Topics in Dynamic Public Finance

John Hassler

Stockholm University and Fudan University

October 28, 2007

1 Optimal unemployment insurance (UI)

There is a large literature of optimal unemployment insurance. The basic issue is how to provide the most efficient unemployment insurance when there is a moral hazard problem. This is arising from an assumption that unemployed individuals can affect the probability they find (and accept) a job offer. However, it is costly for the worker to increase this probability, e.g., because of effort costs, reduced reservation wages or opportunity costs of time.

1.1 The semi-static approach to optimal UI

The basic idea in Baily and Chetty is to simplify the dynamic problem into a static one. This makes the model simple and tractable also when savings is allowed. An important lesson is that when savings is allowed, we can use the drop in consumption at unemployment as a measure of the welfare loss associated with unemployment. In a dynamic model, this does not work when there is no market for savings. Why? The trade-off faced by the planner is to balance the loss of welfare associated with unemployment against the negative effect on search induced by UI.

1.1.1 The simplest model following Baily

- In the first period, the individual works and chooses how much to consume of the income, normalized to unity, and how much to save.
- In the beginning of the second period, the individual becomes unemployed with probability $1 - \alpha$ and otherwise keeps his job.

- During the second period, the individual can determine how long it takes to find a job by choosing the reservation wage y_n and costly search effort c. A share $\beta = \beta(c, y_n)$ of the second period is spent working in the new job.

- While unemployed, the individual gets UI-benefits b.These are paid by taxes on workers.

- Agents have access to a market for precautionary (buffer stock) savings.

Total income in second period if laid off is therefore

$$(1-\beta)(b-c) + \beta y_n (1-\tau) \equiv y_l.$$

In first periods, individuals decide how much to save, s. Interest rate and subjective discount rate is normalized to zero. Welfare is

$$V = u(1-\tau-s) + \alpha u(1-\tau+s) + (1-\alpha)(u(y_l+s)).$$

Government budget constraint is

$$(1+\alpha+(1-\alpha)\beta y_n)t = (1-\alpha)(1-\beta)b.$$

$$\Longrightarrow b = \frac{(1+\alpha+(1-\alpha)\beta y_n)}{(1-\alpha)(1-\beta)}\tau \equiv \mu\tau$$

Denoting the *endogenous* total income by $Y \equiv 1 + \alpha + (1-\alpha)\,\beta y_n$, this implies

$$
\begin{aligned}
b &= \frac{Y}{(1-\alpha)\,(1-\beta)}\tau \\
&\equiv \mu\tau,
\end{aligned}
$$

where we note that μ is not a constant, but depends on individual choices of y_n and β and thus indirectly on taxes and benefits. Given the budget constraint and individual choices, we can therefore write $\mu = \mu(\tau)$ (provided there is a solution, which is not necessarily true for all τ.Explain!)

Note that in first best, c should be chosen to satisfy

$$(y_n + c)\,\beta_c = 1 - \beta$$

since the marginal gain in aggregate income is $(y_n + c)$ and the cost is $1-\beta$. The individual instead gains,

$$y_n\,(1-\tau) + c - b$$

so the private value of search is lower. Similarly, an increase in y_n has benefits β and costs $-(y_n + c)\,\beta_{y_n}$. While private benefits are $(1-\tau)\,\beta$ and private costs $-(y_n\,(1-\tau) + c - b)\,\beta_{y_n}$.

We can now write

$$
\begin{aligned}
V &= u\,(1-\tau-s) + \alpha u\,(1-\tau+s) + (1-\alpha)\,(u\,((1-\beta)\,(\mu\tau - c) + \beta y_n\,(1-\tau) + s)) \\
V &= V\,(c, y_n, s, \mu, \tau)
\end{aligned}
$$

The optimal UI system maximizes solves

$$\max_{\tau} V\left(c, y_n, s, \mu\left(\tau\right), \tau\right)$$

Although, c, y_n, s are affected by τ, these effects need not be taken into account since by individual optimality,

$$V_c = V_{Y_n} = V_s = 0.$$

This is the envelope theorem. Therefore, the first order condition for maximizing V by choosing τ is

$$\frac{dV}{d\tau} = V_\mu \frac{d\mu}{d\tau} + V_\tau = 0,$$

where

$$V_\mu = (1-\alpha)\, u'\left(c_u\right)(1-\beta)\,\tau$$

$$V_\tau = -u'\left(c_1\right) - au'\left(c_2\right) - (1-\alpha)\, u'\left(c_u\right)\beta y_n + (1-\alpha)\, u'\left(c_u\right)(1-\beta)\,\mu,$$

where $c_1 = 1-\tau-s$ is first period consumption, $c_2 = 1-\tau+s$ is second period consumption if the job is retained and $c_u = (1-\beta)\left(\mu\tau - c\right) + \beta y_n\left(1-\tau\right) + s$ is second period consumption if the individual lost his job.

Note that by individual savings optimization (the Euler equation)

$$u'(c_1) = au'(c_2) + (1-\alpha)\,u'(c_u)$$

$$u'(c_1) - (1-\alpha)\,u'(c_u) = au'(c_2)$$

implying

$$V_\tau = -u'(c_1) - (u'(c_1) - (1-\alpha)\,u'(c_u)) - (1-\alpha)\,u'(c_u)\,\beta y_n + (1-\alpha)\,u'(c_u)\,(1-\beta)\,\mu$$

$$= -2u'(c_1) + (1-\alpha)\,(1 - \beta y_n + (1-\beta)\,\mu)\,u'(c_u)\,.$$

Approximating

$$u'(c_1) \approx u'(c_u) + u''(c_u)\,\Delta c$$

where $\Delta c \equiv c_1 - c_u$ is the fall in consumption if becoming unemployed. The first order condition is then

$$\begin{aligned}
0 &= (1-\alpha) u'(c_u)(1-\beta)\tau \frac{d\mu}{d\tau} - 2\left(u'(c_u) + u''(c_u)\Delta c\right) \\
&+ (1-\alpha)(1-\beta y_n + (1-\beta)\mu) u'(c_u) \\
2\left(1+\frac{u''}{u'}\Delta c\right) &= (1-\alpha)(1-\beta)\tau\frac{d\mu}{d\tau} + (1-\alpha)(1-\beta y_n + (1-\beta)\mu) \\
2\left(1+\frac{u''}{u'}\Delta c\right) &= (1-\alpha)(1-\beta)\tau\frac{d\mu}{d\tau} + (1-\alpha)\left(1-\beta y_n + (1-\beta)\frac{Y}{(1-\alpha)(1-\beta)}\right) \\
2\left(1+\frac{u''}{u'}\Delta c\right) &= (1-\alpha)(1-\beta)\tau\frac{d\mu}{d\tau} + (1-\alpha)\left(1-\beta y_n + \frac{Y}{(1-\alpha)}\right) \\
2\left(1+\frac{u''}{u'}\Delta c\right) &= (1-\alpha)(1-\beta)\tau\frac{d\mu}{d\tau} + (1-\alpha)\left(1-\beta y_n + \frac{1+\alpha+(1-\alpha)\beta y_n}{(1-\alpha)}\right) \\
2\left(1+\frac{u''}{u'}\Delta c\right) &= (1-\alpha)(1-\beta)\tau\frac{d\mu}{d\tau} + 2 \\
\frac{u''}{u'}\Delta c &= (1-\alpha)(1-\beta)\tau\frac{d\mu}{d\tau} \\
\frac{u''}{u'}\frac{\Delta c}{Y} &= \frac{\tau}{\mu}\frac{d\mu}{d\tau} \\
\frac{\Delta c}{c} &= \frac{E_{\mu,t}}{-R_r} Y
\end{aligned}$$

where $E_{\mu,t}$ is the elasticity of μ with respect to taxes and R_r the relative risk aversion coefficient. Note that we should not interpret Y as the aggregate *level* of income since we have normalized the pre-unemployment income to unity. Assuming that $y_n \approx 1, Y \approx 1 + \alpha + (1-\alpha)\beta$ which is the time people work. In this simple model, this is value is overstated since no unemplyment occur in the first period. More realistically, it should be close to one.

Without moral hazard, $\frac{d\mu}{d\tau} = 0$, in which case optimality requires $\Delta c = 0$. With moral hazard, higher taxes tends to reduce μ since the tax dependency ratio falls. $\frac{\tau}{\mu}\frac{d\mu}{d\tau} = E_{\mu,t}$ is thus negative. Therefore, $\frac{\Delta c}{c} > 0$. We see that $\frac{\Delta c}{c}$ increases if $\frac{\tau}{\mu}\frac{d\mu}{d\tau}$ is large in absolute terms and falls if risk aversion is large. Baily claims that $E_{\mu,t}$ is in the order .15-.4.

This approach has been generalized by Chetty showing that we can have repeated spells of unemployment, uncertain spells of unemployment, value of leisure, private insurance and borrowing constraints. The model can therefore be extended to evaluate UI reforms. With a more dynamic model, and in particular if capital markets are imperfect, it should be noted that one needs how the whole consumption profile is affected by unemployment. The drop at entering unemployment may not be enough. Shimer and Werning (2007), shows that the *reservation wage* can be used as a summary measure of how bad unemployment is.

