The Climate and the Economy
(Preliminary - do not quote or distribute without permission).

John Hassler, Per Krusell and Conny Olovsson This version: April 2019

April 26, 2019

Contents

1 Integrated Assessment Models – An introduction

The purpose of this book is to describe how the economy and the climate are linked. Before going into any details, let us begin with a very stylized description of the dynamic system we call the earth. We will find it convenient to describe the earth system as consisting of three sub-systems. We call these *the economy, the carbon circulation and the climate.* The economy consists of individuals that make decisions as consumers, producers or perhaps as politicians. These decisions affect emissions and other determinants of climate change as well as responding to current and expected changes in the climate by adaptation. When fossil fuel is burned, carbon dioxide is released and spreads quickly in the atmosphere. The atmosphere is part of the Carbon Circulation sub-system where carbon is transported between different reservoirs and is thus one such reservoir. The biosphere (plants, and to a much smaller extent, animals including humans) and the soil is another. The largest reservoir of carbon consists of the oceans. The Climate is a system that determines the distribution of weather events over time and space. The three subsystems and the way they are interconnected will be described in some detail below. However, let us start with a brief schematic description of some of the ways in which the three parts of the global system affect each other.

First, consider the counter-clockwise relations described in Figure 1. *The economy* uses fossil fuel for energy and in the process, carbon dioxide(CO_2) is emitted into the atmosphere, being part of the *carbon circulation* sub-system. The flows of carbon between the different carbon reservoirs are modeled in the carbon circulation sub-system where the atmospheric concentration of CO_2 over time is determined by the flows between the carbon reservoirs and the additional inflow due to emissions. The CO_2 concentration in the atmosphere, in turn, affects the energy budget in the climate system due to the greenhouse effect. This effect works like a blanket, reducing the long-wave radiation of energy from earth. In the climate system, various aspects of the climate, like the global mean temperature, are then determined as a result of the change in the energy budget. Finally, the climate affects the economy in many ways – a short list of examples may include effects on agricultural productivity, the need for heating and cooling, mortality and the pleasure from outdoor activities. These effects are mitigated or amplified by the actions of agents in the economy, e.g., consumers, producers and politicians.

We can also identify causal effects in the opposite, clockwise, direction. Changes in the climate affect the storage capacity of different carbon reservoirs. Changes in temperature and precipitation affect the biosphere's capacity to store carbon in the form of plants and a higher temperature leads to warmer oceans, which can absorb less CO_2. The amount of CO_2 that is circulating in the atmosphere has a direct effect on agriculture by affecting the photosynthesis. Finally, the economy can affect the climate in other ways than via direct carbon emissions. One example is influence of people on the way the earth reflects incoming sunlight (the albedo effect) by partly changing the color of the surface of the earth. Black roofs and roads hamper the reflection of solar radiation while bright surfaces would reflect more solar radiation. Similarly, the emission of particles and aerosols affects the way the earth reflects incoming sunlight. Moreover, emissions of other greenhouse gases than carbon dioxide also affect the global climate. Methane, for instance, is set free by fossil fuel production and biomass burning but also by animal husbandry such as cattle farming and rice cultivation.

As we can see, these links are bidirectional and dynamic. Naturally, the degree of complexity of the overall model as well as the sub-models may differ. With more complex sub-models, more complex links can also be described. The simplest example of a climate model only describes the global mean temperature as a function of the CO_2 concentration in the atmosphere. Then, the damage caused by climate change is a function of a single climate variable, namely the global mean

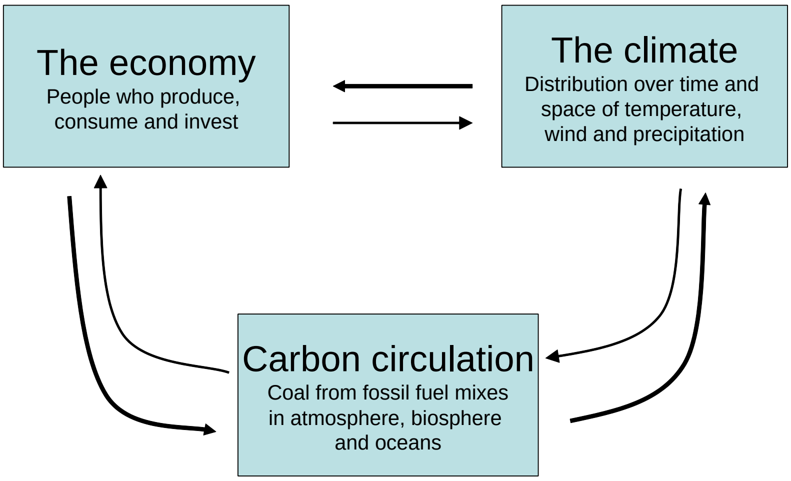


Figure 1: The three sub-systems of an intergrated assessment model.

temperature, although this function may change over time, for example due to various adaptation mechanisms in the economic system. A more complex model of the climate may also predict the regional and temporal distribution of various weather events like severe storms and draughts. Then, the damages this inflicts on the economy can, of course, be modelled in more detail and with a higher degree of realism. In this sense, complexity is good but the simplicity and transparency of more stylized models also have a clear value.

In the remainder of this book, we will describe the three sub-models depicted in Figure 1 and the interaction between them in some detail.

2 The climate

2.1 Forcing and the energy budget

As noted above, the primary effect of higher CO_2 concentration on the climate is due to the fact that the greenhouse effect changes the energy budget of the earth.[1] Let us use a simple example to illustrate the concept of the energy budget. Consider a pot that is placed on a stove. As long as the stove is switched off, the net flow of energy between the stove and the pot is zero – the pot's energy budget is balanced. When the stove is turned on, however, it starts transfer heat to the pot through conduction that warms the pot. The pot's energy budget is now in surplus and heat is accumulated in the pot. Will this continue forever? No, the reason being that as the pot gets hotter, it will itself radiate more and more energy to its surroundings. Therefore, eventually a new balance point will be reached. At the balance point, the pot's energy budget is again in balance – the net flow is zero.

[1] Sometimes the word energy balance is used synonymously with energy budget.

Let us now become a little more formal. Suppose that the earth is in a radiative equilibrium where the incoming flow of short-wave radiation (sun light) is equal to the outgoing flow of (largely infrared thermal) radiation.[2] The energy budget of the earth is then balanced, implying that the heat content and the temperature is constant. Now consider a perturbation of this equilibrium that makes the net flow positive by an amount F. Such an increase could be achieved by an increase in the incoming flow and/or a reduction in the outgoing flow. Regardless of how it is achieved, it implies that the earth's energy budget is in surplus causing an accumulation of heat in the earth. The speed at which the temperature increases is higher the larger is the difference between the inflow and outflow of energy, i.e., the larger the surplus in the energy budget.

As the heat content increases, the temperature rises which, as in the case of the pot, leads to a larger outgoing flow. Sometimes this simple mechanism is referred to as the 'Planck feedback'. As an approximation, let this increase be proportional to the increase in temperature over its initial value. Denoting the temperature perturbation at time t by T_t and the proportionality factor between energy flows and temperature by κ, we can summarize these relations in the following equation[3]

$$\frac{dT_t}{dt} = \sigma\left(F - \kappa T_t\right). \tag{1}$$

The left-hand side of the equation is the rate of change of the temperature at time t. The term in parenthesis on the right-hand side is the net energy flow, i.e., the difference in incoming and outgoing flows. The parameter σ determines how fast the temperature changes, given a net energy flow. When the right hand side is positive, the energy budget is in surplus, heat is accumulated and the temperature increases. Vice versa, if the energy budget had a deficit (a negative term in the parenthesis), heat is lost and the the temperature falls. When discussing climate change, the variable F is typically called *forcing,* and is then defined as the overall change of the energy budget caused by human activities. Clearly, since both F and T_t in (1) are finite, the RHS is finite. Thus, the rate of change in temperature is finite and any change that may occur takes time. In other words, the temperature cannot jump to a new level.

We can use equation (1) to find how much the temperature needs to rise before the system has reached a new equilibrium, i.e., when the temperature has settled down to a constant. Such an equilibrium requires that the energy budget has become balanced, i.e., when the term in parenthesis in (1) again has become zero. Let the steady-state temperature associated with a forcing F be denoted $T\left(F\right)$. At $T\left(F\right)$, the temperature is constant, which requires that the energy budget is balanced, i.e., that $F - \kappa T\left(F\right) = 0$. Thus,

$$T\left(F\right) = \frac{F}{\kappa}. \tag{2}$$

Measuring temperature in degrees Kelvin, and F in Watts per square meter, the unit of κ is $\frac{W/m^2}{K}$.[4] If the earth were just a black ball without atmosphere, we could calculate the exact value of κ from simple laws of physics. In fact, at the earth's current mean temperature $\frac{1}{\kappa}$ would be approximately 0.3 , i.e., an increased in forcing of 1 W/m^2 would lead to an increase in the

[2] We neglect the additional outflow due to the nuclear process in the interior of the earth being in the order of one to ten thousand when compared to the incoming flux from the sun. (e KamLAND Collaboration, (2011), Nature Geoscience 4, 647–651)

[3] Equation (1) is an example of linear difference equations. The difference in the endogenous variable $\left(T_t^A\right)$ is a linear function of the endogenous variable and an exogenous variable (F).

[4] Formally, a flow rate per area unit is denoted flux. However, since we deal with systems with constant areas, flows and fluxes are proportional and the terms are used interchangeably.

global temperature of 0.3 degrees K (an equal amount in degrees Celsius).[5] However, the earth is (luckily) not a black ball without atmosphere. Various feedback mechanisms, like cloud formation and variation in the size of the ice cap around the poles, make a simple calculation of κ impossible. Instead, we need to use more complicated climate models to predict this sensitivity. The conclusion from such models is that $\frac{1}{\kappa}$ is likely to be much larger than for a black body but how much larger is very difficult to say. Often a value two to three times larger than the black ball value of 0.3 is used. Furthermore, we cannot be certain that the linear approximation between F and the change in temperature remains reasonably accurate for large values of F.

So far, we have taken F as given, with the understanding that it is caused by the greenhouse effect. The relationship between the atmospheric CO_2 concentration and forcing can be fairly well approximated by a logarithmic function. Thus, forcing, i.e., the change in the energy budget relative to preindustrial times, can be written as a logarithmic function of the increase in CO_2 concentration *relative* to the preindustrial level or, equivalently, as a logarithmic function of the amount of carbon in the atmosphere relative to the amount in preindustrial times. Let S_t and S_0, respectively, denote the current and preindustrial amount of carbon in the atmosphere. Then, forcing can be well approximated by the equation[6]

$$F = \frac{\eta}{\log 2} \log\left(\frac{S_t}{S_0}\right). \tag{3}$$

The parameter η has a straightforward interpretation; if the amount of carbon in the atmosphere in period t has doubled relative to preindustrial times, forcing is η. If it quadruples, it is 2η and so forth. An approximate value for η is 3.7, implying that a doubling of the amount of carbon in the atmosphere leads to a forcing of 3.7 watts per square meter on earth.[7]

We are now ready to present a relation between the long-run change in the temperature of the earth as a function of the carbon concentration in the atmosphere. Combining equations (2) and (3) we get

$$T(F) = \frac{\eta}{\kappa}\frac{1}{\log 2} \log\left(\frac{S_t}{S_0}\right). \tag{4}$$

As we can see, a doubling of the carbon concentration in the atmosphere leads to an increase in temperature given by $\frac{\eta}{\kappa}$. Using the Planck feedback, $\eta/\kappa \approx 1.1$°C. This is a modest sensitivity, and as already noted very likely a too low estimate of the overall sensitivity of the global climate due to the existence of positive feedbacks.

2.2 Feedbacks

Let us now return to the feedback mechanisms responsible for the relation between forcing and temperature that we mentioned above. Energy flows through the atmosphere are depicted in Figure 2. Averaged over the planet, the incoming energy flow amounts to 340W/m^2. In comparison, we may use a standard domestic electrical heater of 1000 Watts or a light bulb of 60 Watts. The area of earth is 510 trillion m^2 implying that the total energy inflow is around 170 000 TW. The global

[5] See Schwartz et al. (2010) who report that if earth were a blackbody radiator with a temperature of $288K \approx$ 15 degrees Celsius, an increase in the temperature of 1.1 K would increase the outflow by 3.7 W/m^2, implying $\kappa^{-1} = 1.1/3.7 \approx 0.3$.

[6] This relation was first demonstrated by the Swedish physicist and chemist and 1903 Nobel Prize Winner in Chemistry. Therefore, the relation is often referred to as the *Arrhenius Greenhouse Law.* Arrhenius (1896).

[7] See Schwartz et al., (2014). The value 3.7 is,however, not undisputed. Otto et al., (2013) use a value of 3.44 in their calculations.

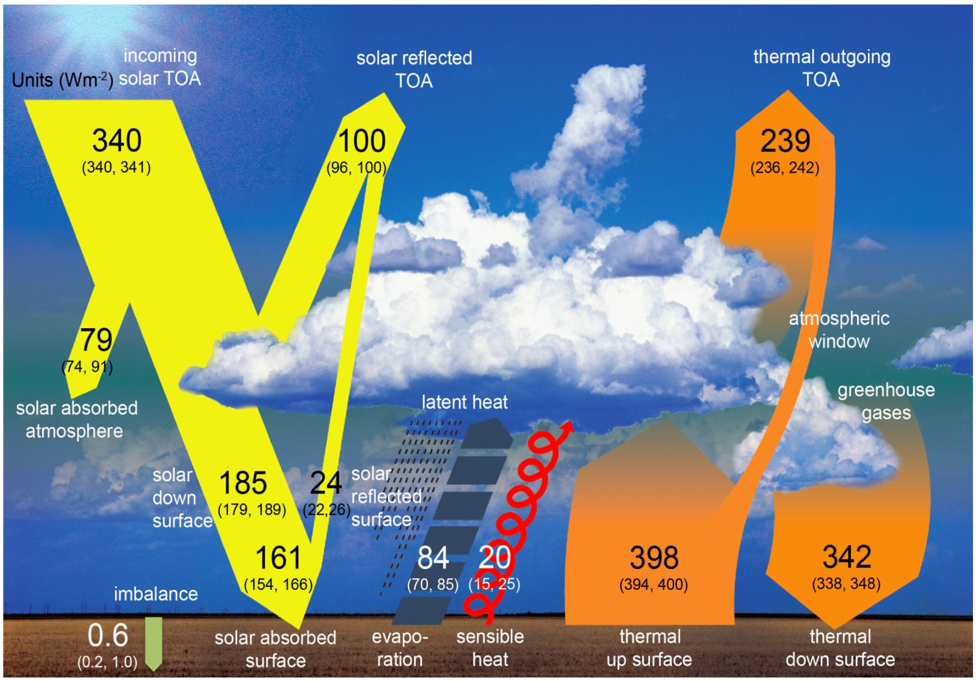


Figure 2: Global mean energy budget under present-day climate conditions. Numbers state magnitudes of the individual energy fluxes in W/m^2, adjusted within their uncertainty ranges to close the energy budgets. Numbers in parentheses attached to the energy fluxes cover the range of values in line with observational constraints. Source: IPCC (2013), Fig 2.11

energy consumption in 2008 was 12.3 Gtoe (Gigaton oil equivalents) equivalent to approximately 140 000$TWhours$.[8]This is thus less than the energy received from the sun in one hour.

Around one third of this incoming energy flow (100/m^2) is directly reflected into space by aerosols (i.e. small particles in the atmosphere), by clouds, atmospheric gases and by bright-colored reflecting parts of the earth's surface such as ice, snow and deserts. The degree to which an object reflects in this way is called its *albedo*. The remainder of the incoming flow, i.e., 240W/m^2 is absorbed by the surface of the earth and the atmosphere. We see that this is almost, but nut fully, balanced by an outgoing flow of long-wave (thermal) radiation at 239W/m^2. The small surplus in the energy budget leads to accumulation of heat in the earth system, i.e., to global warming.

In figure 2, we also see that most of the long-wave radiation emitted from the surface is absorbed by clouds and greenhouse gases and then emitted back into the atmosphere. Hence, it does not immediately leave the atmosphere. That some gases let sunlight pass easily but reflects long-wave radiation is called *the greenhouse effect* and it would, of course, be absent on a planet without atmosphere.[9] The strength of the greenhouse effect depends on the concentration of greenhouse gas in the air. The most important greenhouse gas is water vapor, followed by carbon dioxide (CO_2). Other greenhouse gases are methane, nitrous oxides and ozone. Human activities increase the concentration of these gases in the air and amplify the greenhouse effect. The climate becomes warmer.

A straightforward way of including feedbacks in the energy budget is by adding a term to the

[8]Of this, 70% came from burning fossil fuel. EA World Energy Outlook (2010).

[9]The greenhouse effect is, in fact, necessary for life on earth. Without natural greenhouse gases in the atmosphere, the average ground temperature on earth would be around -19°C.(SOURCE)

energy budget. Suppose initially that feedbacks can be approximated by a linear term xT_t, where x captures the marginal impact on the energy budget due to feedbacks. The energy budget now becomes

$$\frac{dT_t}{dt} = \sigma \left(F + xT_t - \kappa T_t \right), \tag{5}$$

where we think of κ as solely determined by the Planck feedback. The steady state temperature is now given by

$$T(F) = \frac{\eta}{\kappa - x} \frac{1}{\ln 2} \ln \left(\frac{S}{\bar{S}} \right). \tag{6}$$

Since the ratio $\eta/(\kappa - x)$ has such an important interpretation, it is often labelled the *Equilibrium Climate Sensitivity (ECS)*[10] and we will use the symbol λ to denote it. Some feedbacks are positive but not necessarily all and theoretically, we cannot rule out neither $x < 0$ nor $x \geq \kappa$. In the latter case, dynamics would be explosive which appears inconsistent with historical reactions to natural variations in the energy budget. Also $x < 0$ is difficult to reconcile with the observation that relatively small changes in forcing in the history of earth has had substantial impact in the climate. However, within these bands a large degree of uncertainty remains.

According to the IPCC, the ECS is "likely in the range 1.5 to 4.5°C", "extremely unlikely less than 1°C", and "very unlikely greater than 6°C".[11] Another concept, taking some account of the shorter run dynamics, is the *Transient Climate Response (TCR).* This is the defined as the increase in global mean temperature at the time the CO_2 concentration has doubled following a 70 year period of annual increases of 1%.[12] IPCC (2013b, Box 12.1) states that the TCR is "likely in the range 1°C to 2.5°C" and "extremely unlikely greater than 3°C."

The direct effect on forcing of a given increase in the atmospheric concentration of, e.g., CO_2 can be established quite accurately. However, **i**t is much more difficult to establish how large an increase in the average temperature in which this results. As seen in Figure 2, the energy flows are large relative to the direct effect of an increase in the concentration of greenhouse gases. Indirect effects will occur and are typically difficult to quantify accurately. For example, an increased concentration of CO_2 has a direct influence on forcing via the greenhouse effect, but the higher temperature also leads to an increase in air humidity. Water vapor acts as a greenhouse gas and this amplifies the direct effect. Such an amplification of the direct effect is called a *positive feedback mechanism.* Indirect effects may also dampen the initial effect, in which case they are called negative feedback mechanisms. There are many other feedback mechanisms than water vaporization. Higher temperatures lead to melting icecaps and thereby to a decreasing albedo effect (an ice covered area reflects sunlight better than an area not covered by ice, e.g., water). The formation of clouds is also of key importance for the energy flows and indirect effects on cloud formation may cause both positive and negative feedbacks. Because the flows are large relative to the direct effect of emission of greenhouse gases, just a small error in the calculation of the indirect feedback effects will have large impacts on the error in the calculation of the overall effect. This is the reason for the difficulties in establishing an exact value for climate sensitivity $\frac{\eta}{\kappa - x}$ in equation (2).

[10] Note that equilibrium here refers to the energy budget. For an economist, it might have been more natural to call λ the *steady state climate sensitivity.*

[11] (IPCC, 2013a, page 81 and IPCC, 2013b, Box 12.1). The report states that "likely" should be taken to mean a probability of 66-100%, "extremely unlikely" 0-5% and "very unlikely" 0-10%.

[12] This is about twice as fast as current increases in the CO_2 concentration. Over the 5, 10 and 20 year periods ending in 2014, the average increase in the CO_2 concentration has been 0.54, 0.54 and 0.48 percent per year, respectively. However, note that also other greenhouse gases, in particular methane adds to the climate change. For data: see Global Monitor Division of the Earth System Research Labroratory at the U.S. Department of Commerce.

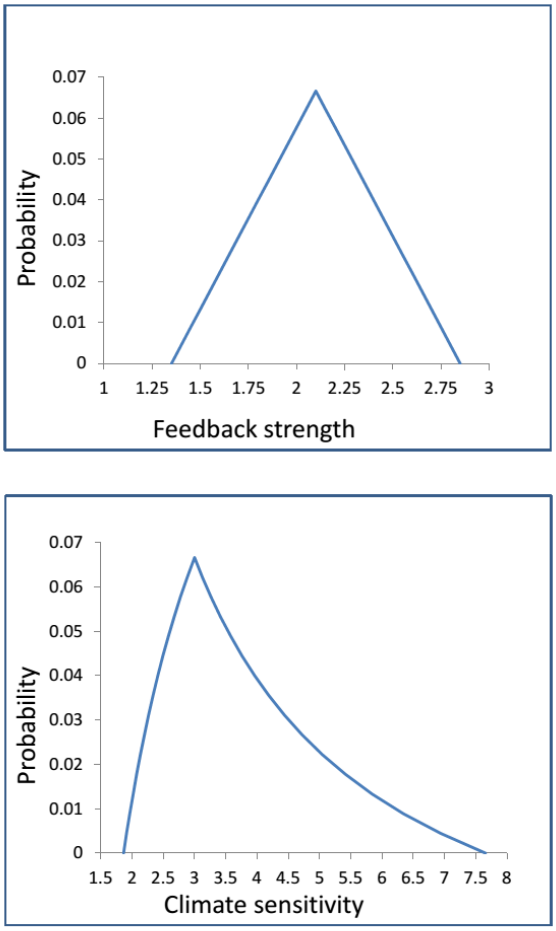


Figure 3: **Figure 5.** Example of symmetric uncertainty of feedbacks producing right skewed climate sensitivity.

2.2.1 Feedbacks and uncertainty

It is important to note that the fact that $\frac{1}{\kappa - x}$ is a non-linear transformation of x has important consequences for how uncertainty about the strength of feedbacks translate into uncertainty about the equilibrium climate sensitivity.[13] Suppose, for example, that the uncertainty about the strength in the feedback mechanism can be represented by a symmetric triangular density function with mode 2.1 and endpoints at 1.35 and 2.85. This is represented by the upper panel of Figure 5. The mean, and most likely, value of x translates into a climate sensitivity of 3. However, the implied distribution of climate sensitivities is severely skewed to the right.[14] This is illustrated in the lower panel, where $\frac{\eta}{\kappa - x}$ is plotted with $\eta = 3.7$ and $\kappa = 0.3^{-1}$.

The models have so far assumed linearity. There are obvious arguments in favor of relaxing this linearity. Changes in the albedo due to shrinking ice sheets and abrubt weaking of the Gulf are possible examples.[15] Such effects could simply be introduced by making x in (5) depend on

[13] The presentation follows Roe and Baker (2007).

[14] The policy implications of the possibility of a very large climate sensitivity is discussed in Weitzman (2011).

[15] Many state-of-the-art climate models feature regional tipping points, see Drijfhouta at al. (2015) for a list. Currently, there is, however, no consensus on the existence of specific global tipping points at particular threshold

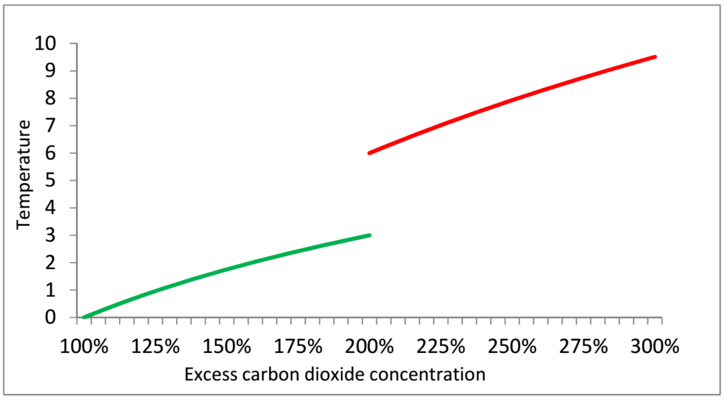


Figure 4: **Figure 6**. Tipping point at 3 K due to stronger feedback.

temperature. This could for example, introduce dynamics with so-called *tipping points*. Suppose, for example, that

$$x = \begin{cases} 2.1 \text{ if } T < 3^o C \\ 2.72 \text{ else} \end{cases}$$

Using the same parameters as above, this leads to a discontinuity in the climate sensitivity. For CO_2 concentrations below two times $\bar{S}$ corresponding to a global mean temperature deviation of 3 degrees, the climate sensitivity is 3. Above that tipping point, the climate sensitivity is 6. The mapping between $\frac{S_t}{\bar{S}}$ and the global mean temperature using equation (4) is shown in Figure 6.

It is also straightforward to introduce irreversibilities, for example by assuming that feedbacks are stronger (higher x) if a state variable like temperature or CO_2 concentration has ever been above some threshold value.

2.3 Total Forcing

As noted above, the increase in CO_2 concentration is a key factor behind a positive forcing. However, it is not the only factor. In Figure 3, the overall change in the energy budget since the beginning of the industrial era is decomposed into a number of different components. The size of the bars indicate expected contributions to overall forcing and the thin black lines that go through the bars indicate a range of uncertainty. We can see that CO_2 has the strongest positive effect among all factors. Other greenhouse gases, including methane and other long-lived gases, as well as ozone, have a positive effect on forcing. The human influence on land use seems to have had a small cooling effect. There are also important potential cooling effects that are caused by aerosols, both directly and by aerosols affecting cloud formation in a way that reduces forcing. However, the uncertainty about these effects, in particular about cloud formation, is particularly large. In comparison to the forcing factors due to humans, the effect of solar irradiance is negligible The sum of the contribution of all factors to forcing is very likely positive, with a point estimate of 2.29 Watts per square meter, but with quite a large range of uncertainty.

2.4 Heating of the oceans

In equation (1), we described a *law-of-motion* for the atmospheric temperature, i.e. an equation that describes what happens to the temperature over time for a given level of forcing. The logic

levels, see Lenton et al. (2008), Levitan (2013) and IPCC (2013b, section 12.5.5).

