MACRO I, 1998 SDPE

Lecture Notes

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I. Consumption, Fiscal Policy and Risk

I.A. Consumption, the Current Account and Risk

1. The Intertemporal consumption choice

Three steps in the development of consumption modeling

1. **Keynesian**

$$C_i = a + bY_i + \varepsilon_i \tag{1.1}$$

worked well in cross-section studies with $a>0$, $0<b<1$. Not consistent with time series evidence. Led to

2. **Permanent income (Friedman, -57) – Life Cycle (Modigliani) hypothesis.**

$$c_t = \frac{A_t + E_t \sum_{s=t}^{T} y_s}{T - t + 1} \tag{1.2}$$

Distinguishes life time wealth and transitory income changes. Relation between income and consumption depends on relation between income changes and lifetime wealth. Compare

$$\begin{aligned} y_t &= y_{t-1} + \varepsilon_t \\ y_t &= kt + \varepsilon_t \end{aligned} \tag{1.3}$$

No account taken for second moments, effects of uncertainty.

3. **Consumption and investment under uncertainty.**

   a. Precautionary savings. Leland (-68), Kimball (-90)

   b. Irreversible investments. Option value of Waiting. Pindyck (-91), McDonald & Siegel (-86).

. Permanent Income, Precautionary Savings and Liquidity Constraints

Consider the problem

$$
\begin{aligned}
& V_0(A_0) \equiv \max_{\{c_t\}} E\left[\sum_{t=0}^{T}(1+\rho)^{-t}U(c_t)\middle|\Omega_0\right] \\
& s.t. \quad A_{t+1} = (A_t + \tilde{y}_t - c_t)(1+\tilde{r}_{t+1}), \\
& \qquad\quad A_{T+1} \geq 0, \\
& \qquad\quad A_0 \text{ given.}
\end{aligned} \tag{1.4}
$$

Use the Bellman equation

$$
\begin{aligned}
& V_t(A_t) = \\
& \max_{c_t} E_t\left[U(c_t) + (1+\rho)^{-1}V_{t+1}\left((A_t + \tilde{y}_t - c_t)(1+\tilde{r}_{t+1})\right)\right]
\end{aligned} \tag{1.5}
$$

FOC:

$$
\begin{aligned}
& FOC\, c_t; \\
& U'(c_t) = (1+\rho)^{-1}E_t\left[V'_{t+1}(A_{t+1})(1+\tilde{r}_{t+1})\right]
\end{aligned} \tag{1.6}
$$

Use the Envelope theorem

$$
\begin{aligned}
& V'_t(A_t) = \\
& (1+\rho)^{-1}E_t\left[V'_{t+1}(A_{t+1}))(1+\tilde{r}_{t+1})\right]
\end{aligned} \tag{1.7}
$$

So

$$
U'(c_t) = E_t\left[(1+\rho)^{-1}U'(c_{t+1})(1+\tilde{r}_{t+1})\right] \tag{1.8}
$$

This is the Euler equation for consumption.

. Some particular Euler Equations

Non-stochastic interest rate.

$$
\begin{aligned}
& U'(c_{t+1}) = \frac{1+\rho}{1+r}U'(c_t) + \varepsilon_{t+1} \\
& E\left[\varepsilon_{t+1}\middle|\Omega_t\right] \equiv E_t\left[\varepsilon_{t+1}\right] = 0.
\end{aligned} \tag{1.9}
$$

Quadratic utility

$$U(c_t) = ac_t - \frac{bc_t^2}{2} \tag{1.10}$$

$$\begin{aligned} E_t(a - bc_{t+1}) &= \frac{1+\rho}{1+r}(a - bc_t) \\ c_{t+1} &= \frac{a}{b}\frac{r-\rho}{1+r} + \frac{1+\rho}{1+r}c_t + \varepsilon_{t+1} \end{aligned} \tag{1.11}$$

This is the Hall equation. No variable known in t is correlated with ε_{t+1}. Can be tested using OLS.

