0.1 What is an Integrated Assessment Model?

An integrated assessment model (IAM) takes all the different areas we have treated so far into account—such as growth, externalities, statistical uncertainties, economic cost minimization, damage measurements and impacts on human activities, carbon cycles and feedback processes from climate change, the carbon cycle—to be able to determine a policy responce outcome, e.g., an optimal carbon taxation. And so, the model seeks to combine knowledge from different academic diciplines. The different parts of the climate change discussion are integrated into *one* numerical and logical model, to be able to say something tangible about how to economically deal with climate change. Finally, the model aims to generate concrete advise for policy making.

Building a model, the scientists need to weigh numerical precision against transparency and simplicity. This is the essence of economic modeling— strip away unnecessary detail and focus on the core structure of the problem. Modelers thus seek to capture the essence of the climate change process without adding complexity. Nordhaus (1994) uses a simple structure, which simplifies the communication about the model's results. This is one reason why the model of Nordhaus (1994) is used by many—including us—as a starting point. [1] We can get important information from this exercise—such as the optimal shape of the curve of the tax rate over time—but we should not have too much confidence in the numerical results, as a basis for policy[2].

Integrated assessment models or IAMs have emerged in recent years as one of the key tools for helping scientists and decision makers assess the problem of global climate change. Climate change has proven to be a very difficult problem for scientists and decision makers to deal with because of the uncertainties about the science of the problem, the affects climate change might have on the environment and human activities, and the potential costs of responding to the problem.[3]

In order to better understand the problem of global climate change, scientists and decision makers have turned to IAMs as a tool for integrating and assessing many of the complex interrelationships between natural and social factors that underlie the climate change problem. Most climate change IAMs, for example, are composed of sub-models that cover the climate, economy, and ecosystems. Indeed, the key strength of an IAM is its ability to combine and integrate information about different systems. Such an effort requires an interdisciplinary approach.[4]

IAMs use a computer program to link an array of component models based on mathematical representations of information from the various contributing diciplines. This approach makes it easier to ensure consistancy among the assumptions input to the various components of the models, but may tend to constrain the type of information that can be used to what is explicitly represented in the model. IAMs can be devided into two broad classes: policy optimization models and policy evaluation models. We will focus on the former type, which seeks to optimize key policy control variables such as

[1] http://www.econ.ucsb.edu/papers/wp31-98.pdf
[2] http://www.econ.ucsb.edu/papers/wp31-98.pdf
[3] (CIESIN, 1995)
[4] (CIESIN, 1995)

carbon emission control rates or carbon taxes, given formulated policy goals (e.g. maximizing welfare or minimizing the costs of meeting a carbon emission target).[5] These IAMs can—according to Weyant et. al. (1995)—be devided into three principal types:

1. *Cost-benefit models*, which attempt to balance the marginal costs of controlling GHG emissions against those of adapting to any climate change. The models vary, but agree that lower control costs, higher damage estimates as well as lower discount rates lead to higher control rates (increases the incentive to act and reduce climate change).

2. *Target-based models*, which optimize responses, given targets for emissions or climate change impacts. A target for e.g. GHG emissions can be set to avoid certain types of risks. Several models try and identify the cost-efficient emission path for reaching a particular CO_2 target, i.e., to identify the path that minimizes abatement costs.

3. *Uncertainty-based models*, which deals with decition-making under conditions of uncertainty and potential policy responses to it. This is a result of the high level of uncertainty about economic factors, natural systems (e.g., the carbon cycle) or the risk of catastrophic events. This can be done by including an uncertainty representation of all key parameters, or by adding a limited number of alternative states to a cost-benefit analysis. Assessments including uncertainty generally find higher optimal rates of abatement than models excluding uncertainty.

Some of the more advanced models can be used for several purposes. Each approach have strenghts and weaknesses and produces particular insights regarding climate change[6] at the expence of some factors, or at the expence of simplicity and transparency.

0.2 The 2 period model

The model used in this section evolves over only 2 periods. This is a simplification, but it will illustrate the point –that an unregulated market will result in too high consumption of fossil fuel in the current period, leaving less than the optimal amunt for the future generation. In section 0.3 we will show that we get the same result using a model with an infinite number of time periods.

