I.B. Fiscal Policy, Life Cycles and OLG Models

1. Ricardian Equivalence with zero birth rate or altruism

Above, in equation (1.33), we assumed that the government financed its spendings by taxing in an amount to generate zero deficits in each period. Now let us discuss whether relaxing this assumption can change our results. In particular, does the timing of taxes matter for consumption and the current account when we hold the stream of government consumption fixed? As before, we assume that the government goods do not interact with other goods in utility or production and disregard exogenous growth in population or technology. Let us do this in continuous time for a change.

The government budget constraint is

$$\begin{aligned}
&\dot{B}_t^G = G_t + r_t B_t^G - \tau_t \\
&e^{-\int_0^t r_s ds}\left(\dot{B}_t^G - r_t B_t^G\right) = \frac{d}{dt} e^{-\int_0^t r_s ds} b_t = e^{-\int_0^t r_s ds}(G_t - \tau_t) \\
&de^{-R(t)} B_t^G = e^{-R(t)}(G_t - \tau_t)dt \\
&e^{-R(t)} B_t^G - e^{-R(0)} B_0 = \int_0^t e^{-R(s)}(G_t - \tau_t)ds
\end{aligned} \tag{1.52}$$

where B^G denotes the government deficit. Now require that the present value of the debt converges to zero (No Ponzi):

$$\begin{aligned}
&\lim_{t\to\infty} e^{-R(t)} B_t^G = 0 \\
&\Rightarrow B_0^G + \int_0^\infty e^{-R(s)}(G_s - \tau_s)ds = 0 \\
&B_0^G + \int_0^\infty e^{-R(s)} G_s ds = \int_0^\infty e^{-R(s)} \tau_s ds
\end{aligned} \tag{1.53}$$

So the present value of all future taxes equals the present value of future government spendings plus current debt stock.

Now look at the consumers. Recall that the slope of the consumption path was given from the Euler equation.

$$\frac{\dot{C}_t}{C_t} = \frac{u'}{-u''C_t}(r_t - \rho) \tag{1.54}$$

As long as the tax does not affect the return on savings, this condition is unaffected by the sequence of taxes.

Now look at the budget constraint. The continuous time version of (1.33) with taxes instead of government consumption is

$$\dot{B}_t = Y_t - C_t - I_t - \tau_t + rB_t \tag{1.55}$$

where *B* is the private sectors holdings of foreign asset. Integrate

$$\begin{aligned} &de^{-R(t)}B_t = e^{-R(t)}(Y_t - C_t - I_t - \tau_t)dt \\ &\lim_{T\to\infty} e^{-R(T)}B_T = 0 = B_0 + \int_0^\infty e^{-R(t)}(Y_t - C_t - I_t - \tau_t)dt \\ &\int_0^\infty e^{-R(t)}C_t dt = B_0 + \int_0^\infty e^{-R(t)}(Y_t - C_t - I_t)dt - \int_0^\infty e^{-R(t)}\tau_t dt \end{aligned} \tag{1.56}$$

Now we can substitute into the last line of (1.53)

$$\int_0^\infty e^{-R(t)}C_t dt = B_0 + \int_0^\infty e^{-R(t)}(Y_t - I_t)dt - B_0^G - \int_0^\infty e^{-R(s)}G_s ds \tag{1.57}$$

so we see that what is important for the households is only the present value of government spending plus initial foreign debt of the government. An implication of this is the following: assume the government change its fiscal policy so for given consumption is borrows more and taxes less. The private sector will then get a stock of assets in the form of government bonds. This stock is, however, of *zero* value in the aggregate. Thus the title of Barro's original paper. "Are government bonds net wealth?". Similarly, if the initial government debt was held by domestic agents, we would add B_0^G to the initial wealth of the consumer. This would cancel with the $-B_0^G$ in (1.57).

What about the current account if the government decides to, say, increase taxes today by $\Delta\tau$? First consider the case when the government borrows or invests abroad. Then from (1.52) we have that

$$\Delta B_t^G = -\Delta\tau \tag{1.58}$$

Similarly, from (1.55)

$$\Delta \dot{B}_t = -\Delta \tau \tag{1.59}$$

The current account in the continuous case is the sum of private and public rates of accumulation of foreign assets

$$CA_t = \dot{B}_t - \dot{B}_t^G \tag{1.60}$$

and from (1.58) and (1.59) we see that the current account is unchanged. If the government borrows and lends to its domestic citizens, then the interpretation of (1.55) changes so B is sum of foreign assets and government bonds (B^G). The current account is still given by the accumulation of foreign assets, which now becomes the accumulation of total assets minus the accumulation of government bonds. Thus, the current account is still given by (1.60), which is unchanged by the change in taxes.

