Carbon Taxing and Alternative Energy - Complements Rather than Substitutes?[1]

Johan Gars[2]

Abstract

The transition to using alternative energy sources instead of fossil fuels will most likely be a very important part of mitigating climate change. Stimulating research on technology for using alternative energy sources is probably a good climate change mitigation policy. It may even seem that if the development of the ability to use alternative energy sources is fast enough, this could be a sufficient policy measure. As will be shown in this paper, however, policy measures aimed at speeding up the use of alternative energy sources and policy measures aimed at the use of fossil fuels (here represented by carbon taxing) are more of complements rather than substitutes. Getting the full benefits of faster development of the use of alternative energy sources, require carbon taxing that takes the development of the use of alternative energy sources into account. The faster the increase of the amount of used alternative energy, the faster the carbon tax rate should change. A faster changing tax rate affects the fossil fuel use to a larger extent. The welfare gains from taxation also tends to increase with the rate of increase of alternative energy use. The conclusion to be drawn from this, is that mitigation policy is complex and that care should be taken in combining different policy measures.

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[2] Institute for International Economic Studies, Stockholm University. This work is part of the Mistra-SWECIA programme, funded by Mistra (the Foundation for Strategic Environmental Research).

1 Introduction

In the IPCC Fourth Assessment report, it can be seen that keeping climate change at manageable levels will require large reductions in the emissions of CO_2 over the coming decades. For instance, keeping further increases in global average mean temperature at about 2 degrees would require about halving the emissions by 2050 compared to the emissions in 2000[3]. In that scenario, the peak in emissions should happen before 2020. It seems that achieving this would require large policy interventions fast. In this paper, I look at two things that policy could be aimed at. One is some kind of stimulation of research on technology for using alternative energy. The other is carbon taxing, which aims more directly at the use of fossil fuels. I argue that these different types of policy measures interact in non-trivial, and perhaps surprising, ways.

In particular, I argue that the different policy measures should be seen as complements rather than substitutes. The full benefits of a faster development of the available alternative energy, can only be realized if there is carbon taxing in place that takes the development of the alternative energy technology into account. On the other hand, the carbon tax should affect the economy to a larger extent, the faster the development of alternative energy use. Also, the welfare gains from taxation tend to be larger the faster the development of alternative energy use.

The intuition behind these results is quite simple. Forward looking owners of fossil fuel resources, realize that the value of their resource will decline, if more alternative energy becomes available. They will therefor want to extract the fuels faster. The faster the development of alternative energy use, the stronger this effect is. From a climate perspective, it is better to spread out emissions more evenly over a longer time period. So the difference between the social planner solution and the decentralized outcome will be larger, the faster the development of the alternative energy.

In order to study these effects, I set up a growth model with energy as a factor of production. I also assume that there are two different energy sources. Fossil fuel based and alternative energy. Fossil fuel based energy causes emissions of CO_2 while alternative energy does not.

Following Hotelling (1931) I model fossil fuels as being costlessly extracted from a given, non-renewable, supply. This is a common way of modeling fossil fuel use and an early example of this in a macro model is Dasgupta and Heal (1974). In the decentralized setting, the fossil fuels are privately owned and

[3]In table SPM.6 in IPCC (2007) it says that temperature changes of between 2.4 and 2.8 compared to pre-industrial levels would require emission reductions of 30-60 percent.

extracted to maximize discounted future profits.

I assume that alternative energy is available as an exogenous, increasing sequence. So in this setting, policy aimed at affecting the rate of increase of available alternative energy is simply a change in the exogenous rate of increase. This simplification allows me to more clearly study the interaction I focus on here.

The climate system is a very complex dynamic system. In order to include it in a manageable model, it must be represented in a simple manner. I suggest a representation that works well in my setting, and that reasonably well can represent the damages given by climate system used in the DICE and RICE models (see Nordhaus and Boyer (2000) and Nordhaus (2008))

I solve the model as a planner problem and as a competitive equilibrium with and without taxation of the use of fossil fuels. In general, a decreasing tax rate is required to shift the fossil fuel use in the right direction. This is because the fuels are used too quickly in the decentralized solution and a decreasing tax rate shifts the fuel use toward the future. I then show that the optimal tax rate should change more quickly the faster the increase in alternative energy use.

By solving the model numerically, I can also look at the welfare effects of taxation. The welfare gain from taxation has a clear (but not completely monotone) tendency to increase with the rate of increase of the amount of available alternative energy.

The rest of the paper is organized as follows. In section 2 I set up the model. In section 3 I solve the model both as a planner problem and as a decentralized equilibrium. In section 4 I show when and how the planner solution can be implemented using taxation of fossil fuels. In section 5 I describe the numerical algorithm used to solve the model, perform a primitive calibration and present the numerical results. The paper is concluded by section 6, where I discuss the results and some possible extensions.

2 The Model

Production in the economy uses capital, labor and energy as inputs. The amount of labor is constant. Energy comes from two different sources. One is fossil fuels that emit CO_2 when used. The other source is alternative energy that does not cause any emissions of CO_2. The amount of fossil fuel based and alternative energy used in period t is E_t and S_t respectively. The different kinds of energy are assumed to be perfect substitutes. The production function is Cobb-Douglas

$$Y_t = D_t K_t^{\alpha} L^{1-\alpha-\gamma}(E_t + S_t)^{\gamma} \tag{1}$$

where $D_t \in [0,1]$ is a damage coefficient that shows to what extent production is decreased by climate related damages.

The climate system is a complicated dynamic system that changes slowly in response to increased concentration of greenhouse gases, in particular CO_2, from emissions. Furthermore, the concentration of CO_2 in the atmosphere depends on how carbon is exchanged between different reservoirs through the carbon cycle. Needless to say, it is difficult to capture all this in a simple way. I will here model climate related damages as depending on a weighted sum of past emissions. In particular, I will assume that climate related damages depend on the following quantity

$$X_t = \sum_{t=0}^{T_E} \sigma_s E_{t-s}$$

This specification includes, as special cases, damages being a pure flow externality ($\sigma_0 = 1$ and $T_E = 0$) and damages depending on total accumulated past emissions (all $s = 1$ and $T_E \to \infty$). A more realistic representation of the actual effects of emitted CO_2 is to have σ_s increase with s initially and then after some time start to decrease. The initial increase comes from that it is not the CO_2 itself that causes the damages but rather the heating that results from increased concentrations of green house gases. But this heating is a slow process. The reason for the eventual decrease is that the emitted CO_2 will, at least partially, be absorbed in the carbon cycle.