Preferences:

$$\log c_1 + \beta \log c_2$$

Technology:

$$y_i = D_i A_i k_i^{\alpha} l_i^{1-\alpha-\gamma} e_i^{\gamma} \tag{0.1}$$

[5]Weyant et. al. (1995)
[6]Weyant et. al. (1995)

where y_i is output per capita in period i, where $i = 1, 2.$, D is damage, A is total factor productivity (TFP), k is capital per worker, l is labor, and e is fossil fuel. The labor supply is assumed to be inelastic, hence $l = 1$ in both periods.

The capital accumulation equation is $k_2 = (1 - \delta) k_1 + i_1$. The capital in period 2 is thus the fraction of the capital in period 1 that has not depreciated plus investments in period 1. We will set $\delta = 1$, i.e. capital depreciates fully across periods. This makes sence if the periods are long.*

Supply of fossil fuel can be written

$$e_1 + e_2 = R \tag{0.2}$$

where we abstract from extraction costs. The problem of economic interest is thus figuring out when to extract, i.e. the optimal extraction between period 1 and period 2.

The damages are represented by the equation

$$D_i = e^{-\nu S_i} \tag{0.3}$$

where S_i is the stock of CO_2 in the atmosphere in period i.

Furthermore, the carbon cycle is

$$S_2 = \rho S_1 + e_2$$

and

$$S_1 = \rho S_0 + e_i$$

where ρ is the fraction of CO_2 that is not stored (sequestered) in oceans, forests, tundra, etc. This implies that $1 - \rho$ is the part of CO_2 that is sequestered.

0.2.1 The Laissez faire case

If the damages from the externalities are not taken into account, the ...function is....

The objective is to find prices and quantities that optimizes the consumer's and the firm's problem.

1. The consumer takes prices as given. The consumer[7] problem is

$$\max_{c_1, c_2, k_2, e_1, e_2} \log c_1 + \beta \log c_2$$

s.t.

$$c_1 + k_2 = r_1 k_1 + w_1 + p_1 e_1 \tag{0.4}$$

$$c_2 = r_2 k_2 + w_2 + p_2 e_2 \tag{0.5}$$

$$e_1 + e_2 = R \tag{0.6}$$

[7]The *representative* consumer is seen as partly a pure consumer and a worker but also an owner of the resources fossil fuel and capital. Hence we have the income $r_1 k_1 + w_1 + p_1 e_1$.

2. The firms problem is

$$\max_{k_i,l_i,e_i} D_iA_ik_i^{\alpha}l_i^{1-\alpha-\gamma}e_i^{\gamma}-r_ik_i-w_il_i-p_ie_i \tag{0.7}$$

where $i=1,2$

3. The labor market is clearing. Hence, $l_i=1$, where $i=1,2$.

Solving it!

1. Substituting c_1 and c_2 into the utility function results in

$$\log\left(r_1k_1+w_1+p_1e_1-k_2\right)+\beta\log\left(r_2k_2+w_2+p_2e_2\right) \tag{0.8}$$

Taking the derivative with respect to k_2 and setting the equation equal to zero yields

$$\frac{1}{c_1}=\beta\frac{1}{c_2}r_2 \tag{0.9}$$

which is the wellknown "Euler equation". This equation states that....
Substituting $e_2=R-e_1$ into the objective function so that we have

$$\log\left(r_1k_1+w_1+p_1e_1-k_2\right)+\beta\log\left(r_2k_2+w_2+p_2\left(R-e_1\right)\right) \tag{0.10}$$

and taking the derivative with respect to e_1 yields

$$\frac{p_1}{c_1}=\beta\frac{p_2}{c_2} \tag{0.11}$$

Combining equation (1.9) and (1.11) results in

$$\frac{p_2}{p_1}=r_2 \tag{0.12}$$

which is the "Hotelling equation". The Hotelling equation tells us the optimal .....

2. The first order conditions for the firm's problem are

$$r_i=\alpha D_iA_ik_i^{\alpha-1}e_i^{\gamma} \tag{0.13}$$

$$w_i=\left(1-\alpha-\gamma\right)D_iA_ik_i^{\alpha}e_i^{\gamma} \tag{0.14}$$

$$p_i=\gamma D_iA_ik_i^{\alpha}e_i^{\gamma-1} \tag{0.15}$$

Substituting y_i with $D_iA_ik^{\alpha}e^{\gamma}$ (l_i is set to 1), the equations (1.13), (1.14) and (1.15) can be rewritten into

$$r_i=\frac{\alpha y_i}{k_i} \tag{0.16}$$

$$w_i=\left(1-\alpha-\gamma\right)y_i \tag{0.17}$$

$$p_i = \frac{\gamma_i y_i}{e_i} \tag{0.18}$$

respectively.