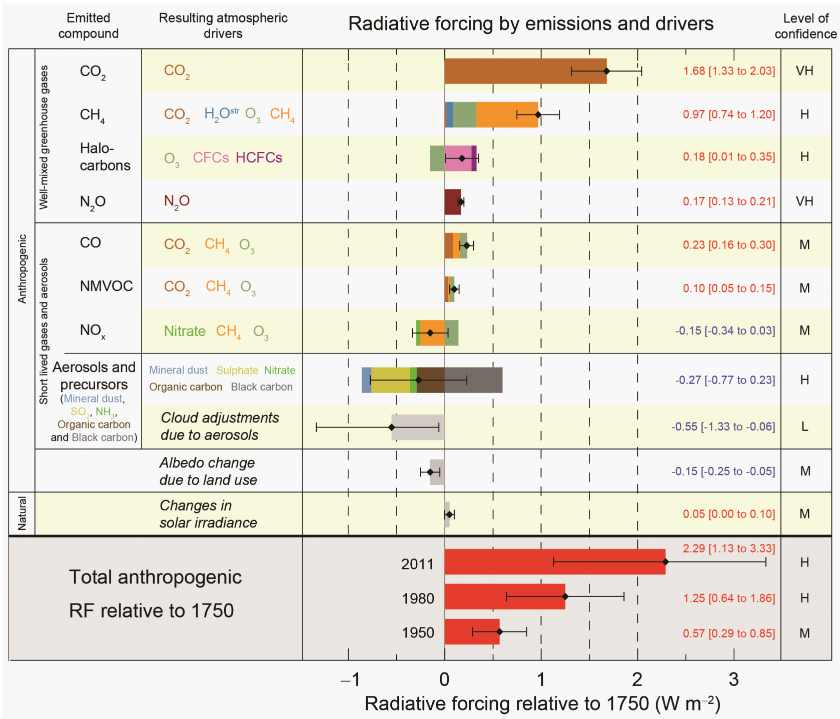


Figure 5: Radiative forcing estimates in 2011 relative to 1750 and aggregated uncertainties for the main drivers of climate change. Source: IPCC, Assessment report 5, Summary for policy makers fig 5.

behind the equation was that of an energy budget. If the budget is in surplus heat is accumulated and the temperature increases (and vice versa). The air heats quite quickly but this is not true for the oceans. Therefore, energy flows between the ocean and the atmosphere may occur and including these in the energy budget can improve our analysis. Let us abstract from the regional differences in temperature discussed in the previous section and let T_t and T_t^L, respectively, denote the atmospheric and ocean temperature in period t. With two temperatures, we will now have energy budgets defined separately for the atmosphere and for the oceans. Furthermore, allow for a variation in forcing over time and let F_t denote the forcing at time t. We then arrive at an extended version of equation (1) given by

$$\frac{dT_t}{dt} = \sigma_1 \left(F_t + xT_t - \kappa T_t - \sigma_2 \left(T_t - T_t^L \right) \right). \tag{7}$$

Comparing (7) to (1), we see that the term $\sigma_2 \left(T_t - T_t^L \right)$ is added. This term represents a new flow in the energy budget (now defined specifically for the atmosphere), namely the net energy flow from the atmosphere to the ocean. To understand this term, note that if the ocean is cooler than the atmosphere, energy flows from the atmosphere to the ocean. This flow is captured in the energy budget by the term $-\sigma_2 \left(T_t - T_t^L \right)$. If $T_t > T_t^L$, this flow has a negative impact on the atmosphere's energy budget and likewise on the rate of change in temperature in the atmosphere (the LHS). The cooler is the ocean relative to the atmosphere, the larger is the negative impact on the energy budget.

To complete this dynamic model, we need to specify how the ocean temperature evolves by using the energy budget of the ocean. If the temperature is higher in the atmosphere than in the oceans, energy will flow to the oceans **thus** causing an increase in the ocean temperature.[16] Writing this as a (linear) equation gives

$$\frac{dT_t^L}{dt} = \sigma_3 \left(T_t - T_t^L \right). \tag{8}$$

Equations (7) and (8) together complete the specification of how the temperatures of the atmosphere and the oceans are affected by a change in forcing.

We can simulate the behavior of the system once we specify the parameters of the system ($\sigma_1, \sigma_2, \sigma_3$,and κ, all positive) and feed in a sequence of forcing F_t. Before that, we should note a few important things about the system.

First, note that the heat capacity of the atmosphere is low relative to that of the oceans. This implies that a warm atmosphere heats the ocean very slowly and the parameters need to be chosen to reflect this. Second, the addition of the ocean temperature affects the dynamics of atmospheric temperature. In particular, it slows down the adjustment process. The ocean acts as a drag by having a negative impact on the atmosphere's energy budget as long as $T_t > T_t^L$. But the long-run effect of a permanent increase in forcing remains identical to that given by equation (4). To see this, assume that both temperatures have settled down to their long-run values as determined by the forcing F at some point in time denoted t^∞. Call these $T(F)$ and $T^L(F)$. By assumption, then $\frac{dT_t^L}{dt} = 0$, which from (8) implies that $T_{t^\infty} = T_{t^\infty}^L$. Since both temperatures were assumed to have settled down to their long-run values, $T(F) = T^L(F)$. Using this in (7), we see that since both $\sigma_2 \left(T_{t^\infty} - T_{t^\infty}^L \right) = 0$ and the left-hand side of (7) are zero, it must be that $F_{t^\infty} - (\kappa - x) T_{t^\infty} = 0$. Thus, $T(F) = \frac{F}{\kappa - x}$remains valid as a description of the long-run implications of a permanent increase in forcing.

[16] We disregard direct effects of forcing on the ocean's energy balance.

The model is easily simulated, for example in a spreadsheet program like Open Office Calc. For this purpose, we first approximate the differential equations (7) and (8) by a system of difference equations. Following Nordhaus and Boyer (2000) , we set

$$\begin{aligned} T_t - T_{t-1} &= \sigma_1 \left(F_{t-1} - (\kappa - x) T_{t-1} - \sigma_2 \left(T_{t-1} - T_{t-1}^L\right)\right) \\ T_t^L - T_{t-1}^L &= \sigma_3 \left(T_{t-1} - T_{t-1}^L\right) \end{aligned} \tag{9}$$

and use the parameters $\sigma_1 = 0.226, \sigma_2 = 0.44, \sigma_3 = 0.02$ and $(\kappa - x) = 1.23$. Now, the left-hand sides are the *change in temperature between discrete periods* rather than the rate of change in continuous time.[17]

The first rows in such a simulation are shown in Table 1. By adding rows in the same fashion, increasing the row number by one for each row (this is done automatically in most spread sheets) we can simulate for any period length. The forcing that is fed into the system is a series of ones in the last column. In comparison, the temperature dynamics without the "drag" from the oceans is included in the last column but one. By feeding in different sequences of forcing, we can experiment with the simulation model.

Table 1. Temperature dynamics

Year	T_t	T_t^L	F_t
=2010	=0	=0	=1
=1+A2	=B2+0.226*(E2-1.23*B2-0.44*(B2-C2))	=C2+0.02*(B2-C2)	=1
=1+A3	=B3+0.226*(E3-1.23*B3-0.44*(B3-C3))	=C3+0.02*(B3-C3)	=1

In Figure 5, the lower curve represents the ocean temperature T_t^L which increases quite slowly. The middle curve is the atmospheric temperature, T_t which increases more quickly. We know that the long-run increase in both temperatures is given by $\frac{1}{\kappa}$ times the increase in forcing, i.e., by 0.75 degrees Celsius. Most of the adjustment to the long-run equilibrium is achieved after a few decades for the atmosphere but takes several hundred years for the ocean temperature. Without the dragging effect of the oceans, the temperature increases faster, as shown by the top curve.

2.5 Global Circulation Models

So far, we have abstracted from the fact that incoming and outgoing radiation are not evenly distributed over the globe. Naturally, the energy flow per unit of area is larger around the equator than it is on the poles due to the fact that the average angle between the sun rays and the surface decreases with the latitude. This tends to create differences in temperatures at different latitudes which cause flows in air and water around the globe which transport heat from the equator towards the poles. This is disregarded in the simplest models of climate change discussed above and can therefore only predict the global mean temperature. In many cases, we also need to make predictions about regional climate changes and other parameters of the climate, like precipitation, frequencies of heavy storms and droughts. For this purpose, *global circulation models*, which explicitly model how flows of air and water transport energy, must be used. Furthermore, global circulation models are needed to predict how several of the feedback mechanisms discussed above respond to changes in the global energy budget.

The uneven inflow of energy causes more heating around the equator than elsewhere. As a consequence, the air around the equator becomes moist and warm. The warm and moist air rises, forming clouds, and flows to higher latitudes where it is once more cooled down and falls to the

[17] The timing assumptions for the right-hand sides may seem somewhat arbitrary but follow the principle that endogenous variables (i.e., those that are determined within the system) on the right-hand side are measured at $t-1$.

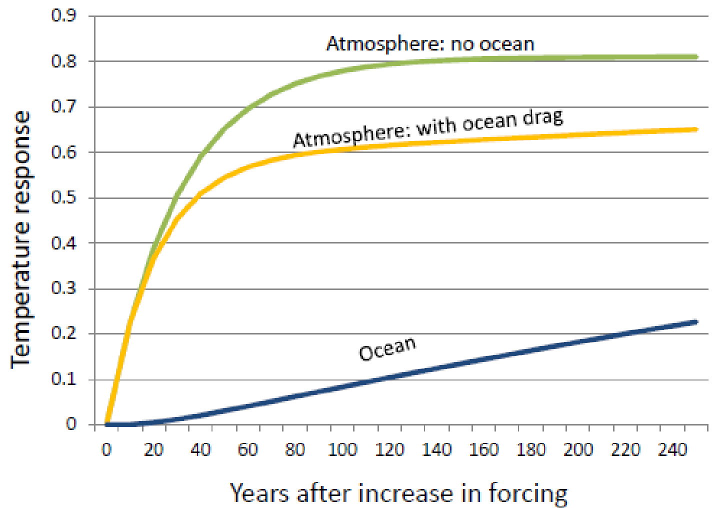


Figure 6: **Figure 5.** Increase in atmospheric and ocean temperature after a permanent forcing of $1W/m^2$.

surface creating high pressures. It then flows back to the equator to fill out the low pressures there. This systematic circular flow is called a Hadley cell and is depicted in Figure 4.

The earth rotates around its axes between the poles and the speed in meters per second is highest at the equator in the same way as the speed in a merry-go-round is highest at the edge. The air that rises at the equator has the same speed as the equator and when it moves towards the poles it therefore tends to have a higher speed than the ground, which moves slower the higher the latitude. For the flow of air along the surface towards the equator in the lower part of the cell, the opposite is true – the wind needs to "catch up" with a faster moving surface. This creates a clear pattern where the wind close to the surface moves to the equator, not as wind from the north and south, but from the northeast north of the equator and southeast south of the equator. These winds have been called the trade winds since the Middle Ages.

The Hadley cell is not the only circular cell where energy is transported. The Ferrel cells and Polar cells help transport energy towards the poles with winds of different temperatures. Also ocean currents like the Gulf stream play a role for the transportation of heat towards the poles and the most advanced global circulation models also model these flows. Finally, we should note that in order to model the flows of air and water in a realistic way, the location of land masses and mountains also needs to be taken into account since they, of course, affect the winds and the currents.

The complexity of general circulation models make them difficult to use in economics. In contrast to systems without human agents, such models do not contain any forward looking agents. Thus, causality runs in one time direction only and the current state of the system cannot depend on expectations of the future state. Therefore, the issue of solving the model, that is key in models with forward looking agents, does not arise.

One way of overcoming this difficulty is statistical downscaling. The output of large-scale dynamic circulation models or historical data is used to derive a statistical relation between aggregate and disaggregated variables. The basic idea here is to treat a small number of state variables as sufficient statistics for a more detailed description of the climate. This works due to the fact that climate change is driven by a common driver, the disruption of the energy balance due to the release

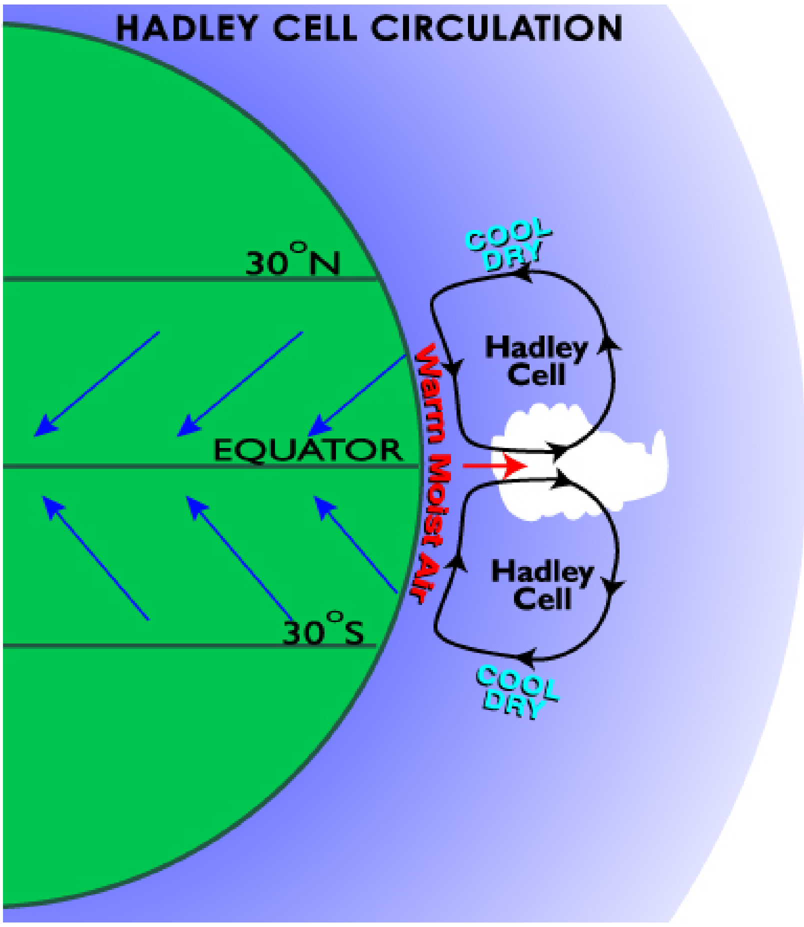
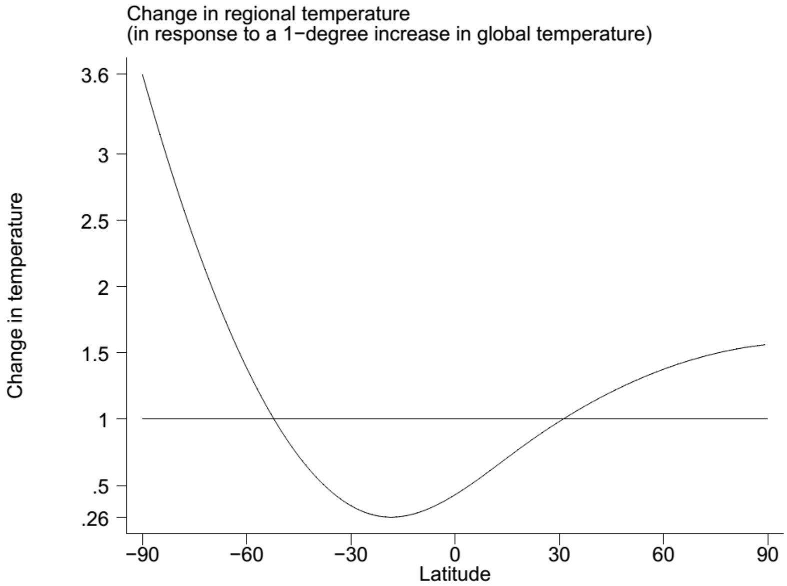


Figure 7: A Hadley cell



of green house gases, in particular CO_2.

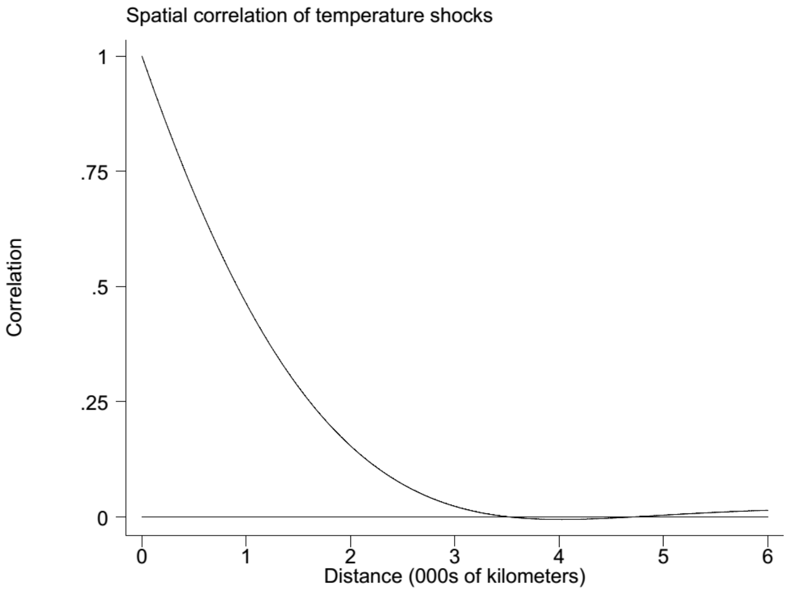
Let $T_{i,t}$ denote a particular measure of the climate, e.g., the yearly average temperature, in region i in period t. We can then estimate a model like

$$\begin{aligned} T_{i,t} &= \bar{T_i} + f(l_i, \psi_1) T_t + z_{i,t} \\ z_{i,t} &= \rho z_{i,t-1} + \nu_{i,t} \\ \mathrm{var}(\nu_{i,t}) &= g(l_i, \psi_2) \\ \mathrm{corr}(\nu_{i,t}, \nu_{j,t}) &= h(d(l_i, l_j), \psi_3) \end{aligned}$$

Here, $\bar{T_i}$ is the baseline temperature in region i. f, g,and g are specified functions parameterized by ψ_1, ψ_2 and ψ_3. $z_{i,t}$ is the prediction error that follows and AR1 process. l_i is some observed characteristic of the region, e.g., latitude and $d(l_i, l_j)$ is a distance measure. Krusell and Smith (2015) estimate such a model on historical data. Figure 5 shows the estimated function f with l_i denoting latitude. We see that an increase in the global mean temperature T_t has an effect on regional temperature levels that depends strongly on the latitude. The effect of a one degree Celsius increase in the global temperature ranges from 0.25 to 3.6 degrees. Figure 5 shows the correlation pattern of prediction errors using d to measure Euclidian distance.

2.6 Historical climate

Looking back around 500 million years in time (which equals around 10 % of the age of the earth), there are actually ways of making statements about how the climate has changed. Naturally, no direct measurements can be used, i.e. scientists have to fall back on proxy methods. A proxy variable is a measurable variable that is known to be correlated with the variable of interest. Usually, proxy data from tree rings, corals, plankton and pollen and other such sources, are applied to draw conclusions about the nature of the climate in the past. Essentially, we can say that the climate has been much warmer in the past than it is today. In the IPCC report of 2007, Solomon



et al. state that "during most of the past 500 million years, earth was probably completely free of ice sheets".[18]

While proxy data constitute the only tool for estimations of past temperatures, atmospheric gas concentrations over the last 650,000 years can actually be measured. These measures are done from bubbles of air that have been trapped in arctic ice cores over the years. Greenhouse gas concentrations can be measured in those air bubbles and conclusions can be drawn about atmospheric greenhouse gas concentrations during the past centuries.

Figure 6 shows how the concentrations of the three important greenhouse gases carbon dioxide, methane and nitrous oxide have varied over the last 650,000 years. Furthermore, the graph also includes a curve called δD which is used as a proxy **for** the temperature.[19] The five gray bars in the graph indicate interglacial periods. Consequently, the white parts in between reflect ice ages. At the moment, we are living in a period between two ice ages. No worries, conditions that may trigger the next ice age will likely not exist for at least the next 30,000 years.

As shown in the figure, the greenhouse gas is closely correlated with temperatures (as proxied by δD). However, the temperature tends to increase before the increase in greenhouse gas concentrations. This, in turn, means that a higher atmospheric CO_2 concentration is not the factor that triggers the turn from a glacial to an interglacial period but, instead, CO_2 concentrations rise in response to an increase in the temperature. It is not yet quite clear where this causality has its origin but a reasonable interpretation is that this is an example of a positive feedback in working. Something caused an increase in the global temperature, this lead to an increase in CO_2 concentration in the atmosphere, which further increased the temperature.

Even though it has not been fully resolved among scientists, we can assume that the mechanism behind the glacial-interglacial cycles on the one hand is a mechanism of variations in how elliptical (eccentric) is the earth's orbit around the sun and of variations in the tilt of the earth's axis relative to its path around the sun, on the other hand. These orbital changes are called Milankovitch cycles

[18]IPCC, 2007, FAQ 6.1

[19]The δD measure uses the fact that a small share of the hydrogen in ocean water is deuterium (hydrogen with an extra neutron). Such *heavy water* has lower vapor pressure and the concentration of heavy water in rainfall can be used to proxy for temperature.

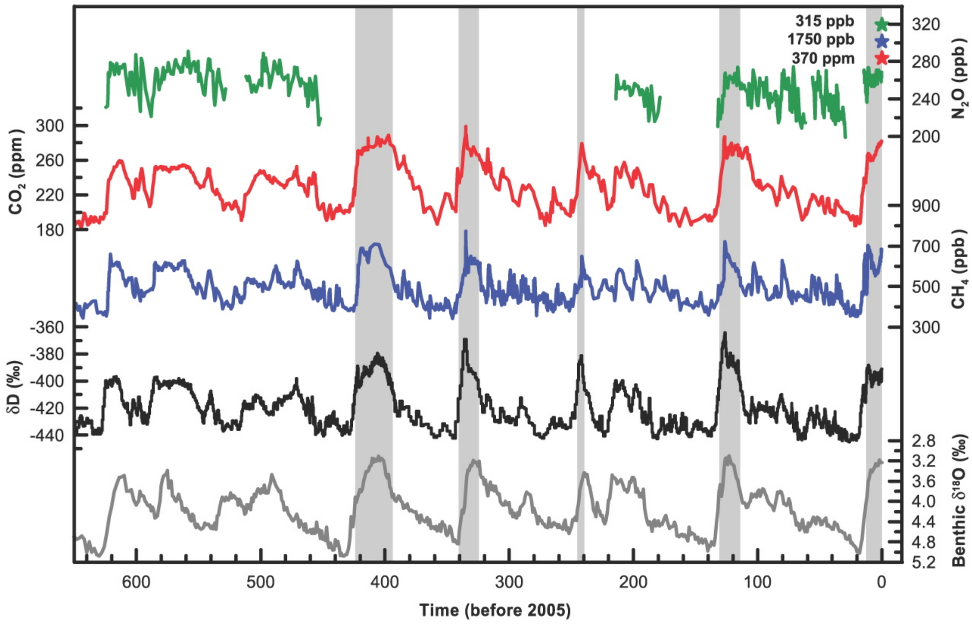


Figure 8: Greenhouse gas concentrations over the last 650,000 years.

and appear to be correlated with slow changes in the global climate. The Milankovitch cycles imply variations in the distribution of sunlight over the globe and can therefore cause changes in the climate. A decisive trigger at the beginning of an ice age may occur when solar radiation during the summer is no longer strong enough to reduce the snow and ice layers in the northern hemisphere that are accumulated during the winter. If this is the case, the blanket of snow can be aggregated over the years which leads to a larger reflection of sunlight from the surface of the earth due to the higher albedo of snow and ice. Such a positive ice-albedo feedback reinforces the initial reduction in the incoming energy flow and an ice age can get started. When the conditions are instead such that summers in the northern hemisphere become warmer, the glaciers start to melt and an interglacial period is about to start. As mentioned above, carbon dioxide concentrations also tend to rise with increasing temperatures. The result is another positive feedback effect. The consequence is feedback effects that are strong enough so that only small variations in solar radiation due to the Milankovitch cycle can cause transitions between ice ages and interglacial periods.

Naturally, a change in received solar radiation on earth is not necessarily caused by a change of the earth's orbit and the obliquity of its axis. Moreover, the energy output of the sun itself is not always equal but varies in cycles and also over longer terms. This variations as well as other factors that are not caused by human beings (such as, for instance, the activity of volcanoes) have an influence on the climate over the centuries.

2.7 More recent changes in the climate

Before the mid 19th century, instrumental records of the temperature are not sufficiently available to allow a re-construction of the global temperature. Instead, various proxies are used, leading to different results depending on the method chosen. However, a reasonable conclusion from judging the overall evidence is that the middle of both the first and second millennium AD was colder than the other periods. The three centuries after 1500 are also often called the "Little Ice Age". Direct measures of the temperature are available from the mid 19th century. The fact that they do not

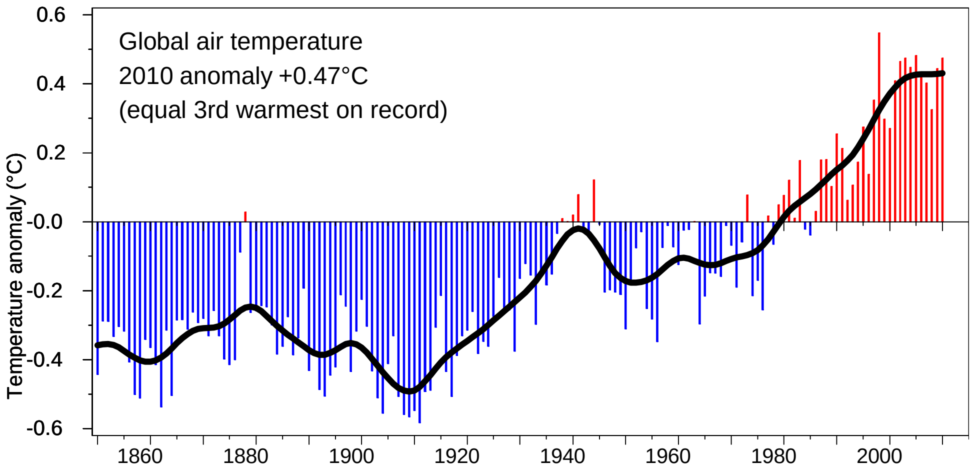


Figure 9: Global mean temperature in excess of the 1961-90 average.

measure the temperature everywhere at all times implies that the calculation of a global mean temperature is open to some discussion. However, a clear consensus has developed that there has been a substantial increase in the temperature over the last 150 years. Over the last 100 years, the increase in the global mean temperature is about 0.7 $^\circ C$.Furthermore, we can be quite confident that such an increase is unprecedented over the last two millennia.[20] The global mean temperature, expressed as deviations from the average over the period 1961-90, is depicted in Figure 7.[21]

2.8 Summary

The interactions between the economy, carbon circulation in the atmosphere and climate change are described in a so-called Integrated Assessment Model. Not only does the economy have effects on ongoing climate change but also the warming climate has substantial impacts on our economy. A bidirectional and dynamic model like the IAM is an approach to capture the reciprocal forces in one consistent framework.

One of the three blocks in the IAM is the climate block, which we have taken a closer look at in this chapter. An important insight is that in a steady state, when the climate is not changing, the incoming flow of radiation energy must equal the outgoing flow of radiation energy. An increase in the amount of greenhouse gases has the direct effect of reducing the outflow of energy, which breaks the balance and the climate is warming. Based on the idea of the energy budget, we constructed a simple model that was capable of explaining the dynamic response of the temperature in the atmosphere and the oceans to an exogenous change in the energy budget (forcing). Due to various feed-back mechanisms that indirectly affect the energy budget, the overall effect on the global temperature is difficult to predict.

Since the sunlight does not hit the earth evenly, energy is transported from the equator towards the poles with winds and water. To predict the regional distribution of climate change and other factors of the climate like precipitation and storm frequencies and analyze various feedback

[20] See Jones and Mann (2004), for an overview.

[21] Source: Brohan, Kennedy, Harris, Tett and Jones, (2006).

mechanisms like cloud formation, these flows need to be modeled in global circulation models.

3 Carbon circulation

As discussed in the introduction, an Integrated Assessment Model contains three blocks; the climate, the carbon circulation and the economy. In the previous chapter, we took a closer look at the climate, the energy budget on earth and a very simple climate model. In this chapter, we will take a closer look at the second block, the carbon circulation, and how to model it for our purposes.

3.1 The stock-flow approach

Burning of fossil fuel leads to emissions of carbon dioxide into the atmosphere. These emissions are very likely to be the largest of all human contributions to climate change. In order to forecast climate change and, for example, calculate the value of the damages caused by emission, we need to understand how the atmospheric concentration of CO_2 is related to past emissions. For this purpose, we will first have a look at a carbon circulation model based on the stock-flow approach. The first fundamental ingredients in the model are the stocks of carbon, which we call carbon reservoirs. In Figure 8, the carbon reservoirs are represented by boxes. The number in black in each box indicates the size of the reservoir in GtC, i.e., billions of tons of carbon. As we can see, the biggest reservoir by far is the Intermediate/deep ocean, with more than 37,000 Gigatons of carbon. Vegetation and the atmosphere are of about the same size although the uncertainty about the former is substantial. Soils represent a larger stock as do carbon embedded in the permafrost.

