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REDISTRIBUTIVE TAXATION IN A SIMPLE PERFECT FORESIGHT MODEL*

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by

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We investigate the redistributive potential of capital taxation in an intertemporal maximizing model of capital formation. First, even unanticipated redistributive capital taxation is severely limited in its effectiveness since it depresses wages. Second, if all pure rates of time preference are equal in the long run, any convergent optimal redistributive capital tax will converge to zero, independent of the factor supply elasticities. Third, if worker-dominated legislatures control only short-term tax rates, the equilibrium of the resulting game among the legislatures will generally result in substantial long-run taxation. These results hold independent of whether workers hold capital.

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1. Introduction

One of the most important questions of public finance is the incidence of a tax. One particularly interesting aspect of this question is the redistributive potential of capital income taxation: how much will the disincentive effects of capital income taxation on capital accumulation and the resulting loss in wages reduce the net benefits of the redistribution for workers, the presumed recipients? In this paper we examine the redistributive potential of capital income taxation in general equilibrium growth models.

Dynamic general equilibrium incidence of capital income taxation has been studied in various versions of the neoclassical growth model by Feldstein (1974), Grieson (1975), Boadway (1979), Bernheim (1981), and Homma (1981). These studies demonstrated that the incidence of a capital income tax may be significantly shifted to labor in the long run, significantly reducing the redistributive potential of capital income taxation, but generally not eliminating it. The major shortcoming of these studies was their concentration on long-run effects, usually ignoring the adjustment process which governs the economy on its approach to the steady state, which is only realized in the limit. When intertemporal incidence calculations are made in such models (as in Boadway and Bernheim) the results are sensitive to the discount rate, that rate being a parameter of intertemporal preferences which, in their models, does not affect savings behavior. In contrast, we examine these issues in a perfect foresight model of growth where capital accumulation is determined by the maximization of a dynamic utility functional for the owners of capital. In such a model we can also examine anticipation effects which are absent in neoclassical growth models, and can calculate the dynamic value of a tax change, taking into account the adjustment process.

We find two recurring themes, one expected and the other surprising. First, the short-run fixity of capital makes both temporary and permanent unexpected increases in the redistributive tax on capital income attractive to the agents who possess less capital than the average holding. This makes it tempting for a relatively poor, but politically powerful, majority to impose unanticipated capital income taxes for the purposes of redistribution. We quantify this in a simple dynamic model of legislative decision-making and argue that high levels of capital income taxation for redistributive purposes are consistent with plausible assumptions concerning the political process and parameters of taste and technology. On the other hand, we find that the outcome of this political process may actually be detrimental to that majority in the long run. In fact, if both workers and capitalists have the same rate of time preference in the steady state, the optimal redistributive tax on capital income from the point of view of any agent is asymptotically zero, <u>independent of long-run factor supply elasticities</u>. This last result stands in stark contrast with the neoclassical models which seem to argue that some redistribution generally benefits workers even in the long run. Together, these results indicate that redistribution of income through capital income taxation is effective only if it is unanticipated and will persist only if workers cannot commit themselves to low taxation in the long run. This result also negates the usual intuition that as long as factor supply is not perfectly elastic there will not be total shifting of the burden and some redistribution would therefore be effective. More generally, these results indicate that the true long-run burden of a factor income tax is not well represented by the long-run impact of the tax on the net-of-tax factor price, the usual index considered when the shifting of a tax burden is studied.

2. The Model

Assume that we have an economy of a large fixed number of identical, infinitely-lived individuals. The common utility functional is assumed to be additively separable in time with a constant pure rate of time preference, ρ:

$$U = \int_0^\infty e^{-\rho t} u(c(t))dt$$

where c(t) is consumption of the single good at time t. To abstract away from differences in taste and to construct a model where we need only examine the evolution of aggregate capital and consumption to determine equilibrium, we will assume that workers and capitalists have the same constant elasticity of marginal utility, β, and the same pure rate of time preference, ρ.

We will assume initially that labor is supplied inelastically by all. This is in keeping with the previous studies, and is appropriate since our concern is with the income inequality due to wealth inequality. L^c units of labor are supplied inelastically at all times t by each capitalist and L^w units of labor are inelastically supplied by workers who do not participate in the capital market, consuming their wages at each moment; all are paid at a wage rate of w(t). This decomposition of the labor force is made so that we may examine the implications of imperfect capital markets. We normalize so that the total labor supply, L, is unity.

There will be one asset in this economy: capital stock. Let F(k) be a standard neoclassical CRTS production function giving output per unit of labor in terms of the aggregate capital-labor ratio, k. At t=0, k_{i0} is the i^{th} capitalists' endowment of capital. Capital is assumed to depreciate at a constant rate of $\delta > 0$ and f(k) shall denote the net national product, that is, gross output minus depreciation. σ will denote the elasticity of substitution between capital and labor in the net production function.

We shall keep the institutional structure simple. Think of each agent as owning his own firm, hiring labor and paying himself a rental of $r(t)$ per unit of capital at t, gross of taxes and depreciation. It is straightforward that the alternative assumption of value-maximizing firms would be equivalent; see Brock and Turnovsky (1981) for formal demonstrations of this. Since there will be no discussion of policies that are sensitive to the institutional structure, we shall use that fact and ignore the institutional detail that firms bring.

The government will play no constructive role: at time t, it taxes capital income net of depreciation at a proportional rate $\tau(t)$, and makes non-negative lump-sum transfers of T^c and T^w to each capitalist and each worker, respectively, and consumes G units of the good, such consumption not affecting the demand of any agent for private consumption goods.

The i'th capitalist will choose his consumption path, $c_i(t)$, capital accumulation, $\dot{k}_i(t)$, subject to the instantaneous budget constraint, taking the wage, rental, and tax rates as given:

$$(1) \qquad \underset{c_i(t),\ k_i(t)}{\text{maximize}} \int_0^\infty e^{-\rho t} u(c_i(t))dt$$

$$\text{s.t.} \quad c_i + \dot{k}_i = w + (r - \delta)(1 - \tau)k_i + T^c$$

$$k_i(0) = k_{i0}$$

(Time arguments are suppressed when no ambiguity results.) The basic arbitrage condition which must hold is

$$(2) \qquad u'(c_i) \equiv \int_t^\infty e^{\rho(t-s)}(r - \delta)(1-\tau)u'(c_i)ds$$

This states that along an optimum path, each capitalist is indifferent between an extra unit of consumption and the extra future consumption that would result from an extra unit of investment. Upon differentiation, this expression yields

$$(3) \qquad \dot{c}_i = -c_i(\rho - (r-\delta)(1-\tau))/\beta$$

We shall also assume that the transversality condition at infinity holds:

$$(4) \qquad (\mathrm{TVC}_\infty) \quad \lim_{t\to\infty} u'(c_i(t))\, k_i(t)\, e^{-\rho t} = 0.$$

This condition is needed to insure that c_i and k_i remain bounded as $t\to\infty$ and is a necessary condition for the agent's problem if $u(\cdot)$ is bounded, which is a harmless assumption here since the net production function is bounded (see Benveniste and Scheinkman (1982)).

To describe equilibrium, impose the equilibrium conditions

$$(5a) \qquad r = f'(k) + \delta$$

$$(5b) \qquad w = f(k) - kf'(k)$$

on (2) and the budget constraint, and sum the resulting individual arbitrage conditions, (3), and capital accumulation equations, thereby yielding the equilibrium equations

$$(6a) \qquad \dot{C} = -C(\rho - (1-\tau)f'(k))/\beta$$

$$(6b) \qquad \dot{k} = f(k) - C - L^c\big(f(k) - kf'(k)\big) + T^c$$

where C is aggregate consumption by capitalists and k is the aggregate capital-labor ratio. The transversality condition implies that

$$(7) \qquad 0 < \lim_{t\to\infty} C(t),\ \lim_{t\to\infty} k(t) < \infty$$

The pair of equations, (6), describes the equlibrium of our economy at any t such that C and k are differentiable. To determine the system's behavior at points where C or k may not be differentiable, we impose the equilibrium conditions on (2), yielding

$$(8) \qquad u'(c_i(t)) = \int_t^\infty e^{-\rho(s-t)}u'(c_i(s))f'(k(s))(1-\tau(s))ds$$

showing that the $c_i(t)$ and $C(t)$ functions are continuous functions of time. The system of relations given by equations (6) and (8) and the inequality (7) will describe the general equilibrium of our economy. This equilibrium is unique, as is demonstrated by the saddle-point structure of equation (6).