Equation (1.16), (1.17) and (1.18) reflects the fact that income shares are constant, i.e. α with regard to capital, $1-\alpha-\gamma$ for labor, and γ for energy. See algorithm (x.x).

1. Going back to the consumer, we have

$$c_1 + k_2 = r_1 k_1 + w_1 + p_1 e_1 = y_1 \tag{0.19}$$

$$c_2 = r_2 k_2 + w_2 + p_2 e_2 = y_2 \tag{0.20}$$

The Euler equation then becomes

$$\frac{1}{y_1 - k_2} = \frac{\beta\alpha \frac{y_2}{k_2}}{y_2} = \frac{\beta\alpha}{k_2}$$

or

$$(y_1 - k_2)\,\beta\alpha = k_2$$

Solving for k_2 yields

$$k_2 = \underbrace{\frac{\beta\alpha}{(1+\beta\alpha)}}_{\text{Saving share}} y_1 \tag{0.21}$$

We know that $c_1 = y_1 - k_2$. Inserting for k_2 gives

$$c_1 = y_1 - k_2 = y_1 - \frac{\beta\alpha}{(1+\beta\alpha)} y_1 = \frac{y_1(1+\beta\alpha) - y_1\beta\alpha}{1+\beta\alpha} = \frac{y_1(1+\beta\alpha-\beta\alpha)}{1+\beta\alpha}$$

and so

$$c_1 = \frac{y_1}{1+\beta\alpha} \tag{0.22}$$

Inserting for r_2, the Hotellings rule, $\frac{p_2}{p_1} = r_2$, may be written

$$\frac{p_2}{p_1} = \frac{\alpha y_2}{k_2}$$

which is the same as

$$\frac{p_2}{p_1}\frac{e_2 e_1}{e_1 e_2} = \frac{\alpha y_2}{k_2} \tag{0.23}$$

Remembering (1.18) we know that $p_2 e_2 = \gamma y_2$, and so (1.23) becomes

$$\frac{\gamma y_2 e_1}{\gamma y_1 e_2} = \frac{\alpha y_2}{k_2}$$

or

$$\frac{e_1}{e_2} = \frac{\alpha y_1}{k_2}.$$

Inserting for k_2 yields

$$\frac{e_1}{e_2} = \frac{\alpha y_1}{\frac{\beta\alpha}{(1+\beta\alpha)} y_1}$$

or

$$\frac{e_1}{e_2} = \frac{1+\beta\alpha}{\beta} > 1 \tag{0.24}$$

, i.e.

$$e_1 > e_2 \tag{0.25}$$

This result may be illustrated in different ways. We know that

$$e_1 + e_2 = R$$

Inserting for e_1 gives us

$$\frac{1+\beta\alpha}{\beta} e_2 + e_2 = R$$

or

$$\left(\frac{1+\beta\alpha+\beta}{\beta}\right) e_2 = R.$$

Solving for e_2 yields

$$e_2 = \frac{R\beta}{1+\beta\alpha+\beta}.$$

Which imply that

$$e_1 = R - \frac{R\beta}{1+\beta\alpha+\beta} = \frac{R\left(1+\beta\alpha+\beta\right) - R\beta}{1+\beta\alpha+\beta} = \frac{R+\beta\alpha R}{1+\beta\alpha+\beta}$$

or

$$e_1 = \frac{R\left(1+\beta\alpha\right)}{1+\beta\alpha+\beta}.$$

Comparing e_1 and e_2 we see that $e_1 > e_2$ if $1 + \beta\alpha > \beta$. Since β and α are both positive, we know that this will always be the case.

Surprisingly, we see that—in the free market case—fossil fuel use e_1 and e_2 and the saving rate do not depend on damage (D_i) or total factor productivity (A_i). This causes externalities and thus efficiencies for society. On the other hand, we see that the prices (p_i, w_i and r_i) do depend on D_i and A_i.