Before turning to the red figures, let us describe the second ingredient of the model, the flows. Black arrows in the figure indicate pre-industrial flows between the stocks measured in GtC per year. In the figure, we can see that for some black arrows from a given reservoir, we find an opposed arrow (or a sum of opposed arrows) that transports similar amounts of carbon back to the given reservoir. So for example, flows between the atmosphere and the ocean was almost balanced. With zero net flows between the stocks, the latter remain constant in size over time. Here, it may be convenient to note the analogy with the model of the energy budget. When net flows of energy between different sub-systems are zero, the heat content and thus the temperature remain constant.

Let us now look more closely at some of the flows between reservoirs. By transforming carbon dioxide into organic substances, vegetation in the earth's biosphere induces a flow of carbon from the atmosphere to the biosphere. This process is well-known as photosynthesis. The reverse process, respiration, is also taking place in plants' fungi, bacteria and animals. This, together with oxidation, fires and other physical processes in the soil, leads to the release of carbon in the form of CO_2 to the atmosphere. A similar process is taking place in the sea, when carbon is taken up by phytoplankton in the sea through photosynthesis and released back into the surface ocean. When phytoplankton sink into deeper layers they take carbon with them. A small fraction of the carbon that is sinking into the deep oceans is eventually buried in the sediments of the ocean floor, but most of the carbon remains in the circulation system between lower and higher ocean water. Between the atmosphere and the upper ocean, CO_2 is directly exchanged. Carbon dioxide reacts with water and forms dissolved inorganic carbon that is stored in the water. When the CO_2 rich surface water cools down in the winter, it falls to the deeper ocean and a similar exchange occurs in the other direction. From the figure, we also note that there are large flows of carbon between the upper layers of the ocean and the atmosphere via gas exchange. These flows are smaller, but of the same order of magnitude as the photosynthesis and respiration. We note that there is a bet flow from the atmosphere to the oceans.

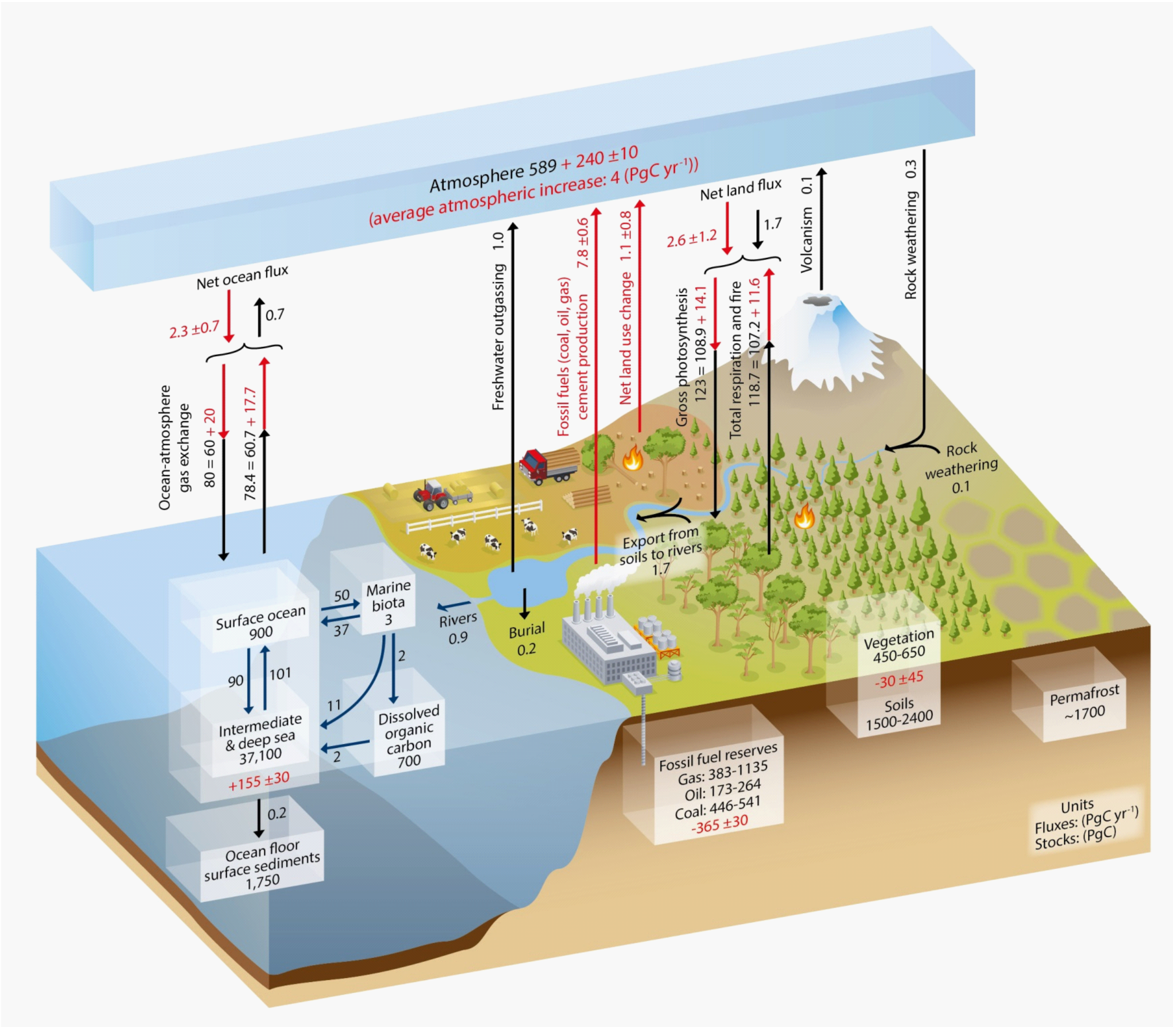


Figure 10: Global carbon cycle. Stocks in GtC (PgC) and flows GtC/year. Source: IPCC (2013), Figure 6.1.

3.2 Human Influence on Carbon Circulation

Before the industrial revolution, human influence on carbon circulation was small. However, atmospheric CO concentration started to rise from the mid 18th century and onwards, mainly due to the burning of fossil fuels and deforestation but also as a result of rising cement production. Other human factors that affect the concentration of carbon dioxide in the atmosphere are the burning of biomass and the conversion of grasslands or forests to croplands.

In Figure 8, the red figures denote changes in the reservoirs and flows over and above pre-industrial values. The figures for reservoirs refer to 2011 while flows are yearly averages during the period 2000-2009. At the bottom of the picture, we see that the stock of fossil fuel in the ground has been depleted by 365 ± 30 GtC since the beginning of industrialization. The flow to the atmosphere due to fossil fuel use and cement production is reported to 7.8 ± 0.6 GtC per year. In addition, changed land use adds 1.1 ± 0.8 GtC per year to the flow of carbon to the atmosphere. In the other direction, the net flows from the atmosphere and to the terrestial biosphere and to the oceans have increased. All in all, we note that while the fossil reserves have shrunk, the amount of carbon in atmosphere has gone from close to 600 to around 840 GtC and currently increases at a rate of 4 GtC per year. A sizeable but somewhat smaller increase has increased in the oceans while the amount of carbon in vegetation has remained largely constant.

Again in close analogy with the energy budget, we see that the gross flows of carbon are large relative to the additions due to fossil fuel burning. Furthermore, the flows may be indirectly affected by climate change. For example, the ability of the biosphere to store carbon is affected by temperature and precipitation. Similarly, the ability of the oceans to store carbon is affected by the temperature. Deposits of carbon in the soil may also be affected by climate change. We will return to these mechanisms below.

3.3 Size of fossil reserves

The extent to which burning of fossil fuel is a problem from the perspective of climate change obviously depends on how much fossil fuel remains to burn. This amount is not known and estimates depends on definitions. The amount of fossil resources that eventually can be used depends on estimates of future findnings as well as on forecasts about technological developments and relative prices. Often, reserves are defined in successively wider classes. For example, the U.S. Energy Information Agency defines four classes for oil and gas. The smallest is *proved reserves*, which are reserves that geologic and engineering data demonstrate with reasonable certainty to be recoverable in future years from known reservoirs under existing economic and operating conditions. As technology and prices change, this stock normally increases over time. Succesively larger ones are *economically recoverable resources*, *technically recoverable resources* and *remaining oil and natural gas in-place.*

Given different definitions and estimation procedures the estimated stocks differ and will change over time. Therefore, the numbers in this section can only be taken as indications. Furthermore, reserves of different types of fossil fuels are measured in different units, often barrels for oil, cubic meters or cubic feet for gas and ton for coal. However, for our purpose, it is convenient to express all stocks in terms of their carbon content. Therefore non-trivial conversion must be undertaken. Given these caveats, we calculate from BP (2015) global proved reserves of oil and natural gas to be approximately 200 GtC and 100 GtC, respectively.[22] At current extraction rates, both these

[22] BP (2015) reports proved oil reserves to 239,8 Gt. For conversion, we use IPCC (2006), table 1.2 and 1.3. From these, we calculate a carbon content of 0.846 GtC per Gt of oil. BP (2015) reports proved natural gas reserves to 187.1 trillion m^3.The same source states an energy content of 35.7 trillion BtU per trillion m^{3}equal to 35. 9 trillion

stocks would last approximately 50 years. Putting this numbers in perspective, we note that the atmosphere currently contains over 800 GtC. Given the results in the previous sections, we note that burning all proved reserves of oil and natural gas would have fairly modest effects on the climate.[23] Again using BP (2015), we calculate proved reserves of coal to around 600 GtC, providing more potential dangers for the climate.

Using wider definitions of reserves, stocks are much larger. Specifically, using data from Mcglade ans Ekins (2015) we calculate ultimately recoverable reserves of oil, natural gas and coal to close to 600 GtC, 400 GtC and 3000 GtC.[24] Rogner (1997), estimates coal reserves to be 3,500GtC with a marginal extraction cost curve that is fairly flat. Clearly, if all these reserves are used, climate change can hardly be called modest.

3.4 A Linear Carbon Circulation Model

Let us now construct a very simple carbon circulation model using the stock-flow approach. To prepare for an implementation in a spread-sheet program, we will write the model in discrete time. Let us begin with a two-stock model, where the variables S_t and S_t^L denote the amount of carbon in the two reservoirs, respectively. Let us think of S_t as the atmosphere and S_t^L as the ocean, for now abstracting from the other reservoirs. Emissions, denoted M_t, enter into the atmosphere. We also need to model the flows between reservoirs. For simplicity, let us assume that a constant share ϕ_1 of S_t flows to S_t^L in every period and, conversely, a share ϕ_2 of S_t^L flows in the other direction. The change in the amount of carbon in the atmosphere between periods t and $t-1$ is then the net flow. This can be written as the following equation

$$S_t - S_{t-1} = -\phi_1 S_{t-1} + \phi_2 S_{t-1}^L + M_{t-1}. \tag{10}$$

The left-hand side is the change in the amount of carbon in the atmosphere, and the right-hand side is the net flow during the previous period, consisting of i) the outflow to the ocean ($-\phi_1 S_{t-1}$), ii) the inflow from the ocean ($\phi_2 S_{t-1}^L$) and iii) emission M_{t-1}.

We can construct a similar equation for the amount of carbon in the ocean;

$$S_t^L - S_{t-1}^L = \phi_1 S_{t-1} - \phi_2 S_{t-1}^L. \tag{11}$$

As we can see, equations (10) and (11) form a linear system of difference equations, quite similar in structure to equation (9). However, there is a key difference; additions of carbon to this system through emissions get "trapped" in the sense that there is no outflow from the system as a whole.[25] This implies that if M settles down to a positive constant, the sizes of reservoirs S and S^L will not approach a steady state, but will grow for ever. If, emissions eventually stop and remain zero, the sizes of the reservoirs will settle down to some steady-state values, but these values will depend on what amount of emissions have been accumulated before that.[26] However, without any further information about the history of emissions, we can calculate the *relative size* of the two reservoirs

kJ. IPCC (2006) reports 15.3 kgC/GJ for natural gas. This means that 1 trillion m^3 natural gas contains 0.546 GtC. For coal, we use the IPCC (2006) numbers for antracit, giving 0.716 GtC per Gt of coal. For all these conversions, it should be noted that there is substantial variation in carbon content depending on the quality of the fuel and the numbers used must therefore be used with sufficient caution.

[23] As we will soon see, a substantial share of burned fossil fuel quickly leaves the atmosphere.

[24] See previous footnote for conversions.

[25] If we were to define also a stock of fossil fuel in ground from which emmissions were taken, total net flows would be zero. Since it is safe to assume that flows to the stock of fossil fuel is neglible, we could simply add an equation $R_t = R_{t-1} - M_{t-1}$ to the others which would capture the depletion of fossil reserves.

[26] Mathematically, we say that the dynamic system has a unit root.

in steady state. Let us denote steady states by omitting the time subscript. Setting the RHS of (10) to zero for zero emissions and assuming the system is in steady state then yields

$$0 = -\phi_1 S + \phi_2 S^L \tag{12}$$

which can be rewritten as

$$\frac{S}{S^L} = \frac{\phi_2}{\phi_1}.$$

An alternative way of deriving this result is to note that in a steady state, it must be the case that the flows to and from each reservoir must be the same. Clearly if the flow into the atmosphere, for example, is larger (smaller) than the outflow, the amount of carbon in the atmosphere must be growing (shrinking). Noting that the inflow to the atmosphere in steady state is $\phi_2 S^L$ and the outflow $\phi_1 S$ we get $\phi_1 S = \phi_2 S^L$ which is equivalent to (12)

Thus, if emissions stop, the ratio of the stocks eventually approach a constant, given by the parameters of the system. We argued above that carbon circulation was approximately in a steady state before fossil fuel burning started. Specifically, we then had 589 GtC in the atmosphere and 37,100 GtC in oceans, yielding a ratio of approximately 1.59%. Regardless of how much carbon is released into the atmosphere, our simplified model implies that, eventually, this ratio will be restored.

We can also use this ratio to calibrate our model; notwithstanding what value we set for ϕ_1 we should set $\frac{\phi_2}{\phi_1} = 1.59\%$. Now, what value should we choose for ϕ_1? Let us think about how different values of ϕ_1 affect the dynamics. Suppose that we set ϕ_1 to quite a large value.[27] Then, flows are large relative to the stocks and the system quickly adjusts to a steady state after a period of emissions. If instead ϕ_1 is very small, flows are small and it takes a long time before the system settles down. We may then calibrate the model by choosing a value of ϕ_1 that implies a reasonable speed of convergence.

BOX 1

The linear systems (10) and (11) can be solved exactly. Suppose that $M_{t+s} = 0$ for all $s \geq 0$, given some fixed t. Then, it is straightforward to show that for all $s \geq 0$,

$$\begin{aligned} S_{t+s} &= \tfrac{\phi_2}{\phi_1+\phi_2}\left(S_t + S_t^L\right) - \tfrac{\phi_2 S_t^L - \phi_1 S_t}{\phi_1+\phi_2}\left(1-\phi_1-\phi_2\right)^s \\ S_{t+s}^L &= \tfrac{\phi_1}{\phi_1+\phi_2}\left(S_t + S_t^L\right) + \tfrac{\phi_2 S_t^L - \phi_1 S_t}{\phi_1+\phi_2}\left(1-\phi_1-\phi_2\right)^s. \end{aligned}$$

The equations show that the sum of carbon is constant (there is a unit root in the system). Furthermore, the term $\frac{\phi_2 S_t^L - \phi_1 S_t}{\phi_1+\phi_2}(1-\phi_2-\phi_1)^s$ determines the convergence. As s approaches infinity, these terms vanish and the stocks of carbon approach the ratio $\frac{\phi_2}{\phi_1}$. The rate of convergence is determined by the sum of the flow parameters (the roots of the system are $(1-\phi_1-\phi_2)$ and 1). Specifically, the closer this sum is to unity, the quicker is the convergence.

3.5 Carbon circulation in a DICE-type model

In the DICE models developed by Nordhaus, the representation of the carbon cycle has three reservoirs. S_t represents the atmosphere in period t, S_t^U is the surface ocean combined with the terrestrial biosphere, and finally S_t^L, which represents the deep oceans. As in the two-stock case, discussed in the previous subsection, the flows of carbon between these reservoirs are assumed to

[27] But it cannot be larger than unity. Why?

be constant fractions of the sizes of the reservoirs expressed by coefficients $\phi_{ij,}$. The flow from reservoirs i to stock j is thus $\phi_{ij}S_t^i$.

The Nordhaus DICE carbon circulation can then be written

$$\begin{aligned} S_t - S_{t-1} &= -\phi_{12}S_{t-1} + \phi_{21}S_{t-1}^U + M_{t-1} \\ S_t^U - S_{t-1}^U &= \phi_{12}S_{t-1} - (\phi_{21} + \phi_{23})\, S_{t-1}^U + \phi_{32}S_{t-1}^L \\ S_t^L - S_{t-1}^L &= \phi_{23}S_{t-1}^U - \phi_{32}S_{t-1}^L. \end{aligned} \tag{13}$$

Note that for every reservoir, all inflows (except emissions) must equal an outflow from another stock. For example, the inflow to the surface ocean from the atmosphere is $\phi_{12}S_{t-1}$ which is the first term in the equation for the change of the size of upper ocean (the middle equation in (13)). The same term but with an opposite sign appears as the first term in in the first equation where it represents the outflow from the atmosphere to the surface ocean. The fact that all outflows are inflows to other reservoirs means that carbon can be transported between the three stocks but no carbon can be lost during the process of exchange. If emissions stop, the sum of carbon in the three reservoirs therefore remains constant for all time. Atmospheric carbon can either stay in the atmosphere or be taken up by the ocean top layer or the biosphere. Carbon in the top layer of the ocean or in the biosphere can either stay there, be given back to the atmosphere or be taken into deeper ocean layers. The carbon in the deep ocean can remain there or be carried up to higher layers by circulation.

With a little creativeness, we can use Figure 8 to calibrate the parameters of the system.[28] In fact, we can do this in several ways. There are four parameters that needs to be given numerical values, so we need to use (at least) four pieces of information from the figure. Let's start with ϕ_{12} , which determines the flow from the atmosphere to the surface ocean. Let us for now disregard vegetation (we will return to that later). Then we note from figure 8 that the pre-industrial flow from was 60 and the stock of carbon in the atmosphere was 589. Thus, we set

$$\phi_{12} = \frac{60}{589} \approx 0.102.$$

Then, let us use that the carbon circulation system was in a steady state (approximately) before industrialization.[29] This means that the flow into the atmosphere equals the flow out of the atmosphere. Thus, we set the inflow to 60 and note that the inflow is given by $\phi_{21}S_{t-1}^U$. Since the stock was 900, this gives us

$$60 = \phi_{21}900 \Rightarrow \phi_{21} = \frac{60}{900} \approx 0.0667.$$

In an exactly analogous way, we use the piece of information that the flow from surface ocean to the deep ocean was 90 out of a stock of 900, giving $\phi_{23} = \frac{90}{900} = 0.100$. Finally, the flow back from the deep ocean to the surface should in a steady state be the same, i.e. 90 out of a stock of 37 100, giving $\phi_{32} = \frac{90}{37100} \approx 0.00243$.

Table 2. Three stock carbon circulation

Year	S_t	S_t^U	S_t^L	M_t
=2011	=589+240	=900+550	=37100+155	=7.8+1.1
=1+A2	=B2-0.102*B2+0.0667*C2+E2	=C2+0.102*B2-(0.0667+0.100)*C2+0.00243*D2	=D2+0.100*C2-0.00243*D2	=7.8+1.1
=1+A3	=B3-0.102*B3+0.0667*C3+E3	=C3+0.102*B3-(0.0667+0.100)*C3+0.00243*D3	=D2+0.100*C3-0.00243*D3	=7.8+1.1

[28] An alternative route, followed by Nordhaus, is to calibrate the model so that it replicates the behavior of a more elaborate model as well as possible.

[29] In the figure, this is not exactly true. Specifically, the inflow to the atmosphere from the surface ocean is 60.7, rather than 60. However, we disregard this small discrepancy.

A spreadsheet implementation of the model is provided in Table 2. By changing the starting values of the stocks and varying the sequence of emissions, we can simulate what happens to the CO_2 stocks over time for different emission scenarios. At this point, we should note that the way we just did the calibration is not the only way we could have done it. We need four pieces of information to pin down the four parameters, but there are more than four pieces in the figure that we can use. For example, we could have used the estimated current flows. In that case, we could have calibrated ϕ_{12} from the ratio of the current flow from atmosphere to the surface ocean divided by the size of the atmosphere, i.e., to $80/(589 + 240) = 0.0965$ If the linear model was an exact representation of the carbon circulation we would get the same parameter in both cases but we do not.

3.6 Depreciation models

Although the stock-flow model has a great deal of theoretical and intuitive appeal, it runs the risk of simplifying complicated processes too much. For example, the ability of the terrestrial biosphere to store carbon depends on temperature and precipitation. Therefore, changes in the climate may have an effect on the flows to and from the biosphere not captured in the model described above. Similarly, the storage capacity of the oceans depends (negatively) on the temperature. These shortcomings could possibly be addressed by including temperature and precipitation as separate variables in the system. Furthermore, also the processes involved in the deep oceans are substantially more complicated than what is expressed in the linear model. In particular, the fact that carbon in the oceans exists in different chemical forms and that the balance between these has an important role for the dynamics of the carbon circulation is ignored but can potentially be of importance.

An important problem with the linear specification (see, Archer, 2005 and Archer et al., 2009) is due to the so-called Revelle buffer factor (Revelle and Suez, 1957). As CO_2 is accumulated in the oceans the water is acidified. This dramatically limits its capacity to absorb more CO_2, making the effective "size" of the oceans as a carbon reservoir decrease by approximately a factor of 15 (Archer, 2005). Very slowly, the acidity decreases and the pre-industrial equilibrium can be restored. This process is so slow, however, that it can be ignored in economic models. The IPCC 2007 report concludes that “About half of a CO2 pulse to the atmosphere is removed over a time scale of 30 years; a further 30% is removed within a few centuries; and the remaining 20% will typically stay in the atmosphere for many thousands of years” and the conclusion of Archer (2005) is that a good approximation is that 75% of an excess atmospheric carbon concentration has a mean lifetime of 300 years and the remaining 25% remain several thousands of years.[30]

The linear model described in the previous sub-section does not capture these predictions very accurately. Specifically, we noted that the ratio of the sizes of the reservoirs would be restored. In pre-industrial times, the share of carbon in the atmosphere was $\frac{597}{597+3200+37100} \approx 1.46\%$ of the total amount in three model reservoirs. Thus, of the total human emissions of fossil carbon to the atmosphere, only 1.46% will eventually remain in the atmosphere. As seen in Figure 9, the rate of convergence to this value is also quite fast. After 100 years, we can see that most of the convergence has been achieved. This clearly puts some doubt on the linear model described above. To some extent, this can be achieved by modifying the parameters in order to get a slower convergence and reduce the size of the deep ocean reservoir. The latter would imply that a larger share of the emitted carbon remains in the atmosphere forever.

[30] Similar findings are reported in IPCC (2013), Climate Change 2013: The Physical Science Basis, Chapter 6, Box 6.1.

An alternative and simple, yet probably reasonable representation of the carbon cycle is to move away from the stock-flow approach and instead use a non-structural approach that tries to directly capture important dynamics. We may then simply assume that a share φ_L of the carbon emitted into the atmosphere stays there forever. Within a decade, a share $1-\varphi_0$ of the remainder has exited the atmosphere into the biosphere and the surface oceans. The remaining part $(1-\varphi_L)\,\varphi_0$ decays at a geometric rate φ. Formally, we can then define a carbon depreciation factor $d\,(s)$ representing the amount of a marginal unit of emitted carbon that remains in the atmosphere s periods into the future as

$$d\,(s) = \varphi_L + (1-\varphi_L)\,\varphi_0\,(1-\varphi)^s\,.$$

Since the amount of carbon remaining in the atmosphere at different horizons is of key importance when calculating optimal CO_2-taxes (an issue to which we will return later in this book), this formulation is convenient.

A similar approach is described in IPCC (2007).[31] There

$$d\,(s) = a_0 + \sum_{i=1}^{3}\left(a_i e^{-\frac{s}{\tau_i}}\right) \tag{14}$$

with $a_0 = 0.217, a_1 = 0.259, a_2 = 0.338, a_3 = 0.186, \tau_1 = 172.9$, $\tau_2 = 18.51$, and $\tau_3 = 1.186$ where s and the $\tau_i's$ are measured in years. With this parametrization, 50% of an emitted unit of carbon has left the atmosphere after 30 years, 75% after 356 years and 21.7% stays for ever. It is important to note that IPCC states that this depreciation model is appropriate for an initial CO_2 concentration equal to the current.[32] The parameters of the depreciation function should be allowed to depend on initial conditions and inframarginal future emissions. If emissions are very large. a larger share will remain in the atmosphere for a long time. An upper limit, based on an emissions 10 times the current accumulated, can be set at around 40% remaining after 2000 years, rather than the 22% if (14) is used.

3.7 A linear relation between emissions and temperature

As discussed above, it may be to simplistic to analyze the carbon circulation in isolation. The storage capacity of the various carbon sinks depends on how the climate develops. One might think that including these interactions would make the model more complicated. However, this may not necessarily be the case. In fact, there is evidence that various feed-backs and nonlinearities in the climate and carbon-cycle systems tends to cancel each other making the combined system behave in much simpler and linear way.[33] In order to demonstrate this, let us defined the variable CCR_m (Carbon-Climate Response) as the change in the global mean temperature over some specified time interval m per unit of emissions of fossil carbon into the atmosphere over that same time interval

$$CCR \equiv \frac{T_{t+m} - T_t}{\sum_{s=t}^{t+m-1} M_s}.$$

The numerator is the change in global mean temperature over the time interval t to $t+m$. The denominator is total accumulated carbon emissions over the same time interval. Given our previous discussion in this and the previous chapter, one would think that this variable is far from a constant. The dynamic behavior of the climate and the carbon cycle could make CCR_m depend

[31] Climate Change 2007: Working Group I: The Physical Science Basis, table 2.14.

[32] 378 ppm equivalent to 805 GtC.

[33] This subsectionis based on Matthews et al. (2009).

on the length of the time interval considered. For example, since it takes time to heat the oceans, the temperature response could depend on whether the time interval over which we measure is a decade or a century. Similarly, since also the carbon dynamics is slow, the extra CO_2-concentration induced by a unit of emission tends to be lower the longer the time interval considered. Furthermore, CCR_m might depend on how much emissions have already occurred. Higher previous emissions can reduce the effectiveness of carbon sinks and even turn them into net contributors. The marginal effect on temperature from an increase in the CO_2-concentration also depends on the level of CO_2 concentration.

Quite surprisingly, Matthews et al. (2009) show that all these dynamic and non-linear effects tend to cancel, making it a quite good approximation that CCR_m is a constant, independent of both the time interval considered and the amount of previous emissions. Of course, there are uncertainties about the value of CCR. Matthews et al. (2009) pin down the uncertainty, arguing that a 95% confidence interval is between 1 and 2.1 degrees Celsius per 1000 GtC. Both model simulations and historical data suggest a best estimate of approximate one and a half degrees warming per 1000 GtC. This means that we can write the (approximate) linear relationship

$$T_{t+m} = T_t + CCR \sum_{s=t}^{t+m-1} M_s$$

To get some understanding for this surprising result, first consider the time independence. We have shown in the previous chapter that when the ocean is included in the analysis, there is a substantial delay in the temperature response of a given forcing. Thus, if the CO_2-concentration permanently jumps to a higher level, it takes many decades before even half the final change in temperature has occurred. On the other hand, if carbon is released into the atmosphere, a large share of if is removed quite slowly from the atmosphere. It happens to be the case that these dynamics cancel each other, at least if the time scale is from a decade up to a millennium. Thus, in the shorter run, the CO_2 concentration and thus forcing is higher but this is balanced by the cooling effect of the oceans.