Some critical assumptions for the Ricardian Equivalence result

1. Perfect capital markets (so the individual can save or borrow to offset any changes in the time path of taxes and deficits. Same interest for government and individuals.

2. Taxes are lump sum.

3. Infinite horizons and no new-coming generations, not necessarily infinite lives.

. Ricardian Equivalence with Finite lives and altruism

Consider the simplest example of this. Each generation lives for one period, cares about their kids, who have the same per period utility function.

$$\begin{aligned} V(A_t) &= \max_{c_t} U(c_t) + \beta V\left(A_{t+1}\right) \\ s.t.\ A_{t+1} &= (1+r)(A_t - c_t) \end{aligned} \tag{1.61}$$

Now we can substitute from future generations problems

$$
\begin{aligned}
&V(A_t) = \max_{c_t} U(c_t) + \beta\left[\max_{c_{t+1}} U\left(c_{t+1}\right) + \beta V\left(A_{t+2}\right)\right] \\
&s.t. \quad A_{t+1} = (1+r)(w_t + A_t - c_t) \\
&\qquad\;\; A_{t+2} = (1+r)(w_{t+1} + A_{t+1} - c_{t+1}) \\
&\Rightarrow \\
&V(A_t) = \max_{\{c_t\}} \sum_{s=0}^{\infty} \beta^s U(c_{t+s}) + \lim_{T\to\infty} \beta^T V\left(A_{t+T}\right) \\
&s.t. \sum_{s=0}^{\infty} (1+r)^{-s}\left(c_{t+s} - w_{t+1}\right) = A_t + \lim_{T\to\infty} (1+r)^{-T} A_{t+T}
\end{aligned}
\tag{1.62}
$$

As you see, this is mathematically equivalent to dynamic optimization problem with infinite horizons. So the solution to this problem must be unaffected by the financing path of the governments consumption. Note that we require perfect capital markets. In this context it in particular requires that A_t may be negative or that households choose positive values (bequests) voluntarily.

2. Fiscal Policy in a Small Open Economy OLG Model

Consider a very simple version of the Samuelson-Diamond-Blanchard OLG model.

. Households

Live for two periods and solve

$$\begin{aligned} &\max_{C_{1,t},C_{2,t+1}} U\left(C_{Y,t}\right)+\frac{1}{1+\rho}U\left(C_{O,t+1}\right) \\ &s.t. \frac{C_{O,t+1}}{1+r}+C_{Y,t}=W_t-\tau_t \end{aligned} \tag{1.63}$$

where τ_t is taxes levied on the young in period *t.* The household consumption function is increasing in both income and the interest rate since each young generation consist of net savers.

$$C_{Y,t}=C(W_t-\tau_t,r) \tag{1.64}$$

The economy is small and open so factor prices are exogenous. Assume for simplicity that the economy's net assets are invested abroad. Without a government, we then have

$$\begin{aligned} &B_{t+1}=S_t^Y\equiv W_t-C_{Y,t} \\ &S_t^O=rB_t-C_{O,t}=-S_{t-1}^Y=-B_t \end{aligned}. \tag{1.65}$$

So, the current account is

$$CA_t=B_{t+1}-B_t=S_t^Y+S_t^O \tag{1.66}$$

Now assume there is a government that taxes and transfers money. If there is a deficit it borrows from the current young generation. Now consider the following fiscal policy experiment.

First, the government unexpectedly transfers *T* in period *t* to the old and finance it with a deficit, i.e., it borrows from the currently young. It then rolls over the debt but issues taxes on the young to raise money for the interest. The consumption of the old generation in period *t* increases by *T*. What about the young in *t*?. By assumption, τ_t=0, so their budget constraint is unchanged so their consumption must be unchanged.