In any case, this the model is not suitable to analyze

1. General equilibrium effects like impacts on wages, search spillovers and job creation.

2. Interaction with other taxes-fiscal spillovers.

3. Time varying benefits.

1.2 The dynamic approach with observable savings

The seminal paper by Shavel & Weiss (1979) focuses on the optimal time profile of benefits. It is a simple infinite horizon discrete time model where the aim is to maximize utility of a representative unemployed subject to a government budget constraint. Utility is given by

$$\sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^{t}\left(u\left(c_{t}\right)-e_{t}\right)$$

where c_t is period t consumption and e_t is a privately chosen unobservable effort associated with job search. The subjective discount rate is r, which is assumed to coincide with an exogenous interest rate.

It is assumed that the individual has no access to capital markets so $c_t = b_t$ when the individual is unemployed. After regaining employment, the wage is w forever.

When the individual becomes employed he stays employed for ever for simplicity. Agents have no access to credit markets (or equivalently, savings is perfectly monitored and benefits can be made contingent on them) so the planner can perfectly control the consumption of the individual. The mortal hazard problem is that individuals can affect the probability of finding a job. As in Baily (1978), the individual controls both the search effort (here called e_t) and the reservation wage (here w_t^*).

Given an effort level e_t, the individual receives one job offer per period with an associated wage drawn from a distribution with a time invariant probability density $f\left(w_t, e_t\right)$. The probability of finding an acceptable job in period t is thus

$$p\left(w_{t}^{*}, e_{t}\right)=\int_{w_{t}^{*}}^{\infty} f\left(w_{t}, e_{t}\right) d w_{t}$$

with

$$p_w\left(w_t^*, e_t\right) = -f\left(w_t, e_t\right) \leq 0 \text{ and}$$

$$p_e\left(w_t^*, e_t\right) > 0$$

where the latter is by assumption.

Let E_t be the expected utility of an unemployed individual that choose optimally a sequence $\left\{e_{t+s}, w_{t+s}^*\right\}_{s=0}^{\infty}$. Define

$$u_t = \tilde{u}\left(w_t^*, e_t\right) \equiv \frac{1+r}{r} \int_{w_t^*}^{\infty} u\left(w_t\right) \frac{f\left(w_t, e_t\right)}{p\left(w_t^*, e_t\right)} dw_t$$

This is the expected utility from next period, *conditional* on finding a job this period, which starts next period. We note that

$$\tilde{u}_w\left(w_t^*, e_t\right) \geq 0$$

$$\tilde{u}_e\left(w_t^*, e_t\right) \geq 0.$$

The first inequality follows from the fact that *conditional* on finding a job, wages are higher for higher reservation wages. The second inequality is by assumption, higher search effort leads to no worse acceptable job offers.

E_t satisfies the standard Bellman equation

$$E_t = \max_{e_t, w_t^*} u\left(b_t\right) - e_t + \frac{1}{1+r}\left(p\left(w_t^*, e_t\right) \tilde{u}\left(w_t^*, e_t\right) + \left(1 - p\left(w_t^*, e_t\right)\right) E_{t+1}\right)$$

The first-order conditions are

$$e_t; \frac{1}{1+r}\left(p_e\left(w_t^*, e_t\right)\left(\tilde{u}\left(w_t^*, e_t\right)-E_{t+1}\right)+p\left(w_t^*, e_t\right)\tilde{u}_e\left(w_t^*, e_t\right)\right)=1$$

$$w_t^*; -p_w\left(w_t^*, e_t\right)\left(\tilde{u}\left(w_t^*, e_t\right)-E_{t+1}\right)=p\left(w_t^*, e_t\right)\tilde{u}_w\left(w_t^*, e_t\right).$$

In the first equation, the LHS is the marginal benefit of higher search effort, coming from a higher probability of finding a job and better jobs if found. These balances the cost which is 1. In the second equation, the LHS is the marginal cost of higher reservation wages, coming from a lower probability of finding a job. The RHS is the gain, coming from better jobs if accepted.

By the envelope theorem

$$\frac{dE_t}{dE_{t+1}}=\frac{\partial E_t}{\partial E_{t+1}}=\frac{1-p\left(w_t^*, e_t\right)}{1+r}$$

Now, anything that reduce E_{t+1} will reduce $1-p\left(w_t^*, e_t\right)$, i.e., make hiring more likely. To see this, note that if E_{t+1} falls.

$$p_e\left(w_t^*, e_t\right)\left(\tilde{u}\left(w_t^*, e_t\right)-E_{t+1}\right)+p\left(w_t^*, e_t\right)\tilde{u}_e\left(w_t^*, e_t\right), \text{ and}$$

$$-p_w\left(w_t^*, e_t\right)\left(u\left(w_t^*, e_t\right)-E_{t+1}\right)$$

both becomes larger if choices are unchanged. In words, the marginal benefit of searching higher and the marginal cost of setting higher reservation wages both increase. Thus, a reduction in E_{t+1} increase search effort and reduce the reservation wage increasing p.

Now, we can show that benefits should have a decreasing profile.

Proof:

Suppose contrary that $b_t = b_{t+1}$. Then consider an infinitessimal increase in b_t financed by an actuarially fair reduction in b_{t+1}, that is

$$db_t = -\frac{1-p}{1+r} db_{t+1} > 0$$

where $p(w_t^*, e_t)$ is calculated at the initial (constant) benefit levels. The direct effect on felicitity levels (period utilities) is

$$\begin{aligned} & u'(b_t)\, db_t + \frac{1-p}{1+r} u'(b_{t+1})\, db_{t+1} \\ & -u'(b_t) \frac{1-p}{1+r} db_{t+1} + \frac{1-p}{1+r} u'(b_{t+1})\, db_{t+1} \\ & = 0 \end{aligned}$$

since $u'(b_t) = u'(b_{t+1})$ .By the envelope theorem, we need not take into account changes in endogenous variables when calculating welfare. Therefore, E_t is unchanged. Since $u(b_t)$ has increased, E_{t+1} must have fallen. When calculating the budgetary effects we need to into account the endogenous changes on p.

Let

$$B_t = b_t + \frac{1-p}{1+r} b_{t+1}$$

Then,

$$\begin{aligned} dB_t &= db_t + \frac{1-p}{1+r} db_{t+1} - \frac{dp}{1+r} b_{t+1} \\ &= -\frac{dp}{1+r} b_{t+1} \end{aligned}$$

Since E_{t+1} has fallen, $dp > 0$. Thus $dB_t < 0$. I.e., the cost of providing utility E_t has fallen. Equivalently, the insurance is more efficient than the starting point $b_t = b_{t+1}$.

1.2.1 Extensions

Hopenhayn and Nicolini extend the model by Shavel & Weiss in an important dimension – it enriches the policy space of the government by allowing taxation of workers to be contingent on their unemployment history. It is shown that the government should use this extra way of ”punishing” unemployment. The intuition is that relative to the first best, which is a constant unemployment benefit, the government must ”punish” unemployment. Doing this by only reducing unemployment benefits is suboptimal, by spreading the punishment of unsuccessful search over the entire future of the individual, a more efficient insurance can be achieved. I.e., lower cost of providing a given utility level. It is shown that this may be quantitatively important. Another contribution is to show that the problem can be formulated in a recursive way with the *promised utility* as state variable.

Using H&N's notation, we assume that individuals can choose an unobservable effort level a_t that positively effect the hiring probability. In H&N 1997, it is assumed that $p(a_t)$ is an concave and increasing function and hiring is an absorbing state with a wage w forever. In H&N 2005, it is instead assumed that spells are repeated, with an exogenous separation probability s and

$$p(a) = \begin{cases} p \text{ if } a = 1 \\ 0 \text{ otherwise} \end{cases}$$

which is the assumption we make here.

The individual has a utility function

$$E \sum_{t=0}^{\infty} \left(\frac{1}{1+r} \right)^{t} (u(c_t) - a_t).$$

Let $\theta_t \in \{0, 1\}$ be the employment status of the individual in period t, where $\theta_t = 1$ represents employment. Let $\theta^t = (\theta_0, \theta_1, ...\theta_t)$ be the history of the agent up until period t. The history of a person that is unemployed in period t is therefore $\theta^{t-1} \times 0 = (\theta_0, \theta_1, ...\theta_t, 0) \equiv \theta^t_u$, and similarly, $\theta^{t-1} \times 1 \equiv \theta^t_e$).

An allocation is now defined as a rule that assigns consumption and effort as a function of θ^t at every point in time and for every possible history, $c_t = c(\theta^t)$. We focus on allocations where $a_t = 1$. Individuals must be induced to volontarily choose $a_t = 1$. Allocations that satisfies this are called incentive compatible allocations.

Given an allocation we can compute the expected discouted utililty at every point in time for every possible history, $V_t = V(\theta^t)$. The problem is now to choose the allocation that minimizes the cost of giving some fixed initial utility level to the representative individual. This problem can be written in a recursive way. In period zero, the planner gives a consumption level c_0, prescribes an effort level a_0 (=1) and promised continuation utilities V^e_1 and V^u_1. The problem of the planner in period zero is to minimize costs of providing a given expected utility level V_0 subject to the incentive constraint the individual voluntarily chooses a_0. The problem is recursive and at any node, costs of providing promised utilities are minimized given incentive constraints

The problem of the unemployed individual is also recursive. – as unemployed, maximized utility is (the agent only controls a_t)

$$V\left(\theta_u^t\right) = u\left(c_t\right) - 1 + \frac{1}{1+r}\left(pV\left(\theta_u^t \times 1\right) + \left(1-p\right)V\left(\theta_u^t \times 0\right)\right)$$

with the incentive constraint

$$\frac{1}{1+r}p\left(V\left(\theta_e^{t+1}\right) - V\left(\theta_u^{t+1}\right)\right) \geq 1.$$

Define $W\left(V_t\right)$ as the minimum cost for the planner to provide a given amount of utility V_t to an employed. Similarly, let $C\left(V_t\right)$ denote the minimal cost of providing utility V to an unemployed (are these function changing over time?). W satisfies

$$\begin{aligned} W\left(V_t\right) &= \min_{c_t, V_{t+1}^e, V_{t+1}^u} c_t - w + \frac{1}{1+r}\left(\left(1-s\right)W\left(V_{t+1}^e\right) + sC\left(V_{t+1}^u\right)\right) \\ s.t. V_t &= u\left(c_t\right) + \frac{1}{1+r}\left(\left(1-s\right)V_{t+1}^e + sV_{t+1}^u\right), \end{aligned}$$

where$V_t = V\left(\theta_e^t\right)$, $c_t = c\left(\theta_e^t\right)$, $V_{t+1}^e = V\left(\theta_e^t \times 1\right)$ and $V_{t+1}^u = V\left(\theta_e^t \times 0\right)$.