The climate damages in period t are a function of X_t

$$D_t = D(X_t) = D\left(\sum_{t=0}^{T_E} \sigma_s E_{t-s}\right) \tag{2}$$

As a particular functional form I will use

$$D(X_t) = e^{-X_t} = e^{-\sum_{s=0}^{T_E} \sigma_s E_{t-s}} \tag{3}$$

When using this damage function, the quantity

$$\kappa \equiv \sum_{s=0}^{T} \beta^s \sigma_s \tag{4}$$

will be convenient.

This representation of damages can be compared to that used by Nordhaus in the RICE and DICE models (see e.g. Nordhaus and Boyer (2000) and

Nordhaus (2008)). There, the climate system is represented by a dynamic system with five state variables (three carbon reservoirs in the carbon cycle and two temperatures). In section 5 I will show that this specification can, at least for relatively similar emission paths, reproduce the damages given by that system well.

The fossil fuels can be extracted costlessly from a total resource that contains R_t at time t. So, the restriction on total fossil fuel use is given by

$$\sum_{t=0}^{\infty} E_t \leq R_0 \tag{5}$$

The amount of available alternative energy is a given, exogenous, sequence. This sequence will be increasing over time.

Given the time scales at which climate change operates it seems reasonable to assume full depreciation. I will also assume that the households have logarithmic utility.

To simplify the notation below, I will sometimes use the Heavyside function $H : \mathbb{R} \to \{0, 1\}$ defined by

$$H(x) = \begin{cases} 1 \text{ if } x > 0 \\ 0 \text{ if } x \leq 0 \end{cases} \tag{6}$$

3 Solving the model

In this section I will solve the model both as a planner solution and as a decentralized equilibrium. The difference between them comes from the climate externality.

3.1 Planner Solution

The planner chooses investment and use of fossil fuels to maximize the utility of the representative household. The planner problem is

$$\max_{\{C_t, K_{t+1}, E_t, R_{t+1}\}} \sum_{t=0}^{\infty} \beta^t \ln(C_t), \text{ s.t. } C_t + K_{t+1} = Y_t,\ R_{t+1} = R_t - E_t \in [0, R_t] \tag{7}$$

The lagrangian of this problem can be written

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \ln(Y_t - K_{t+1}) + \lambda \left(R_0 - \sum_{t=0}^{\infty} E_t \right) - \sum_{t=0}^{\infty} \mu_t E_t \tag{8}$$

Consumption/investment decision Taking the FOC wrt K_{t+1} in (8) gives

$$\frac{C_{t+1}}{C_t} = \beta\alpha\frac{Y_{t+1}}{K_{t+1}} \tag{9}$$

This gives the consumption/investment rule

$$C_t = (1-\beta\alpha)Y_t \text{ and } K_{t+1} = \beta\alpha Y_t \tag{10}$$

Fossil fuel use The planner chooses how to allocate the use of fossil fuels over time. In this setting, the planner will not necessarily exhaust all the fuels. The following proposition specifies the conditions under which the constraint on total available fossil fuels (5) is binding in the planner solution.

Proposition 1. With damages given by (3), the constraint on total available fossil fuel resources (5) binds if and only if

$$\sum_{t=0}^{\infty}\left(\frac{\gamma}{\kappa}-S_t\right)H\left(\frac{\gamma}{\kappa}-S_t\right) > R_0 \tag{11}$$

When the constraint does not bind, the fossil fuel use is given by

$$E_t = \left(\frac{\gamma}{\kappa}-S_t\right)H\left(\frac{\gamma}{\kappa}-S_t\right) \tag{12}$$

Proof. Taking FOC wrt E_t in (8) gives

$$\lambda + \mu_t = \sum_{s=0}^{T_E}\frac{\beta^{t+s}}{C_{t+s}}\frac{\partial Y_{t+s}}{\partial E_t} = \frac{\beta^t}{C_t}\gamma\frac{Y_t}{E_t+S_t} + \sum_{s=0}^{T_E}\frac{\beta^{t+s}}{C_{t+s}}\frac{Y_{t+s}}{D_{t+s}}\frac{\partial D_{t+s}}{\partial E_t}$$

From the consumption rule (10), the ratio between production and consumption can be substituted for

$$\lambda + \mu_t = \frac{\beta^t}{1-\alpha\beta}\frac{\gamma}{E_t+S_t} + \frac{\beta^t}{1-\alpha\beta}\sum_{s=0}^{T_E}\frac{\beta^s}{D_{t+s}}\frac{\partial D_{t+s}}{\partial E_t}$$

Using the assumed form of the damage function (2) this can be rewritten

$$\lambda + \mu_t = \frac{\beta^t}{1-\alpha\beta}\frac{\gamma}{E_t+S_t} + \frac{\beta^t}{1-\alpha\beta}\sum_{s=0}^{T_E}\beta^s\sigma_s\frac{D'_{t+s}}{D_{t+s}} \tag{13}$$

where $D'_{t+s} = D'(X_{t+s})$.

With the specific functional form (3) the sum in this expression becomes

$$\sum_{s=0}^{T_E} \beta^s \sigma_s \frac{D'_{t+s}}{D_{t+s}} = -\sum_{s=0}^{T_E} \beta^s \sigma_s = \{(4)\} = -\kappa$$

Substituting this in (13) gives

$$\lambda + \mu_t = \frac{\beta^t}{1-\alpha\beta} \frac{\gamma}{E_t + S_t} - \frac{\beta^t \kappa}{1-\alpha\beta} \tag{14}$$

Assuming that the constraint on total available fuels (5) does not bind, $\lambda = 0$. In any period where $E_t > 0$, $\mu_t = 0$ and (14) gives

$$E_t = -\frac{\gamma}{\sum_{s=0}^{T} \beta^s \sigma_s \frac{D'_{t+s}}{D_{t+s}}} - S_t$$

or using (3)

$$E_t = \frac{\gamma}{\kappa} - S_t$$

In (11) this is summed for all t where this expression is positive. So if the inequality in (11) is not fulfilled, the constraint on total available fossil fuels does not bind and fossil fuel use in the planner solution is given by (12). $\square$

Looking at (11), the constraint depends in a reasonable way on the parameters. Increasing γ (higher productivity of energy), decreasing κ (smaller climate damages) or decreasing S_t (less alternative energy available) tends to lead to exhaustion of the fossil fuels. And the other way around.

