# 1 Non-rival productive inputs.

Consider a general production technology:

$$Y_t = Y(A_t, K_t, L_t) \tag{1}$$

We conventionally label $K_t$ as "capital", $L_t$ as labor and $A_t$ as an index of technical knowledge. I will now try to be more fundamental, and single out a classification of the productive inputs which is not based on some statistical aggregate, but, rather, on some *economic* properties which are relevant for the problems which we study in the theory of growth.

Let us recall some basic notions of public economics.

- Private good $\equiv$ RIVAL + EXCLUDABLE
- Public (collective) good $\equiv$ NON-RIVAL + NON-EXCLUDABLE
- EXCLUDABILITY - problem of property rights. A good is perfectly excludable when the holder can withhold the benefits associated with the commodity from others without incurring significant costs (an example of non excludable good is fish in a certain segment of the ocean).
- RIVALROUSNESS - A good is *rival* in nature when the use of that good by one agent preclude the simultaneous use of the same good by other agents. An example of non-rival good is radio broadcast.

Let me add a notion which is relevant in growth theory.

- REPRODUCIBILITY - An input is *reproducible* in nature if it can be accumulated in time by (directly or indirectly) foregoing present consumption. The stock of land is the most obvious example of non-reproducible factor. However, also goods which are accumulated according to some exogenous (e.g. demographic) dynamics enter this group.

In general, $K_t$ is a reproducible, private input, while $L_t$ is a non-reproducible private input. The critical feature of technical knowledge is that it intrinsically entails some public good elements, and this is what makes it a very different input in the process of production. Let us examine them.

- Non-rivalrousness $\rightarrow$ when one agent uses technical information to produce a good or a service, his action does not materially prevent others from using the same information.
- Non-excludability $\rightarrow$ creators or owners of technical information often have difficulty in preventing others from making unauthorized use of it.

When we introduced non-rival inputs, the technology becomes non-convex, and standard replication arguments which we apply when we assume a world of constant returns to scale cease to be valid.

An example. A firm pays 2 engineers to study the optimal set-up of her production sector. Using the output (knowledge) of their activity the firm can organize production and use 10 L and 10 K to produce 1 unit of output. Using the same organizational structure, it can also replicate the process and use 20 L and 20 K to produce 2 units of output.. Had the firm also doubled the number of engineers (from 2 to 4) these could have set up some even better organization, which would have allowed to obtain, say, 2.5 units of output out of 20 L plus 20 K.

So, there are increasing returns to scale overall. The traditional replication argument cannot be applied here, because it is not necessary to replicate non-rival inputs. The logic of this argument suggests instead that the world is characterized by CRS to the private inputs alone.

More formally, we have a prod. function of the type:

$$F(A, \lambda K, \lambda L) = \lambda F(A, K, L) < F(\lambda A, \lambda K, \lambda L) \tag{2}$$

where $\lambda > 1$. This raises the well-known problem that with IRS factors cannot be paid the value of their marginal contribution (i.e. competitive solution is not sustainable). In fact:

$$F(A, K, L) < F_A A + F_K K + F_L L \tag{3}$$

and a competitive firm would suffer a loss.

## 1.1 Exogenous technical change.

The neoclassical growth model model (NGM) solves the problem introduced by the non-rival nature of $A$ by treating it as an exogenously provided public input, both non excludable and non-rival, which receives no compensation. Every individual firm is free to exploit the entire stock of $A$. The time evolution of $A$ is determined by an exogenous law of motion and does not respond any market incentive. In other words, in the NGM *knowledge is not a reproducible factor*, in the sense that its accumulation does not depend on economic decisions.

Here is an interpretation of the model. Growth is led by innovation, and scientific discoveries are the primary force behind innovation. Scientific advances, however, largely reflect the interest and resources of a community – the scientists – which operates outside the profit sector of the economy. Thus, we do not need to bother with looking for interaction between the actions taken by profit-maximizing firms and utility-maximizing consumers on the one hand, and the activity of scientists on the other hand. In fact, we can even ignore how the scientists get rewarded for the benefits which their inventions bring about. Technical progress falls as "manna from heaven" on productive firms.

