Journal of Monetary Economics 21 (1988) 195–232. North-Holland

PRODUCTION, GROWTH AND BUSINESS CYCLES
I. The Basic Neoclassical Model

Robert G. KING, Charles I. PLOSSER and Sergio T. REBELO*

University of Rochester, Rochester, NY 14627, USA

Received September 1987, final version received December 1987

This paper presents the neoclassical model of capital accumulation augmented by choice of labor supply as the basic framework of modern real business cycle analysis. Preferences and production possibilities are restricted so that the economy displays steady state growth. Then we explore the implications of the basic model for perfect foresight capital accumulation and for economic fluctuations initiated by impulses to technology. We argue that the neoclassical approach holds considerable promise for enhancing our understanding of fluctuations. Nevertheless, the basic model does have some important shortcomings. In particular, substantial persistence in technology shocks is required if the model economy is to exhibit periods of economic activity that persistently deviate from a deterministic trend.

1. Introduction and summary

Real business cycle analysis investigates the role of neoclassical factors in shaping the character of economic fluctuations. In this pair of essays, we provide an introduction to the real business cycle research program by considering the basic concepts, analytical methods and open questions on the frontier of research. The focus of the present essay is on the dynamic aspects of the basic neoclassical model of capital accumulation. This model is most frequently encountered in analyses of economic growth, but we share Hicks' (1965, p. 4) perspective that it is also a basic laboratory for investigating more general dynamic phenomena involving the choice of consumption, work effort and investment.

Our use of the neoclassical model of capital accumulation as the engine of analysis for the investigation of economic fluctuations raises a number of central issues. First, what role does economic growth play in the study of

*The authors acknowledge financial support from the National Science Foundation. King and Plosser have joint affiliations with the Department of Economics and the W.E. Simon Graduate School of Business, University of Rochester. Rebelo is affiliated with the Department of Economics, University of Rochester and the Department of Economics, Portuguese Catholic University. We have benefited from the comments of Andrew Abel and Larry Christiano, as well as from those of seminar participants at the Federal Reserve Bank of Richmond, Brasenose College, Oxford, Institute for International Economic Studies, University of Stockholm, Northwestern University, Yale University, and Columbia University.

economic fluctuations? More precisely, does the presence of economic growth restrict the preference and production specifications in ways that are important for the analysis of business cycles? Second, what analytical methods can be employed to study the time series implications of the neoclassical model? Third, what are the dynamics of the neoclassical model in response to technology shocks? Finally, does the neoclassical model – driven by technology shocks – replicate important features of macroeconomic time series? The analysis of these issues forms the core of the present paper and establishes the building blocks of real business cycle theory.

Real business cycle theory, though still in the early stages of development, holds considerable promise for enhancing our understanding of economic fluctuations and growth as well as their interaction. The basic framework developed in this essay is capable of addressing a wide variety of issues that are commonly thought to be important for understanding business cycles. While we focus here on models whose impulses are technological, the methods can be adapted to consider shocks originating from preferences or other exogenous factors such as government policies and terms of trade. Some of these extensions to the basic framework are developed in the companion essay.

To many readers it must seem heretical to discuss business cycles without mentioning money. Our view, however, is simply that the role of money in an equilibrium theory of economic growth and fluctuations remains an open area for research. Further, real disturbances generate rich and neglected interactions in the basic neoclassical model that may account for a substantial portion of observed fluctuations. The objective of real business cycle research is to obtain a better understanding of the character of these real fluctuations. Without an understanding of these real fluctuations it is difficult *a priori* to assign an important role to money.

The organization of the paper follows the sequence of questions outlined above. We begin in section 2 by describing the preferences, endowments and technology of the basic (one-sector) neoclassical model of capital accumulation.[1] In contrast to the familiar textbook presentation of this model, however, work effort is viewed as a choice variable. We then discuss the restrictions on production possibilities and preferences that are necessary for steady state growth. On the production side, with a constant returns to scale production function, technical progress must be expressible in labor augmenting (Harrod neutral) form. In a feasible steady state, it follows that consumption, investment, output and capital all must grow at the exogenously specified rate of technical change. On the other hand, since the endowment of time is constant, work effort cannot grow in the steady state. Thus, preferences must be restricted so that there is no change in the level of effort on the steady state

[1]A more detailed and unified development of the material is presented in the technical appendix, available from the authors on request.

growth path despite the rise in marginal productivity stemming from technical progress, i.e., there must be an exact offset of appropriately defined income and substitution effects.

Section 3 concerns perfect foresight dynamic competitive equilibria, which we analyze using approximations near the steady state. Using a parametric version of the model, with parameters chosen to match the long-run U.S. growth experience, we study the interaction between intertemporal production possibilities and the equilibrium quantity of labor effort. Off the steady state path, we find that capital and effort are negatively related despite the fact that the marginal product of labor schedule is positively related to the capital stock. That is, in response to the high real rate of return implied by a low capital stock, individuals will substitute intertemporally to produce additional resources for investment.

Working from a certainty equivalence perspective, section 4 considers how temporary productivity shocks influence economic activity, generating 'real business cycles' in the terminology of Long and Plosser (1983). Again there is an important interaction between variation in labor input – this time in response to a productivity shock – and the intertemporal substitution in production permitted by capital accumulation. Purely temporary technology shocks call forth an expansion of labor input once the Long and Plosser (1983) assumption of complete depreciation is replaced by a more realistic value,[2] since more durable capital increases the feasibility of intertemporal substitution of goods and leisure. Nevertheless, with purely temporary productivity shocks, we find that there are important deficiencies of the basic neoclassical model. Although there is substantial serial correlation in consumption and capital as a consequence of consumption smoothing, there is effectively no serial correlation in output or employment. This lack of propagation reflects two basic properties of the parameterized model: (i) a negative relation between capital and effort along the transition path and (ii) the minor effect of a purely temporary technology shock on a large and durable capital stock. Thus, the basic neoclassical capital accumulation mechanism is important for permitting intertemporal substitution of goods and leisure, but it does not generate serial correlation in output and employment close to that exhibited by macroeconomic data.

It is necessary, therefore, to incorporate substantial serial correlation in productivity shocks [as in Kydland and Prescott (1982), Long and Plosser (1983), Hansen (1985), and Prescott (1986)] if the basic neoclassical model is to generate business fluctuations that resemble those in post-war U.S. experience. Since serial correlation involves movements in productive opportunities that are more persistent in character, labor input responds less elastically to a

[2] By a purely temporary shock, we mean one that lasts for a single time period, which is taken to be a quarter in our analysis.

given size shock, but its response remains positive. On the other hand, with more persistent productivity shocks, consumption responds more elastically in accord with the permanent income theory.

In section 5, we show that the basic neoclassical model – with persistent technology shocks – captures some key features of U.S. business cycles. For example, the model replicates observed differences in volatility across key series. Measured as a percentage of the standard deviation of output, there is an identical ordering of the model's implications for investment, wages, consumption and hours, and the U.S. time series: investment is most volatile, followed by wages, consumption and then hours. But there are also aspects of the data that are poorly captured by the single-shock model. For example, consumption, investment and hours are much more highly correlated with output in the model than in the data.

Professional interest in real business cycle analysis has been enhanced by the comparison of moments implied by neoclassical models with those of U.S. time series, as initiated by Kydland and Prescott (1982). Our implications for moments differ from those of Hansen (1985) and Prescott (1986), principally because we do not filter actual and model-generated time series to remove slow-moving components. For example, in Hansen's and Prescott's analyses, filtered hours and output have virtually identical volatilities, in both the model and the transformed data. By contrast, in our analysis, the volatility of hours is about half that of output (both in our model and post-war detrended U.S. data). These differences occur despite the fact that there is little economic difference in the models under study.

Section 6 provides a brief summary and some concluding remarks.

2. The basic neoclassical model

Our analysis of economic growth and fluctuations starts by summarizing the key features of the basic one-sector, neoclassical model of capital accumulation. Much of the discussion in this section will be familiar to readers of Solow (1956), Cass (1965), Koopmans (1965) and subsequent textbook presentations of their work, but it is important to build a base for subsequent developments.

2.1. Economic environment

We begin by considering the preferences, technology and endowments of the environment under study.

Preferences. We consider an economy populated by many identical infinitely-lived individuals with preferences over goods and leisure represented by

$$U = \sum_{t=0}^{\infty} \beta^t u(C_t, L_t), \qquad \beta < 1, \tag{2.1}$$

where C_t is commodity consumption in period t and L_t is leisure in period t. Consumption and leisure are assumed throughout to be goods so that utility is increasing in C_t and L_t.[3]

Production possibilities. There is only one final good in this economy and it is produced according to a constant returns to scale neoclassical production technology given by

$$Y_t = A_t F(K_t, N_t X_t), \tag{2.2}$$

where K_t is the predetermined capital stock (chosen at $t-1$) and N_t is the labor input in period t.[4] We permit temporary changes in total factor productivity through A_t. Permanent technological variations are restricted to be in labor productivity, X_t, for reasons that we discuss below.

Capital accumulation. In this simple neoclassical framework the commodity can be either consumed or invested. The capital stock evolves according to

$$K_{t+1} = (1-\delta_K)K_t + I_t, \tag{2.3}$$

where I_t is gross investment and δ_K is the rate of depreciation of capital.[5]

Resource constraints. In each period, an individual faces two resource constraints: (i) total time allocated to work and leisure must not exceed the endowment, which is normalized to one, and (ii) total uses of the commodity must not exceed output. These conditions are

$$L_t + N_t \leq 1, \tag{2.4}$$

$$C_t + I_t \leq Y_t. \tag{2.5}$$

Naturally, there are also the non-negativity constraints $L_t \geq 0$, $N_t \geq 0$, $C_t \geq 0$ and $K_t \geq 0$.

[3] Momentary utility, $u(\cdot)$, is assumed to be strictly concave and twice continuously differentiable. Further, it satisfies the Inada conditions, namely that $\lim_{c\to 0} \mathrm{D}_1 u(c, L) = \infty$ and $\lim_{c\to\infty} \mathrm{D}_1 u(c, L) = 0$, $\lim_{L\to 0} \mathrm{D}_2 u(c, L) = \infty$ and $\lim_{L\to 1} \mathrm{D}_2 u(c, L) = 0$, where $\mathrm{D}_i u(\cdot)$ is the first partial derivative of $u(\cdot)$ with respect to the function's ith argument.

[4] By neoclassical, we mean that the production function is concave, twice continuously differentiable, satisfies the Inada conditions, and that both factors are essential in production.

[5] We abstract from adjustment costs to capital accumulation throughout the analysis, as these seem to us to be basically a restricted formulation of the two-sector neoclassical model.

2.2. Individual optimization and competitive equilibrium

The standard neoclassical analysis focuses on the optimal quantities chosen by a 'social planner' or representative agent directly operating the technology of the economy. Since our setup satisfies the conditions under which the second welfare theorem is valid, optimal capital accumulation will also be realized in a competitive equilibrium.[6] In the companion essay, we discuss departures from the strict representative agent model including government expenditures and distorting taxes, productive externalities and heterogeneity of preferences and productivities. In these contexts, we will need to be more precise about distinguishing between individual choices and competitive outcomes.

2.3. Steady state growth

A characteristic of most industrialized economies is that variables like output per capita and consumption per capita exhibit sustained growth over long periods of time. This long-run growth occurs at rates that are roughly constant over time within economies but differ across economies. We interpret this pattern as evidence of steady state growth, by which we mean that the levels of certain key variables grow at constant – but possibly different – rates, at least some of which are positive. Additional restrictions on preferences and technologies are required if the system is to exhibit steady state growth.

Restrictions on production. For a steady state to be feasible, Swan (1963) and Phelps (1966) show that permanent technical change must be expressible in a labor augmenting form, which rationalizes our specification in (2.2) above. To make for an easier comparison with other studies, we adopt the Cobb–Douglas production process for the bulk of our analysis,

$$Y_t = A_t K_t^{1-\alpha}(N_t X_t)^{\alpha}, \tag{2.6}$$

where the quantity $N_t X_t$ is usually referred to as effective labor units.[7]

Since variation in A_t is assumed temporary, we can ignore it for our investigation of steady state growth. The production function (2.6) and the accumulation equation (2.3) then imply that the steady state rates of growth of output, consumption, capital and investment per capita are all equal to the

[6] The basic reference is Debreu (1954). See also Prescott and Lucas (1972).