0.2.2 Social planner case

The social planner problem is somewhat different. In this case we also take the carbon in the atmosphere into account when we maximize. Equation (1.4) and (1.5) has become (1.26) and (1.27).

$$\max_{c_1,c_2,k_2,e_1,e_2,S_1,S_2} \log c_1 + \beta \log c_2$$

s.t. the constraints

$$c_1 + k_2 = D_1 A_1 k_1^{\alpha} e_1^{\gamma} \tag{0.26}$$

$$c_2 = D_2 A_2 k_2^{\alpha} e_2^{\gamma} \tag{0.27}$$

$$e_1 + e_2 = R \tag{0.28}$$

In addition, we have constraints stating the carbon cycle. The carbon in the atmosphere equals the amount of carbon not stored by the carbon sequesters, plus the carbon emissions in the current period. That is,

$$S_1 = \rho S_0 + e_1 \tag{0.29}$$

$$S_2 = \rho S_1 + e_2 \tag{0.30}$$

Furthermore, we include the damage constraints, where damage is a function of the carbon emissions as well as a factor, ν, representing the externality. The higher the ν, the greater is the effect of *

s.t.

$$D_1 = e^{-\nu S_1} \tag{0.31}$$

$$D_2 = e^{-\nu S_2} \tag{0.32}$$

Inserting for equation (1.31) and (1.32) into (1.26) and (1.27) respectively, and (1.28) into (1.30) and (1.27), as well as setting $l = 1$, we end up with the constraints

$$c_1 + k_2 = e^{-\nu S_1} A_1 k_1^{\alpha} e_1^{\gamma} \equiv y_1 \tag{0.33}$$

$$c_2 = e^{-\nu S_2} A_2 k_2^{\alpha} \left(R - e_1\right)^{\gamma} \equiv y_2 \tag{0.34}$$

$$S_1 = \rho S_0 + e_1 \tag{0.35}$$

$$S_2 = \rho S_1 + R - e_1 \tag{0.36}$$

where we have defined the two former equations as y_1 and y_2. To solve the model, we set up the Lagrange problem.

$$\mathcal{L} = \log c_1 + \beta \log c_2 - \lambda_1 \left(c_1 + k_2 - e^{-\nu S_1} A_1 k_1^{\alpha} e_1^{\gamma}\right) - \lambda_2 \left(c_2 - e^{-\nu S_2} A_2 k_2^{\alpha} \left(R - e_1\right)^{\gamma}\right)$$

$$-\mu_1 \left(S_1 - \rho S_0 - e_1\right) - \mu_2 \left(S_2 - \rho S_1 - R + e_1\right) \tag{0.37}$$

The multipliers may be interprated in the following way:

$\lambda_1 =$ Marginal utility of resources in period 1

$\lambda_2 =$ Marginal utility of resources in period 2

$\mu_1 =$Marginal utility of CO_2 in the atmosphere in period 1

μ_2 =Marginal utility of CO_2 in the atmosphere in period 2
where μ_1 and μ_2 will be negative values.

Taking first order conditions
From $\frac{\partial \mathcal{L}}{\partial c_1}$:

$$\frac{1}{c_1} = \lambda_1 \tag{0.38}$$

From $\frac{\partial \mathcal{L}}{\partial c_2}$:

$$\frac{\beta}{c_2} = \lambda_2 \tag{0.39}$$

$$\frac{\partial \mathcal{L}}{\partial k_2} = -\lambda_1 - \lambda_2 \left(-e^{-\nu S_2} A_2 \alpha k_2^{\alpha-1} e_2^{\gamma}\right) = 0$$

which becomes

$$\lambda_2 e^{-\nu S_2} A_2 \alpha k_2^{\alpha-1} e_2^{\gamma} = \lambda_1 \tag{0.40}$$

i.e.,

$$\frac{\lambda_1}{\lambda_2} = \frac{y_2 \alpha}{k_2} \tag{0.41}$$

$\frac{\lambda_1}{\lambda_2}$ is thus the real return to capital. Equation (1.41) may be viewed as the Eulers equation in the social planner case, i.e. the social planner equivalent to equation (1.x).

$$\frac{\partial \mathcal{L}}{\partial e_1} = -\lambda_1 \left(-e^{-\nu S_1} A_1 k_1^{\alpha} \gamma e_1^{\gamma-1}\right) - \lambda_2 \left(-e^{-\nu S_2} A_2 k_2^{\alpha} \gamma e_2^{\gamma-1} (-1)\right) - \mu_1 (-1) - \mu_2 = 0$$

which becomes

$$\lambda_1 \left(e^{-\nu S_1} A_1 k_1^{\alpha} \gamma e_1^{\gamma-1}\right) - \lambda_2 \left(e^{-\nu S_2} A_2 k_2^{\alpha} \gamma e_2^{\gamma-1}\right) + \mu_1 - \mu_2 = 0$$