## 1.2 Knowledge as spillover.

In attempt to make the evolution of $A$ responsive to market incentives, Arrow (1962), Sheshinsky (1967) and more recently Romer (1986) assumed that knowledge creation was a by-product of investment.[1]

[1] The idea that growth is led by "intentional innovative activity" stretches back to the intuition of some earlier authors such as Schumpeter and Kaldor, who argued that a virtuous circle links investment and innovation.

The basic idea of these authors is that the acquisition of knowledge (learning) is related to experience. In a sense, new machines "educate" workers, who learn "by doing" and "by using" the newly introduced equipment (e.g. they are induced to study the principles of functioning etc.). Like in the neoclassical model, knowledge is treated as a *public good*. Formally, $A_t$ is assumed to depend on the cumulative investment or capital stock accumulated in the whole economy, formally $A_t = A(K_t)$. Investment in physical capital from any firm determines as by-product accumulation of knowledge which spread costlessly to all firms in the economy (*knowledge spillover*). All firms are assumed to be small and to take the aggregate stock of capital as given, each ignoring the effect of its investment on the stock of public good $A$.

In this model, knowledge is *non-rival* and *non-excludable*, but, differently from the neoclassical model, it is a *reproducible* input, since it is the by-product of the accumulation of physical capital which is a reproducible factor. Since knowledge is a public input which receives no compensation – and there are non-increasing returns to private inputs – standard competitive equilibria are sustainable, although the presence of externalities makes the competitive equilibrium inefficient. The novelty of this approach is that policies affecting saving rates have permanent effects on growth, and one can study the effect of alternative policies and relate their effects to the growth performance of different economies.

## 1.3 Intentional technical change and non-competitive endogenous growth models: non-rival, (partially) excludable, reproducible knowledge.

The theory developed in the 90's builds up on the idea that technological progress is not just a random process, nor a mere spillover, but rather one guided by market forces. The models of endogenous technical change *cum R&D* abandon the assumption – which was maintained by all models previously mentioned – that technical knowledge is a pure public good, although they insist on the *partially* excludable nature of the benefits from innovation.

In these models to find new ideas has some cost. New ideas are *non-rival* in nature, since as a new machine, intermediate input or productive technique is discovered, all firms in the economy could in principle benefit from that discovery. However, there exists a legal system which protects the property rights of the innovators: the *patents*. Since it is possible to patent innovations, innovating firms can enjoy, for a finite period or forever, some monopoly power over the production of a certain good or the use of a certain production technique. The expectation of future monopoly rents justifies the sunk cost associated with the initial investments in R&D.

Technical progress has been modeled in two different ways:

1. a continuous expansion of the varieties of inputs that the firms producing consumption goods can use;

2. a progressive improvement in the quality of a limited number of products or intermediate goods.

According to the former, the difference between a developed and a less developed economy is not that a larger stock of hammers or screwdrivers are available to final producers in the former than in the in the latter economy. Rather, in the advanced country, final good firms can also use scissors, pincers, etc.; namely a larger variety of capital goods is available. To put a more realistic example, the set of real and financial services which are available to a firm in the US is much wider than the set of services which the same firm could use in a poor country. Note that this approach does not stress any difference between the productivity of inputs introduced at earlier or later stages of development. It is only variety which matters.

The second approach (which will not be analyzed in this introductory course) emphasizes that process of innovation makes available inputs and machines which are more and more productive. In the technology of personal computers, for instance, the Pentium processor is more efficient than its predecessor 486, which in turn represented an improvement over the 386, etc. This process of innovation can be defined as *vertical*. An important feature of *vertical innovation* is what Schumpeter defined *destructive creation*: an innovation causes the obsolescence of the prior vintages of machines and allows to a firm to capture markets which were previously controlled by other firms. But the firm which invests in R&D knows that the monopoly rents it can gain are temporary, and destined to be disrupted by future innovations. Hence the expectation about the pace of future innovations is a determinant of the profitability of today's innovation.