[7] We note, however, that if technological change is labor augmenting, then the observed invariance of factor shares to the scale of economic activity cannot be used to rationalize the restriction to the Cobb–Douglas form. In the presence of labor augmenting technological progress the factor shares are constant for *any* constant returns to scale production function.

growth rate of labor augmenting technical progress.[8] Denoting one plus the growth rate of a variable Z as γ_Z (i.e., Z_{t+1}/Z_t), then any feasible steady state requires

$$\gamma_Y = \gamma_C = \gamma_K = \gamma_I = \gamma_X, \tag{2.7a}$$

and the growth rate of work effort to be zero, i.e.,

$$\gamma_N = 1. \tag{2.7b}$$

Since time devoted to work N is bounded by the endowment, it cannot grow in the steady state (2.7b). Thus, the only admissible constant growth rate for N is zero.

In any such feasible steady state, the marginal product of capital and the marginal product of a unit of labor input in efficiency units are constant. The levels of the marginal products, however, depend on the ratio of capital to effective labor, which is not determined by the restriction to a feasible steady state.

Restrictions on preferences. Eqs. (2.7a) and (2.7b) describe the technologically feasible steady state growth rates. If these conditions are not compatible with the efficiency conditions of agents in the economy, then they are of little interest since they would never be an equilibrium outcome. We can insure that the feasible steady state is compatible with an (optimal) competitive equilibrium, however, by imposing two restrictions on preferences: (i) the intertemporal elasticity of substitution in consumption must be invariant to the scale of consumption and (ii) the income and substitution effects associated with sustained growth in labor productivity must not alter labor supply.

The first condition must hold because the marginal product of capital, which equals one plus the real interest rate in equilibrium, must be constant in the steady state. Since consumption is growing at a constant rate and the ratio of discounted marginal utilities must equal one plus the interest rate, the intertemporal elasticity of substitution must be constant and independent of the level of consumption.

The second condition is required because hours worked cannot grow ($\gamma_N = 1$) in the steady state. To reconcile this with a growing marginal productivity of labor – induced by labor augmenting technical change (X_t) – income and substitution effects of productivity growth must have exactly offsetting effects on labor supply (N).[9]

[8] This result, in fact, holds for any constant returns to scale production function.

[9] Effective labor (NX_t) will continue to grow at rate γ_X.

These conditions imply the following class of admissible utility functions:[10]

$$u(C, L) = \frac{1}{(1-\sigma)} C^{1-\sigma} v(1-N) \tag{2.8a}$$

for $0 < \sigma < 1$ and $\sigma > 1$, while for $\sigma = 1$,

$$u(C, L) = \log(C) + v(1-N). \tag{2.8b}$$

Some additional restrictions are necessary to assure that (i) consumption and leisure are goods and (ii) that utility is concave.[11] The constant intertemporal elasticity of substitution in consumption is $1/\sigma$ for these utility functions. For the remainder of our analysis we restrict ourselves to utility functions of this class.

The requirement that preferences be compatible with steady state growth has important implications for the study of economic fluctuations. If there is no capital [i.e., if the production function is just of the form $A_t(N_t X_t)^\alpha$], then there will be no response of hours to variation in X_t or A_t in general equilibrium. This arises because (i) utility implies that the income and substitution effects of wage changes just offset and (ii) with no intertemporal substitution in production, income effects must be fully absorbed within any decision period [as in Barro and King (1984)]. Thus, in all of the parameterizations of the neoclassical model that we consider, variations in work effort are associated with intertemporal substitution made possible in equilibrium by capital accumulation.

2.4. *Stationary economies and steady states*

The standard method of analyzing models with steady state growth is to transform the economy into a stationary one where the dynamics are more amenable to analysis. In the context of the basic neoclassical model, this transformation involves dividing all variables in the system by the growth component X, so that $c = C/X$, $k = K/X$, $i = I/X$, etc. This economy is identical to a simple 'no-growth' economy with two exceptions. First the capital accumulation equation, $K_{t+1} = (1-\delta_K)K_t + I_t$, becomes $\gamma_X k_{t+1} = (1-\delta_K)k_t + i_t$. Second, transforming consumption in the preference specifica-

[10] See the technical appendix for a demonstration of the necessity of these conditions and that they imply (2.8a) and (2.8b).

[11] When momentary utility is additively separable (2.8b), all that we require is that $v(L)$ is increasing and concave. When momentary utility is multiplicatively separable, then we require that $v(L)$ is (i) increasing and concave if $\sigma < 1$ and (ii) decreasing and convex if $\sigma > 1$. Defining $D^n v(L)$ as the nth total derivative of the function $v(L)$, we further require that $-\sigma[LD^2 v(L)/Dv(L)] > (1-\sigma)[LDv(L)/v(L)]$ to assure overall concavity of $u(\cdot)$.

tion generally alters the effective rate of time preference. That is,

$$U = \left(X_0^{1-\sigma}\right) \sum_{t=0}^{\infty} (\beta^*)^t \left[\frac{1}{(1-\sigma)} c_t^{1-\sigma} v(L_t)\right] \quad \text{for} \quad \sigma \neq 1, \tag{2.9a}$$

$$U = \sum_{t=0}^{\infty} (\beta^*)^t [\log(c_t) + v(L_t) + \log(X_t)] \quad \text{for} \quad \sigma = 1, \tag{2.9b}$$

where $\beta^* = \beta(\gamma_X)^{1-\sigma}$ and $\beta^* < 1$ is required throughout to guarantee finiteness of lifetime utility. Thus, unless $\sigma = 1$, $\beta^* \neq \beta$. By suitable selection of X_0, we can in either case make the objective $\sum_{t=0}^{\infty}(\beta^*)^t u(c_t, L_t)$. Combining the resource constraints, we form the Lagrangian

$$\begin{aligned}\mathscr{L} = {} & \sum_{t=0}^{\infty} (\beta^*)^t u(c_t, 1 - N_t) \\ & + \sum_{t=0}^{\infty} \Lambda_t [A_t F(k_t, N_t) - c_t - \gamma_X k_{t+1} + (1 - \delta_K) k_t].\end{aligned} \tag{2.10}$$

The efficiency conditions for the transformed economy are (2.11)–(2.15). In these expressions, D_i is the first partial derivative operator with respect to the ith argument. For convenience, we discount the Lagrange multipliers, i.e., $\lambda_t = \Lambda_t/(\beta^*)^t$.

$$\mathrm{D}_1 u(c_t, 1 - N_t) - \lambda_t = 0, \tag{2.11}$$

$$\mathrm{D}_2 u(c_t, 1 - N_t) - \lambda_t A_t \mathrm{D}_2 F(k_t, N_t) = 0, \tag{2.12}$$

$$\beta^* \lambda_{t+1} [A_{t+1} \mathrm{D}_1 F(k_{t+1}, N_{t+1}) + (1 - \delta_K)] - \lambda_t \gamma_X = 0, \tag{2.13}$$

$$A_t F(k_t, N_t) + (1 - \delta_K) k_t - \gamma_X k_{t+1} - c_t = 0, \tag{2.14}$$

$$\lim_{t \to \infty} (\beta^*)^t \lambda_t k_{t+1} = 0, \tag{2.15}$$

where (2.11)–(2.14) must hold for all $t = 1, 2, \dots \infty$ and (2.15) is the so-called transversality condition. The economy's initial capital stock, k_0, is given.

Optimal per capita quantities for this economy – for a given sequence $\{A_t\}_{t=0}^{\infty}$ of technology shifts – are sequences of consumption $\{c_t\}_{t=0}^{\infty}$, effort $\{N_t\}_{t=0}^{\infty}$, capital stock $\{k_t\}_{t=0}^{\infty}$, and shadow prices $\{\lambda_t\}_{t=0}^{\infty}$ that satisfy the efficiency conditions (2.11)–(2.15). Under our assumptions about preferences and production possibilities, conditions (2.11)–(2.15) are necessary and sufficient for an optimum.[12]

The prices that decentralize the optimal solution as a competitive equilibrium can be computed using the technology shifts $\{A_t\}_{t=0}^{\infty}$ and the optimal sequences $\{N_t\}_{t=0}^{\infty}$, $\{k_t\}_{t=0}^{\infty}$ and $\{\lambda_t\}_{t=0}^{\infty}$. For instance, in a complete initial date markets framework the sequence of equilibrium prices of labor and the final good are, respectively, $\{\lambda_t A_t D_2 F(k_t, N_t)\}_{t=0}^{\infty}$ and $\{\lambda_t\}_{t=0}^{\infty}$. Under perfect foresight (rational expectations), a regime of sequential loan markets and spot markets in labor services also supports the optimal solution as a competitive equilibrium. In this market structure, the relevant prices are the real interest rate between t and $t+1$, r_t, and the real wage rate, w_t. It is easy to demonstrate that these are given by $(1+r_t) = \gamma_X \lambda_t / \lambda_{t+1} \beta^*$ and $w_t = A_t D_2 F(k_t, N_t)$.

3. Perfect foresight capital accumulation

A major feature of the basic one sector neoclassical model with stationary technology is that the optimal capital stock converges monotonically to a stationary point.[13] While qualitative results such as the monotonicity property are important, we wish to undertake quantitative analyses of capital stock dynamics. This requires that we exploit the fact that (2.11)–(2.14) can be reduced to a non-linear system of first-order difference equations in k and λ or a second-order equation in k only. The two boundary conditions of this system are the transversality condition (2.15) and the initial capital stock, k_0. We focus on approximate linear dynamics in the neighborhood of the steady state denoted by (A, k, N, c and y).[14]

3.1. Approximation method

The initial step in obtaining the system of linear difference equations is to approximate (2.11)–(2.14) near the stationary point. To do this, we express each condition in terms of the percentage deviation from the stationary value, which we indicate using a circumflex [e.g., $\hat{c}_t = \log(c_t/c)$, $\hat{k}_t = \log(k_t/k)$, etc.]. Then, we linearize each condition in terms of deviations from the stationary

[12] See Weitzman (1973) and Romer and Shinotsuka (1987).

[13] In the fixed labor case, which is the most thoroughly studied, this property has been shown to derive principally from preferences, in that the concavity of $u(\cdot)$ is sufficient for monotonicity so long as there is a maximum sustainable capital stock [Boyd (1986) and Becker et al. (1986)]. In environments such as ours, where the production function is strictly concave in capital (for fixed labor), monotonicity also insures that capital approaches a unique stationary point.

[14] The technical appendix discusses solution methods in considerable detail. The linear approximation method, it should be noted, rules out certain phenomena that may arise in the basic neoclassical model, such as a humped shaped transition path for investment [see King (1987)].

point. The results for the first two conditions can be written as follows:

$$\xi_{cc}\hat{c}_t - \xi_{cl}\frac{N}{1-N}\hat{N}_t - \hat{\lambda}_t = 0, \tag{3.1}$$

$$\xi_{lc}\hat{c}_t - \frac{N}{1-N}\xi_{ll}\hat{N}_t - \hat{\lambda}_t - \hat{A}_t - (1-\alpha)\hat{k}_t + (1-\alpha)\hat{N}_t = 0, \tag{3.2}$$

where ξ_{ab} is the elasticity of the marginal utility of a with respect to b.[15]

Approximation of the intertemporal efficiency condition (2.13) implies that

$$\hat{\lambda}_{t+1} + \eta_A\hat{A}_{t+1} + \eta_k\hat{k}_{t+1} + \eta_N\hat{N}_{t+1} = \hat{\lambda}_t, \tag{3.3}$$

where η_A is the elasticity of the gross marginal product of capital with respect to A evaluated at the steady state, etc.[16] Finally, approximation of the resource constraint (2.14) implies

$$\begin{aligned}\hat{y}_t &= \hat{A}_t + \alpha\hat{N}_t + (1-\alpha)\hat{k}_t \\ &= s_c\hat{c}_t + s_i\phi\hat{k}_{t+1} - s_i(\phi-1)\hat{k}_t,\end{aligned} \tag{3.4}$$

where s_c and s_i are consumption and investment shares in output and $\phi = K_{t+1}/I_t = \gamma_X/[\gamma_X - (1-\delta_K)] > 1$.

As in other linear optimal control settings, expressions (3.1) and (3.2) can be solved to give optimal decisions $\hat{c}_t$, $\hat{N}_t$ as functions of the state variables $\hat{k}_t$, $\hat{A}_t$ and the co-state (shadow price) $\hat{\lambda}_t$. Further, given these (conditionally) optimal decisions, expressions (3.3) and (3.4) imply a first-order dynamic system in $\hat{\lambda}$ and $\hat{k}$,

$$\begin{bmatrix}\hat{k}_{t+1}\\ \hat{\lambda}_{t+1}\end{bmatrix} = W\begin{bmatrix}\hat{k}_t\\ \hat{\lambda}_t\end{bmatrix} + R\hat{A}_{t+1} + Q\hat{A}_t, \tag{3.5}$$

where W is a 2×2 matrix and R and Q are 2×1 vectors. To compute the solution to this difference equation and to examine its properties, we use the decomposition $W = P\mu P^{-1}$, where P is the matrix of characteristic vectors of W and μ is a diagonal matrix with the characteristic roots on the diagonal. Ordering the roots (μ_1, μ_2) in increasing absolute value, it can be shown that

[15]When the utility function is additively separable, it follows that $\xi_{cc} = -1$, $\xi_{cl} = \xi_{lc} = 0$ and $\xi_{ll} = LD^2v(L)/Dv(L)$. When the utility function is multiplicatively separable, it follows that $\xi_{cc} = -\sigma$, $\xi_{cl} = LDv(L)/v(L)$, $\xi_{lc} = 1-\sigma$ and $\xi_{ll} = LD^2v(L)/Dv(L)$.