Second, for the independence of CCR with respect to previous emissions note that the Arrenhius law discussed in the previous chapter implies a logarithmic relation between CO_2 concentration and the temperature. Thus, at higher CO_2 concentrations, an increase in the CO_2 concentration has a smaller effect on the temperature. On the other hand, existing carbon cycle models tends to have the property that the storage capacity of the sinks diminish as more CO_2 is released into the atmosphere. These effects also balance — at higher levels of CO_2 concentration, an additional unit of emissions increase the CO_2 concentration more but the effect of CO_2 concentration on temperature is lower in the same proportion.

Mathematically, we can express our reasoning by decomposing CCR as follows

$$CCR_m = \frac{\Delta S}{\sum M} \times \frac{\Delta T}{\Delta S},$$

where $\frac{\Delta S}{\sum M}$ is the change in the CO_2 concentration per unit of emissions while $\frac{\Delta T}{\Delta S}$ is the change in temperature per unit of change in the CO_2 concentration. The equation says that the change in temperature per unit of emission is the product of the change in CO_2 concentration per unit of emission and the change in temperature per unit of increase in the CO_2 concentration. For short time intervals and at high levels of CO_2-concentrations (past emissions), the first term is large and the second small and vice versa.

Given a value of CCR, it is immediate to calculate how much more emissions can be allowed in order to limited global warning to a particular value. Suppose, for example, we use a value of CCR=1.5. Then, to limit global warming to 3 degrees Celsius, we cannot emit more than $(3/1.5) \times 1000 = 2000$ GtC. Since we have already emitted about 500 GtC, 1500 GtC remains.[34] If, on the other hand, we use the upper limit of the 95% confidence interval (CCR=2.1) and aim to reduce global warming to 2 degrees Celsius, accumulated emissions can not be more than $(2/2.1) \times 1000 = 950$ GtC of which 500 is already emitted. Clearly, the latter means that current trends must quickly be reversed. To see this, note that average growth rate of fossil fuel emissions over the period 1990 to 2009 was 1.8%. With emissions in 2009 being around 9 GtC and assuming a yearly growth rate of 1.8% it takes around 35 years to emit 450 GtC.

3.8 References

Arrhenius, Svante, (1896), "On the Influence of Carbonic Acid in the Air upon the Temperature of the Ground", *Philosophical Magazine and Journal of Science,* 41:5, p237-276.

Archer, David (2005), "The Fate of Fossil Fuel CO_2 in Geologic Time", *Journal of Geophysical Research*, 110(C9).

Brohan, Kennedy, Harris, Tett and Jones, (2006), "Uncertainty estimates in regional and global observed temperature changes: a new dataset from 1850", *J. Geophysical Research* 111. Updated and published at http://http://www.cru.uea.ac.uk/

Drijfhouta, Sybren, Sebastian Bathiany, Claudie Beaulieu, Victor Brovkin, Martin Claussen, Chris Huntingford, Marten Scheffer, Giovanni Sgubin, and Didier Swingedouw, (2015), "Catalogue of abrupt shifts in Intergovernmental Panel on Climate Change climate models", *Proceedings of the National Academy of Sciences*, vol. 112, no. 43.

IPCC (Intergovernmental Panel on Climate Change) (2007), "Climate Change 2007: The Physical Science Basis", Contribution of Working Group I to the Fourth Assessment Report of the Intergovernmental Panel on Climate Change, Eds. Bert Metz, Ogunlade Davidson, Peter Bosch, Rutu Dave, and Leo Meyer.

IPCC (2013), "Climate Change 2013: The Physical Science Basis", Contribution of Working Group I to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change, eds. Stocker, T.F., D. Qin, G.-K. Plattner, M. Tignor, S.K. Allen, J. Boschung, A. Nauels, Y. Xia, V. Bex and P.M. Midgley, Cambridge University Press, Cambridge, United Kingdom and New York, NY, USA.

IPCC (2013b), Stocker, T.F., D. Qin, G.-K. Plattner, M. Tignor, S.K. Allen, J. Boschung, A. Nauels, Y. Xia, V. Bex and P.M. Midgley, *Climate Change 2013: The Physical Science Basis. Contribution of Working Group I to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change,* Cambridge University Press, Cambridge, United Kingdom and New York, NY, USA.

Jones and Mann, (2004), "Climate over past millennia", REVIEWS OF GEOPHYSICS, VOL. 42.

Lenton, Timothy M., Hermann Held, Elmar Kriegler, Jim W. Hall, Wolfgang Lucht, Stefan Rahmstorf and Hans Joachim Schnellnhuber, (2008), "Tipping elements in the Earth's climate system", *Proceedings of the National Academy of Sciences*, vol. 105, no. 6, pages 1786-1793.

Levitan, Dave (2013), "Quick-Change Planet: Do Global Climate Tipping Points Exist?", *Scientific American* 25, March 2013.

Matthews, H. Damon, Nathan P. Gillet, Peter A. Stott and Kirsten Zickfeld (2009), "The proportionality of global warming to cumulative carbon emissions", *Nature,* Vol 459 June, p829-33.

[34] The 500 GtC consits of around 350 GtC from fossil fuels and 150 GtC from changes in land use. See Carbon Dioxide Information Analysis Center (http://cdiac.ornl.gov) for long data series on emissions

Nordhaus, William and Joseph Boyer (2000), *Warming the World: Economic Modeling of Global Warming*, MIT Press, Cambridge, MA.

Otto, A, Otto, FEL, Allen, MR, Boucher, O, Church, J, Hegerl, G, Forster, PM, Gillett, NP, Gregory, J, Johnson, GC, Knutti, R, Lohmann, U, Lewis, N, Marotzke, J, Stevens, B, Myhre, G and Shindell, D, (2013), "Energy budget constraints on climate response", *Nature Geoscience*, 6 (6). 415 - 416.

Schwartz, Stephen, Robert Charlson, Ralph Kahn, John Ogren, and Henning Rodhe (2010), "Why Hasn't Earth Warmed as Much as Expected?", *Journal of Climate,* 23

Schwartz, S. E., R. J. Charlson, R. Kahn, and H. Rodhe (2014), "Earth's Climate Sensitivity: Apparent Inconsistencies in Recent Assessments", *Earth's Future*, 2

Rogner, H.-H. (1997), "An Assessment of World Hydrocarbon Resources", *Annual Review of Energy and the Environment*, 22, 217-262.

4 Public economics

More than a decade ago, most countries joined an international treaty – the United Nations Framework Convention on Climate Change (UNFCCC) – with the ambition of beginning to consider how global warming can be reduced and how to cope with whatever temperature increases that are inevitable. More recently, a number of nations approved an additional treaty: the Kyoto Protocol which is an international and legally binding agreement that aims at reducing greenhouse gases.

These international collaborations raise several questions. First, why are they necessary in the first place? Are the markets not able to allocate the resources in an efficient way? Why cannot each individual country just do what is best by itself? In addition, if policy is necessary, then what exactly does this policy look like?

In order to analyze these questions and/or evaluate the welfare implications of different policies, we first need a crash course in consumer theory, i.e., we first need to assume something about how consumers value different alternatives. Second, we need to understand that global warming is a different type of "good" than a private good like, for instance, a computer that is traded on a market. We start with consumer theory and consumer preferences.

4.1 Consumer theory

Preferences are the building block of consumer choice. A good way of beginning is to think about preferences in terms of comparisons between different market baskets. A market basket is simply a collection of one or more commodities. It might, for example, consist of various forms of food and clothing that a consumer buys on a regular basis. The consumer is assumed to be able to rank how one basket is preferred to another basket. In fact, if we assume that the preferences satisfy a number of assumptions – called axioms – we can represent the consumer's preferences by a mathematical function, a so-called utility function. First we need to define the notation. Suppose that x and y are two market bundles. Then, we will use the following notation

$$\begin{array}{rl} x \succsim y & \text{is } x \text{ is at least as good as } y \\ x \succ y & \text{is } x \text{ is preferred to } y \\ x \sim y & \text{is} \quad x \text{ is indifferent to } y. \end{array}$$

The theory of consumer behavior then begins with four axioms for consumer preferences for one market basket versus another.

1. Preferences are complete, which means that a consumer can compare and rank any and all bundles. Specifically, if x and y are two distinct bundles then either $x \succsim y$ or $y \succsim x$ must be true.

2. Preferences are reflexive. This means that $x \sim x$ must be true. This is an axiom that might seem obvious but it is necessary for rational behavior.

3. Preferences are transitive, which means that if $x \succ y$ and $y \succ z$, it must be true that $x \succ z$.

4. Preferences are continuous so that if x is preferred to y then bundles "sufficiently close" to x must also be preferred to y. This last axiom is often made to rule out certain discontinuous behavior.

If a consumer's preferences over bundles satisfy the four axioms, there exists a continuous utility function such that if $x \succ y$ then $u(x) > u(y)$. The utility function is then a way of describing the choices that the consumer makes. Potentially, the utility function will contain all "goods" that the consumers value and care about. Hence, the arguments in the function do not only have to be market goods, but could, for instance, equally well be clean air, biodiversity and other people's well being and their consumption. In most cases, however, economists abstract from most of these arguments in unless they are perceived to be necessary for the analysis.

With the utility function we can then evaluate how a consumer will react to an imposed tax on the consumption bundle she consumes. In addition, by comparing the utility she gets from the two bundles (with and without the tax) we can evaluate how her welfare is affected. However, before answering the questions about why policy might be needed on issues related to climate change, we need to know for what goods markets can be expected to work efficiently. This is the topic of the next section.

4.2 The first theorem of welfare economics

Consider an economy with two consumers: consumers 1 and 2 that can both be completely described by their preferences, $\succeq_1$ and $\succeq_2$, and their initial endowments of the two commodities, ω_1^1, ω_1^2, ω_2^1 and ω_2^2, where subscripts denote the agent and superscripts denote the goods. The concept of a good is very broadly defined and goods are distinguished by time, location and states of the world. These consumers are allowed to trade the goods among them to become better off and there is assumed to exist a market for each good. The agents are assumed to behave competitively, i.e., they take prices as given. An allocation is a collection of consumption bundles that describes what each of the two agents is holding. The prices p_1, p_2, are endogenously determined in the trading process.

In this economy, each agent $i = 1, 2$ is solving the following problem

$$\max_{x_i^1, x_i^2} u\left(x_i^1, x_i^2\right)$$

s.t.

$$p_1 x_i^1 + p_2 x_i^2 = p_1 \omega_i^1 + p_2 \omega_i^2.$$

The outcome of this individual problem is the consumer's demand function. Formally, let $x_1^1\left(p_1, p_2\right)$ be agent 1's demand function for good 1 and $x_2^1\left(p_1, p_2\right)$ be agent 2's demand function for good 1, and define the analogous expressions for good 2.

What are the features of the equilibrium outcome resulting from such a process? What is the optimal outcome? Which allocative mechanisms can be used to achieve the optimal outcome? To answer these questions, we will use the concept of a Walrasian equilibrium. An allocation is a Walrasian equilibrium if

$$x_1^1\left(p_1^*, p_2^*\right) + x_2^1\left(p_1^*, p_2^*\right) = \omega_1^1 + \omega_2^1,$$

and

$$x_1^2\left(p_1^*, p_2^*\right) + x_2^2\left(p_1^*, p_2^*\right) = \omega_1^2 + \omega_2^2,$$

Hence, in words, a Walrasian equilibrium is an allocation where aggregate demand (here consisting of the sum of the two consumers' demand) for each good is exactly equal to the supply of the good. Or, put differently, an allocation where all markets clear. A Walrasian equilibrium can be shown to exist.

Theorem 1 *First theorem of welfare economics. If the allocation is a Walrasian equilibrium, then it is also Pareto efficient.*

Proof. Assume that the equilibrium is not efficient, i.e., that there is some other feasible allocation $\left(y_1^1, y_2^1, y_1^2, y_2^2\right)$ such that

$$y_1^1 + y_2^1 = \omega_1^1 + \omega_2^1 \tag{15}$$

and

$$y_1^2 + y_2^2 = \omega_1^2 + \omega_2^2, \tag{16}$$

and

$$\left(y_1^1, y_1^2\right) \succsim_1 \left(x_1^1, x_1^2\right) \tag{17}$$

$$\left(y_2^1, y_2^2\right) \succsim_2 \left(x_2^1, x_2^2\right), \tag{18}$$

where at least one of the comparisons in (17) and (18) is strict so that one agent strictly prefers the $\left(y^1, y^2\right)$-bundle. Assume now that it is agent 1 who strictly prefers the y-bundle so that the comparison in (17) is strict. The two equations (15)-(16) then say that the y-allocation is feasible and the next two that it is strictly preferred by agent 1 and weakly preferred by agent 2 to the x-allocation. But the hypothesis is that we have a market equilibrium where each agent purchases the best bundle she can afford. If $\left(y_1^1, y_1^2\right)$ is better than the bundle agent 1 is choosing, it must be that it costs more than agent 1 can afford since by assumption it is preferred and affordable. Similarly, if agent 2 is indifferent between $\left(x_2^1, x_2^2\right)$ and $\left(y_2^1, y_2^2\right)$, it must be that the bundle $\left(y_2^1, y_2^2\right)$ costs at least at much as the $\left(x_2^1, x_2^2\right)$, otherwise agent 2 would not be indifferent between the two bundles. From axiom 4 above, the agent could then have spent more on any of the two goods which would have increased the utility. Hence, we have

$$p_1 y_1^1 + p_2 y_1^2 > p_1 \omega_1^1 + p_2 \omega_1^2$$

$$p_1 y_2^1 + p_2 y_2^2 \geq p_1 \omega_2^1 + p_2 \omega_2^2.$$

Now add these two equations to get

$$p_1 \left(y_1^1 + y_2^1\right) + p_2 \left(y_1^2 + y_2^2\right) > p_1 \left(\omega_2^1 + \omega_1^1\right) + p_2 \left(\omega_2^2 + \omega_1^2\right).$$

Finally, substitute equation (15)-(16) into the above equation to get

$$p_1 \left(\omega_1^1 + \omega_2^1\right) + p_2 \left(\omega_1^2 + \omega_2^2\right) > p_1 \left(\omega_2^1 + \omega_1^1\right) + p_2 \left(\omega_2^2 + \omega_1^2\right),$$

which is clearly a contradiction, since the left-hand side and the right-hand side are the same. ■

The theorem states that if the assumptions are satisfied, the market equilibrium is Pareto efficient, which means that it is not possible to make all agents better off. In other words, it is not possible for policy to increase the efficiency in the economy. Here, we have a simple endowment economy with only two agents and two goods, but the theorem generalizes to an arbitrary number of agents and goods. In addition, recall that goods are very broadly defined and can be distinguished by time (i.e., different time periods), location and states of the world (i.e., when consumption of the goods is subject to uncertainty). It also applies to economies with firms and production of the goods. The theorem is thus very strong.

However, the outcome completely depends on the initial distribution of endowments and therefore it need not be considered as "fair". Thus, there could be room for a policy to change the distribution of resources.

This brings us back to the question raised at the beginning of the chapter: are the markets able to allocate the resources in an efficient way when it comes to global warming (when abstracting from redistributional issues)? Can we apply the first theorem of welfare economics and just trust that when firms, consumers and countries trade, this will result in a Pareto efficient outcome? The answer is no and the reason is that one of the important assumptions that are required for the theorem to be true is not fulfilled. We now turn to study the effects of market failure, i.e., when some of the assumptions of the welfare theorem do not hold and hence, we can no longer rely on market equilibria to get Pareto optimal outcomes.

4.3 Externalities

There are many cases where the actions of an individual or a firm directly affect other individuals or firms, where one firm imposes a cost on other firms but does not compensate the other firm or, alternatively, where one firm receives benefits from other firms but does not pay for the benefit. Instances where one individual's actions impose a cost on others are called negative externalities and instances where one individual's actions yield a benefit on others are called positive externalities. Externalities are labeled consumer externalities or producer externalities depending on the source of the externality. Highway congestion is an example of a consumer externality and industrial pollution is an example of a producer externality. In the presence of externalities, the first theorem of welfare economics does not hold because there are things that people care about that are not priced. Specifically, the assumption that there is a market for every good is not true. In the case of a negative externality, more of the product is produced and sold than the efficient amount. Since the marginal benefit is not equal to the marginal cost, a deadweight welfare loss arises.

4.3.1 An example with a consumer externality

Consider now once more the example in the first section and assume that the utility function for agent 2 is the same as before, while the utility function of agent 1 also depends on agent 2's consumption of good 1, i.e.,

$$u_1\left(x_1^1, x_1^2, x_2^1\right)$$

and

$$u_2\left(x_2^1, x_2^2\right).$$

Examples could, for instance, be tobacco smoke or alcohol. Now, it no longer needs to be true that

$$p_1 y_1^1 + p_2 y_1^2 > p_1 \omega_1^1 + p_2 \omega_1^2.$$

In fact, the y-allocation may be affordable and preferred if it has a y_2^1 that is significantly different and better for agent 1 than x_2^1. Both agents are still choosing allocations that are best for them but their choices no longer reflect the social benefits. In this case, agent 1 cares about the consumption of agent 2 but cannot affect this choice and agent 2 ignores it. In other words, there are things that the agents care about that are not priced.

4.3.2 An example with a production externality

Consider an economy with two firms labeled 1 and 2. Firm 1 produces an output X which is sold on a competitive market. However, the production of also imposes a cost $\gamma(X)$ on firm 2. We may consider this as a technology used by firm 1, i.e. for each unit of output X it produces it also produces X units of pollution which harm firm 2. We call $\gamma(X)$ an *externality* since it is external from the point of view of the agent who controls X.

If we denote the price of output by p, the profits of the two firms are

$$\pi_1 = \max_X pX - c(X)$$

$$\pi_2 = -\gamma(X).$$

Both cost functions are assumed to be increasing and convex. For simplicity, additional profits that firm 2 might have are, for simplicity, abstracted from.

The equilibrium amount of output in this economy is

$$p = c'(X).$$

However, this amount is too large from a social point of view. The reason is that firm 1 only takes the private costs into account, while ignoring the social costs which are the private costs plus the costs imposed on the other firm.

So what is the efficient amount of output in this economy? To find this out, we set up the social planner's problem. This implies maximizing profits while taking all costs into account. Formally:

$$\pi = \max_X pX - c(X) - \gamma(X).$$

Note that this is the same problem as the problem of a merged firm. The first-order condition is

$$p = c'(X) + \gamma'(X) \tag{19}$$

which is evaluated at the optimal value of X.

The output that satisfies equation (19) is the efficient amount of output. It is characterized by that price being equal to the social cost, being equal to the sum of the marginal private cost and the marginal externality. By internalizing the externality, the inefficiency is taken care of.

4.4 An example with $\mathbf{CO_2}$ emissions

Here, we consider the example with a production externality and also discuss different solutions to that problem. The setting is one where a representative firm uses an input like fossil fuel to produce an output. The use of fossil fuels is bad since it produces CO_2-emissions which have a negative effect on the climate. The result is an increased frequency of weather shocks such as heat waves, droughts, flooding etc. These costs can be measured as a share of final output (GDP). Here, we abstract from the exact mechanism through which the input affects the climate and just assume that the use of fossil fuels reduces GDP (we will explicitly add the climate later). Since agents are not facing the full costs of their actions, they use too much fossil fuel.

We start out without externalities to get an understanding of the model and to see what features the equilibrium has without externalities. There are two sectors in the economy: one that produces fossil fuels E, and one that produces final output Y. Both firms in these sectors are assumed to behave competitively. Moreover, the resources required to produce one additional unit of fossil fuels μ, is assumed to be constant (and denoted in terms of final output). This last assumption is not based on any estimated features of reality but is made purely for simplicity. In reality, it could very well be the case that it is more and more costly to extract additional units of fossil fuel (implying a convex cost function).

4.4.1 Case 1: no externality

Suppose there is a perfect market where suppliers of fossil fuel supply fossil fuel E that they can produce at marginal cost μ.[35] The assumption of perfect competition implies that

$$p = \mu,$$

i.e., the price equals the marginal cost.

The representative firm in the final goods sector uses fossil fuels and labor as inputs for producing an output y. Specifically, the production function for output is given by

$$Y \equiv f(E, L) = E^{\nu} L^{1-\nu}, \tag{20}$$

with $0 < \nu < 1$, $0 < \mu < 1$ and $\nu > \mu$. Finally, the total supply of labor is fixed at one unit, i.e., $L = 1$.

Using the fact that $p = \mu$, the profit maximization problem for the firm is

$$\pi = \max_{E} E^{\nu} L^{1-\nu} - \mu E - wL.$$

The first-order condition with respect to E and L is respectively given by

$$\nu E^{\nu-1} L^{1-\nu} = \mu \tag{21}$$

and

$$(1-\nu) E^{\nu} L^{-\nu} = w.$$

Now use $L = 1$ in (21) giving

$$E = \left(\frac{\mu}{\nu}\right)^{\frac{1}{\nu-1}},$$

or

$$E = \left(\frac{\nu}{\mu}\right)^{\frac{1}{1-\nu}}. \tag{22}$$

Final output is then given by

$$Y = \left(\left(\frac{\nu}{\mu}\right)^{\frac{1}{1-\nu}}\right)^{\nu},$$

which can be written as

$$Y = \left(\frac{\nu}{\mu}\right)^{\frac{\nu}{1-\nu}}. \tag{23}$$

[35] We now disregard the possibility that the total supply is finite. We will return to this in later chapters.

4.4.2 Case 2: adding an externality

Let us now assume that the use of fossil fuels causes costs in the sense that it reduces final output. The representative firm does not take the negative effect on production into account when deciding how much fossil fuel to use. We maintain perfect markets, requiring that we have a large number of identical firms. From the individual firm's perspective, the average level of fossil energy use in the economy, denoted by $\overline{E}$, that has a negative effect on production is exogenously given, i.e., the individual firm cannot do anything about it.

The production function for final output is then given by

$$Y \equiv f\left(\overline{E}, E, L\right) = \overline{E}^{-\gamma} E^{\nu} L^{1-\nu},$$

with $\gamma > 0$ and where the term $\overline{E}^{-\gamma}$ captures the damages caused by the use of fossil fuels. Hence, final output is decreasing in $\overline{E}$. In equilibrium, it must be true that $\overline{E} = E$ since all firms are the same, but the important thing is that the representative firm does not view $\overline{E}$ as a choice variable. We also assume that $\nu > \gamma$ so that when E increases, and consequently $\overline{E}$ increases by the same amount, output also increases.

Since nothing has changed in the sector producing fossil fuels, we still have that $p = \mu$. The profit maximization problem for the firm is then

$$\pi = \max_{E} \overline{E}^{-\gamma} E^{\nu} L^{1-\nu} - \mu E - wL.$$

The first-order condition with respect to E and L is, respectively, given by

$$\nu \overline{E}^{-\gamma} E^{\nu-1} L^{1-\nu} = \mu, \tag{24}$$

and

$$(1-\nu)\,\overline{E}^{-\gamma} E^{\nu} L^{-\nu} = w. \tag{25}$$

Once more, using the facts that $L = 1$, and that in equilibrium $\overline{E} = E$ in (24), we get

$$\nu E^{\nu-\gamma-1} = \mu,$$

which can be solved for E:

$$E_{lf} = \left(\frac{\nu}{\mu}\right)^{\frac{1}{1+\gamma-\nu}}, \tag{26}$$

where the index lf denotes the outcome in *laissez faire* (as opposed to the amount of E (denoted E_{sp}) in the social planning problem below). Final output is given by

$$Y = \left(\left(\frac{\nu}{\mu}\right)^{\frac{1}{1-\nu+\gamma}}\right)^{\nu-\gamma} L^{1-\nu},$$

or, since $L = 1$,

$$Y_{lf} = \left(\frac{\nu}{\mu}\right)^{\frac{\nu-\gamma}{1+\gamma-\nu}}. \tag{27}$$

The level of output is lower than without the externality. This is expected since the externality is causing losses. In addition, when the cost of the externality goes to zero, Y_{lf} approaches the quantity produced in the case without the externality (see equation 23). Now, multiply both sides of (24) by E and note that

$$\nu \underbrace{\overline{E}^{-\gamma} E^{\nu} L^{1-\nu}}_{Y} = \mu E.$$

Similarly, multiply both sides of (25) by L to get

$$(1-\nu) \underbrace{\overline{E}^{-\gamma} E^{\nu} L^{1-\nu}}_{Y} = wL.$$

The total profit is then

$$\pi = Y - \underbrace{\nu Y}_{\mu E} - \underbrace{(1-\nu) Y}_{wL} = 0-$$

4.4.3 The social planning problem

The social planning problem can be considered as an answer to the question of what the optimal thing is to do. We imagine that there is a benevolent planner that is maximizing profits while taking all costs into consideration. Hence, the planner does not take $\overline{E}$ as given, but realizes that $\overline{E} = E$. Assuming that the marginal value of output equals the price of output, the maximization problem is

$$\max_{E} E^{-\gamma} E^{\nu} L^{1-\nu} - \mu E. \tag{28}$$

This can be written as

$$\max_{E} E^{\nu-\gamma} L^{1-\nu} - \mu E. \tag{29}$$

The first-order condition with respect to E is

$$(\nu - \gamma) E^{\nu-\gamma-1} L^{1-\nu} = \mu. \tag{30}$$

Using the fact that $L = 1$, the above equation can be solved for E:

$$E_{sp} = \left(\frac{\nu-\gamma}{\mu}\right)^{\frac{1}{1+\gamma-\nu}} < E_{lf} = \left(\frac{\nu}{\mu}\right)^{\frac{1}{1+\gamma-\nu}}.$$

Production is then finally given by

$$Y_{sp} = \left(\left(\frac{\nu-\gamma}{\mu}\right)^{\frac{1}{1+\gamma-\nu}}\right)^{\nu-\gamma},$$

or

$$Y_{sp} = \left(\frac{\nu-\gamma}{\mu}\right)^{\frac{\nu-\gamma}{1+\gamma-\nu}}.$$

Note once more that we can multiply both sides of (30) by E to obtain

$$(\nu - \gamma) \underbrace{E^{\nu-\gamma} L^{1-\nu}}_{Y_{sp}} = \mu E. \tag{31}$$

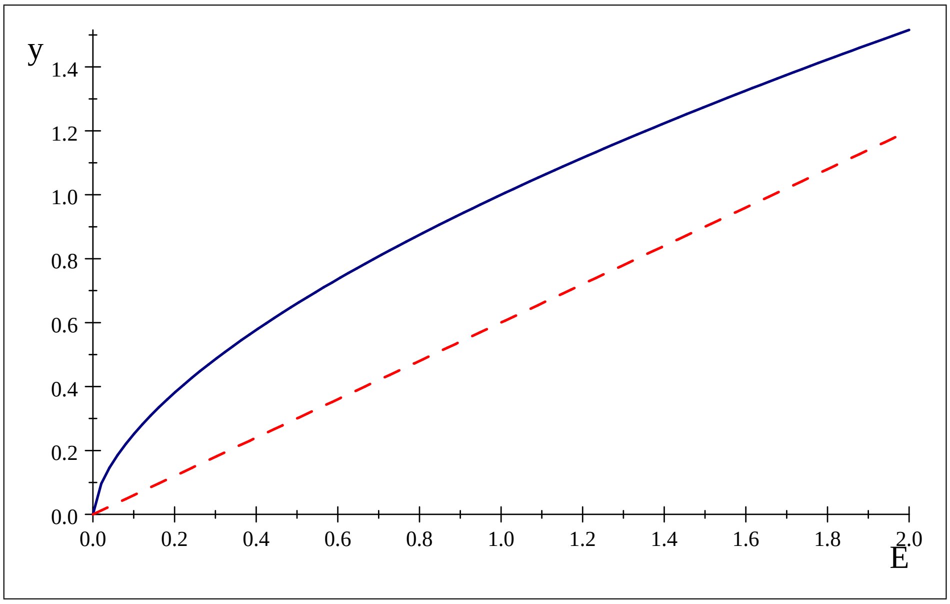


Figure 11: Planners problem

In equilibrium, the value of production is strictly larger than the costs. Combining (29) and (31), we get

$$Y_{sp} - \underbrace{(\nu - \gamma)\, Y_s}_{\mu E},$$

or

$$Y_{sp}\,(1 - (\nu - \gamma)) > 0, \tag{32}$$

As can be seen in the figure, total production (the solid blue line) minus the costs employed (the red dashed line) is maximized when the distance between the two curves is the largest. This occurs at $E_{sp} = 1$ in the figure while the market solution is above 2. [36] For $E > 1$,additional units of fossil fuel increase the production but the marginal increase in production is then less than the marginal cost and this is inefficient.