A sufficient (but far from necessary) condition for (11) to hold is that

$$\lim_{t\to\infty} S_t < \frac{\gamma}{\kappa}$$

When looking at the fossil fuel use, in a case where the constraint on total amount of available fuels (5) does bind, the following lemma will be useful.

Lemma 1. For $E_0 \in [0, \frac{\gamma}{\kappa} - S_0)$ the sequence given by

$$\frac{\beta^t \gamma (E_0 + S_0)}{\gamma - (E_0 + S_0)\kappa(1-\beta^t)} - S_t \tag{15}$$

is decreasing over time and becomes negative in finite time. For each t, it is also continuous and strictly increasing in E_0. As E_0 goes to $\frac{\gamma}{\kappa} - S_0$, (15) goes to $\frac{\gamma}{\kappa} - S_t$ for all t.

Proof. To see that it is decreasing, rewrite the first term as

$$\frac{\beta^t \gamma(E_0+S_0)}{\gamma-(E_0+S_0)\kappa(1-\beta^t)} = \frac{\gamma}{\kappa}\frac{\beta^t(E_0+S_0)}{\frac{\gamma}{\kappa}-(E_0+S_0)+\beta^t(E_0+S_0)}$$

Since, by assumption, $E_0 + S_0 < \frac{\gamma}{\kappa}$ this expression is increasing in β^t and consequently decreasing in t. It can quite easily be seen that it is continuous. Furthermore, the denominator is strictly larger than zero, while the numerator goes to zero, making the fraction go to zero as t goes to infinity. Given that S_t is a positive and increasing sequence, (15) becomes, and stays, negative in finite time. For any t, $(1-\beta^t) > 0$ which means that for any given t, the first term of (15) is increasing in E_0. Finally, for any t, $\beta^t \in (0,1)$. The denominator of (15) can be rewritten

$$\gamma-(E_0+S_0)\kappa(1-\beta^t) = \gamma-(E_0+S_0)\kappa+(E_0+S_0)\kappa\beta^t$$

As $E_0 \to \frac{\gamma}{\kappa} - S_0$, this goes to $(E_0+S_0)\kappa\beta^t$ and (15) goes to $\frac{\gamma}{\kappa} - S_t$. □

This lemma can now be used to prove the following proposition about the constrained fossil fuel use

Proposition 2. With damages given by (3), the fossil fuel use in the planner solution when the constraint on total available fuels (5) binds (that is, when (11) is fulfilled) is given by

$$\tilde{E}_t(E_0) = \left(\frac{\beta^t\gamma(E_0+S_0)}{\gamma-(E_0+S_0)\kappa(1-\beta^t)} - S_t\right) H\left(\frac{\beta^t\gamma(E_0+S_0)}{\gamma-(E_0+S_0)\kappa(1-\beta^t)} - S_t\right) \tag{16}$$

for the unique E_0 that solves

$$\sum_{t=0}^{\infty} \tilde{E}_t(E_0) = R_0 \tag{17}$$

Proof. I will start from the FOC wrt E_t with damages given by (3), (14), and assume that $E_0 > 0$. In any other period where $E_t > 0$, $\mu_t = 0$. Combining (14) for 0 and t gives

$$\frac{\gamma}{E_0+S_0} - \kappa = \beta^t\frac{\gamma}{E_t+S_t} - \beta^t\kappa$$

or, after rearrangement

$$E_t = \frac{\beta^t\gamma(E_0+S_0)}{\gamma-\kappa(1-\beta^t)(E_0+S_0)} - S_t$$

So the fossil fuel use must be given by $\tilde{E}_t(E_0)$ in (16). By assumption, the constraint on total fossil fuels is binding. This implies (17). What remains to show is that there is a unique E_0 that solves (17). Relying on lemma 1, the sum in (17) is convergent for all $E_0 \in [0, \frac{\gamma}{\kappa} - S_0)$ since only a finite number of terms are non-zero. Since it is a sum of finitely many non-zero terms (where the number of non-zero terms is weakly increasing in E_0), each of which is continuous and strictly increasing E_0, the sum must also be strictly increasing and continuous in E_0. For $E_0 = 0$, the sum is zero. As $E_0 \to \frac{\gamma}{\kappa} - S_t$ the sum becomes at least R_0. The sum converges term by term to the corresponding unconstrained sum in (11). If the sum in (11) is convergent, (16) converges to this sum which, by assumption, is at least R_0. If that sum is divergent, it can be truncated to give a finite sum that is at least R_0. The corresponding truncation of (16) will then converge to that sum. This implies that there is some $E_0 \in (0, \frac{\gamma}{\kappa} - S_0)$ so that the sum in (17) is at least R_0. So, the sum in (17) is a strictly increasing function of E_0 that starts at 0 and ends up at at least R_0. This implies that (17) has a unique solution. $\square$

A property of the planner solution that will be interesting when discussing taxation is how E_0 depends on how fast the available alternative energy increases.

Proposition 3. Consider two sequences of available alternative energy $\{S_t\}_{t=0}^{\infty}$ and $\{S'_t\}_{t=0}^{\infty}$, such that $S'_0 = S_0$, $S'_t > S_t$ for all $t > 0$ and the constraint on total available fossil fuels (5) is binding for both sequences. Then E_0 induced by $\{S_t\}_{t=0}^{\infty}$ will be smaller than E'_0 induced by $\{S'_t\}_{t=0}^{\infty}$.

Proof. If E_0 was the same in both cases, then the sum in (11) would be smaller for $\{S'_t\}$ than for $\{S_t\}$. But since, by assumption, (11) should be fulfilled in both cases and since the sum is increasing in E_0, E'_0 induced by $\{S'_t\}$ must be larger than E_0 induced by $\{S_t\}$. $\square$

Summarizing the planner solution In the planner solution with damages given by (3), the consumption/investments choice is in each period given by (10). If (11) does not hold, the choice of fossil fuel use is given by (12). If (11) does hold, the choice of fossil fuel use is given by (16) for the unique $E_0 \in (0, \frac{\gamma}{\kappa} - S_0]$ such that $\sum_{t=0}^{\infty} \tilde{E}_t(E_0) = R_0$

3.2 Decentralized Solution

In the decentralized setting, all decisions are made by price taking agents. The households choose between consumption and investment based on the return to saved capital. They derive income from labor, returns to capital and dividends from shares in fossil fuel extracting and alternative energy providing firms. I will not explicitly model the trade in these shares. I will assume that all households have equal shares in these companies. In equilibrium, the share prices must be such that no trade takes place and I will abstract from that trade. I will also assume that these companies pay out all their profits (equal to their income) as dividends.