# 2 Endogenous technical change. Romer (JPE 1990).

This model of endogenous growth with expanding variety of inputs is a simplified version of Romer (JPE 1990).

## 2.1 Households.

Households decide the plan of consumption/saving in order to maximize utility, subject to an intertemporal budget constraint and a No-Ponzi game condition. Households' income consists of capital and labor income. Labor is supplied inelastically, with the workforce being constant and equal to $L$. Assets consists of loans only (debt being negative asset holding). There is no physical capital.[2] In equilibrium there will be no intra-household borrowing. However, there will be firms which borrow from household, thus, in equilibrium, households will hold a positive stock of wealth. In particular, as we will see, there will be new intermediate firms which borrow in order to finance the cost of entering the market (product innovation). For simplicity, we will assume that each new firm issue unredeemable bonds and sell them to the households.

Formally, we assume an isoelastic utility function:

$$U = \int_0^\infty e^{-\rho t} \frac{C_t^{1-\theta} - 1}{1-\theta} dt. \tag{4}$$

[2] This is my simplification. In the original article, there is physical capital, as well.

Households maximize (4) subject to the intertemporal budget constraint:[3]

$$\dot{B}_t = r_t B_t + w_t L - C_t, \tag{5}$$

a *no-Ponzi game* condition, and $B_0$ given.[4] As usual, the maximization problem yields the following Euler condition:

$$\dot{C}_t = \frac{r_t - \rho}{\sigma} \cdot C_t \tag{6}$$

## 2.2 Production.

The production side of the economy consists of two sectors of activity:

1. final good firms, which employ labor and a set of intermediate goods to produce a unique consumption good;

2. intermediate good firms. When it decides entry, each intermediate producer borrows and invests a certain amount of resources to pay researchers who create a new product. Formally, this works like a set-up entry cost. Once a new product has been created (which occurs instantaneously) by the R&D department, the firm can costlessly patent the innovation and acquire a perpetual monopoly power over the production of the corresponding input. Note that this set-up cost makes the global technology non-convex at the firm level, thus a competitive equilibrium cannot exist in the intermediate industry. To manufacture the intermediate good, each intermediate firm has to pay one unit of output per unit of intermediate product manufactured. In other terms, the final good in this economy is both a consumption good and an input to the productive process of intermediate goods.[5]

[3] To avoid confusion with technical knowledge, denoted by $A_t$, I will in this section denote assets by $B_t$.

[4] Note that equilibrium requires that every firm's $i$ debt amounts to $\frac{w_{s_i}}{\delta A_{s_i}}$ where $s_i$ is the time when firm $i$ was born. Since $\frac{w_{s_i}}{\delta A_{s_i}} = \frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}}{\delta}$ for all $s_i$, then the total stock of debt of all existing firms is $B_t = \frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}}{\delta} A_t$ for all $t \in (0, \infty)$. Hence $B_0$ is immediately determined once an initial condition $A_0$ is specified.

[5] In Romer (1990) the variable input is physical capital. This makes the original model

### 2.2.1 Final good sector.

The technology to produce final goods is represented by the following production function:

$$Y_t = DL_{y,t}^{1-\alpha} \int_0^{A_t} x_{j,t}^{\alpha} dj \tag{7}$$

where $x_j$ is the quantity of the intermediate good $j$, $A_t$ is the measure of intermediate goods available at $t$, $L_y$ is labor and $D$ is a technological parameter. This specification stretches back to Spence (1976) and Dixit and Stiglitz (1977) where it captured the idea that consumers have a preference for variety of goods.

