Labor supply in the past, present, and future: a balanced-growth perspective

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December 17, 2015

Abstract

We argue that a stable utility function of consumption and hours worked for which income effects are slightly stronger than substitution effects can rationalize the long-run data for the main macroeconomic quantities. In these long-run data, in the U.S. as well as in other countries, as productivity grows at a steady rate, hours worked fall slowly and at an approximately constant rate. We narrow down the set of preferences consistent with balanced growth under constant (negative) hours growth. The resulting class amounts to a slight enlargement of the well-known "balanced-growth preferences" that dominate the macro literature and are based on requiring constant hours worked. Thus, hours falling at a constant rate is not inconsistent with the remaining balanced-growth facts but merely requires a slight broadening of the preference class considered. From this perspective, we interpret the recent decades of stationary hours worked in the U.S. as a temporary departure from a long-run pattern, and to the extent productivity will keep growing, we predict that hours will fall further.

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1 Introduction

The purpose of this paper is to propose a choice- and technology-based theory for the long-run behavior of the main macroeconomic aggregates. Such a theory—standard balanced-growth theory, specifying preferences and production possibilities along with a market mechanism to be consistent with the data—already exists, but what we argue here is that it needs to be changed. A change is required because of data on hours worked that we document at some length: over a longer perspective—going back a hundred years and more—and looking across many countries, hours worked are falling at a remarkably steady rate: at a little less than half a percentage point per year. Figure 1 illustrates this for a collection of countries. This finding turns

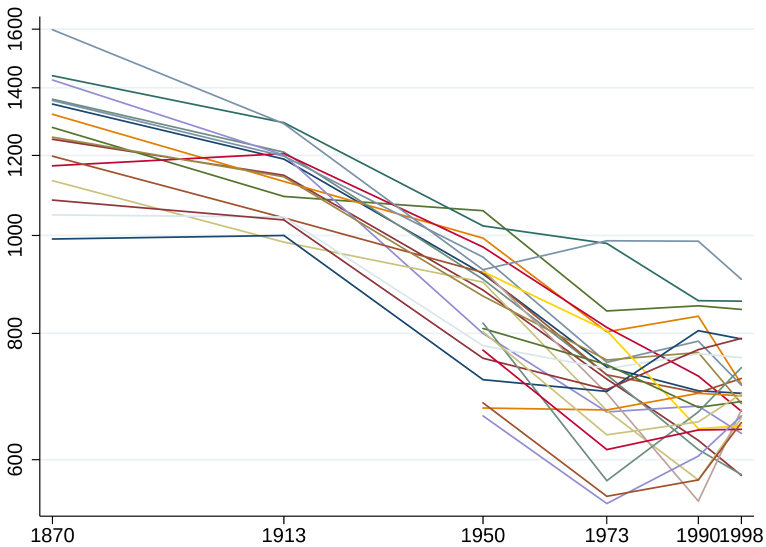


Figure 1: **Yearly hours worked per capita 1870–1998**

Source: Maddison "The world economy: a millennial perspective", 2001. The sample includes the following 25 countries: Austria, Belgium, Denmark, Finland, France, Germany, Italy, Netherlands, Norway, Sweden, Switzerland, United Kingdom, Ireland, Spain, Australia, Canada, United States, Argentina, Brazil, Chile, Colombia, Mexico, Peru, Venezuela, Japan. Regressing the log of hours on a country fixed effect and year gives a slope coefficient of -0.00462 in the full sample (and -0.00398 for the period 1950–1998). Huberman and Minns (2007) provide similar data.

out to contrast the data in the postwar U.S., where hours are well described as stationary, but going back further in time and looking across countries leads one to view the recent U.S. data rather as an exception.

Since the persistent fall in hours worked is not consistent with the preferences and technology used in standard framework, we alter this framework. Our alteration is very simple and, on a general level, obvious: to rationalize decreasing hours worked we point to steadily increased productivity over a very long periods and preferences over consumption and leisure with the feature that income effects exceed substitution effects. As in the case of the standard setting, we however also impose additional structure by summarizing the long-run data as (roughly, at least) having been characterized by *balanced growth*. So on a balanced growth path, our main economic aggregates—hours worked, output, consumption, investment, and the stock of capital—all grow at constant rates. Characterizing the data as fluctuations around such a path may be viewed as a poor approximation, but here we nevertheless do maintain the position that such a characterization is roughly accurate, at least for the last 150 years of data for many developed countries. Hence, we ask: is there a stable utility function such that consumers choose a balanced growth path, with constant growth for consumption, and constant (negative) growth for hours, given that labor productivity grows at a constant rate? We restrict ourselves to time additivity and constant discounting, in line with the assumptions used to derive the standard preference framework. We find that there indeed are preferences that do deliver the desired properties and our main result is a complete characterization of the class of such preferences.

The modern macroeconomic literature is based on versions of a framework featuring balanced growth with constant hours worked, to a large extent motivated with reference to U.S. data on postwar hours worked; see, e.g., Cooley and Prescott (1995). Our main point here is not to take fundamental issue with this practice; in fact, our proposed utility specification in some ways is quantitatively very similar to the preferences normally used. However, for some issues the distinction may be important. As for discussion of hours historically, there is significant recognition in the macroeconomic literature that from a longer historical perspective, hours have indeed fallen. For example, several broadly used textbooks actually do point to the fact that hours worked have fallen significantly over the longer horizon, often with concrete examples of how hard our grand-grandfathers (and -mothers) worked; see, e.g., Barro's (1984) book and Mankiw's (2010) latest intermediate text. In a discussion of some significant length, Mankiw actually reminds us of a very well-known short text wherein John Maynard Keynes speculates that hours worked would fall dramatically in the future—from the perspective he had back then (see Keynes, 1930). Keynes thus imagines a 15-hour work week for his grandchildren, in particular, supported by steadily rising productivity. As it turned out, Keynes was wildly off quantitatively, but we would argue that he was right qualitatively (on this issue. . .). Finally, in his forthcoming chapter on growth facts, Jones (2015) also points to the tension between the typical description of hours as stationary and the actual historical data.[1]

From Keynes's U.K. perspective, over the postwar period, and in contrast to the U.S. experience, hours worked actually fell steadily until as recently as circa 1980, at which point they appear to have stabilized; we review the data in some detail in Section 2 below. But perhaps more importantly, the picture that arises from looking at a broader set of countries strengthens the case for falling, rather than constant, hours, and going further back in time reinforces this conclusion. With our eye-balling, at least, a reasonable approximation is actually even more stringent: hours worked are falling at a rate that appears roughly constant over longer periods (though, of course, with swings over business cycles, etc.). This rate is slow—somewhere between 0.3% and 0.5% per year—so shorter-run data will not suffice for detecting this trend, to the extent we are right; to halve the number of

[1] Jones writes *A standard stylized fact in macroeconomics is that the fraction of the time spent working shows no trend despite the large upward trend in wages. The next two figures show that this stylized fact is not really true over the longer term, although the evidence is somewhat nuanced.*

hours worked at this rate requires around 200 years.

Turning back to the case of the U.S., over the last more than 150 years, thus, as hours have fallen, output has grown at a remarkably steady rate, mainly interrupted only by the Great Depression and World War II. Moreover, over this rather long period, all the other macroeconomic balanced-growth facts also hold up very well; we review these data briefly in Section 3. Thus, as output is growing at a steady rate, hours are falling slowly at a steady rate. The interpretation of these facts that we adopt here is that preferences for consumption and hours belong to the class we define. This preference class is, in fact, very similar to that used ubiquitously in the macroeconomic literature: that defined in King, Plosser, and Rebelo (1988). King, Plosser, and Rebelo showed that the preferences they put forth, often referred to as KPR or, perhaps more descriptively, *balanced-growth* preferences, were the only ones consistent with an exact balanced growth path for all the macroeconomic variables with the restriction to constant hours worked. The class of preferences that we consider in the present paper is thus strictly larger in that it also allows hours worked to shrink over time at a constant rate along a balanced path.

In compact terms, one can describe the period utility function under KPR as a power function of $cv(h)$, where c is consumption and h hours worked and v is an arbitrary (decreasing) function. What we show in our main Theorem 1 is that the broader class has the same form: period utility is a power function of $cv(hc^{\frac{\nu}{1-\nu}})$, where $\nu < 1$ is the preference parameter that guides how fast hours shrink relative to productivity. In terms of gross rates, if productivity grows at rate γ, then hours grow at rate $\gamma^{-\nu}$, whereas consumption grows at $\gamma^{1-\gamma}$. For $\nu > 0$, the factor $c^{\frac{\nu}{1-\nu}}$ captures the stronger income effect: as consumption grows, there is an added "penalty" to working (since v is decreasing). Our preference class obviously nests KPR: KPR corresponds to $\nu = 0$.