[16]With the Cobb–Douglas assumption, it follows that $\eta_A = [\gamma_X - \beta^*(1-\delta_K)]/\gamma_X$, $\eta_k = -\alpha\eta_A$ and $\eta_N = \alpha\eta_A$.

$0 < \mu_1 < 1 < \beta^{*-1} < \mu_2$. The general solution to the difference equation for specified initial conditions $\hat{\lambda}_0$ and $\hat{k}_0$ is given by

$$\begin{bmatrix} \hat{k}_t \\ \hat{\lambda}_t \end{bmatrix} = W^t \begin{bmatrix} \hat{k}_0 \\ \hat{\lambda}_0 \end{bmatrix} + \sum_{h=0}^{t} W^h R \hat{A}_{t-h+1} + \sum_{h=0}^{t} W^h Q \hat{A}_{t-h}. \tag{3.6}$$

Since $W^t = P\mu^t P^{-1}$ and the root μ_2 exceeds $(\beta^*)^{-1} > 1$, it follows that the system is on an explosive path and thus violates the transversality condition for arbitrary $\hat{\lambda}_0$. There is a specific value of the initial shadow price $\hat{\lambda}_0$, however, that results in (3.6) satisfying the transversality condition (2.15). This particular solution specifies the unique optimal (and competitive equilibrium) time path of capital accumulation $\{\hat{k}_t\}_{t=0}^{\infty}$ and shadow prices $\{\hat{\lambda}_t\}_{t=0}^{\infty}$. Given these optimal sequences, consumption $\{\hat{c}_t\}_{t=0}^{\infty}$ and effort $\{\hat{N}_t\}_{t=0}^{\infty}$ can be computed from (3.1) and (3.2). It is also direct to compute variations in output, investment, real wages and real interest rates. For example, output variations are given by $\hat{y}_t = \hat{A}_t + \alpha \hat{N}_t + (1-\alpha)\hat{k}_t$ in (3.4). With Cobb–Douglas production, real wages are proportional to labor productivity, so that $\hat{w}_t = \hat{y}_t - \hat{N}_t$.

In general, optimal decisions for consumption, capital, effort, etc. depend on the entire sequence $\{\hat{A}_t\}_{t=0}^{\infty}$. As demonstrated in the technical appendix, the time path of efficient capital accumulation may be written in the form

$$\hat{k}_{t+1} = \mu_1 \hat{k}_t + \psi_1 \hat{A}_t + \psi_2 \sum_{j=0}^{\infty} \mu_2^{-j} \hat{A}_{t+j+1}, \tag{3.7}$$

where ψ_1 and ψ_2 are complicated functions of the underlying parameters of preferences and technology. The dynamics of capital accumulation depend on the previous period's capital stock with coefficient μ_1. In addition, with time-varying total factor productivity, the optimal solution for capital accumulation depends on the current productivity level $(\hat{A}_t)$ and on the entire future time path of displacements to productivity 'discounted' by μ_2.

3.2. Transition path dynamics

In order to provide a quantitative evaluation of the dynamic properties of the neoclassical model we choose a set of parameters values that match the average U.S. growth experience. The properties of the transition paths to the steady state capital stock (k) can then be numerically investigated by setting $A_t = A$ for all t. In this case, the (approximately) optimal sequence of transformed capital stocks described by (3.7) reduces to the first-order difference equation $\hat{k}_{t+1} = \mu_1 \hat{k}_t$, with $|\mu_1| < 1$. Given an initial condition $k_0 = K_0/X_0$, the transformed economy's capital stock approaches its steady state value more quickly the closer μ_1 is to zero. In addition, the variations in consumption, investment, output, work effort, the real wage and the real interest rate are determined according to linear relations:

$$\hat{c}_t = \pi_{ck}\hat{k}_t, \qquad \hat{i}_t = \pi_{ik}\hat{k}_t, \qquad \hat{y}_t = \pi_{yk}\hat{k}_t,$$

$$\hat{N}_t = \pi_{Nk}\hat{k}_t, \qquad \hat{w}_t = \pi_{wk}\hat{k}_t, \qquad r_t - r = \pi_{rk}\hat{k}_t, \tag{3.8}$$

where r is the steady state real interest rate, $r = \gamma_X/\beta^* - 1$. Except for π_{rk}, the π coefficients should be interpreted as the elasticities of the flow variables with respect to deviations of the capital stock from its stationary value. The transition paths of these flow variables, therefore, are simply scaled versions of the capital stock's transition path. In general, the values of μ_1 and the π coefficients are complicated functions of the underlying parameters of the model, i.e., α, σ, δ_K, β and γ_X.

3.2.1. A fixed labor experiment

Within the neoclassical model with fixed labor, variations in σ alter substitution over time. Table 1 summarizes the quantitative effects of varying σ on the adjustment parameter μ_1 and the π coefficients.[17] The values of the underlying parameters assume that the time interval is a quarter and are summarized in the table. Labor's share $\alpha = 0.58$ is the average ratio of total employee compensation to GNP for the period 1948 to 1986; γ_X is one plus the common trend rate of growth of output, consumption and investment, which is 1.6% per year in the post-war era.[18] The value for $\beta^* = \gamma_X/(1+r)$ is chosen to yield a return to capital of 6.5% per annum, which is the average real return to equity from 1948 to 1981.[19] Finally, the depreciation rate is set at 10% per annum, which leads to a share of gross investment of 0.295.

In the fixed labor model, some of the π coefficients are invariant to σ. The elasticities of output and real wages with respect to capital are simply determined by $\pi_{yk} = \pi_{wk} = (1-\alpha)$ which is 0.42 in our table 1 example. The value of $\pi_{rk} = \eta_k$ is also invariant to σ and takes the value -0.024. This means that output and real wages move directly with capital and real interest rates inversely with capital.

In the case of log utility ($\sigma = 1$), the table shows that the adjustment coefficient (μ_1) is 0.966 which implies that one-half of any initial deviation from the stationary state is worked off in 20 quarters or 5 years. If the capital

[17] Tedious algebra shows that $\pi_{ck} = [(1-\alpha) - (\gamma_X\mu_1 - (1-\delta_K))(k/y)]/s_c$ and $\pi_{ik} = [(\gamma_X\mu_1 - (1-\delta_K))(k/y)]/s_i$. It is direct that $\pi_{Nk} = 0$ and $\pi_{yk} = (1-\alpha)$. Finally, μ_1 is the smaller root of the quadratic equation $\mu^2 - [1/\beta^* - s_c\eta_K/\sigma s_i\phi + 1]\mu + 1/\beta^*$.

[18] Details of this computation and the data used are discussed in section 5.2.

[19] Note that while β^* is invariant with respect to σ under the assumption that $\beta^* = \gamma_X/(1+r)$, β is not since $\beta^* = \beta\gamma_X^{1-\sigma}$.

Table 1

Effects of intertemporal substitution in consumption on near steady state dynamics (fixed labor model).

	Lower substitution						Higher substitution		
σ	10	5	2	1.5	1.0	0.67	0.5	0.2	0.1
μ_1	0.992	0.987	0.977	0.973	0.966	0.958	0.950	0.919	0.886
π_{ik}	0.726	0.557	0.213	0.066	−0.179	−0.474	−0.729	−1.798	−2.963
π_{ck}	0.292	0.363	0.507	0.568	0.670	0.793	0.900	1.346	1.832
Half-life[a] (in quarters)	86	54	30	25	20	16	14	8	6

Baseline parameter values:		*Implied value of:*	
Labor's share (α)	0.58	$-\eta_k$	0.024
Rate of depreciation (δ_K)	0.025	c/k	0.069
Utility discount rate (β^*)	0.988	i/y	0.295
Technological growth rate ($\gamma_X - 1$)	0.004	$r = [\gamma_X/\beta^* - 1]$	0.016

[a] Half-life is defined by the solution to $\mu_1^n = 0.5$, rounded to the nearest integer.

stock is initially below its steady state value, then investment is above its steady state rate ($\pi_{ik} = -0.176 < 0$) and consumption is below its steady state rate ($\pi_{ck} = 0.670 > 0$).

Alternative values of σ change π_{ck}, π_{ik} and μ_1 in intuitive ways. For example, when σ is large the representative agent is less willing to substitute intertemporally and thus desires very smooth consumption profiles. Hence, there is little reaction of consumption to a shortfall in capital (π_{ck} small). Consequently the adjustment to the steady state is slower (μ_1 closer to 1.0) than when $\sigma = 1.0$. When σ is small, there is more willingness to substitute consumption intertemporally and thus a given capital shortfall occasions a larger reduction in consumption. There is thus a more rapid adjustment of capital (μ_1 further from 1.0) than with $\sigma = 1$.

3.2.2. *Varying work effort*

We are also interested in the pattern of efficient variation in work effort along the transition path, how the labor–leisure margin alters the speed of capital stock adjustment (μ_1) and the responses of the price and quantity variables. To investigate these effects quantitatively, we reinstate labor as a choice variable and suppose that the utility function has the simple form $u(c, L) = \log(c) + \theta_l \log(L)$. The parameter θ_l is chosen so that stationary hours are 0.20.[20] Our choice of this value is based on the average percentage of time devoted to market work in the U.S. during the period 1948–1986.[21]

The resulting value of μ_1 is 0.953, implying a half-life of just under 14 quarters for deviations of the capital stock from its stationary level. This is a slightly more rapid pace of adjustment than the comparable fixed labor case with $\sigma = 1$ in table 1, since work effort provides an additional margin along which agents can respond. The values of the elasticities are $\pi_{ck} = 0.617$, $\pi_{ik} = -0.629$, $\pi_{Nk} = -0.294$, $\pi_{yk} = 0.249$, $\pi_{wk} = 0.544$ and $\pi_{rk} = -0.029$. Transition paths of the key variables are plotted in fig. 1. Starting from an initially low capital stock, there is a sustained period in which output and consumption are low, but rising, while work effort and investment are high, but declining. Temporary variation in work effort is efficient even though steady state hours are invariant to growth.

The economic mechanisms behind these transition paths are important. The initially low capital stock has three implications for the representative consumer in the transformed economy. First, non-human wealth is low relative to its stationary level. Second, the marginal product of labor (shadow real wage)

[20] In our computations, we directly specify that $N = 0.20$ in the linear expressions (3.1) and (3.2), noting that logarithmic utility implies zero cross elasticities and unitary own elasticities. This implicitly specifies the utility function parameter θ_l.

[21] This value is equal to the average work week as a fraction of total weekly hours for the period 1948 to 1986.