3. Case I: Workers Don't Save

In this section we turn our attention to a simple class model similar to neoclassical savings models--only capitalists save and only workers work. Both assumptions are consistent with basic economic theory: we may assume that capitalists are on a corner of their labor supply decision due to their wealth, leisure being a normal good, and workers find neither saving nor borrowing valuable because of the transactions costs associated with small transactions. The crucial difference between this model and neoclassical savings models is that capitalists' behavior is governed by the maximization of an intertemporal utility function. (Some neoclassical models allow for the possibility of workers saving. That case will be analyzed separately since the results are different.) Note that we make the usual assumption of inelastic labor supply. Therefore the two models differ only in their specification of investment.

We will analyze only the case where all capital income tax receipts are redistributed uniformly among the workers. This is the only interesting case since there is no point in this model to taxing capital income and returning it to capitalists. Therefore, the capitalists' sole source of income is their return to capital, and the equilibrium of this economy is described by

$$\dot{C} = -C(\rho - f'(k)(1-\tau)/\beta \tag{9a}$$

$$\dot{k} = (1-\tau)kf'(k) - C \tag{9b}$$

We first should note that the steady state capital stock, k^∞, is a function of the tax rate τ, that relation being the solution to

$$f'(k^\infty) = \rho/(1-\tau) \tag{10}$$

In particular, the long-run capital supply curve is perfectly elastic, the net-of-tax return being ρ. We shall later see, however, that this is not the cause of our results. We examine this case because the essential points may be easily illustrated, and where it appears that these may not be robust to more general utility functionals, we shall prove the desired general result.

We will generally be concerned with the desirability of a tax on capital from the point of view of the workers who will receive the revenue in the form of lump-sum transfers. The first way in which we will address this question is to assume that the economy is in the steady state associated with a constant tax rate of τ on capital income tax and ask if the workers want to increase the current tax, to increase tax in the near future, and/or to increase the tax rate in the distant future by a small increment. More precisely, if we consider $t = 0$ to be the present, we want to compute the net impact on worker welfare of increasing the capital income tax rate at time $t > 0$ by $\varepsilon h(t)$ for small ε. That policy change is to be enacted today and made known to all. The new equilibrium would be the solution to

$$\dot{C} = -C(\rho - f'(k)(1-\tau-\varepsilon h(t)))/\beta \tag{11a}$$

$$\dot{k} = (1-\tau-\varepsilon h(t))\, kf'(k) - C \tag{11b}$$

For any ε, the solutions of C and k in the system (11) can be expressed as $C(t,\varepsilon)$ and $k(t,\varepsilon)$, respectively. We are interested in the impact of a change, modelled as a change in ε. Therefore, the initial impacts of a change in ε are denoted

$$k_\varepsilon(t) \equiv \frac{\partial k}{\partial \varepsilon}(t,0) \qquad C_\varepsilon(t) \equiv \frac{\partial C}{\partial \varepsilon}(t,0)$$

$$\dot{k}_\varepsilon(t) \equiv \frac{\partial}{\partial \varepsilon}\left(\frac{\partial k}{\partial t}\right)(t,0) \qquad \dot{C}_\varepsilon(t) = \frac{\partial}{\partial \varepsilon}\left(\frac{\partial k}{\partial t}\right)(t,0)$$

To determine the initial impact of such a policy change, we differentiate this system with respect to ε and evaluate the derivative at $\varepsilon = 0$ and at the initial steady-state level of capital. (For a general treatment of this perturbation technique, see Judd (1982a and 1982b).)

$$(12) \qquad \begin{pmatrix} \dot{C}_\varepsilon \\ \dot{k}_\varepsilon \end{pmatrix} = J \begin{pmatrix} C_\varepsilon \\ k_\varepsilon \end{pmatrix} - \begin{pmatrix} Cf' h(t)/\beta \\ h(t)kf' \end{pmatrix}$$

where

$$J = \begin{pmatrix} 0 & (1-\tau)f''(k)C/\beta \\ -1 & (1-\tau)(f'(k) + kf''(k)) \end{pmatrix}$$

where k and C are evaluated at their steady-state values corresponding to τ. Taking Laplace transforms of this linear system and solving yields

$$(13) \qquad \begin{pmatrix} \Psi_\varepsilon(s) \\ K_\varepsilon(s) \end{pmatrix} = (sI-J)^{-1} \begin{pmatrix} -Cf' H(s)/\beta + C_\varepsilon(0) \\ -H(s)\ kf' \end{pmatrix}$$

where Ψ_ε, K_ε and H are Laplace transforms[1] of C_ε, k_ε, and h, respectively and $C_\varepsilon(0)$ is the initial change in C due to ε. C will generally jump to ensure stability of the system. Our analysis will make extensive use of the eigenvalues of J, which are

$$(14) \qquad \mu,\lambda = \frac{\rho}{2}\left(1 - \frac{\theta_L}{\sigma} \pm \sqrt{\left(1 - \frac{\theta_L}{\sigma}\right)^2 + \frac{4}{\beta}\frac{\theta_L}{\sigma}}\right)$$

Note that they are independent of τ and that $\mu > 0 > \lambda$.[2] Also note that $\mu \gtrless \rho$ as $\beta \lessgtr 1$.

The transversality condition of the capitalist's choice assures the stability of capital accumulation and the boundedness of $k_\varepsilon(\cdot)$. This allows us to determine $C_\varepsilon(0)$ since boundedness of $k_\varepsilon(\cdot)$ implies boundedness of $K_\varepsilon(\mu)$, implying after some manipulation[3] that

$$(15) \qquad C_\varepsilon(0) = H(\mu)\,\frac{\rho}{1-\tau}\left(1 - \frac{\mu}{\rho}\beta\right)\frac{C}{\beta}$$

Determination of $C_\varepsilon(0)$ therefore leads to a complete solution for $\Psi_\varepsilon(s)$ and $K_\varepsilon(s)$, as expressed in (13).

From (14), we see that $\mu\beta/\rho \lessgtr 1$ as $\beta \lessgtr 1$. (15) shows that capitalists may increase or decrease their consumption in response to a tax increase. This is not surprising since the income effect of lower future income on demand for goods today and the substitution effect due to today's goods becoming cheaper relative to tomorrow's goods, act in different directions. If $\beta > 1$, then the capitalist has a strong preference for a smooth consumption path due to the high curvature of the utility function and the income effect dominates, resulting in less consumption today. If $\beta < 1$, the price effect dominates and consumption jumps up. In either case, the change in consumption is proportional to $H(\mu)$, the tax rate change discounted at the rate μ. Since $H(\mu)$ is greater as the tax increase continues for a longer time, we see that the magnitude of the change in consumption is greater for tax increases of greater duration, whereas the sign depends only on β.

(i) Impact Effects of Tax Changes

The impact on the workers' utility of this tax change can be calculated from the solutions for $\Psi_\varepsilon(s)$ and $K_\varepsilon(s)$. Let c^w denote consumption of the representative worker. Workers consume their wages and the subsidy from the government:

$$c^w(t,\varepsilon) = f(k) - kf'(k) + (\tau + \varepsilon h(t))kf'(k) \tag{16}$$

Since we are initially at the steady state associated with $\varepsilon = 0$, we may differentiate as before, and evaluating at $\varepsilon = 0$ yields,

$$c_\varepsilon^w(t,0) = h(t)\ kf' - kf''k_\varepsilon + \tau k_\varepsilon(f' + kf'') \tag{17}$$

This expression decomposes the impact on worker consumption into its separate components. The first term, $h(t)kf'$, is the increment to tax revenues and resulting rebate to workers. The second term, $-kf''k_\varepsilon$, is the impact on the typical worker's wage of a charge of k_ε in capital stock. The last term is the impact of the induced capital accumulation on tax revenues collected.