Having argued that preferences in our class—with a ν that is slightly larger than zero—provide a good account of the longer-run data, what do we then make of the postwar U.S. experience with stationary hours? The purpose here is not to propose a full account of the shorter-run data, but it seems relevant in this context to also revisit the papers by Prescott (2004) and Rogerson (2006, 2008), who argue that relative tax-rate changes account for the fact that Europeans work less hard now than Americans compared to the early postwar period. If one looks at France and Germany, it is clear that hours decline at a rather fast rate—faster than the rate at which hours decline in the broad cross-section of countries. Our perspective here, then, is that perhaps these countries and the U.S. are temporarily departing from a long-run trend, given that productivity has been growing at a rather constant rate throughout this period. One explanation may well be relative tax-rate changes. Another possible explanation is sharply increased wage inequality, with median wage growth close to zero for many decades; for any worker who does not experience wage growth, the optimal response is of course not to decrease hours. Finally, female labor-force participation took off sooner in the U.S. and has reached a level above that in most European economies, and to the extent this phenomenon represents the loosening of constraints (such as discrimination) it could indeed affect hours in a way that is not fully offset by the reduction of male hours.

Interestingly, our class encompasses some utility functions that are often used in the literature (both in macroeconomics and in other fields). One is another famous functional form of the same vintage as KPR: Greenwood, Hercowitz, and Huffman's (1988) proposed utility function, often referred to as GHH preferences. The GHH class assumes a quasi-linear utility function where utility can be written as a function of c minus an increasing (and convex) function of h. This formulation implies that there is no income effect at all on hours worked. With a judicious choice of v and a $\nu < 0$ we obtain a frequently used case within the GHH class in which the convex function of hours is restricted to be a power function. Clearly, without an income effect, hours grow under this formulation (so long as productivity grows). GHH preferences are often used in applied contexts (see, e.g., Chetty, xyz) because they allow simple comparative statics.

Another well-known case is a utility function displaying a constant Frisch elasticity of labor supply (see MaCurdy, 1981). This elasticity is the derivative of log hours with respect to log wages keeping the marginal utility of consumption constant and it is obtained when the period utility function is additive in a power function of c and a power function of h. However, unless the function of consumption is logarithmic (a special case of the power function), these preferences are well-known not to be consistent with constant hours worked. We show, again by a judicious choice of v, that our preference class actually includes the constant-Frisch case. That is, this class of utility functions is consistent with balanced growth—if one admits that hours can change over time along a balanced path. For shrinking hours, one needs the curvature to be high enough (higher than log curvature), since otherwise the marginal utility value of working an hour will grow: if productivity doubles, the marginal utility of consumption must more than halve, because otherwise it will not be optimal to lower hours.

A case that is new relative to the literature is one where the "relative risk aversion" to consumption (RRA)—the inverse of the intertemporal substitution elasticity of consumption—is an increasing (or decreasing) function of the $hc^{\frac{\nu}{1-\nu}}$ composite. Under KPR preferences, the RRA must equal a constant: a preference parameter (usually labeled σ). So, in particular, it is possible that the RRA under our preference specification moves countercyclically, thus displaying higher risk aversion in recessions than in booms. In the cross-section, by the same token, richer households would then be less averse to risk (in the relative sense) and choose riskier portfolios. We briefly discuss this and other possible applications (to growth and business cycles) in Section 6 of our paper.

The paper begins with two data sections. In Section 2 we look at hours worked over different time horizons and in different countries. In Section 3, we then motivate our balanced-growth perspective on longer-run data by revisiting the long-run facts for aggregates, with a focus on the United States. The theory section of the paper is contained in Section 4 where we lay out the precise balanced-growth restrictions. Then we go on to state our Theorem 1 about what utility function is needed in order for consumers to *choose* balanced-growth consumption and labor sequences. The proof of the theorem is in the Appendix. However, the proof relies heavily on two lemmata—one characterizing the implications of balanced-growth choices for the consumption-hours indifference curves and one for consumption curvature—and we discuss those results in some detail in the main text. The theory section also has a Theorem 2, which is straightforward, showing sufficiency of the stated preference class for balanced-growth choices. The theory section finally contains a sequence of illustrations with examples of utility functions in this preference class. Section 5 comments on consumer heterogeneity, a relevant issue since our theory relies on representative-consumer analysis. This section also briefly discusses the cross-sectional wage-hours-wealth data. Section 6 looks at the Prescott-Rogerson Europe vs. the U.S. postwar comparison of hours worked from the perspective of our theory, and Section 7 concludes.

2 Hours worked over time and across countries

We now go over the hours data from various perspectives: across time and space.

2.1 Hours over time

Figure 2 is the main justification for the assumption constant hours worked maintained in the macro literature. At least in post-war U.S. data this seems to be a good approximation.

What if we look at some other developed countries? Figure 3 shows hours worked for other selected countries on a logarithmic scale. Now we see that a horizontal line is no longer the best approximation of the data. A country-fixed effect regression

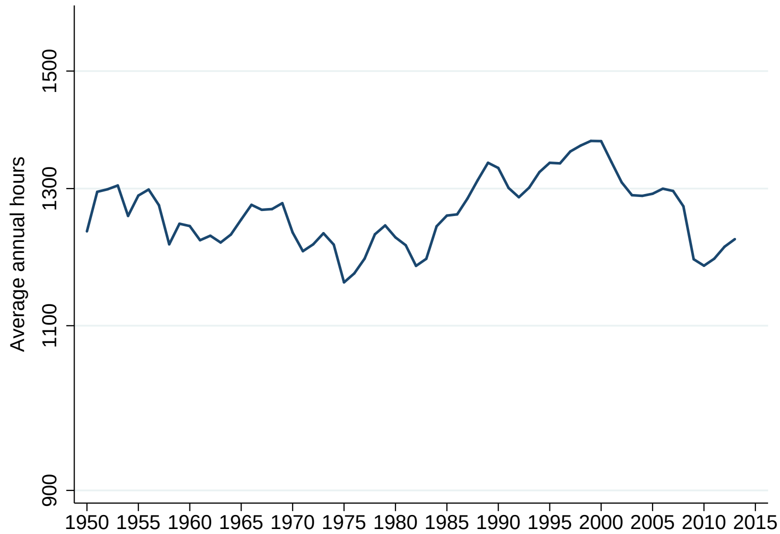


Figure 2: **U.S. average annual hours per capita aged 15–64, 1950–2013**

Notes: Source: GGDC Total Economy Database for total hours worked and OECD for the data on population aged 15–64. The figure is comparable to the ones in Rogerson (2006). Regressing the logarithm of hours worked on time gives an insignificant slope coefficient.

suggests that hours fall at 0.45% per year. To be sure, however, there is significant heterogeneity; Canada, for example, has stationary hours quite like those in the United States.

The falling hours in Figure 3 tell the same picture. The result is not due to the selection of countries. A complementary Figure B.1 in the Appendix B.1 shows the corresponding data for all countries available in the OECD database. Average hours are declining clearly in this unrestricted sample, at roughly 0.36% per year. Hence in the cross-country data of the post-war period the United States and Canada overall rather look like outliers. Interestingly, as B.2 in Appendix B.1 shows, a time-use survey shows decreasing hours worked even for the post-war United States.

From a longer-run perspective, the U.S. hours have also clearly been falling (see Figure B.3). We also see that once one abstracts from the Great Depression and WWII, hours have been falling at a rather steady rate. Only the period 1980–2000

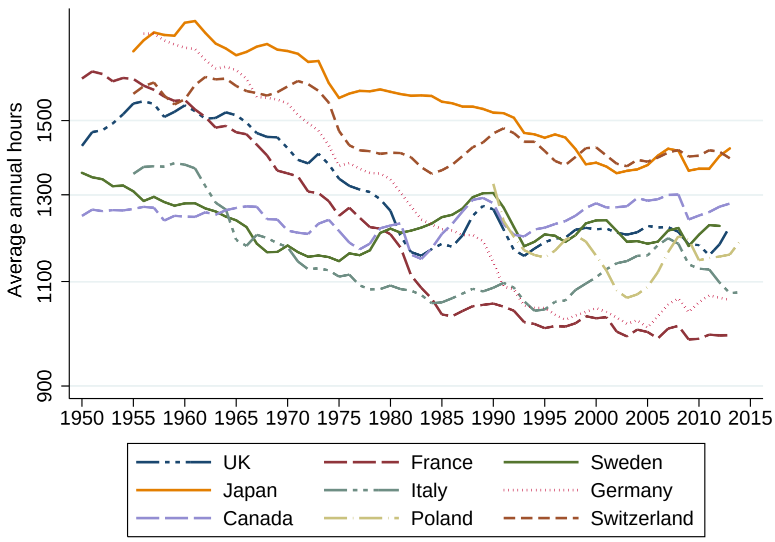


Figure 3: **Selected countries average annual hours per capita aged 15–64, 1950–2015**

Notes: Source: GGDC Total Economy Database for total hours worked and OECD for the data on population aged 15–64. The figure is comparable to the ones in Rogerson (2006). Regressing the logarithm of hours worked on time gives a slope coefficient of -0.00455.

looks exceptional.

Ramey and Francis (2009) also provide data on schooling (time attending school and studying at home). As Figure B.3 in Appendix B.1 shows, average weekly hours of schooling increased by less than two hours in total over the period 1900–2005 and cannot, therefore, account for the drop in hours worked (hence: leisure has increased).