The owners of fossil fuel resources sell the fuels at market prices to maximize discounted profits over an infinite time-horizon.

3.2.1 Competitive equilibrium

I will denote the factor prices of capital, labor and energy by ρ_t, w_t and p_t^E respectively.

A competitive equilibrium consists of sequences of quantities $\{C_t, K_{t+1}, E_t\}_{t=0}^{\infty}$ and prices $\{\rho_{t+1}, w_t, p_t^e\}_{t=0}^{\infty}$ such that

1. The households solve

$$\max_{\{C_t, K_{t+1}\}} \sum_{t=0}^{\infty} \beta^t \ln(C_t), \text{ s.t. } C_t + K_{t+1} = \rho_t K_t + w_t + p_t^E (E_t + S_t) \quad (18)$$

2. The fossil fuel producing firms maximize discounted profits where the profits are discounted by the market return to capital (where I set $\rho_0 = 1$), that is they solve

$$\max_{\{E_t\}} \sum_{t=0}^{\infty} p_t^E E_t \left(\prod_{s=0}^{t} \rho_s \right)^{-1} \quad (19)$$

3. Factor prices are competitive.

3.2.2 Characterization of the equilibrium

The competitive price of capital and labor is that they are paid their marginal product. The competitive price of energy is that it is paid its marginal cost ignoring any externalities. So in a competitive equilibrium

$$\rho_t = \alpha \frac{Y_t}{K_t}, \; w_t = (1 - \alpha - \gamma) \frac{Y_t}{L} \text{ and } p_t^E = \gamma \frac{Y_t}{E_t + S_t} \quad (20)$$

Households Turning to the households' problem, the FOC wrt K_{t+1} gives

$$\frac{C_{t+1}}{C_t} = \beta\rho_{t+1} = \{(20)\} = \beta\alpha\frac{Y_{t+1}}{K_{t+1}} \tag{21}$$

The budget constraint from (18) can be rewritten

$$C_t + K_{t+1} = \rho_t K_t + w_t + p_t^E(E_t + S_t) = \{(20)\} = Y_t$$

which is expected given the CRS production function.

So (21) is the same equation as in the planner problem (9). The solution is also the same. That is

$$C_t = (1 - \beta\alpha)Y_t \text{ and } K_{t+1} = \beta\alpha Y_t \tag{22}$$

Fossil fuel extracting firms The owners of fossil fuel resources maximize the discounted value of the income stream from selling the fuels. The discount rate they use is the market return on capital. The maximization problem is given by

$$\max_{\{E_t\}} \sum_{t=0}^{\infty} \left(\prod_{s=0}^{t} \rho_s \right)^{-1} p_t^E E_t \quad \text{s.t. } E_t \geq 0 \text{ and } \sum_{t=0}^{\infty} E_t \leq R_0$$

where $\rho_0 \equiv 1$

Proposition 4. All fossil fuels will be exhausted in any competitive equilibrium

Proof. The energy price is positive in all periods. This means that a price taking fossil fuel extraction firm always can increase profits by extracting more in any period. Therefore profit maximization must require that all resources are exhausted. □

Proposition 5. In any competitive equilibrium, the energy price must follow the Hotelling rule

$$p_t^e = \left(\prod_{s=0}^{t} \rho_s \right) p_0^e \tag{23}$$

for any periods where aggregate fossil fuel extraction is positive.

Proof. The lagrangian for profit maximization is given by

$$\mathcal{L} = \sum_{t=0}^{\infty} \left(\prod_{s=0}^{t} \rho_s \right)^{-1} p_t^E E_t + \lambda \left(r_0 - \sum_{t=0}^{\infty} E_t \right) - \mu_t E_t$$

The FOC wrt E_t gives

$$\lambda + \mu_t = \left(\prod_{s=0}^{t} \rho_s\right)^{-1} p_t^E \tag{24}$$

Since, from proposition 4, the constraint on available resources must be binding, $\lambda \neq 0$. In any period t where $E_t > t$, $\mu_t = 0$. Assuming that $E_0 > 0$, in any other period t where $E_t > 0$ (24) gives

$$p_0^E = \left(\prod_{s=0}^{t} \rho_s\right)^{-1} p_t^E$$

(23) follows. □

Proposition 6. In any competitive equilibrium, the aggregate fossil fuel use follows the sequence

$$E_t = \big(\beta^t(E_0 + S_0) - S_t\big)\, H\left(\beta^t(E_0 + S_0) - S_t\right) \tag{25}$$

with E_0 as the unique value such that

$$\sum_{t=0}^{\infty} \big(\beta^t(E_0 + S_0) - S_t\big)\, H\left(\beta^t(E_0 + S_0) - S_t\right) = R_0 \tag{26}$$

Proof. Substituting the factor prices (20) in the Hotelling rule (23) gives

$$\frac{Y_t}{E_t + S_t} = \left(\prod_{s=1}^{t} \alpha \frac{Y_s}{K_s}\right) \frac{Y_0}{E_0 + S_0}$$

Using the investment rule (22) in this expression gives

$$\frac{Y_t}{E_t + S_t} = \left(\prod_{s=1}^{t} \alpha \frac{Y_s}{\beta \alpha Y_{s-1}}\right) \frac{Y_0}{E_0 + S_0} = \frac{\beta^{-t} Y_t}{E_0 + S_0} \Rightarrow E_t = \beta^t(E_0 + S_0) - S_t$$

So in any period where $E_t > 0$ it must be given by (25). The next step is to show that here is a unique E_0 such that (26) is fulfilled. Each (positive) term in the sum is (strictly) increasing in E_0. For each $E_0 \in [0, R_0]$ the sum is convergent ($\sum_{t=0}^{\infty} \beta_t(E_0 + S_0)$ is convergent). For $E_0 = 0$ the sum is zero and for $E_0 = R_0$ the sum is at least R_0. The sum is also continuous in E_0. So the sum is continuous and strictly increasing in E_0. Since it starts at zero for $E_0 = 0$ and becomes at least R_0 there is a unique E_0 such that (26) is fulfilled. □

Note that the equilibrium condition on the fossil fuel use only determines the aggregate fuel use. As long as that is fulfilled, any individual fossil fuel owner is indifferent between any extraction patterns, over the periods where aggregate fossil fuel use is positive, such that their resource is exhausted.

So, summing up, the competitive equilibrium can be summarized in the following proposition

Summarizing the competitive equilibrium The competitive equilibrium is characterized by the households' investment/consumption decision (22) and the fossil fuel use is given by (25) for the unique E_0 that fulfills (26).