$$\lambda_2 \left(e^{-\nu S_2} A_2 k_2^{\alpha} \gamma e_2^{\gamma-1}\right) - \mu_2 = \lambda_1 \left(e^{-\nu S_1} A_1 k_1^{\alpha} \gamma e_1^{\gamma-1}\right) + \mu_1$$

$$\frac{\lambda_2 e^{-\nu S_2} A_2 k_2^{\alpha} \gamma e_2^{\gamma-1} - \mu_2}{\lambda_1 e^{-\nu S_1} A_1 k_1^{\alpha} \gamma e_1^{\gamma-1} + \mu_1} = 1$$

$$\frac{\frac{\lambda_2 y_2 \gamma}{e_2} - \mu_2}{\frac{\lambda_1 y_1 \gamma}{e_1} + \mu_1} = 1 \tag{0.42}$$

Multiplying both sides with $\frac{\lambda_1}{\lambda_2}$ yields

$$\frac{\frac{y_2 \gamma}{e_2} - \frac{\mu_2}{\lambda_2}}{\frac{y_1 \gamma}{e_1} + \frac{\mu_1}{\lambda_1}} = \frac{\lambda_1}{\lambda_2} \tag{0.43}$$

Equation (1.40) is the *modified* Hotelling formula illustrating that

$$\frac{\text{The marginal social value of } e_2}{\text{The marginal social value of } e_1} = \text{Return to capital} \tag{0.44}$$

Compared to equation (1.12) in the free market case in section 1.1., the Hotelling formula in the social planner case includes external effects from fossil fuel, γ, and not only market prices. Hence, the social value is measured, in contrast to in equation (1.12), where costs exeeding market price are ignored.*[8]

$$\frac{\partial \mathcal{L}}{\partial s_2} = -\nu\left(-\lambda_2\right)\left(-e^{-\nu S_2}\right) A_2 k_2^{\alpha}\left(R - e_1\right)^{\gamma} - \mu_2 = 0$$

$$-\nu\lambda_2 y_2 = \mu_2$$

or

$$-\nu y_2 = \frac{\mu_2}{\lambda_2} \tag{0.45}$$

$$\mu_2 = -\nu y_2 \lambda_2 \tag{0.46}$$

$$\frac{\partial \mathcal{L}}{\partial s_1} = -\lambda_1\left(-\nu\right)\left(-e^{-\nu S_1}\right) A_1 k_1^{\alpha} e_1^{\gamma} - \mu_1 - \mu_2\left(-\rho\right) = 0$$

$$-\lambda_1 \nu y_1 = \mu_1 - \rho\mu_2$$

$$-\nu y_1 = \frac{\mu_1}{\lambda_1} - \frac{\rho\mu_2}{\lambda_1}$$

$$\frac{\mu_1}{\lambda_1} = -\nu y_1 + \frac{\rho\mu_2}{\lambda_1} \tag{0.47}$$

Inserting equation (1.46) into (1.47)

$$\frac{\mu_1}{\lambda_1} = -\nu y_1 - \rho\frac{\lambda_2}{\lambda_1} y_2 \nu$$

Inserting (1.45) and (1.47) into the modified Hotelling equation (1.43) yields

$$\frac{\frac{y_2\gamma}{e_2} - \nu y_2}{\frac{y_1\gamma}{e_1} + \left(-\nu y_1 + \frac{\rho\mu_2}{\lambda_1}\right)} = \frac{\lambda_1}{\lambda_2} \tag{0.48}$$

Next, inserting (1.46) again into (1.48) results in

$$\frac{\frac{y_2\gamma}{e_2} - \nu y_2}{\frac{y_1\gamma}{e_1} - \nu\left(y_1 + \frac{\rho\lambda_2 y_2}{\lambda_1}\right)} = \frac{\lambda_1}{\lambda_2} \tag{0.49}$$

[8] Include something about the last sentence on page 9 in Per's notes

According to the Euler equation (1.41) we know that $\frac{\lambda_1}{\lambda_2} = \frac{\alpha y_2}{k_2}$. We may thus combine equation (1.49) and (1.41) to get

$$\frac{\frac{y_2\gamma}{e_2} - \nu y_2}{\frac{y_1\gamma}{e_1} - \nu\left(y_1 + \frac{\rho\lambda_2 y_2}{\lambda_1}\right)} = \frac{\alpha y_2}{k_2} \tag{0.50}$$