The constraint can be called promise keeping constraint and has a Lagrange multiplier δ_t^e.

C satisfies

$$C\left(V_t\right)=\min_{c_t,V^e_{t+1},V^u_{t+1}} c_t+\frac{1}{1+r}\left(pW\left(V^e_{t+1}\right)+\left(1-p\right)C\left(V^u_{t+1}\right)\right)$$
$$\text{s.t. } \frac{1}{1+r}p\left(V^e_{t+1}-V^u_{t+1}\right)\geq 1,$$
$$V_t=u\left(c_t\right)-1+\frac{1}{1+r}\left(pV^e_{t+1}+\left(1-p\right)V^u_{t+1}\right).$$

where $V_t=V\left(\theta_u^t\right)$, $c_t=c\left(\theta_u^t\right)$, $V^e_{t+1}=V\left(\theta_u^t\times 1\right)$ and $V^u_{t+1}=V\left(\theta_u^t\times 0\right)$.

The first constraint is the incentive constraint, with an associated Lagrange multiplier γ_t and the second is the promised utility with Lagrange multiplier δ_t^u.[1] Given that $u\left(c_t\right)$ is concave and $u^{-1}\left(V_t\right)$ therefore is convex, it is straightforward to show that C and W are convex functions.

First order conditions when the agent is employed are

$$1=\delta_t^e u'\left(c_t\right) \tag{1}$$
$$W'\left(V^e_{t+1}\right)=\delta_t^e$$
$$C'\left(V^u_{t+1}\right)=\delta_t^e.$$

The envelope condition is

$$W'\left(V_t\right)=\delta_t^e=\frac{1}{u'\left(c_t\right)}=W'\left(V^e_{t+1}\right)=C'\left(V^u_{t+1}\right).$$

The fact that $W'\left(V_t\right)=W'\left(V^e_{t+1}\right)$ implies that nothing change for the employed indi-

[1]Note that the Lagrange multipliers depends on the history θ_t.

vidual as long as his remains employed. In fact, his consumption does not upon loosing his job either. This is due to the fact that there is no moral hazard problem on the job and full insurance is therefore optimal.[2]

When the agent is unemployed, the FOC and envelope conditions are

$$
\begin{aligned}
1 &= \delta_t^u u'\left(c_{t+1}\right) \\
W'\left(V_{t+1}^e\right) &= \gamma_t + \delta_t^u \\
(1-p)\, C'\left(V_{t+1}^u\right) &= -\gamma_t p + \delta_t^u (1-p) \\
C'\left(V_t\right) &= \delta_t^u .
\end{aligned}
$$

Giving

$$
\begin{aligned}
C'\left(V_t\right) &= \frac{1}{u'\left(c_t\right)} \\
W'\left(V_{t+1}^e\right) &= \frac{1}{u'\left(c_t\right)} + \gamma_t \\
C'\left(V_{t+1}^u\right) &= \frac{1}{u'\left(c_t\right)} - \gamma_t \frac{p}{1-p}
\end{aligned}
\tag{2}
$$

Results

Since the incentive constraint will bind[3], $\gamma_t > 0$ and therefore

$$
W'\left(V_{t+1}^e\right) > C'\left(V_t\right) > C'\left(V_{t+1}^u\right).
$$

[2] From now, I will mostly skip writing out the explicit dependence on history, hopefully without creating confusion.

[3] Prove that it must by assuming that it doesn't and derive the implications of that.

The result $C'(V_t) > C'\left(V_{t+1}^u\right)$ and the convexity of C implies that the unemployed should be made successively worse off ($V_{t+1}^u < V_t$) as long as he remains unemployed. Since $C'(V_t) = \frac{1}{u'(c_t)}$ this means that consumption must fall. Furthermore, as the IC-constraint $\frac{1}{1+r}p\left(V_{t+1}^e - V_{t+1}^u\right) \geq 1$ binds, if V_{t+1}^u keeps falling as long as the unemployed remains unemployed, so must V_{t+1}^e implying that consumption when becoming employed is lower the lower the agent has been unemployed.

1.2.2 The inverse Euler equation.

Multiplying the second line of (2) by p and the third by $(1-p)$ and adding them yields,

$$\frac{1}{u'(c_t)} = pW'\left(V_{t+1}^e\right) + (1-p)\,C'\left(V_{t+1}^u\right). \tag{3}$$

Recall that V_{t+1}^e is the utility next period if the agent becomes employed, in which case, by (1), $W'\left(V_{t+1}^e\right) = \frac{1}{u'(c_{t+1})}$, where $c_{t+1} = c\left(\theta_{t+1}^e\right)$ denotes consumption in period $t+1$ conditional on the getting a job in $t+1$ (and the history that led to consumption in t being $c_t = c(\theta_t)$). Similarly, V_{t+1}^u is next periods utility if the agent remains unemployed. By (2), $C'\left(V_{t+1}^e\right) = \frac{1}{u'\left(c_{t+1}|_{\theta_{t+1}=9}\right)}$, where $c_{t+1}|_{\theta_{t+1}=0}$ denotes consumption if the agent remains unemployed. Equation (3) can therefore be written

$$\begin{aligned}
\frac{1}{u'(c_t)} &= p\frac{1}{u'\left(c_{t+1}|_{\theta_{t+1}=1}\right)} + (1-p)\frac{1}{u'\left(c_{t+1}|_{\theta_{t+1}=0}\right)} \\
\frac{1}{u'(c_t)} &= E_t\frac{1}{u'(c_{t+1})}.
\end{aligned}$$

This is the famous "Inverse Euler Equation" (Rogerson, -85 Econometrica)[4]. Note the difference between this and the standard Euler equation.

$$u'(c_t) = E_t u'(c_{t+1}).$$

The inverse Euler equation has an important implication. To see this, first note that Jensen's inequality,

$$E_t \frac{1}{u'(c_{t+1})} > \frac{1}{E_t u'(c_{t+1})} \Rightarrow \frac{1}{E_t \frac{1}{u'(c_{t+1})}} < E_t u'(c_{t+1})$$

since the inverse function is convex. Using this with the Inverse Euler equation gives,

$$u'(c_t) = \frac{1}{E_t \frac{1}{u'(c_{t+1})}} < E_t u'(c_{t+1}).$$

The fact that $u'(c_t) < E_t u'(c_{t+1})$ in the optimal allocation means that the agent would like to save more, i.e., he is savings constrained. The incentive constraint implies that it is optimal to prevent the individual to save as much as he would like to. Suppose, for example, that utility is logarithmic, then we have

$$\frac{1}{c_t} = \frac{1}{E_t c_{t+1}} \Rightarrow c_t = E_t c_{t+1},$$

[4]With a difference between subjective and market discount rates (ρ and r, respectively), we would get

$$\frac{1}{u'(c_t)} \frac{1+r}{1+\rho} = E_t \frac{1}{u'(c_{t+1})}.$$

while the Euler equation, guiding private preferences, implies the privately optimal consumption c_t^* given future consumption is

$$c_t^* = \frac{1}{E_t\left(\frac{1}{c_{t+1}}\right)} < E_t c_{t+1}.$$

The intuition is that with more wealth and higher consumption, it is more costly to implement the incentive constraint. Thus, the benevolent planner want to prevent some wealth accumulation. The standard interpretation of this is that when there are incentive constraints, it may be optimal to tax the returns to savings. However, it may turn out that this tax is nevertheless zero in expectation, thus not creating any revenue for the planner/government (Kocherlakota 2005, Econometrica). How can such a tax discourage savings? Hint: risk premium depends on covariance with marginal utility. Explain!