Technical progress takes the form of the increase over time of the range (formally, the measure) of inputs which final good firms can use. The additive separability of the function implies that new inputs are *different* (imperfect substitutes) from the existing ones, although they are neither intrinsically better nor worse. The most remarkable feature is that the marginal product of each input is decreasing but there are constant returns to the number (measure) of intermediate goods, $A_t$, which can be regarded as the level of technical knowledge. In order to appreciate better this fact, assume that the firm employs the same amount of all available intermediate inputs (this will be true in equilibrium). In this case the production function can be written as:

$$Y = L_y^{1-\alpha} A^{1-\alpha} (Ax)^{\alpha} \tag{8}$$

which shows that for a given value of $A$, production exhibits decreasing returns to the total amount of intermediate goods, $Ax$ (the reproducible factor in this economy), while for a given value of $x$ there are constant returns to $Ax$.

a two-state variable models, as there are two assets, capital and knowledge. Following our approach we do not need to be concerned with transitional dynamics (in the original paper Romer restrict his analysis to the balanced growth trajectory). The main results are identical under the two alternative specifications

Let $w_y$ denote the salary perceived by workers in the final sector, and $p_j$ be the price of the variety $j$ of intermediate input. The price of the final product is normalized to 1. The representative firm in the competitive final sector maximizes profits, given by:

$$\pi_Y = L_{y,t}^{1-\alpha} \int_0^{A_t} {x_{j,t}}^{\alpha} dj - w_{y,t} L_{y,t} - \int_0^{A_t} p_{j,t} x_{j,t} dj \tag{9}$$

The first order conditions provide the following factor demand functions:

$$p_{j,t} = \alpha L_{y,t}^{1-\alpha} {x_{j,t}}^{\alpha-1} \qquad \forall j \in [0, A_t] \tag{10}$$

and

$$w_{y,t} = (1-\alpha) L_{y,t}^{-\alpha} \int_0^{A_t} {x_{j,t}}^{\alpha} \tag{11}$$

#### 2.2.2 Intermediate sector.

We assume that an entrant firm, in order to invent (and patent) at time $t$ new ideas which enable it to produce a measure $dj$ of new intermediate products, has to employ $\frac{1}{\delta A_t} dj$ labor units. Note that we are assuming that the number of researchers which is needed to invent a new idea **decreases as the mass of intermediate goods invented grows**. However, the innovating firms neglect the effect of their entry decisions on the technology of future innovators. In other words, there is a **spillover** from present to future research activity. This can be rationalized by the idea that today's researchers have free access to the stock of previous applications for patents and can study the features of existing inputs to receive inspiration for future innovations. The process of innovation is therefore characterized by *imperfect excludability.* On the one hand the benefits of innovations are excludable thanks to the patent system. On the other hand the innovator cannot appropriate the benefits which spillover to future R&D activity. Knowledge is not a pure public good, but is not a perfect private good either.

More formally, we can write:

$$\dot{A}_t = \delta A_t L_{x,t} \tag{12}$$

where $\dot{A}_t$ is the flow of innovations at $t$,and $L_{x,t}$ denotes the hours of work employed in R&D at $t$. The growth rate of the varieties of intermediate goods is a linear function of the number of workers employed in R&D in the economy.

Let us calculate the profits of active intermediate firms. Recall that the marginal cost of producing any intermediate good is constant and equal to one unit of final product.

$$\pi_{j,t} = p_{j,t}x_{j,t} - x_{j,t} \tag{13}$$

which is maximized subject to the demand function (10). The solutions for quantity and price are, respectively:

$$x_{j,t} = x_t = \alpha^{\frac{2}{1-\alpha}} L_{y,t} \tag{14}$$

and

$$p_{j,t} = p = \frac{1}{\alpha} \tag{15}$$

Hence, the profit is:

$$\pi_{j,t} = \pi_t = (p-1)x_t = \frac{1-\alpha}{\alpha}\alpha^{\frac{2}{1-\alpha}} L_{y,t} \tag{16}$$

### 2.2.3 Equilibrium factor prices.

We will characterize equilibrium factor prices assuming steady-state. By steady-state we mean a state in which consumption and production grow at the same constant rate, which we will denote by $\gamma$, and the two sectors employ constant proportions of the workforce. In this economy, no path other than the steady-state is an equilibrium (I will not prove this fact). In other words, this model has no transitional dynamics, and exhibits dynamics analogous to the $AK$ model.