The Hall Equation as a First Order Linear Approximation

$$\begin{aligned} E_t U'(c_{t+1}) &= \frac{1+\rho}{1+r}U'(c_t) \\ E_t U'(c_{t+1}) &\approx E_t\left[U'(c_t) + U''(c_t)(c_{t+1} - c_t)\right] \\ &= U'(c_t) + U''(c_t)E_t(c_{t+1} - c_t) \\ &= \frac{1+\rho}{1+r}U'(c_t) \\ \Rightarrow \Delta c_{t+1} &= \frac{r-\rho}{1+r}\frac{U'(c_t)}{-U''(c_t)} + \varepsilon_{t+1} \end{aligned} \tag{1.12}$$

Note that low relative risk aversion and $r>\rho$ gives high consumption growth. Why? Note also that by taking a first order approximation we disregard third moments in utility an3d second moments in consumption – certainty equivalence.

Examples of analytical solutions.

A. Quadratic utility, constant interest rate, only income risk, finite horizon.

Simplify $\rho=r$.

$$\begin{aligned} &c_{t+1} = c_t + \varepsilon_{t+1} \\ &\Rightarrow E_t c_{t+s} = c_t \forall s \geq 0. \end{aligned} \tag{1.13}$$

From intertemporal (collapsed) budget we know that

$$\begin{aligned} \sum_{s=t}^{T} \frac{c_s}{(1+r)^{s-t}} &= A_t + \sum_{s=t}^{T} \frac{y_s}{(1+r)^{s-t}} \\ E_t \sum_{s=t}^{T} \frac{c_s}{(1+r)^{s-t}} &= A_t + E_t \sum_{s=t}^{T} \frac{y_s}{(1+r)^{s-t}} \equiv A_t + H_t \end{aligned} \tag{1.14}$$

Then using (1.13) we get

$$c_t = \frac{r}{(1+r)\left(1-(1+r)^{-T+t-1}\right)} \left(\overbrace{\{A_t + H_t\}}^{W_t} \right) \tag{1.15}$$

Certainty equivalence.

Look at $\rho=r=0$. Then perfect smoothing

$$c_t = \frac{W_t}{T-t+1} \tag{1.16}$$

This is the *Modigliani Life Cycle Hypothesis.*

Let $T=\infty$ and $\rho=r>0$. Then

$$c_t = \frac{rW_t}{1+r} = \bar{y}_t^P \tag{1.17}$$

the *Friedman Permanent Income Hypothesis.*

B. No labor income, interest rate risk (*multiplicative*), CRRA (e.g. log), infinite horizon, time autonomous problem (z i.i.d.).

For example, take the problem in (1.4), assume income is zero always, utility is log and time is infinite. Guess that the value function is given by

$$V(A_t) = a \ln A_t + B \tag{1.18}$$

Substitute this into (1.6).

$$\begin{aligned} \frac{1}{c_t} &= (1+\rho)^{-1} E_t \left[\frac{a}{A_{t+1}} (1+\tilde{r}_{t+1}) \right] \\ &= (1+\rho)^{-1} E_t \left[\frac{a}{A_t - c_t} \right] \end{aligned} \tag{1.19}$$

which gives

$$c_t = \frac{1+\rho}{1+\rho+a} A_t \tag{1.20}$$

Substituting this into the Bellman equation yields

$$\begin{aligned}
a \ln A_t + B &= \ln \frac{1+\rho}{1+\rho+a} A_t + \frac{1}{1+\rho} E_t\left(a \ln A_{t+1} + B\right) \\
&= \ln A_t + \ln \frac{1+\rho}{1+\rho+a} + \frac{1}{1+\rho} E_t\left(a \ln\left(\left(A_t - c_t\right)\left(1+\tilde{r}_{t+1}\right)\right) + B\right) \\
&= \ln A_t + \ln \frac{1+\rho}{1+\rho+a} + \frac{1}{1+\rho} E_t\left(a \ln\left(\frac{a}{1+\rho+a} A_t \left(1+\tilde{r}_{t+1}\right) \right) + B \right) \\
&\Rightarrow a = \frac{1+\rho}{\rho}, \qquad c_t = \frac{\rho}{1+\rho} A_t
\end{aligned} \tag{1.21}$$

where $1/(1+\rho)$ is discount factor. Note that log W_t follows a random walk. A kind of certainty equivalence since for log utility income and substitution effects cancel.