In the logarithmic example, suppose individuals can save and borrow a gross interest rate r. Consider a marginal tax rate that depends on employment status and last period individual asset holdings, $\tau_{t+1}^e = \tau^e(a_t)$ and $\tau_{t+1}^u = \tau^u(a_t)$. For notational simplicity, let $c_{t+1}|_{\theta_{t+1}=1} \equiv c_{t+1}^e$ and $c_{t+1}|_{\theta_{t+1}=0} \equiv c_{t+1}^u$.Then, to have the individual Euler equation satisfied, we need

$$\begin{aligned} u'(c_t) &= \beta E_t u'(c_{t+1})(1+r)(1-\tau(a_t)) \qquad (4) \\ \frac{1}{c_t} &= \left(p\frac{1}{c_{t+1}^e}\left(1-\tau_{t+1}^e\right) + (1-p)\frac{1}{c_{t+1}^u}\left(1-\tau_{t+1}^u\right)\right) \end{aligned}$$

The inverse Euler equation requires

$$c_t = pc_{t+1}^e + (1-p)\, c_{t+1}^u \tag{5}$$

Suppose we consider a zero expected tax rate, i.e., $p\tau_{t+1}^e = -(1-p)\,\tau_{t+1}^u$.Then,

$$\tau_{t+1}^e = \frac{-(1-p)}{p}\tau_{t+1}^u. \tag{6}$$

Using (3) to replace c_t (4) together with (6) yields

$$\begin{aligned}
\tau_{t+1}^u &= \frac{p\left(c_{t+1}^e - c_{t+1}^u\right)}{pc_{t+1}^e + c_{t+1}^u(1-p)} = \frac{p\Delta c_{t+1}}{E_t c_{t+1}} \\
\tau_{t+1}^e &= -\frac{(1-p)\left(c_{t+1}^e - c_{t+1}^u\right)}{pc_{t+1}^e + c_{t+1}^u(1-p)} = -\frac{(1-p)\,\Delta c_{t+1}}{E_t c_{t+1}}
\end{aligned}$$

These tax rates eads to both the Euler and the inverse Euler equation being satisfied. Possibly together with lump sum transfers, they can *implement* the optimal allocation as a private choice of the agents. Note that the tax is *negative* in case the agent becomes employed, while positive if he remains unemployed. That is, it creates a net return that is negatively correlated with marginal utility. N

Result: Rendahl (2007)

Consider the repeated H&N economy but where individuals have access to a safe observable bond. A tax/transfer that only depends on last period asset holdings and employment status can implement the second-best allocation. Unemployment benefits falls in the asset position of the agent. Over an unemployment spell, unemployment benefits increase but consumption falls.

1.3 The Dynamic approach with unobservable saving

2 Optimal taxation – the Ramsey approach

2.1 Optimal taxation under commitment – the Ramsey problem

Consider a simple two period model, where individuals choose how much labor to supply and how much to consume in the two periods. The government must tax consumption, savings and/or labor to finance its spending needs. There will be three margins that can be distorted, the labor leisure choice in the two periods and the relative level of consumption in the two periods. Perhaps, on might think that optimal taxation should imply that all three trade-offs should be distorted. As we will see, that turns out not to be the case. This result can provide some understanding of the important Chamley & Judd result which we will derive later.

Preferences

The representative agent has an additively separable utility function in consumption and leisure,

$$U\left(c_1, c_2, l_1, l_2\right) = \sum_{t=1}^{2} \beta^{t-1} u\left(c, l\right).$$

Technology

Output is produced by labor only on a competitive labor market. One unit of labor produces w units of the consumption good. The consumption good can be stored between periods. One unit of the good stored gives $1+r$ units of the second period, where r is positive or negative. Individuals have one unit of labor each period to split between work and leisure l.

Budget constraints

The government needs to finance its consumption by tax revenues. For simplicity, we have already assumed that its consumption does not interfere with the individuals private problem. We will assumed that the government cannot finance its consumption by lump sum taxation. We do this without providing an explicit reason within the model. Instead, the government has at its disposal, a linear labor income tax $\tau_{l,t}$, a consumption tax $\tau_{c,t}$ and a tax on savings, τ_s. Individual budget constraints are therefore

$$c_1 (1 + \tau_{c,1}) + i + b = w (1 - l_1) (1 - \tau_{l,1})$$

$$c_2 (1 + \tau_{c,2}) = w (1 - l_2) (1 - \tau_{l,2}) + (i + b) (1 + r) (1 - \tau_s),$$

where i is physical investments (stored goods) and b is government borrowing assumed to require a return $1 + r$ before taxes to be held. We can collapse this to

$$c_1 (1 + \tau_{c,1}) + \frac{c_2}{1 + r} \frac{1 + \tau_{c,2}}{(1 - \tau_s)} = w (1 - l_1) (1 - \tau_{l,1}) + \frac{w (1 - l_2) (1 - \tau_{l,2})}{(1 + r) (1 - \tau_s)}.$$

It turns out that it is convenient to divide this by $(1 + \tau_{c,1})$ and multiply the last term in the RHS by $\frac{1+\tau_{c,2}}{1+\tau_{c,2}}$. We can then write the budget constraint as

$$c_1 + \frac{c_2}{1+r}\frac{1+\tau_{c,2}}{(1-\tau_s)(1+\tau_{c,1})} = w(1-l_1)\frac{1-\tau_{l,1}}{1+\tau_{c,1}} + \frac{w(1-l_2)}{1+r}\frac{1-\tau_{l,2}}{1+\tau_{c,2}}\frac{1+\tau_{c,2}}{(1-\tau_s)(1+\tau_{c,1})}.$$

The aggregate resource constraint of the economy is

$$c_1 + \frac{c_2}{1+r} + G = w(1-l_1) + \frac{w(1-l_2)}{1+r}. \tag{7}$$

Do we need to bother about the government budget constraint in addition to the private and the aggregate?

Individual optimality

The first order conditions of the individual problem are[5]

$$\begin{aligned} c_1; u_c(c_1, l_1) &= \lambda \\ l_1; u_l(c_1, l_1) &= \lambda w \frac{1-\tau_{l,1}}{1+\tau_{c,1}} \\ c_2; \beta u_c(c_2, l_2) &= \lambda \frac{(1+\tau_{c,2})}{(1+r)(1-\tau_s)(1+\tau_{c,1})} \\ l_2; \beta u_l(c_2, l_2) &= \lambda \frac{w}{1+r}\frac{1-\tau_{l,2}}{1+\tau_{c,2}}\frac{1+\tau_{c,2}}{(1-\tau_s)(1+\tau_{c,1})} \end{aligned} \tag{8}$$

[5]We disregard the constraint that $i_1 \geq 0$, otherwise, we could have corner soluitions.

2.1.1 A simple example with a labor tax and consumption taxes.

Let us now assume that the government only has access to a constant labor tax and a consumption tax that is allowed to vary. Also assume for tractability that $u(c,l) = \ln c + \ln l$

The first order conditions of the individual problem are then

$$\begin{aligned}
\frac{1}{c_1} &= \lambda \\
\frac{1}{l_1} &= \lambda w \frac{1-\tau_l}{1+\tau_{c,1}} \\
\beta \frac{1}{c_2} &= \lambda \frac{1+\tau_{c,2}}{(1+r)(1+\tau_{c,1})} \\
\beta \frac{1}{l_2} &= \frac{\lambda w}{1+r} \frac{1-\tau_l}{1+\tau_{c,2}} \frac{1+\tau_{c,2}}{1+\tau_{c,1}}
\end{aligned}$$

Eliminating λ, the individual optimality constraints are

$$\begin{aligned}
\frac{l_1}{c_1} w &= \frac{1+\tau_{c,1}}{1-\tau_l} \\
\frac{c_2}{c_1 \beta (1+r)} &= \frac{1+\tau_{c,1}}{1+\tau_{c,2}} \\
\frac{l_2}{c_2} w &= \frac{1+\tau_{c,2}}{1-\tau_l} \\
c_1 + \frac{c_2}{1+r} \frac{1+\tau_{c,2}}{1+\tau_{c,1}} &= w(1-l_1) \frac{1-\tau_l}{1+\tau_{c,1}} + \frac{w(1-l_2)}{1+r} \frac{1-\tau_l}{1+\tau_{c,2}} \frac{1+\tau_{c,2}}{1+\tau_{c,1}}
\end{aligned}$$

with the solution

$$c_1 = \frac{(1-\tau_l)}{(1+\tau_{c,1})} \frac{w\,(2+r)}{2\,(1+\beta)\,(1+r)} \tag{9}$$

$$l_1 = \frac{2+r}{2\,(1+r)\,(1+\beta)} \tag{10}$$

$$c_2 = \beta \frac{(1-\tau_l)}{(1+\tau_{c,2})} \frac{w\,(2+r)}{2\,(1+\beta)} \tag{11}$$

$$l_2 = \frac{\beta\,(2+r)}{2\,(1+\beta)}$$

The Ramsey problem is now to maximize utility over the tax rates, τ_l, $\tau_{c,1}$ and $\tau_{c,2}$, subject to the resource constraint. Disregarding constants, this is

$$\max_{\tau_l,\tau_{c,1},\tau_{c,2}} \ln(1-\tau_l) - \ln(1+\tau_{c,1}) + \beta\left(\ln(1-\tau_l) - \ln(1+\tau_{c,2})\right)$$

subject to the resource constraint (7) where (9) is used to replace the private choice variables. First order conditions are

$$\begin{aligned}
\tau_{c,1};\, 1+\tau_{c,1} &= \lambda \frac{w\,(2+r)}{2\,(1+\beta)\,(1+r)}\,(1-\tau_l) \\
\tau_{c,2};\, 1+\tau_{c,2} &= \lambda \frac{w\,(2+r)}{2\,(1+\beta)\,(1+r)}\,(1-\tau_l) \\
\tau_l;\, \frac{1+\beta}{1-\tau_l} &= \lambda \frac{w\,(1+\tau_2+\beta\,(1+\tau_1))\,(2+r)}{2\,(1+\beta)\,(1+r)\,(1+\tau_1)\,(1+\tau_2)}
\end{aligned}$$

As we see, the first order conditions for the consumption taxes are symmetrical – implying that it is optimal to set consumption taxes equal in the two periods. To what level does not matter, as long as τ_l is properly adjusted. For example, we could choose $\tau_{c,1} = \tau_{c,2} = 0$, in which case

$$\frac{1}{1-\tau_l} = \lambda \frac{w\left(1+\beta\right)\left(2+r\right)}{\left(1+\beta\right)2\left(1+\beta\right)\left(1+r\right)}.$$