The time trend in total hours worked can be split up into trends in participation rates and trends in hours per employed. Figure 5 shows that hours per employed was declining at a remarkably constant rate, including during the post-war period. (That hours worked per employed are falling is a remarkably robust fact over time and across countries though the rate of decline differs across countries.) In other words: hours in the post-war U.S. are only relatively stable because the participation rate increased steeply.

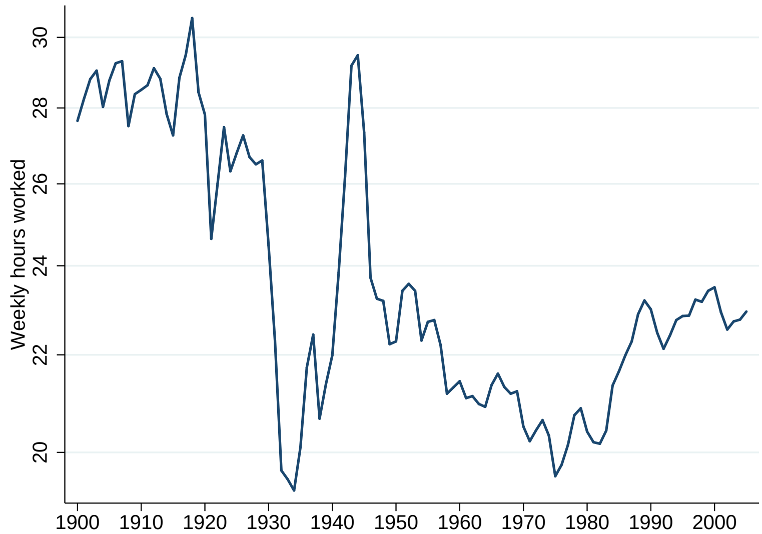


Figure 4: **Weekly U.S. hours worked per population aged 14+, 1900–2005**

Notes: Source: Ramey and Francis (2009). Regressing the logarithm of hours worked on time gives slope coefficient of -0.00285.

Figure B.4 in Appendix B.1 shows this split again for the post-war U.S.

To sum up: over 100+ years, hours have been falling in all developed countries. In the post-war data hours are still falling in most countries. In countries where they are rather stable, like Canada or the U.S., they are stable only because the participation rate increased quite dramatically. Hours per worker show a clear downward trend in all countries. Participation rates do not show a clear trend over time in developed countries. Hence we conclude that if the participation rate does not increase further in future in the U.S., hours will continue to fall. In fact since the Great Recession, the participation rate fell, as did hours worked per working-age population.

2.2 Hours worked in the cross-section

In the cross-section of countries, our theory predicts that labor productivity (or GDP per capita) should be negatively correlated with hours worked. A negative correlation of the logarithm of hours worked and the logarithm of wages or per-

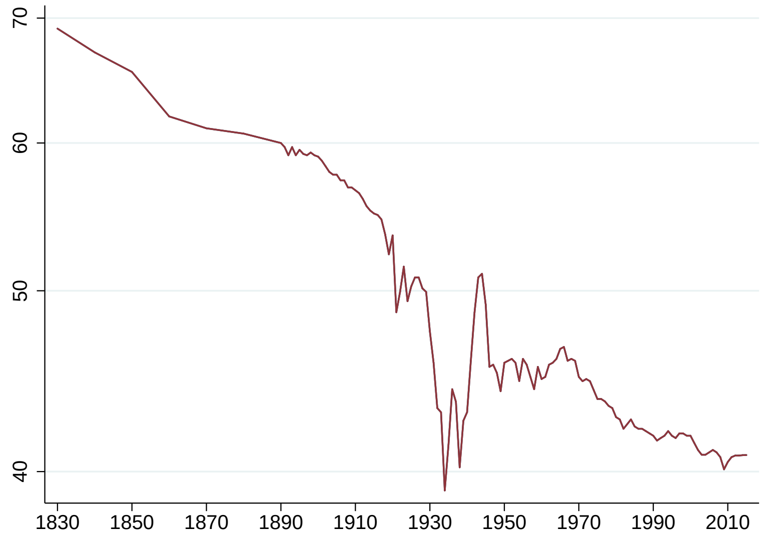


Figure 5: **U.S. weekly hours worked per employed in nonfarm establishments 1830–2015**

Source: Average weekly hours data for 1830–80: Whaples (1990, Table 2.1). 1890-1970: Historical Statistics of the United States: Colonial Times to 1970 (Series D765 and D803). 1970–2015: Statistical Abstract of the United States the number for nonfarm establishments. This graph shows an updates series of the data in Greenwood and Vandenbroucke (2008). Regressing the log of hours on a constant and year gives a slope coefficient of -0.00315 in the full sample (and -0.00208 for the years 1970–2015).

capita income has already been established in Winston (1966). (See also Bick, Fuchs-Schuendeln and Lagakos, 2015, for a more recent documentation of this correlation.) Figure 7 shows this negative correlation in our data set. Figure B.5 in Appendix B.1 shows this correlation for the years 1955 and 2010 separately.

Finally, in Figure 7 we focus on the 21 countries with data for 1955–2010 and look at the correlation in the growth rates in labor productivity and hours worked over these 55 years. The figure shows again that hours clearly fell for most of the countries. Moreover, with the exception of South Korea, labor productivity growth is clearly negatively related with growth in hours worked.

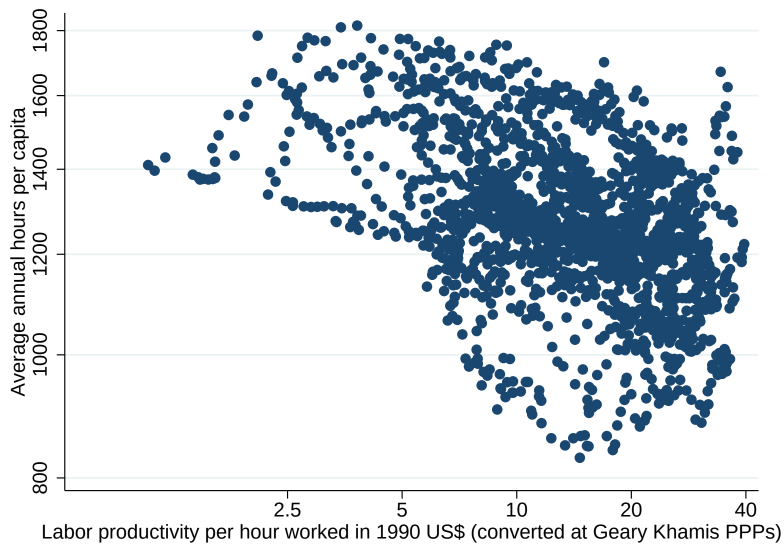


Figure 6: **Hours worked vs. labor productivity**

Source: Source: GGDC Total Economy Database for total hours worked and labor productivity and OECD for the data on population aged 15–64. Regression the logarithm of hours worked on the logarithm of labor productivity and a country fixed effect gives a slope coefficient of -0.13 and an R^2 of 0.69..

3 Balanced-growth facts and theory

For completeness, we now review the basic "stylized facts of growth" for the United States. These data have been instrumental in guiding the technology and preference specifications in macroeconomic theory. Figure 8a and 8b show how output and consumption grew over the decades at a very steady rate.

Figure 8c and 8d show that the consumption-output ratio and the capital-output ratio remained remarkably stable. (Figure B.6 in Appendix B.1 shows some additional balanced-growth facts often imposed in the macro literature, like constant hours worked or constant factor income shares.)

Our main take-away message from Figure 8 is that—in the style of Kaldor (1961)—we would like to impose restrictions on our macro framework in a manner that is consistent with these facts. We define accordingly a balanced growth path.

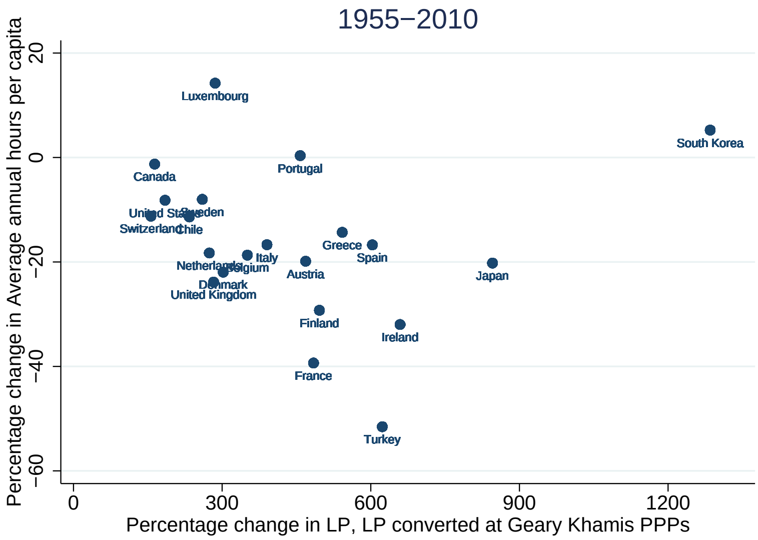


Figure 7: **Changes in hours worked vs. labor productivity**

Source: Source: GGDC Total Economy Database for total hours worked and labor productivity and OECD for the data on population aged 15–64. Regression the logarithm of hours worked on the logarithm of labor productivity and a country fixed effect gives a slope coefficient of -0.13 and an R^2 of 0.69.