4.5 Solutions

It has now been illustrated that when firms are faced with a negative externality, optimal output is lower than without an externality. This implies that the use of fossil fuel is lower in the centralized first best solution than in the decentralized solution. The question is then if it is possible to implement some policy that ensures that the first best allocation is achieved also in the decentralized economy. In this section, we will discuss different solutions to the problem, including assigning property rights, taxes and quantity restrictions.

4.5.1 Property rights

One way of viewing the problem is to say that the property rights are not well defined. In particular, the Coase theorem (after Ronald Coase) states that if trade in an externality is possible and there

[36] The parameters in the example in the figure are set to $\alpha = 0.8$, $\beta = 0.2$ and $\phi = 0.6$.

are no transaction costs, bargaining will lead to an efficient outcome. This is regardless of the initial allocation of property rights. However, in the setting described here, the bargaining process is unlikely to be efficient. If the right to pollute the atmosphere is given to an individual (or a group of individuals), this individual effectively controls fossil fuel use. This agent is then a monopolist and cannot be expected to behave in a way that is socially optimal. Second, it is not realistic to give some agent the property rights to the air and the atmosphere. The Coase theorem is thus of less use when it comes to externalities related to climate change, since the solutions suggested above do not realistically apply in a global setting. A related problem is that the atmosphere really is a public good (it is hard to exclude people from using it). The problems of pricing the externality then also have all the characteristics that are associated with the pricing of public goods.

4.5.2 Merging

According to the First Welfare Theorem, whenever a market allocation is not Pareto efficient, then there is some way of increasing the aggregate surplus. As noted above (in section 4.3.2), if the externality of one firm has a negative effect on the profits of another firm, it will always be efficient for the firms to merge. In other words, by coordinating the actions of both firms, more profits can be generated than when the firms act separately. If the representative firm was just one firm, then it would internalize the costs on production from using fossil fuel since it would then bear the full cost from fossil fuel use. This firm would then set $E_{lf} = E_{sp}$ (convince yourself of this!). However, in a global setting it is not realistic or desirable that all firms that are using fossil fuel merge. In addition, even if they would do so, there would be other problems since that firm then could not be expected to behave competitively. Another, more promising solution is to use taxes.

4.5.3 Pigouvian taxes

As shown in section 2, in the presence of negative externalities, the social cost of a market activity is not covered by the private cost of the activity. The market outcome is then not efficient and it may result in over-consumption of the good. One solution to this problem is to impose a so-called Pigouvian tax on the activity that generates negative externalities. The tax should then be set so that it exactly reflects the size of the negative externality. In this way, the corrective tax increases the price so that it includes both the private and the social cost.

A tax on fossil fuel carbon tax can enhance efficiency if it corrects the market distortions that arise when people do not take the external effects of their energy consumption into account. To implement the first best in the decentralized economy, the planner can impose a proportional tax on the use of energy. The profit maximization problem is then

$$\pi = \max_E \overline{E}^{-\gamma} E^{\nu} L^{1-\nu} - (\mu + \tau) E - wL,$$

where τ is the Pigouvian tax. The first-order conditions with respect to E and L are then, respectively, given by

$$\nu \overline{E}^{-\gamma} E^{\nu-1} L^{1-\nu} - \tau = \mu \tag{33}$$

and

$$(1-\nu) \overline{E}^{-\gamma} E^{\nu} L^{-\nu} = w. \tag{34}$$

From (30), it follows that the social planner should set

$$\tau = \gamma \overline{E}^{-\gamma} E^{\nu-1} L^{1-\nu}, \tag{35}$$

since this implies that

$$(\nu - \gamma) \overline{E}^{-\gamma} E^{\nu-1} L^{1-\nu} = \mu,$$

which in equilibrium, i.e., with $\overline{E} = E$, gives the same condition as in the first best:

$$E = \left(\frac{\nu - \gamma}{\mu} \right)^{\frac{1}{1+\gamma-\nu}}.$$

Plugging in the expression for E_{sp} into (35), we get that the tax is given by

$$\tau = \frac{\gamma \mu}{\nu - \gamma}. \tag{36}$$

Note that an effective result requires that all nations agree on implementing the optimal tax. However, if they do, an individual country has incentives to deviate and increase E, because profits are positive at the optimal allocation (see equation 32). Hence, in a competitive setting the government has incentives to try to undercut other governments by offering lower tax rates to increase production and/or to attract firms to its country. Note that this incentive is true for all countries. Unless the tax can be combined with other international sticks and/or carrots, output is likely to increase and return to the inefficient market allocation (equation 22).

Another practical problem is that for the tax authorities to derive the efficient tax rate, they must actually know the externality cost function. However, a key problem is that in many cases, the authorities do not have this information. In fact, Pigou himself wrote that "It must be confessed, however, that we seldom know enough to decide in what fields and to what extent the State, on account of [the gaps between private and public costs] could interfere with individual choice".[37] This is particularly true when it comes to issues of climate change. First, there is a really large uncertainty about how much the temperature can be expected to increase and second, what damages different temperatures will cause.

4.5.4 Quantity restrictions

If the tax authorities actually know the externality cost function, they can simply just directly tell the firms how much to produce. That is, another option for the government is to directly restrict the quantity. A variant of quantity regulation of emissions is the cap-and-trade system, which involves tradable emissions permits. A number of emission licenses for a specified pollutant are then issued. If someone wants to emit more than what it is licensed for, it may buy additional licenses from other owners of licenses. Conversely, a firm that has more licenses than it intends to use can sell its surplus. The advantage of allowing trade is that some firms can reduce emissions more cheaply than others. For a firm with high costs of reducing emissions, it is more efficient to buy permits from firms that are able to reduce emissions more cheaply. This gives everyone an incentive to reduce pollution.

[37] Pigou (1954).

4.5.5 Taxes vs quantities

As shown above, the regulator can control pollution either by regulating the price, i.e., by imposing a Pigouvian tax or by directly setting emission quantities. In the absence of uncertainty, these two policy instruments are completely equivalent. Hence, with complete knowledge and perfect information about the costs of reducing the pollution and the benefits of cleaner air, there is a formal identity between the use of prices and quantities as planning instruments. However, in the presence of uncertainty this may no longer be the case. In reality, there is likely to be some uncertainty about both the exact specification of the cost function and the benefit function. The question is which of the two instruments that is then preferred? This problem is studied by Weitzman (1974). Specifically, he considers the case where an amount q of a certain good can be produced at a cost $C(q)$, yielding benefits $B(q)$. However, the costs also depend on a disturbance term (i.e., a random variable): $C(q, \mu)$. Similarly, the benefits also depend on another random variable: $B(q, \eta)$. Weitzman suggests that we could consider q as the cleanliness of air being emitted by a certain type of source. The costs then depend on q but they may be uncertain because the technology quantified by μ is uncertain. At a given level of q, the benefits may also be uncertain since they, among other things, depend on the weather, measured by η.

Specifically, he finds that price instruments are favoured when the marginal benefit schedule is relatively flat and quantity instruments are favoured when the marginal cost schedule is relatively flat. Price controls are preferred in the former case because if the social marginal benefit is approximately constant in some range, the best policy is to set the price and let the producers find the optimal output themselves (after eliminating the uncertainty from costs). This result in the latter case comes from the fact that when marginal costs are nearly flat, the smallest miscalculation or change results in either much more or much less than the desired quantity.

Despite the close resemblance between the two systems in theory, they are somewhat different in practice. Specifically, they produce different types of uncertainty. When the regulator chooses to use taxes, the price is clear for the polluters, but the amount of pollution generated is not. When restricting quantity, the government is aware of the amount of pollution, but the price of emissions will not be clear.

There is an ongoing debate about whether price or quantity restrictions are the better. Nordhaus argues in favor of the carbon tax by stating that quantitative limits may produce volatility in the market price of carbon.[38] This arises because of the inelasticity of both supply and demand of permits. The volatility might be economically costly for firms and provides inconsistent signals to private-sector decision makers. A carbon tax, on the other hand would, according to Nordhaus, provide consistent signals and would not vary so severely. He also thinks that the tax approach provides less opportunity for corruption than do quantitative limits, because the tax approach does not create artificial scarcities and thus encourages rent-seeking behavior. All in all, Nordhaus advocates the conceptual simplicity of the tax approach and views the cap-and-trade approach embodied in the Kyoto model as a poor choice of mechanism.

However, the tax-based approach also has shortcomings. First of all, even though taxes are a well-used tool in general, it is on unfamiliar ground in international environmental agreements. Moreover, there are some concerns regarding the price elasticity on carbon: how well do carbon emissions respond to increased prices?

In addition, since taxes cannot guarantee a specific emission level, a quantity restriction may be desirable because it better allows the regulator to ensure a specific level of emissions, which makes the system more predictable regarding climate change. However, knowing the correct quantity level

[38] Nordhaus (2009).

of emissions that will not cause excessive climate change is challenging or even impossible seeing that we do not have sufficient knowledge of all the direct and indirect effects on the climate, as well as the carbon system or the economic costs of climate change. Moreover, while a tax generates costs on the firms as well as revenue for the government, cap and trade often involve handing out licenses to existing agents, so that the revenue goes to the industry instead of the government. This might make quantity regulations more feasible politically speaking, since the firms are partly compensated for their loss. Hence, according to Krugman, it will probably be more challenging to achieve agreements in national parliaments regarding taxes, considering the powerful interests of lobbying energy-intensive firms affected by the regulation.[39] Even though a market-based policy instrument is a necessary condition for giving disincentives on carbon emissions, Krugman still advocates supplementing them with a direct regulation (legislation) on coal use.

4.6 Carbon Leakage

Since it is difficult to agree on an international policy, some countries might just decide to start reducing emissions by themselves. In this way, they will "move first" and then hope that others will follow. What is the consequence of this policy? Is it good for the environment or will it just hurt the countries that implement the policy with no effects on global aggregate emissions? One potential problem is carbon leakage, i.e., the fact that if one country imposes a tax on carbon, this might just shift the use of carbon to another country (that has a less strict climate policy). Below, we consider two cases to illustrate some determinants of whether carbon leakage is likely to occur.

Assume that there are two identical countries where the production function of the representative firm is given by

$$Y = AE^{\nu}.$$

For now, we abstract from any externalities.

4.6.1 First case: constant fuel prices

In the first case, the price of for fossil fuel is assumed to be constant, i.e., the cost for a firm of buying an amount E is simply μE. A representative firm in country 1 then solves the following problem

$$\max_{E_1} AE_1^{\nu} - \mu E_1.$$

The first-order condition with respect to E_1 is

$$A\nu E_1^{\nu-1} = \mu.$$

with the familiar interpretation that the marginal product of fuel is set equal to the price.

This can be solved for E_1:

$$E_1 = \left(\frac{A\nu}{\mu}\right)^{1/(1-\nu)}.$$

Similarly, since the two countries are identical, the output of firm 2 is

$$E_2 = \left(\frac{A\nu}{\mu}\right)^{1/(1-\nu)}.$$

[39] Krugman's article in the *New York Times*, April 5, 2010.

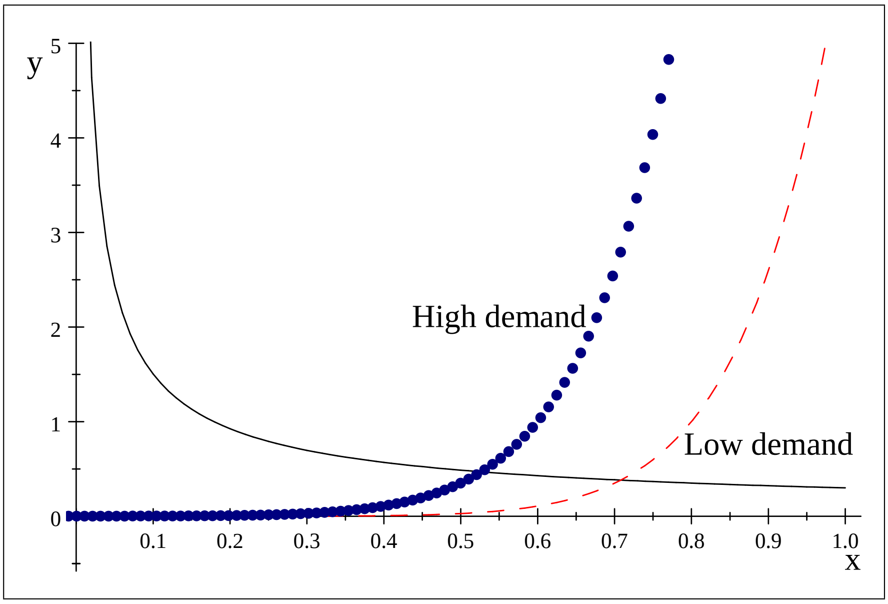


Figure 12: Supply and demand, country 1, $\psi = 10$

Now, note that the output of the two firms only depends on exogenous parameters. Hence, the output of firm 1 is independent of the output of firm 2 and vice versa. This implies that there will be no carbon leakage at all. There will be no increase in the production of firm 2 if country 1 decreases its production.

4.6.2 Second case: non-constant marginal costs

Now, instead assume that the price faced by each firm is affected by the demand of all other firms in the world. Higher average demand leads to a higher price. Let's take the example that the price of oil satisfies

$$\left(\bar{E}_1 + \bar{E}_2\right)^{\psi}.$$

The larger is ψ, the more convex is the pricing function.

A representative firm in country 1 then solves the following problem:

$$\max_{E_1} AE_1^{\nu} - \left(\bar{E}_1 + \bar{E}_2\right)^{\psi} E_1.$$

As above, we assume that the firm takes the average demand in the two countries as given. The first-order condition with respect to E_1 is

$$A\nu E_1^{\nu-1} = \left(\bar{E}_1 + \bar{E}_2\right)^{\psi}.$$

In equilibrium, all firms are doing the same so if we focus on country 1, we have

$$A\nu \bar{E}_1^{\nu-1} = \left(\bar{E}_1 + \bar{E}_2\right)^{\psi}$$

Let us now in figure 12 plot the LHS and the RHS against $\bar{E}_1$. Furthermore, we plot the RHS for two values of foreign average demand $\bar{E}_2$, one high and one low. We first do this for a high

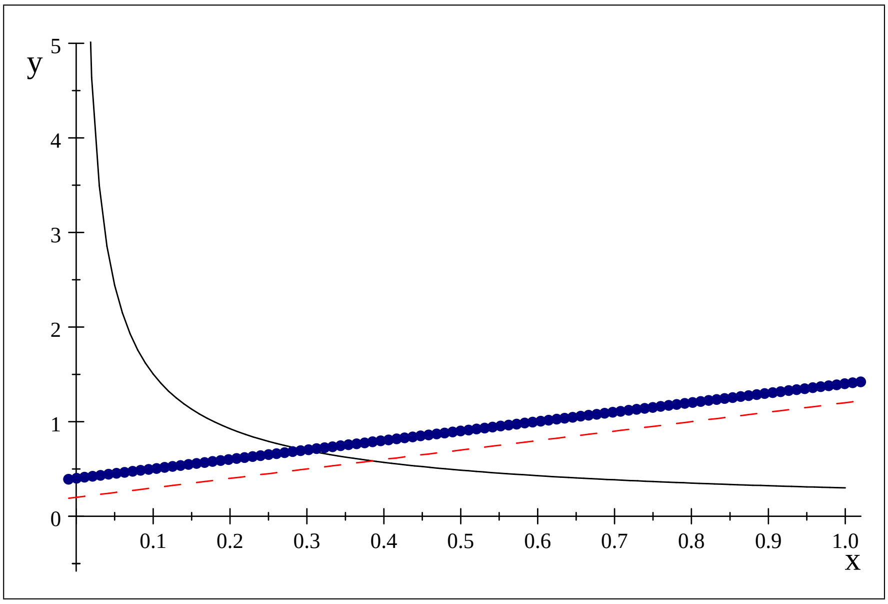


Figure 13: Supply and demand, country 1, $\psi = 1$

value of ψ, namely 10.[40]The downward-sloping curve is $A\nu\bar{E}_1^{\nu-1}$, the marginal product of fossil fuel in country 1. The leftmost upward-sloping function is the price as a function of domestic demand given high foreign demand and the rightmost curve is price given low demand. As we see, the latter curve is the same as the former, but shifted to the right by the amount $\bar{E}_2$ has fallen. In fact we, see leakage in this case is almost complete.

Let's now in figure 13 do the same experiment but instead set $\psi = 1$. In this case, the price is much less sensitive to aggregate global demand. Despite the fact that we change foreign demand by the same amount as in the previous figure, domestic demand changes much less. In other word, leakage is much smaller.

4.7 Problems with implementing the policy

4.7.1 Universal participation

As is clear from the above, it can be highly problematic to implement the efficient policy in practice. Even though all countries in our examples were identical, it was problematic because each of them had an incentive to deviate from the efficient tax rate. In reality, there are many more features that add to the difficulties because countries are heterogeneous. For this reason, they might disagree on how the policy should be divided between different countries.

Countries may also have different incentives to reduce emission because of different perceptions of damages, income levels, political structures, environmental attitudes and country sizes. Russia could, for instance, argue that it might be less affected by a temperature increase. A poor country may advocate that it had not yet contributed to much of the damage as well as point out that its low income level makes it challenging to contribute.

For an outcome to be self-enforcing, it must be viewed as being fair to all parties. There are ways of deterring incentives to non-participation when the issues regard unfairness. One alternative is side payments—i.e. countries that gain most from an agreement compensate those who would

[40]We use the parameters $A = 1, \alpha = 0.3$, and $\bar{E}_2 = 0.2, 0.4$, respectively.

lose or gain least — which can implicitly be provided through the international tradable permit system. In general, market-based instruments can be considered as providing positive incentives in the sense that they can reduce costs overall and potentially for all parties.

Still, these positive incentives might not be sufficient to overcome the severe free-riding problems. Hence, negative incentives such as punishment for non-participation might also be necessary. However, ensuring that such incentives provide credible threats is difficult given the lack of a powerful world government with coercive powers.

4.8 References

Dell, M., Benjamin F. Jones and Benjamin A. Olken. "Climate change and economic growth: evidence from the last half century", NBER Working Paper No.14132.

Krugman, P., "Building a Green Economy, the New York Times, April 5, 2010.

Nordhaus, W., A question of balance, Weighing the options on global warming policies", Yale University press, New Haven & London, 2008.

Nordhaus, W., "Economic issues in designing a global agreement on global warming." Keynote address prepared for Climate Change: Global Risks, Challenges and Decisions. Copenhagen, 2009.

Pigou, A. C., Some Aspects of the Welfare State, 1954.

Weitzman M, "Prices *vs.* quantities", *Review of Economic Studies* 1974.

5 Growth theory in climate research

The relationship between climate and economic growth has been of interest in social research for a considerate amount of time. The subject of growth is relevant for climate research in a number of ways. First of all the climate is—indirectly, via the carbon cycle—influenced by the burning of fossil fuels. Clearly, the use of fossil fuel is linked to economic activity and therefore projections of future fossil fuel use is interlinked with aspects of economic growth.

Secondly, there are different views on the reversed effect, namely the effect from climate change on the economy. One perspective is that long-run average temperature affects economic performance negatively, through a variety of channels, which may have implications on economic decisions on actions against climate change.

Clearly, there are several factors that connect growth and climate economics. Throughout this chapter, however, we will mainly lay our focus on the demand side of the economy as opposed to the supply side.

5.1 Empirics

There is large variation in per capita income as well as growth rates across countries' economies and growth rates do not remain constant over time. The following graphs are in log scales and illustrate both the distribution of prosperity across countries as well as its evolution over time. The following figure shows the development of GDP per capita in a number of countries over close to two hundred years. As we see, the curves for the US and Britain are fairly close to linear implying the growth rate has been fairly stable during this long time period.[41] In some of the other countries, growth is more unstable with stagnant periods and growth "miracles".

[41] The y-axis uses a log scale, implying that a constant growth rate gives a straight line.

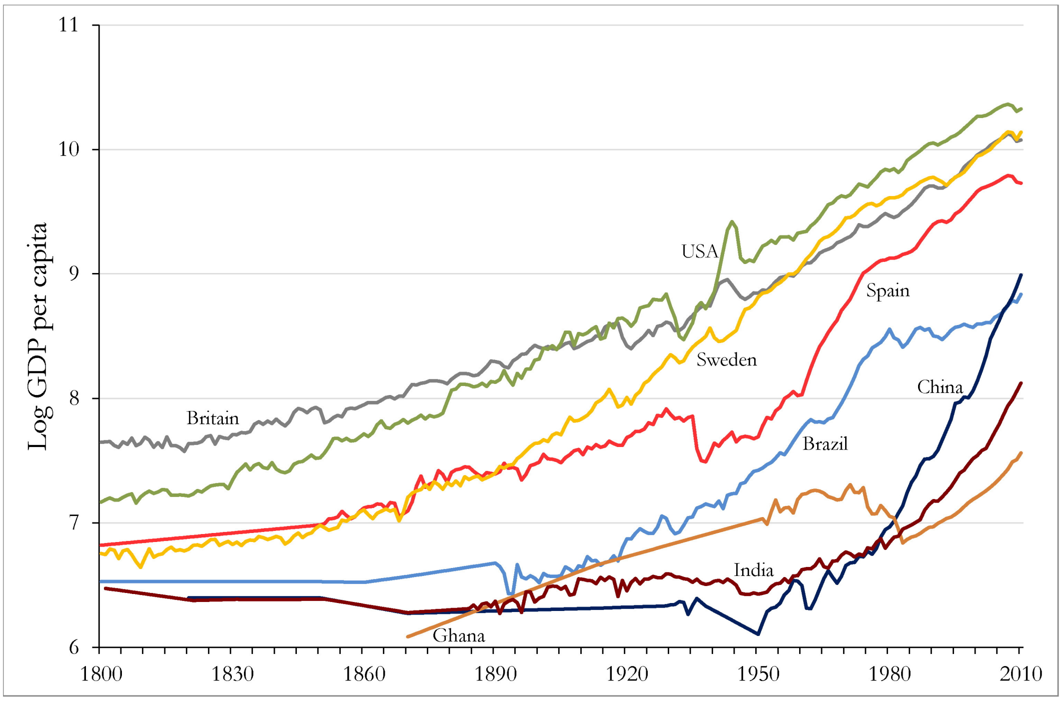


Figure 14: Historic growth

The next figure shows the distribution of countries with regard to their GDP per capita in 1960, 1980 and 2000. The distribution has shifted to the right over time but there is no clear tendency towards a wider distribution.

Using population weights, the shift to the right is more pronounced.

From the perspective of this book, it is of particular interest that there is a negative correlation between the average temperature in a country and its GDP per capita. This is illustrated in the following graph, which also shows how climate change has affected the countries so far (rings vs crosses) and the variability of the climate. The graph also shows the large variation in GDP per capita – in the order of 4-5 log points, implying a ratio between the richest and the poorest country in the order of $50-150$.[42] We will make an attempt to use growth theory to account for these differences below.

[42] $e^4 \approx 55, e^5 \approx 148$.

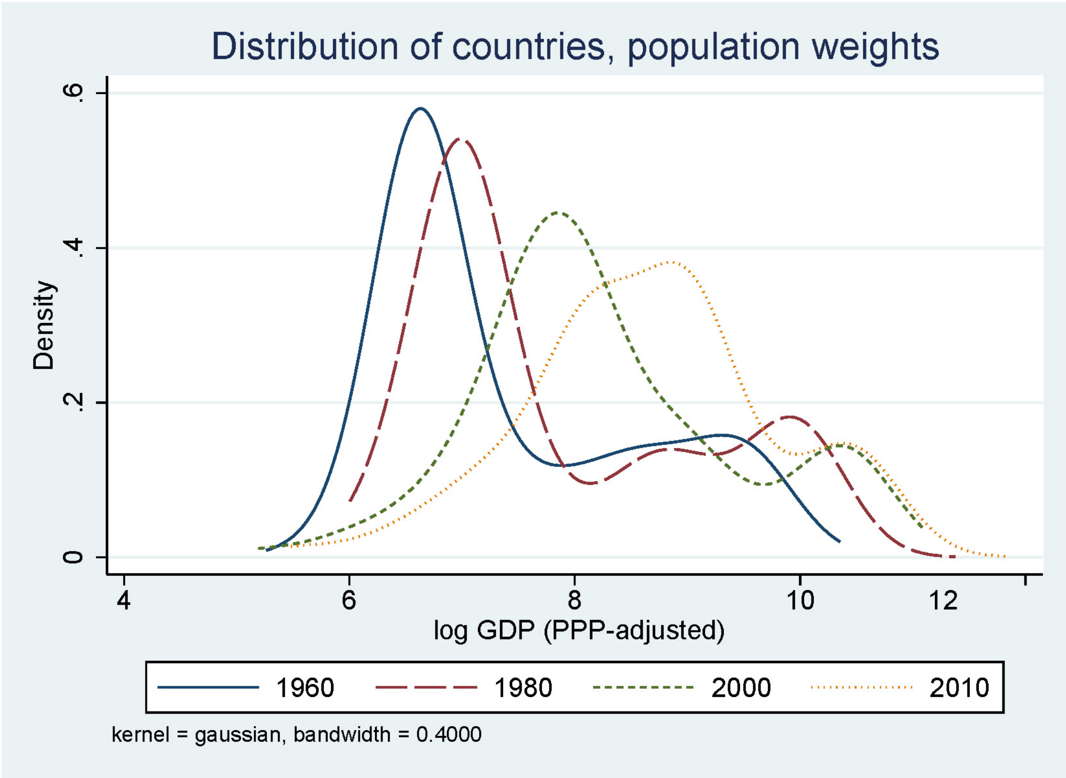
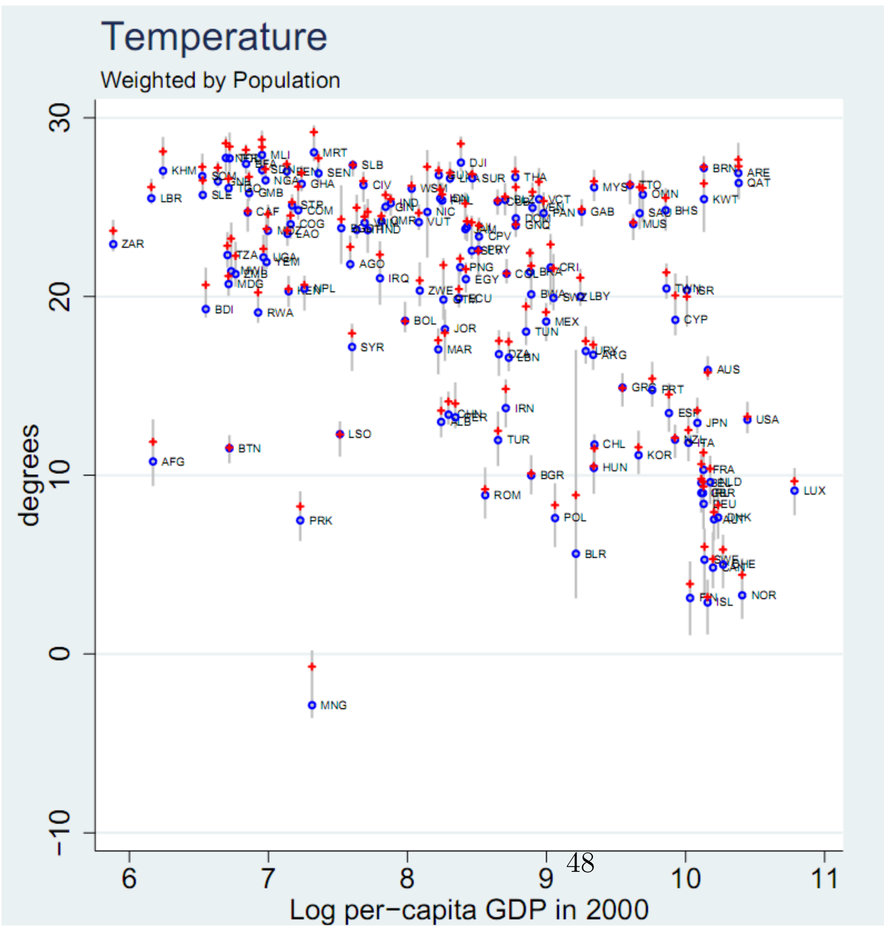


Figure 15: Distribution of countries according to PPP-adjusted GDP per capita.