The change in utility of workers in terms of the good at t=0, y_ε^w, is equal to the discounted change in utility divided by the current marginal utility of consumption. That change is

$$y_\varepsilon^w = kf'H(\rho)\left(1 + \left\{1 - \frac{H(\mu)}{H(\rho)}\left(\frac{\mu\beta}{\rho} - 1\right) - \beta\right\}\left(\frac{\tau}{1-\tau}\frac{\sigma}{\theta_L} + 1\right)/(1 - \beta)\right) \tag{18}$$

where θ_L is labor's share of the net product, and θ_K will denote the capital share.

For tax increases of very short duration, $H(\mu)H(\rho)^{-1}$ is unity. Then

$$y^w_\varepsilon = kf'H(\rho)\{\frac{\beta}{1-\beta}(1-\frac{\mu}{\rho})(\frac{\tau}{1-\tau}\frac{\sigma}{\Theta_L}+1)+1\} \tag{19}$$

Since $\mu > \rho$ if and only if $\beta < 1$, utility increases if $\tau = 0$, but falls for some positive τ. Hence, a tax increase of short duration will always be desired by the workers if the economy is in the untaxed steady state, but will not be desirable if τ is sufficiently large. This is because of the assumed capital market imperfections: workers are not able to save any of the proceeds from a tax increase, and at high tax rates prefer to keep the capital producing and in the capitalists' hands rather than consume it.

Second, if $h(t) = 1$, i.e., a permanent tax increase is enacted, then $H(\mu)H(\rho)^{-1} = \rho/\mu$ and

$$y^w_\varepsilon = \frac{kf'}{\rho}(\frac{\rho/\mu - 1}{1-\beta}(\frac{\tau}{1-\tau}\frac{\sigma}{\Theta_L}+1)+1) \tag{20}$$

Again, utility increases if $\tau = 0$, but falls for large τ.

The third case, that of a permanent tax increase which begins at some future time T is more complex. Such a tax increase is represented by

$$h(t) = \begin{cases} 0, & t < T \\ 1, & t \geq T \end{cases}$$

Note from our expressions for the eigenvalues that $\mu > \rho$ if and only if $\beta < 1$. This is important for our net gain calculation because if $\mu > \rho$, then $H(\mu)H(\rho)^{-1}$ goes to zero as the imposition of the tax is pushed into the future, whereas if $\mu < \rho$ then $H(\mu)H(\rho)^{-1}$ diverges to infinity as the tax is delayed. Hence, if $\mu > \rho$, $H(\mu)H(\rho)^{-1}$ is essentially zero for tax increases taking effect in the distant future, whereas if $\mu < \rho$, that term dominates.

These observations immediately lead to the determination of the desirability of imposing a tax which shall come into effect only in the distant future. First, if $\tau = 0$ initially, and if $\beta < 1$, then $\mu > \rho$ and for distant tax increases the $H(\mu)H(\rho)^{-1}$ term becomes negligible. Using the formulae for μ and λ it is straightforward to compute that utility is unchanged for distant tax increases if $\tau = 0$ initially, and falls if $\tau > 0$. We therefore see that if capitalists have a small elasticity of marginal utility, workers today will not want to have an anticipated tax increase imposed on the capitalists in the distant future, even if the revenues are distributed to the workers. Note that this is also the case where capitalists will increase current consumption in response to an increase in expected future taxation. This capital decumulation in response to future taxation leads to a decline in wages in the near term, offsetting the revenue gain of the tax increase.

On the other hand, if $\beta > 1$, implying that $\mu < \rho$, then workers will want anticipated redistributive taxation in the distant future. This can be seen from (18) by noting that for distant tax increases, $H(\mu)H(\rho)^{-1}$ will be large and dominate (18). Also, when $\beta > 1$, capitalists save in response to the anticipated tax increase, such immediate capital accumulation raising wages immediately. Hence, if currently $\tau = 0$, this short-run wage effect is an additional benefit of the distant tax increase. Since $-\mu\beta/\rho + 1 < 0$ in this case, utility will increase for distant tax increases if $\tau = 0$ initially.

In summary, we have proved

Theorem 1: In the steady state corresponding to no taxation, workers will want a perfectly anticipated increase in the capital income tax in the distant future if and only if $\beta > 1$. Also they will always want either an immediate temporary or immediately enacted permanent tax increase. In steady states associated with sufficiently high tax rates, workers will desire immediate temporary and permanent tax decreases.

Theorem 1 tells us exactly when the revenue gains from these kinds of tax increases will be exactly offset by the wage losses due to the induced capital decumulation. It is not surprising that β be an important factor since it influences the rate at which the capitalists respond to tax increase and how they respond immediately to a tax change. What is curious about Theorem 1 is that there are both cases where no distant tax increase will be desired by the workers and where distant tax increases will not be desired. No general presumption may be made.

We should note that these results depend only on the curvature of the capitalists' instantaneous utility function. The only critical parameter of the workers' utility function was the pure rate of time preference. We assumed that both workers and capitalists have additively separable utility functions discount utility at the same rate. This is not done because of any belief in its validity, but rather because the effects of heterogeneous discount rates are well known and produce the obvious effects.

The neoclassical model which has the greatest similarity to our model is the simple two-class model with workers saving nothing and capitalists saving a fixed proportion of disposable income. Not only does this neoclassical model look like ours with regard to workers' savings but also the steady state is similar in that the after-tax rate of return on capital is fixed and independent of the tax rate. In both models there is 100 percent shifting of the capital income tax in that the long-run net return to capital is unaffected. However, in our intertemporal optimization model, this does not imply that workers do not gain from a redistributive tax. Given the fixity of capital in the short run, it is not surprising that a short-run tax increase is desirable for workers. What is surprising is that under some conditions workers will want a tax increase which comes only in the very distant future where this 100 percent shifting presumably occurs. This effect is absent in neoclassical growth models since they have no anticipation effects. The basic condition, that the capitalist utility function be sufficiently concave, is intuitive because it leads to a low rate of adjustment of the capital stock to tax changes, effectively reducing the rate at which the tax is shifted to labor.

(ii) The Optimal Redistributive Tax for Workers

These impact analyses of long-range tax changes leads us to inquire as to the long-run nature of an optimal tax on capital imposed by workers. Let $\bar{r}(t)$ be the rate of return net of both taxes and depreciation realized by capitalists at t under a tax law. What would the chosen tax law look like? For any such tax law, the laws of motion for the capitalist class are given by

$$\dot{k} = \bar{r}k - C \tag{21}$$

$$\dot{C} = -C(\rho - \bar{r})/\beta$$

$$\lim_{t\to\infty} k(t),\ C(t) < \infty$$

Suppose that a government concerned only with the welfare of the workers determines a tax policy for all future time and that such revenues must cover a constant stream of government consumption, G, as well as lump-sum transfers to the workers. Furthermore, suppose that it has only two instruments for raising revenue and redistributing income: capital income taxation and lump-sum transfers to workers. It is more convenient to formulate the problem in terms of the after-tax return on capital, $\bar{r}$, rather than in terms of the tax rate. The two approaches are equivalent since by appropriate taxation, the government can achieve an arbitrary time path of factor returns. Since an equal amount of labor is supplied inelastically by all workers, there would be no difference between a labor subsidy or tax and a lump-sum transfer, so this formulation is valid when we allow labor taxes. In particular, if the capital income tax does not raise enough to finance G, then lump-sum taxes are imposed on the workers. The optimal control problem for the worker-controlled government then becomes

$$\max_{\bar{r}} \int_0^\infty e^{-\rho t} u(f(k) - \bar{r}k - G)dt$$

$$\text{s.t.} \quad \dot{k} = \bar{r}k - C$$

$$\dot{C} = C(\rho - \bar{r})/\beta$$

$$\lim_{t\to\infty} C(t),\ k(t) < \infty$$

The current-value Hamiltonian for this problem is

$$H = u(f - \bar{r}k - G) + q_1(rk - C) + q_2\, C(\rho - \bar{r})/\beta \tag{22}$$

where q_1 and q_2 are the current-value multipliers of the state variables k and C, respectively. The laws of motion for solutions to this problem are

$$\begin{aligned} \dot{q}_1 &= \rho q_1 - u'(f' - \bar{r}) - q_1\bar{r} \\ \dot{q}_2 &= \rho q_2 + q_1 - q_2(\rho - \bar{r})/\beta \\ 0 &= -u'k + q_1 k - q_2 C/\beta \end{aligned} \tag{23}$$