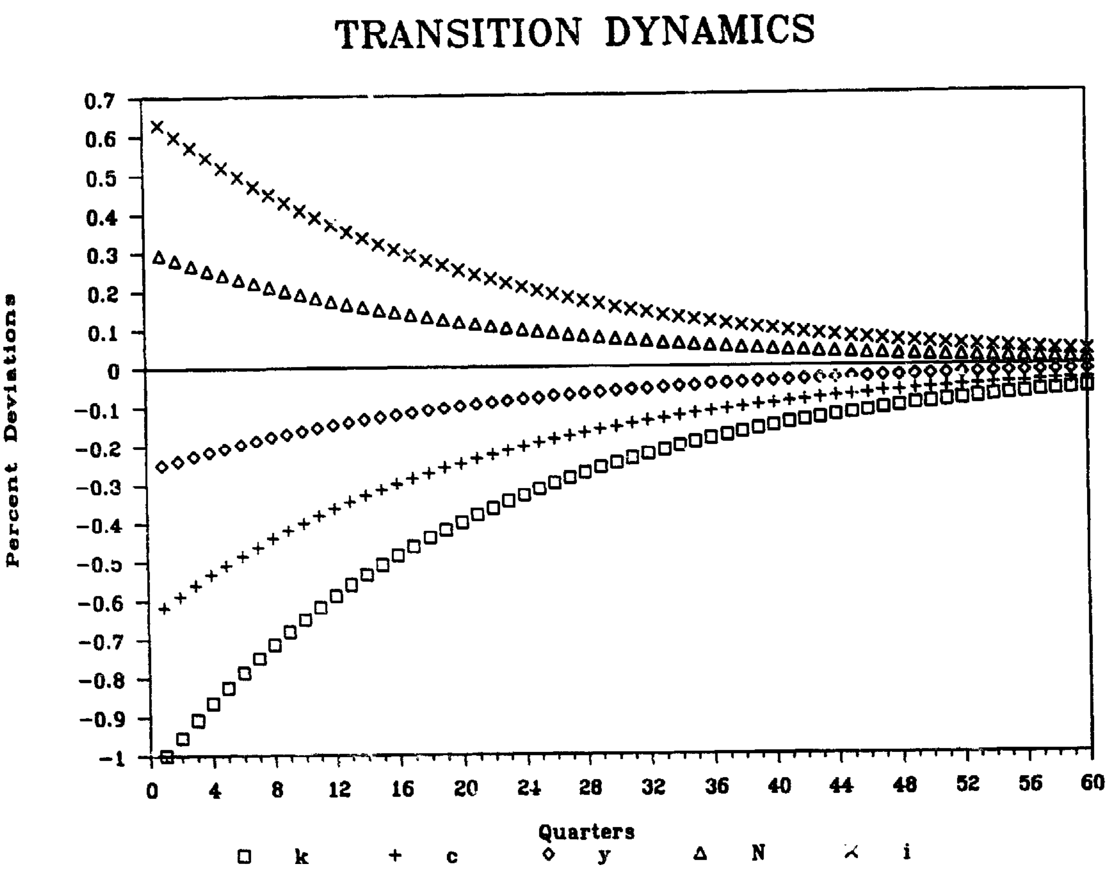


Fig. 1

is low relative to the stationary level. Third, the marginal product of capital (shadow real interest rate) is high relative to its stationary level. The first and third factors induce the representative consumer to work additional hours; the second factor exerts the opposite influence. With the particular preferences and technology under study, the former factors dominate, resulting in hours that are high – relative to the stationary level – along the transition path from a low initial capital stock.

It is beyond the scope of this paper to undertake a detailed sensitivity analysis of how the μ and π coefficients change with parameters of the environment. However, we have studied how the root μ_1 depends on a list of parameter values by computing an elasticity of μ_1 with respect to each parameter.[22] The elasticities are quite small ranging from -0.11 for labor's share (α) to -0.001 for the rate of technological progress ($\gamma_X - 1$).[23] Our

[22]We thank Adrian Pagan for pushing us to conduct these experiments.

[23]The elasticity for steady state hours (N) is 0.003; for depreciation (δ_K) is -0.03 for the intertemporal elasticity of substitution (σ) is 0.03, and for the elasticity of the marginal utility of leisure ($LD^2v(L)/Dv(L)$) is 0.003.

conclusion is that the speed of adjustment is not highly sensitive to the choice of parameter values.

4. Real business cycles

This section follows the pioneering work of Kydland and Prescott (1982) and Long and Plosser (1983) by incorporating uncertainty – in the form of temporary productivity shocks – into the basic neoclassical model. Although other aspects of the underlying economic environment are identical to those of the preceding section, the business cycle analysis is in marked contrast to the standard 'growth theory' analysis, in which time variation in technology is taken to be smooth, deterministic and permanent.

4.1. Linear business cycle models

In principle, quantitative analyses of stochastic elements should follow Brock and Mirman's (1972) seminal analysis of the basic neoclassical model under uncertainty. One would begin by postulating a specific stationary stochastic process for technology shocks, calculate the equilibrium laws of motion for state variables (the capital stock) and related optimal policy functions for controls (consumption, investment and work effort). It would then be natural to interpret observed business fluctuations in terms of the economy's stationary distribution. The principle barrier to the execution of this strategy is computational. The equilibrium laws of motion for capital and for flows cannot be calculated exactly for models of interest, but must be approximated with methods that are computationally burdensome.[24] Furthermore, computational strategies for approximate suboptimal equilibria are not well developed.

In our analysis we invoke certainty equivalence, employing a linear systems perspective. Our use of certainty equivalence methods in the study of real business cycles builds on prior work by Kydland and Prescott (1982), but the details of our procedures are different.[25] An advantage of our method is that it

[24] Examples include Sargent (1980) and Greenwood, Hercowitz and Huffman (1986).

[25] Kydland and Prescott (1982) eliminate non-linearities in constraints (such as the production function) by substituting resource constraints into the utility function and taking a quadratic approximation to the resulting return function. We derive efficiency conditions under certainty and approximate these to obtain linear decision rules. These two procedures are equivalent for the class of models we consider when standard Taylor series approximations are used with each procedure. The only substantive difference between our approximation method and Kydland and Prescott's is that while they search for an approximation based on a likely range of variation of the different variables, we center our linearizations on the steady state. According to Kydland and Prescott (1982, p. 1357, 11) this difference in approximation techniques has little impact on their results. Our procedure yields formulas that have a transparent economic interpretation and allows us to replicate exactly the Long and Plosser (1983) closed form solution.

is readily extended to the study of suboptimal dynamic equilibria, as we show in our second essay. Nevertheless, a detailed analysis of the overall accuracy of these approximation methods in a business cycle context remains to be undertaken.

For the basic neoclassical model, our strategy works as follows. We develop approximate solutions for capital and other variables near the stationary point of the transformed economy as in the previous section. Then, working from a certainty equivalence perspective, we posit a particular stochastic process for $\hat{A}$ and replace the sequence $\{\hat{A}_{t+j}\}_{j=0}^{\infty}$ with its conditional expectation given information available at t. In particular, suppose that $\hat{A}_t$ follows a first-order autoregressive process with parameter ρ. Then, given (3.7), the state dynamics are given by the linear system

$$s_{t+1} \equiv \begin{bmatrix} \hat{k}_{t+1} \\ \hat{A}_{t+1} \end{bmatrix} = \begin{bmatrix} \mu_1 & \pi_{kA} \\ 0 & \rho \end{bmatrix} \begin{bmatrix} \hat{k}_t \\ \hat{A}_t \end{bmatrix} + \begin{bmatrix} 0 \\ \varepsilon_{A,t+1} \end{bmatrix} = Ms_t + \varepsilon_{t+1}, \tag{4.1}$$

where $\pi_{kA} = \psi_1 + \psi_2\rho/(1-\rho\mu_2^{-1})$ and $s_t' \equiv (\hat{k}_t, \hat{A}_t)$ is the state vector.

Additional linear equations specify how consumption, work effort, investment, shadow prices and output depend on the state variables s_t. Let the vector $z_t' = (\hat{c}_t, \hat{N}_t, \hat{y}_t, \hat{i}_t)$ be a vector of controls and other flow variables of interest. Then the linear equations relating flows to states are

$$z_t = \begin{bmatrix} \hat{c}_t \\ \hat{N}_t \\ \hat{y}_t \\ \hat{i}_t \\ \hat{w}_t \\ r_t - r \end{bmatrix} = \begin{bmatrix} \pi_{ck} & \pi_{cA} \\ \pi_{Nk} & \pi_{NA} \\ \pi_{yk} & \pi_{yA} \\ \pi_{ik} & \pi_{iA} \\ \pi_{wk} & \pi_{wA} \\ \pi_{rk} & \pi_{rA} \end{bmatrix} \begin{bmatrix} \hat{k}_t \\ \hat{A}_t \end{bmatrix} = \Pi s_t, \tag{4.2}$$

where the π coefficients are determined, as in section 3, by requiring that the shadow prices and elements of z_t satisfy the linearized first-order conditions.[26]

[26] This state space formulation (4.1) and (4.2) can be solved to obtain the vector autoregressive-moving average (ARMA) representation of the endogenous variables z. In the basic neoclassical model with persistent technology shocks ($\rho \neq 0$), each element of z_t is ARMA (2,1) with common autoregressive but different moving average polynomials. Following Zellner and Palm (1974) and Orcutt (1948), the evolution of states can be expressed as follows

$$\det(I - MB)s_t = \operatorname{adj}(I - MB)\varepsilon_t,$$

where B is the backshift operator, $\det(I-MB)$ is the determinant of the 2×2 matrix defined by $I - MB$, and $\operatorname{adj}(I-MB)$ is the adjoint of $I - MB$. From inspection of (4.1) it is clear that, for $\rho \neq 0$, the determinant of $(I - MB)$ is a second-order polynomial $(1-\mu_1 B)(1-\rho B)$. There are moving average terms of at most order 1 in $\operatorname{adj}(I - MB)$. Further, since $z_t = \Pi s_t$, the elements of z_t inherit the ARMA (2,1) structure of the state vector. The relatively simple ARMA structure of the individual elements of z is a result of the dimensionality of the state vector. In a model with many state variables the order of the polynomial $\det(I - MB)$ could become quite large, implying more complex ARMA representations for the elements of z.

This formulation facilitates computation of (i) impulse response functions for the system and (ii) population moments of the joint (z_t, s_t) process.

Impulse responses. Impulse response functions provide information on the system's average conditional response to a technology shock at date t, given the posited stochastic process. The response of the system in period $t+k$ to a technology impulse at date $t+1$ is

$$s_{t+k} - \mathrm{E}s_{t+k}|s_t = M^{k-1}\varepsilon_{t+1},$$

$$z_{t+k} - \mathrm{E}z_{t+k}|s_t = \Pi M^{k-1}\varepsilon_{t+1},$$

where $\varepsilon'_{t+1} = (0, \varepsilon_{A,t+1})$.

Population moments. Population moments provide additional, unconditional properties of the time series generated by the model economy. We stress that although there is a single shock to the economic model under study, the dynamic character of the model means that the unconditional time series will, in general, not be perfectly correlated. The linear character of the system implies that it is relatively straightforward to calculate population moments. For example, given the variance–covariance matrix of the states, $\Sigma_{ss} = \mathrm{E}(s_t s_t')$, it is easy to calculate the autocovariance of z at lag j, $\mathrm{E}(z_t z'_{t-j}) = \Pi M^j \Sigma_{ss} \Pi'$. In our analysis below, we will be concerned with how these properties of the model change as we alter parameters of preferences and technology.

4.2. Alternative parameterizations of the basic neoclassical model

We explore four alternative parameterizations of the basic neoclassical model, obtained by varying certain aspects of preferences and technology. Though far from exhaustive, these parameterizations shed some light on important aspects of neoclassical models. Table 2 summarizes the parameter values that are employed in our four versions of the neoclassical model. Throughout, as in table 1, we use production parameter values for labor's share as $\alpha = 0.58$ and the growth of exogenous technical progress as $(\gamma_X - 1) = 0.004$ per quarter. In all specifications, we take the momentary utility function to be of the additively separable form, $u(c, L) = \log(c) + \theta_l v(L)$. This specification implies zero cross-elasticities ($\xi_{lc} = \xi_{cl} = 0$) and unitary elasticity in consumption ($\sigma = -\xi_{cc} = 1$), while leaving the elasticity of the marginal utility of leisure with respect to leisure (ξ_{ll}) as a parameter to be specified. The parameter θ_l in all parameterizations is adjusted so as to yield a steady state value for N equal to 0.20, the average time devoted to market work in the U.S. during the period 1948–1986. In all of these formulations, the values of σ, γ_X and β combine to yield a steady state real interest rate of 6.5% per annum.

Table 2

Alternative model parameterizations.

Key parameters	(1) Long–Plosser with 100% depreciation	(2) Long–Plosser with realistic depreciation	(3) Panel data labor supply elasticity	(4) Infinite labor supply elasticity
Preference parameters				
Elasticity of marginal utility of consumption with respect to:				
Consumption (ξ_{cc})	−1	−1	−1	−1
Leisure (ξ_{cl})	0	0	0	0
Elasticity of marginal utility of leisure with respect to:				
Consumption (ξ_{lc})	0	0	0	0
Leisure (ξ_{ll})	−1	−1	−10	0
Steady state fraction of time worked (N)	0.20	0.20	0.20	0.20
Utility discount rate (β)	0.988	0.988	0.988	0.988
Technological parameters				
Labor's share of production (α)	0.58	0.58	0.58	0.58
Technological growth rate ($\gamma_X - 1$)	0.004	0.004	0.004	0.004
Depreciation rate of capital (δ_K)	1	0.025	0.025	0.025

Our point of departure is the parameterization of Long and Plosser (1983). The key features of this specification are additively separable, logarithmic preferences, a Cobb–Douglas production function and 100% depreciation. This specification is instructive because there is an exact closed-form solution that enables us to establish a benchmark for judging our approximation methods. The second specification alters the Long–Plosser formulation by assuming less than 100% depreciation. This alteration is sufficient to obtain stochastic properties for key variables that are more compatible with common views of economic fluctuations. We refer to this case as the 'baseline' model – it is closely related to the divisible labor economy studied by Hansen (1985).[27] The next two experiments consider some perturbations of the elasticity of labor supply. The third parameterization uses an 'upper bound' labor supply elasticity from the panel data studies reviewed by Pencavel (1986). This elasticity is ten times smaller than that imposed by the logarithmic preferences of the baseline mode.[28] The fourth parameterization illustrates the consequences of infinite intertemporal substitutability of leisure or, equivalently, the indivisibility of individual labor supply decisions stressed by Rogerson (1988) and Hansen (1985).

