Macro I Gothenburg

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Labor - leisure trade-off – the RBC model

- Any reasonable model of business cycles arguably needs fluctuations in labor use.
- One way; search – it takes time to find a job/worker and varying amounts of job-creation - destruction – Mortensen-Pissarides – Shimer Puzzle.
- A principally different way, assume perfect labor markets and add a labor/leisure choice.
- Later, we will add frictions and imperfections to this model -> the New Keynesian model.

A growth model with a labor-leisure choice

- Problem of planner/representative household in period 0:

$$\max_{\{C_t, K_{t+1}, L_t, N_t\}_{t\geq 0}} E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t)$$

$$\text{s.t. } C_t + K_{t+1} = F(K_t, N_t, Z_t) + (1-\delta) K_t \forall t \geq 0$$

$$N_t = 1 - L_t$$

$$K_0 \text{ given.}$$

- Z_t is a stochastic productivity shock, perhaps autocorrelated and non-stationary.
- Decisions are taken every period after seeing the current period shock. C_t, K_{t+1}, L_t determined in period t.

Planner solution vs. Markets

- Here, no market imperfections. Planner solution equals a decentralized equilibrium where:
  - households supply labor and rent capital to a competitive factor market to maximize PDV of utility (dynamic problem?)
  - firms rent capital and labor and produce with a CRS production technology to maximize profits in every period. (dynamic problem?)
  - Firms owned by households (profits?)

Optimality

- Substitute N_t for $1 - L_t$. Lagrange objective in period t then:
  $E_t \sum_{s=0}^{\infty} \beta^s \{U(C_{t+s}, L_{t+s})$
  $+\lambda_{t+s}(F(K_{t+s}, 1 - L_{t+s}, Z_{t+s}) + (1-\delta) K_{t+s} - C_{t+s} - K_{t+s+1})\}$
- First-order conditions for L_t; $U_L(C_t, L_t) - \lambda_t F_N(K_t, 1 - L_t, Z_t) = 0$
- For K_{t+1}; $-\lambda_t + E_t \beta \lambda_{t+1} (F_K(K_{t+1}, 1 - L_{t+1}, Z_{t+1}) + 1 - \delta) = 0$
- For C_t; $U_C(C_t, L_t) - \lambda_t = 0$
- Define $R_{t+1} \equiv F_K(K_{t+1}, 1 - L_{t+1}, Z_{t+1}) + 1 - \delta$.
- Condition for K_{t+1} gives $U_C(C_t, L_t) = E_t[\beta U_C(C_{t+1}, L_{t+1}) R_{t+1}]$ (Euler equation, intertemporal tradeoff). Note expectation of product!
- Define $w_t \equiv F_N(K_t, 1 - L_t, Z_t)$
- Condition for L_t gives $\frac{U_L(C_t, L_t)}{U_C(C_t, L_t)} = w_t$.(Intratemporal tradeoff)

Balanced growth

- Over time, we know productivity (Z_t) has grown (very much). Over long periods:
  - interest rate has not trended but wages have.
  - labor supply has not changed (much).
  - capital has grown parallel to wages, consumption and output (*balanced growth)*
- Puts requirements on utility functions (and that technical change is labor augmenting).

1: Constant labor supply in balanced growth

- Suppose the wage increases by a factor X, and consumption also increases by the same factor X. Use intratemporal FOC. Then, if U_L / U_C *increases* by a factor X, labor supply should be unchanged.
- Mathematically, $\frac{U_L(XC,L)}{U_C(XC,L)} = Xw \forall X$.
- This is satisfied if (in fact iff, KPR-88 and Broer&Krusell, -18) utility is of the form $U(C,L) = U(Cv(L)) \Rightarrow \frac{U_L(C,L)}{U_C(C,L)} = C\frac{v'(L)}{v(L)}$
- Then is $\frac{U_L(C,L)}{U_C(C,L)}$ proportional to something only depending on L, with a proportionality factor C.