The steady-state conditions for this problem are therefore

$$\begin{aligned} &\dot{k} = 0 \\ &\dot{C} = 0 \Rightarrow \bar{r} = \rho \\ &\dot{q}_1 = \dot{q}_2 = 0 \Rightarrow f' = \bar{r} = \rho \Rightarrow \tau = 0 \end{aligned} \tag{24}$$

We have therefore proved Theorem 2:

Theorem 2: If the redistributive capital taxation program which maximizes worker utility converges, if both classes have the same pure rate of time preference, and only capitalists save, then the otpimal that tax vanishes aymptotically. Specifically, there is no redistribution desired by the workers in the limit and any government consumption is financed by lump-sum taxation of workers.

One obvious weakness of Theorem 2 is the assumption of global asymptotic stability of the optimal program. This assumption is common in intertemporal taxation models, although there is no basis for this other than some hope that the system settles down. Initially this seems especially problematic in light of Theorem 1, i.e., workers sometimes like tax increases in the long run when they are initially in the untaxed steady state. While we have no proof of convergence for our problem, we should note one reassuring fact. If the workers were limited to choosing a policy which left capitalists' shadow value of capital, $u'(c)$, initially unchanged, then there is no possible first-order gain to workers' utility if they are in the untaxed steady state. This is clear since if $C_\varepsilon(0) = 0$, then $H(\mu)(-\mu\beta/\rho + 1)$ must be zero, implying that y^w_ε equals $-H(\rho)kf'\sigma\tau/(\theta_L(1-\tau))$, which is negative if τ is positive, and zero in the untaxed steady state. This is a test of the asymptotic optimality of $\tau = 0$ since if the optimal program does have τ converge to zero, then no alteration which leaves C initially unchanged should increase the objective, this test being the Bellman optimality condition which must hold along an optimal path. This also shows that the gain from imposing a tax in the distant future is solely due to the unanticipated nature of that change. Had that change been anticipated, there would be no jump in the marginal utility of consumption of capitalists since C must be continuous along any anticipated path.

The neoclassical analysis closest in spirit to this exercise was carried out by Hamada (1967). He examined the optimal transfer from capitalists to workers when workers can't save and capitalists have a fixed savings rates, s. He showed that if the initial capital stock was small then the workers would accept a small transfer until a critical level of capital stock, k^*, was reached at which point the transfer is increased to $\rho k^*/s > 0$ and capital stock becomes stationary. While confirmation awaits numerical analysis, we feel safe in conjecturing that the optimal program here has a quite different character, with large transfers initially and no transfer asymptotically. Theorem 2 shows that there would be no transfer asymptotically assuming convergence and Theorem 1 indicates that in the short run some transfer is desirable for the workers. We therefore see that the intertemporal pattern of transfers is very different in the intertemporal maximization framework when compared to the neoclassical savings framework.

The obvious weakness of our optimal redistributive tax analysis is that we have no idea as to how long it will take to reach the zero tax on capital income. Also, we do not know how much redistribution is accomplished in terms of lifetime utility. It is very doubtful that there are any tractable examples where we can explicity calculate the optimal tax schedule. Resolution of these questions await numerical analysis which could give some insight.

(iii) Equilibrium Redistributive Taxation in a Simple Political Model

In Theorem 2 we implicitly assumed that a worker-controlled legislaturea could determine all future taxes. It is, however, a fact of political life that legislatures set only current tax rates and those in the near future. That leads us to ask what the equilibrium government policy would be under these constraints. Consider the following game. At periodic intervals a legislature dominated by workers meets and determines the constant level of capital income taxation which will prevail until the next meeting. We make the (realistic) assumption that today's legislatures cannot commit future legislatures to future tax rates. This institutional restriction leads to the standard dynamic consistency problems; that is, the optimal program studied in Theorem 2 may not be realized as the equilibrium of the legislative process, since future legislatures may not find it optimal to follow the program considered optimal from today's point of view.

We will first analyze a tractable, yet sound, version of this problem: we assume that the time interval between legislative deliberations is so large that the current legislature does not need to take into account the impact its decisions has on future legislatures. This is obviously unrealistic, but more realistic assumptions lead to games that are intractable. Hence, this game, which presumably is the limit of the more realistic model as the interval between legislature meetings diverges, is studied in the hope that its behavior is indicative of more realistic models.

We should observe that even with this restriction of a constant tax rate between legislatures, the desired policy from any legislature's viewpoint is to have no tax on capital asymptotically. We will not go through the details here, but this is clear from above where it was shown that any tax increase which left the capitalists' shadow value for capital initially unchanged is undesirable if the economy is in the steady state associated with a positive tax. This observation shows that the nonzero steady-state tax rates below are due to the dynamic inconsistency of the problem and not due to the restriction of constant rax rates between legislative meetings.

The feature of the game which we will study is its steady state.[4] We define the steady state of the game as that tax rate, τ^{∞}, such that if the current tax rate is τ^{∞} and we are currently in the steady state associated with

τ^{∞}, then a legislature meeting today would want to neither increase nor decrease the tax rate.

This steady state is similar in spirit to the game equilibrium growth model of Phelps and Pollak (1968). There, each generation chooses its savings rate to maximize its utility subject to a prediction of a fixed savings rate for future generations. While the individuals change from generation to generation in Phelps and Pollak and they don't here, the situations are comparable due to the dynamic consistency problems associated with a government, or ruling class, making decisions while having to take into account the behavior of individuals who have expectations concerning its future behavior.

To solve for the steady state of the game, we need to compute the change in workers' utility which results from a permanent tax increase which begins immediately, that is, when h(t) = 1. Then the change in worker utility is found to be (after using (20) and footnote 2)

$$y_{\varepsilon}^{w} = \frac{kf'}{\rho} \left[1 - \frac{\rho}{1-\tau} \frac{\rho}{\rho-\lambda} \frac{1}{\beta\mu} \left(\tau + (1-\tau) \frac{\theta_L}{\sigma}\right)\right]$$

τ^{∞} is that tax rate such that this gain is zero and is given by

$$\tau^{\infty} = 1 - \frac{1}{1 + \beta \frac{\mu}{\rho}} \tag{25}$$

We would like to have some idea as to what are reasonable values for τ^{∞}. To do so requires information about reasonable values of σ, β, and factor shares. Throughout we will assume $\theta_K = .25$; plausible changes in θ_K yield no substantial changes. Empirical anlysis of production and consumption (e.g., Weber (1970), (1975), Lucas (1969), Hansen and Singleton (1982), Ghez and Becker (1975), and Berndt and Christensen (1973)) indicate that σ is between .3 and 1.3 with the most reasonable values being between these extremes, and that β is between .5 and 15.0, with values around 1.0 being estimated by the most recent analysis of Hansen and Singleton (1982).

