Problem set 3: OLG models and fiscal policy

Due: Tuesday February 16

0.1. Problem 1: The Overlapping Generations Model

Assume an OLG-economy where there are two types of agents (A and B) in each generation. Let N_{it} be the number of agents of type i ($i = A, B$) in period t. All agents live for 2 periods. An agent born at time t consumes c_t^Y when young and c_{t+1}^O when old. The endowment profile for type A agents is $(1, 0)$ and for type B agents $(0, 1)$. Both agents have utility functions of the form

$$U\left(C_t^Y, C_{t+1}^O\right) = \ln C_t^Y + \beta \ln C_{t+1}^O$$

(a) Determine the optimal consumption profile, for a given interest rate, for agent i ($i = A, B$).

(b) What is the equilibrium rate of interest?

(c) Determine the consumption profile in equilibrium for the two types of agents.

Problem 2: A small open OLG economy

Consider the following small open OLG economy which faces a constant interest rate r. Agents live for two periods. In the first period they work and get an income y_t, and in the second period they are retired. An agent born at time t consumes c_t^Y when young and c_{t+1}^O when old. Consumer preferences are given by the utility function

$$U\left(c_t^Y, c_{t+1}^O\right) = \ln c_t^Y + \beta \ln c_{t+1}^O.$$

Let N_t denote the number of young agents at time t. The population growth rate is $(1 + n)$ and the per capita income growth rate is $(1 + g)$.

The government sets a labor-income tax rate, τ. Tax revenues are used for lump-sum transfers to the old agents, h_t^O. In each period the government budget must be balanced, i.e.

$$N_t \tau y_t = N_{t-1} h_t^O.$$

(a) Derive the savings function, i.e. derive an agent's optimal choice of savings as a function of his income y_t, the tax rate τ, and the transfer h_{t+1}^O.

(b) Assume that $\beta(1+r) = 1$.

Calculate the agent's reaction to tax rate changes, i.e. calculate $\frac{\partial c_t^Y}{\partial \tau}$, and $\frac{\partial c_{t+1}^O}{\partial \tau}$.

Under what conditions will there be Ricardian equivalence in this economy?

Under what conditions will all agents in the economy benefit from positive transfers from young to old? Explain why.

Problem 3: A closed OLG economy

Consider an economy similar to that in the previous problem, but now assume that the economy is closed. There is no government activity, so $\tau = h_t^O = 0$ for all t. Also, there is no productivity growth ($g = 0$).

Production is

$$f(K_t) = k_t^\alpha,$$

where k_t denotes the per capita capital stock in the beginning of period t.

Perfect markets ensure that the interest rate is

$$r_t = f'(k_t),$$

and that labor income is

$$w_t = f(k_t) - r_t k_t.$$

(a) Derive the savings function, i.e. derive an individual's optimal choice of savings as a function of his wage.

(b) Show that the dynamics of k_t is given by

$$k_{t+1} = \frac{(1-\alpha)\beta}{(1+n)(1+\beta)} k_t^\alpha.$$

(c) Determine the steady state capital stock. Check its stability properties.

0.2. Problem 4: Pension systems

Let us now use the model in problem 3 to compare two pension systems.

(a) First, consider a fully funded pension system. In period t, the government collects lump sum taxes τ_t from the young agents. The tax revenues are invested in physical capital and returned to the old generation the next period. The old at $t+1$ thus get $(1 + r_{t+1})\,\tau_t$ from the government.

Compared to the model in problem 2, how will savings, the interest rate, and the capital stock be affected by this pension system?

(b) Next, consider a pay-as-you-go pension system. Again, in period t, the government collects lump sump taxes τ_t from the young, but now the tax revenues are immediately transferred to the currently old. Assume that the per capita tax is constant over time, i.e. $\tau_t = \tau$ for all t.

How will savings, the interest rate, and the capital stock be affected by this pension system?