Inserting for our expression for k_2 from equation (1.21) results in

$$\frac{\gamma\frac{y_2}{e_2} - \nu y_2}{\gamma\frac{y_1}{e_1} - \nu\left(y_1 + \rho\frac{\lambda_2}{\lambda_1}y_2\right)} = \frac{\alpha y_2}{\frac{\alpha\beta}{(1+\alpha\beta)}y_1} = \frac{1}{\beta}\cdot\frac{y_2}{y_1}(1+\alpha\beta) \tag{0.51}$$

Simplifying by substituting the y_2 in the denominator of (1.51) with equation (1.41) solved for y_2 where k_2 is replaced with the expression for k_2 from equation (1.21). Furthermore, we multiply (1.51) with $\frac{y_1}{y_2}$ on both sides. The result is

$$\frac{\gamma\frac{1}{e_2} - \nu}{\gamma\frac{1}{e_1} - \nu\left(1 + \rho\frac{\beta}{(1+\alpha\beta)}\right)} = \frac{1}{\beta}\cdot(1+\alpha\beta) \tag{0.52}$$

Substituting e_2 with $R - e_1$ to be able to solve for e_1 as a function of only exogenous factors

$$\gamma\frac{1}{R-e_1} - \nu = \frac{1}{\beta}\cdot(1+\alpha\beta)\left[\gamma\frac{1}{e_1} - \nu\left(1+\rho\frac{\beta}{(1+\alpha\beta)}\right)\right]$$

Multiplying with $(R-e_1)\,e_1$, factorizing e_1 out, structuring the expression as a second degree function[9]

$$\underbrace{e_1^2\left[\nu - \frac{1}{\beta}\cdot(1+\alpha\beta)\,\nu\left(1+\rho\frac{\beta}{(1+\alpha\beta)}\right)\right]}_{=\text{constant}}$$
$$+e_1\underbrace{\left[\gamma\text{-}\nu R + \frac{1}{\beta}\cdot(1+\alpha\beta)\,\gamma + \nu\frac{1}{\beta}\cdot(1+\alpha\beta)\left(1+\rho\frac{\beta}{(1+\alpha\beta)}\right)R\right]}_{=\text{ constant}}$$
$$\underbrace{-\frac{1}{\beta}\cdot(1+\alpha\beta)\,R\gamma}_{=\text{ constant}} = 0$$

Despite its complex look, this equation is simply a normal second degree equation, seeing as we only have one endogenous variable, e_1. Inserting values for the parameters or solving the equation in a mathematical program yields two possible solutions for e_1. One of these solutions is negative. Since we cannot have a negative amount for fossil fuel, there is only one possible solution for the equation. We already know that $e_1 + e_2 = R$. Adding the fact that e_1 must be positive, we know that $e_1$must be in the interval $[0, R]$.

[9]Will remove the following equation if they do not disagree. Unnecessary detailed level.

From equation (x.x) we can also calculate that e needs to be$< \frac{\gamma}{\nu}$ (or something. Where was this from?).[10]

We see that expression (1.50) could be rearranged to be the same as in the Laissez faire case in equation (1.24), if we ignore the ν-terms.

We would like to know if the social planner case results in a different optimal consumption path compared to the Laisez faire case. If this is the case, it is optimal for society to avoid the laissez faire situation, seeing as the social planner case includes the social costs ignored by the inividual decision-makers in the free market. We thus want to compare the $\frac{e_1}{e_2}$ ratios for the two cases.

The Laissez faire solution was (as we see in equation (1.24)

$$\frac{e_1}{e_2} = \frac{1+\beta\alpha}{\beta} > 1$$

In the social planner case we have the equation

$$\frac{\gamma\frac{1}{e_2} - \nu}{\gamma\frac{1}{e_1} - \nu\left(1+\rho\frac{\beta}{(1+\alpha\beta)}\right)} = \frac{(1+\alpha\beta)}{\beta}$$