4.3. Quantitative linear business cycle models

The reference point for our discussion is table 3, which summarizes the linear systems representation given in eqs. (4.1) and (4.2). That is, table 3 provides the coefficients, μ_1, ρ, π_{kA} of the matrix M and the coefficients of the Π matrix under two assumptions about persistence of technology shocks ($\rho = 0$ and $\rho = 0.9$).

Long–Plosser with complete depreciation. Applying the exact solutions found in Long and Plosser (1983), the capital stock for this parameterization evolves

[27]There are at least three differences between our methodology and that employed by Hansen (1985) which make our results not directly comparable. First, we use a different linearization technique, as discussed above. Second, we compute the population movements rather than estimate them through Monte Carlo simulation. Third, we do not filter the series with the Hodrick and Prescott (1980) filter. See footnote 31 for a discussion of differences in parameter values and of the effects of the Hodrick and Prescott filter.

[28]For preferences separable in consumption and leisure, the elasticity of labor supply is $(1 - 1/N)/\xi_{ll}$, where N is the steady state fraction of time devoted to work. Thus if the elasticity of labor supply is 0.4 and $N = 0.20$, then $\xi_{ll} = -10.0$.

We are reluctant to adopt this economy as our benchmark given the difficulty in interpreting the disparity between the elasticity of labor supply of women and men in the context of our representative agent economy. Furthermore, Rogerson (1988) has demonstrated that, in the presence of indivisibility in individual labor supply decisions, an economy with finite elasticity of labor supply may behave as if this elasticity were infinite. Hence, our fourth parameterization has preferences consistent with an infinite elasticity of labor supply ($\xi_{ll} = 0$).

Table 3

Parameter values of the linear system (4.1)–(4.2).

	ρ	μ_1	π_{kA}	π_{ck}	π_{cA}	π_{Nk}	π_{NA}	π_{yk}	π_{yA}	π_{ik}	π_{iA}	π_{wk}	π_{wA}	π_{rk}	π_{rA}
Long–Plosser with complete depreciation	0	0.420	1.000	0.420	1.000	0.000	0.000	0.420	1.000	0.420	1.000	0.420	1.000	−0.244	−0.580
	0.9	0.420	1.000	0.420	1.000	0.000	0.000	0.420	1.000	0.420	1.000	0.420	1.000	−0.244	0.320
Long–Plosser with realistic depreciation	0	0.953	0.166	0.617	0.108	−0.294	1.332	0.249	1.773	−0.629	5.747	0.544	0.441	−0.029	−0.005
	0.9	0.953	0.137	0.617	0.298	−0.294	1.048	0.249	1.608	−0.629	4.733	0.544	0.560	−0.029	0.055
Panel data labor supply elasticity	0	0.963	0.111	0.654	0.075	−0.080	0.317	0.374	1.184	−0.296	3.830	0.454	0.867	−0.025	−0.003
	0.9	0.963	0.097	0.654	0.235	−0.080	0.262	0.374	1.152	−0.296	3.341	0.454	0.890	−0.025	0.040
Rogerson–Hansen infinite labor supply elasticity	0	0.947	0.206	0.598	0.130	−0.424	2.071	0.174	2.201	−0.838	7.143	0.598	0.130	−0.032	−0.007
	0.9	0.947	0.164	0.598	0.337	−0.424	1.579	0.174	1.916	−0.838	5.683	0.598	0.337	−0.032	0.065

according to the stochastic difference equation,

$$\hat{k}_{t+1} = (1-\alpha)\hat{k}_t + \hat{A}_t = \sum_{j=0}^{\infty} (1-\alpha)^j \hat{A}_{t-j}, \tag{4.3}$$

which indicates that in our approximation it should be the case that $\mu_1 = (1-\alpha)$ and $\pi_{kA} = 1.0$. As emphasized by Long and Plosser, (4.3) illustrates that even without long-lived commodities, capitalistic production enables agents to propagate purely transitory productivity shocks forward in time in keeping with their preferences for smooth consumption.

The solutions of Long and Plosser also imply that there are simple log-linear relations for the flow variables ($\hat{y}$, $\hat{c}$, $\hat{i}$ and $\hat{N}$),

$$\hat{y}_t = \hat{i}_t = \hat{c}_t = (1-\alpha)\hat{k}_t + \hat{A}_t = \sum_{j=0}^{\infty} (1-\alpha)^j \hat{A}_{t-j}, \tag{4.4}$$

$$\hat{N}_t = 0. \tag{4.5}$$

In percent deviations from steady state, output, consumption, and investment all share the stochastic structure of the capital stock. Work effort, on the other hand, is constant (i.e., $\hat{N}_t = 0$). With work effort constant, real wages (proportional to output per man hour) move just like output. With $\sigma = 1$, interest rates are equal to the expected change in consumption ($r_t - r = \mathrm{E}_t\hat{c}_{t+1} - \hat{c}_t$). Thus, in terms of (4.2), $\pi_{yk} = \pi_{ck} = \pi_{ik} = \pi_{wk} = (1-\alpha)$, $\pi_{yA} = \pi_{cA} = \pi_{iA} = \pi_{Nk} = 1$, and $\pi_{Nk} = \pi_{NA} = 0$. Finally, $\pi_{rk} = -\alpha(1-\alpha)$ and $\pi_{rA} = (\rho - \alpha)\hat{A}_t$.

Turning to the approximate solutions reported in table 3, we see that these match the exact solutions (4.3)–(4.5) for the parameter values in table 2. For example, with $\alpha = 0.58$, the coefficient $\mu_1 = (1-\alpha) = 0.42$ as required by eq. (4.1) above. Further, we see that there are two special features of this parameterization previously noted by Long and Plosser in their consideration of multi-sector, log-linear business cycle models. First, the solution involves no influence of expected future technological conditions on the properties of the endogenous variables. This conclusion follows from the observation that the linear systems coefficients linking quantity variables to technology (π_{kA}, π_{cA}, π_{NA}, etc.) are invariant to the persistence (ρ) in the technology shock process. Second, the relation between work effort and the state variables (π_{Nk} and π_{NA}) indicates that the approximation method preserves the other special implications of complete depreciation, namely that effort is unresponsive to the state of the economy ($\pi_{NA} = \pi_{Nk} = 0$).

Fundamentally, each of these invariance results reflects a special balancing of income and substitution effects. For example, more favorable technology conditions ($\hat{A}_{t+j} > 0$) exert two offsetting effects on accumulation: (i) an income effect (since there will be more outputs at given levels of capital input) that operates to lower saving and capital accumulation and (ii) a substitution effect (arising from an increased marginal reward to accumulation) that operates to raise saving. With complete depreciation and logarithmic utility, income and substitution effects exactly offset.

With respect to real interest rates, the complete depreciation model also helps indicate how serial correlation in $\hat{A}$ alters the model's implications. The coefficient $\pi_{rA} = (\rho - \alpha)$, so that with $\rho = 0$ diminishing returns predominates and an impulse to A lowers the rate of return. But with high persistence ($\rho > \alpha$), interest rates rise due to the shift up in the future marginal reward to investment.

Long-Plosser with realistic depreciation. Adopting a more realistic depreciation rate ($\delta_K = 0.025$ or 10% per year) dramatically alters the properties of the basic neoclassical model. The adjustment parameter μ_1 rises from 0.42 to 0.953, indicating that the capital stock adjusts more slowly. Second, π_{kA} falls from 1.0 to 0.166 when $\rho = 0$ and is no longer invariant to serial correlation properties of $\hat{A}$.

These responses can be explained in terms of the basic economics of lowering the depreciation rate. First, when there is a lower depreciation rate, it follows that there is a higher steady state capital stock and a lower output–capital ratio. As δ_K goes from 1.0 to 0.025, y/k falls from 2.4 to 0.10. This suggests a substantial decline in the elasticity π_{kA}. Second, the change in μ_1 and the sensitivity of π_{kA} to ρ reflect implications that δ_K has for the relative importance of wealth and intertemporal substitution effects. With lower depreciation, the intertemporal technology – linking consumption today and consumption tomorrow – becomes more linear near the stationary point.[29] This means that the representative agent faces less sharply diminishing returns in intertemporal production possibilities and will choose a temporally smooth consumption profile that requires more gradual elimination of deviations of the capital stock from its stationary level (μ_1 rises from 0.42 when $\delta_K = 1$ to 0.953 when $\delta_K = 0.025$). The depreciation rate also impinges on the relative importance of substitution and wealth effects associated with future shifts in technology ($\hat{A}_{t+j}$ for $j > 0$). In particular, the dominance of the wealth effect is indicated by a comparison of purely temporary ($\rho = 0$) with more persistent technology shocks. Capital accumulation is less responsive to technological conditions when the shocks are more persistent (i.e., π_{kA} falls from 0.166 to 0.137 when ρ rises from 0 to 0.9). For the same reason, more persistent technology shocks imply that consumption is more responsive ($\pi_{cA} = 0.108$ when $\rho = 0$ and $\pi_{cA} = 0.298$ when $\rho = 0.9$) and investment is less responsive

[29] There is a marked decline in the elasticity of the gross marginal product of capital schedule, $AD_1F(k, N) + (1 - \delta_K)$, with respect to capital. It falls from $-\eta_k = 0.58$ to 0.023.

($\pi_{iA} = 5.747$ when $\rho = 0$ and $\pi_{iA} = 4.733$ when $\rho = 0.9$). The persistence of shocks also has implications for the response of relative prices to technology shifts. Real wages respond more elastically, since there is a smaller variation in effort when shifts are more permanent. As in the model with complete depreciation, real interest rates respond positively to technology shifts when these are highly persistent.

Altering the character of intertemporal tradeoffs also has implications for labor supply via intertemporal substitution channels. When technology shifts are purely temporary ($\rho = 0$), a one percent change in total factor productivity calls forth a 1.33 percent change in hours. This impact is attenuated, but not eliminated, when shifts in technology are more persistent ($\pi_{NA} = 1.05$ when $\rho = 0.9$). The nature of these intertemporal substitution responses is perhaps best illustrated by examining impulse response functions, which are derived from the coefficients presented in table 3. The two parts of fig. 2 contain impulse responses under our alternative assumptions about the persistence of shocks. In panel A, when technology shifts are purely temporary, intertemporal substitution in leisure is very evident. In the initial period, with positive one percent technology shock, there is a major expansion of work effort. The initial period output response is more than one-for-one with $\hat{A}$ ($\pi_{yA} = 1.77$) because of the expansion in work effort. The bulk of the output increase goes into investment with a smaller percentage change in consumption.

In subsequent periods, after the direct effect of the technology shift has dissipated, the only heritage is a capital stock higher than its steady state value. The change in the capital stock induced by the initial period technology shock is 'worked off' via a combination of increased consumption and reduced effort. The impacts on output are smaller, in percentage terms, than the impacts on consumption or capital, because the transition path back toward the stationary point is associated with negative net investment and negative response of effort. This means that the response function after one period in fig. 2, panel A, is determined by the internal transition dynamics given in fig. 1. The only difference is that in fig. 2 the experiment is a positive increment to the capital stock of 0.166 instead of the negative increment of -1.0. in fig. 1.

In panel B of fig. 2, when technology shifts are more persistent, the impulse responses involve a combination of exogenous ($\hat{A}$) and endogenous dynamics ($\hat{k}$). There is now a protracted period in which technology shocks serve to introduce positive comovements of hours, output, consumption and investment. The magnitudes of these responses are altered by the fact that agents understand the persistent character of technological shifts. In comparison with the case where technology shifts are purely temporary, consumption is more responsive to $\hat{A}$ while effort is less.