4 Optimal Taxation

A possible, and often discussed, instrument for affecting the use of fossil fuels is carbon taxing. I will here assume that the tax revenues are paid as lump sums to the households. In a setting with some government expenditures that must be financed by taxation, an additional advantage of carbon taxing is that it decreases the need for other, potentially distortionary, taxation.

Let τ_t and g_t denote the tax rate and lump sum transfers respectively.

I will assume that the tax is payed by the fossil fuel extracting firms. This means that the tax revenues in period t is given by $\tau_t p_t^E E_t$. While the income (and profit) of the extracting firms is given by $(1-\tau_t)p_t^E E_t$.

The optimal taxation problem is to choose tax rates $\{\tau_t\}$ that implements a competitive equilibrium with taxation that gives the highest utility for the representative household.

4.1 Competitive equilibrium with taxation

A competitive equilibrium with taxation consists of sequences of quantities $\{C_t, K_{t+1}, E_t\}_{t=0}^{\infty}$, prices $\{\rho_{t+1}, w_t, p_t^e\}_{t=0}^{\infty}$ and tax rates and lump sum transfers $\{\tau_t, g_t\}_{t=0}^{\infty}$ such that

1. The households solve

$$\max_{\{C_t, K_{t+1}\}} \sum_{t=0}^{\infty} \beta^t \ln(C_t), \text{ s.t. } C_t + K_{t+1} = \rho_t K_t + w_t + (1-\tau_t)p_t^E E_t + p_t^E S_t + g_t \tag{27}$$

2. The fossil fuel producing firms maximize discounted profits where the profits are discounted by the market return to capital (where I set

$\rho_0 = 1$), that is they solve

$$\max_{\{E_t\}} \sum_{t=0}^{\infty} (1-\tau_t) p_t^E E_t \left(\prod_{s=0}^{t} \rho_s \right)^{-1} \tag{28}$$

3. Factor prices are competitive.

4. The government's budget constraint is fulfilled for each t

$$g_t = \tau_t p_t^E E_t \tag{29}$$

4.2 Characterization of the equilibrium

The competitive factor prices are the same as without taxation

$$\rho_t = \alpha \frac{Y_t}{K_t},\ w_t = (1-\alpha-\gamma)\frac{Y_t}{L} \text{ and } p_t^E = \gamma \frac{Y_t}{E_t + S_t} \tag{30}$$

Substituting the government's budget constraint in the households' budget constraint gives

$$C_t + K_{t+1} = \rho_t K_t + w_t + (1-\tau_t) p_t^E E_t + p_t^E S_t + \tau_t p_t^E E_t = \rho_t K_t + w_t + p_t^E (S_t + E_t)$$

Using the factor prices this becomes

$$C_t + K_{t+1} = Y_t$$

So the households' problem is the same as without taxation. This relies on all income eventually going to the households' in some form and in equal shares.

Turning to the fossil fuel use, the following proposition can be stated.

Proposition 7. There can be a competitive equilibrium with taxation where all fossil fuels are not exhausted if and only if $\tau_t = 1$ for all t.

Proof. This result is closely related to that in proposition 4. There is always a strictly positive energy price. If the tax rate is lower than 1 in some period, fossil fuel extracting firms can increase their profit by extracting more in that period and therefore all fuels must be exhausted in equilibrium. If, on the other hand, $\tau_t = 1$ for all t, the fossil fuel extracting firms are indifferent over any extraction patterns including those that do not exhaust all fuels. □

So if all fossil fuels are not exhausted in the planner solution, it can only be implemented using taxation with the, for practical purposes uninteresting, tax scheme $\tau_t = 1$ for all t. On the contrary, if all fossil fuels are exhausted in the planner solution, it can be implemented.

Proposition 8. As long as the planner solution exhausts the fossil fuel resources completely, it can be implemented as a competitive equilibrium with taxation using any tax sequence $\{\tau_t\}$ that fulfills

$$\frac{1-\tau_t}{1-\tau_0} = \frac{\gamma}{\gamma - (E_0 + S_0)\kappa(1-\beta^t)} \tag{31}$$

in any period where the planner solution prescribes using positive amounts of fossil fuels.

Proof. To begin with, note that the consumption/investment decision is the same in the decentralized and planner solution for any sequence of tax rates. So the taxation need only change the pattern of fossil fuel use.

The lagrangian of the fossil fuel extracting firms profit maximization with taxation is given by

$$\mathcal{L} = \sum_{t=0}^{\infty} \left(\prod_{s=0}^{t} \rho_s \right)^{-1} (1-\tau_t) p_t^E E_t + \lambda \left(r_0 - \sum_{t=0}^{\infty} E_t \right) - \mu_t E_t$$

The FOC wrt E_t gives

$$\lambda + \mu_t = \frac{(1-\tau_t)p_t^E}{\prod_{s=0}^{t} \rho_s}$$

Assuming that $E_0 > 0$ and combining this with the FOC in any other period where $E_t > 0$ ($\mu_t = 0$) gives

$$(1-\tau_0)p_0^E = \frac{(1-\tau_t)p_t^E}{\prod_{s=1}^{t} \rho_s}$$

Substituting the factor prices from (30) and simplifying gives

$$E_t = \beta^t \frac{1-\tau_t}{1-\tau_0}(E_0 + S_0) - S_t$$

To find the tax sequence that implements the planner solution, the RHS can be set equal to the constrained fossil fuel use in the planner solution (15) giving

$$\frac{1-\tau_t}{1-\tau_0} = \frac{\gamma}{\gamma - (E_0 + S_0)\kappa(1-\beta^t)}$$

□

Two things about the optimal tax scheme can be noted

Proposition 9. The tax scheme that implements a planner solution where all fossil fuels are exhausted should be such that the tax rate is decreasing over time.

Proof. This follows from inspection of the optimal tax scheme (31). Since $1-\beta^t$ is increasing in t the RHS is increasing in t. So the ratio in the LHS should be increasing in t which means that τ_t should be decreasing. □

Proposition 10. Consider two sequences of available alternative energy $\{S_t\}_{t=0}^{\infty}$ and $\{S_t'\}_{t=0}^{\infty}$, such that $S_0' = S_0$, $S_t' > S_t$ for all $t > 0$ and the constraint on total available fossil fuels (5) is binding for both sequences. The optimal tax sequence should then change more quickly when available alternative energy is given by $\{S_t'\}_{t=0}^{\infty}$ compared to when it is given by $\{S_t\}_{t=0}^{\infty}$.