Removing the ν terms yields the same solution as in the laissez faire case, i.e. $\frac{\gamma\frac{1}{e_2}}{\gamma\frac{1}{e_1}} = \frac{(1+\alpha\beta)}{\beta}$ which becomes $\frac{e_1}{e_2} = \frac{(1+\alpha\beta)}{\beta}$. We compare $\frac{\frac{1}{e_{2LF}}}{\frac{1}{e_{1LF}}} = \frac{(1+\alpha\beta)}{\beta}$ with $\frac{\frac{1}{e_{2SP}}-\frac{\nu}{\gamma}}{\frac{1}{e_{1SP}}-\frac{\nu\left(1+\rho\frac{\beta}{(1+\alpha\beta)}\right)}{\gamma}} = \frac{(1+\alpha\beta)}{\beta}$ where LF denotes laissez faire and SP is social planner. For simplicity we set $\rho\frac{\beta}{(1+\alpha\beta)} \equiv X$, where $X > 0$ since β , ρ and α are all positive parameters.
Hence, we have

$$\frac{\frac{1}{e_{2SP}} - \frac{\nu}{\gamma}}{\frac{1}{e_{1SP}} - \frac{\nu}{\gamma}(1+X)} = \frac{(1+\alpha\beta)}{\beta} \tag{0.53}$$

We want to decide if the ν terms make the social planner $\frac{e_1}{e_2}$ smaller or larger than the laissez faire eqvivalent.

We know that $\frac{\frac{1}{e_{2SP}}-\frac{\nu}{\gamma}}{\frac{1}{e_{1SP}}-\frac{\nu}{\gamma}(1+X)} = \frac{\frac{1}{e_{2LF}}}{\frac{1}{e_{1LF}}}$ since both the LHS and the RHS are equal to $\frac{(1+\alpha\beta)}{\beta}$. $\rho\frac{\beta}{(1+\alpha\beta)} \equiv X$, where $X > 0$ since β , ρ and α are all positive parameters. Since X must be positive, the ν term in the denominator is larger than the ν term in the numerator in the expression $\frac{\frac{1}{e_{2SP}}-\frac{\nu}{\gamma}}{\frac{1}{e_{1SP}}-\frac{\nu}{\gamma}(1+X)}$. Hence, the denominator is reduced relative to if it simply said $\frac{\frac{1}{e_{2SP}}}{\frac{1}{e_{1SP}}}$. For the LHS to still be equal to the RHS with only e_1 and e_2 as endogenous variables, we see that $\frac{1}{e_{1SP}}$ needs to be larger than $\frac{1}{e_{1LF}}$. Hence, e_1^{SP}

[10]This paragraph is probably unnecessary. Can be shortened.

needs to be smaller than e_1^{LF} for the LHS to be equal to the RHS. In the social planner case (the optimum for society), the fossil fuel use in the first period is reduced relative to the following period. I.e., $\frac{e_1^{LF}}{e_1^{LF}} > \frac{e_1^{SP}}{e_2^{SP}}$. This result means that in the Laissez faire case we will end up with a higher e_1 compared to in the social planner case. Hence, without market regulation, we will consume too much fossil fuel in the first period. Generally, we may say that this illustrates that market regulation ought to slow/postpone the use of fossil fuel.

To be able to achieve the optimal result (i.e., the social planner solution)–knowing that the laissez faire solution results in too much consumption of fossil fuel too soon– we need to find a way to create incentives to postpone and decrease current consumption. This could be done by adding taxes to fossil fuel that are high in the current period. In the second period the tax rate should decline seeing as [nice formulation, blabla] (or perhaps no tax in the latter period? In the infinite horizon model the tax rate should similarly start off high and then decline in the following periods).

0.3 An infinite time horizon

This section changes the assumption we have made about the time horizon. In stead of two periods, we now assume an infinite number of periods. Consequently, we operate with integrals. Do we? Do we go from discrete to continous values? No i don't think so. Still discrete time periods.

0.3.1 The Laissez faire case

blabla

0.3.2 The social planner case

blabla

0.4 Comparision with other IAM's[11]

There are many attempts to answer the question *How much and how fast should we react to the threat of global warming?* The early 90s debate between William R. Cline (The economics of global warming, 1992. AvWilliam R. Cline) and Nordhaus (year), two early pioneers of modeling the economic effects of climate change, has strong resemblances with the later debate between Nordhaus (year) and Stern (year). British climate expert Sir Nicholas Stern (The "Stern Review on the Economics of Climate Change", year) emphasized that actions against climate change could be made in a cost efficient mannor. The review portraites the damages from climate change as large and demands sharp and immediate reduction in green house gases. Stern's results are strongly opposed by

