1 Real business cycle

1.1 The program:

- Use a modified version of the neoclassical growth (Ramsey) model to explain a number of regularities associated with about business cycle fluctuations (high frequency phenomena, 3 to 5 years).
- Modifications:
    1. Introduce sources of disturbances
    2. Allow for variations in employment (labor supply)
- Understand aggregate fluctuations using a competitive Walrasian model. Fluctuations do not necessarily call for inefficient resource allocation.

1.2 The facts:

- Detrending data using a Hodrick-Prescott filter.
- Characterize an observed time series, y_t, as the sum of a cyclical component, y_t^c, and a growth component, y_t^g

$$y_t = y_t^c + y_t^g$$

Given a parameter λ, y_t^g is chosen to minimize the loss function:

$$\sum_{t=0}^{\infty} (y_t^c)^2 + \lambda \cdot \sum_{t=0}^{\infty} \left(\left(y_{t+1}^g - y_t^g \right) - \left(y_t^g - y_{t-1}^g \right) \right)^2 .$$

The higher λ, the smoother the growth component is forced to be. If $\lambda \to \infty$, the growth component is just a linear trend. If $\lambda = 0$, the growth component is the whole series, and there is no cyclical component.

- Standard approach with quarterly data is to choose $\lambda = 1,600$. If the original series were stationary, this choice would take away fluctuations at frequencies lower than about 8 years. We want to suppress low frequency fluctuations.

- Facts for the US economy:

  1. Total hours worked fluctuate about as much as output, with a very high contemporary correlation
  2. Employment fluctuates almost as much as output and total hours of work, while average weekly hours fluctuate considerably less.
  3. Consumption of non-durable and services, fluctuate significantly less than output.
  4. Investments, fluctuate much more than output and consumption. Changes in inventories exhibits the largest fluctuations.
  5. Real wages are procyclically (in contradiction with the Keynesian theory with nominal rigidities) according to establishment survey data. According to national income accounts, however, wages are uncorrelated with output. Wage fluctuations are, in any case, less pronounced than output fluctuations.
  6. Productivity is slightly procyclical, but fluctuates less than output.

1.3 The choice of hours worked and leisure.

- Elementary choice in one and two-period model.

  1. One-period model:

$$\max_{\{c,h\}} \frac{c^{1-\theta} - 1}{1 - \theta} + \psi \frac{(1-h)^{1-\theta} - 1}{1 - \theta}$$

subject to $c = wh$ ($1 - h$ denotes leisure). The solution is:

$$\frac{h}{1-h} = \psi^{-\frac{1}{\theta}} \cdot w^{\frac{1-\theta}{\theta}}.$$

Thus, when $\theta < 1$ ($\theta > 1$), the hours worked increase (decrease) with the real wage. When $\theta = 1$ (logarithmic utility), income and substitution effect cancel and the choice of hours is independent of the real wage.

2. Two-period model:

$$\max_{\{c_1, c_2, h_1 h_2\}} \left[\frac{c_1^{1-\theta} - 1}{1-\theta} + \psi \frac{(1-h_1)^{1-\theta} - 1}{1-\theta} \right] + \\ +\beta \left[\frac{c_2^{1-\theta} - 1}{1-\theta} + \psi \frac{(1-h_2)^{1-\theta} - 1}{1-\theta} \right]$$

subject to:

$$c_1 + \frac{1}{1+r} c_2 = w_1 h_1 + \frac{1}{1+r} w_2 h_2$$

Calculating FOCs and rearranging yields the solutions:

$$\frac{c_2}{c_1} = \left[\beta (1+r)\right]^{1/\theta} \quad (1)$$

$$\frac{1-h_2}{1-h_1} = \left[\beta \cdot (1+r) \cdot \frac{w_1}{w_2}\right]^{1/\theta} \quad (2)$$

The latter shows that:

(a) the relative labor supply in the two periods respond to the relative wage. Assume, to fix ideas, that $(1 + r) = 1/\beta$. Then, if w_1 is larger than w_2 (agents expect that future productivity will be lower than current productivity), agents supply more labor today than in the next period. And viceversa.