C. Only labor income risk (*additive*) and normal i.i.d. innovations, finite horizon, CARA (exponential) utility. Simplify and set ρ=r=0. The consumer solves

$$\begin{aligned}
&\max_{\{c_t\}} E_0\left[\sum_{t=0}^{T} \frac{e^{-\gamma c_t}}{-\gamma}\right] \\
&s.t. \quad A_{t+1} = A_t + y_t - c_t \\
&A_0 = A_T = 0.
\end{aligned} \tag{1.22}$$

Assume a process for y_t , for example

$$y_{t+1} = y_t + \varepsilon_{t+1}. \tag{1.23}$$

with ε_{t+1} N(0,σ^2). Guess that

$$c_{t+1} = c_t + \frac{\gamma\sigma^2}{2} + \varepsilon_{t+1}. \tag{1.24}$$

By using that if $c \overset{d}{=} N\left(\bar{c}, \sigma^2\right)$ then $E\left[e^{-\gamma c}\right] = e^{-\gamma \bar{c} + \frac{\gamma^2\sigma^2}{2}}$ we can check that this satisfies the Euler equation. The budget constraint implies

$$\sum_{s=t}^{T} c_s = A_t + \sum_{s=t}^{T} y_t \tag{1.25}$$

With the expressions for expected consumption and income given by (1.24) and (1.23) (1.25), after taking expected values as of *t*, simplifies to

$$c_t = \frac{1}{T-t+1} A_t + y_t - \frac{\gamma(T-t)\sigma^2}{4} \tag{1.26}$$

Note the problem with long and infinite horizons, consumption may be negative.

Quantifying Precautionary Savings

Take the Euler equation

$$E_t U'(c_{t+s}) = \left(\frac{1+\rho}{1+r}\right)^s U'(c_t) \qquad \forall s \geq 0. \tag{1.27}$$

Note that if U' is convex the LHS is increasing in a mean preserving risk increase. Increases in risk thus has to be matched by decreasing consumption today and increasing expected consumption tomorrow. Both helps restore (1.27).

Do Taylor approximation of (1.27) letting ρ=r

$$\begin{aligned} &U'(c_t) \\ &= E_t[U'(c_{t+1})] \approx U'(c_t) + U''(c_t) E_t[c_{t+1} - c_t] + \frac{1}{2} U'''(c_t) E_t\left[(c_{t+1} - c_t)^2\right] \\ &\Rightarrow E_t[c_{t+1} - c_t] \approx \underbrace{\frac{-U'''(c_t)}{U''(c_t)}}_{p_a} \frac{1}{2} E_t\left[(c_{t+1} - c_t)^2\right] \end{aligned} \tag{1.28}$$

or

$$E_t\left[\frac{c_{t+1} - c_t}{c_t}\right] \approx \underbrace{\frac{-U'''(c_t) c_t}{U''(c_t)}}_{p_r} \frac{1}{2} E_t\left[\left(\frac{c_{t+1} - c_t}{c_t}\right)^2\right] \tag{1.29}$$

p_a and p_r are the absolute and relative coefficients of prudence.

2. A Stochastic Model of a Small Open Economy

. Some preliminaries:

Let us define the stock of *net* foregin assets at the *end* of period t as B_{t+1}. Equivalently, B_{t+1} is the stock of foreign assets at the beginning of period t *net of* interest paid between period t and t+1. Then:

$$B_{t+1} = Y_t - C_t + B_t(1 + r_t) \tag{1.30}$$

where Y_t is aggregate income (except income on foreign assets), C_t is aggregate consumption and investments by domestic residents and the government and $r_t B_t$ is the interest received on the stock of foreign assets that was held between period t-1 and t.

We now define the current account

$$CA_t = B_{t+1} - B_t \tag{1.31}$$

which thus is the change in the holdings of foreign assets that took place during period t.

Let us define *the permament value* of a variable from the relation

$$\sum_{s=t}^{\infty}(1 + r_s)^{s-t}\,\overline{x}_t = E_t \sum_{s=t}^{\infty}(1 + r_s)^{s-t}\,x_s \tag{1.32}$$

so, for example, the permanent value of a variable income stream is the for ever fixed income level that would have the same expected PDV as the variable income stream.

Consider a representative individual that spends money on consumption C_t and investements I_t. There is also a government that consumes G_t. If this good enters the utility function of the individual, it enters additively, so the Euler equation is not changed.

