Macro I - Problem set 2: Consumption and the dynamics of a small open economy

Due: Tuesday February 12

Problem 1

Exercise 2.1 in Obstfeld and Rogoff:

Suppose that a country has negative net foreign assets and adopts a policy of running a trade balance surplus sufficient to repay a constant small fraction of the interest due each period. It rolls over the remaining interest. That is, suppose it sets its trade balance according to the rule $TB_s = -\xi r B_s \quad , \xi > 0$.

(a) Using the current account identity and the definition of the trade balance, show that under this policy, net foreign assets follow the equation

$$B_{s+1} = [1 + (1 - \xi) r] B_s$$

(b) Show directly that the intertemporal budget constraint is satisfied for any $\xi > 0$. [Hint: show why

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} TB_s = -\sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \xi r B_s = -(1+r) B_t \qquad]$$

Note that even if ξ is very small, so that trade balance surpluses are very small, debt repayments grow over time as B_s grows more negative.

Added: Show that the claim holds not only for $\xi > 0$, (as claimed in the original exercise) but also for any $0 < \xi < \frac{1}{r}$. (If $\xi < \frac{1}{r}$, then the country never pays back all of its debt)

(c) Have you now proved that current account sustainability requires only that countries pay an arbitrarily small constant fraction of interest owed each period, rolling over the remaining debt and interest? [Hint: consider the case of an endowment economy with $G = I = 0$ and constant output Y. How big can B get before the country owes all his future output to creditors? Will this bound be violated if ξ is not big enough? If so, how can the intertemporal budget constraint have held in part (b) ?]

Additional hint: What is the implied path of consumption?

Problem 2

Exercise 2.4 in Obstfeld and Rogoff:

Consider the linear-quadratic stochastic consumption model, in which $(1+r)\beta = 1$ and $G = I = 0$ on all dates.

(a) Use the current account identity together with consumption equation

$$c_t = \frac{r}{1+r}\left[(1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\mathrm{E}_t Y_s\right]$$

to show that the change in consumption is the present value of changes in expected future output levels,

$$c_{t+1} - c_t = \frac{r}{1+r}\sum_{s=t+1}^{\infty}\left(\frac{1}{1+r}\right)^{s-(t+1)}(\mathrm{E}_{t+1} - \mathrm{E}_t)\,Y_s$$

In this equation, for any variable X_s, $(\mathrm{E}_{t+1} - \mathrm{E}_t)\,X_s$ denotes the amount by which expectations of X_s are revised as a result of information that arrives between dates t and $t+1$. (Here, the change in consumption automatically equals the unexpected change or innovation in consumption, due to Hall's random walk result.)

(b) Suppose output follows a nonstationary process like

$$Y_{t+1} - Y_t = \rho\,(Y_t - Y_{t-1}) + \varepsilon_{t+1} \tag{0.1}$$

where $0 < \rho < 1$. Show that for $s > t$,

$$(E_{t+1} - E_t)\,Y_s = \left(1 + \rho + ... + \rho^{s-(t+1)}\right)\varepsilon_{t+1} = \frac{1-\rho^{s-t}}{1-\rho}\varepsilon_{t+1}$$

(c) Conclude that for the permanent-income consumption equation

$$c_t = \frac{r}{1+r}\left[(1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\mathrm{E}_t Y_s\right]$$

the consumption innovation on date $t+1$ is

$$c_{t+1} - c_t = \frac{1+r}{1+r-\rho}\varepsilon_{t+1} = \frac{1+r}{1+r-\rho}\,(Y_{t+1} - \mathrm{E}_t Y_{t+1})$$

Because $\rho > 0$, consumption innovations now are more variable than output innovations when individuals desire smooth consumption.

(d) Compute the current account response to output innovations in this case. Verify the claim at the end of section 2.3.3:

Because (0.1) makes all future output levels rise by more than ε_t, permanent output fluctuates now more than current output (except in the special case $\rho = 0$). Consumption smoothing now implies that an unexpected increase in output causes an even greater increase in consumption. As a result, a positive output innovation now implies a current account *deficit*, in sharp contrast to the prediction for the stationay case.

Problem 3

Exercise 2.6 in Obstfeld and Rogoff:

$$CA_t = B_{t+1} - B_t = \left(Y_t - \mathrm{E}_t\widetilde{Y}_t\right) - \left(I_t - \mathrm{E}_t\widetilde{I}_t\right) - \left(G_t - \mathrm{E}_t\widetilde{G}_t\right) \qquad (0.2)$$

(0.2) implies that $CA_t = Z_t - \mathrm{E}_t\widetilde{Z}_t$, where $Z = Y - G - I$ is net output. Show that an equivalent equation is

$$CA_t = -\sum_{s=t+1}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\mathrm{E}_t\triangle Z_s$$

Hint: define the lead operator L^{-1} by $L^{-1}\mathrm{E}_tX_t = \mathrm{E}_tX_{t+1}$ for any variable X, and use the ,ethodology described in Supplement C to chapter 2, noting that

$$\mathrm{E}_t\widetilde{Z}_t = \frac{r}{1+r}\left(1 - \frac{1}{1+r}L^{-1}\right)^{-1}\mathrm{E}_tZ_t$$