Blue circle (red plus) is mean temp in 1950-1959 (1996-2005). Gray lines is range of annual temperature over sample period.

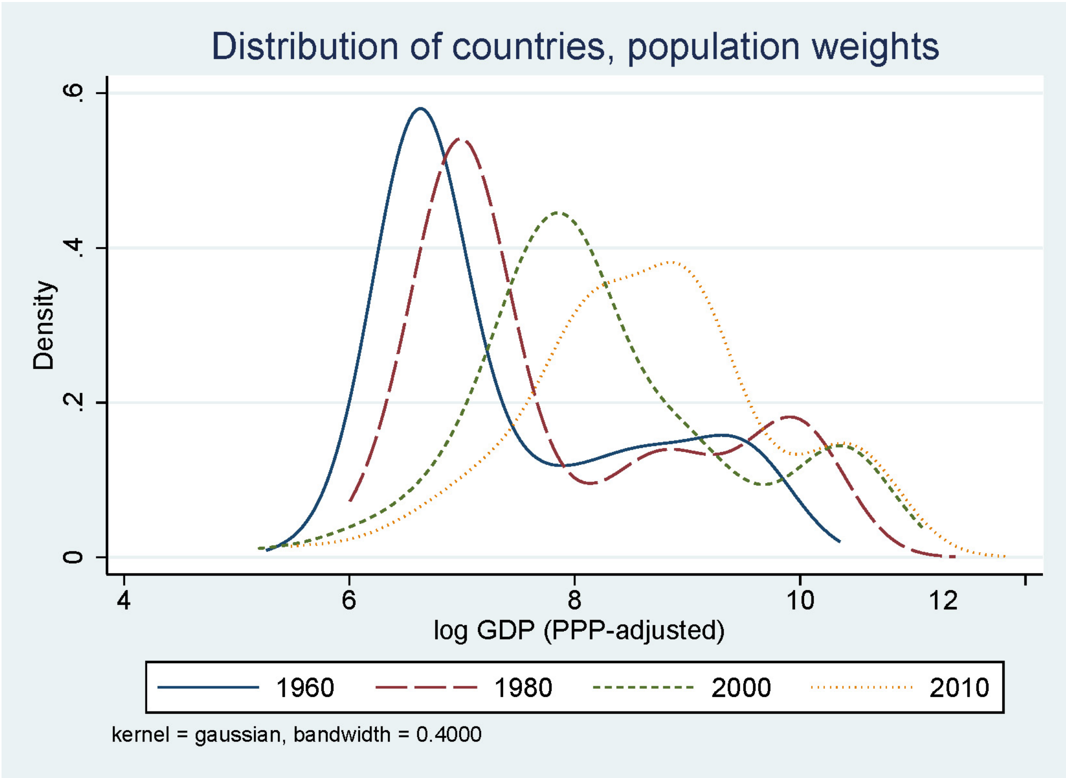


Figure 16: Distribution of countries according to PPP-adjusted GDP per capita.

5.2 Growth accounting

On a general level output is determined by different factors. One is the availability of factors of production (capital, labor, raw materials) and another is their "efficiency levels". Other factors are the quality of the capital (a broad notion of capital includes infrastructure); the amount of human capital/knowledge embodied in people; as well as how efficiently the inputs are used. Are the factors matched in the most productive way?

Under some simplifying assumptions, we can think of output in a country as a function of capital and labor

$$Y = f(K, L)$$

where for simplicity we abstract from raw materials, such as fossil fuel. Capital and labor are denoted *factors of production* and the relation between output and these factors is described by the *aggregate production function* f.

The amount inputs differs between countries due to differences in labor force participation and perhaps more importantly, due to difference in skill and education. By measuring K and L one can then assess how each of these factors matters.

To decompose differences in output between countries, we may use a method to measure the contribution of different factors to economic growth. Indirectly we thus compute the rate of technological progress, measured as the "*Solow residual*", in an economy. Under the assumptions that outputs and inputs can be measured in comparable ways across countries; the existence of an aggregate production function that is identical across countries except for a scalar "productivity" level (A) allows us to account for differences in income between countries. We assume output in country

i is given by

$$Y_i = A_i K_i^{\alpha} l_i^{1-\alpha},$$

If a value of α is assigned to this equation, we may perform a simple accounting exercise to break down growth in output into growth in capital, growth in labor and growth in technology, respectively.

Capital/labor share of income

We can show that α is capital's share of output and that labor share of output is $1-\alpha$ by looking at the first-order conditions in the firms maximization problem to obtain expressions for prices, w and r, as functions of capital stocks;

$$\max_{K,L} \pi = AK^{\alpha}L^{1-\alpha} - rK - wL$$

Taking the derivatives

$$\begin{aligned} \frac{\partial \pi}{\partial K} &= \alpha A K^{\alpha-1} L^{1-\alpha} = r \\ \frac{\partial \pi}{\partial L} &= (1-\alpha) A K^{\alpha} L^{-\alpha} = w \end{aligned}$$

If markets are perfect, firms will make sure that the rental rate of capital r and the wage w is equal to the marginal productivities. Multiplying the two equations above K and L, respectively and dividing by Y, gives capital's share and labor's share of firm revenues , respectively.

$$\begin{aligned} \alpha &= \frac{rK}{Y} \\ 1-\alpha &= \frac{wL}{Y} \end{aligned}$$

By using this relation, we can directly find the parameter α by measuring the the income share of capital in GDP. Over time and across countries, it turns out that this number has been quite stable at around 0.3.

Consider two countries i and j, with output Y_i and Y_j. Using the production function, we have

$$\frac{Y_i}{Y_j} = \frac{A_i K_i^{\alpha} L_t^{1-\alpha}}{A_j K_j^{\alpha} L_j^{1-\alpha}}$$

Taking natural logs of this ratio yields:

$$\log Y_i - \log Y_j = [\log A_i - \log A_j] + \alpha[\log K_i - \log K_j] + (1-\alpha)[\log L_i - \log L_j] \tag{37}$$

The second (third) term on the right hand side in the equation is the importance of capital (labor) in explaining output differences between i and j. The first term on the right hand side is a "*residual*" and "explains" what cannot be accounted for by the other factorsand is said to be a measure of *total factor productivity.* Total factor productivity is often referred to as "*technology*", but in this case one should keep in mind that it is technology in the widest possible sense. It is serving as a catch-all for anything else that is left unexplained by the other two factors, labor and capital. Specifically, the residual can be interpreted as the extent to which the assumptions are off; mismeasurements of inputs and outputs; an inappropriate assumption regarding the aggregate technology (e.g., it would look different in an agricultural economy, a manufacturing economy, and a service economy); inefficiencies in production which could be caused by poor institutions, poor infrastructure, etc.; or differences in technology across countries.

5.2.1 Measuring factor inputs

Countries differ a lot regarding how educated their workers are as well as in terms of the marginal value of a year of education in terms of productivity. However, we assume that a year of basic education is worth the same everywhere. Furthermore, each additional year gives the same "premium". In the U.S., a multitude of studies indicate that a year of additional education raises a worker's wage by $6-10\%$ and studies in other countries give similar results.[43] Lastly, we assume that there are no externalities to schooling, i.e., all returns to schooling accrue to the individual. Assuming negligible training and education of the workers in the poorest countries and normalizing their quality to 1, workers in the richest countries then have 12 years more of schooling than the workers in the poorest countries. It then follows that L_i is $1.08^{12} \approx 2.5$ and $L_j = 1$ if consider a rich and a poor country of the same size in terms of worked hours.

$$\log L_{top} - \log L_{bottom} \approx \log 2.5 \approx 0.9.$$

Multiplying this number by 0.7, we have that the last term on (37) is 0.63 if we use a very rich and very poor country.

We will return to how to measure capital inputs later, but as an example, let us use the fact that capital/output ratios tends to be fairly stable along a economic development. Then, if we want to account for a log difference $\log(Y_i) - \log(Y_j)$ of 4 (implying an income ratio of around 50), the contribution from differences in capital stocks is given by $0.3 * 4 = 1.2$. In sum, we can therefore account for $0.63 + 1.2 = 1.83$ of the 4 log point difference, i.e., a bit less than half, by differences in quality adjusted labor and capital.

5.3 The Solow growth model

Solow's growth model provides an important foundation for understanding growth and has become the work-horse model that modern macroeconomic models are built on. In the simple Solow model without technology growth, economies may grow for a while, but not perpetually. It will gradually move towards its *steady state* level, which in the simplified model is a point where there is no permanent economic growth. An economy that starts off with a stock of capital per worker below its steady state level will experience growth along the transition path to steady state, seeing as investment per worker exceeds the amount needed to keep capital per worker constant. Eventually, however, growth slows down as the economy approaches its steady state and ultimately growth in capital per worker stops altogether. This is a stable point and will be maintained because capital and labor will continue to grow in the same ratio. However, recalling the graphs, the countries in fact grow at a roughly constant rate, i.e. growth does not seem to decline according to empirical data.

Adding technology growth the model, however, enables the Solow model to explain *sustained* growth. This addition makes the model more in line with empirical data from many countries which so far shows continued growth. A general prediction of the model is that an economy will always converge towards a *balanced growth rate*, which depends only on the rate of technological progress.

First, we present the simpler version of the model, i.e. without technology growth. Before going into the model, there are several simplifying assumptions that need to be made. *Constant returns to scale* entails that a proportional change in all inputs results in an increase in output by

[43] PER'S SOURCE

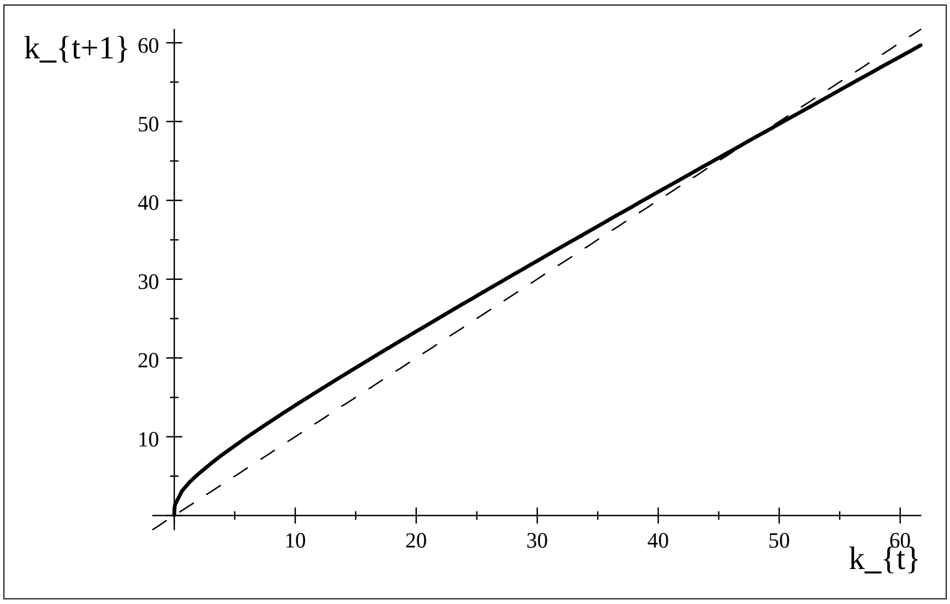


Figure 17: $K_{t+1} = (1-\delta)\,K_t + sAK_t^{\alpha}$

that same proportional change. *Inada conditions*[44] concern the shape of the production function, whereas *diminishing returns on all inputs* means that the **marginal** production of a factor starts to progressively decrease as the factor is increased. Furthermore, we assume some degree of substitution between the inputs as well as exogenous rates of technology. Lastly, a constant fraction, δ, of the capital stock depreciates every period and a fraction, s, of output each period is invested/saved.

A key equation in the Solow's exogenous growth model is the one that describes how capital accumulates in a country and it is given by

$$K_{t+1} = (1-\delta)\,K_t + sAK_t^{\alpha}L_t^{1-\alpha} \tag{38}$$

where the key variable, s, is treated as exogenous (to be endogenized later). This gives us a law of motion for capital. According to this equation, the capital stock in the following period $(t+1)$ is equal to the fraction of capital that has not yet depreciated in period t plus the fraction saved in period t (including some technology parameter).

We can express this equation per unit of labor. Denote capital per unit of labor by K and output per unit of labor by Y. We then have

$$K_{t+1} = (1-\delta)\,K_t + sAK_t^{\alpha} \tag{39}$$

The dynamic equation (39) has a unique steady state. This is easily seen by noting that K_{t+1} is a concave function of K_{t+1} with a slope at zero that tends to infinity as K_t approach zero as shown in the figure below, where we have set $\delta = 0.2, \alpha = 0.3$ and $sA = 3$.The graph also has a dashed 45 degree line and where there curve $(1-\delta)\,K_t + sAK_t^{\alpha}$ crosses the 45 degree line is the steady state.

Denoting the steady state by K_{ss}, we can easily find it from by substituting K_{ss} for K_t and K_{t+1} and solving., i.e.,

$$K_{ss} = (1-\delta)\,K_{ss} + sA_iK_{ss}^{\alpha},$$

[44] The value of the function at 0 is 0; the function is continuously differentiable; the function is strictly increasing in inputs; the derivative of the function is decreasing (thus the function is concave); the limit of the derivative towards 0 is positive infinity; the limit of the derivative towards positive infinity is 0.

implying that

$$K_{ss} = \left(\frac{sA_i}{\delta}\right)^{\frac{1}{1-\alpha}} \tag{40}$$

The dynamic equation (39) is stable in the sense that for any $K_0 > 0$, it path converges over time to K_{ss}.

Output per capita, y, and capital per worker, K, are directly related. A higher capital per worker entails a higher output per worker. Note from the equation (40) that a higher savings rate or a higher level of technology thus makes an economy richer, ceteris paribus. We can check this by inserting the expression for K_{ss} into the production function to get an expression for output per unit of labor, y_{ss}.

Using the production function

$$y_i = A_i K_i^{\alpha}$$

where $L = 1$. Inserting for K_{ss} from equation (40) yields the steady state level of output

$$Y_{ss} = A_i \left(\frac{sA_i}{\delta}\right)^{\frac{\alpha}{1-\alpha}}.$$

From this result we can see that a higher savings rate will make the economy richer.

5.3.1 Balanced growth with exogenous technological change

Let us now add technological growth to the model, which will allow for long term continuous growth. Technology is assumed to grow at an exogenous rate g_A. In other words, Solow's model is an *exogenous* growth model. Technological progress is thus treated as exogenous—like "manna from heaven—, which means that we do not model from where it originates.

Suppose that technology at a constant rate g_A and assume for now that labor input is constant normalized to unity. The production function is then

$$Y_t = A_0 e^{g_A t} K_t^{\alpha}.$$

If we define $\widetilde{K}_t = \frac{K_t}{e^{\frac{g_A}{1-\alpha}t}}$,

$$\begin{aligned}
\widetilde{K}_{t+1} e^{\frac{g_A}{1-\alpha}(t+1)} &= (1-\delta)\widetilde{K}_t e^{\frac{g_A}{1-\alpha}t} + sA_0 e^{g_A t}\left(\widetilde{K}_t e^{\frac{g_A}{1-\alpha}t}\right)^{\alpha} \\
&= (1-\delta)\widetilde{K}_t e^{\frac{g_A}{1-\alpha}t} + sA_0 \widetilde{K}_t^{\alpha} e^{\frac{g_A}{1-\alpha}t} \\
\widetilde{K}_{t+1} &= \frac{(1-\delta)}{e^{\frac{g_A}{1-\alpha}}}\widetilde{K}_t + \frac{sA_0}{e^{\frac{g_A}{1-\alpha}}}\widetilde{K}_t^{\alpha}
\end{aligned}$$

Noting that $e^{\frac{g_A}{1-\alpha}} \approx 1 + \frac{g_A}{1-\alpha}$yields

$$\widetilde{K}_{t+1} = \frac{1-\delta}{1+\frac{g_A}{1-\alpha}}\widetilde{K}_t + \frac{sA}{1+\frac{g_A}{1-\alpha}}\widetilde{K}_t^{\alpha}.$$

As we see, this is the same equation as (38)) except for the term $1+\frac{g_A}{1-\alpha}$ which affects the slope of the curve. We can again derive the formula for the steady state of $\widetilde{K}$,denoted;

$$\widetilde{K}_{ss} = \left(\frac{sA}{\frac{g_A}{1-\alpha}+\delta}\right)^{\frac{1}{1-\alpha}}$$

The interpretation is that there is a steady state in the adjusted variable $\widetilde{K}_t$. Clearly when $\widetilde{K}_t$ has reached its steady state $\widetilde{K}_{ss}$, and K_t grows at the rate $\frac{g_A}{1-\alpha}$ so that $K_{t+1} = e^{\frac{g_A}{1-\alpha}}K_t$. What about output?

$$\begin{aligned}
\frac{y_{t+1}}{y_t} &= \frac{A_{t+1}K_{t+1}^{\alpha}}{A_tK_t^{\alpha}} \\
&= \frac{A_0e^{g_A(t+1)}\left(e^{\frac{g_A}{1-\alpha}}K_t\right)^{\alpha}}{A_0e^{g_At}K_t^{\alpha}} \\
&= \frac{e^{g_A(t+1)}\left(e^{\frac{\alpha}{1-\alpha}g_A}\right)}{e^{g_At}} = e^{\frac{g_A}{1-\alpha}}
\end{aligned}$$

That is, output also grows at the rate $\frac{g_A}{1-\alpha}$. Using the fact that labors income share is constant, it also follows that the wage rate grows at $\frac{g_A}{1-\alpha}$ while the rental rate of capital r is constant since capital and output grows at the same rate. Such a growth path, where capital and output grows at a common and constant rate is called a *balanced growth path.* Here, it is important to note that savings enter the level of $\widetilde{K}_{ss}$ but not the growth rate along the balanced growth path. Thus, a higher savings shifts up the balanced growth path but does not change it's slope.

If we also have growth in labor input, we can find the balanced growth rate in the following way. Take the logarithm of the production function

$$\ln Y_t = \ln A_t + \alpha \ln K + (1-\alpha)\ln L_t.$$

Noting the time derivative of a log of variable is equal to its growth rate and setting the growth rate of K_t equal to the growth rate of Y_t, we have

$$\begin{aligned}
g_Y &= g_A + \alpha g_Y + (1-\alpha)\, g_L \\
&\Rightarrow\; g_Y = \frac{g_A}{1-\alpha} + g_L
\end{aligned}$$

Thus, the balanced growth rate of output is $\frac{g_A}{1-\alpha} + g_L$ and consequently, the balanced growth rate of per capita output is $\frac{g_A}{1-\alpha}$.

5.4 The determinants of saving

People have preferences over consumption goods at different points in time. The graphic outline of the preferences looks like the standard indifference curves. Yet, in *this* case the two "goods" are consumption between two periods. For two periods, a utility function could look like

$$u\left(c_{2010}\right) + \beta u\left(c_{2011}\right) \tag{41}$$

Here, u is a concave function (decreasing marginal utility) and $\beta < 1$ is a *discount factor* representing how much less the individual value consumption in the future compared to the present.

The key determinants of a consumer's inter-temporal preferences involve smoothing and impatience. *Smoothing,* since the indifference curves are convex toward the origin.[45] Just how *concave* u is illustrates the level of indifference between current and future consumption. One would rather have constant than volatile consumption. The more concave the curve, the less indifferent is the

[45] A function is *convex* if the function lies below or on the straight line segment connecting two points (i.e. in two dimensions), for any two points in the interval.

individual. The preferences involve *impatience* for the reason that if the real interest rate was zero—so that consumption today cost the same as consumption in the future—people would consume more now than in the future, simply because people value consumption today more than consumption tomorrow.

Consider a two-period model with logarithmic utility. A representative consumer thus wants to maximize

$$\log C_1 + \beta \log C_2.$$

Using a production function $Y_t = AK_t^\alpha$ (setting the labor endowment to unity) and assuming an initial capital endowment of K_1, consumption has to satisfy the aggregate resource constraint

$$\begin{aligned} C_1 + K_2 &= (1-\delta) K_1 + AK_1^\alpha \\ C_2 &= (1-\delta) K_2 + AK_2^\alpha \end{aligned}$$

Substituting the consumption levels into the utility function and taking first-order conditions with respect to K_2, one can in principle solve for savings. In the special case where $\delta = 1$, the problem becomes particularly easy. Substituting from the resource constraints, the problem becomes

$$\max_{K_2} \log\left(AK_1^\alpha - K_2\right) + \beta \log\left(AK_2^\alpha\right).$$

The first order condition is

$$\frac{1}{Y_1 - K_2} = \frac{\beta\alpha}{K_2}$$

with the solution

$$K_2 = \frac{\alpha\beta}{1+\alpha\beta} Y_1.$$

5.5 The optimal saving rate with an infinite horizon

Now consider an infinite horizon model where production is given by

$$Y_t = A_t K_t^\alpha. \tag{42}$$

Note that here, we are allowing an arbitrary path for the technology variable A_t. Relying on the finding above that assuming full depreciation simplifies the analysis we set $\delta = 1$ and set the representative agent's preferences to be logarithmic in consumption. To maximize utility we then solve the following inter-generational maximization problem

$$\max_{\{C_t, K_{t+1}\}_{t=0}^\infty} \sum_{t=0}^\infty \beta^t \log\left(C_t\right)$$

subject to the resource constraint

$$C_t + K_{t+1} = A_t K_t^\alpha. \tag{43}$$

Note that this problem can be written as

$$\max_{\{C_t, K_{t+1}\}_{t=0}^\infty} \sum_{t=0}^\infty \beta^t \log\left(A_t K_t^\alpha - K_{t+1}\right),$$

given K_0

The first order condition with respect to K_{t+1} is

$$\begin{aligned}
-\frac{1}{A_t K_t^{\alpha} - K_{t+1}} + \beta \frac{\alpha A_{t+1} K_{t+1}^{\alpha-1}}{A_{t+1} K_{t+1}^{\alpha} - K_{t+2}} &= 0 \\
\frac{1}{C_t} &= \beta \frac{1}{C_{t+1}} \alpha \frac{Y_{t+1}}{K_{t+1}}
\end{aligned} \tag{44}$$

Now let s_t denote the savings rate so that

$$K_{t+1} = s_t Y_t = s_t A_t K_t^{\alpha}$$

and

$$C_t = (1 - s_t) Y_t.$$

Using this in (44) gives

$$\frac{1}{(1 - s_t) Y_t} = \beta \frac{1}{(1 - s_{t+1}) Y_{t+1}} \frac{\alpha Y_{t+1}}{s_t Y_t}.$$

Simplifying yields,

$$s_{t+1} = 1 + \beta\alpha - \frac{\beta\alpha}{s_t}.$$

This is an unstable difference equation. Therefore, the only way it can be satisfied at all times is if savings is always equal to its steady state given by $s = \beta\alpha$ at any time.

Certainly, the result that the savings rate is constant relies on the particular assumptions. Specifically, it is due to the fact that when i) there is full depreciation so that consumption and investments need to sum to aggregate output and ii) utility is logarithmic, the income and substitution effect of changes in the return to savings cancel. This is a very convenient result that we will use below. We should also note that although the savings rate does vary substantially over the business cycle, it is more stable over time and across countries. An example of the latter is shown in the following figure.

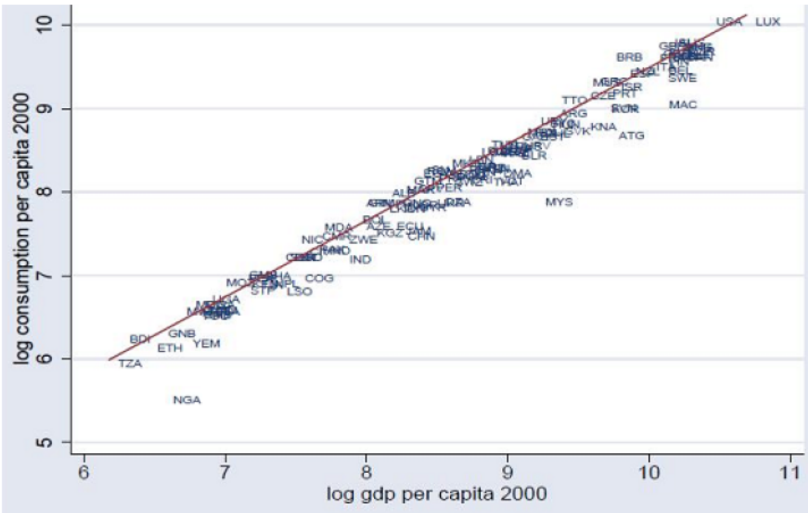


Figure 18: Consumption vs income.

5.6 References

Acemoglu, D., Johnson, S. and Robinson, J., 2001. The colonial origins of comparative development: An empirical investigation. *The American Economic Review*, 91(5), pp. 1369-1401

Acemoglu, D., 2009. *Introduction to Modern Economic Growth.* New Jersey: Princeton University Press

Barro, R. and Sala-i-Martin, X., 1992. Convergence. *The Journal of Political Economy*, 100(2), pp. 223-251

Dell, M., Jones, B. and Olken, B., 2009. Does climate change affect economic growth? Available through Vox *http://www.voxeu.org/index.php?q=node/3633* Accessed 18 March 2011

Diamond, J., 1997. *Guns, Germs, and Steel: The Fates of Human Societies.* New York: W.W. Norton & Company

Dollar, D., and Kraay, A., 2002. Institutions, Trade, and Growth. In: The World Bank. *Carnegie-Rochester Conference Series on Public Policy.* July 2002

Gallup, J., Sachs, J., and Mellinger, A., 1999. Geography and economic development. *International Regional Science Review*, 22(2)

Glaeser, E., Porta, R., Lopez-de-Silanes, F. and Schleifer, A., 2004. Do institutions cause growth? *Journal of Economic Growth*, 9(3). Available at http://www.economics.harvard.edu/faculty/glaeser/fi

Hall, R. and Jones, C., 1999. Why Do Some Countries Produce So Much More Output Per Worker Than Others? *The Quarterly Journal of Economics*, 114(1): 83-116

Jones, C., 2002. *Introduction to Economic Growth.* 2nd ed. New York: W. W. Norton & Company

Lindbeck, A. and Persson, M., 2000. What are the Gains from Pension Reform? *Working Paper Series* 535, Research Institute of Industrial Economics

North, D., 1989. Institutions and economic growth: An historical introduction. *World Development*, 17(9). pp. 1319-1332

Rodrik, D., Subramanian, A. and Trebbi, F., 2004. Institutions Rule: The Primacy of Institutions Over Geography and Integration in Economic Development. *Journal of Economic Growth.*

Springer, 9(2), pp. 131-165

Rodrik, D., 2005. Growth Strategies. In: P. Aghion & S. Durlauf (ed.). *Handbook of Economic Growth,* 1(1), Ch. 14

Rodrik, D., 2008. *What use is sources-of-growth accounting?* Professor of International Political Economy Harvard Kennedy School. Weblog, Feb. 11th 2008. Available at http://rodrik.typepad.com/dani_rod use-is-sou.html Accessed 19 March 2011.