Assume for now that the government applies taxes so its budget balances each period. The representative individual thus has a budget constraint given by

$$B_{t+1} = Y_t - C_t - G_t - I_t + (1 + r)B_t \tag{1.33}$$

Collapsing this as in (1.14), taking expectations, assuming the no Ponzi condition $\lim_{s\to\infty}(1 + r)^{-s} B_s$ and using the definition in (1.32)

$$
\begin{aligned}
&\sum_{s=t}^{\infty}\frac{C_s}{(1+r)^{s-t}}=(1+r)B_t+\sum_{s=t}^{\infty}\frac{Y_s-G_s-I_s}{(1+r)^{s-t}}\\
&E_t\sum_{s=t}^{\infty}\frac{C_s}{(1+r)^{s-t}}=(1+r)B_t+E_t\sum_{s=t}^{\infty}\frac{Y_s-G_s-I_s}{(1+r)^{s-t}}\\
&E_t\sum_{s=t}^{\infty}\frac{C_s}{(1+r)^{s-t}}=(1+r)B_t+\frac{1+r}{r}\left(\overline{Y}-\overline{G}_t-\overline{I}_{tt}\right)
\end{aligned}
\tag{1.34}
$$

The RHS can be thought of as the (net) wealth of the representative individual.

Now use the result from the previous section that in the risk-free case or under certainty equivalence (recall what could generate that) and when $\rho=r>0$, the Euler equation for consumption implies a consumption with a level that is expected to be constant. In this case this means that,

$$
\begin{aligned}
&\frac{1+r}{r}C_t+\frac{1+r}{r}\overline{G}_t+\frac{1+r}{r}\overline{I}_t=B_t+\frac{1+r}{r}\overline{Y}_t\\
&C_t=rB_t+\overline{Y}_t-\overline{G}_t-\overline{I}_t
\end{aligned}
\tag{1.35}
$$

Now use this in the definition of the current account to get

. The fundamental current account equation

$$
\begin{aligned}
CA_t=B_{t+1}-B_t&=Y_t-G_t-I_t+rB_t-C_t\\
&=Y_t-G_t-I_t+rB_t-\left(rB_t+\overline{Y}_t-\overline{G}_t-\overline{I}_t\right)\\
&=\left(Y_t-\overline{Y}_t\right)-\left(G_t-\overline{G}_t\right)-\left(I_t-\overline{I}_t\right)
\end{aligned}
\tag{1.36}
$$

So the current account is in surplus if Y_t is above its permanent level or if government consumption or investments is below their permanent levels. Can you provide the intuition for this?

Now let the effects of output shocks. Assume that income follows the stochastic process

$$
A(L)y_t=\varepsilon_t \tag{1.37}
$$

We can then use result that

$$
\begin{aligned}
&A(L)y_t=\varepsilon_t\\
&\Rightarrow E_t\sum_{s=0}^{\infty}\left(\frac{1}{1+r}\right)^s y_{t+s}=\frac{y_t}{A\left(\frac{1}{1+r}\right)}
\end{aligned}
\tag{1.38}
$$

So, for example, if the

$$Y_t - \rho Y_{t-1} = \varepsilon_t$$

$$E_t \sum_{s=0}^{\infty} (1+r)^{-s} Y_{t+s} = \frac{1}{1-\rho\frac{1}{1+r}} Y_t = \frac{1+r}{1+r-\rho} Y_t \tag{1.39}$$

$$\overline{Y}_t = \frac{r}{1+r-\rho} Y_t$$

The effect on the current account then becomes,

$$\frac{\partial CA_t}{\partial Y_t} = \frac{\partial Y_t}{\partial Y_t} - \frac{\partial \overline{Y}_t}{\partial Y_t} = 1 - \frac{r}{1+r-\rho} = \frac{1-\rho}{1+r-\rho} \tag{1.40}$$

Similarly, when

$$\Delta Y_t - \rho \Delta Y_{t-1} = \varepsilon_t \Rightarrow A(L) = 1-(1+\rho)L + \rho L^2$$

$$\overline{Y}_t = \left(\frac{r}{1+r}\right) \frac{1}{1-(1+\rho)\frac{1}{1+r} + \rho\left(\frac{1}{1+r}\right)^2} Y_t = \frac{r}{1+r} \frac{(1+r)^2}{r(1+r-\rho)} Y_t \tag{1.41}$$

$$= \frac{1+r}{1+r-\rho} Y_t$$

So, if ρ larger than zero, permanent income fluctuates more than temporary. In this case,

$$\frac{\partial CA_t}{\partial Y_t} = 1 - \frac{1+r}{1+r-\rho} = \frac{-\rho}{1+r-\rho} \tag{1.42}$$

so the effect on the current account by an unexpected increase in income is negative. The intuition is that if ρ larger than zero, an increase in income that is noted today, signals even larger income in the future. So, consumption increases to take also that increase into account.