Proof. From proposition 3 it follows that if E_0 and E_0' are initial fossil fuel use induced by $\{S_t\}_{t=0}^{\infty}$ and $\{S_t'\}_{t=0}^{\infty}$ respectively, then $E_0' > E_0$. Now, compare the tax rate at time t and time $t+1$. From proposition 9, the tax rate is decreasing over time

$$\tau_t > \tau_{t+1} \Rightarrow \frac{1-\tau_t}{1-\tau_{t+1}} < 1$$

Using (31), the ratio is given by

$$\frac{1-\tau_t}{1-\tau_{t+1}} = \frac{\gamma - (E_0+S_0)\kappa(1-\beta^{t+1})}{\gamma - (E_0+S_0)\kappa(1-\beta^{t})}$$

Taking the derivative with respect to E_0 gives

$$\frac{\mathrm{d}}{\mathrm{d}E_0}\frac{1-\tau_t}{1-\tau_{t+1}} = -\frac{\gamma\kappa\beta^t(1-\beta)}{\left(\gamma - (E_0+S_0)\kappa(1-\beta^{t})\right)^2} < 0$$

So the tax rate should change more quickly for $\{S_t'\}_{t=0}^{\infty}$ than for $\{S_t\}_{t=0}^{\infty}$ □

5 Numerical Solution

I have solved the model numerically, varying the rate of increase of the available amount of alternative energy. These numerical solutions should not be taken too seriously as attempts to give quantitative answers, but rather as examples of how it could look.

In both the planner and the decentralized solution, what remains to be determined for a complete solution, is the initial fossil fuel use, E_0. Once this has been found, the solutions follow.

For the planner solution, it can first be tested whether or not fossil fuels are exhausted. This is done by summing (12) and seeing whether this is larger than R_0 or not. If it is smaller, the planner solution is given by (12) and (10).

If the restriction on total fossil fuel use (5) is binding, fossil fuel use follows (16). What need to be found is the E_0 that leads to exact exhaustion of the fossil fuels. I find this using a search algorithm based on iterative halving of the interval containing the correct E_0. The solution is then given by (16), the found E_0 and the consumption rule (10).

In the decentralized solution, the restriction on total fossil fuel use is always binding and the sequence of fuel use is characterized by (25). The E_0 that exactly exhausts the fossil fuels is, again, found using a search algorithm. The solution is then given by (25), the found E_0 and (22).

5.1 Setting parameters

A number of parameters need to be determined. Following Nordhaus (2008) I set the time step to be 10 years.

I have used a yearly discount factor of 0.98.

The production function parameters α and γ are set to the factors' respective income shares. I use $\alpha = 0.3$ and $\gamma = 0.1$.

The initial capital stock was set to half the asymptotic steady-state capital stock.

The initial fossil fuel stock is more problematic to set. Again following Nordhaus I have set it to $R_0 = 6000$ even though, in this model, this leads to a too large initial fossil fuel use.

For the sequence of available alternative energy use I have used the shape

$$S_t = S_\infty - (S_\infty - S_0)(d_S)^t$$

I have set $S_0 = 20$ and $S_\infty = 1000$. So in all simulations the available amount of alternative energy starts from S_0 and goes asymptotically to S_∞. I vary d_S, which determines the rate of convergence between 0.93 and 0.99. In total, I use ten different values. Some resulting sequences can be seen in figure 1. Note that a higher value of d_S corresponds to a slower rate of convergence.

The coefficients $\{\sigma_s\}$ in the damage function (3) are calibrated to give the

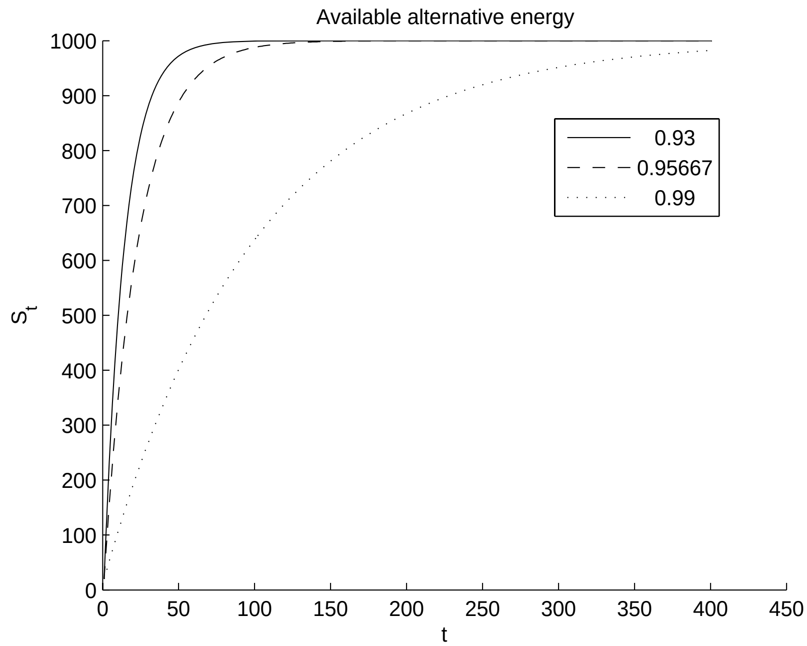

Figure 1: Available amount of alternative energy

same damages as the climate system used in the DICE and RICE models[4]. I set T_E (the number of time periods during which current emissions affect climate related damages) to 150. For the calibration, I then use the result that in the decentralized solution, fossil fuel use (25) is independent of the damage function. This means that I can solve for the competitive outcome, including the emissions path, without specifying the damage function. I can then simulate the climate system from DICE 2007 with these emissions and calculate the associated climate damages.