Steady-state immediately implies that in (14), (16) we must have $x_t = x$ and $\pi_t = \pi$, namely the production and profits of intermediate firms are time-invariant. The aggregate final production is then:

$$Y_t = L_y^{1-\alpha} A_t {x_t}^{\alpha} = \alpha^{\frac{2\alpha}{1-\alpha}} L_y A_t \tag{17}$$

where the second equality follows from (14). By log-differentiating (17) we obtain that $\gamma = \frac{\dot{Y}_t}{Y_t} = \frac{\dot{A}_t}{A_t}$ and, using (12):

$$\gamma = \delta L_{x,t} \tag{18}$$

We turn now to determine the assignment of the workforce to the two sectors of the economy. Agents are indifferent between working in either sector. Therefore, for employment to be strictly positive in both sectors, the salary paid by final and intermediate firms must be identical. Formally:

$$w_{x,t} = w_{y,t} = w_t = (1-\alpha) L_y^{-\alpha} A_t x^{\alpha} = (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} A_t \tag{19}$$

In the intermediate sector there is free entry. This implies that in each period new firms will enter until the present discounted value of future profits exceeds the cost of innovations, represented by wages paid to researchers. Formally, the free-entry condition requires that for a positive rate of innovation:

$$\pi \int_t^{\infty} e^{-\left[\int_t^{\tau} r_s ds\right]} d\tau = \frac{w_t}{\delta A_t} \tag{20}$$

where $r$ is the interest rate. Recall here that new firms need to borrow in order to pay their researchers, and pay the service of the debt at the market interest rate. In a steady-state, the interest rate must be constant at steady-state (otherwise consumption growth rate cannot be constant along the equilibrium path), hence (20) simplifies to:

$$\frac{\pi}{r} = \frac{w_t}{\delta A_t} \tag{21}$$

Substituting salaries and profits by their expressions using (19) and (16), this free-entry condition becomes:

$$\frac{\left(\frac{1-\alpha}{\alpha}\right) \alpha^{\frac{2}{1-\alpha}} L_y}{r} = \frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}}{\delta} \tag{22}$$

The right hand-side expression, i.e. the cost of innovation, is independent of $A_t$ (in fact it is time-independent). The reason is clear: although the salary of researchers grows linearly with $A_t$, (as labor productivity in the final sector grows) this is perfectly offset by the increasing productivity of researchers. This clarifies the fundamental role of the knowledge spillover. Without externalities, the cost of innovation would grow over time, and technical progress and growth would come to a halt, like in the neoclassical model.[6]

Simplifying, we can now rewrite (22) as:

$$r = \alpha\delta L_y = \alpha\delta(L - L_x) \tag{23}$$

This is the equilibrium relationship between interest rate and employment in R&D which is derived from the profit maximization of all firms in this economy. As one expects, the higher the interest rate which firms have to pay to consumers, the lower the number of innovating firms which employ researchers, and, therefore, the lower the share of the workforce which is employed in innovative activity.

## 2.3 General Equilibrium.

We start by noting that every intermediate firm issues upon entry a stock of debt equal to $\frac{w}{\delta A} = \frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}}{\delta}$, and keeps it constant throughout its perpetual lifetime. Since there is, at time $t$, a measure $A_t$ of active intermediate firms, then the total stock of debt of all existing firms is $B_t = \frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}}{\delta}A_t$ for all $t \in (0, \infty)$, and this will be, in equilibrium, the asset holding of the households in the economy.