(b) the relative labor supply in the two periods respond to the interest rate. A higher interest rate induces agents to increase their labor supply today as the return to savings is higher.

(c) the sensitivity of the responses decreases with θ (i.e., increases with the intertemporal elasticity of substitution, defined as $1/\theta$). In the logarithmic case ($\theta = 1$), there is intertemporal substitution of labor supply, and the elasticity is unity.

1.4 The baseline RBC model.

- The economy consists of a large number (measure one) of identical, price taking firms and a large number of identical, price taking infinitely lived households. I will restrict attention to a simple and tractable parameterization, although many variations can be found in the literature.

- Technology:

$$Y_t = e^{z_t} \cdot K_t^{\alpha} \left(A_t N_t H_t\right)^{1-\alpha} \tag{3}$$

where

$$z_t = \rho z_{t-1} + \varepsilon_t \tag{4}$$

and ε_t is iid normally distributed disturbance,

$$A_t = (1+g)^t ; \quad N_t = (1+n)^t .$$

- Preferences. We restrict analysis to log-preferences:[1]

$$U_0^e = E_0 \left[\sum_{t=0}^{\infty} (1+n)^t \beta^t \cdot \left[\log\left(c_t\right) + \psi \cdot \log\left(1-h_t\right)\right] \right]$$

U_0^e is maximized by choosing sequences $\{c_t\}_{t=\{0,1,..,\infty\}}$, $\{h_t\}_{t=\{0,1,..,\infty\}}$, $\{a_t\}_{t=\{0,1,..,\infty\}}$ subject to:

$$(1+n) \cdot a_{t+1} = w_t h_t + (1+r_t) a_t - c_t$$

and a No-Ponzi game condition.

[1] Lower case denotes per capita variables.

- In a competitive equilibrium, factor prices are equal to value of the marginal products, thus:

$$a_t = k_t$$

$$w_t h_t + (r_t + \delta) k_t = y_t = e^{zt} \cdot k_t^{\alpha} \cdot \left((1+g)^t \cdot h_t\right)^{1-\alpha}$$

- We can then exploit the standard equivalence results and reformulate the problem in terms of an economy of household-managers (or as the social planner solution).

- The intertemporal b.c. of a household-manager (or the resource constraint of the planner) is given by:[2]

$$(1+n) \cdot k_{t+1} = (1-\delta) k_t + e^{z_t} \cdot k_t^{\alpha} \cdot \left((1+g)^t \cdot h_t\right)^{1-\alpha} - c_t \qquad (5)$$

- In order to simplify expressions, we set $g = n = 0$ in the analysis which follows (we will, however, allow for positive g and n in the quantitative analysis).

- This problem admits a recursive formulation, such that:[3]

$$\begin{aligned} V(z_t, k_t) &= \max_{\{c_t, h_t, k_{t+1}\}} \left\{ [\log(c_t) + \psi \cdot \log(1-h_t)] + \beta \cdot E_t [V(z_{t+1}, k_{t+1})] \right\} \\ &= \max_{\{c_t, h_t, k_{t+1}\}} \{ [\log(c_t) + \psi \cdot \log(1-h_t)] + \\ &\quad + \beta \cdot E_t \left[V\left(z_{t+1}, (1-\delta) k_t + e^{z_t} \cdot k_t^{\alpha} h_t^{1-\alpha} - c_t\right) \right] \} \end{aligned}$$

[2] If we express the i.b.c. in terms of variables per effective units of labor, we have:

$$(1+n) \cdot (1+\gamma) \cdot \hat{k}_{t+1} = (1-\delta) \hat{k}_t + e^{zt} \cdot \hat{k}_t^{\alpha} \cdot \hat{h}_t^{1-\alpha} - c_t$$

[3] Note that in order to characterize equilibrium in a genuinely decentralized economy (as opposed to the household-manager shortcut adopted here) one needs some attention about the notation. In that case, one has to distinguish between individual and aggregate variables, when writing the Bellman equations. See Cooley and Prescott, p. 14 for details.