We have so far treated investments as unaffected by the shock to *Y*. In reality, there may, of course, a correlation. For example, if a shock to Y_t increase the marginal productivity of capital, investments may increase in order to increase the capital stock. A way to model this is that

$$Y_t = A_t F(K_t) \tag{1.43}$$

Now, if A_t increases permanently, this raises marginal productivity, which raises investments. Then, the current account must be negative in *t*. This since investments are above their permanent level and output is no higher than its permanent level.

3. The Lucas Critique Some Empirical Consumption Puzzles

The Lucas Critique

Sample moments between observed macro variables – like consumption, disposable income and output – change when policy change. This since optimum decision rules change with policy. Econometric models can thus only be used in short-term forecasting and can “provide *no* useful information as to the actual consequences of alternative economic policies”.

MPC example

Assume that disposable income follows

$$Y_t = \rho Y_{t-1} + \varepsilon_t \tag{1.44}$$

Then

$$\bar{Y}_t = \frac{r}{1+r-\rho} Y_t \tag{1.45}$$

use (1.17) then we may calculate MPC

$$\frac{\partial C_t}{\partial Y_t} = \frac{\partial C}{\partial \bar{Y}_t}\frac{\partial \bar{Y}_t}{\partial Y_t} = \frac{r}{1+r-\rho} \tag{1.46}$$

If ρ is close to unity, MPC is close to unity. Now let there be a temporary lump sum transfer τ to the house hold. A naive Keynesian would say that this increase consumption almost one for one. But,

$$C_t = \frac{r}{1+r} A_t + \bar{Y}_t \tag{1.47}$$

Calculate MPC out of the transfer

$$\frac{\partial C_t}{\partial \tau} = \frac{r}{1+r} \tag{1.48}$$

So if MPC is estimated on income and then used to predict effects of fiscal policy we get wrong results (if $\rho \neq 0$). Must have an *economic* model to understand the effects of policy changes.

. Tests and Puzzles

Hall -78

$$C_{t+1} = \alpha + \lambda C_t + \varepsilon_{t+1}$$
$$\varepsilon_{t+1} \perp \Omega_t \tag{1.49}$$

Can be tested by adding variables known in *t* to an OLS regression. Finds no influence from c_{t-1-s} and y_{t-s}. *S&P* stock market index has a significant influence. Suggested explanation – slow adjustments.

Carrol and Summers -89

Strong correlation long run growth in aggregate income and consumption in cross-country study. Also at individual level. Appears that consumption grows one for one also with expected growth in income.

Potential explanations;

Liquidity constraints – must in such case be almost everybody. Most people have only very low financial savings.

Flavin -81, JPE

"Excess Sensitivity" to predicted changes in income.

Consumption change to predicted changes in income, e.g., when new pensions are paid out not when they are decided upon.

Campbell and Deaton -89

"Excess Smoothness".

C&D estimates a second order process for income

$$\begin{aligned} \Delta Y_t &= \alpha + \rho \Delta Y_{t-1} + \varepsilon_t, \\ A(L) Y_t &= \alpha + \varepsilon_t, \\ A(L) &\equiv 1 - (1+\rho) L + \rho L^2. \end{aligned} \tag{1.50}$$

with ρ=0.442. An increase in growth signals future high growth. Then a shift in income today has a very large effect on permanent income so consumption should change very much. In this case

$$\sigma_{\Delta c_t} = \frac{1+r}{1+r-\rho} \sigma_{\varepsilon_t} \approx 1.8 \sigma_{\varepsilon_t} \tag{1.51}$$

Instead they find that the coefficient is around 1 and only 1/2 for non-durables. So consumption is *excessively smooth*. This relies on non-stationary income.

Excess sensitivity and excess smoothness is two sides of the same coin. When expected future income increases consumption rise less then permanent income but responds when expectations are realized.

Caballero QJE -90 discusses precautionary savings as an explanation for excess sensitivity and excess smoothness. Assume that expected volatility of future earnings increase in the level (e.g., if y is a log random walk). Then a positive shock to expected future earnings increase precautionary savings so consumption does not respond one for one. When realized risk disappears so consumption increase.