Table 1 gives examples of the size of τ^{∞} for various values of β and σ. We note that these values for τ^{∞} are neither trivial nor unreasonably large relative to existing capital taxation. It is straightforward to show that τ^{∞} increases as β increases in magnitude. This relation has an intuitive explanation: as β is larger, the income effects of a tax increase dominate the price effects, as seen in (15) above, leading capitalists to respond to a tax increase more by cutting consumption and less by rapid decumulation of capital. Therefore the decrease in wages is slower for larger β, making it more tempting to workers to raise the tax. The dependence of τ^{∞} on σ is more complex. For large β, τ^{∞} increases as σ increases, whereas for small β, τ^{∞} is larger as σ decreases.

In reading Table 1 (and similar tables below), keep in mind that the elasticity of substitution and factor shares cannot both be constant unless σ is unity. Since estimates of capital share less controversial than the estimates of factor substitutability, and the steady-state tax rate is relatively insensitive to reasonable variations in capital share, we choose to set $\theta_K = 0.25$ and vary σ. The precise way of reading Table 1 is "if the current steady state is that associated with a tax rate less than the τ^{∞} of the current β and σ, and if $\theta_K = 0.25$, then a permanent unanticipated increase in the capital income tax is desirable for the workers". We are implicitly assuming that this political game will converge.

Ideally, we would like to solve for the steady state of the game where the period between successive legislatures is of a more realistic duration. Reinganum and Stokey (1981) is an example in industrial organization theory where this "period of commitment" has substantial impact on the equilibrium. At this time, there is no useful practical solution to the game where the current legislature takes into account the reactions of future legislatures, with such reactions being rational. However, since the duration of a legislature's influence is certainly an important factor, and we want some idea of its impact, we next assume that the legislature meets every T periods to set a tax rate and it makes the (unsound) assumption of zero conjectural variation in the future legislative decisions: that is, current legislatures' expectations of tax decisions by later legislatures are insensitive to the tax policy the current legislature chooses. This zero reaction may nearly be rational. If β is small, simple phase diagram manipulations show that a small change in the tax rate for the next T periods will only slightly change the capital stock at the time of the next legislative meeting if it is expected that that next legislature will not react, which in turn is a reasonable expectation if the next legislature's policy is a continuous function of the capital stock existing at the time of its deliberation. We therefore conjecture that this formulation is a reasonable approximation. Even if future legislatures do react and the equilibrium is stable, those future reactions would decay in response to perturbations away from equilibrium, in which case T could be viewed as an approximation to how persistent the effects of a tax change are. In such a case, one could interpret our results below as indicating what the steady-state equilibrium tax rate is when the equilibrium duration of perturbations is effectively T.

Again we examine the steady state of this game. Define τ_T^∞ to be that tax rate such that if the current legislature finds itself meeting when the capital stock is at the steady-state level corresponding to τ_T^∞, then τ_T^∞ will be chosen by that legislature to be the constant tax rate for the next T periods, where it assumes that future legislatures will choose τ_T^∞ no matter what tax rate is imposed currently. It follows from (18) that

(26) $$\tau_T^\infty = X_T^\infty/(1 + X_T^\infty)$$

where

(27) $$X_T^\infty = \frac{[H(\mu)/H(\rho)](\mu\beta/\rho + 1)}{1 + \beta - [H(\mu)/H(\rho)](\mu\beta/\rho + 1)} \; \frac{\theta_L}{\sigma}$$

Table 2 gives τ_T^∞ for various values of σ, β, and T. The dependence of τ_T^∞ on T is particularly interesting. We see that if $\beta < 1$, τ_T^∞ decreases as T increases, whereas τ_T^∞ increases in T if $\beta > 1$. This is due to the initial response of capitalists to an unanticipated tax increase. If $\beta < 1$, they increase their consumption if τ is increased for T periods with the resulting decumulation being greater as T increases. Thus, for large T, the legislature faces more decumulation in response to a tax increase for T periods than it would if T were small, reducing the incentive to increase the redistributive tax. However, if $\beta > 1$, capitalists reduce their consumption in response to a tax increase, such a decrease being larger as T is greater. Here there is greater incentive to raise τ as T increases, since capitalists decumulate less as T increases.

The final thing to note about Tables 1 and 2 is that the tax rates for reasonable values of σ, β, and T are similar to the current tax rates on capital income. Only in the extreme cases of $\sigma = .3$ and $\beta = 10.0$ and 5.0 do we find low values of capital taxation, and otherwise the equilibrium tax rates are largely in the 40% to 60% range. While not proving the validity of our model, it does show that the predictions which arise are not unrealistic.

(iv) Equilibrium Redistribution in More Sophisticated Political Models

In the political model analyzed above, we concentrated on an equilibrium concept which implicitly assumed away the possibility of reputation effects. To illustrate the importance of that fact let us consider a much more sophisticated political equilibrium.

Suppose that the legislature meets frequently and that each legislature knows that if it pursued any policy other than that consistent with the intertemporal optimum given in Theorem 2, then all future governments would also pursue their noncooperative equilibrium policies instead of the intertemporally optimal policies. Since the intertemporal optimum is generally dynamically inconsistent, those noncooperative equilibrium policies will be different leading to an intertemporally inferior outcome. The belief by the current legislature that this inferior outcome will occur is rational if all future legislatures also believe it, since once this is a common belief, the rational choice for all legislatures would be to pursue those noncooperative equilibrium policies if the current legislature deviates from the optimum. Hence, no legislature of sufficiently small duration would trigger the fall into the inferior dynamically consistent noncooperative equilibrium policies since it cares about the future as well as the present. This demonstrates that with the appropriate beliefs, the intertemporal optimum is an equilibrium.

This type of argument has been used several times before. Good examples include McMillan(1979), where this argument is used to "solve" the free-rider problem, Stokey (1981), where it is shown that a durable goods monoplist may achieve its intertemporal optimum, and Green (1980), where we find that noncooperative games may achieve collusive outcomes. Here this argument leads us to conclude that the intertemporal optimum may be achieved even if no one legislature has the power to set policy for all future time _if the structure of beliefs and expectations are just right_. Of course, once we allow these types of equilibria and strategies we also have the problem of a continuum of multiple equilibria, as exemplifed in the "Folk Theorem" of repeated game theory. In this context, any time path of tax rates is an equilibrium as long as the payoff from deviation is smaller than the loss due to triggering the descent to an inferior dynamically consistent equilibrium.

The comments show that the precise results we obtain are due to the restrictions we put on the legislatures' strategy spaces--they are allowed to consider only the current capital stock and how their decisions affect its future evolution, and are not allowed to condition their decision on whether previous legislatures obeyed the policy prescribed by the optimum. We also are not allowing agents to observe calendar time since that would allow them to partially infer past policies from the current state of the world.

These restrictions were imposed in the belief that this is an interesting case, free of the subtle and fragile nature of reputation-based equilibria. This case gives us some idea about how inferior the results may be when reputation forces are not present.

4. Case II: All Agents Own Capital

We next examine the case where all agents participate in the capital market and differ only in their endowments of capital. In particular, all agents supply one unit of labor. Minor adjustments in the above equilibrium analysis show that the equilibrium equations will aggregate to

$$\begin{aligned} \dot{C} &= -C(\rho - (1-\tau)f'(k))/\beta \\ \dot{k} &= f(k) - C \end{aligned} \tag{28}$$

where C is now mean consumption of all agents and k is the capital-labor ratio. Again we suppose that we are in the steady state associated with some constant rate of taxation and that agents will have to evaluate the desirability of various tax changes. The perturbation analysis conducted above when applied here yields

$$K_{\varepsilon}(\rho) = \frac{C}{\beta} f' \frac{H(\rho) - H(\mu)}{(\rho-\lambda)(\rho-\mu)} \tag{29}$$

where $\varepsilon h(t)$ again is the increase in the tax rate at t, and where now λ and μ are the negative and positive eigenvalues of the linearization of (28) around its steady state. One feature of this system which differs from the case of no saving by workers is that $\rho < \mu$ always[5]; hence $H(\rho)$ exceeds $H(\mu)$ in magnitude whenever $h(t)$ is of one sign. More intuitively, this fact says that the rate of divergence away from the steady state along any divergent path exceeds the rate of discount. If taxes are increased, then $H(\rho) > H(\mu)$. Since $\lambda < 0 < \rho < \mu$, (29) then shows that $K_{\varepsilon}(\rho)$ would be negative, that is, a tax increase causes a decline in the discounted value of the capital stock through time.