What happens to the current account? The savings of the young is now taking two forms, one part is in foreign assets and one in bonds. (Why are the bonds net wealth here?) So,

$$B_{t+1} = W_t - C_{Y,t} - T \tag{1.67}$$

Alternatively, if the government borrowed abroad, (1.67) would still be true if we let B_{t+1} include the government holdings of foreign assets. And

$$CA_t = B_{t+1} - B_t = -T \tag{1.68}$$

Note that this is equal to private savings plus public savings

$$CA_t = S_t^Y + S_t^O + S_t^G \tag{1.69}$$

since the savings of the young is unchanged, the savings of the old is unchanged (their income including transfers increases with the same amount as the consumption),. while the government have a deficit (negative saving) of *T*. Thus, the expansionary fiscal policy, leads to a current account deficit.

What happens after t+1? Each new young generation lends T to the government, gets it back when they are old and are taxed to pay the interest rate. Thus, $\tau_s = rT$, for all s>0. So their consumption must be

$$C_{Y,t+s} = C(W_{t+s} - rT, r) \tag{1.70}$$

Clearly, consumption smoothing implies that the consumption falls relative to $C_{Y,t}$ by an amount inside the interval (0, *Tr*). The lower limit is consumption is not affected, i.e., all consumption reduction by the generation born in t+1 is done when they are old. The upper limit if is all consumption reduction is done when they are young. Consumption smoothing implies that the true effect is in-between these two limits. So:

$$\begin{aligned} &B_{t+2} = W_{t+1} - rT - C_{Y,t+1} - T \\ &CA_{t+1} = B_{t+2} - B_{t+1} = -rT + C_{Y,t+1} - C_{Y,t+2} \\ &-rT < CA_{t+1} < 0 \end{aligned} \tag{1.71}$$

Note, that the negative effect on the current account in t+1 can be understood as the government taxing people with a low propensity to consume (young) and gives the receipts to people with high propensity to consume (old). Again, an expansionary fiscal policy reduces the current account.

What happens after $t+2$? Each new young generation lends T to the government, gets it back when they are old and are taxed to pay the interest rate to the currently old. So they face the same budget constraint and the current account is zero. So clearly, the policy experiment has reduced the stock of foreign assets.

The intuition for the breakdown of Ricardian Equivalence is that a budget deficit is a way to transfer resources between generations. In general, a government spending plan should be financed by a mortgage with the same pay-back period as the life span of what is financed not to cause intergenerational transfers. E.g., a bridge that lasts 50 years should be financed with a 50 year mortgage.

Note that we can interpret the experiment as a *Pay-as-you-go* pension system.

3. A Large Economy OLG model

Let us now consider a closed economy, where factor prices are determined as equilibrium outcomes at domestic factor markets.

. Households

Live for two periods and solve

$$\begin{aligned} &\max_{C_{1,t},C_{2,t+1}} U\left(C_{1,t}\right)+\frac{1}{1+\rho}U\left(C_{2,t+1}\right) \\ &s.t.\frac{C_{2,t+1}}{1+r_{t+1}}+C_{1,t}=\text{labor income in } t. \end{aligned} \tag{1.72}$$

. Firms

Hire labor and capital on competitive market and combine them in a CRS production function to produce the only good. This is sold on a competitive market to the households.

Firms solve

$$\max_{K_t,L_t} F(K_t,A_tL_t)-(w_tA_tL_t+r_tK_t) \tag{1.73}$$

where A_t is a productivity index that grows geometrically at rate g. We can think of AL as the number of *effective* units of labor. Also L grows at a geometric rate n. So

$$\begin{aligned} A_{t+1}&=(1+g)A_t \\ L_{t+1}&=(1+n)L_t \end{aligned} \tag{1.74}$$

The competitive market imply that factors are paid their marginal products. $r = F_K$ and $w=F_{AL}$.

Since F is H(1) we have

$$\frac{1}{A_tL_t}F\left(K_t,A_tL_t\right)=F\left(\frac{K_t}{A_tL_t},1\right)\equiv f\left(k_t\right) \tag{1.75}$$

So f is production per effective labor unit and k is capital per effective labor unit. Note that

$$F(K_t, A_t L_t) \equiv A_t L_t f\left(\frac{K_t}{A_t L_t}\right)$$
$$r_t = F_{K_t} = A_t L_t f'(k_t)\frac{1}{AL} = f'(k_t) \tag{1.76}$$

Since we CRS and competitive markets firms make zero profits. We can write this as

$$w_t A_t L_t = F(K_t, A_t L_t) - r_t K_t$$
$$w_t = f(k_t) - r_t k_t \tag{1.77}$$