The damage coefficient from the DICE climate system in period t is called Ω_t. Ideally I would like to have my damage function D_t equal to Ω_t in each period. To approximate this, I use a least squares regression. Taking logarithms, the regression equation is

$$\ln(\Omega_t) \approx \ln(D_t) = -\sum_{s=0}^{T_E} \sigma_s E_{t-s}$$

[4] I have used the climate system from DICE 2007 (Nordhaus 2008) with a very small change of the temperature measure to make the damage function go asymptotically to one (in my numerical simulation of the original formulation, it approached 0.9999)

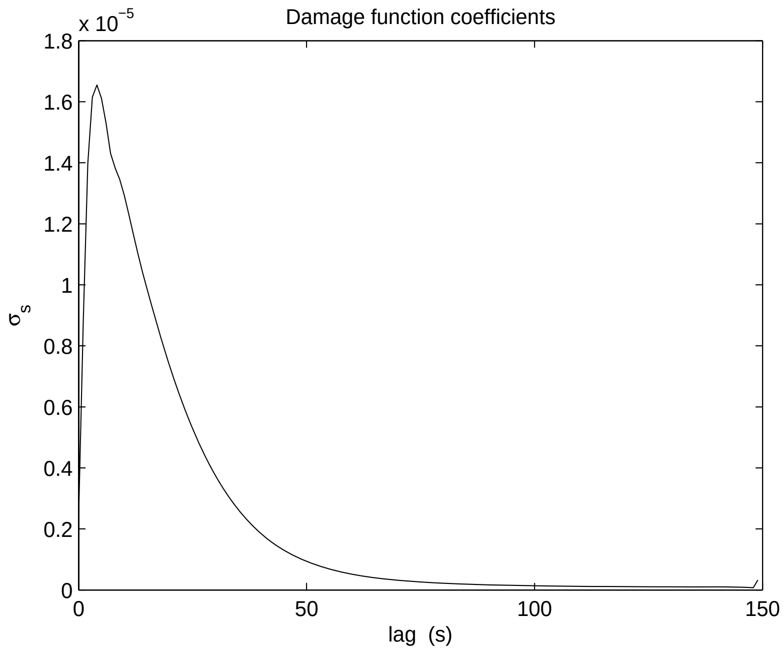

Figure 2: Damage coefficients as a function of the time lag

for each of the simulated time periods[5]. I have calibrated the damage function for the decentralized solution for the first case ($d_S = 0.93$). The resulting coefficients are shown in figure 2. In order to see how well the damages are reproduced, I have calculated both the damages given by the DICE climate system, and those given by the damage function used here, for two cases apart from the one it was calibrated for. These are the corresponding planner solution and also a quite different case with the same amount of total emissions spread out over the same time but with constant emissions per period. As can be seen in figure 3, the damage function used here does a relatively good job at representing the damages for both of these cases. For radically different emission patterns, however, the fit is significantly worse. So it seems that the damage function I use is good enough for the present purpose.

In figure 4, the emission paths in the planner and decentralized solutions can be seen for some simulations.

Since the tax sequences that induce the efficient fossil fuel use are not unique, I have calculated them so that when the fossil fuels are exhausted, the tax rate becomes zero. The resulting tax rates for three different cases