We assume that the revenues are lump-sum rebated uniformly to all agents. The discounted value of the change in wages plus the change in rebate is equal to

$$kf'H(\rho) + K_{\varepsilon}(\rho)f'(\tau + (1-\tau)\theta_L/\sigma)$$

However, now an agent possessing k^W units of capital loses $H(\rho)f'\ k^W$ on the capital he holds. The net gain is therefore equal to

$$y_{\varepsilon}^W = (k - k^W)f'(H(\rho) + (1-\tau)\frac{K_{\varepsilon}(\rho)}{k}\frac{\theta_L}{\sigma}) + \tau K_{\varepsilon}(\rho)f' \tag{30}$$

Certain features of this model are seen directly. First, if an agent's holding of capital, k^W, equals the mean holding of capital, k, then he does not desire any incremental redistributive tax on capital income. Second, since $\mu > \rho$, no worker, even those owning no capital, will want a tax increase in the distant future if the economy is currently in the untaxed steady state.

Next we examine the redistributive potential of unanticipated capital income taxation here, just as we did above, by computing τ_T^∞ for various values of T and from the viewpoint of various agents distinguished by their holdings of capital. Table 3 displays values of τ_T^∞ for various values of σ, β, T and α, where α is k^w/k, i.e., the capital holdings of the "pivotal voter" expressed as a fraction of the average capital holding. If $\tau < \tau_T^\infty$ and α is k^w/k for the pivotal voter, then he will want an unanticipated increase in the capital income tax for T periods. We focus on a pivotal voter here since our legislature has a one-dimensional decision, changes in τ do not affect the ranking of individuals in the distribution of wealth, and, locally, there is a critical level of wealth such that an individual wants a tax increase if and only if he has less capital than that level. To fully justify the pivotal analysis conducted here we would have to solve global problems which are beyond the scope of this paper. We are therefore examining only one necessary condition for a long-run equilibrium, that there is not sufficient political power to cause small changes in policy. If the tax rate were determined by majority rule, then the pivotal voter would be one who held the median amount of capital. However, if more than a majority was needed to raise the tax level or if voting requirements--such as the holding of a minimal amount of property--were enforced, then the pivotal voter may be some agent who held more capital than the median holding. We assume throughout that the pivotal voter holds less capital than the average, since this is a realistic focus.

From the values of τ_T^∞ in Table 3 we see that τ_T^∞ is less as the pivotal voter holds more capital, but still may be substantial even when the pivotal voter holds 90 percent of the mean level of capital. τ_T^∞ declines as T increases, as β decreases, and as σ increases. The dependence on T is unambiguous here since $\mu > \rho$ implies that temporary tax increases always lead to an increase in consumption, with the decumulation being greater for large T. Again, the equilibrium tax rates are realistic. The dependence on α is intuitive, with taxes being greater when the pivotal voter holds relatively less capital. Note the sensitivity of the steady-state tax rate to this parameter of income distribution, indicating that small changes in political structure may lead to substantial changes in redistributive taxation. Table 3 also shows that the equilibrium tax rate is small only when the pivotal voter holds almost as much capital as the average, utility is not too concave, and when the legislature is long-lived.

Next, we consider the tax program optimal for the workers, each of whom holds k^w units of capital. Assume that k^c is the holding of each capitalist, where we assume $k^w < k^c$. Also, let $k = k^w + k^c$ be the aggregate capital holdings and c^w and c^c be worker and capitalist consumption, respectively. The workers' government problem is to choose $\bar{r}(t)$ to finance lump-sum transfers to workers and government consumption, G, so as to maximize worker welfare subject to the constraints that the economy is in equilibrium and that at each moment government revenues cover government consumption of G per worker and the net transfers (possibly negative) to workers. Formally, this problem is:

$$\max_{\bar{r},x,y} \int_0^\infty e^{-\rho t} u(c^w) dt$$

$$\text{s.t.} \quad \dot{k}^w = f(k) - \bar{r}k^c - G - c^w$$

$$\dot{c}^w = c^w(\rho - \bar{r})/\beta$$

$$\dot{k}^c = \bar{r}k^c - c^c$$

$$\dot{c}^c = c^c(\rho - \bar{r})/\beta$$

The current-value Hamiltonian for this problem is

$$(31)\qquad H = u(c^w) + p_1(f - \bar{r}K^w - c^w - G) + p_2 x + p_3(\bar{r}K^c - c^c) + p_4 y + p_5(x - c^w(\rho - \bar{r})/\beta) + p_6(y - c^c(\rho - \bar{r})/\beta)$$

By the Pontryagin Maximum principle, the laws of motion for the optimal program are characterized by

$$(32)\qquad \begin{aligned} \rho p_1 - \dot{p}_1 &= p_1 f' \\ \rho p_2 - \dot{p}_2 &= u'(c^w) - p_1 - p_5(\rho - \bar{r})/\beta \\ \rho p_3 - \dot{p}_3 &= (p_3 - p_1)\bar{r} \\ \rho p_4 - \dot{p}_4 &= -p_3 - p_6(\rho - \bar{r})/\beta \\ 0 &= -p_1 k^w + p_3 k^c + p_5 c^w \\ 0 &= p_2 + p_5 \\ 0 &= p_4 + p_6 \end{aligned}$$

Steady-state again implies $\bar{r} = f'$, hence no tax on capital is desired in the limit if the optimal program is stable.

Theorem 3: If workers and capitalists have the same constant rate of time preference, and both have access to perfect capital markets, then the optimal redistributive tax on capital for the workers is asymptotically zero if it converges.

Theorem 3 should be compared to the analysis of Pestieau and Possen (1978). They analyze a model where a social planner has an instantaneous utility function over average consumption and the distribution of income, which is discounted at a constant rate. They assume however that private investment is described by a constant savings rate. They find that with labor taxation, capital taxation and bonds, there will be no income distribution asymptotically if the marginal value of income equality is positive in the steady state. It is straightforward to show that Theorem 3 remains true even if bonds are allowed; also, asymptotic income equality will not generally be assured.[6] If bonds are not available to the planner, then Pestieau and Possen show that some redistribution will generally be desired asymptotically, whereas Theorem 3 contradicts this. Again we see that the character of optimal redistribution changes substantially when the analysis is conducted in an intertemporal maximizing framework.

A paper which reaches a similar conclusion is Brito (1981). He shows in an intergenerational model that the optimal tax program will eventually not tax capital life-cycle capital but will eliminate bequests, independent of how the planner values different generations. Brito adopts a utilitarian social welfare function when valuing the utility of individuals within a generation. Here we get the no capital tax result when we have an arbitrary <u>intragenerational</u> social welfare function. Together, these papers indicate the generality of the asymptotic inefficiency of capital income taxation.

5. Pareto-Efficient Taxation

We have concentrated so far on the case of constant rates of time preference and inelastically supplied labor. This was clearly appropriate for our comparative dynamics and our political equilibrium analysis. However, the results of the optimal taxation analyses may look special and sensitive to these specifications. Therefore we next turn to the problem of Pareto-efficient taxation with two classes of infinitely-lived agents with elastic labor supplies and heterogeneous flexible time preferences. We demonstrate that the results of Theorems 2 and 3 are due to neither the inelastic labor supply nor the infinitely elastic long-run supply of capital. We find that in any convergent Pareto-efficient tax program, the tax rate on capital income is asymptotically zero, showing that the capital income tax has no role for either redistributive or efficiency purposes in the long run.