Note that we now see that both wages and interest rates are determined by the current capital to effective labor ratio. We thus write

$$w_t = w(k_t)$$
$$r_t = r(k_t) \tag{1.78}$$

. Capital market

The labor income of a young generation in *t* equals $w_t A_t L_t$. Part of this is consumed ($c_{1,t}$) and the rest is saved. Let s_t denote the share of labor income that is saved. This will, in general depend on labor income and the interest rate. The savings in period *t* is next periods capital stock

$$s_t w_t A_t L_t = K_{t+1}$$
$$s_t w_t = \frac{K_{t+1}}{A_t L_t} = \frac{K_{t+1}}{A_{t+1}L_{t+1}/(1+n)(1+g)} \tag{1.79}$$
$$k_{t+1} = \frac{s_t w_t}{(1+n)(1+g)}$$

Now assume that s_t only depends on r_{t+1}. (Which class of utility functions produce this result? What are the effects of higher *r*?) Then we can write the last line of (1.79) as

$$k_{t+1} = \frac{s(r_{t+1}(k_{t+1}))w(k_t)}{(1+n)(1+g)} \tag{1.80}$$

This (implicitly) defines a difference equation for *k*, i.e., a relation between k_t and k_{t+1} that has to be satisfied in this model. Note that if we made the Solow assumption of a constant savings rate, the difference equation takes an explicit form. Since in this case the RHS contains no k_{t+1}. Fortunately there is a utility function for which the income and substitution effects of higher interest rates cancel so that the household will choose a constant savings rate regardless of the interest rate.

. Functional specification

Let us look at a particularly simple specification. Assume U is the log function and that that production is (Wicksell-) Cobb-Douglas, $f(k) = k^{\alpha}$.

Now we can write the consumption decision of the consumer as follows.

$$\max_{s_t} = \ln(1-s_t)w_t A_t L_t + \frac{1}{1+\rho}\ln s_t(1+r_{t+1})w_t A_t L_t \tag{1.81}$$

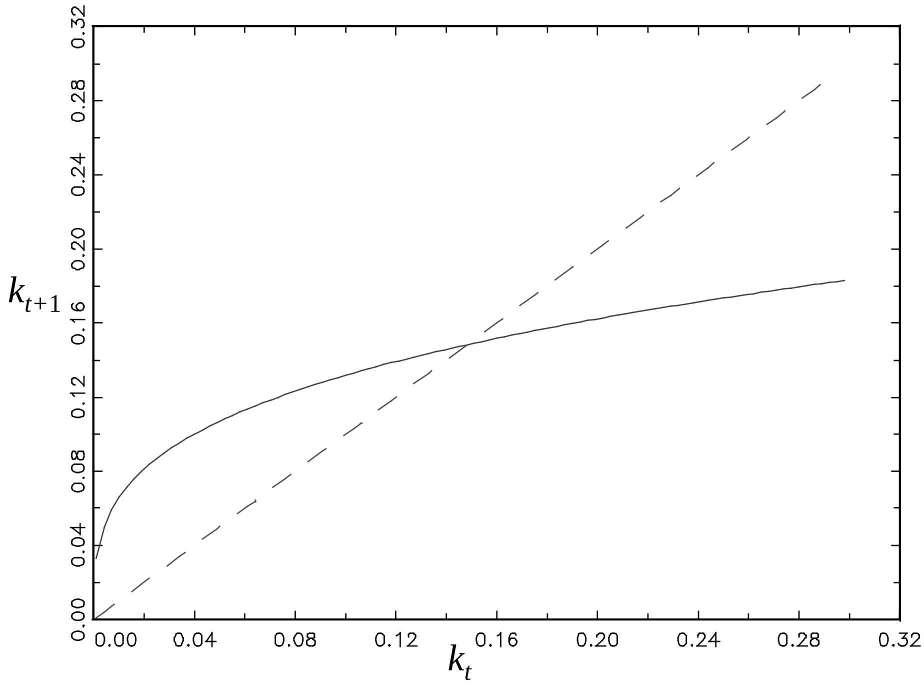
with foc

$$\begin{aligned} \frac{1}{1-s_t} &= \frac{1}{(1+\rho)s_t} \\ \Rightarrow s_t &= \frac{1}{2+\rho} \end{aligned} \tag{1.82}$$

Now we only need to specify the wage function to have an explicit version of (1.80). We see immediately that $w = (1-\alpha)k^{\alpha}$. So (1.80) becomes

$$\begin{aligned} k_{t+1} &= \frac{(1-\alpha)k_t^{\alpha}}{(2+\rho)(1+n)(1+g)} \\ \ln k_{t+1} &= \ln\frac{1-\alpha}{(2+\rho)(1+n)(1+g)} + \alpha \ln k_t \end{aligned} \tag{1.83}$$

Can you solve this difference equation and is it stable? Here is an example of a plot of (1.83).