Substituting this into the FOC for or we could set $\tau_{c,1}$ and $\tau_{c,2}$ yields:

$$\begin{aligned} 1 &= \lambda \frac{w\left(2+r\right)}{2\left(1+\beta\right)\left(1+r\right)\left(1\right)^2} \frac{\left(1+\beta\right)2\left(1+\beta\right)\left(1+r\right)}{\lambda w\left(1+\beta\right)\left(2+r\right)} \\ &= 1 \end{aligned}$$

Alternatively, we could set $\tau_l = 0$, in which case,

$$1+\tau_{c,1} = 1+\tau_{c,2} = \lambda \frac{w\left(2+r\right)}{2\left(1+\beta\right)\left(1+r\right)}$$

which implies that the FOC for τ_l also is satisfied at $\tau_l = 0$,

$$\begin{aligned} \frac{1+\beta}{1} &= \lambda \frac{w\left(\left(\lambda \frac{w(2+r)}{2(1+\beta)(1+r)}\right) + \beta\left(\lambda \frac{w(2+r)}{2(1+\beta)(1+r)}\right)\right)\left(2+r\right)}{2\left(1+\beta\right)\left(1+r\right)\left(\lambda \frac{w(2+r)}{2(1+\beta)(1+r)}\right)^2} \\ &= 1+\beta \end{aligned}$$

EXPLAIN! This provides *distortion smoothing* – the labor leisure choice is distorted equally much in both period. Perhaps, this is not surprising.

Formally, we know $\tau_{c,1} = \tau_{c,2} = \tau_c$. Given this, the first order conditions are

$$\begin{aligned} \tau_c ; \frac{1+\tau_c}{1-\tau_l} &= \lambda \frac{w\left(2+r\right)}{2\left(1+\beta\right)\left(1+r\right)} \\ \tau_l ; \frac{1+\tau_c}{1-\tau_l} &= \lambda \frac{w\left(2+r\right)}{2\left(1+\beta\right)\left(1+r\right)} \end{aligned}$$

2.1.2 The general case

Let us now define

$$\begin{aligned}
\frac{1-\tau_{l,1}}{1+\tau_{c,1}} &\equiv W_1, \\
\frac{1-\tau_{l,2}}{1+\tau_{c,2}} &\equiv W_2, \\
\frac{(1+\tau_{c,2})}{(1+\tau_{c,1})(1-\tau_s)} &\equiv W_i.
\end{aligned} \tag{12}$$

We can then write the system as

$$\begin{aligned}
\frac{u_l(c_1,l_1)}{u_c(c_1,l_1)\,w} &= W_1 \\
\frac{u_l(c_2,l_2)}{u_c(c_2,l_2)\,w} &= W_2 \\
\frac{u_c(c_2,l_2)}{u_c(c_1,l_1)}\beta(1+r) &= W_i \\
c_1+\frac{c_2}{1+r}W_i &= w(1-l_1)\,W_1+\frac{w(1-l_2)}{1+r}W_2W_i \\
c_1+g+\frac{c_2+g}{1+r} &= w(1-l_1)+\frac{w(1-l_2)}{1+r}
\end{aligned} \tag{13}$$

The first two equations are the FOC for labor supply and the third is the Euler equation (FOC for savings (or c_1, c_2). The fourth is private budget constraints and the last the aggregate resource constraint.

Provided g is not too high, this gives a solution for c_1, c_2, l_1, l_2 and one of the tax wedges, as a function of two of the other wedges and parameters.

Result 1 Although the government has access to 5 different taxes, the distortion relative to the first best is a function of the three wedges W_1, W_2 and W_i.

Using result 1, we conclude that all tax systems that provide the same wedges as the one with a constant labor tax and a constant consumption tax gives the same utility. Provide some examples. Furthermore, the restriction we imposed, namely $\tau_{l,1} = \tau_{l,2}$ and $\tau_s = 0$, does not reduce welfare. Explain!

Finally, in the optimal allocation $W_i = 1$,i.e., there is no intertemporal wedge. With a constant consumption tax, this requires a zero tax on savings.

2.2 The primal approach

An often used way of solving the problem is the *primal approach.* The idea here is to write the problem as the planner directly choosing the consumption and labor of the individual. With access to lump sum taxes, the only constraint for the planner is the resource constraint and first best will be achieved. With only proportional taxes, incentive compatibility must be respected. It turns out that we can write this constraint without any taxes or prices. We do this by substituting the first order constraints of the individual (first three equations of (13) into the private budget constraint (the fourth equation of (13)). This yields,

$$c_1 + c_2 \frac{u_c(c_2, l_2)}{u_c(c_1, l_1)} \beta = (1 - l_1) \frac{u_l(c_1, l_1)}{u_c(c_1, l_1)} + (1 - l_2) \frac{u_l(c_2, l_2)}{u_c(c_2, l_2)} \frac{u_c(c_2, l_2)}{u_c(c_1, l_1)} \beta. \tag{14}$$

The Ramsey problem can then be expressed as

$$\max_{c_1, c_2, l_1, l_2} \sum_{t=1}^{2} \beta^{t-1} u(c, l)$$

s.t. (14) and (7). As we see, no taxes or prices (interest rate) enter this problem except through the aggregate resource constraint.

In our logarithmic example, (14) becomes

$$
\begin{aligned}
c_1(1+\beta) &= c_1\left(\frac{1-l_1}{l_1}+\frac{(1-l_2)}{l_2}\beta\right) \\
\Longrightarrow l_1 &= \frac{l_2}{2l_2(1+\beta)-\beta}
\end{aligned}
$$

since the $c_1 = 0$ root is irrelevant. Substituting this into the objective function and taking first order conditions w.r.t. c_1 and c_2 yields,

$$
\max_{c_1,c_2,l_2} \ln c_1 + \ln \frac{l_2}{2l_2(1+\beta)-\beta} + \beta(\ln c_2 + \ln l_2)
$$

$$
\begin{aligned}
\frac{1}{c_1} &= \lambda \\
\beta\frac{1}{c_2} &= \lambda\frac{1}{1+r} \\
\Rightarrow \frac{c_2}{c_1} &= \beta(1+r)
\end{aligned}
$$

$$
s.t. \left(w\left(1-\frac{l_2}{2l_2(1+\beta)-\beta}\right)+\frac{w(1-l_2)}{1+r}-\left(c_1+\frac{c_2}{1+r}+G\right)\right)
$$

again confirming that the intertemporal margin should be zero, requiring $W_i = 0$.

Often, the focus is on the allocation, i.e., how consumption and leisure is allocated over time. To go further, one might want to find a tax system that implements this allocation. As we have seen, there are often many such systems.

2.3 The Chamley-Judd result

There is an infinitely lived representative agent with preferences

$$\sum_{t=0}^{\infty} \beta^t u\left(c_t, l_t\right)$$

The household has one unit of labor per period, to be split between leisure l and work n.The aggregate budget constraint is

$$c_t + g_t + k_{t+1} = F\left(k_t, n_t\right) + \left(1-\delta\right) k_t \tag{15}$$

The production function is constant returns to scale and factor markets are competitive. Profit maximization of the representative firm implies

$$w_t = F_n\left(k_t, n_t\right)$$

$$r_t = F_k\left(k_t, n_t\right)$$

The government needs to finance an exogenous stream of expenditures $\{g_t\}_t^{\infty}$ using taxes on labor and capital and can smooth taxes by using a bond. Thus,

$$\begin{aligned} g_t + b_t &= \tau_t^k r_t k_t + \tau_t^n w_t n_t + \frac{b_{t+1}}{R_t} \\ &= F\left(k_t, n_t\right) - \left(1-\tau_t^k\right) r_t k_t - \left(1-\tau_t^n\right) w_t n_t + \frac{b_{t+1}}{R_t} \end{aligned}$$

where b_t is government borrowing and R_t is the interest rate on government bonds.

Households have budget constraints

$$c_t + k_{t+1} + \frac{b_{t+1}}{R_t} = (1 - \tau_t^n)\, w_t n_t + \left(1 - \tau_t^k\right) k_t r_t + (1 - \delta)\, k_t + b_t$$

First order conditions are:

$$\begin{aligned}
c_t ; u_c (c_t, l_t) &= \lambda_t \\
l_t ; u_l (c_t, l_t) &= \lambda_t (1 - \tau_t^n)\, w_t \\
k_{t+1} ; \lambda_t &= \beta \lambda_{t+1} \left(\left(1 - \tau_{t+1}^k\right) r_{t+1} + (1 - \delta)\right) \\
b_{t+1} ; \lambda_t \frac{1}{R_t} &= \beta \lambda_{t+1}
\end{aligned}$$

Clearly, the first three implies

$$\begin{aligned}
\frac{u_l (c_t, l_t)}{u_c (c_t, l_t)} &= (1 - \tau_t^n)\, w_t \\
u_c (c_t, l_t) &= \beta u_c (c_{t+1}, l_{t+1}) \left(\left(1 - \tau_{t+1}^k\right) r_{t+1} + (1 - \delta)\right)
\end{aligned}$$

and the last two the no arbitrage condition

$$R_t = \left(1 - \tau_{t+1}^k\right) r_{t+1} + (1 - \delta)$$

Transversality conditions are

$$\lim_{T\to\infty}\left(\prod_{i=0}^{T-1} R_i^{-1}\right) k_{T+1} = 0$$
$$\lim_{T\to\infty}\left(\prod_{i=0}^{T-1} R_i^{-1}\right) \frac{b_{T+1}}{R_T} = 0$$