[11] A lot of copying and pasting. Working on it now

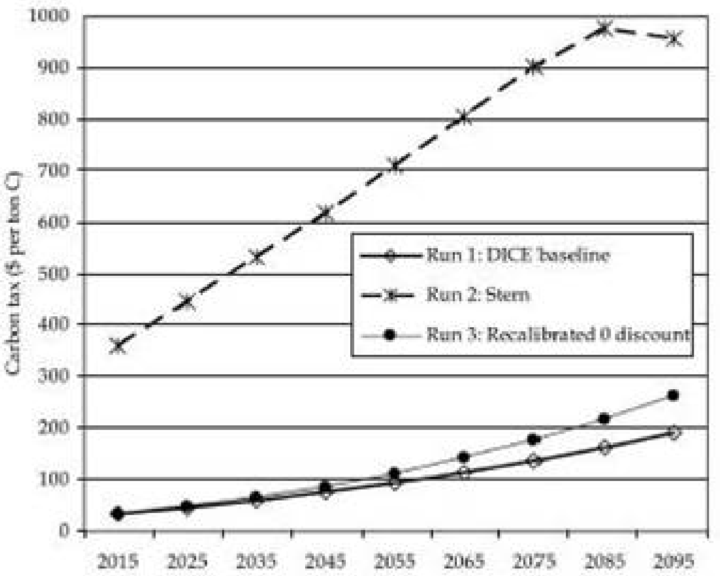

Figure 0.1:

Nordhaus (year), who believes Stern's results depend deceisively on the use of—among other assumptions—a near-zero time discount rate.

Assigning weight to benefits received far in the future is quite difficult. The most debated critical parameter is thus perhaps the rate of time discount (for example, see Arrow, et. al., 1996, Cline, 1992, and Nordhaus, 1994, Stern, year). Most of the damages from climate change takes place far in the future. And so, the more weight that is placed on the future, the optimal price on climate change rise.[12] Treating future generations symmetrically with present generations, would imply a zero discount rate. As the discount rate increases, the value of the welfare for the future generations decrease. Stern uses a discount rate of 0.1% whereas Nordhaus uses a rate of 3 %.

Nordhaus (year) recommends an optimal carbon tax that is low in the beginning but increases as the damages relative to output increase. According to this theory, the return on alternative investments as the optimal benchmark for climate investments. "The return on alternative investments" is—in Nordhaus opinion—the risk free market real interest rate. This view is seen as "consumption smoothing" or as the "climate policy ramp" because of the smooth and upwards sloping shape of the curve of the recommended carbon taxes, see the lowest curve in figure 0.1.

In our model, the optimal solution is high taxes in the beginning, followed by lower taxes. That is, we end up with the opposite result. The intuitive reason is [what??? the following is only suggestions] that the damage function escalate if we postpone our actions. That is, the return from taking actions against climate change is higher now than later. The formal reason for why we end up with such a different result using the same basic framework (allthough simplified—but these simplifications do not change the

[12] http://www.econ.ucsb.edu/papers/wp31-98.pdf

result), is because of our inclusion of backstop technology as well as [X*w*h*a*t*?*X] into the model. [Risk? Lagged externalities? Noooo.]

IAMs vary widely in several key areas. IAMs differ in the complexity of the economic and climate sectors. An important difference between IAMs is the treatment of uncertainty, a fundamental concern in climate change policy. Finally, IAMs differ in the responsiveness of agents, i.e., the policy implications .[13]

Probably the most high profile European models are the IMAGE models (IMAGE 1.0 and IMAGE 2.0) developed by the National Institute of Public Health and the Environment (RIVM) of The Netherlands. The European Community supported the development the ESCAPE model which was a joint project between the Climatic Research Unite (CRU) at the University of East Anglia in the UK and RIVM in The Netherlands. Other examples of IAMs developed in Europe include the PAGE model developed at Cambridge University and MAGICC model developed at CRU at the University of East Anglia in the UK. The Dutch National Institute of Public Health and the Environment or RIVM is a leader in the development of climate change IAMs. A large interdisciplinary team of researchers at RIVM has developed several IAMs including IMAGE (Integrated Model to Assess the Greenhouse Effect) 1.0, IMAGE 2.0[14]

US

Pacific Northwest Laboratory is developing the MiniCAM model and its larger, and more complex cousin ProCAM. Researchers at the Electric Power Research Institute (EPRI) and Stanford University have developed the MERGE model,

In addition, individual researchers or small teams of researchers have also developed IAMs. Notable among these efforts is the DICE model developed by William Nordhaus at Yale University [15].

0.5 References

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[13] http://www.econ.ucsb.edu/papers/wp31-98.pdf

[14] (CIESIN, 1995)

[15] (CIESIN, 1995)

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