Next, consider the following two equations:

1. we rewrite the Euler equation, (6), as:

$$r = \rho + \sigma\gamma$$

[6] Other versions of the model (see the book of Barro and Sala-i-Martin) assume that the cost of innovation is a constant amount of final output rather than labor. In this case no spillover is needed.

which represents the relationship between interest rate and growth from consumers' utility maximization. The willingness of consumers to save and finance R&D is an increasing function of the market interest rate.

2. Using (18), we can rewrite (23) as:

$$r = \alpha\delta L - \alpha\gamma \tag{24}$$

which was interpreted earlier.

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The plot shows that for an interior solution to exist, it is necessary and sufficient that $\alpha\delta L > \rho$. When this condition fails to be satisfied, all workers are employed in the production of consumption goods. In this case the number of inputs remains constant and production is subject to a regime of decreasing returns with zero growth. Romer (p. S96) writes that "... *civilization, and hence growth, could not begin until human capital could be spread from the production of goods for immediate consumption*".

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We can find analytical expressions for growth and employment in R&D:

$$\begin{aligned} \gamma^e &= \frac{\delta\alpha L - \rho}{\alpha + \sigma} \\ L_x^e &= \gamma^e/\delta \end{aligned} \tag{25}$$

Since the model has no transitional dynamics (like the simple $AK$ model) a closed form solution can be found for the entire path of any variable of the model, once an initial condition $A_0$ is specified. We have seen that $Y_0^e$ and $x^e$ are linear functions of $A_0$, while $C_0 = Y_0^e - A_0 x^e$. Consumption, therefore grows linearly with $A_t$, like in the $AK$ model it grows linearly with the stock of capital.

We now turn to the efficiency properties of the solution. The decentralized equilibrium is inefficient for two reasons:

1. intermediate firms exert monopoly power, and charge a price which is above the marginal cost of production for each unit sold;

2. the accumulation of ideas produces externalities which are not internalized in the laissez-faire economy. An innovating firm compares the cost of finding new ideas, $\left(\frac{w_t}{\delta A_t}\right)$, with the present discounted value of its profits $\left(\frac{\pi}{r}\right)$,and ignores the effect of its decisions on the future productivity of innovation.

The details of the welfare analysis and the discussion of policies which can restore efficiency are discussed as an exercise.

## 2.4 Economic Integration and Endogenous Growth

The model just seen has a dependence on scale that is crucial to analyze the effects of trade and economic integration. Take two identical countries with identical labor endowment, $L = L^*$. Both countries would grow at the same rate $\gamma = \frac{\delta\alpha L-\rho}{\alpha+\sigma}$. But if they join, the growth rate of the integrated country would be $\gamma^I = \frac{\delta\alpha(L+L^*)-\rho}{\alpha+\sigma} = \frac{2\alpha\delta L-\rho}{\alpha+\sigma}$.

An interesting question is whether trade is a substitute of economic integration. To answer this question, we will consider two isolated, completely identical economies that are growing at the same growth rate. We will study two cases:

1. Trade in goods $\rightarrow$ knowledge spillovers do not cross borders;

2. Trade in both goods and ideas $\rightarrow$ knowledge spillovers do cross borders;

We assume for simplicity that before trade the two countries are producing two disjoint intervals of intermediate goods (the more realistic case in which some intermediate goods are produced by both countries would only introduce some transitional dynamics without altering the main results). Opening **trade in goods** has no permanent effect on the growth rates of the two economies in this model. To see why, we start with the observation that the equilibrium growth rate of output would remain unchanged if each country keeps devoting $L_x^e$ workers to the research activity (recall that $\left(\gamma = \frac{\dot{A}}{A} = \delta L_x\right)$ since there are no cross-border spillovers). It is then sufficient to establish that the split of the workforce between research and manufacturing remains is not affected by trade. The intuitive reason is that trade increases the productivity of workers in production and the profitability of research by the same factor. Hence the number of workers employed in research activity remains unchanged. After trade is opened, the number of inputs used in each countries will be twice the number that has been produced and designed domestically. This will determine *level* effects on output and consumption, but no growth effect.