which yields the following First Order conditions:

$$\begin{aligned} \frac{1}{c_t} &= \beta \cdot E_t \left[\frac{\partial V}{\partial k_{t+1}} \left(z_{t+1}, k_{t+1} \right) \right] && (6) \\ \frac{\psi}{1 - h_t} &= \beta \cdot E_t \left[\frac{\partial V}{\partial k_{t+1}} \left(z_{t+1}, k_{t+1} \right) \right] \cdot (1 - \alpha) \frac{y_t}{h_t} && (7) \end{aligned}$$

and the following "Benveniste-Scheinkman" condition:[4]

$$\begin{aligned} \frac{\partial V}{\partial k_t} \left(z_t, k_t \right) &= \beta \cdot E_t \frac{\partial V}{\partial k_{t+1}} \left(z_{t+1}, k_{t+1} \right) \cdot \left[\alpha \cdot \frac{y_t}{k_t} + (1 - \delta) \right] && (8) \\ &\equiv \beta \cdot E_t \frac{\partial V}{\partial k_{t+1}} \left(z_{t+1}, k_{t+1} \right) \cdot R_t \end{aligned}$$

where we have defined $R_t \equiv \left(\alpha \cdot \frac{y_t}{k_t} + 1 - \delta \right)$.

- Next, by plugging (6) into (8), we get:

$$\frac{\partial V}{\partial K_t} \left(z_t, k_t \right) = \frac{1}{c_t} \cdot R_t$$

Hence, taking one period ahead and expectations:

$$E_t \left[\frac{\partial V}{\partial k_{t+1}} \left(z_t, k_{t+1} \right) \right] = E_t \left[\frac{1}{c_{t+1}} \cdot R_{t+1} \right] \tag{9}$$

- Similarly, by plugging (7) into (8), we get:

$$\frac{\partial V}{\partial K_t} \left(z_t, k_t \right) = (1 - \alpha)^{-1} \cdot \frac{\psi}{1 - h_t} \cdot \frac{h_t}{y_t} \cdot R_t$$

Hence:

$$E_t \left[\frac{\partial V}{\partial k_{t+1}} \left(z_t, k_{t+1} \right) \right] = E_t \left[(1 - \alpha)^{-1} \cdot \frac{\psi}{1 - h_{t+1}} \cdot \frac{h_{t+1}}{y_{t+1}} \cdot R_{t+1} \right] \tag{10}$$

[4] This is obtained by differentiating $V(z_t, k_t)$ with respect to k_t, and using the envelope theorem implying that the partial derivatives with respect to c_t and h_t equal to zero.

- Finally, using (9) and (10):

$$1 = \beta \cdot E_t \left[\frac{c_t}{c_{t+1}} \cdot \left(\alpha \cdot \frac{y_{t+1}}{k_{t+1}} + 1 - \delta \right) \right] \tag{11}$$

$$\frac{h_t}{1-h_t} = \beta \cdot E_t \left[\frac{h_{t+1}}{1-h_{t+1}} \frac{y_t}{y_{t+1}} \cdot \left(\alpha \cdot \frac{y_{t+1}}{k_{t+1}} + 1 - \delta \right) \right] \tag{12}$$

  which, together with the technology (3)-(4) and the intertemporal budget constraints (5), defines a system of stochastic difference equations which characterize the equilibrium solution.

- Two observations:

  1. In a decentralized economy, the two conditions are equivalent to:

  $$\frac{1}{c_t} = \beta \cdot E_t \left[\frac{(1 + r_{t+1})}{c_{t+1}} \right] \tag{13}$$

  $$1 = \beta \cdot E_t \left[\frac{1-h_t}{1-h_{t+1}} \frac{w_t}{w_{t+1}} (1 + r_{t+1}) \right] \tag{14}$$

     - (13) is the familiar optimality condition in consumption theory (since, here, $\frac{u'(c_{t+1})}{u'(c_t)} = \frac{c_t}{c_{t+1}}$).
     - (14) is the multiperiod correspondent (with uncertainty) of (2), showing that agents increase (reduce) their current labor supply when the current state of productivity (wage) is high (low) relative to the expected level next period, and when the expected interest rate is high (low).