2: Constant interest rate in balanced growth

- In a steady state with constant interest rate, and constant growth rate g of consumption the Euler equation is

$$\frac{U_C(C,L)}{U_C((1+g)C,L)} = \beta R.$$

- For this to be true for all C, we need a function with constant intertemporal elasticity of substitution utility function (equivalently, CRRA). The only class of functions satisfying this are of the form
$U(C,L) = u(Cv(L)) = \frac{\sigma(Cv(L))^{\frac{\sigma-1}{\sigma}}-1}{\sigma-1}$,or with $\sigma = 1$,
$U(C,L) = \ln C + v(L)$.
- The parameter $\sigma > 0$ measures how much consumption needs to change (in percent) per percentage change in marginal utility i.e., $-\left(\frac{d\ln(U_c)}{d\ln c}\right)^{-1} = \sigma$. With time-additive utility, this is also the intertemporal substitution. Also inverse of risk aversion. Why?

What happens with CARA?

- Assume constant *absolute risk aversion*(and disregard labor for now), $U = -\frac{e^{-\sigma C}}{\sigma}$, with $U_C = e^{-\sigma C_t}$.Then

$$\frac{U_C(C)}{U_C((1+g)C)} = \frac{e^{-\sigma C}}{e^{-\sigma(1+g)C}} = e^{\sigma C g}$$

- In words, as the level of consumption increases, the rate of interest required to support a constant growth rate g *increases.*
- This is since with a CARA utility, the ratio of the marginal utilities between two consumption levels depends on the *difference* between them, not the ratio.
- Therefore, with CARA utility, constant interest rate can support growth that is constant in absolute value (linear, not exponential growth), something that doesn't seem in accordance with empirics.

Shocks and labor supply

- Key task of the RBC model is to be able to produce variations in labor supply.
- Consider the log-case. Then, the intertemporal Euler condition is $\frac{1}{C_t} = \beta E_t \frac{R_{t+1}}{C_{t+1}}$and the intratemporal $v'(L_t) = \frac{w_t}{C_t}$.
- If a permanent technological shock changes wages and consumption by the same proportion (like along a balanced growth path), the RHS of intratemporal is unchanged and so should therefore labor supply be. By construction!
- On the other hand, a *temporary* technological should shift w_t *more* than C_t proportionally, since individuals want smooth consumption. Therefore, a temporary shock should affect labor supply more the more temporary it is.
- What happens to labor supply response of temporary productivity shock if there is no savings?

Role of labor elasticity

- Suppose $U(C_t, L_t) = \ln C_t + v(L_t) = \ln C_t + \frac{\nu}{\nu-1}\phi L_t^{\frac{\nu-1}{\nu}}$, then the intratemporal condition is

$$\phi L_t^{-\frac{1}{\nu}} = \frac{w_t}{C_t} \Rightarrow C_t = \frac{w_t L_t^{\frac{1}{\nu}}}{\phi}$$

- Use in Euler with log utility,
$1 = \beta E_t \left[R_{t+1} \frac{U'(C_{t+1})}{U'(C_t)} \right] = \beta E_t \left[R_{t+1} \frac{C_t}{C_{t+1}} \right]$;

$$1 = \beta E_t \left[R_{t+1} \frac{w_t}{w_{t+1}} \left(\frac{L_t}{L_{t+1}} \right)^{\frac{1}{\nu}} \right]$$

- Again, if w_t is high relative to w_{t+1} should expect leisure to be relatively low (work a lot) in period t (given that R_{t+1} does not change). How much depends on labor supply elasticity, i.e., ν.
- Microevidence suggests low ν, macro a higher value. Problem?

Solving the model

- What does solving the model mean?
- Optimality conditions give us relations between C_t and C_{t+1}, and between C_t and L_t. Not enough.
- Solving the model is to find C_t and L_t as functions of the state variables (K_t and Z_t).
- Then we know the complete dynamics of the model and can simulate it, for instance.
- With log consumption utility and full depreciation, we can solve the model analytically. Set $U(C_t, L_t) = \ln C_t + v(L_t)$ and $K_{t+1} + C_t = Z_t K_t^\alpha (1 - L_t)^{1-\alpha}$.