Let us examine the point more formally:

1. On the one hand the increase of $A$ doubles the marginal product of capital in the manufacturing sector, increasing it from

$$\frac{\partial Y}{\partial L_y} = (1-\alpha) L_y^{-\alpha} x^{\alpha} A$$

to:

$$\frac{\partial Y}{\partial H} = (1-\alpha) L_y^{-\alpha} x^{\alpha} (A + A^*)$$

where $A^*$ is the measure of the set of intermediate goods produced abroad (hence, $A + A^* = 2A$).

2. As far as the intermediate sector is concerned, opening trades implies that the market for any new design is twice as large as it was in the

absence of trade. This doubles the profit stream of intermediate firms. After trade each firm maximizes

$$\pi_{j,t} = p_{j,t}x_{j,t} + p^*_{j,t}x^*_{j,t} - x_{j,t} - x^*_{j,t}$$

subject to two independent demand equations like (10). The resulting sale price of the variety $j$ will be the same in both countries, and equal to the autarchy price $p_j = \frac{1}{\alpha}$. Moreover, the firm will sell in each market the quantity given by (14), hence the profit accruing to the firm will be

$$\pi_{j,t} = \pi_t = (p-1)(x_t + x^*_t) = \frac{1-\alpha}{\alpha}\alpha^{\frac{2}{1-\alpha}}(L_{y,t} + L^*_{y,t}).$$

Since by assumption $A_t = A^*_t$ and, by symmetry, $L_{y,t} = L^*_{y,t}$ in equilibrium, the equilibrium condition $\frac{\pi}{r} = \frac{w_t}{\delta A_t}$ guaranteed by free entry becomes:

$$\frac{\frac{1-\alpha}{\alpha}\alpha^{\frac{2}{1-\alpha}}2L_{y,t}}{r} = \frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}2A_t}{\delta A_t} \tag{26}$$

which is, after simplifying, identical to (21). Thus, the split of the workforce between production and research does not change and there are no permanent growth effects. However, trade increases output, consumption and welfare in both countries, since final producers can now use a larger set of intermediate goods.

Alternatively, suppose that **flows of ideas** between the two countries are permitted. Now research in each country depends on the total worldwide stock of ideas contained in the union between $A$ and $A^*$. We maintain the simplifying assumption that before trade the set of ideas used in the two countries was disjoint. Then, when trade is allowed:

$$\dot{A} = \dot{A}^* = \delta L_x(A + A^*) = 2\delta L_x A$$

Even if the change did not affect the allocation of the workforce between manufacturing and research, the rate of growth of $A$ would double. But there is an additional effect coming from the fact that the increase in the set of ideas available for use in research increases labor productivity in research. The effect of freeing flows of ideas is equivalent to doubling the parameter $\delta$. In the geometrical analysis, trade in goods plus flow of ideas shows up as an upward shift of the $r_{tech}$. This shift is equivalent to that which would be observed in the case of perfect economic integration (note that doubling $\delta$ is equivalent to doubling $L$). With trade in goods plus flows of ideas we have both level and growth effects, exactly like in the case of economic integration.

Although we could also analyze the case in which free flows of ideas are allowed, but trade in goods is not, this case seems of little relevance. The reason is twofold. First, even though the two countries are perfectly specialized at time zero in the production of disjoint subsets of intermediate goods, there is no reason why this perfect specialization should persist for ever. Second, and more important, once flows of information are allowed, there would be a positive incentive for researchers in one country to copy designs from the other, and little offsetting incentive for the countries to enforce property rights. In the extreme case in which identical knowledge is created in each country, opening flows of information has no effect at all on production.