  2. By taking the ratio between (6) and (7), we obtain:

  $$\frac{1-h_t}{h_t} = \frac{\psi}{(1-\alpha)} \frac{c_t}{y_t} = \frac{\psi}{(1-\alpha)} \left(1 - \frac{s_t}{y_t} \right) \tag{15}$$

     which shows that if the current propensity to savings exceeds (falls short) its steady-state level, the labor supply will also be above (below) the steady-state.

- Unfortunately, it is in general impossible to find explicit analytical solutions, and there exists a vast literature which deals with numerical methods and algorithms of solution (see Cooley and Prescott for an introductory discussion, and K. Judd "*Numerical Methods in Economics*", MIT Press 1998, for a more detailed analysis) .

1.5 A simple case.

- There is, however, a particular case for which an analytical solution can be found. Assume $\delta = 1$, i.e., full depreciation. An interpretation is that

$$Y_t^{net} \equiv Y_t - \delta K_t = K_t^{\alpha} (AH_t)^{1-\alpha}$$

namely, the production function gives the output net of capital depreciation.

- We can characterize the equilibrium by a guess-and-verify method. We guess:

  1. $c_t = \pi_1 \cdot e^{z_t} \cdot k_t^{\alpha}$
  2. $k_{t+1} = \pi_2 \cdot e^{z_t} \cdot k_t^{\alpha}$
  3. $h_t = \bar{h}$.

- Substitute the guesses into (11), and obtain:

$$\begin{aligned} 1 &= \beta \cdot \left[\frac{c_t}{c_{t+1}} \cdot \alpha \cdot \frac{y_{t+1}}{k_{t+1}} \right] \\ &= \beta \cdot E_t \left[\frac{\pi_1 \cdot e^{z_t} \cdot k_t^{\alpha}}{\pi_1 \cdot e^{z_{t+1}} \cdot k_{t+1}^{\alpha}} \cdot \alpha \cdot \frac{e^{z_{t+1}} \cdot k_{t+1}^{\alpha} \bar{h}^{1-\alpha}}{\pi_2 \cdot e^{z_t} \cdot k_t^{\alpha}} \right] \\ &= \beta \cdot \alpha \cdot E_t \left[\frac{\bar{h}^{1-\alpha}}{\pi_2} \right] \end{aligned}$$

hence:

$$\pi_2 = \alpha\beta\bar{h}^{1-\alpha}$$

- Rewrite the intertemporal budget constraint:

$$k_{t+1} = e^{z_t} \cdot k_t^\alpha \bar{h}^{1-\alpha} - c_t$$

as:

$$\pi_2 \cdot e^{z_t} \cdot k_t^\alpha = e^{z_t} \cdot k_t^\alpha \bar{h}^{1-\alpha} - \pi_1 \cdot e^{z_t} \cdot k_t^\alpha$$

hence:

$$\pi_1 = \bar{h}^{1-\alpha} - \pi_2 = \bar{h}^{1-\alpha} (1 - \alpha\beta)$$

- Finally, use (15):

$$\frac{1-h}{h} = \frac{\psi}{(1-\alpha)} \frac{c_t}{y_t} = \frac{\psi (1-\alpha\beta)}{(1-\alpha)}$$

which shows that there exist constants $\bar{h}, \pi_1, \pi_2$ which verify the guess.

1.5.1 Some features of the equilibrium in the simple model.

- Features:

  1. The propensity to saving is constant along the business cycle, since:

  $$c_t = (1 - \alpha\beta) \cdot e^{z_t} \cdot k_t^\alpha \cdot \bar{h}^{1-\alpha} = (1 - \alpha\beta) \cdot y_t$$

  This implies that the model predicts that consumption and investment should exhibit fluctuations of the same magnitude, in contradiction with the observation that investments fluctuate much more than consumption.[5]

[5] Note that variability is measured here by normalized (percentage) standard deviations. Let x, y be two random variables. Let s_x,s_y denote the respective standard deviations. Thus, the normalized standard deviations are defined as $\sigma_x \equiv s_x/\bar{x}$ and $\sigma_y \equiv s_y/\bar{y}$. Clearly, if $x = Z \cdot y$ (where Z is a constant), then $\bar{x} = Z \cdot \bar{y}$, $s_x = Z \cdot s_y$ and $\sigma_x = \sigma_y$.