Romer, P., 1990. Endogenous Technological Change. *The Journal of Political Economy,* 98 (5), pp. 71-102

Weitzman, M., 2007. A Review of The Stern Review on the Economics of Climate Change. *Journal of Economic Literature.* 45(3): 723-724

(Chapter head:)

6 Natural Resource Economics

Natural resource economics concerns the supply, demand, and allocation of natural resources. In any economy, there could be a number of commodities that are used as inputs in the production function, whose available stock cannot be increased implying that they are depleted over time. One example is fossil fuels. These conditions raises several questions. First, at what rate should these resources be depleted? Second, given that these resources are important inputs, what does it imply for the economy's growth rate? If some inputs are essential[46] and there is no technical progress, must feasible output eventually have to decline to zero? These problems received attention in the 1970s after the oil shocks. Examples of leading research contributions are Dasgupta and Heal (1974), as well as Stiglitz (1974).

Another important question concerns the effects of taxes when some inputs are in fixed supply. Furthermore, this chapter provides an analysis of the effect of increased investments into cleaner and more sustainable energy sources—backstop technology—on fossil fuel use.

Ultimately, we will treat the issue regarding renewable resources. We will illustrate that the optimization analysis is somewhat different in this case. The primary economic challenge in this case is maintaining an efficient, sustainable flow across time. This goes for resources such as fisheries, forests, agriculture, etc. We will analyze the management of forests—seeing as trees are important for carbon storage—and determine what the effect on optimal forest rotation is when carbon uptake in trees is included in the model.

6.1 A cake-eating problem

6.1.1 One region and one period

We will start by a very simple example where oil is used in production and exists in finite supply. Assume that the total supply of oil is fixed at the amount R. Assume also that the cost providing the units of E is equal to zero (i.e., there are no extraction costs). The sector that owns the oil is then willing to sell everything at any price larger than zero.

The problem in the final goods sector is then

$$\pi = \max_{E} E^{\nu} L^{1-\nu} - pE - wL \tag{45}$$

The first-order condition w.r.t E is:

$$p = \nu \left(\frac{L}{E} \right)^{1-\nu} = \frac{\nu Y}{E}.$$

where $Y = E^{\nu} L^{1-\nu}$ denotes aggregate output. The equation implies that the price is equal to the marginal product of oil. We get a positive price and hence, all oil is sold:

$$p = \frac{\nu Y}{R} \tag{46}$$

Let us now consider the effect of taxes in this model. Assume that we want to impose a tax rate to reduce the amount of E used. The problem is then with taxes:

$$\pi = \max_{E} E^{\nu} L^{1-\nu} - (p + \tau) E - wL.$$

[46] By essential we mean that the input in necessary for production, i.e. if E is essential we have $F(K, E) = 0$ when $E = 0$.

The first order condition with respect to E is

$$p+\tau=\nu\left(\frac{L}{E}\right)^{1-\nu}=\frac{\nu Y}{E},$$

The sector that owns the oil is willing to sell all oil as long as the price is larger than zero. In fact, selling all oil at a strictly positive price is optimal for them. Hence, we have

$$p+\tau=\frac{\nu Y}{R} \tag{47}$$

Since, the right hand side (47) is independent of the tax it must be that if the tax rate goes up, the price will go down by the same amount. The effect of the tax is to reduce profits for the supplying sector and to generate tax revenues without affecting anything else in the economy.

What happens if $\tau>\frac{\nu Y}{R}$. Clearly the equation then implies a negative price. In such a case the private marginal product of oil is lower than the tax, *when all existing oil* is sold. Clearly, oil sellers then loose money by selling. This cannot be an equilibrium unless we can force oil owners to sell at a negative price. Instead, the quantity E has to adjust downwards until the private marginal product equals the tax

$$\tau=\nu\left(\frac{L}{E}\right)^{1-\nu}\Rightarrow E=\left(\frac{\nu L^{1-\nu}}{\tau}\right)^{\frac{1}{1-\nu}}$$

which will be strictly positive. In this case, the return to the oil owners is zero. They would like to constrain the oil supply further since that would generate profits. Without an ability to collude this is not possible (here by assumption).

A key conclusion from this is that only if the price received by oil owners is pushed to zero can the tax affect aggregate oil use.

6.2 Two-regions and one period

Assume now that there are two regions labeled 1 and 2. Furthermore, assume that oil is a traded commodity with a common world market price p, excluding taxes. Region 1 imposes the tax rate :

$$\pi_1=\max_{E_1}E_1^{\nu}L_1^{1-\nu}-(p+\tau)E_1-wL_1,$$

with $E_1+E_2=R$. In region 2, the tax rate is 0. Hence the maximization problem for that region is again given by:

$$\pi_2=\max_{E_2}E_2^{\nu}L_2^{1-\nu}-pE_2-wL_2,$$

The first order conditions with respect to E_1 and E_2 are respectively

$$\begin{aligned} p+\tau &= \nu\left(\frac{L_1}{E_1}\right)^{1-\nu} \\ p &= \nu\left(\frac{L_2}{E_2}\right)^{1-\nu} \end{aligned}$$

Now, take the difference between the the first and the second equation above and use the fact that $E_2=R-E_1$ as long as $p>0$. We then arrive at

$$\tau=\nu\left[\left(\frac{L_1}{E_1}\right)^{1-\nu}-\left(\frac{L_2}{R-E_1}\right)^{1-\nu}\right].$$

We thus see that if region 1 chooses a high positive tax rate $\tau > 0$, the left hand side increases. For the equation to hold, the right hand side must then also increase, which it will do if E_1 decreases and E_2 (given by $R - E_1$) increases by the same amount. The only result of the higher tax rate in region 1 is thus to decrease the production in region 1, while increasing the production in region 2. The aggregate effect is zero. This effect is in the popular debate referred to as *carbon leakage.* In addition, it is inefficient, since the oil should be allocated in a way so that the marginal product is the same in the two regions. Hence aggregate production could be increased.

6.2.1 Two periods

Dasgupta and Heal (1974) consider a problem of how to deplete a natural resource over time in an infinite horizon setting. We will for simplicity consider a problem with only two periods, i.e., the world ends in period two. The two periods are labeled 1 and 2. We will also, for simplicity, abstract from externalities and from taxes.

Suppose we have a given stock R of a resource, which can not be increased and we want to find out how to use it over time. This is generally referred to as a "cake-eating" problem where R is the cake. In our setting we may view R as the total stock of oil. Utility is logarithmic in consumption of the resource. Formally, the problem is

$$\max_{E_1, E_2} \log(E_1) + \beta \log(E_2) \tag{48}$$

subject to

$$E_1 + E_2 \leq R,$$

where β is the subjective discount factor. If all oil is used (which it of course will), the last equation implies that

$$E_2 = R - E_1$$

The problem can then thus be written

$$\max_{E_1} \log(E_1) + \beta \log(R - E_1)$$

The first order condition with respect to E_1 is:

$$\frac{1}{E_1} - \beta \frac{1}{R - E_1} = 0.$$

The above equation can be solved for E_1:

$$E_1 = \frac{1}{1+\beta} R.$$

implying

$$E_2 = \frac{\beta}{1+\beta} R,$$

Hence, the consumer will eat the share $\frac{1}{1+\beta}$ of the cake in period 1, and the rest $\frac{\beta}{1+\beta}$ in period 2. Under the assumption β is less than one, the consumer chooses to consume less in period 2 than in period 1. Note that the rule for E does not depend on anything but the discount rate. Hence, it does not matter whether a country is rich or poor for the share that they eat.

Again, taxes cannot reduce the total amount used, but it may affect the timing of the energy use and this might be important.

6.3 Adding capital and a Cobb-Douglas production function

Let us now add capital. Specifically assume that capital depreciates fully between periods and that the production function is of the Cobb-Douglas form

$$Y_t = F(K_t, E_t, L_t) = K_t^{\alpha} E_t^{\nu} L_t^{1-\nu-\alpha}.$$

Since labor is not going to be important for the analysis, we normalize it to unity. Let us also assume that the utility function is logarithmic in consumption, i.e.,

$$U(C_1, C_2) = \log(C_1) + \beta \log(C_2)$$

With K_1 given, the problem is then to choose the variables to maximize the discounted utility in the two periods subject to the resource constraints. For the non-renewable resource, the relevant constraint is still given by $E_1 + E_2 \leq R$. In addition, there are now more constraints governing consumption and savings. Specifically, they are

$$\begin{aligned} C_1 + K_2 &= K_1^{\alpha} E_1^{\nu} \\ K_3 + C_2 &= K_2^{\alpha} E_2^{\nu} \end{aligned}$$

These two equations make sure that consumption plus savings equals (does not exceed) income. In order to make the two periods a bit more symmetric, we force the consumers to put aside an exogenous amount K_3 from period 2 output. Using $E_2 = R - E_1$ (will this be true as above), we have

$$C_1 = K_1^{\alpha} E_1^{1-\alpha} - K_2 \quad \text{and} \quad C_2 = K_2^{\alpha} (R - E_1)^{1-\alpha} - K_3$$

The maximization problem is then

$$\max_{K_2, E_1} \log\left(K_1^{\alpha} E_1^{\nu} - K_2\right) + \beta \log\left(K_2^{\alpha} (R - E_1)^{\nu} - K_3\right)$$

The first order conditions with respect to K_2 and E_1 respectively, are

$$\begin{aligned} \frac{1}{C_1} &= \beta \frac{1}{C_2} \alpha \frac{Y_2}{K_2} \\ \frac{1}{C_1} \nu \frac{Y_1}{E_1} &= \beta \frac{1}{C_2} \nu \frac{Y_2}{E_2} \end{aligned}$$

Rewriting the first of the equations above, we get

$$\frac{C_2}{\beta C_1} = \frac{\alpha Y_2}{K_2} \tag{49}$$

which is called the Euler equation. Note that LHS is the ratio of marginal discounted utilities in the two periods, and the RHS is the marginal gross return on investing in capital. To see the latter note that by investing one more unit of capital to be used in period 2, output increases by the marginal product of capital given by the RHS.

Rewriting the second equation yields

$$\frac{C_2}{\beta C_1} = \frac{\nu Y_2 / E_2}{\nu Y_1 / E_1}. \tag{50}$$

Combining these two equations we get

$$\frac{\nu Y_2/E_2}{\nu Y_1/E_1} = \frac{\alpha Y_2}{K_2}$$

To understand these two important equations, it is important to also understand that in this economy with competitive input markets, the gross return to capital will be given by the marginal product of capital in that period, i.e., $r_2 = \frac{\alpha Y_2}{K_2}$. As noted above, if one unit of consumption is saved in period 1 and instead invested into capital next period, this generates additional output (and consumption opportunities) given by period 2 marginal product of capital. Similarly, the price of energy in a period will be given by the marginal product of energy in that period, i.e.,

$$p_1^E = \nu Y_1/E_1, \quad \text{and} \quad p_2^E = \nu Y_2/E_2.$$

Using these facts, the equations (49) and (50) can be written

$$\frac{C_2}{\beta C_1} = r_2$$

$$\frac{p_2^E}{p_1^E} = r_2 \tag{51}$$

Equation (51) is *The Hotelling formula* (after Hotelling, 1931). The equation says that the gross price growth of the natural resource (oil) equal the gross return on capital. The intuition for this result is straightforward. In this economy, there are two ways of saving (i.e., transferring consumption opportunities) between the periods. The first is to save in the form of capital. The second is to save in the form of the resource – not using a unit of oil today and instead using it next period. It cannot be optimal to use two ways of saving in a way that one gives more returns than the other – they must thus in an optimal allocation have the same return.

The Hotelling formula is key to understanding the economics of resources in finite supply, or to put it differently, resources that have the dynamic property that using a unit today reduces available resources in the future. It can be extended to allow for positive extraction costs, externalities and market power.

Nevertheless, according to the theorem, the oil price should grow at the rate of the interest rate. The Hotelling rest is thus the maximum rent that could be obtained while depleting the non-renewable resource.

6.4 Backstop technology - the green paradox

Local and global pollution control efforts, if uncoordinated, may exacerbate environmental externalities. For example, a stricter cap on emission flows may actually increase the global pollution stock and hasten the date when the global pollution cap is reached. These results suggest that pollution regulation at the local and global levels may need to be coordinated. In the absence of coordination, a policy change at one level may worsen pollution problems at another level, because of the dynamic effects of the policy change. This is because pollution regulation leads to a decline in the value of the polluting resource and therefore under some conditions, consumption may actually increase. These insights are somewhat contrary to the impressions obtained from a static model, where different types of regulation may serve as substitutes. In a dynamic model, the effect of a policy intervention on the entire dynamic path is relevant and limiting pollution in one period may lead to an increase in another.

We will analyze the effect of increased investments into cleaner and more sustainable energy sources—backstop technology—on the economy and on the fossil fuel use. Intuitively, we might expect that as oil resources get depleted, they become more expensive, and alternative energy sources will become relatively cheaper, increasing the demand for the alternative energy sources. However, we illustrate another result, namely that the presence of a backstop technology could also lower the spot prices of the oil and in fact accelerate its extraction and depletion.

Consider now again the problem from section 7.2.3 with two periods, and a fixed amount of fossil fuel $\overline{R}$. Assume also that a clean energy N_2, becomes available in period 2. Production is Cobb-Douglas, i.e.,

$$Y_1 = K_1^{\alpha} E_1^{\nu} \quad \text{and} \quad Y_2 = K_2^{\alpha} \left(E_2 + N_2\right)^{\nu}.$$

The resource constraints is

$$E_1 + E_2 \leq R$$

Since the world ends in period 2, nothing is saved for the future.

Under the assumption all fossil fuel will be used, the budget constraints are

$$C_1 = K_1^{\alpha} E_1^{\nu} - K_2 \quad \text{and} \quad C_2 = K_2^{\alpha} \left(R - E_1 + N_2\right)^{\nu} - K_3.$$

With log utility, the maximization problem is

$$\max_{K_2, E_1} \log\left(K_1^{\alpha} E_1^{\nu} - K_2\right) + \beta \log\left(K_2^{\alpha} \left(R - E_1 + N_2\right)^{\nu} - K_3\right).$$

The first-order condition with respect to K_2 can be written

$$\frac{C_2}{\beta C_1} = \alpha \frac{Y_2}{K_2}$$

which again, is identical to the case without backstops.

This is not surprising given the intuition for the Euler equation. Using the notation $C_2 = (1 - s_2) Y_2$ and $K_2 = s_1 Y_1$,we have

$$\frac{(1 - s_2) Y_2}{\beta (1 - s_1) Y_1} = \alpha \frac{Y_2}{s_1 Y_1} \Rightarrow s_1 = \beta \frac{\alpha}{1 - s_2 + \beta\alpha}.$$

Without loss of generality, let us focus on a case when $s_1 = s_2$ in order to make the model as similar as possible to an infinite horizon model.

Then

$$s_1 = \beta\alpha$$

The FOC with respect to E_1 can again be written

$$\frac{C_2}{\beta C_1} = \frac{\nu Y_2 / \left(E_2 + N_2\right)}{\nu Y_1 / E_1}$$

Again, these equations can be combined to get the Hotelling equation:

$$\frac{\nu Y_2 / \left(E_2 + N_2\right)}{\nu Y_1 / E_1} = \alpha \frac{Y_2}{K_2}. \tag{52}$$

Now use the expression for K_2 in the Hotelling equation (52).

$$\frac{\nu Y_2/\left(E_2+N_2\right)}{\nu Y_1/E_1}=\alpha\frac{Y_2}{s_1 Y_1}$$

Simplifying this and using that as long as all fossil oil is used $E_2 = R - E_1$ yields

$$\begin{aligned}\frac{1/\left(E_2+N_2\right)}{1/E_1} &= \frac{\alpha}{s_1}=\beta\\ \Rightarrow & \frac{R-E_1+N_2}{E_1}=\beta\end{aligned}$$

Clearly, LHS of the last equation is a constant that is independent of N.. Then, we see that as N_2 increases, E_1 must decrease in order for the equation to hold. The intuition of the result is that increased future supply of clean energy decreases the price of fossil fuel in the future. Since resource owners are forward-looking, they realize this and increase current extraction. This reduces the price today and reduces it tomorrow which restores the equilibrium where the rate of price growth is equal to the interest rate.

6.5 References

Black, G., 1995. Is Hotelling's Rule Relevant to Domestic Oil Production? *Journal of Environmental Economics and Management*, 36(2), pp. 149-169

Bulte, E., Demania, R. and Deacon, R., 2004. Resource Abundance, Poverty and Development, *ESA Working Paper,* No. 04-03.

Boschini, A., Pettersson, J. and Roine, J., 2007. Resource curse or not: A question of appropriability. *Scandinavian Journal of Economics*, 109(3), pp. 593-617

Dasgupta P. and Heal, G., 1974. The Optimal Depletion of Exhaustible Resources. *Review of Economic Studies,* 41 (Symposium on the Economics of Exhaustible Resources), pp. 3-28

Faustmann, M. 1849. On the determination of the value which forest land and immature stands possess for forestry. Journal of Forest Economics, 1, pp. 7-44. [Reprinted 1995]

Hartman, R., 1976. The harvesting decision when a standing forest has a value. *Economic Inquiry* 14: 52-58

Hassler, Krusell and Olovsson, 2009. Energy-saving technical change. *IIES Working paper.*

Hoen, H. and Solberg, B., 1997. CO2-taxing, timber rotations, and market implications. Critical Reviews in *Environmental Science and Technology*, 27(1), pp. 151-162

Hotelling, H., 1931. The Economics of Exhaustible Resources. *Journal of Political Economy*, 39, pp. 137-175

van Kooten, G., Binkley, C. and Delcourt, G., 1995. Effect of carbon taxes and subsidies on optimal forest rotation age and supply of carbon services. *American Journal of Agricultural Economics,* 77(2), pp. 365-374

Matsuyama, K., 1992. Agricultural Productivity, Comparative Advantage, and Economic Growth. *Journal of Economic Theory,* 58, pp. 317-334.

Romero, C., Ros, V. and Diaz-Balteiro, L., 1998. Optimal Forest Rotation Age When Carbon Captured is Considered: Theory and Applications. *The Journal of the Operational Research Society*, 49(2), pp. 121-131

Sachs, J. and Warner, A., 2001. The curse of natural resources. *European Economic Review,* 45(4-6), pp. 827-838

Sohngen, B. and Mendelsohn, R., 2003. An Optimal Control Model of Forest Carbon Sequestration. *American Journal of Agricultural Economics.* 85(2), pp. 448-457

Stiglitz, J., 1974. Growth with Exhaustible Natural Resources: Efficient and Optimal Growth Paths. *Review of Economic Studies,* 41(Symposium on the Economics of Exhaustible Resources), pp. 123-137

Torvik, R., 2009. Why do some resource-abundant countries succeed while others do not? *Oxford Review of Economic Policy,* 25(2) pp. 241-256

7 Economic Damages

In this section, we discuss how the economy is affected by climate change. Certainly, this is a complicated issue since there is an almost infinite number of ways climate change can affect the economy. Emitting fossil carbon affects the climate and the effects are independent of whom emitted the carbon. This creates an externality as we discussed in the chapter on Public Economics. To be able to set a price on this externality we need to calculate all potential effects of a marginal extra unit of carbon emissions everywhere in the world for every time period some share of the emitted carbon unit remains in the atmosphere. Then, we need to summarize these effects in a meaningful way. This may seems as an impossible task and we certainly immediately need to acknowledge that a perfect answer is impossible to find. However, we can do better than just giving up!

7.1 Two approaches

We can think of two qualitatively different methods of measuring the economic impact of climate change. One can be labeled a *bottom-up* approach and the other *aggregate reduced form.* Before going into details, let us sketch the two methods in order to highlight their differences and complementarity.

The idea behind the bottom-up approach is straightforward. We first catalogue all possible ways changes in the climate can affect the economy. There are obvious items that should be included in the list – like various effects on agriculture, flooding due to sea level rise and changes in health. However, it is also clear that it is difficult to know when it is reasonable to stop adding items to the list. In any case, at some point this stage must be left for the next. Then, then information is gathered about how climate affect the economy for each of the items of the list. In some cases, there is amble studies but obviously there is much less for others. The final step is to put all effects together in an aggregate function that describes the sum of all the effects.

If we follow the second approach, we go directly to the relation between aggregate measures like GDP, consumption and investments and climate. The idea is here to associate natural historical variation in climate to changes in the aggregate variables of interest, e.g., GDP. For example, is it the case that countries that have a hot climate or have been hit by unusually hot climate for some period of time tend to have lower GDP of experience lower growth rates? An ideal way to address the issue of how the climate affects the economy would be to conduct a randomized experiment, where some randomly chosen regions of the world was given a changed climate and the others not.[47] Obviously, this is not possible, but by using natural variation and argue that this variation is not caused by differences in the economy, we can construct what is called "natural experiments".[48] Thus, if we find two economies that appear similar in some ways at some point in time and then find out that one of the economies for an extended period was hit by an unusual change in weather outcomes we can treat this as a substitute for a true experiment and draw similar conclusions as if it had been a true experiment.

Clearly, the two approaches have different pros and cons. A main pro of the bottom up approach is that since we specify each specific mechanisms behind the relation between climate and the economic outcome we can study that in detail and become fairly certain about how it works. At best, we can get to a credible identification of cause and effect. A careful study of for example how forestry in Sweden is affected by changes in temperature, the length of the growing season and

[47] Arguably, since we should expect spillovers between regions, we might need a large number of Earths and change the climate in some of them to construct a perfect experiment.

[48] There is now a large and quickly expanding literarure on nartural experiments in economics. See, e.g., DiNardo (2008).

precipitation can provide a credible answer to the very specific question at hand. In fact, we might trust our description of the mechanism so well that we can use it to extrapolate our results outside the range of observed climate variables. The con of the bottom up approach is that we can never be sure that we have captured all important mechanisms.

The pros and cons of aggregate reduced form is more or less flipped around. Since we here study the relation between climate change and aggregate variables like GDP, it should be less likely that we miss and important effect. On the other hand, since we do not specify particular mechanisms we do attempt to identify exactly how the temperature affects e.g., GDP. Therefore, we risk picking up spurious effects. Perhaps more importantly, since we do not identify what is behind the relations in the data, it is hard to judge how stable they are and if the results can be extrapolated.

The observation that the pros and cons are flipped between the two approaches clearly suggests that they are complementary. Thus, both should be used in order to increase our understanding about how the climate might effect the economy.

7.2 Damage functions

In order to make the formal analysis of the relation between climate change and the economy tractable, it must be simplified. This simplification needs to be done in several ways. As we have noted above, the definition of climate as a set of defining characteristics of the distribution of weather events makes it multidimensional covering both different types of weather events, like temperature, wind speed and precipitation and different geographical locations. Nevertheless, we need to reduce his high level of dimensionality drastically. In fact, in many models, including e.g., Nordhaus' DICE and RICE models and the model we will develop below, we will use a one variable representation of the climate, namely the *global mean temperature* T. It may seem like we then discard almost all information about the climate. However this is not really true since a given change in the global mean temperature can be used to infer a lot about how climate has changed in other dimensions and in particular regions.[49] The global mean temperature is a *good statistic* for climate in the omitted dimensions.

The second simplification we need to do regards the way the climate, represented by a small number of statistics, affects the economy. Given the way we have represented the economy above, we can think of a number of different ways of modelling the link between the climate and the economy First, we may introduce an effect of the climate in the aggregate production function. For example we could set $Y_t = F(K_t, E_t, L_t, T_t)$ so that, given capital, energy and labor inputs, the global mean temperature affects output.

Second, we could allow the climate to have an effect on the factors of production. A different climate might increase the probability of damages on capital and structures which could be captured by letting depreciation be a function of climate. Also we could argue that labor productivity is affected by the temperature.

Third, we could allow a direct effect of climate on utility. As noted in chapter 4, the utility function should include all goods the individuals value. We could therefore include an effect of climate in the utility function. Letting C_t denote aggregate consumption, we may assume that utility is given by the function $U(C_t, T_t)$.

All these ways of describing the effect of the climate on the economy exist in the literature. However, if done in a proper way, it is reasonable to argue that it may not matter much which way

[49] A way to this is to use simulations from large climate models and simply use statistical methods to compute the relation between aggregate variables like the global mean temperature and disaggregated variables. This is a coarse form of *statistical downscaling.* For an economic application, see Krusell and Smith (2013).

it is done.[50] The logic behind this statement is that since we describe welfare by a utility function, we can put a price on everything that let us compare "apples and pears". For example, if climate affect utility directly, we may state that a given change in utility is equivalent to a given change in consumption and vice versa. For this reason, we proceed using the first way to model climate effects.

A convenient way to describe the effects of climate change on the economy is to define a *damage function*. Specifically, if we let

$$Y_t = D(T_t) F(K_t, E_t, L_t)$$

$D(T_t)$ describes how much is left of output after climate damages. Under this formulation, damages are proportional to output with a proportionality factor given by $1 - D(T_t)$, which thus depends on climate as measured by T (and perhaps other measures of the climate).

7.3 Bottom-up calibration

After having decided to represent damages as a (net of) damage function $D(T)$, we need to specify a functional form and calibrate the parameters. Taking the bottom-up approach, we would collect as many studies as possible on the effects of climate change. A good example of this approach is Nordhaus and Boyer (2000). They separate the effects into seven different types of mechanisms and divide the world into 13 regions.[51] For each mechanism i and region j they use existing studies to specify a region specific damage function specified as $Q_{ij}(T)$.[52] The functional form of $Q_{ij}(T)$ depends on the mechanism i with parameters that depends on the region. For example, the damage function for health is set to

$$Q_{health,j}(T) = 0.002721 T_j(T)^{0.2243}$$

where $T_j(T)$ is the temperature increase in region j associated with an increase in the global mean temperature T. For costal damages, the function is assumed to be

$$Q_{costal,j}(T) = \alpha_{costal,j} T^{1.5}$$

For agriculture, the damage function in a particular region depends on its initial temperature, due to the finding that regions with an initial average temperature below 11.5 degrees Celsius tends to gain from global warming. Nordhaus summarize various studies, getting e.g., a doubling of CO_2 (increasing$T$ around 3 degrees) reduces agricultural output as a share of GDP by 0.07% in the U.S., -0.51% in China, -0.87% in Russia, 1.54% in India, and 0.06% in Africa.

For the risk of a climate catastrophe, less scientific evidence is available. Here, Nordhaus instead uses a survey of expert opinions. The survey ask experts about their assessment of the probability of a dramatic permanent loss of output (between 22% and 44% of GDP depending on the region)

[50] An important consideration here is that the effective discount rate used for future utility and future consumption may differ. See e.g., Sterner and Persson (2008).

[51] The types of effects are agriculture, sea-level rise, other market sectors, health, non-market amenity impacts, human settlements and eco-systems, catastrophes. The regions are the U.S., OECD Europe, Eastern Europe, Japan, Russia, China, Africa, India, other high income, other middle, other low middle income, Low income, and high income OPEC.

[52] They also include a term $\left(\frac{Y_{j,t}}{Y_{j,1995}}\right)^{\eta_i}$ intended to allow for damages for a given temperature to depend on ouput. If $\eta_i > 0$ a given temperature has a damage that becomes larger the higher is GDP relative to the base year 1995. For agriculture, a value of -0.1 is used, capturing the observation that as economies grow, they tend to become less dependent on the agriculture and less sensitive to climate and weather events. In the discussion here, we disregard this term. See Nordhaus and Boyer (2000) for details.

associated with an increase in the global mean temperature of 2.5 and 6%. Based on the expert opinions, Nordhaus and Boyer (2000) sets these probabilities to 1.2% and 6.8% Finally, they do a risk adjustment which implies that the willingness to pay to avoid a damage is larger than the expected loss. Specifically, a 1% risk of loosing 40% of GDP is worth 1.21% of GDP to avoid which is about three times as large as the expected loss of 0.4%.

Having specified all $Q_{i,j}(T)$, Nordhaus and Boyer (2000) sum the effects over mechanisms and calculate damages at T equal 2.5 and 6 degrees Celsius for each region. This gives the numbers $D_j(2.5)$ and $D_j(6)$ for each region. Finally, the damage functions is specified as

$$D_j(T) = \frac{1}{1 + \alpha_{1,j}T + \alpha_{2,j}T^2}$$

and this is calibrated by solving the equations

$$\begin{aligned} D_j(2.5) &= \frac{1}{1 + \alpha_{1,j}2.5 + \alpha_{2,j}(2.5)^2} \\ D_j(6) &= \frac{1}{1 + \alpha_{1,j}6 + \alpha_{2,j}(6)^2} \end{aligned}$$

for each region.