Simple solution

- The Euler equation is $1 = \beta E_t \frac{U_C(C_{t+1}, L_{t+1})}{U_C(C_t, L_t)} R_{t+1} = \beta E_t \frac{C_t}{C_{t+1}} R_{t+1}$
- Defining the savings ratio s_t, we have $C_t = (1 - s_t) Z_t K_t^{\alpha} (1 - L_t)^{1-\alpha}$. The RHS of $1 = \beta E_t \left(\frac{C_t}{C_{t+1}} \right) R_{t+1}$ becomes
- $= \beta E_t \left(\frac{(1-s_t) Z_t K_t^{\alpha} (1-L_t)^{1-\alpha}}{(1-s_{t+1}) Z_{t+1} K_{t+1}^{\alpha} (1-L_{t+1})^{1-\alpha}} \right) Z_{t+1} \alpha K_{t+1}^{\alpha-1} (1 - L_{t+1})^{1-\alpha}$
- $= \beta E_t \left(\frac{1-s_t}{1-s_{t+1}} Z_t K_t^{\alpha} (1 - L_t)^{1-\alpha} \right) \alpha \frac{1}{K_{t+1}}$
- $= \beta E_t \left(\frac{1-s_t}{1-s_{t+1}} Z_t K_t^{\alpha} (1 - L_t)^{1-\alpha} \right) \alpha \frac{1}{s_t Z_t K_t^{\alpha} (1-L_t)^{1-\alpha}}$
- $= \beta E_t \left(\frac{1-s_t}{1-s_{t+1}} \right) \alpha \frac{1}{s_t}$

Solving the model:2

- $\beta E_t \left(\frac{1-s_t}{1-s_{t+1}} \right) \alpha \frac{1}{s_t}$ is independent of Z_t and thus a non-stochastic non-linear difference equation.
- It has a steady state at $\alpha\beta$ with a linearized explosive root $\frac{1}{\alpha\beta} > 1$ and no initial condition for s_t so the only solution is to jump immediately to the steady state. Thus, we can conclude that only $s_t = \alpha\beta \forall t$ is consistent with the Euler equation. (non-linear solution?)
- Thus, the consumption function is

$$\begin{aligned} C_t &= (1-\alpha\beta)\, Y_t \\ &= (1-\alpha\beta)\, Z_t K_t^{\alpha} (1-L_t)^{1-\alpha} \end{aligned}$$

Labor-leisure choice

- The intratemporal FOC says

$$U_L(C_t, L_t) = U_C(C_t, L_t)\, w_t$$

$$v'(L_t) = \frac{Z_t(1-\alpha)K_t^{\alpha}(1-L_t)^{1-\alpha}}{(1-L_t)\,C_t}$$

$$v'(L_t) = \frac{Z_t(1-\alpha)K_t^{\alpha}(1-L_t)^{1-\alpha}}{(1-L_t)(1-s)Z_tK_t^{\alpha}(1-L_t)^{1-\alpha}}$$

$$\rightarrow v'(L_t)(1-L_t) = \frac{1-\alpha}{1-s}$$

- So, labor supply is constant. If, for example, $v(L_t) = \phi \ln L_t$, we get $L_t = \frac{1}{1+\frac{1-\alpha}{\phi(1-\alpha\beta)}} \forall t$. This is not particularly useful, right?

Why does this not work as a business-cycle model?

1. A change in next periods expected productivity changes the return to saving between t and $t+1$ and next periods marginal utility in opposite directions.
2. With log consumption utility and full depreciation the effects exactly cancel. Future does not matter for current consumption. Z_{t+1} cancels.
3. Similarly, a shock today, Z_t increases the wage and consumption proportionally when the savings rate is constant.The ratio of wages and marginal utility is thus not affected and marginal utility of leisure does not need to be changed.
4. Seems fine for long-run changes, but not for business cycle.
5. A fix: relax full full depreciation. Intuitively, an additional resource in the budget constraint (stock of non-depreciated capital) that does not change one-for-one with productivity makes consumption respond less than one-for-one. Income effects are smaller. Income and substitution effects don't cancel.