2. Employment is constant. Another bad feature (related to the former, see eq. (15)), as we saw that employment in the data fluctuate almost as much as output. The counterpart of this observation is that wages fluctuate too much in this model. Observe that:

$$w_t = (1 - \alpha) \frac{y_t}{h_t}.$$

But, since we found that h does not respond to technological shocks, then wages should fluctuate as much as output, in contradiction with the empirical observation that the former fluctuate much less than the latter.

3. Log-output follows a second-order autoregressive process (AR2). This has been argued to be a good prediction. If output is detrended linearly (but not if other methods are used), AR2 is a good representation of the log-GDP process. Recall that:

$$\begin{aligned} \log(k_{t+1}) &= \log(\pi_2) + \alpha \cdot \log(k_t) + z_t \\ z_t &= \rho \cdot z_{t-1} + \varepsilon_t \end{aligned}$$

Hence:

$$z_{t-1} = \log(k_t) - \log(\pi_2) - \alpha \cdot \log(k_{t-1})$$

Thus, substituting to z_{t-1} and z_t their expressions:

$$\log(k_{t+1}) = (1 - \rho) \cdot \log(\pi_2) + (\alpha + \rho) \cdot \log(k_t) - \alpha \cdot \rho \cdot \log(k_{t-1}) + \varepsilon_t$$

Next, observe that

$$\log(y_t) = (1 - \alpha) \log(\bar{h}) + \alpha \log(k_t) + z_t = \xi + \log(k_{t+1})$$

where ξ is a constant. Hence:

$$\log(y_t) = \zeta + (\alpha + \rho) \cdot \log(y_{t-1}) - \alpha \cdot \rho \cdot \log(y_{t-2}) + \varepsilon_t$$

where ζ is a constant. Or, expressing this in terms of deviations from the steady-state:

$$\tilde{y}_t = (\alpha + \rho) \cdot \tilde{y}_{t-1} - \alpha \cdot \rho \cdot \tilde{y}_{t-2} + \varepsilon_t$$

where $\tilde{y}$ denotes the difference between $\log(y)$ and the steady-state. This is an AR2 process with a positive coefficient on the first lag, and a negative coefficient on the second lag.

- Why do we get constant labor supply, despite households' willingness to substitute labor supply intertemporally? This is due to the offsetting impact of technology and capital movement. A positive technology shock, if we hold fixed K and r, induces agents to work more today, as it increases current wages relative to expected future wages. But it also increase savings (next period's capital), and this lowers the expected interest rate, which acts to reduce labor supply. More formally, $E_t\left[\frac{w_t}{w_{t+1}}(1 + r_{t+1})\right]$ remains constant after a positive shock, since the first term goes up and the second goes down in such a way that the product remains constant.

- With the AR2 representation output can have a "hump-shaped response to disturbances. Suppose that $\alpha = 1/3$ and $\rho = 0.9$. Consider an economy which was at its steady-state ($y = 0$) and is hit by a unitary shock ($\varepsilon_t = 1$). The impulse response function will look as follows

GRAPH

- Note that the hump-shape depends on the assumption of that technological shocks are very persistent ($\rho = 0.9$). If the process were iid ($\rho = 0$), for example, the output process would be AR1, and exhibit very little persistence. Almost 9/10 of the initial effect is gone after two periods. Thus, this very simple model does not have a mechanism which translates transitory technology disturbances into long lasting output movements.

1.6 The general case.

1.6.1 Calibration.

- Calibration: assign value to parameters in order for the model to match some long-run (growth) observations and micro (panel data) estimates.

- Cooley and Prescott assume the capital share (α) to be 0.4 (since it includes government capital). The long-run annual growth rate of GDP per capita and population are set to match the corresponding empirical observations (annual rates of 1.56% and 1.2%, respectively).