Other labor supply elasticities. First, when we restrict preferences to be consistent with an 'upper bound' labor supply elasticity of 0.4 for prime age

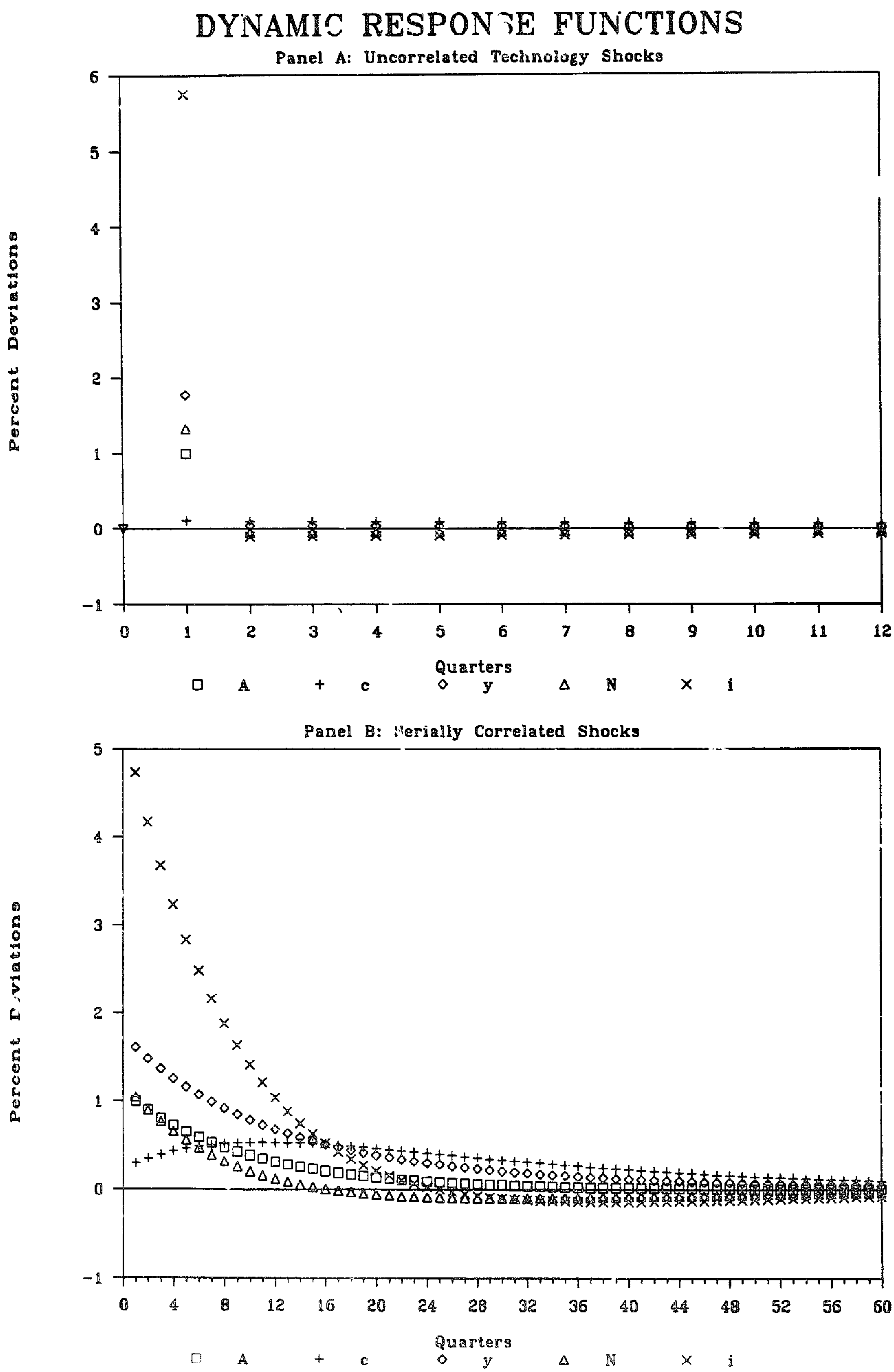

DYNAMIC RESPONSE FUNCTIONS
Panel A: Uncorrelated Technology Shocks
Percent Deviations
Quarters
A
c
y
N
i
Panel B: Serially Correlated Shocks
Percent Deviations
Quarters
A
c
y
N
i

Fig. 2

males reported by Pencavel (1986), we obtain an economy whose dynamics are broadly similar to those of the baseline model except for the amplitude of response to technology shocks. In the case of purely temporary production shocks ($\rho = 0$), the elasticity of response of labor to shocks in technology (π_{NA}) is 0.317, roughly one fourth of the value of π_{NA} for the baseline model. This reduced variability in labor is accompanied by smaller variability in consumption and investment.[30]

Second, when the labor supply elasticity is infinite, we have an economy that is the mirror image of the previous one in terms of amplitude of response to shocks. In the case of purely temporary shocks, the values of π_{cA} and π_{iA} are roughly 1.2 times those of the baseline model, while π_{NA} is fifty percent higher.

5. Implications for time series

This section develops some of the predictions that the basic neoclassical model makes about economic time series when it is driven by a single technology shock. Using the model's organizing principles, we also present some summary statistics for post-war quarterly U.S. time series.

5.1. *Variability of components of output*

A major feature of economic fluctuations is the differential variability in the use of inputs (labor and capital) and in the components of output (consumption and investment). Table 4 presents some selected population moments for the four alternative parameterizations that summarize the models' implications for relative variability.

The specification with complete depreciation has implications that are readily traced to the simple structure of (4.3) and (4.4). First, output, consumption and investment have identical variances. Second, with complete depreciation, investment and capital differ only in terms of timing, so that capital and output are equally variable.

When realistic depreciation is imposed ($\delta_K = 0.025$), differences in the relative variability of the components of output are introduced. Further, these implications depend on the stochastic process for the technology shifts, since the moments of time series depend on the linear system coefficients reported in table 3 (which are dependent on the persistence parameter ρ). With purely temporary shocks, consumption is much less variable than output (about two tenths as variable) and investment is far more variable (more than three times

[30] A productivity shock induces intertemporal substitution of leisure by raising the productivity of current versus future labor and intratemporal substitution by increasing the opportunity cost of leisure in terms of consumption. Both the elasticity of intertemporal substitution of leisure and the elasticity of intratemporal substitution are smaller in this economy than in the baseline model. The reduction in the degree of substitution contributes to a reduced variability of consumption.

Table 4

Selected population moments for four alternative parameterizations.

		Standard deviation relative to technology ($\hat{A}$)				Standard deviation relative to output ($\hat{y}$)			Correlation with output ($\hat{y}$)		
	ρ	$\hat{y}$	$\hat{c}$	$\hat{i}$	$\hat{N}$	$\hat{c}$	$\hat{i}$	$\hat{N}$	$\hat{c}$	$\hat{i}$	$\hat{N}$
Long–Plosser with complete depreciation	0	1.10	1.10	1.10	0.0	1.0	1.0	0.0	1.0	1.0	—
	0.9	1.64	1.64	1.64	0.0	1.0	1.0	0.0	1.0	1.0	—
Long–Plosser with realistic depreciation	0	1.78	0.35	5.76	1.34	0.20	3.24	0.75	0.38	0.99	0.98
	0.9	1.86	0.19	4.28	0.89	0.64	2.31	0.48	0.82	0.92	0.79
Panel data labor supply elasticity	0	1.19	0.28	3.83	0.32	0.23	3.21	0.27	0.39	0.99	0.97
	0.9	1.46	1.01	3.17	0.23	0.69	2.17	0.16	0.85	0.92	0.75
Infinite labor supply elasticity	0	2.20	0.41	7.16	2.09	0.18	3.25	0.95	0.37	0.99	0.98
	0.9	2.11	1.30	5.01	1.32	0.62	2.38	0.63	0.80	0.93	0.81

as variable). Labor input is much more variable than consumption and about three fourths as variable as output.

When shifts in technology become more persistent ($\rho = 0.9$), there are important changes in implications for relative variabilities. Consumption is now six tenths as volatile as output, which accords with the permanent income perspective and with the altered linear systems coefficients discussed previously. Labor input is less than half as volatile as output, which fundamentally reflects diminished desirability of intertemporal substitution of effort with more persistent shocks.[31]

Alterations in the labor supply elasticity exert predictable effects on relative variability of labor input and output, while having relatively minor implications for the relative variability of the components of output. Relative to the baseline model, the reduction in labor supply elasticity to the level suggested by the panel data studies results in a decline of the variability of labor both in absolute terms and in relation to the variability of output. The relative volatility of the labor input in terms of output implied by the model is 0.27, roughly half of the standard deviation of hours relative to detrended output in the U.S. for the period 1948–1986.[32]

In table 5 we present some additional time series implications of our baseline neoclassical model. One notable feature is that $\hat{y}$, $\hat{i}$ and $\hat{N}$ exhibit almost no serial correlation in the absence of serially correlated technology shocks. This is not true for consumption, wages or interest rates, however, which are smoother and correlated with lagged values of output.

[31]The baseline model is structurally identical to the divisible labor economy studied by Hansen (1985). It differs, however, in values assigned to parameters. In our notation, Hansen's economy involves $\alpha = 0.64$, $\beta^* = 0.99$, $\gamma_X = 1.00$; $N = 0.33$ and $\delta_K = 0.025$. These alternative parameter values have implications for the moments reported in tables 4 and 5. Using a persistence parameter $\rho = 0.90$, the model's relative volatility measures (standard deviations of variables relative to standard deviation of output) are as follows: consumption (0.62), investment (2.67) and hours (0.41). Basically, relative to table 4 these results reflect the decline in labor supply elasticity implied by $N = 1/3$ rather than $N = 1/5$. The contemporaneous correlations with output are as follows: consumption (0.81), investment (0.92) and hours (0.81). If we filter the population moments with the Hodrick–Prescott (HP) filter, then the relative variabilities and correlations are altered. For consumption these are (0.25) and (0.80), respectively, for investment they are (3.36) and (0.99) and for hours they are (0.55) and (0.98). These alterations occur because the effect of the HP filter is to give less weight to low frequencies, downplaying persistent but transient aspects of the series in question. [See the graph of the transfer function of the HP filter in Singleton (1988).] For example, the correlation of output at the yearly interval (lag 4) is 0.72 in the unfiltered Hansen parameterization and it is 0.08 in the filtered version. It is this sensitivity of results to filtering that makes us hesitant to undertake detailed comparisons with results reported by Hansen.

[32]The inability of the model to generate a sufficiently high variation in labor when the elasticity of labor supply is restricted to be consistent with panel data studies has stimulated several extensions to the basic neoclassical model. Kydland (1984) demonstrates that introducing agent heterogeneity in the model can increase the relative volatility of the average number of hours worked with respect to the volatility of labor productivity. Rogerson (1988) establishes that, in the presence of indivisibility in individual labor supply, an economy with finite elasticity of labor supply behaves as if it had an infinite elasticity of labor supply. This motivates our interest in the fourth parameterization. As Hansen (1985), we find that in this economy labor is too volatile relative to output.

Table 5

Population moments: Baseline model.

Variable	Std. dev.	Std. dev. relative to $\hat{y}$	Autocorrelations			Cross-correlations with $\hat{y}_{t-j}$									
			1	2	3	12	8	4	2	1	0	−1	−2	−4	−8	−12
Panel A: $\rho = 0.0$, $\sigma(\hat{A}) = 1.0$																
$\hat{y}$	1.78	1.0	0.03	0.03	0.03	0.02	0.02	0.02	0.03	0.03	1.0	0.03	0.03	0.02	0.02	0.02
$\hat{c}$	0.35	0.20	0.95	0.91	0.87	0.21	0.26	0.31	0.34	0.36	0.38	0.08	0.07	0.07	0.05	0.05
$\hat{i}$	5.76	3.24	−0.01	−0.01	−0.01	−0.01	−0.02	−0.02	−0.02	−0.02	0.99	0.02	0.02	0.02	0.01	0.01
$\hat{N}$	1.34	0.75	−0.02	−0.02	−0.02	−0.03	−0.03	−0.04	−0.04	−0.05	0.98	0.01	0.01	0.01	0.01	0.01
$\hat{w}$	0.53	0.30	0.44	0.42	0.40	0.12	0.15	0.18	0.20	0.21	0.87	0.06	0.06	0.05	0.04	0.04
r	0.02	0.01	0.95	0.91	0.87	−0.21	−0.26	−0.31	−0.34	−0.36	−0.38	−0.08	−0.07	−0.07	−0.05	−0.05
Panel B: $\rho = 0.9$, $\sigma(\hat{A}) = 2.29$																
$\hat{y}$	4.26	1.0	0.93	0.86	0.80	0.42	0.55	0.74	0.86	0.93	1.0	0.93	0.86	0.74	0.55	0.42
$\hat{c}$	2.73	0.64	0.99	0.98	0.97	0.76	0.82	0.86	0.85	0.84	0.82	0.76	0.71	0.61	0.47	0.36
$\hat{i}$	9.82	2.31	0.88	0.77	0.67	0.11	0.26	0.52	0.70	0.80	0.92	0.85	0.79	0.68	0.50	0.38
$\hat{N}$	2.04	0.48	0.86	0.73	0.62	−0.11	0.04	0.32	0.52	0.65	0.79	0.73	0.67	0.57	0.42	0.31
$\hat{w}$	2.92	0.69	0.98	0.96	0.94	0.69	0.78	0.85	0.88	0.90	0.90	0.84	0.78	0.67	0.51	0.39
r	0.11	0.03	0.87	0.76	0.66	−0.47	−0.34	−0.07	0.14	0.28	0.43	0.40	0.36	0.30	0.22	0.16

5.2. Some empirical issues and observations

Since the early part of this century, with the NBER studies of business cycles and economic growth under the leadership of Wesley Mitchell and Simon Kuznets, it has become commonplace for macroeconomic researchers to design models to replicate the principal features of the business cycles isolated by the NBER researchers. More recently, the development of statistical and computing technology has led individual researchers to define analogous sets of 'stylized facts' about economic fluctuations that models are then designed to emulate.