The steady state of (1.83) is its fixed point

$$k^* = \left(\frac{1-\alpha}{(2+\rho)(1+g)(1+n)} \right)^{\frac{1}{1-\alpha}} \tag{1.84}$$

The interest rate in steady state is

$$r^* = \alpha (k^*)^{\alpha-1} = \left(\frac{\alpha(2+\rho)(1+g)(1+n)}{1-\alpha} \right) \tag{1.85}$$

. Dynamic Inefficiency

In each time period the amount of resources is $K_t + F(K_t, A_t L_t)$. This has to be split into aggregate consumption and next periods capital stock. We can than write

$$K_t + F\left(K_t, A_t L_t\right) = K_{t+1} + C_t \tag{1.86}$$

where C_t is total consumption of young and old in period *t*. Now divide by $A_t L_t$

$$\begin{aligned} k_t + f\left(k_t\right) &= \frac{K_{t+1}}{A_t L_t} + c_t \\ &= k_{t+1}(1+g)(1+n) + c_t . \end{aligned} \tag{1.87}$$

c_t is aggregate consumption per unit of current effective unit of labor.

Now consider an economy in a steady state. We then have

$$\begin{aligned} k^* + f\left(k^*\right) &= k^*(1+g)(1+n) + c^* \\ \Rightarrow c^* &= f\left(k^*\right) - k^*(g+n+gn) \end{aligned} \tag{1.88}$$

Let us find the value of the steady state capital stock that maximizes aggregate consumption per effective unit of labor. The FOC for this problem is

$$f'\left(k^{gr}\right) = n + g + ng \tag{1.89}$$

This is a variant of the Ramsey Golden Rule. A steady state capital stock above k^{gr} is not Pareto efficient. If we are above k^{gr} we could increase aggregate consumption in all periods now and in the future by reducing forcing the steady state capital stock to be lower.

Now compare (1.89) and (1.85)

$$\begin{aligned} r^* &= \left(\frac{\alpha(2+\rho)(1+g)(1+n)}{1-\alpha}\right) \lesseqgtr n + g + ng \\ \left(\frac{\alpha(2+\rho)}{1-\alpha}\right) &\lesseqgtr 1 - \frac{1}{(1+g)(1+n)} \end{aligned} \tag{1.90}$$

So it is possible that the economy is not on the Pareto frontier and thus dynamically inefficient. What goes wrong here and why does not the first welfare theorem hold? One way of understanding this is to realize that the only way a young generation can reduce the capital stock is by saving less but then consumption would have to be lower for them in the second period. If, however, there was a mechanism that specified that young people saved less but in return got some transfers from the young next period, everybody could be made better off. What does this remind you off?

. Borrowing versus Taxing in the standard OLG model

Now let us look at whether Ricardian Equivalence holds in a standard OLG model without altruism. To focus on the path of financing rather than the spending side we consider a government which wants to finance a constant spending level of 0 for convenience. Consider a government that issues debt equal to T_t, gives the proceeds to the currently young, rolls over the debt for ever and the tax the young in all future periods to pay the interest rate minus the growth rate of the economy. This means that t_t will be held constant at a level *t*. For simplicity let us use the log utility, Cobb-Douglas production specification.

The saving decision of the currently young given in is unaffected but the transfer has to be added to the wage

$$\begin{aligned} C_{1,t} &= (1-s_t)\left(w_t A_t L_t + T_t\right) \\ &= \frac{1+\rho}{2+\rho}\left((1-\alpha)k_t^{\alpha} A_t L_t + T_t\right) \end{aligned} \tag{1.91}$$

From this we see that current consumption increases when the policy is started up. Total savings also increase and become

$$S_t = \frac{1}{2+\rho}\left((1-\alpha)k_t^{\alpha} A_t L_t + T_t\right) \tag{1.92}$$