4 Characterization

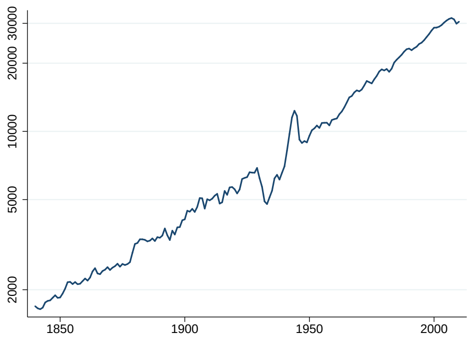
We now provide our formal analysis. The workhorse macro framework has a resource constraint given by

$$K_{t+1} = F\left(K_t, A_t h_t L_t\right) + (1-\delta)K_t - L_t c_t, \tag{1}$$

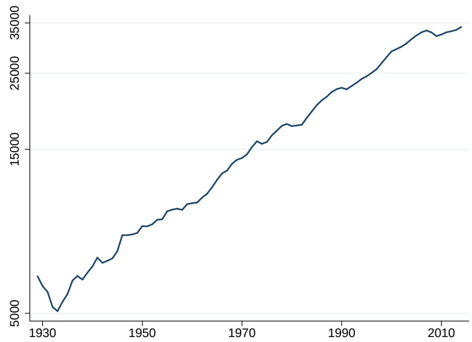
where capital letters refer to aggregates and lower-case letters per-capita values, and $F\left(K_t, A_t h_t L_t\right)$ is a neoclassical production function. Here, L is population, h is hours worked per-capita and δ the depreciation rate. Growth is of the labor-augmenting kind, because of the Uzawa theorem.[2] We thus assume constant exogenous technology and population growth, i.e.,

$$A_t = A_0 \gamma^t, \text{ and } L_t = L_0 \eta^t. \tag{2}$$

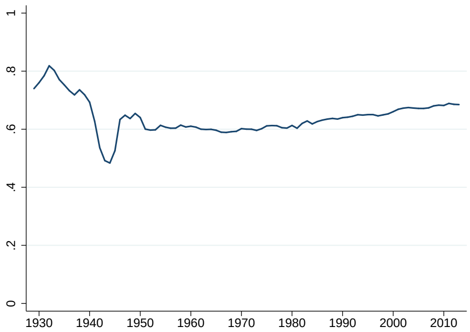
[2]Recently, Grossman, Helpman, Oberfield, and Sampson (2015) discusses an interesting exception to the Uzawa theorem.



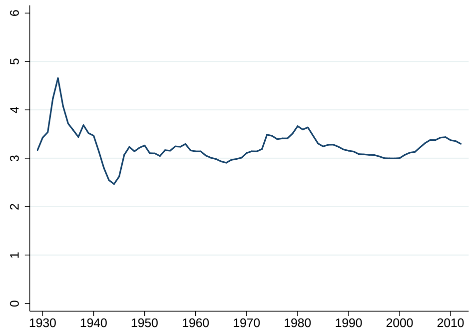
(a) GDP per capita



(b) Consumption per capita



(c) Consumption-output ratio



(d) Capital-output ratio

Figure 8: Balanced growth

Turning to preferences, we assume that they are additively separable over time with a constant discount factor β. Quite importantly, and in line with the KPR setting, the instantaneous utility, $u(c_t, h_t)$, is assumed to be stationary. There is a time constraint

$$h_t + l_t = 1. \tag{3}$$

A balanced-growth path for this economy is a time path along which K and c grow at constant rates. Such a path thus requires $\frac{K_{t+1}}{K_t} = \frac{A_{t+1}}{A_t}\frac{h_{t+1}}{h_t}\frac{L_{t+1}}{L_t} = \frac{L_{t+1}}{L_t}\frac{c_{t+1}}{c_t}$.

This in turn implies

$$\gamma \frac{h_{t+1}}{h_t} = \frac{c_{t+1}}{c_t} \tag{4}$$

and

$$\frac{K_{t+1}}{K_t} = \gamma \frac{h_{t+1}}{h_t} \eta. \tag{5}$$

Hence, a balanced-growth path requires $\frac{h_{t+1}}{h_t}$ to be constant.

Turning to preferences, we assume that households (whether infinitely or finitely lived) maximize

$$\cdots + u(c_t, h_t) + \beta u(c_{t+1}, h_{t+1}) + \ldots \tag{6}$$

subject to a time constraint and a budget constraint

$$a_{t+1} = (1 + r_t)a_t + h_t w_t - c_t. \tag{7}$$

Assuming an interior solution ($c_t > 0$, $1 > h_t > 0$), we can thus base our analysis on the first-order conditions to this maximization problem.

On a balanced growth path where labor productivity (alternatively, the real wage per hour) changes at constant gross rate $\gamma > 0$, we need to have consumption grow at the same rate as labor income. The derivations above led to $g_c = \gamma g_h$, where g_c is the gross growth rate of consumption and g_h that of hours worked. We thus seek preferences such that g_c and g_h are determined uniquely as a function of the growth rate in (real) wages. Thus, we parametrize preferences with a constant ν so that $g_c = \gamma^{1-\nu}$ and $g_h = \gamma^{-\nu}$.[3] The special case $\nu = 0$ is of interest but we will mainly focus on $\nu \neq 0$; $\nu = 0$ is the standard case, where hours will be constant on a balanced growth path.

Thus, a balanced growth path is one where, for all t, $c_t = c_0 \gamma^{(1-\nu)t}$ and $h_t = h_0 \gamma^{-\nu t}$, for some values c_0 and h_0. One can think of c_0 as a free variable here, determined by the economy's, or the consumer's, overall wealth, with h_0 pinned down by a labor-leisure choice given c_0.

[3] With $\nu \geq 1$ the theory would predict decreasing (or constant) consumption as the wage rate increases; we rule this case out.

In the following we are interested in an interior solution of the consumption and labor supply decision that is consistent with a balanced growth path. Such an interior solution requires that utility is strictly increasing in consumption and strictly decreasing in hours worked. Two first-order conditions are relevant for the consumer's optimization. The labor-leisure choice is characterized by

$$-\frac{u_2(c_t, h_t)}{u_1(c_t, h_t)} = w_t,$$

where w_t, the return from working one unit of time, thus grows at rate γ: $w_t = w_0\gamma^t$.[4] On a balanced growth path we thus need this condition to hold for all t. In our theorem below, we will also require that preferences admit a balanced growth path for all $w_0 > 0$. That is, we are looking for preferences that will admit a balanced path for consumption and hours at growth rates $\gamma^{1-\nu}$ and $\gamma^{-\nu}$, respectively, regardless of the (initial) level of the wage rate relative to consumption.

The intertemporal (Euler) equation reads

$$\frac{u_1(c_t, h_t)}{u_1(c_{t+1}, h_{t+1})} = \beta(1 + r_{t+1}),$$

where r is the return on saving and $\beta > 0$ the discount factor. If the economy grows along a balanced path, then we would like this condition to hold for all t, and we need the right-hand side to be equal to an appropriate constant, a constant that moreover depends on the rate of growth of consumption and hours. We will denote this constant R and discuss its dependence on c, h, and γ below. In the analysis below, we will switch from sequence to functional notation. Thus we leave out t subscripts and instead specify the balanced-growth conditions as a requirement that the paths of all the variables start growing from arbitrary positive values (save for those nonlinear restrictions relating the variables to each other that are implied by the equilibrium conditions): they can be scaled arbitrarily.

[4]In a decentralized equilibrium, this return denotes the individual wage rate including potential taxes and transfers. Similarly, the return on saving we discuss below should be taken to be net of taxes and transfers.

4.1 Balanced growth using functional language

So note that our balanced-growth path requirements on the utility function can be expressed as follows.

Assumption 1. *For any $w > 0$, $c > 0$, and $\gamma > 0$, there exists an $h > 0$ and an $R > 0$ such that, for any $\lambda > 0$,*

$$-\frac{u_2\left(c\lambda^{1-\nu}, h\lambda^{-\nu}\right)}{u_1\left(c\lambda^{1-\nu}, h\lambda^{-\nu}\right)} = w\lambda, \tag{8}$$

and

$$\frac{u_1\left(c\lambda^{1-\nu}, h\lambda^{-\nu}\right)}{u_1\left(c\lambda^{1-\nu}\gamma^{1-\nu}, h\lambda^{-\nu}\gamma^{-\nu}\right)} = R, \tag{9}$$

where $\nu < 1$.