Our perspective is that the development of stylized facts outside of a circumscribed class of dynamic models is difficult at best.[33] First, models suggest how to organize time series. Further, it is frequently the case that stylized facts are sensitive to the methods of detrending or prefiltering. In this investigation we take the perspective that the basic neoclassical model has implications for untransformed macroeconomic data and not some arbitrary or prespecified transformation or component that is defined outside the context of the model [cf. Hicks (1965 p. 4)]. Although we do not perform formal statistical tests of model adequacy, the manner in which we proceed with data analysis is dictated by the models under study.

We have considered deterministic labor augmenting technological change that grows at a constant proportionate rate as the source of sustained growth (trend). The neoclassical model then predicts that all quantity variables (with the exception of work effort) grow at the same rate γ_X. The non-deterministic components of consumption, output and investment ($\hat{y}$, $\hat{c}$ and $\hat{i}$) are then

$$\begin{aligned} \hat{y}_t &= \log(Y_t) - \log(X_t) - \log(y), \\ \hat{c}_t &= \log(C_t) - \log(X_t) - \log(c), \\ \hat{i}_t &= \log(I_t) - \log(X_t) - \log(i), \end{aligned} \tag{5.1}$$

where y, c and i are the steady state values in the transformed economy. Labor augmenting technical progress, $\log(X_t)$, can be expressed as the simple linear trend

$$\log(X_t) = \log(X_0) + t \cdot \log(\gamma_X). \tag{5.2}$$

Thus, in the language of Nelson and Plosser (1982), the implied time series are trend stationary. Moreover, they possess a common deterministic trend. Therefore, the model instructs us to consider deviations of the log levels of GNP, consumption and investment from a common linear trend as empirical counterparts to $\hat{y}$, $\hat{c}$ and $\hat{i}$. Work effort, on the other hand, possess no trend and, thus, $\hat{N}$ is simply deviation of the log of hours from its mean.

[33] See also Koopmans (1947) and Singleton (1988).

In order to provide some perspective on the models' properties, we summarize some of the corresponding sample moments of the U.S. time series. The series we consider are the quarterly per capita values of real GNP, consumption of non-durables and services (CNS), gross fixed investment (GFI) and average weekly hours per capita.[34] Following the structure (5.1) and (5.2), we detrend the log levels of each of the first three series by computing deviations from a common estimated linear time trend. The estimated common trend, which corresponds to an estimate of $\log(\gamma_X) \approx (\gamma_X - 1)$, is 0.4% per quarter.[35] The real wage is the gross average hourly earnings of production or non-supervisory workers on non-agricultural payrolls. We chose not to study interest rates because of the well-known difficulties of obtaining measures of expected real interest rates.

Plots of our empirical counterparts to $\hat{y}$, $\hat{c}$, $\hat{i}$ and $\hat{N}$ are presented in fig. 3. Their properties are summarized in table 6 in a manner analogous to the summary of the baseline model in table 5. Our sample period is the first quarter of 1948 (1948.1) to the fourth quarter of 1986 (1986.4). Deviations of output from the common deterministic trend, which are plotted as a benchmark in each of the panels in fig. 3, have a standard deviation of 5.6% and range in value from −13.0% to 10%. The sample autocorrelations in table 6 indicate substantial persistence, suggesting that there may be a non-stationary component to the series not eliminated by removing a common deterministic trend.

The panels A and B show empirical counterparts to $\hat{c}$ and $\hat{i}$, plotted against the reference variable $\hat{y}$. Consumption and investment are highly correlated with output. Table 6 reports estimated correlation coefficients of 0.85 for consumption and 0.60 for investment over the 1948.1–1986.4 sample period. Consumption is less volatile than output, with a sample standard deviation of 3.9% (versus 5.6% for output) and a sample range of −7.8% to 7.4%. Investment is more volatile than output, with a sample standard deviation of 7.6% and sample range of −20.7% to 16.3%. Further, the autocorrelation statistics in table 6 indicate substantial serial correlation in both consumption and investment.

Panel C of fig. 3 contains a plot of the empirical counterpart of per capita hours as well as that of output. This labor input measure has a standard deviation of 3.0%, with a maximum value of 6.5% and a minimum value of

[34]All series are taken from the CITIBASE database. GNP, CNS and GFI are quarterly values. Population (P) is the total civilian non-institutional population 16 years of age and older. Employment (E) is total workers employed as taken from the Household Survey, Bureau of Labor Statistics. Average weekly hours of all workers (H) is also from the Household Survey. Average hours per capita is then calculated as $E \cdot H/P$ and averaged for the quarter. The wage rate is gross average hourly earnings of production workers.

[35]This is the source of the estimate of γ_X we use to parameterize the basic model in section 3. We choose not to impose the common trend assumption on wage rates because it involves a specific assumption about market structure.

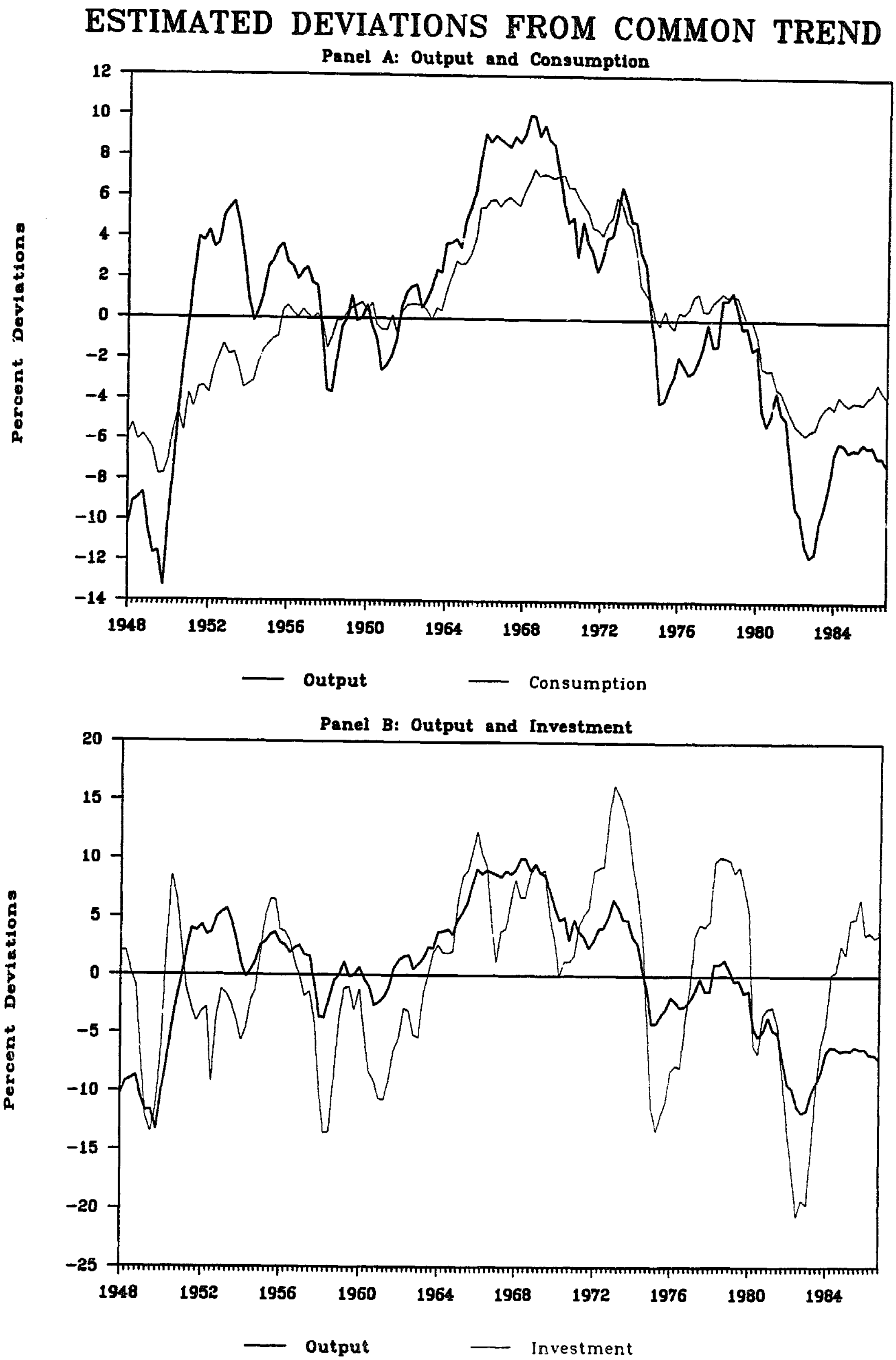
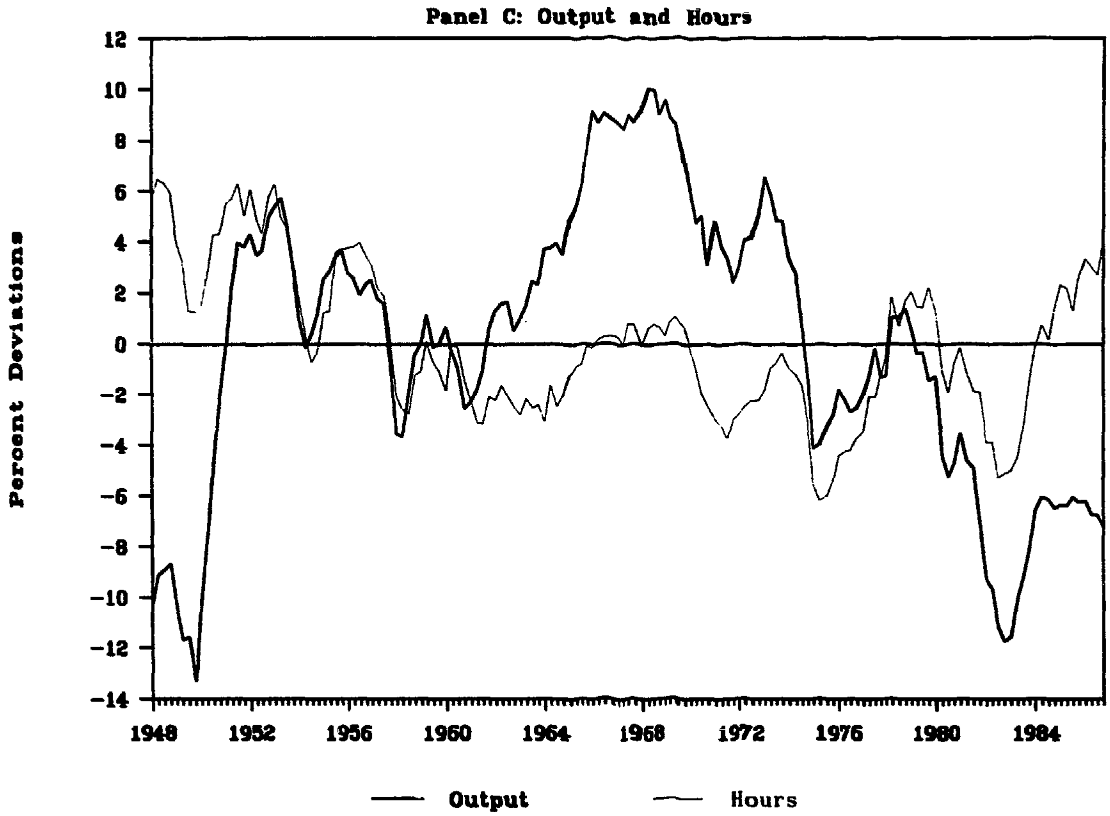

ESTIMATED DEVIATIONS FROM COMMON TREND
Panel A: Output and Consumption
Percent Deviations
12
10
8
6
4
2
0
-2
-4
-6
-8
-10
-12
-14
1948
1952
1956
1960
1964
1968
1972
1976
1980
1984
Output
Consumption
Panel B: Output and Investment
Percent Deviations
20
15
10
5
0
-5
-10
-15
-20
-25
1948
1952
1956
1960
1964
1968
1972
1976
1980
1984
Output
Investment