We can now make the following definitions:

Definition 2 *A* feasible allocation *is a sequence* $\{k_t, c_t, l_t, g_t\}_{t=0}^{\infty}$ *that satisfies the aggregate budget constraint (15).*

Definition 3 *A* price system *is a sequence of prices* $\{w_t, r_t, R_t\}_{t=0}^{\infty}$ *that is bounded and non-negative.*

Definition 4 *A* government policy *is a sequence* $\left\{\tau_t^n, \tau_t^k, b_t\right\}_{t=0}^{\infty}$ *and perhaps* $\{g_t\}_{t=0}^{\infty}$ *if that can be chosen.*

Definition 5 *A* competitive equilibrium *is a feasible allocation, a price system and a government policy such that*

1. *Given the price system and the government policy, the allocation solves the maximization problem of the individual and of the firm.*
2. *The government budget constraints are satisfied.*

Definition 6 *The* Ramsey problem *is to choose a competitive equilibrium (i.e.,pick a particular government policy) that maximizes the welfare of the representative individual.*

The Lagrangean of the Ramsey problem can be written

$$L = \sum_{t=0}^{\infty} \beta^t \{ u\left(c_t, 1 - n_t\right)$$

$$+ \psi_t \left(F\left(k_t, n_t\right) - \left(1 - \tau_t^k\right) r_t k_t - \left(1 - \tau_t^n\right) w_t n_t - b_t - g_t + b_{t+1}/R_t \right)$$

$$+ \theta_t \left(F\left(k_t, n_t\right) + (1 - \delta) k_t - c_t - g_t - k_{t+1} \right)$$

$$+ \mu_{1,t} \left(u_l\left(c_t, l_t\right) - u_c\left(c_t, l_t\right)\left(1 - \tau_t^n\right) w_t \right)$$

$$+ \mu_{2,t} \left(u_c\left(c_t, l_t\right) - \beta u_c\left(c_{t+1}, l_{t+1}\right)\left(1 - \tau_{t+1}^k\right) r_{t+1} + (1 - \delta) \right)$$

Now, the first order condition for k_{t+1} is

$$\theta_t = \beta \psi_{t+1} \left(F_k\left(k_{t+1}, n_{t+1}\right) - \left(1 - \tau_{t+1}^k\right) r_{t+1} \right) - \theta_{t+1} \left(F_k\left(k_{t+1}, n_{t+1}\right) + (1 - \delta) \right)$$

and for c_t

$$u_c\left(c_t, 1 - n_t\right) = \theta_t$$

giving

$$u_c\left(c_t, 1 - n_t\right) = \beta \psi_{t+1} \left(F_k\left(k_{t+1}, n_{t+1}\right) - \left(1 - \tau_{t+1}^k\right) r_{t+1} \right)$$

$$+ \beta u_c\left(c_{t+1}, 1 - n_{t+1}\right) \left(F_k\left(k_{t+1}, n_{t+1}\right) + (1 - \delta) \right).$$

Suppose there is a steady state of the model, then

$$
\begin{aligned}
u_c &= \beta \left(\psi \left(F_k - \left(1 - \tau^k \right) F_k \right) + u_c \left(F_k + (1 - \delta) \right) \right) \\
&= \beta \left(\psi \tau^k F_k + u_c \left(F_k + (1 - \delta) \right) \right).
\end{aligned}
$$

Private optimality (the Euler equation), implies in steady state

$$
\begin{aligned}
u_c &= \beta u_c \left(\left(1 - \tau^k \right) F_k + (1 - \delta) \right) \\
1 &= \beta \left(F_k + (1 - \delta) - \tau^k F_k \right) \\
\frac{1}{\beta} + \tau^k F_k &= F_k + (1 - \delta)
\end{aligned}
$$

giving

$$
\begin{aligned}
u_c &= \beta \left(\psi \tau^k F_k + u_c \left(\frac{1}{\beta} + \tau^k F_k \right) \right) \\
&= \beta \left((\psi + u_c) \tau^k F_k + u_c \left(\tau^k F_k \right) \right) + u_c \\
0 &= \beta (\psi + u_c) \tau^k F_k
\end{aligned}
$$

requiring $\tau^k = 0$.

2.4 Discussion

We have shown that also in this simple economy, tax smoothing implies that the intertemporal margin should not be distorted. We have also found an equivalence between constant consumption taxes and an investment tax. In an infinite horizon model, a positive investment tax in steady state has implications identical to ever increasing consumption taxes. This can thus provide some intuition for Chamley & Judd's result that investment taxes should not be used in the long run. The result is quite robust. For example it extends to the case of heterogeneity, if the government wants to use it's revenues to support some capital poor individuals, it should not tax capital accumulation in steady state. Here intuition could be that the capital stock in steady state is elastic enough to imply the tax incidence of capital taxes is on workers.

The result also extends to the stochastic case, in which case *expected taxes* should be zero and not distort savings.

However, it does not go through in some cases:

1. If there are untaxed factors of production that generate profits and these factors are strict complements to capital. Then capital should be taxed (negatively if they are substitutes).

2. If market incompleteness makes people save too much for precautionary reasons.

In the short run, capital income taxes also collect revenue from sunk investments. Then, the tax is partly lump sum, which provides an argument for such taxes early in the planning horizon. But when is that zero? Has it already occurred a long time ago? In any case, we see a time consistency problem here.

Not also that the long-run maybe quite far out and people alive today might loose by a policy that maximizes the welfare of a constructed infinitely lived

2.5 Time consistent taxation

TBW.

3 New Public Finance – the Mirrlees approach

Let us now consider the dynamic Mirrlees approach to optimal taxation. Here, individuals are assumed to be different. These differences can be either in their productivity or in their value of leisure. Such differences imply that there is differences between individuals in their trade-off between leisure and work. It is assumed that the government cannot directly observe this differences, only observe the individuals market choices. For example, governments observe income, but not the effort exerted to get this income.

Consider a simple two-period example from GTW.

Individual preferences are:

$$E\left(u\left(c_1\right)+v\left(n_1\right)+\beta\left(u\left(c_2\right)+v\left(n_2\right)\right)\right)$$

where c_t is consumption and n_t is labor supply/work effort. u is increasing and concave and v decreasing and concave. Individuals differ in their ability, denoted θ. It is assumed that there is a finite number $i \in \{1,2,...,N\}$ of ability levels and ability might change over time. We will interchangeably use type and ability to denote θ. Output is produced in competitive firms using a linear technology where each individual i produces

$$y_t\left(i\right)=\theta\left(i\right)n_t\left(i\right).$$

There is a continuum of individuals of a unitary total mass. In the first period, individuals are given abilities by nature according to a probability function $\pi_1(i)$. The ability can then change to the second period. Second period ability is denoted $\theta(i,j)$ and the transition probability is $\pi_2(j|i)$.

There is a storage technology with return R. Finally, the government needs to finance some spendings G_1 and G_2. At first, we analyze the case of no aggregate uncertainty.

The aggregate resource constraint is

$$\sum_i \left(y_1(i) - c_1(i) + \sum_j \frac{y_2(i,j) - c_2(i,j)}{R} \pi_2(j|i) \right) \pi_1(i) + K_1 = G_1 + \frac{G_2}{R} \tag{16}$$

where K_1 is an aggregate initial endowment.

The problem is now to maximize the utilitarian welfare function subject to the resource constraints and the incentive constraints, i.e., that individuals themselves choose labor supply and savings. A way of finding the second best allocation is to let the planner provide consumption and work conditional on the ability an individual claims to have (and if relevant, the aggregate state). Here this is in the first period $c_1(i), y_1(i)$ and in the second, $c_2(i,j), y_1(i,j)$. Individuals then report their abilities to the planner. The strategy of an individual is his first period report and then a reporting plan as a function of the realized period 2 ability. Let's call the report i_r and $j_r(j)$, where the latter is the report as a function of the true ability. The incentive constraint is then that individuals voluntarily report their true ability. According to the *revelation principle,* this always yields the best incentive compatible allocation. The *truth-telling* constraint is then that

$$
\begin{aligned}
&u\left(c_1\left(i\right)\right)+v\left(\frac{y_1\left(i\right)}{\theta_1\left(i\right)}\right)+\beta\sum_j\left(u\left(c_2\left(i,j\right)\right)+v\left(\frac{y_2\left(i,j\right)}{\theta_2\left(i,j\right)}\right)\right)\pi_2\left(j|i\right) \\
&\geq u\left(c_1\left(i_r\right)\right)+v\left(\frac{y_1\left(i_r\right)}{\theta_1\left(i\right)}\right)+\beta\sum_j\left(u\left(c_2\left(i_r,j_r\left(j\right)\right)\right)+v\left(\frac{y_2\left(i_r,j_r\left(j\right)\right)}{\theta_2\left(i,j\right)}\right)\right)\pi_2\left(j|i\right)
\end{aligned}
\tag{17}
$$

for any possible reporting strategy $i_r, j_r\left(j\right)$. Note that the θ_s are the true ones in both sides of the inequality. Note also that *truth-telling* implies that

$$
u\left(c_2\left(i,j\right)\right)+v\left(\frac{y_2\left(i,j\right)}{\theta_2\left(i,j\right)}\right)\geq u\left(c_2\left(i_r,j_r\left(j\right)\right)\right)+v\left(\frac{y_2\left(i_r,j_r\left(j\right)\right)}{\theta_2\left(i,j\right)}\right)\forall j,
\tag{18}
$$

otherwise utility could be increased by reporting j_r if the second period ability is j. The planning problem is to maximize

$$
\sum_i\left(u\left(c_1\left(i\right)\right)+v\left(\frac{y_1\left(i\right)}{\theta_1\left(i\right)}\right)+\beta\sum_j\left(u\left(c_2\left(i,j\right)\right)+v\left(\frac{y_2\left(i,j\right)}{\theta_2\left(i,j\right)}\right)\right)\pi_2\left(j|i\right)\right)\pi\left(i\right)
$$

subject to (16) and (17).