That is, we must be able to scale variables arbitrarily, but of course consistently with the balanced rates, and still satisfy the two first-order conditions. The scaling is accomplished using λ (for wages/productivity), $\lambda^{1-\nu}$ (for consumption), and $\lambda^{-\nu}$ (for hours) in these conditions. Our main theorem below will thus characterize the class of utility functions u consistent with these conditions. Our theorem will not provide conditions on convexity of the associated maximization problem (of the consumer, or a social planner); obviously, however, conditions must be added such that the first-order conditions indeed characterize the solution. We briefly discuss this issue in the applied contexts below.

4.2 The main theorem

Our main theorem states what restrictions are necessary on the utility function to generate balanced growth.

Theorem 1. *If $u(c, h)$ is twice continuously differentiable, strictly increasing in c and strictly decreasing in h, and satisfies Assumption 1, then (save for additive and* *multiplicative constants) it must be of the form*

$$u(c,h) = \frac{\left(c \cdot v\left(hc^{\frac{\nu}{1-\nu}}\right)\right)^{1-\sigma} - 1}{1-\sigma},$$

for $\sigma \neq 1$*, or*

$$u(c,h) = \log(c) + \log\left(v(hc^{\frac{\nu}{1-\nu}})\right),$$

where v *is an arbitrary, twice continuously differentiable function satisfying, for all* $x \equiv hc^{\frac{\nu}{1-\nu}} > 0$*,* $v'(x) < 0$ *and* $v(x) + \frac{\nu}{1-\nu}v'(x)x > 0$*.*

The proof relies crucially on two lemmata, one characterizing the marginal rate of substitution (MRS) function between c and h and one characterizing the curvature with respect to consumption: the relative risk aversion (RRA) function. The proof then uses these lemmata to derive the final characterization. The proofs of the lemmata and of how to use them to complete the proof of the theorem are contained in Appendix A.1. However, we will state and comment on the lemmata, as they are of some independent interest, as well as on the overall method of proof.

4.2.1 The consumption-hours indifference curves

We thus begin with the following lemma:

Lemma 1. *If* $u(c,h)$ *satisfies (8) for all* $\lambda > 0$*, and for an arbitrary* $c > 0$ *and* $w > 0$*, then its marginal rate of substitution (MRS) function, defined by* $u_2(c,h)/u_1(c,h)$*, must be of the form*

$$\frac{u_2(c,h)}{u_1(c,h)} = c^{\frac{1}{1-\nu}} v_1(hc^{\frac{\nu}{1-\nu}}), \tag{10}$$

for an arbitrary function v_1*.*

This lemma characterizes the shape of the within-period indifference curves. Notice here that, in the long run, $hc^{\frac{\nu}{1-\nu}}$ will be constant so that the argument of v_1 will not change over time.

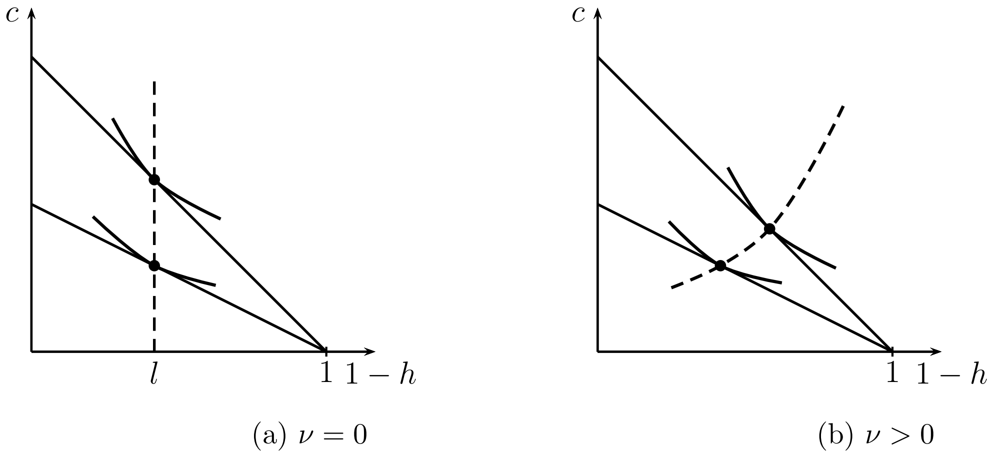


Figure 9: The consumption-leisure trade-off, I

The proof of Lemma is very similar to that for Euler's theorem, but with manipulating the first-order condition for the labor-leisure choice.

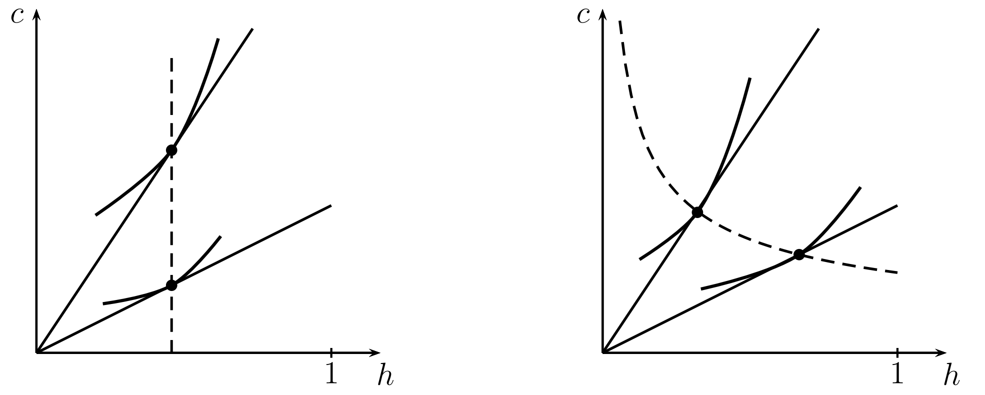
The indifference curves are illustrated with the following sequence of graphs. In Figure 9, we see the KPR indifference curves to the left and our case to the right, with consumption and leisure on the axes. Clearly, in our case, higher income implies more leisure.

These same preferences can equivalently be depicted with consumption and hours on the axes, as in Figure 10. As in the previous figure, the KPR case is to the left and has constant hours worked, whereas in the right-hand side panel hours decline with higher income.

Finally, Figure 11 takes the right-hand side graph from the previous figure and puts it on the left. On the right, now, we see that same combination of points but on log scales for both the axis. Here, the indifference curves are linear, and that is the the defining characteristic of the indifference curves in Lemma 1.

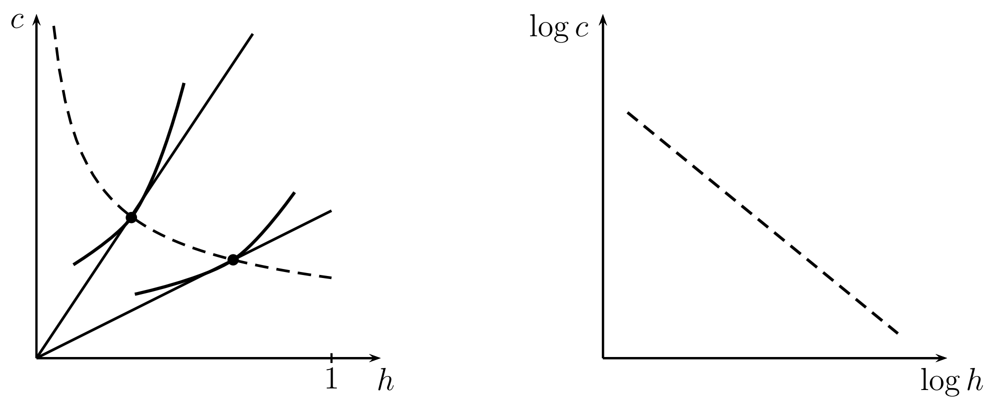
4.2.2 Curvature

Next, let us characterize curvature of u with respect to c with Lemma 2.



(a) $\nu = 0$ (b) $\nu > 0$

Figure 10: The consumption-leisure trade-off, II



(a) in levels (b) log scale

Figure 11: The consumption-leisure trade-off, III

Lemma 2. *Under Assumption 1, the relative risk aversion (RRA) function,* $\frac{-cu_{11}(c,h)}{u_1(c,h)}$*, must satisfy*

$$-\frac{cu_{11}(c,h)}{u_1(c,h)} = v_2(hc^{\frac{\nu}{1-\nu}})$$

for an arbitrary function v_2*.*

As for the previous lemma, let us point out that in the long run, i.e., along a balanced path, $hc^{\frac{\nu}{1-\nu}}$ is constant. Thus, the RRA will be constant. However, its long-run level is endogenous, and over shorter time horizons, in general it will not be constant.

The proof of the lemma is straightforward: it involves differentiation of the Euler equation with respect to λ, the use of Lemma 1, and some manipulations.

4.2.3 The proof structure and some comments

The structure of the overall proof, based on the lemmata, is as follows. Our description is in two steps that are similar in nature. First, use Lemma 2 to integrate over c to obtain a candidate for u_1; this can be accomplished straightforwardly since the left-hand side of the lemma can be expressed as the derivative of $\log u_c$ with respect to $\log c$. Now note that integration with respect to one variable delivers an unknown function (a "constant") of the other variable. This function can then be restricted by comparison with the characterization in Lemma 1 (a "cross-check").