We assume that an individual in class i, i=1,2, has a utility functional of the Uzawa form:

$$U = \int_0^\infty e^{-R^i} u^i(c^i,\ell^i)dt$$

where

$$\dot{R}^i = \phi^i(c^i,\ell^i)$$

is the instantaneous rate of time preference as a function of representative class i consumption, c^i, and class i labor, ℓ^i. Suppose that the representative class i agent holds $k^i(t)$ units of capital at t. Then he solves the following problem (for convenience, we drop the i superscripts at this point):

$$\max_{c,\ell} \int_0^\infty e^{-R}u(c,\ell)$$

$$\text{s.t.} \quad \dot{k} = \bar{r}k - c + \bar{w}\ell$$

$$\dot{R} = \phi(c,\ell)$$

The present-value Hamiltonian for the problem is

$$H(k,R,c,\bar{r},q_1,q_2) = e^{-R}u(c,\ell) + q_1(\bar{r}k + w\ell - c) + q_2\phi(c,\ell) \tag{33}$$

where q_1 and q_2 are the costates for the state variables, R and k, respectively. The equations of motion for the optimal path are

(34)
$$\dot{q}_1 = -q_1\bar{r}$$
$$\dot{q}_2 = e^{-R}u(c,\ell)$$
$$0 = e^{-R}u_c - q_1 + q_2\phi_c$$
$$0 = e^{-R}u_\ell + q_1\bar{w} + q_2\phi_\ell$$

We can transform this system into current value terms, where Q_1 and Q_2 are the current-value costates:

(35) $$Q_i = q_i e^R, \quad i=1,2$$

Using (35), we may rewrite (34) in terms of the current values:

(36a) $$\dot{Q}_1 = Q_1(\phi(c,\ell) - \bar{r})$$

(36b) $$\dot{Q}_2 = u(c,\ell) + Q_2\phi(c,\ell)$$

(36c) $$0 = u_c - Q_1 + Q_2\phi_c$$

(36d) $$0 = u_\ell + Q_1\bar{w} + Q_2\phi_\ell$$

Note that in the steady state of this system, $\phi = \bar{r}$, showing that the steady state net return to capital may vary with steady state consumption and labor without causing capital holdings to diverge. Hence, the long-run factor prices are not fixed, and long-run supply curves of both factors may have finite and nonzero elasticities. (See Uzawa for a more complete analysis of such utility functionals.)

We assume that the government has a social welfare function which is a positively weighted average of individual utilities. We also assume that wage taxes may be imposed, and that a uniform lump-sum rebate is allowed, but no lump-sum taxes. Let $\bar{w}$ be the after-tax wage and S the uniform lump-sum rebate. The government's problem is then to choose $\bar{r}(t)$, $S(t)$ and $\bar{w}(t)$ so as to maximize social welfare, subject to the constraint that the economy is in equilibrium, and that current revenues cover current rebates and current government consumption, G. (The addition of a bond market changes no asymptotic result and is assumed away to eliminate the possibility that our results hold because no revenue is being raised asymptotically.) That problem is:

$$\max_{S,\bar{r},\bar{w},\{Q_j^i\}_{i=1,j=1}^{2,\ \ 2}} \alpha \int_0^\infty e^{-R^1} u^1(c^1,\ell^1)dt + (1-\alpha) \int_0^\infty e^{-R^2} u^2(c^2,\ell^2)dt$$

$$\begin{aligned}
\text{s.t.}\quad \dot{k}^i &= \bar{r}k^i + \bar{w}\,\ell^i - c^i + S \\
\dot{R}^i &= \phi^i(c^i,\ell^i) \\
\dot{Q}_1^i &= Q_1^i(\phi^i(c^i,\ell^i) - \bar{r}) \\
\dot{Q}_2^i &= u^i(c^i,\ell^i) \\
0 &= u_c^i - Q_1^i + Q_2^i\phi_c^i \\
0 &= u_\ell^i - Q_1^i w + Q_2^i\phi_\ell^i \\
0 &= (f' - \bar{r})k + (f - \frac{kf'}{\ell^1 + \ell^2} - \bar{w})(\ell^1 + \ell^2) - S \\
S &\geq 0
\end{aligned}$$

where i=1,2 and α is between 0 and 1.

The equations of motion for the optimal problem include

$$(37) \qquad \dot{\lambda}_i = -\bar{r}\lambda_i + (f' - \bar{r})\mu$$

where λ_i is the costate of k^i, i=1,2, and μ is the shadow price of the balanced budget constraint, hence nonzero.

Define the current values of λ_1, λ_2, μ:

$$\Lambda_i = \lambda_i e^R;\ i=1,2; \qquad M=\mu e^R$$

Then the equations of motion in (37) can be expressed

$$(38) \qquad \dot{\Lambda}_i = (\phi^i - \bar{r})\Lambda_i + (f' - \bar{r})M$$

In a steady state of the optimal problem,

(39) $$\dot{Q}_1^i = 0 \Rightarrow \bar{r} = \phi^i(c^i, \ell^i),\ i=1,2$$

(40) $$\dot{\Lambda}_i = 0 \Rightarrow (f' - \bar{r})M = 0$$

which implies that $\bar{r} = f'$ since M is positive. This demonstrates Theorem 4:

Theorem 4: If the optimal program converges to a steady state, then the tax rate on capital is zero in the long run.

Theorem 4 shows that redistributive capital taxation is not desired in anyone's optimal program, independent of long-run factor supply responses, as long as the optimal program converges to a steady state of consumption, leisure and assets for all. The assumption of equal discount rates asymptotically is appropriate in this context since it is a necessary condition for both groups to hold capital in the long run. Otherwise, one group would face a net return on investment different from its discount rate causing either investment or decumulation.

Chamley (1980) came to the same conclusion for the special case of ϕ being constant, i.e., an infinitely elastic supply curve of capital, and of a single class, thereby examining efficient taxation only. We see that the zero long-run interest tax result is quite robust, even when we allow the possibility of redistribution, heterogeneity in tastes, and arbitrary long-run elasticities of supply for both factors. Given the arbitrary nature of the u^i and ϕ^i functions, it is clear that the asymptotic distribution of wealth and income could be highly unequal. Yet the steady state of the optimal tax program involves no capital income tax. This forcefully shows that capital income taxation is useless as a redistributive tool in the long run for this broad class of utility functions, under the assumption of stability.

This result for infinitely-lived agents model should also be compared to comparable analysis for the two-period life overlapping generation model. Atkinson and Sandmo (1980), among others, have shown (assuming global asymptotic stability) that in the steady state of the optimal policy, the tax on interest depends crucially on cross-elasticities among consumption goods and leisure. Here we have shown that the interest tax should be zero without imposing the usual separability assumptions on preferences. These models differ in the extent of intergenerational bequest motives and the amount of intertemporal aggregation. Examination of continuous time overlapping generation models is needed to indicate which approach better approximates the real world of finite lives and frequent transactions.

5. Conclusions

In this paper we have studied the redistributive potential of capital income taxation in a model where investment behavior is based on the maximization of some intertemporal utility function. First, if the economy will converge to a steady state where all agents have a common rate of time preference, no agent will asymptotically choose redistributive capital income taxation, independent of his initial and asymptotic level of wealth. This holds even when agents have a non-additive utility functional where the long-run supply curve of capital may not be perfectly elastic.

The second basic issue examined is the determination of capital income taxation in a simple political model. We studied a simple model of legislative decision-making where the current legislature cannot bind future legislatures' tax decisions, and examined the long-run nature equilibrium of the resulting game. We found that a significant rate of capital income taxation will generally result as long as the pivotal voter holds less capital than the social average and the period of commitment is not too long.

Furthermore, the levels predicted are within the range of observed tax rates when we make reasonable assumptions concerning tastes and technology.

We also showed that these results are independent of the ability of workers to participate in the capital markets.

In summary, we have seen that redistribution through capital income taxation may be ineffective in the long run in a utility maximizing model of capital accumulation, but likely to persist at substantial levels due to the limited ability of agents to commit themselves to future political decisions.