Fig. 3



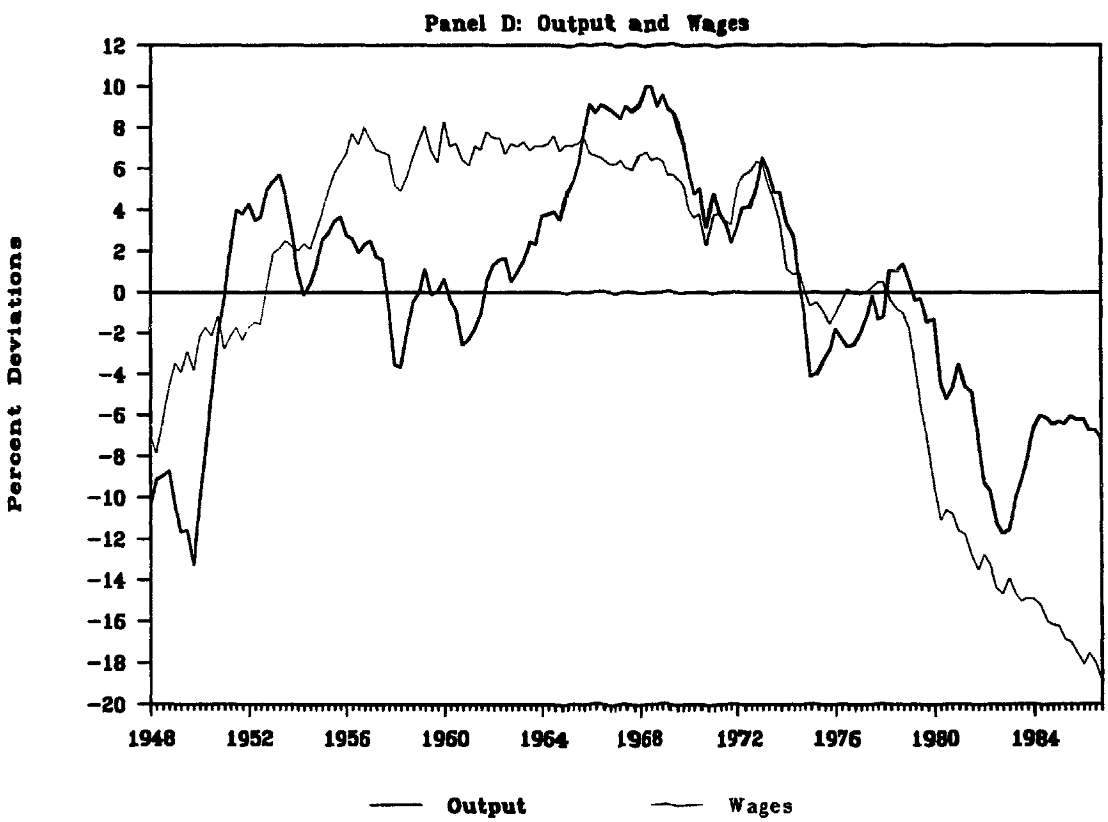


Fig. 3 (continued)

Table 6

Sample moments: Quarterly U.S. data, 1948.1–1986.4.[a]

Variable	Std. dev.	Relative std. dev.[b]	Autocorrelations 1	2	3	Cross-correlations with y_{t-j} 12	8	4	2	1	0	−1	−2	−4	−8	−12
$\hat{y}$	5.62	1.00	0.96	0.91	0.85	0.35	0.53	0.79	0.91	0.96	1.0	0.96	0.91	0.79	0.53	0.35
$\hat{c}$	3.86	0.69	0.98	0.95	0.93	0.59	0.68	0.78	0.83	0.84	0.85	0.82	0.78	0.69	0.43	0.26
$\hat{i}$	7.61	1.35	0.93	0.78	0.62	0.18	0.20	0.38	0.51	0.57	0.60	0.59	0.55	0.43	0.22	0.08
$\hat{N}$	2.97	0.52	0.94	0.85	0.74	−0.44	−0.31	−0.07	0.03	0.06	0.07	0.07	0.05	−0.01	0.01	0.08
$\hat{w}$	6.49	1.14	0.97	0.93	0.89	0.60	0.63	0.68	0.72	0.74	0.76	0.72	0.69	0.62	0.42	0.25

[a]All variables are taken from the National Income Accounts.
[b]Relative standard deviation of z is $\sigma(\hat{z})/\sigma(\hat{y})$.

−6.2% over the post-war period. The correlation between output and hours reported in table 6 is essentially zero! Inspection of the plot, however, appears to suggest that this relation is closer if one visually corrects for the periods in which output is on average high or low. In fact, if one splits the sample into subperiods of approximately 5 years each, the correlation between output and hours is never less than 0.30 and averages 0.77. Thus, when we permit the sample mean to vary (which is what looking at subperiods effectively does), the correlation between hours and output appears much higher.[36] It is important to stress that there is no theoretical justification for looking at data in subperiods. The basic neoclassical model that we have been discussing has a single source of low frequency variation (the deterministic trend in labor productivity) which has been removed from the time series under study. The sensitivity of these results to the sample period suggests the possibility of a low frequency component not removed by the deterministic trend. This is consistent with the highly persistent autocorrelation structure of output noted above.

The practice of removing low frequency variation in economic data plays an important role in empirical research on business fluctuations. NBER business cycle research has generally followed Mitchell's division of time series into cyclical episodes, removing separate cycle averages for individual series. Our belief is that this methodology is likely to remove important low frequency aspects of the relations between time series, in a manner broadly similar to the computation of correlations over subperiods. Most modern empirical analyses of cyclical interactions have also followed the practice of removing low frequency components from actual and model-generated time series.[37] Studying the impact of such low frequency filtering on economic time series generated by our baseline model, King and Rebelo (1987) find that there are major distortions in the picture of economic mechanisms presented by low frequency filtering. Among these are two that are particularly relevant to the labor–output relation. First, in the theoretical economy analyzed by King and Rebelo, application of a low frequency filter raises the correlation between output and labor input. Second, a low frequency filter dramatically reduces the correlation between output and capital.

Panel D of fig. 3 contains a plot of our empirical measure of $\hat{w}$. While the correlation with output is positive (0.76), it is not as strong as predicted by

[36] The subperiod results for the other variables are qualitatively similar to the overall sample. We have also explored the use of another hours series to insure that this finding was not an artifact of our data. Using an adjusted hours series developed by Hansen (1985), which covers only the 1955.3 to 1984.1 period, the correlation is 0.28 compared to 0.48 for our series for the same period. Breaking this shorter sample into subperiods also yields higher correlations than those for the overall period for the Hansen data.

[37] For example, Kydland and Prescott (1982) filter both the data *and* the output of their model using a filter proposed by Hodrick and Prescott (1980). Hansen (1985) follows this practice as well.

the model. Moreover, the positive correlation seems to arise primarily from the association at lower frequencies.

There are two main conclusions we draw from this cursory view of the data. The first, and most important, is that the one sector neoclassical model that we use as our baseline specification is not capable of generating the degree of persistence we see in the data without introducing substantial serial correlation into the technology shocks. The second is that the data suggest the related possibility of a low frequency component not captured by the deterministic trend. This motivates our interest in models with stochastic growth in the companion essay.

6. Conclusions

This paper has summarized the growth and business cycle implications of the basic neoclassical model. When driven by exogenous technical change at constant rates, the model possesses a steady state growth path under some restrictions on preferences for consumption and leisure. Although these restrictions imply that labor effort is constant in the steady state, they do not imply that effort is constant along transition paths of capital accumulation or in response to temporary technology shocks. Rather, the intertemporal substitution made feasible by capital accumulation applies to both consumption and effort in general equilibrium.

When driven by highly persistent technology shocks, the basic neoclassical model is capable of replicating some stylized facts of economic fluctuations. First, the model generates procyclical employment, consumption and investment. Second, the model generates the observed rankings of relative volatility in investment, output and consumption. But along other dimensions, the basic model seems less satisfactory. In particular, the principle serial correlation in output – one notable feature of economic fluctuations – derives mainly from the persistence of technology shocks. On another level, as McCallum (1987) notes, the model abstains from discussing implications of government and the heterogeneity of economic agents.

Perhaps the most intriguing possibility raised by the basic model is that economic fluctuations are just a manifestation of the process of stochastic growth. In the companion essay, we discuss current research into this possibility, along with issues concerning the introduction of government and heterogeneity.

References

Barro, R. and R. King, 1984, Time separable preferences and intertemporal substitution models of business cycles, Quarterly Journal of Economics 99, 817–839.

Becker, R., J. Boyd III and C. Foias, 1986, The existence of Ramsey equilibrium, Working paper (University of Rochester, Rochester, NY).

Boyd III, J., 1986, Recursive utility and the Ramsey problem, Working paper no. 60 (Center for Economic Research, University of Rochester, Rochester, NY).

Brock, W. and L. Mirman, 1972, Optimal economic growth and uncertainty: The discounted case, Journal of Economic Theory 4, 479–513.

Cass, D., 1965, Optimum growth in an aggregative model of capital accumulation, Review of Economic Studies 32, 223–240.

Debreu, G., 1954, Valuation equilibrium and Pareto optimum, Proceedings of the National Academy of Sciences of the U.S.A. 38, 886–893.

Greenwood, J., Z. Hercowitz and G. Huffman, 1986, Investment, capacity utilization and the real business cycle, Working paper (University of Western Ontario, London, Ont.).

Hansen, G., 1985, Indivisible labor and the business cycle, Journal of Monetary Economics 16, 309–327.

Hicks, J., 1965, Capital and growth (Oxford University Press, New York).

Hodrick, R. and E. Prescott, 1980, Post-war U.S. business cycles: An empirical investigation, Working paper (Carnegie-Mellon University, Pittsburgh, PA).

King, R., 1987, Business cycles and economic growth, Lecture notes on macroeconomics (University of Rochester, Rochester, NY).

King, R. and S. Rebelo, 1987, Low frequency filtering and real business cycles, in progress (University of Rochester, Rochester, NY).

Koopmans, T., 1947, Measurement without theory, Review of Economics and Statistics 29, 161–172.

Koopmans, T., 1965, On the concept of optimal economic growth, in: The econometric approach to development planning (Rand-McNally, Chicago, IL).

Kydland, F., 1984, Labor force heterogeneity and the business cycle, Carnegie–Rochester Conference Series on Public Policy 21, 173–208.

Kydland, F. and E. Prescott, 1982, Time to build and aggregate fluctuations, Econometrica 50, 1345–1370.

Long, J. and C. Plosser, 1983, Real business cycles, Journal of Political Economy 91, 1345–1370.

McCallum, B.T., 1987, Real business cycles, Unpublished manuscript (Carnegie-Mellon University, Pittsburgh, PA).

Nelson, C. and C. Plosser, 1982, Trends and random walks in macroeconomic time series: Some evidence and implications, Journal of Monetary Economics 10, 139–167.

Orcutt, G., 1948, A study of the autoregressive nature of the time series used for Tinbergen's model of the economic system of the United States, 1919–1932, Journal of the Royal Statistical Society B 10, 1–45.

Pencavel, J. 1986, Labor supply of men: A survey, in: Orley Ashenfelter and Richard Layard, eds., Handbook of labor economics (North-Holland, Amsterdam).

Phelps, E., 1966, Golden rules of economic growth (Norton, New York).

Prescott, E., 1986, Theory ahead of business cycles measurement, Carnegie–Rochester Conference Series on Public Policy 25, 11–66.

Prescott, E. and R. Lucas, 1972, A note on price systems in infinite dimensional space, International Economic Review 13, 416–422.

Rogerson, R., 1988, Indivisible labor, lotteries and equilibrium, Journal of Monetary Economics 21, 3–16.

Romer, P. and T. Shinotsuka, 1987, The Kuhn–Tucker theorem implies the transversality condition at infinity, Unpublished paper (University of Rochester, Rochester, NY).

Sargent, T., 1980, Tobin's 'q' and the rate of investment in general equilibrium, Carnegie–Rochester Conference Series on Public Policy 12, 107–154.

Singleton, K., 1988, Econometric issues in the analysis of equilibrium business cycle models, Journal of Monetary Economics, this issue.

Solow, R., 1956, A contribution to the theory of economic growth, Quarterly Journal of Economics 70, 65–94.

Swan, T., 1963, On golden ages and production functions, in: Kenneth Berril, ed., Economic development with special references to southeast Asia (Macmillan, London).

Weitzman, M., 1973, Duality theory for infinite time horizon convex models, Management Science 19, 738–789.

Zellner, A. and F. Palm, 1974, Time series analysis and simultaneous equations models, Journal of Econometrics 2, 17–54.