Minneapolis Q. Rev.* 10 (Fall 1986): 9–22.

Rebelo, Sergio. "Long-Run Policy Analysis and Long-Run Growth." *J.P.E.* 99 (June 1991): 500–521.

Rosen, Sherwin. "A Theory of Life Earnings." *J.P.E.* 84, no. 4, pt. 2 (August 1976): S45–S67.

Stokey, Nancy L. "Credible Public Policy." *J. Econ. Dynamics and Control* 15 (October 1991): 627–56.

Yuen, C. W. "Taxation, Human Capital Accumulation and Economic Growth." Manuscript. Chicago: Univ. Chicago, 1990.

Zhu, X. "Optimal Taxation with Endogenous Government Expenditures." Manuscript. Chicago: Univ. Chicago, 1991.