Second, once the first integration and cross-checking, with its implied restrictions, is completed, integrate again with respect to c, from the obtained u_1, to deliver a candidate for u. Then, as in the previous step, another function of h appears, and it too needs to be cross-checked with Lemma 1 and thus further restricted. This, then, completes the proof.

Notice that, although we were motivated by data displaying increasing productivity growth and falling hours, the proof does not assume $\gamma > 1$ or $\nu \geq 0$. Potentially, the model could thus generate an increasing h at a constant rate, and we shall see an example of this below.

Second, to our surprise, we did not see a full proof of the KPR result in the literature. In particular, in the proofs we have looked at, the fact that the RRA is constant along a balanced path is taken to mean that this constant is exogenous/only a function of preferences (σ). This is a correct presumption but nontrivial to prove, and it is dealt with in our proof in the Appendix A.1.[5]

4.2.4 Sufficiency

Of course, the v in the theorem has to be such that a characterization based on first-order conditions is valid. Thus, v has to be such that the indifference curves defined by $u_0 = cv(hc^{\frac{\nu}{1-\nu}})$ have the right shape for all u_0. I.e., $v^{-1}(u_0/c)c^{-\frac{\nu}{1-\nu}}$ has to be strictly increasing and concave in c for all u_0.

Under these restrictions, we thus also have the following theorem, guaranteeing sufficiency.

Theorem 2. *Assume that $\nu < 1$. If $u(c, h)$ is given by*

$$u(c,h) = \frac{\left(c \cdot v\left(hc^{\frac{\nu}{1-\nu}}\right)\right)^{1-\sigma} - 1}{1-\sigma},$$

for $\sigma \neq 1$, or

$$u(c,h) = \log(c) + \log\left(v(hc^{\frac{\nu}{1-\nu}})\right),$$

where v is an arbitrary, twice continuously differentiable function with $v(x) > -\frac{\nu}{1-\nu}v'(x)x$ and $v'(x) < 0$ for all x and the above-stated concavity requirements, then it satisfies Assumption 1.

Since this proof is much less cumbersome than that for the main theorem, and since it involves the manipulations necessary in applied work based on the preference class we identify here, we include it in the main text.

[5] We would be very grateful if someone could point us to a proof somewhere, because we may well have missed it.

Proof. Straightforward differentiation delivers

$$u_1(c,h) = \frac{1}{c}\left(1 + \frac{\nu}{1-\nu}\frac{v'(hc^{\frac{\nu}{1-\nu}})}{v(hc^{\frac{\nu}{1-\nu}})}hc^{\frac{\nu}{1-\nu}}\right)\left(c\cdot v\left(hc^{\frac{\nu}{1-\nu}}\right)\right)^{1-\sigma}$$

and

$$u_2(c,h) = \frac{1}{h}\frac{v'(hc^{\frac{\nu}{1-\nu}})}{v(hc^{\frac{\nu}{1-\nu}})}hc^{\frac{\nu}{1-\nu}}\left(c\cdot v\left(hc^{\frac{\nu}{1-\nu}}\right)\right)^{1-\sigma}.$$

Dividing the latter by the former we obtain

$$\frac{u_2(c,h)}{u_1(c,h)} = \frac{c}{h}\frac{\frac{v'(hc^{\frac{\nu}{1-\nu}})}{v(hc^{\frac{\nu}{1-\nu}})}hc^{\frac{\nu}{1-\nu}}}{1+\frac{\nu}{1-\nu}\frac{v'(hc^{\frac{\nu}{1-\nu}})}{v(hc^{\frac{\nu}{1-\nu}})}hc^{\frac{\nu}{1-\nu}}}.$$

By multiplying c by $\lambda^{1-\nu}$ and h by $\lambda^{-\nu}$ we obtain that this expression increases by a factor λ. We have thus reproduced the first part of Assumption 1, i.e., the first-order condition for labor on a balanced growth path.

By evaluating $u_1(c,h)/u_1(c\gamma^{1-\nu}, h\gamma^{-\nu})$, we obtain $\gamma^{\sigma(1-\nu)}$, i.e., an expression that is independent of c and h and hence c and h can be scaled arbitrarily. By letting $R = \gamma^{\sigma(1-\nu)}$ we therefore see that also the second condition of Assumption 1 is verified. Finally, it is easy to see that $v(x) > -\frac{\nu}{1-\nu}v'(x)x$ and $v'(x) < 0$ ensure that utility is strictly decreasing in h and strictly increasing in c.

■

4.2.5 The elasticity of intertemporal substitution

The IES—the intertemporal elasticity of substitution of consumption—is a key object in some macroeconomic analyses. In the time-additive setting considered here, it is also directly related to the coefficient of relative risk aversion.[6] Here we wish to point out that in our framework, although the EIS remains constant on a balanced

[6]Swanson (2012) argues that, in the presence of leisure, the appropriate intertemporal notion of relative risk aversion is a different object, defined based on the value function. Hence, what we refer to here is not risk aversion in that sense but in the static sense.

growth path, it can be endogenously determined. In contrast, in the standard KPR setting, the EIS is exogenous. We have the following.

Proposition 1. *Given the preferences specified in Theorem 1, with $\nu = 0$, the intertemporal elasticity of substitution is independent of c and h: it equals $1/\sigma$. With $\nu \neq 0$, however, the intertemporal elasticity of substitution can depend on, and be both increasing or decreasing in, $hc^{\frac{\nu}{1-\nu}}$.*

Proof. For the KPR class this is verified straightforwardly. For the case $\nu \neq 0$ the verification builds on working out special cases. See Section 4.3.4. ■

4.3 Special cases and relations to the literature

We now look at special cases of interest.

4.3.1 King-Plosser-Rebelo (1988): $\nu = 0$

With $\nu = 0$ we get the following class of preferences

$$u(c,h) = \begin{cases} \frac{(c \cdot v(h))^{1-\sigma} - 1}{1-\sigma} & \text{if } \sigma \neq 1 \\ \log(c) + \log v(h), & \text{if } \sigma = 1. \end{cases} \tag{11}$$

This is the most general preference class that is consistent with a balanced growth path along which h is constant. These preferences were first specified by King, Plosser, and Rebelo (1988). In the KPR class the income and substitution effects of changes in the wage rate precisely cancel each other and households choose to supply constant hours. Due to this feature the KPR class is dominating the macro literature. Sometimes the KPR class is also referred to as "balanced-growth preferences" and often their use is justified because it seems appealing to restrict attention to a framework that is consistent with a balanced growth path. However, the KPR class is not only characterized by the balanced growth restriction but also by the requirement that labor supply is constant.

Within the KPR class two special cases stand out. One is the Cobb-Douglas case with $v(h) = (1-h)^{\kappa}$ and $\sigma \neq 1$ (or with $\sigma = 1$ and $v(h) = \kappa \log(1-h)$). Thus, $u(c,h) = (c(1-h)^{\kappa})^{1-\sigma}/(1-\sigma)$ for $\sigma \neq 1$ and otherwise $u(c,h) = \log c + \kappa \log(1-h)$. The Cobb-Douglas case restricts the elasticity of substitution between consumption and leisure to be one. Furthermore, the Cobb-Douglas case is part of the Gorman class, which implies that the labor supply is independent of wealth (and non-labor income).

The second often-used case of KPR preferences is

$$u(c,h) = \log c - \psi \frac{h^{1+\frac{1}{\theta}}}{1+\frac{1}{\theta}},$$

which is obtained by setting $\sigma = 1$ and $v(h) = \exp\left(-\psi \frac{h^{1+\frac{1}{\theta}}}{1+\frac{1}{\theta}}\right)$. The parameter $\theta > 0$ controls the (constant) Frisch elasticity whereas the relative risk aversion (and the intertemporal elasticity of substitution) is one.

4.3.2 A case of the Greenwood-Hercowitz-Huffman (1988) preferences

With $v(x) = 1 - x^{-\frac{1-\nu}{\nu}}$ and $\nu < 0$ with $\sigma \neq 1$ (and $v(x) = \log\left(x^{-\frac{1-\nu}{\nu}}\right) + \log\left(x^{\frac{1-\nu}{\nu}} - 1\right)$ with $\sigma = 1$), we obtain the quasi-linear preferences

$$u(c,h) = \begin{cases} \frac{\left(c - h^{1-\frac{1}{\nu}}\right)^{1-\sigma} - 1}{1-\sigma} & \text{if } \sigma \neq 1, \\ \log\left(c - h^{1-\frac{1}{\nu}}\right) & \text{if } \sigma = 1. \end{cases} \tag{12}$$

with $\nu < 0$. This is an often used case of the Greenwood-Hercowitz-Huffman (1988) preferences in which the Frisch elasticity is constant and equal to $-\nu$. These preferences are non-homothetic but they are part of the Gorman class. GHH preferences preclude any income effect on hours worked. Clearly, with a substitution effect alone, GHH preferences imply increasing hours as the wage rate increases. Consequently, we have $\nu < 0$ and there is no overlap with the KPR class. In fact, preferences (12) imply a relative risk aversion which depends on $hc^{\frac{\nu}{1-\nu}}$.