Table 1

β \ σ	.3	.5	.8	1.0	1.3
.5	.45	.43	.41	.40	.40
1.0	.50	.50	.50	.50	.50
2.0	.56	.58	.59	.60	.61
5.0	.58	.64	.70	.73	.75
10.0	.60	.68	.77	.81	.84

The entry corresponding to each σ-β pair is τ^{∞} in equation (25). The calculations assume that $\theta_K = 0.25$.

Table 2

σ / β	.5	.8	1.0	1.3
.50	.58 .56 .51 .46	.56 .54 .49 .45	.56 .53 .49 .44	.55 .53 .48 .43
1.00*	.50	.50	.50	.50
2.0	.40 .45 .45 .49	.43 .47 .47 .52	.44 .47 .50 .54	.45 .48 .50 .54
5.0	.27 .29 .34 .41	.33 .35 .40 .48	.36 .38 .44 .52	.40 .42 .47 .56
10.0	.18 .20 .24 .31	.26 .29 .34 .43	.31 .33 .39 .49	.36 .38 .45 .55

The column of numbers associated with each σ–β pair is τ_T^{∞} from equations (26) and (27) for T = 16, 40, 100, 200, respectively. Again, $\theta_K = 0.25$.

*When $\beta = 1.00$, $\tau_T^{\infty} = .5$ for all T and σ.

Table 3

β	σ / α	.4 / 0.0	.5	.9	.65 / 0.0	.5	.9	1.0 / 0.0	.5	.9
.5		.72	.62	.33	.69	.59	.30	.67	.55	.27
		.61	.49	.21	.58	.45	.18	.55	.42	.16
		.42	.29	.09	.39	.26	.08	.36	.24	.07
		.35	.20	.05	.28	.17	.04	.25	.15	.04
1.0		.77	.68	.40	.74	.65	.37	.72	.62	.34
		.67	.57	.28	.65	.53	.25	.62	.51	.23
		.51	.37	.13	.48	.34	.11	.45	.31	.10
		.40	.27	.07	.36	.23	.06	.33	.21	.05
3.0		.83	.76	.52	.81	.74	.49	.79	.72	.46
		.76	.67	.40	.74	.65	.37	.72	.63	.35
		.64	.51	.22	.62	.49	.20	.60	.47	.19
		.55	.41	.13	.53	.38	.12	.51	.35	.11

The column of numbers corresponding to each σ-β-α triple is τ_T^∞ in the model where all have access to the capital market for T = 16, 32, 100, and ∞, respectively. Again, $\theta_K = .25$.

ENDNOTES

[1]The Laplace transform of $g(t)$ is $G(s)$ where $G(s) = \int_0^\infty e^{-st} g(t)dt$.

[2]Straightforward calculations also show that the following useful identities hold for the model of section 3:

$$\Delta \equiv (\rho-\mu)(\rho-\lambda) = \frac{\rho^2\theta_L}{\sigma}(1 - \beta^{-1})$$

$$\frac{f'^2(1-\tau)}{\Delta\beta} = \frac{-\sigma}{(1-\tau)\theta_L(1 - \beta)}$$

[3]For details, see Judd (1982b).

[4]Brock and Turnovsky (undated) have also examined related questions in a similar manner.

[5]The eigenvalues in this model are

$$\mu, \lambda = \frac{\rho}{2(1-\tau_K)}\left(1 \pm \sqrt{1 + 4(1-\tau_K)\theta_L/(\sigma\theta_K\beta)}\right)$$

In the interest of keeping the notation clean, we have defined μ and λ twice. This is excusable here because it will always be clear from context which μ (positive eigenvalue) and which λ (negative eigenvalue) is meant.

[6]Since this point will be made forcefully in the more general model of section 6, we will not elaborate on it here.

REFERENCES

Atkinson, A. B.and A. Sandmo, "Welfare Implications of the Taxation of Savings", Economic Journal, 90 (Sept. 1980).

Benveniste, L. and J. Scheinkman, "Duality Theory for Dynamic Optimization Models: The Continuous Time Case," Journal of Economic Theory (1982).

Berndt, E. R., and Christensen, L. R., "The Translog Function and the Substitution of Equipment, Structures and Labor in U.S. Manufacturing, 1929-1968", Journal of Econometrics, (1973).

Bernheim, B. D., "A Note on Dynamic Tax Incidence," Quarterly Journal of Economics, Vol. 96 (1982), 705-723.

Boadway, R. "Long-Run Tax Incidence: A Comparative Dynamic Approach," Review of Economic Studies, Vol. 46 (1979), 505-511.

Brito, D. L., "Optimal Taxation, the Distribution of Wealth and the Ownership of Capital," Murphy Institute Discussion Paper no. 81/2, (1981).

Brock, W. and S. Turnovsky, "The Analysis of Macroeconomic Policies in Perfect Foresight Equilibrium," International Economic Review (1981).

———, unpublished notes.

Chamley, C., "Optimal Intertemporal Taxation and the Public Debt", Cowles Foundation Discussion Paper No. 554, (1980).

Feldstein, M. S., "Incidence of a Capital Income Tax in a Growing Economy with Variable Savings Rate," Review of Economic Studies (1974), 505-513.

Ghez, G. and G. S. Becker, The Allocation of Time and Goods over the Life Cycle, (1975).

Green, E.."Noncooperative Price Taking in Large Dynamic Markets", Journal of Economic Theory 22 (1980), 155-182.

Grieson, R. E., "The Incidence of Profit Taxes in a Neo-classical Growth Model," Journal of Public Economics, Vol. 4 (1975), 75-85

Hamada, K., "On the Optimal Transfer and Income Distribution in a Growing Economy,"Review of Economic Studies (1967), 295-299.

Hansen, L., and K. Singleton,"Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models", Econometrica 50(1982), 1269-1286.

Homma, M., "A Dynamic Analysis of the Differential Incidence of Capital and Labor Taxes in a Two-Class Economy," Journal of Public Economics, Vol. 15 (1981), 363-378.

Judd, K. L., "An Alternative to Steady State Comparisons in Perfect Foresight Models," Economics Letters, 1982a (forthcoming).

______,"Short-run Analysis of Fiscal Policy in a Simple Perfect Foresight Model", mimeo, 1982b.

Lucas, R. E., "Labor-Capital Substitution in U.S. Manufacturing", in The Taxation of Income from Capital, A. C. Harberger and M. J. Bailey (eds.), Brookings, 1969.

McMillan, "Individual Incentives in the Supply of Public Inputs", Journal of Public Economics 12 (1979), 87-98.

Oniki, H., "Comparative Dynamics (Sensitivity Analysis) in Optimal Control Theory", Journal of Economic Theory (1971).

Pestieau, P., and U. M. Possen, "Optimal Growth and Distribution Policies," Journal of Public Economics 9 (1978), 357-372.

Phelps, E. S. and R. A. Pollak, "On Second-Best National Saving and Game-Equilibrium Growth," Rewiew of Economic Studies 35 (1968), 185-199.

Reinganum, J. and N. Stokey, "Oligopoly Extraction of a Nonrenewable, Common Property Resource: The Importance of the Period of Commitment in Dynamic Games", Northwestern University, CMSEMS Discussion Paper No. 508, October, 1981.

Solow, R. M. "A Contribution to the Theory of Economic Growth," Quarterly Journal of Economics, 70 (1956), 65-94.

Stokey, N., "Rational Expectations and Durable Goods Pricing", Bell Journal of Economics, 12 (Spring 1981), 112-128.

Uzawa, H., "Time Preference, the Consumption Function and Optimum Asset Holdings", in Value, Capital and Growth: Papers in Honour of Sir John Hicks, J.N. Wolfe(ed.). Edinburgh: University Press, 1968, 485-504.

Weber, W., "The Effect of Interest Rates on Aggregate Consumption", American Economic Review (1970).

______, "Interest Rates, Inflation and Consumer Expenditures", American Economic Review (1975).