Note that part of the savings is in form of government bonds. But it is only real savings that translate into next periods capital stock. To put it differently next periods capital stock is equal to private savings plus government savings, the latter being equal to minus B_t

$$\begin{aligned} K_{t+1} &= S_t - T_t \\ &= \frac{1}{2+\rho}\left((1-\alpha)k_t^{\alpha} A_t L_t + B_t\right) - T_t \\ &= \frac{(1-\alpha)}{2+\rho} k_t^{\alpha} A_t L_t - \frac{1+\rho}{2+\rho} T_t \\ \Rightarrow k_{t+1} &= \frac{(1-\alpha)k_t^{\alpha} - (1+\rho)t_t}{(2+\rho)(1+n)(1+g)} \end{aligned} \tag{1.93}$$

Alternatively we could derive (1.93) from noting that next periods capital stock equals production plus the capital stock minus consumption.

$$
\begin{aligned}
K_{t+1} &= K_t + Y_t - C_{1,t} - C_{2,t} \\
&= K_t + k_t^{\alpha} A_t L_t - C_{1,t} - K_t - \alpha k_t^{\alpha} A_t L_t \\
&= (1-\alpha)k_t^{\alpha} A_t L_t - \frac{1+\rho}{2+\rho}\Big((1-\alpha)k_t^{\alpha} A_t L_t + T_t\Big)\cdot \\
&= \frac{1}{2+\rho}\Big((1-\alpha)k_t^{\alpha} A_t L - (1+\rho)T_t\Big)
\end{aligned}
\tag{1.94}
$$

From (1.82) we see that it is clear that the deficit financed transfer causes next periods capital stock to decrease. This means that the interest rate goes up. This in turn means that the old in the next period can consume more for each dollar saved when young. Since total savings (the sum of real and financial savings) increased, the old in $t+1$ will consume more. The generation born in time t when the debt was issued will this clearly benefit and we do not have Ricardian Equivalence.

From next period the government takes away an amount equal to the debt plus interest rate from the young and gives them the rolled over debt. Let H_{t+1} denote total wealth of a generation born at $t+1$. We then have

$$
\begin{aligned}
H_{t+1} &= w(k_{t+1})A_{t+1}L_{t+1} - tA_t L_t (1 + r(k_{t+1})) + tA_{t+1}L_{t+1} \\
&= w(k_{t+1})A_{t+1}L_{t+1} - tA_{t+1}L_{t+1}\frac{1+r(k_{t+1})}{(1+n)(1+g)} + tA_{t+1}L_{t+1} \\
&= A_{t+1}L_{t+1}\left(w(k_{t+1}) - t\left(\frac{r(k_{t+1}) - (n+g+ng)}{(1+n)(1+g)}\right)\right)
\end{aligned}
\tag{1.95}
$$

We see that wealth decrease (increase) in b if the economy is dynamically efficient (inefficient). This is for given k_{t+1} Since the introduction of a government debt necessarily implies a lower capital stock this has negative effects on the wealth and utility of the young in $t+1$. For sufficiently large dynamic inefficiencies it is possible that that the first effect dominates for small debt levels.

Now consider the consumption of the young;

$$
C_{1,t+1} = \frac{1+\rho}{2+\rho} A_{t+1}L_{t+1}\left(w(k_{t+1}) - t\left(\frac{r(k_{t+1}) - (n+g+ng)}{(1+n)(1+g)}\right)\right)
\tag{1.96}
$$

and consumption of the old is

$$C_{2,t+1} = (1+r_{t+1})(k_{t+1}A_{t+1}L_{t+1} + tA_t L_t)$$
$$= (1+r_{t+1})\left(k_{t+1}A_{t+1}L_{t+1} + t\,\frac{A_{t+1}L_{t+1}}{(1+n)(1+g)} \right) \tag{1.97}$$

Now consider the capital accumulation equation. This equals total resources available minus consumption

$$K_{t+2} = K_{t+1} + Y_{t+1} - C_{2,t+1} - C_{1,t+1} \tag{1.98}$$

Normalizing and simplifying we get

$$K_{t+2} = A_{t+1}L_{t+1} - A_{t+1}L_{t+1}t\,\frac{1+\rho}{2+\rho}$$
$$k_{t+2} = \frac{(1-\alpha)k_t^{\alpha} - t(1+\rho)}{(2+\rho)(1+n)(1+g)} \tag{1.99}$$

This follows from the fact that next periods capital stock is equal to the share of production that goes to the young minus their consumption. We see that this difference equation implies that next periods *k* is lower the larger is *t*. This implies that the new steady state capital stock is necessarily smaller when a debt roll-over is introduced.