The following figure shows damages $(1 - D(T))$ as calibrated by Nordhaus and Boyer (2000) for four different regions.

It is important to note that the damages vary substantially by region. At three degrees, damages in the US is .7% while it is over 5% in low income countries.

7.4 Aggregate reduced form

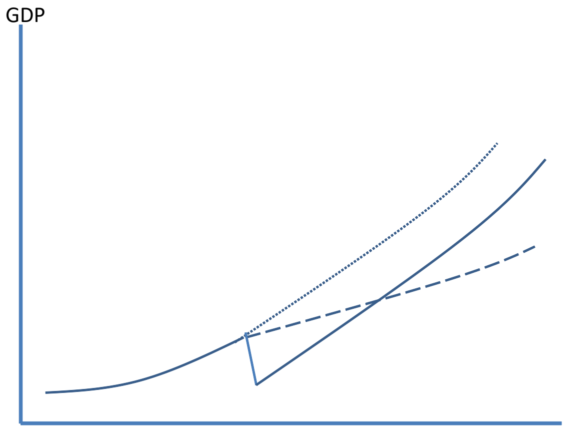
Few studies have so far used the aggregate reduced form approach. A recent example, however, is Dell, Jones and Olken (2008). They use natural variation in climate and investigate the correlation between temperature and precipitation on the one hand and economic growth on the other. .The authors construct historical temperature and precipitation data for 136 countries from 1950 to 2003 and combine this dataset with historical growth data from the *Penn World Tables.* The main identification strategy uses year-to-year fluctuations in temperature and precipitation within countries to estimate the impact of temperature and precipitation on economic growth. Specifically, they assume that

$$\begin{aligned} Y_{it} &= e^{\beta T_{j,t}} A_{j,t} L_{j,t} \\ \frac{A_{j,t} - A_{j,t-1}}{A_{j,t}} &= g_j + \gamma T_{j,t} \end{aligned}$$

where $T_{i,t}$ is temperature or precipitation, $A_{j,t}$ is productivity and $L_{j,t}$ labor input, all measured for country j in period t. β, g_j and γ are parameters that are estimated from the data.

In the first equation, a negative β implies that a higher temperature (or precipitation) has a negative effect on the *level* of GDP. In the second, we see that a negative γ on the other hand, implies that a higher temperature (or precipitation) leads to lower *growth rates*.

We illustrate the difference between a level effect (negative β) and a growth effect (a negative γ) in the following figure. We assume that the temperature increases permanently. In this case, a negative β leads to a parallel shift down in the path for GDP (the solid line). A negative γ leads to lower growth, as illustrated by the dashed line. In absence of a change in the temperature, GDP would follow the dotted line.



Dell, Jones and Olken also allow rich and poor countries to have different coefficients. The results of the study is that there is a substantial negative effect on growth, but only in countries that are poorer than the median. The effect is strong, with one degree Celsius leading to a fall in the growth rate by 1%. Since the length of the dataset is 50 years, it is not possible to say anything about growth rates in the very long run, but the results indicate that the effect does not die out quickly. Regarding precipitation, the results are substantially weaker.

7.5 Conclusion

After reviewing the evidence on damages from climate change a main conclusion is that our knowledge is very limited. The evidence we have point in the direction of some damages, in particular for poor countries. The work by Nordhaus implies damages that are substantial if the global mean temperature increases more than a few degrees, in particular in low income countries and in Europe. These results are at least not fully inconsistent with the aggregate reduced form results, which suggests negative effects on poor countries. The value of further quantitative studies on the effects of global warming is arguable very high.

7.6 References

Dell, M., B.F. Jones and B. A Olken, (2008), "Climate Change and Economic Growth: Evidence from the Last Half Century",

NBER Working Paper No. 14132

DiNardo, J., (2008), "Natural Experiments and Quasi-Natural Experiments", in Durlauf, S.N. and L.E. Blume (eds), *The New Palgrave Dictionary of Economics*, Second Edition.

Krusell, P and T. Smith, (2013),

Sterner, T. and U. M. Persson, (2008), "An Even Sterner Review: Introducing Relative Prices into the Discounting Debate", *Review of Environmental Economics and Policy*, volume 2:1, pp. 61–76

8 Integrated Assessment Models

We will now build a simple integrated assessment model (IAM), building on what we have done so far in the courses.

In the following subsection, we formulate a simplified two-period model. In the next subsection we expand the model into one with an infinite number of periods.

We embed a simple linear model of the carbon cycle in a standard neoclassical growth model where one input to the production function, oil, is non-renewable.

8.1 A 2-period model

As shown to be convenient above, we assume logarithmic utility given by

$$\log c_1 + \beta \log c_2$$

Furthermore, we have the Cobb-Douglas production function

$$Y_t = D_t A_t K_t^{\alpha} L_t^{1-\alpha-\nu} E_t^{\nu} \tag{53}$$

where Y_t is output per capita in period t. D denotes net of damage output, A is total factor productivity (TFP), K is capital per worker, L is labor, and E is fossil fuel. Labor supply is assumed to be fixed to unity for simplicity.

The resource constraint is as above that

$$\begin{aligned} K_2 + C_1 &= Y_1 \\ K_3 + C_2 &= Y_2 \end{aligned}$$

where K_3 and K_1 are exogenous.

This makes (more) sense if the periods are long, e.g. a decade. Total supply of fossil fuel is given by R. Hence,

$$E_1 + E_2 = R \tag{54}$$

where we abstract from extraction costs.

The damages are represented by the equations

$$\begin{aligned} D_1 &= e^{-\nu S_1} \\ D_2 &= e^{-\nu S_2} \end{aligned}$$

which describes the mapping from CO_2 concentrations via climate change to damages. Note that the higher D, the lower the damages, and so, D may be viewed as "what is left". We see that an increased stock of S increases the damages (lowers D).

Furthermore, the carbon cycle is assumed to be

$$S_2 = \rho S_1 + E_2$$

and

$$S_1 = E_1$$

where we can think of $1 - \rho$ as the share of the stock of CO_2 that is removed between periods due to uptake from oceans and other carbon sinks.

First, we will model the free market (also called the decentralized) solution. We will show that this situation leads to excessive use of fossil fuel relative to the social optimum which will be analyzed thereafter.

8.1.1 Laissez faire

In the free market case there are no government interventions, and the externalities are not taken into account. The representative consumer takes prices as given. As a consumer she maximizes utility given her income. She is owns the firms and receives any profit they earn in addition to her income from savings and working. The consumer problem is

$$\max_{C_1,C_2,K_2} \log C_1 + \beta \log C_2$$

s.t.

$$\begin{aligned} C_1 + K_2 &= r_1 K_1 + w_1 + \pi_1 \\ C_2 + K_3 &= r_2 K_2 + w_2 + \pi_2 \end{aligned} \tag{55}$$

where the RHS's of the equations are the income, consisting of capital returns, wage income and profits from the fossil fuel selling firms. The representative consumption good firm's problem is in each period

$$\max_{K_t} D_t A_t K_t^{\alpha} L_t^{1-\alpha-\nu} E_t^{\nu} - r_t K_t - w_t L_t - p_t E_t$$

where $t = 1, 2$.

The fossil fuel owning firm solves

$$\begin{aligned} &\max p_1 E_1 + \frac{p_2 E_2}{r_2} \\ s.t.\ R_2 \;&\geq\; E_1 + E_2 \end{aligned}$$

i.e., it maximizes the discounted value of profits (revenues).

Substituting from the budget constraint into the objective function of the consumer yields

$$\log\left(r_1 K_1 + w_1 + \pi_1 - K_2\right) + \beta \log\left(r_2 K_2 + w_2 + \pi_2 - K_3\right)$$

The first order condition with respect to K_2 yields

$$\frac{C_2}{\beta C_1} = r_2 \tag{56}$$

which is the Euler equation. Knowing that r is the price of capital (i.e., the interest rate), the equation states how consumption should be allocated between the two periods.

The consumption good producing firm's first order conditions are that

$$\begin{aligned} \frac{\alpha Y_t}{K_t} &= r_t \\ \frac{(1-\alpha-\nu) Y_t}{L_t} &= w_t \\ \frac{\nu Y_t}{E_t} &= p_t \end{aligned} \tag{57}$$

i.e., the marginal products of capital, labor and oil should be set equal to their respective prices.

Profit maximization of the oil firm requires that it sells all (no) oil in period 1 if the return on keeping oil in the ground $\left(\frac{p_2}{p_1}\right)$ is smaller (larger) than the interest rate r_2. Clearly, neither of these outcomes is possible since oil is necessary for production. In particular, the price would be infinite if no oil is sold. Thus, we are left the conclusion that oil owners make sure that the Hotelling equation is satisfied

$$\frac{p_2}{p_1} = r_2 \tag{58}$$

Going back to the consumer, we use (56) and (57) and note that $C_2 = (1 - s_2) Y_2$, giving

$$\frac{1 - s_2}{\beta (1 - s_1)} = \frac{\alpha}{s_1}.$$

Again focusing on the case when K_3 is chosen so that savings is constant, we get $s_1 = s_2 = \alpha\beta$ like in the previous chapter.

Using the savings rate in the Hotelling equation (58) together with (57) yields

$$\frac{\nu Y_2/E_2}{\nu Y_1/E_1} = \frac{\alpha Y_2}{s Y_1} \Rightarrow \frac{E_2}{E_1} = \frac{s}{\alpha} = \beta$$

Thus oil use fall over time and the rate is *fully* determined by the subjective discount factor.[53] In particular, the path of oil use is independent of damages! Intuitively, we may understand that this is not optimal from an overall social perspective. To formally show this, we turn to the next subsection.

8.1.2 Social planner

In the social planner problem we treat the carbon stock in the atmosphere as a choice variable. Hence, the problem is

$$\max_{C_1,C_2,K_2,E_1,E_2,S_1,S_2} \log C_1 + \beta \log C_2$$

s.t. the constraints

$$\begin{aligned} C_1 + K_2 &= D_1 A_1 K_1^{\alpha} E_1^{\nu} \\ C_2 + K_3 &= D_2 A_2 K_2^{\alpha} E_2^{\nu} \\ E_1 + E_2 &\leq R \end{aligned}$$

In addition, we have constraints stating the carbon cycle. The carbon in the atmosphere equals the amount of carbon not stored by the carbon sequesters, plus the carbon emissions in the current period. I.e.,

$$\begin{aligned} S_1 &= E_1 \\ S_2 &= \rho S_1 + E_2 = \rho E_1 + E_2 \end{aligned}$$

Furthermore, we include the damage constraints, where damage is a function of the carbon emissions as well as a factor, ν, representing the externality. Keep in mind that the damage variable, D, is low for high levels of damage. And so, the higher the ν, the lower is D, i.e., the damage increases as the external effect grows.

[53] This result hinges on the constant savings rate, but not crucially. Allowing variations in savings by changing K_3 also α enters the equation for E_2/E_1. In any case, we know that savings is rather constant over time except for business cycle variations.

$$\begin{aligned} D_1 &= e^{-\gamma S_1} \\ D_2 &= e^{-\gamma S_2} \end{aligned}$$

Using the constraints, we can write the objective as

$$\begin{aligned} &\log\left(e^{-\gamma E_1} A_1 K_1^\alpha E_1^\nu - K_2\right) \\ &+\beta \log\left(e^{-\gamma(\rho E_1 + E_2)} A_2 K_2^\alpha E_2^\nu - K_3\right) \end{aligned}$$

subject to the constraint

$$E_1 + E_2 \leq R.$$

Consider first the first order condition with respect to K_2. This is

$$\frac{1}{C_1} = \beta \frac{1}{C_2} \frac{\alpha Y_2}{K_2} \Rightarrow \frac{C_2}{C_1 \beta} = \alpha \frac{Y_2}{K_2}$$

which is the familiar Euler equation. In the decentralized equilibrium we know this is equal to the interest rate.

Consider now the derivative of the objective with respect to E_1. This is

$$\frac{1}{C_1} \frac{\nu Y_1}{E_1} - \frac{1}{C_1} \gamma Y_1 - \beta \frac{1}{C_2} \rho \gamma Y_2 \tag{59}$$

There are three terms. The first, is the marginal utility of consumption times the marginal product of oil in the first period *not taking into account any externalities!* The second term is externality in the first period. This consists of the output loss due to damages in the first period by using one more unit of oil (γY_1) times the marginal utility of consumption $\left(\frac{1}{C_1}\right)$. The third is the externality in the second period. This term is has the following interpretation; γY_2 is the output loss from having one more unit of S_2 in the atmosphere, ρ is the share of a unit of emissions in period 1 that remains in period 2. Thus, the second period output loss by increasing E_1 by a marginal unit is $\rho \gamma Y_2$. By multiplying by the discounted marginal utility of consumption $\left(\beta \frac{1}{C_2}\right)$ we get the marginal utility loss of this.

If we divide (59) by the marginal utility of consumption in period 1, we get the *social value of a unit of oil used in period 1*, expressed in terms of period 1 consumption. This yields

$$\frac{\nu Y_1}{E_1} - \gamma Y_1 - \beta \frac{C_1}{C_2} \rho \gamma Y_2.$$

Recalling that in the decentralized equilibrium (without taxes on savings) interest rate $r_2 = \frac{C_2}{C_1 \beta}$, we can write this as

$$\frac{\nu Y_1}{E_1} - \gamma Y_1 - \frac{\rho \gamma Y_2}{r_2}.$$

That is, the social value of oil in consumption units in the first period is the *private value* $\frac{\nu Y_1}{E_1}$ minus the sum of current marginal damages (γY_1) and the discounted value of next periods damages. Let us denote the total discounted value of the damages induced by a unit of oil used in period 1 by

$$\Gamma_1 = \gamma Y_1 - \frac{\rho \gamma Y_2}{r_2}$$

Consider now the derivative with respect to E_2. This is

$$\beta \frac{1}{C_2} \frac{\nu Y_2}{E_2} - \beta \frac{1}{C_2} \gamma Y_2.$$

This consists of the discounted utility value of the marginal product of oil in the second period times the discounted utility value of the damage in the second period caused by an extra unit of S_2.

If we as above measure this in period 1 consumption units by dividing by period one marginal utility, we get

$$\frac{1}{r_2} \left(\frac{\nu Y_2}{E_2} - \gamma Y_2 \right) \tag{60}$$

We can then define the value of damages induced by emissions in period 2 by

$$\Gamma_2 = \gamma Y_2$$

Suppose now that the planner would like to use all oil. Then the optimality condition for choosing E_1 and E_2 is to set the marginal values of E_2 and E_1 equal, i.e.,

$$\frac{\nu Y_1}{E_1} - \Gamma_1 - \frac{\rho \gamma Y_2}{r_2} = \frac{1}{r_2} \left(\frac{\nu Y_2}{E_2} - \Gamma_2 \right)$$

giving

$$\frac{\frac{\nu Y_2}{E_2} - \Gamma_2}{\frac{\nu Y_1}{E_1} - \Gamma_1} = r_2. \tag{61}$$

This is the *modified Hotelling equation.* It states that ratio of the *social* value of oil in period 2 and 1 should equal the interest rate. In which way does this change the oil use? This obviously depends on how the marginal damage evolves over time. In (Golosov et al. 2011), it is sown that in an infinite horizon version of this model Γ_t is proportional to Y_t. Call this proportionality Λ so that $\Gamma_t =$. Then we have

$$\frac{Y_2 \left(\frac{\nu}{E_2} - \Lambda \right)}{Y_1 \left(\frac{\nu}{E_1} - \Lambda \right)} = r_2.$$

In this case, it is easy to verify that the LHS is *increasing* in Λ. Thus, higher damages tends to increase the value of postponing the use of oil.

8.2 Infinite Horizon

Let us now turn to a more realistic infinite horizon model of the economy and the climate. Doing this, we may calibrate also the carbon cycle and damages in a more realistic way. We build on the lessons from above and postulate an aggregate production function given by

$$Y_t = e^{-\gamma_t (S_t - \bar{S})} A_t K_t^{\alpha} E_t^{\nu} \tag{62}$$

Here S_t is the stock of carbon in the atmosphere and $\bar{S}$ is the pre-industrial level. A_t measures the state of technology, K_t the stock of capital and E_t energy input. We assume full depreciation of capital and utility given by

$$\sum_{t=0}^{\infty} \beta^t \ln (C_t)$$

Energy E_t comes from a stock in finite supply so that

$$\sum_{t=0}^{\infty} E_t \leq R_0.$$

It turns out the addition of oil to the problem does not change savings behavior and the optimal savings rate in the model is constant and given by $a\beta$ so that

$$K_{t+1} = \alpha\beta Y_t.$$

Let us now generalize the description of the carbon cycle. Specifically, we assume that

$$S_t = \sum_{s=0}^{t+T} (1 - d_s) E_{t-s} \tag{63}$$

where $d_s \in [0, 1]$ for all s. Here $1 - d_s$ represents the amount of carbon that is left in the atmosphere s periods into the future.

The IPCC 2007 report concludes that *"About half of a CO_2 pulse to the atmosphere is removed over a timescale of 30 years; a further 30% is removed within a few centuries; and the remaining 20% will typically stay in the atmosphere for many thousands of years" and the conclusion of Archer (2005) is that a good approximation is that 75% of an excess atmospheric carbon concentration has a mean lifetime of 300 year and the remaining 25% stays forever".* For our purposes, as shown above, what is key is the rate of depreciation of the atmospheric carbon concentration in excess of the pre-industrial level. Thus, rather than develop a nonlinear version of Nordhaus's three-reservoir system, we just make direct assumptions on these depreciation rates, which we allow to change over time. We do this by setting

$$1 - d_s = \varphi_L + (1 - \varphi_L)\,\varphi_0\,(1 - \varphi)^s\,. \tag{64}$$

From our perspective, thus, a simple, yet reasonable, representation of the carbon cycle is therefore that we describe in equation (64), where (i) a share φ_L of carbon emitted into the atmosphere stays there forever; (ii) another share, $1 - \varphi_0$, of the remainder exits the atmosphere into the biosphere and the surface oceans within a decade; and (iii) a remaining part, $(1 - \varphi_L)\,\varphi_0$, decays (slowly) at a geometric rate φ. We use the approximation of Archer (2005) to yield a half-life of carbon dioxide in the atmosphere of 30 periods. Hence, φ_L is set to 20%, as in the IPCC report. The remaining parameter φ_0 is set so that $d_2 = \frac{1}{2}$, giving $\varphi_0 = 0.393$. Thus, we have

$$\begin{aligned} \varphi &= 0.0228 \\ \varphi_L &= 0.2, \\ \varphi_0 &= 0.393. \end{aligned}$$

As before, we use an exponential damage function to approximate the current state-of-the-art damage function which is given in Nordhaus (2007). Nordhaus uses a proportional damage function specified as

$$1 - D_N(T_t) = \frac{1}{1 + \theta_2 T_t^2},$$

where T is the mean global increase in temperature above the preindustrial level, with $\theta_2 = 0.0028388$. The damage function D_N is, due to the square of temperature in the denominator,

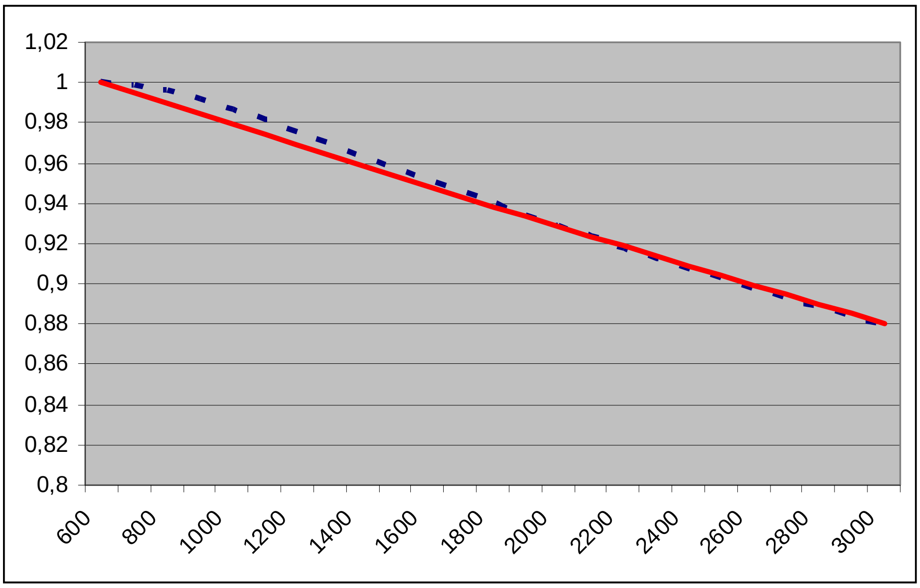


Figure 19: Net of damage function $(1 - D(T(S)))$ Nordhaus (dashed) and exponential (solid).

convex for a range of values up to some high temperature after which it is concave (naturally, since it is bounded above by 1).

For our purposes, however, we need to express the damage function in terms of the stock of atmospheric carbon, S_t. The standard assumption in the literature (say, as used in RICE) is to let the global mean temperature be a logarithmic function of the stock of atmospheric carbon:

$$T_t = T(S_t) = \lambda \log\left(1 + \frac{S_t}{\bar{S}}\right) / \ln 2, \tag{65}$$

where $\bar{S} = 581$ GtC (Gigatons of carbon) is the pre-industrial atmospheric CO_2 concentration. A standard value for the climate sensitivity parameter λ here is 3.0 degrees Celsius. Thus, we assume that a doubling of the stock of atmospheric carbon leads to a 3-degree Celsius increase in the global mean temperature. As noted above, there is substantial discussion and, perhaps more importantly, uncertainty, about this parameter, among other things due to imperfect understanding of feedback effects. Therefore, it is important to allow uncertainty, as we do in this paper.

In summary, to obtain a mapping from the carbon dioxide concentration in the atmosphere to damages as a percent of GDP, one needs to combine $D_N(T)$ and $T(S)$. This amounts to a composition of a convex and a concave function (for low values of S). In the figure below, we show the mapping according to Nordhaus's calibration by plotting his $1 - D_N(T(S_t))$ (dashed) together with the damage function assumed in our analysis (solid): an exponential function with parameter $\gamma = 5.3 \times 10^{-5}$.

The range of the x axis is from 600 GtC, which corresponds to preindustrial levels, to 3,000 GtC, which corresponds to the case when most of predicted stocks of fossil fuel are burned over a fairly short period of time. Nordhaus's formulation implies an overall convexity for a range of values of S, which our function does not exhibit. This convexity is not quantitatively large, however, and the two curves are quite close. We thus conclude that our exponential approximation appears rather reasonable.

Given these assumption, we can now use our model to evaluate the social cost of carbon. As in the previous section, the damages will depend on three factors (see equation (60): i) how big damage on output a marginal unit of carbon in the atmosphere does, ii) how long-lived an emitted unit of carbon is and iii) how we discount future damages. Specifically this is

$$\Gamma_t = -\sum_{j=0}^{\infty} \frac{\partial Y_{t+j}}{\partial S_{t+j}} \frac{\partial S_{t+j}}{\partial E_t} \beta^j \frac{U'(C_{t+j})}{U'(C_t)}.$$

This formula computes the discounted present value of the damages induced by a marginal unit of carbon emitted in period t. It is a sum of terms that in each period consists of three factor: i) $\frac{\partial Y_{t+j}}{\partial S_{t+j}}$ is the marginal effect on output in period $t+j$, ii) $\frac{\partial S_{t+j}}{\partial E_t}$is how much of a unit of carbon that was emitted in period t remains in the atmosphere in period $t+j$ and iii) $\beta^j \frac{U'(C_{t+j})}{U'(C_t)}$ is the relative value of consumption at period $t+j$ and t (the marginal rate of substitution).

Give our assumptions, we can express this in closed form: $\frac{\partial Y_{t+j}}{\partial S_{t+j}} = \gamma Y_{t+j}$, $\frac{\partial S_{t+j}}{\partial E_t} = \varphi_L + (1-\varphi_L)\,\varphi_0\,(1-\varphi)^j$ and $\beta^j \frac{U'(C_{t+j})}{U'(C_t)} = \beta^j \frac{(1-s_t)Y_t}{(1-s_t)Y_{t+j}}$ which since savings is constant is equal to $\beta^j \frac{Y_t}{Y_{t+j}}$. Using this in the formula gives

$$\begin{aligned}
\Gamma_t &= \sum_{j=0}^{\infty} \gamma Y_{t+j} \left(\varphi_L + (1-\varphi_L)\,\varphi_0\,(1-\varphi)^j\right) \beta^j \frac{Y_t}{Y_{t+j}} \\
&= \gamma Y_t \sum_{j=0}^{\infty} \left(\varphi_L + (1-\varphi_L)\,\varphi_0\,(1-\varphi)^j\right) \beta^j \\
&= \gamma Y_t \left(\frac{\varphi_L}{1-\beta} + \frac{(1-\varphi_L)\varphi_0}{1-(1-\varphi)\,\beta}\right)
\end{aligned}$$

Since we now that introducing a tax on oil equal to the externality – we now have a formula for the optimal tax. From our results, we can draw the following conclusions

1. *The tax per unit of fossil fuel should be exactly indexed to output; other than that, only primitive parameters appear!*

2. *Only subjective discounting, damages, carbon depreciation parameters matter.*

3. *In particular, the optimal tax is independent of technology, population, details of energy supply, etc.*

Plugging number to calibrate, we get the tax in dollars. Let us use a yearly GDP of 70 trillion \$US, and a yearly discount factor of $\beta = 0.985$. Recalling that we have a model where a period is a decade, we then get that the tax should be

$$\begin{aligned}
&\left[\gamma Y \left(\frac{\varphi_L}{1-\beta} + \frac{(1-\varphi_L)\varphi_0}{1-(1-\varphi)\,\beta}\right)\right]_{\gamma=5.3\times10^{-5},\varphi=0.0228,\varphi_L=0.2,\varphi_0=0.393,Y=700*10^{12},\beta=0.985^{10}} \\
&= \;1.258\,6\times 10^{11}\$/GtC.
\end{aligned}$$

: Since a Gigaton equals 10^9 tons, the tax per ton is 126.8 dollars. The model can also easily be used for predictions but we leave that for now.

8.3 References

Center for International Earth Science Information Network (CIESIN), 1995. *Thematic Guide to Integrated Assessment Modeling of Climate Change* [online]. University Center, Mich. Available at http://sedac.ciesin.org/mva/iamcc.tg/TGHP.htm [Accessed 21.03.2011]

Cline, W., 1992. *The economics of global warming.* Washington DC: Institute of International Economics

Golosov, M., Hassler, J., Krusell, P. and Tsyvinsky, A., 2011. Optimal taxes on fossil fuel in general equilibrium. *NBER Working paper 17348.*

Kelly, D., and Kolstad, C., 1998. Integrated Assessment Models for Climate Change Control. In: Folmer, H. and Tietenberg, T. (Eds.), *International Yearbook of Environmental and Resource Economics 1999/2000:* A Survey of Current Issues, ch. 4

Nordhaus, W., 1993. *The American Economic Review*, 83(2), pp. 313-317

Stern, N., 2007. *The Economics of Climate Change: The Stern Review.* Cambridge: Cambridge University Press. Available through
The Treasury /http://www.hm-treasury.gov.uk/independent_reviews/stern_review_economics_climate_cha

Weitzman, M., 2007. A Review of The Stern Review on the Economics of Climate Change. *Journal of Economic Literature* 45(3), pp. 703–724

Weyant, J., Davidson, O., Dowlatabadi, H., Edmonds, J., Grubb, M., Richels, R., Rotmans, J., Shukla, P., Cline, W., Fankhauser, S. and Tol, R., 1996. Integrated assessment of climate change: An overview and comparison of approaches and results. In: Working group III of the IPCC. 1996. *Climate Change 1995—Economic and Social Dimensions of Climate Change.* Cambridge: Cambridge University Press. Ch. 10