Quasi-linear preferences are widely used in the applied labor literature, where the household problem is often assumed to be static and σ can be set to zero without loss of generality. However, the quasi-linear formulation does preclude income effects.

4.3.3 Constant Frisch elasticity à la MaCurdy (1981)

With $v(x) = \left(1 - \frac{\psi\nu(1-\sigma)}{(1-\nu)(\sigma-1)} x^{\frac{(1-\nu)(\sigma-1)}{\nu}}\right)^{\frac{1}{1-\sigma}}$ for $\sigma \neq 1$, we obtain the constant Frisch elasticity of labor supply à la MaCurdy (1981) with

$$u(c,h) = \frac{c^{1-\sigma} - 1}{1-\sigma} - \psi \frac{h^{\frac{(1-\nu)(\sigma-1)}{\nu}}}{\frac{(1-\nu)(\sigma-1)}{\nu}}, \text{ if } \sigma \neq 1. \tag{13}$$

The attractiveness of this functional form is that two important elasticities are controlled by two separate parameters: the EIS is constant and equal to $1/\sigma$ and the constant Frisch elasticity is $\theta = \frac{\nu}{(1-\nu)(\sigma-1)-\nu}$. As is well known, with $\sigma \neq 1$, preferences of the form (13) are not part of the KPR class. For this reason, as already discussed in subsection 4.3.1, a significant part of the macroeconomic literature restricts itself to the case with a unitary EIS by setting $\nu = 0$, $\sigma = 1$ and $v(h) = \exp\left(-\psi \frac{h^{1+\frac{1}{\theta}}}{1+\frac{1}{\theta}}\right)$.[7] Then preferences become $u(c,h) = \log c - \psi \frac{h^{1+\frac{1}{\theta}}}{1+\frac{1}{\theta}}$ and are part of the KPR class.

Figure 12 below illustrates how σ and ν have to be restricted on a balanced path with falling hours: $\nu > 0$ requires $\sigma > \frac{1}{1-\nu} > 1$. Thus, any point on the downward-sloping curve is admissible (in the figure ν is set at a quantitatively reasonable value).

[7]For instance, Shimer (2010) proposes this preference specification in chapter 1 of his textbook and then writes *This formulation imposes that preferences are additively separable over time and across states of the world. It also imposes that preferences are consistent with balanced growth—doubling a household's initial assets and its income in every state of the world doubles its consumption but does not affect its labor supply. [...] I maintain both of these assumptions throughout this book.*

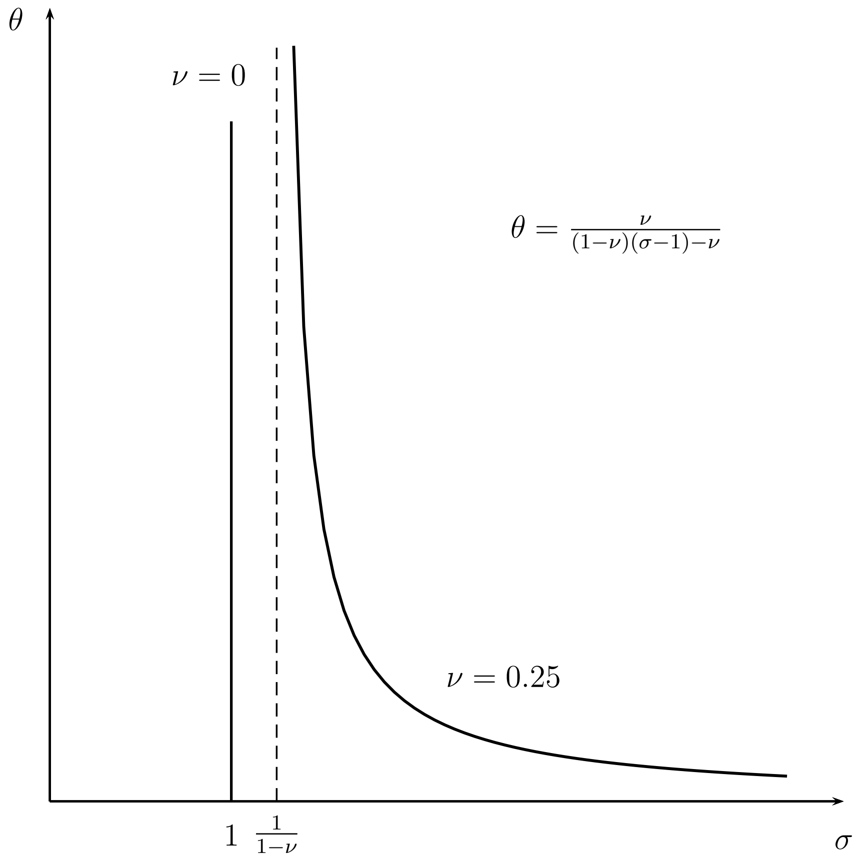


Figure 12: Combinations of elasticities

The figure shows combinations of relative risk aversion σ and Frisch elasticity θ in the functional form (13) that are consistent with (i) constant hours ($\nu = 0$) and (ii) hours falling at rate $\gamma^{-0.25}$. With two percent productivity growth, i.e., $\gamma = 1.02$ and $\nu = 0.25$ hours worked decline at roughly 0.5 percent per year.

4.3.4 An endogenous IES

The present section connects with Proposition 1 and discusses the EIS or, alternatively, the (static) RRA implied by our preference class. As already pointed out, under GHH—which is part of the present class, though that utility function is not consistent with falling hours if productivity grows—the RRA is nontrivially determined. For many applications, perhaps particularly in asset pricing, it may be interesting to consider preferences where the RRA in particular is decreasing in the consumption-hours aggregate $ch^{\frac{\nu}{1-\nu}}$: in this case booms involve lower risk aversion.

We have not pursued a general investigation into how the RRA may vary under different assumptions on v. It may however be instructive to simply show that a formulation with a decreasing RRA is possible. So let $v(x) = \left(1 - \frac{\psi(1-\sigma)}{\epsilon}x^{\epsilon}\right)^{\frac{1}{1-\sigma}}$ and $\epsilon \equiv \frac{1-\nu}{\nu}$. We then obtain the functional form

$$u(c,h) = \frac{c^{1-\sigma}}{1-\sigma} - \psi\frac{h^{\epsilon}c^{2-\sigma}}{\epsilon}, \tag{14}$$

for $\psi > 0$, $\sigma > 2$ and $\epsilon > \sigma - 1$. In this case, we obtain

$$\text{RRA} = \sigma + \frac{(2-\sigma)\psi\frac{h^{\epsilon}c}{\epsilon}}{1-(2-\sigma)\psi\frac{h^{\epsilon}c}{\epsilon}}, \tag{15}$$

which is decreasing in $x \equiv hc^{\frac{\nu}{1-\nu}}$.

4.3.5 Departing from time invariance or time separability

Our preferences rely on there being a stationary utility function $u(c,h)$ characterizing choice. It is not altogether uncommon in the literature that people use utility functions that are either not stationary or not time-separable. As for non-stationarity, the typical assumption is that some elements of the period utility function shift with labor productivity. Such functions are often motivated by (but not derived from) some form of home-production structure where the same productivity growth as in final-goods production occurs. As an example, one can make (13) consistent with constant hours in the long run by adding a time-varying term in front of ψ that is growing at the appropriate rate (see, e.g., Mertens and Ravn, 2011). Such a formulation has been deemed useful when one wants to consider free curvature in consumption and hours separately and yet not violate the balanced-growth conditions.

Reconciling constant hours worked in the long run with a small (or inexistent) income effect has also attracted some attention in the macro literature, since small income effects are sometimes appealing when it comes to fluctuations around a balanced growth path. Hence, the literature has extended the KPR class by giving up the assumption of time separability. A particularly well-known case is Jaimovich and Rebelo (2009). Our analysis shows that even the GHH utility function (12) is part of the general balanced-growth preferences specified in Theorem 1 though, as discussed above, they would imply increasing hours worked as wages grow. However, by adding a "habit" term $X_t = c_t^{\rho} X_{t-1}^{1-\rho}$ in front of $h^{1-\frac{1}{\nu}}$ in (12), we can also obtain the preferences studied in Jaimovich and Rebelo (2009).

The purpose here is not to take issue with preference formulations that depart from time invariance or time separability. It suffices to say that the "tricks" that have been employed in the literature are still possible to employ under our preference class.

5 Consumer heterogeneity and cross-sectional facts

We now comment briefly on two important concerns.

5.1 Models of consumer heterogeneity

Our theory of labor supply in the long run, strictly speaking, holds only for a representative-agent economy. Is it relevant, then, in cases when aggregation does not hold? Whereas it is beyond the scope of the present paper to provide a full answer to this question, let us still conjecture in the affirmative. More precisely, we conjecture that in an environment with a stationary distribution of agents heterogeneous in assets, wages, utility-function parameters, etc., preferences in our class are needed to match the aggregate growth facts (including aggregate hours shrinking at a constant rate).