- We then need to determine β, δ and ψ. We use the long-run properties of the model. Let $z_t = 0$, and no uncertainty. Consider balanced growth path.

  1. Dividing both sides of the i.b.c., (5), by k:

  $$(1+n) \cdot (1+g) = (1-\delta) + \frac{y-c}{k} = (1-\delta) + \frac{i}{k},$$

     This is used to calibrate δ. Since the invt. to capital ratio is estimated to be about 0.076, then $\delta = 0.012$ (4.8% per year).

  2. From (11):

  $$\frac{c_{t+1}}{c_t} = (1+g) = \beta \cdot \left(\alpha \cdot \frac{y}{k} + 1 - \delta\right)$$

     This is used to calibrate β. Since the capital output ratio is estimated to be 3.32, then $\beta = 0.987$ (annual discount rate 5.3%).

3. From (15):

$$\frac{1-h}{h} = \frac{\psi}{(1-\alpha)}\left(1 - \frac{s}{y}\right)$$

This is used to calibrate ψ. Since the saving rate is estimated to be 0.25, and $h = 0.31$ (people work a third of their discretionary time to market activity), then $\psi = 1.78$.

- Finally, we need to calibrate the shocks. Recall that $z_t = \rho z_{t-1} + \varepsilon$, where ε is iid normal. We need to assign a value to ρ (persistence of the shocks) and one to σ_ε (volatility of the shocks). Along a balanced growth:

$$z_t - z_{t-1} = \ln\left(Y_t - Y_{t-1}\right) - \alpha\left(\ln K_t - \ln K_{t-1}\right) - (1-\alpha)\left(\ln L_t - \ln L_{t-1}\right)$$

which gives us a series for the z_t and their difference. The resulting process is close to being a random walk. Cooley and Prescott, propose, then, a value of $\rho = 0.95$. The resulting standard deviation for the innovations to technology is 0.007.

1.6.2 Results.

- Simulate a large number of the realizations of the model economy, and calculate a number of statistics (variances, covariances, etc.). Then, compare them with the corresponding statistics calculated using the data.

- See table Cooley and Prescott, p. 34.

GRAPH

- 1. Output fluctuates less in the model than in the US economy (the volatility predicted by the model is 70-75% of that in the data). Technological shocks alone cannot account for the total variability of GDP (but, it is claimed, can account for a large share of it).
  2. Labor input fluctuates only about half as much as in the US economy (implying that wages fluctuate too much).
  3. Investments fluctuate much more than does output and consumption fluctuate much less than does output. This is consistent with the US data. However, relative to GDP, consumption fluctuates significantly more in the data than in the model.
  4. The correlation of all variables with output is very high, and higher than in the data.
  5. The model predicts a strong positive correlation between the labor input and labor productivity, whereas these are practically uncorrelated in the US data.

x	σ_x (m)	σ_x (d)	σ_x/σ_y (m)	σ_x/σ_y (d)	$\rho_{x,,y}$ (m)	$\rho_{x,,y}$ (d)
Output	1.351	1.72	1	1	1	1
Consumption	0.329	1.27	0.244	0.738	0.84	0.83
Investment	5.954	8.24	4.407	4.791	0.99	0.91
Hours	0.769	1.65	0.569	0.930	0.99	0.86

1.6.3 Impulse-response functions

- See Romer, p. 169-71.

GRAPH

- Comments:

  1. The technology shock is, by assumption, very persistent. Thus, wage adjusts very smoothly → little intertemporal substitution in labor supply due to wage dynamics.
  2. The movements in labor supply are, instead, mainly associated with changes in the interest rate. r jumps up, then falls, and eventually grows again a little. Why? Imagine that labor supply were inelastic. Then the increase in productivity would tend to raise the expected interest rate. But savings also react, and the change in the stock of capital, in principle, undo or even revert the effect on r (recall that this was happening in the simple model with full depreciation). Since depreciation occurs slowly, however, current savings are only a small fraction of the capital stock, and the increase of savings and capital is not sufficient to offset the positive effect of the productivity shock on the marginal product of capital. The reversion, in fact, eventually occurs also in this model, but it is only after 14 quarters that r falls below its long-run value. This happens for the combination of the dying effect of the productivity shock and the cumulated increases of capital stock over the previous 3 and a half years.