Letting stars *,denote optimal allocations. We can now define three wedges (distortions) that the informational friction may cause. These are the consumption-leisure (intratemporal) wedges

$$
\begin{aligned}
\tau_{y_1}\left(i\right)&\equiv 1+\frac{v'\left(\frac{y_1^*(i)}{\theta_1(i)}\right)}{\theta_1\left(i\right)u'\left(c_1^*\left(i\right)\right)}, \\
\tau_{y_2}\left(i,j\right)&\equiv 1+\frac{v'\left(\frac{y_2^*(i,j)}{\theta_2(i,j)}\right)}{\theta_2\left(i,j\right)u'\left(c_2^*\left(i,j\right)\right)},
\end{aligned}
$$

and the intertemporal wedge

$$\tau_k(i) \equiv 1 - \frac{u'(c_1^*(i))}{\sum_j \beta R u'(c_2(i,j))\,\pi_2(j|i)}.$$

Clearly, in absence of government interventions, these wedges would be zero by perfect competition and the first-order conditions of private optimization.

3.1 The inverse Euler equation

We will now show that if individual productivities are not always constant over time, the intertemporal wedge will not be zero. The logic is as follows and similar to what we have done above. In an optimal allocation, the resource cost (expected present value of consumption) of providing the equilibrium utility to each type, must be minimized. Consider the following peturbation around the optimal allocation for a given first period ability type i. Increase utility by a marginal amount Δ for all possible second period types $\{i, j\}$ the agent could become. To compensate, decrease utility by $\beta\Delta$ in the first period. Clearly, the objective function is not changed. What about the thruth-telling constraint?.

First, note that expected utility is not changed.

Second, since utility is changed in parallel for all ability levels the individual could have in the second period, there relative ranking cannot change. In other words, if we add Δ to both sides of (18) it must still be satisfied.

Thus, the incentive constraint is unchanged. However, the resource constraint is not necessarily invariant to this peturbation. Let

$$\tilde{c}_1(i;\Delta) = u^{-1}\left(u\left(c_1^*(i)\right) - \beta\Delta\right),$$

$$\tilde{c}_2(i,j;\Delta) = u^{-1}\left(u\left(c_2^*(i,j)\right) + \Delta\right)$$

denote the perturbed consumption levels. The resource expected resource cost of these are

$$\begin{aligned}
&\tilde{c}_1(i;\Delta) + \sum_j \frac{1}{R}\tilde{c}_2(i,j;\Delta)\,\pi_2(j|i) \\
&= u^{-1}\left(u\left(c_1^*(i)\right) - \beta\Delta\right) + \sum_j \frac{1}{R}u^{-1}\left(u\left(c_2^*(i,j)\right) + \Delta\right)\pi_2(j|i).
\end{aligned}$$

The first-order condition for minimizing the resource cost over Δ must be satisfied at $\Delta = 0$, for the * consumption levels to be optimal.

Thus,

$$\begin{aligned}
0 &= \\
&= \frac{-\beta}{u'\left(c_1^*(i)\right)} + \sum_j \frac{1}{R}\frac{1}{u'\left(c_2^*(i,j)\right)}\pi_2(j|i) \\
&\Rightarrow \frac{1}{u'\left(c_1^*(i)\right)} = E_1\frac{1}{\beta R u'\left(c_2^*(i,.)\right)}
\end{aligned}$$

From Jensen's inequality, we find that

$$\begin{aligned}
u'\left(c_1^*(i)\right) &< E\beta R u'\left(c_2^*(i,.)\right) \\
&\Rightarrow \tau_k(i) > 0,
\end{aligned}$$

if and only if there is some uncertainty in c_2^*. Note that this uncertainty would come from second period ability being random and the allocation implying that second period consumption depends on the realization of ability. If second period ability is non-random, i.e., $\pi_2(j|i) = 1$ for some j, then $\tau_k(i) = 0$.

3.2 A simple logarithmic example: insurance against low ability.

Suppose in the first period, ability is unity and in the second $\theta > 1$ or $\frac{1}{\theta}$ with equal probability.Disregard government consumption – set $G_1 = G_2 = 0$, although non-zero spending is quite easily handled. The problem is therefore to provide a good insurance against a low-ability shock when this is not observed.

The first best allocation is the solution to

$$\max_{c_1, y_1, c_h, c_l, y_h, y_l} u(c_1) + v(y_1) + \beta\left(\frac{u(c_h) + v\left(\frac{y_h}{\theta}\right)}{2} + \frac{u(c_l) + v\left(\frac{y_l}{\frac{1}{\theta}}\right)}{2}\right)$$

$$s.t. 0 = y_1 + \frac{y_h + y_l}{2R} - c_1 - \frac{c_h + c_l}{2R}$$

First order conditions are

$$u'(c_1) = \lambda$$

$$v'(y_1) = -\lambda$$

$$\beta u'(c_h) = \frac{\lambda}{R}$$

$$\beta u'(c_l) = \frac{\lambda}{R}$$

$$\beta v'\left(\frac{y_h}{\theta}\right)\frac{1}{\theta} = -\frac{\lambda}{R}$$

$$\beta v'(\theta y_l)\,\theta = -\frac{\lambda}{R}$$

3.2.1 A simple example

Suppose for example that $u(c) = \ln(c)$ and $v(n) = -\frac{n^2}{2}$ and $\beta R = 1$. Then, we get

$$\frac{1}{c_1} = \lambda$$

$$\frac{1}{c_h} = \lambda$$

$$\frac{1}{c_l} = \lambda$$

$$y_1 = \lambda$$

$$\frac{y_h}{\theta^2} = \lambda$$

$$y_l\theta^2 = \lambda$$

$$c_1 + \frac{c_h + c_l}{2} - y_1 - \frac{y_h + y_l}{2} = 0$$

We see immediately that $c_1 = c_h = c_l$ while $y_h = \theta^2 y_1$ and $y_l = \frac{y_1}{\theta^2}$ and $y_1 = \sqrt{\frac{2}{\left(1+\frac{1}{2}\left(\theta^2+\theta^{-2}\right)\right)}} =$

n_1. Therefore, $n_h = \frac{y_h}{\theta} = \theta^2 n_1$ and $n_l = y_l \theta = \frac{n_1}{\theta}$. Thus, if the individual becomes of high ability in the second period, he should work more but don't get any higher consumption. Is this incentive compatible?

We conjecture that the binding incentive constraint is for the high ability type. High has to be given sufficient consumption to make him voluntarily choose not to report being low ability. If he misreports, he gets c_l and is asked to produce y_l. The constraint is therefore

$$
\begin{aligned}
& u(c_1) + v(y_1) + \beta \left(\frac{u(c_h) + v\left(\frac{y_h}{\theta}\right)}{2} + \frac{u(c_l) + v(\theta y_l)}{2} \right) \\
& \geq u(c_1) + v(y_1) + \beta \left(\frac{u(c_l) + v\left(\frac{y_l}{\theta}\right)}{2} + \frac{u(c_l) + v(\theta y_l)}{2} \right)
\end{aligned}
$$

$$
\begin{aligned}
u(c_h) + v\left(\frac{y_h}{\theta}\right) &\geq u(c_l) + v\left(\frac{y_l}{\theta}\right) \\
\ln c_h - \ln c_l &\geq \frac{y_h^2 - y_l^2}{2\theta^2}
\end{aligned}
$$

We conjecture this is binding. The problem is then

$$
\max_{c_1, y_1, c_h, c_l, y_h, y_l} \ln(c_1) - \frac{y_1^2}{2} + \left(\frac{\ln c_h - \frac{\left(\frac{y_h}{\theta}\right)^2}{2}}{2} + \frac{\ln c_l - \frac{(\theta y_l)^2}{2}}{2} \right)
$$

$$
\begin{aligned}
s.t. 0 &= y_1 + \frac{y_h + y_l}{2} - c_1 - \frac{c_h + c_l}{2} \\
0 &= \ln c_h - \ln c_l - \frac{y_h^2 - y_l^2}{2\theta^2}.
\end{aligned}
$$

Denoting the shadow values by λ_r and λ_I the FOCs for the consumption levels are

$$
\begin{aligned}
c_1 &= \frac{1}{\lambda_r} \\
c_h &= \frac{1+2\lambda_I}{\lambda_r} \\
c_l &= \frac{1-2\lambda_I}{\lambda_r}
\end{aligned}
$$

from which we see

$$
\frac{c_h^*}{c_1^*} = 1+2\lambda_I, \frac{c_l^*}{c_1^*} = 1-2\lambda_I
$$

and

$$
\tau_k\left(i\right) \equiv 1 - \frac{\lambda_r}{\frac{\lambda_r}{1+2\lambda_I}\frac{1}{2} + \frac{\lambda_r}{1-2\lambda_I}\frac{1}{2}} = \left(2\lambda_I\right)^2,
$$

: implying a positive intertemporal wedge if the IC constraint binds.