[5] I used 400 periods

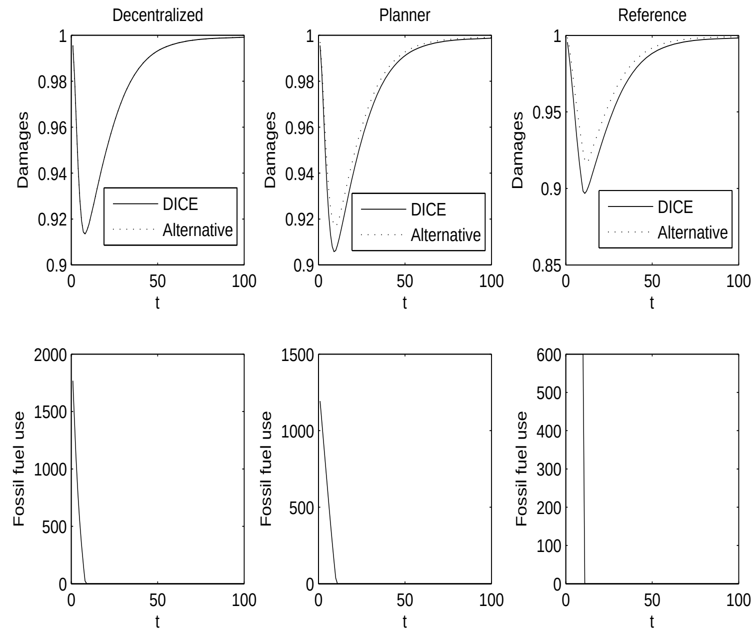

Figure 3: Comparisons of damage function for different emission paths

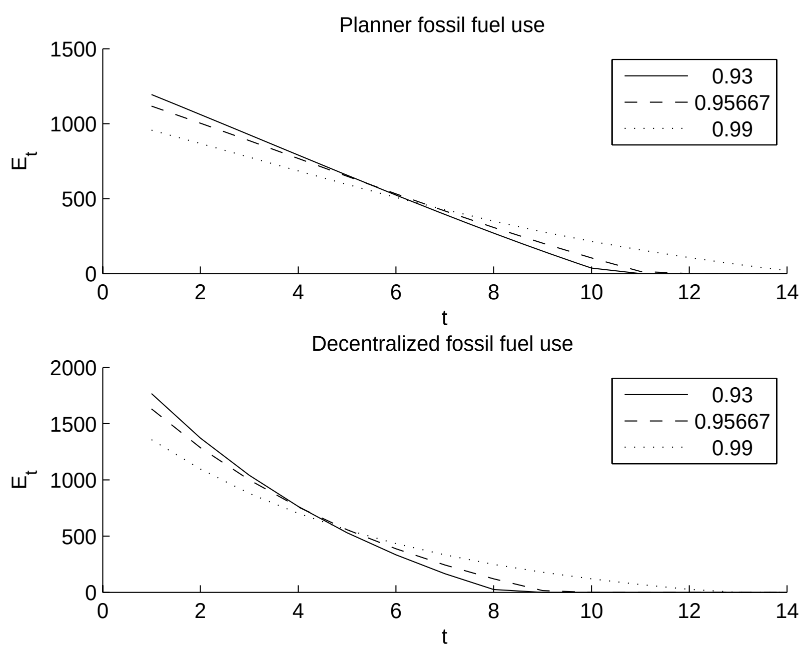

Figure 4: Fossil fuel use in planner and decentralized solution

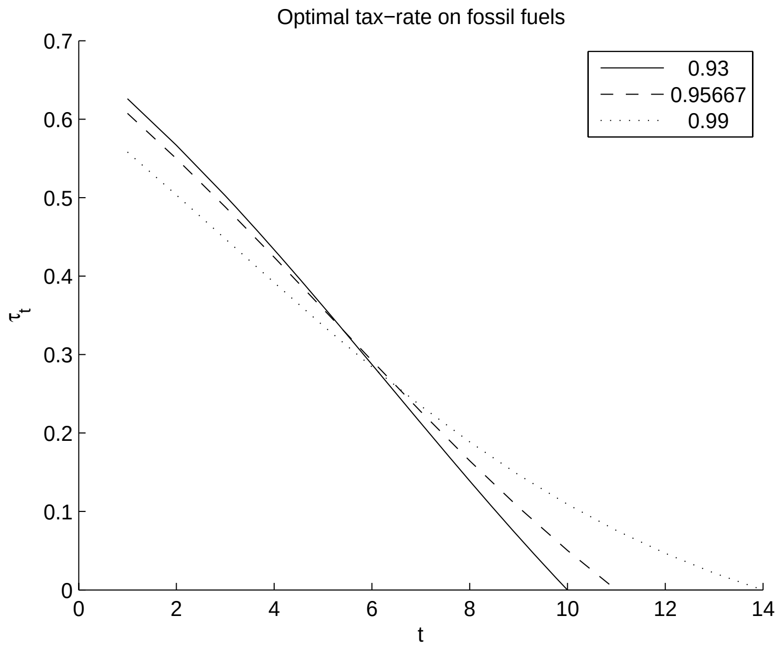

Figure 5: Optimal tax rates

can be seen in figure 5.

In figure 6 the welfare gains from taxation are shown as a function of the rate of increase of the available amount of alternative energy. The measure used is by what share consumption in each period in the decentralized solution should be increased to give the same utility as in the planner solution. That is, I find a Δ_C such that the consumption sequence in the planner solution $\{C_t^{plan}\}$ gives the same utility as $\{(1+\Delta_C)C_t^{dec}\}$. So from the figure it can be seen that consumption in each period should be increased by approximately 0.4% each period in the decentralized solution to give the same utility as the planner solution. Recalling that a higher value of d_S corresponds to lower growth rate of the alternative energy, it can be seen that for this calibration, there is a clear upwards trend (although not completely monotone) in the welfare gains from taxation as the rate of development of alternative energy increases.

6 Discussion

So, I have demonstrated in this paper that, seen as climate change mitigation policy measures, investments in technology for alternative energy use and

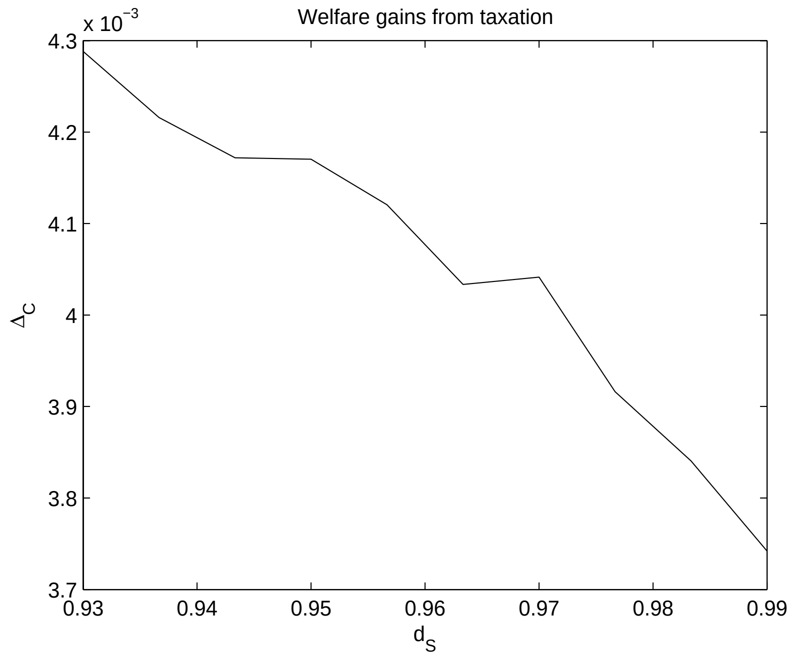

Figure 6: Welfare gains from taxation as a function of rate of increase of available alternative energy

carbon taxing should be seen as complements rather than substitutes. In order to get the full benefits from investments in alternative energy, the carbon taxing scheme should be adjusted to take this rate of development into account. And when the development of alternative energy is faster there is a larger role for carbon taxing. The taxation scheme should affect the economy to a larger extent and if anything, the welfare gains from taxation are larger.

I get relatively small welfare gains of taxation. This may be related to the fact that I have mainly calibrated my model to the DICE model. Some other authors have found larger damages than the DICE model for different reasons. For example, the Stern Review (Stern 2007) estimates the value of the damages to be at least equivalent to 5% of GDP from now and forever. These higher estimates relies to a large extent on the use of much less discounting. If I set the yearly discount factor to $\beta = .999$ which is the same as in the Stern Review, I get that consumption should be increased by a few percent each year to get the same utility in the decentralized equilibrium as in the planner solution[6]. So I then get much closer to the estimates that the

[6]In my model, the total constraint on available fossil fuels is then no longer binding

Stern Review gets. Even though the comparison I make is not the same as that Stern does, it is still conforting that I get the same order of magintude.

Others have argued that the damage function used in DICE underestimates the damages. Sterner and Persson (2008) argue that looking at the effects of climate change on the composition of production would lead to larger damage estimates. Weitzman (2009) have a very different argument, but one that in my context could be interpreted similarly. He argues that the value of mitigation is potentially severely underestimated by most Integrated Assessment Models (including DICE) since they do typically not adequately address the low probability catastrophic events that could occur. In my model, both of these issues could be interpreted as an increase of the coefficients in the damage function. Doing that would increase the welfare gains from taxation. Both increasing the damages and decreasing the amount of discounting also seems to make the connection between welfare gains and the rate of increase of available alternative energy more pronounced.

It deserves to be mentioned that there still are welfare gains from faster development of alternative energy use even without proper carbon taxing. Faster development of alternative energy use increases welfare in the decentralized equilibrium without taxation[7]. In a richer model, however, faster development of technology for alternative energy use comes at a cost and that cost must be weighed against the benefits.

I think that the general conclusion to be drawn from this work is that climate change mitigation policy is complex. Different policy measures interact in non-trivial ways. This indicates the need for a richer model, that captures these interactions, so that the combined effect of different policy measures can be analyzed. Such a model should endogenize technology and explicitly model policies that stimulate research on technology for alternative energy use.

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which menas that the planner solution can no longer be implemented using taxation

[7]It is actually possible to get decreasing welfare from faster development of alternative energy use in this model. But it requires increasing the damages so much that it does not seem at all relevant for practical purposes

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