- Persistence in this model is due to:

  1. persistent technology shocks;
  2. sluggish dynamics of capital.

1.6.4 Problems and potential solutions.

- Main failure: too little employment fluctuations.

- Way outs:

  1. Reduce the persistence of technology shock. This would increase the intertemporal substitution in labor supply associated with wage changes. The problem is fluctuations become soon unrealistically sharp and short (if $\rho = 0.5$ only 30% of the initial impulse survives after two periods).
  2. Abandon the logarithmic specification and allow the intertemporal elasticity of substitution of labor supply to take on values larger than one. E.g.:

  $$u(c_t, h_t) = \log(c_t) + \psi \cdot \frac{(1 - h_t)^{1-\theta} - 1}{1 - \theta}$$

  In this case, with $\theta > 1$, the model would predict larger fluctuations to all variables (good), with a particularly strong effect on the fluctuations in the labor input (good). Problem: microeconomic estimates find vary low values of the elasticity of substitution (Altonji (1984), MaCurdy (1981)). Namely, the micro-evidence suggests that $\theta > 1$, whereas we would need $\theta < 1$ to improve. Some recent papers (Greenwood, Hercowitz and Huffman, 1991) explore with some success different preference specification, which allow the instantaneous elasticity of substitution to differ from the intertemporal elasticity of substitution.

3. Recognize explicitly the fact that most of the change in the labor input come from changes along the *extensive margin* (movements into and out of employment) rather than along the *intensive margin* (hours per employee). As we will see, this change increases significantly the responsiveness of labor input to shocks.

1.7 The indivisible-labor RBC model.

- See: Gary Hansen: "Indivisible Labor and the Business Cycle" *Journal of Monetary Economics* (1985), 309-327.
- Assume that h can only take on two values: $h = h_0$ (employed) or $h = 0$ (not employed).
- No adjustment can occur along the intensive margin (hours).
- Wage income is perfectly insured: every agent receives wage w_t irrespective of whether or not they work.
- Agents can buy lotteries:

  1. with probability π they work h_0 and receive a wage w_t
  2. with probability $1 - \pi$ they are unemployed and, yet, receive a wage w_t

- Although all agents perceive the same ex-ante utility, the unemployed are better off than the employed ex-post.
- Preferences:

$$
\begin{aligned}
u^e(c_t, h_t) &= E\left[\log(c_t) + \psi \cdot \log(1 - h_t)\right] = \\
&= \log(c_t) + \psi \cdot (\pi \cdot \log(1 - h_0) + (1 - \pi) \cdot \log 1) = \\
&= \log(c_t) + \psi \cdot \log(1 - h_0) \cdot \pi_t
\end{aligned}
$$

  where agents choose sequences of c_t and π_t rather than sequences of c_t and h_t.

- In equilibrium, if we denote average labor by h_t, we have $h_t = \pi_t \cdot h_0$, and $\pi_t = h_t / h_0$.

- Thus, agents' preferences have the following features:

$$u^e(c_t, h_t) = \log(c_t) + \left[\frac{\psi}{h_0} \cdot \log(1 - h_0) \right] \cdot h_t \equiv \log(c_t) - Z \cdot h_t$$

  where $Z > 0$.

- Now, the instantaneous utility is linear in hours worked, namely, the intertemporal elasticity of substitution of labor supply is infinite. However, this comes from the presence of lotteries to convexify discrete employment choice, and not from assuming, in contradiction with the micro evidence, that agents' preference exhibit a large intertemporal elasticity of substitution.

- With a sample 1955-1984, we have the following standard deviations in percent:

	US data	RBC	H-R
Output	1.76	1.35	1.76
Consumption	1.29	0.42	0.51
Investment	8.60	4.24	5.71
Hours	1.66	0.70	1.35
Productivity	1.18	0.68	0.87