The intratemporal wedges are found by analyzing the FOC's for the labor supplies

$$
\begin{aligned}
y_1^* &= \lambda_r \\
y_h^* &= \frac{\lambda_r}{1+2\lambda_I}\theta^2 \Rightarrow y_h^* c_h^* = \frac{\lambda_r}{1+2\lambda_I}\theta^2 \frac{1+2\lambda_I}{\lambda_r} = \theta^2 \\
y_l^* &= \frac{\lambda_r}{\theta^4 - 2\lambda_I}\theta^2 \Rightarrow y_l^* c_l^* = \frac{\lambda_r}{\theta^4 - 2\lambda_I}\theta^2 \frac{1-2\lambda_I}{\lambda_r} = \frac{1-2\lambda_I}{\theta^2\left(1-2\lambda_I\theta^{-4}\right)}
\end{aligned}
$$

$$
\begin{aligned}
\tau_{y_1} &= 1 - \frac{y_1^*}{\frac{1}{c_1^*}} = 0, \\
\tau_{y_2}(h) &= 1 + \frac{v'\left(\frac{y_h^*}{\theta}\right)}{\theta u'(c_h^*)} = 1 + \frac{-\frac{y_h^*}{\theta}}{\theta \frac{1}{c_h^*}} \\
&= 1 + \frac{-\frac{\frac{\lambda_r}{1+2\lambda_I}\theta^2}{\theta}}{\theta \frac{1}{\frac{1+2\lambda_I}{\lambda_r}}} = 0
\end{aligned}
$$

and

$$
\begin{aligned}
\tau_{y_2}(l) &= 1 + \frac{v'(\theta y_l^*)}{\frac{1}{\theta} u'(c_l^*)} = 1 + \frac{-\theta y_l^*}{\frac{1}{\theta} \frac{1}{c_h^*}} \\
&= 1 + \frac{-\theta \frac{\lambda_r}{\theta^4 - 2\lambda_I}\theta^2}{\frac{1}{\theta} \frac{1}{\frac{1-2\lambda_I}{\lambda_r}}} = 2\lambda_I \frac{\theta^4 - 1}{\theta^4 - 2\lambda_I} > 0
\end{aligned}
$$

As we see, the wedge for the high ability types is zero, but positive for the low ability type.[6] For later use, we note that

$$
\begin{aligned}
y_1^* c_1^* &= 1 \\
y_h^* c_h^* &= \frac{\lambda_r}{1+2\lambda_I}\theta^2 \frac{1+2\lambda_I}{\lambda_r} = \theta^2 \\
y_l^* c_l^* &= \frac{\lambda_r}{\theta^4 - 2\lambda_I}\theta^2 \frac{1-2\lambda_I}{\lambda_r} = \frac{1-2\lambda_I}{\theta^2 (1 - 2\lambda_I \theta^{-4})}
\end{aligned} \tag{19}
$$

[6]The wedge, asymptotes to infinity as λ_I approach $\frac{\theta^4}{2}$. Can you explain?

3.3 Implementation

It is tempting to interpret the wedges as taxes and subsidies. However, this is not entirely correct since the wedges in general are functions of all taxes. Furthermore, while there is typically a unique set of wedges this is generically not true for the taxes. As we have discussed above, many different tax systems might implement the optimal allocation. One example is the draconian, use 100% taxation for every choice except the optimal ones.

Only by putting additional restrictions is the implementing tax system found. Let us consider a combination if linear labor taxes and savings taxes that together with type specific transfers implement the allocation in the example. To do this, consider the individual problem,

$$\max_{c_1,y_1,s,y_h,y_l,c_h,c_l} \ln(c_1) - \frac{y_1^2}{2} + \left(\frac{\ln c_h - \frac{\left(\frac{y_h}{\theta}\right)^2}{2}}{2} + \frac{\ln c_l - \frac{(\theta y_l)^2}{2}}{2} \right)$$

$$s.t. 0 = y_1(1-\tau_1) - c_1 - s + T$$

$$0 = y_h(1-\tau_h) + s(1-\tau_{s,h}) - c_h + T_h$$

$$0 = y_l(1-\tau_h) + s(1-\tau_{s,l}) - c_l + T_l$$

with Lagrange multipliers λ_1, λ_h and λ_r.

First order conditions for the indviduals are;

$$\begin{aligned}
\frac{1}{c_1} &= \lambda_1 \\
y_1 &= \lambda_1 (1 - \tau_1) \\
\lambda_1 &= \lambda_h (1 - \tau_{s,h}) + \lambda_l (1 - \tau_{l,h}) \\
\frac{y_h}{2\theta^2} &= \lambda_h (1 - \tau_h) \\
\frac{\theta^2 y_l}{2} &= \lambda_l (1 - \tau_l) \\
\frac{1}{2c_h} &= \lambda_h \\
\frac{1}{2c_l} &= \lambda_l
\end{aligned} \tag{20}$$

Using this, we see that

$$\frac{1}{c_1} = \frac{1}{2c_h} (1 - \tau_{s,h}) + \frac{1}{2c_l} (1 - \tau_{l,h})$$

Setting,

$$\tau_{s,h} = -2\lambda_I$$

$$\tau_{s,l} = 2\lambda_I.$$

this gives

$$\frac{1}{c_1} = \frac{1}{2c_h} (1 + 2\lambda_I) + \frac{1}{2c_l} (1 - 2\lambda_I)$$

which is satisfied if we plug in the optimal allocation $c_h^* = c_1^* (1 + 2\lambda_I)$ and $c_l^* = c_1^* (1 - 2\lambda_I)$

$$\frac{1}{c_1^*} = \frac{1 + 2\lambda_I}{2c_1^* (1 + 2\lambda_I)} + \frac{1 - 2\lambda_I}{2c_1^* 1 - 2\lambda_I}$$

Note that the expected capital income tax rate is zero, but it will make savings lower than without any taxes. Why?

Similarly, by noting from (19) that in the optimal second best allocation, we want

$$y_1 c_1 = y_1^* c_1^* = 1,$$

which is implemented by $\tau_1 = 0$. For the high ability type, the second best allocation in (19) is that $y_h^* c_h^* = \theta^2$, which is implemented by $\tau_h = 0$ since (20) implies that $y_h c_h = \theta^2 (1 - \tau_h)$.

For the low ability type, we want $y_l^* c_l^* = \frac{1-2\lambda_I}{\theta^2(1-2\lambda_I \theta^{-4})}$. From (20), we know $y_l c_l = \frac{1-\tau_l}{\theta^2}$,so we solve

$$\begin{aligned} \frac{1 - \tau_l}{\theta^2} &= \frac{1 - 2\lambda_I}{\theta^2 (1 - 2\lambda_I \theta^{-4})} \\ &\Rightarrow \tau_l = 2\lambda_I \frac{\theta^4 - 1}{\theta^4 - 2\lambda_I}. \end{aligned}$$

Note that if $\lambda_I = \frac{1}{2}$, $\tau_l = 1$. I.e., the tax rate is 100%. There is no point going higher than that, so λ_I cannot be higher than $\frac{1}{2}$.

Finally, to find the complete allocation, we use the budget constraints of the private individual and the aggregate resource constraint. This will recover the transfers T, T_h and T_l. We should note that $T_l > T_h$ is consistent with incentive compatibiity. Why? Because if you claim to be a low ability type you will have to may a high labor income tax which is bad if you are high ability and earn a high income. Thus, by taxing high income lower, we can have a transfer system that transfers more to the low ability types.

3.3.1 Third best – laizzes faire

The allocation in without any government involvements is easily found by setting all taxes to zero.

$$
\begin{aligned}
\frac{1}{c_1} &= \lambda_1 \\
y_1 &= \lambda_1 \\
\lambda_1 &= \lambda_h + \lambda_l \\
\frac{y_h}{2\theta^2} &= \lambda_h \\
\frac{\theta^2 y_l}{2} &= \lambda_l \\
\frac{1}{2c_h} &= \lambda_h \\
\frac{1}{2c_l} &= \lambda_l
\end{aligned} \tag{21}
$$

Using these and the budget constraints, we get

$$
\begin{aligned}
y_1 &= \frac{1}{c_1} \\
\frac{1}{c_1} &= \frac{1}{2c_h} + \frac{1}{2c_l} \\
\frac{y_h}{2\theta^2} &= \frac{1}{2c_h} \\
\frac{\theta^2 y_l}{2} &= \frac{1}{2c_l}
\end{aligned}
$$

$$y_1 = c_1 + s$$

$$y_h + s = c_h$$

$$y_l + s = c_l$$

which implies

$$
\begin{aligned}
c_1 + s &= \frac{1}{c_1} \\
\frac{1}{c_1} &= \frac{1}{2c_h} + \frac{1}{2c_l} \\
c_h &= \frac{1}{2}s + \frac{1}{2}\sqrt{s^2 + 4\theta^2} \\
c_l &= \frac{\frac{1}{2}s\theta + \frac{1}{2}\sqrt{s^2\theta^2 + 4}}{\theta}
\end{aligned}
$$

I did not find an analytical solution to this, but setting $\theta = 1.1$ I found the solution $\{c_1 = 0.997\,75, c_h = 1.102\,3, s = 4.504\,5 \times 10^{-3}, c_l = 0.911\,35\}$.

3.4 Time consistency

Under the Mirrlees approach, the government announces a menu of taxes or of consumption baskets. People then make choices that in equilibrium reveal their true types (abilities) to the government. Suppose the government could then re-optimize. Would it like to do this?

The problem is more severe in a dynamic setting provided abilities are persistent. Why?

In a finite horizon economy, there might only be very bad equilibria (Roberts, 84). But better equilibria might arise in infinite horizon.