The reason for this conjecture is perhaps best explained with an example. So consider the modern macro-style models of inequality: the Bewley-Huggett-Aiyagari model. This model by now exists in a vast variety of versions, with the common element being that there are incomplete markets for consumer-specific idiosyncratic shocks of different kinds and implied differences in wealth and consumption. Many of these models also consider substantial additional heterogeneity, such as in preferences (see, e.g., the multiple-discount factor model in Krusell and Smith, 1998), and yet others consider life-cycle versions with and without bequest motives (see, e.g., Huggett, 1996).

These modern-macro models of inequality, then, don't display aggregation (at the very least due to incomplete markets) and they are typically analyzed in steady state. A key question, then, is: for standard KPR preferences, and more generally for the broader class of preferences considered here, do these models admit balanced growth? The answer is yes. It is straightforward to transform variables and verify this assertion, just like a representative-agent model would be rendered stationary by variable transformation. Of course, growth makes a difference—the discount rate(s), for example, would need to be transformed—so that some aspects of the aggregate variables (such as the capital-output ratio) will depend on the rate of growth, as will the moments of the stationary distribution of wealth. Why does the transformation of variables work? It is straightforward to see that it is precisely because the preferences are in the pre-specified class, and for this reason we have trouble seeing that it would work outside of this class: balanced growth is, by definition, a set of paths for the economy's different variables that can be rendered constant by standard transformation.

5.2 Cross-sectional data

Another concern one might have from a perspective of heterogeneity is that perhaps the model with an income effect that is larger than the substitution effect might be inconsistent with what we know from cross-sectional data on households. In particular, there seems to be a view that consumers with higher wages work more not less, as would be implied by our theory.

First, we are not entirely sure of what the data says. Ideally, one would want a life-time, all-inclusive hours measure and then ceteris-paribus experiments where a permanent wage is changed across households. Arguably, convincing such studies are hard to come by. Interestingly, there is in fact a recently published study that claims that the wage-hours correlation is negative, not positive. In particular, the study of the intensive margin in Heathcote, Storesletten, and Violante (2014) reports such a correlation, after taking out time dummies and age effects. As for the extensive margin, it is well documented that the highly educated work longer, but they also start working later.

If, however, the perception that high-skilled people work more, not less, is correct, then we must point out that such a fact would be difficult to explain also with the standard model, i.e., with KPR preferences: our generalization would merely make the challenge slightly more difficult. There are studies in the literature that have attempted to address this issue, using a combination of assumptions. One is that the high wages that are observed—and are observed to be associated with higher working hours—represent a temporary window of opportunity. For such a situation, our preference class is consistent with a positive correlation. Another possibility is a non-convexity of the budget set of consumers in the form of a wage rate that depends on the amount of hours worked (see Erosa, Fuester and Kambourov, 2015). Thus, there are promising ways to generate a positive wage-hours correlation, though it remains to address the balanced-growth facts with these theories.

Finally, we note that based on the consumer data they look at, Heathcote, Storesletten and Violante (2014), actually use a MaCurdy preference formulation—thus, one in our class—that implies a strong income effect, even a stronger one that we need to account for the long-run fall in hours: $\nu = 0.184 > 0$.

6 Hours worked in Europe and the U.S.

Prescott (2004) and Rogerson (2006, 2008) use preferences that imply constant hours worked in the long run. They argue that U.S. hours worked relative to those in Europe have gone up because of upward movements of tax rates in Europe relative to those in the U.S. What does the present theory of long-run labor supply have to say about these comparisons?

The present theory does not present a problem for the Prescott-Rogerson argument. The perspective offered by our model only calls for a slightly different interpretation of the data. The stable hours in the postwar U.S. may or may not be difficult to explain from the perspective of our theory, but regardless of this, the falling hours in the main European economies may well be because of higher taxes (in relative terms). It is beyond the scope of the present paper to investigate this issue in detail, but let us briefly look at the data and make some remarks.

So Figure 13 shows postwar hours in the U.S., in Germany, and in France. Clearly, hours fall at a fast rate in the European economies, indeed at a much faster rate than in the broader cross-section of countries we looked at in the data section of the paper. Thus, the cross-country average is a rate of decline in hours that lies in between that of the U.S. and those in Germany and France. But to the extent one wanted to explain the U.S. experience from the perspective of our model, what could be said? We believe that there are at least three reasons why the U.S. experience may have represented a temporary departure from a long-run trend of

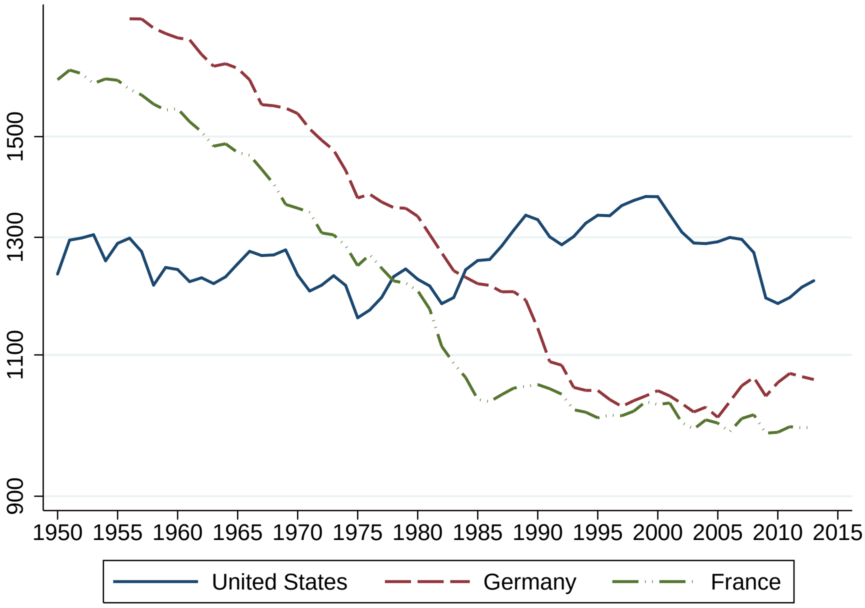


Figure 13: **Hours worked in the U.S., Germany, and France, 1950–2013**

Notes: Source: GGDC Total Economy Database for total hours worked and OECD for the data on population aged 15–64. The figure is comparable to the ones in Rogerson (2006).

falling hours. One is the tax cuts of the Reagan years, that appear to have been permanent: the Prescott-Rogerson argument. Another is the fact that median wages have not grown much at all in the U.S., and per-capita hours are un-weighted by productivity/wages, so if the vast majority of the population does not experience wage growth, constant hours is of course consistent with our theory. Thus, the well-documented increase in wage inequality from the late 1970s—which is also when hours were rising within the postwar period—is factor two. Factor three is women's increased labor-force participation, which clearly is a transitional phenomenon, and the point here is that women may formerly have been constrained and not able to work (at appropriate wages) so that one cannot simply take a unitary perspective on the household and say that male hours should decline at a very high rate to compensate for women's higher hours. It would be interesting to try to account for these factors together, along with other drivers, but again this would be a project in itself and the comments here just mean to signal how our preference formulation would fit into such an investigation.

Finally, what about the developments of hours worked currently in the U.S., and what about their future path? Very recently, participation rates have fallen, and from the perspective of our theory, at least if the bulk of households have begun to experience higher wages, such a fall in the participation rates would be consistent with our theory and would also imply that one would not expect participation to bounce back up.[8] Similarly, for the future, if productivity keeps growing, our theory of course predicts that hours worked will keep falling.

7 Conclusions

We have presented an extension to the standard preference framework used to account for the balanced-growth facts. The new preference class admit that hours

[8]This is of course not to say that there are no cyclical reasons for the participation movements.

worked fall at a constant rate when labor productivity grows at a constant rate, as we have also documented the data to show across history and space. The new preference class intuitively involves an income effect that exceeds the substitution effect. What are alternative theories that could explain why hours fall? Could an alternative theory explain the past without contradicting the constant-hours presumption of the standard macroeconomic model? We suspect that some other form of non-homotheticity would work, at least qualitatively, and that it would be possible to formulate a non-homotheticity such that it vanishes with increasing income/productivity, as with Stone-Geary formulation for consumption. It seems challenging to confront such a theory with data in a systematic way, but perhaps it could be done. An advantage with our framework is that it does incorporate (non-vanishing) non-homotheticity in a tractable, intuitive way.

We believe that our new preference class has potentially interesting implications in a range of contexts. As for growth theory and growth empirics, note that on our balanced path, the main macro aggregates (output, investment, consumption) grow at the rate $\gamma^{1-\nu} > 1$ (ignoring population growth), i.e., at a rate lower than productivity and in a way that is determined by the preference parameter ν. Notice also that from a development perspective, falling hours worked is not a sign of economic malfunctioning but rather the opposite: it is the natural outcome given preferences and productivity growth, and it rather instead illustrates clearly how output is an incomplete measure of welfare (see Jones and Klenow, 2015): leisure grows. Interestingly, our theory says that growth theory probably should not abstract from labor supply (which is typically set to "1" in models); rather, it seems an important variable to model in conjunction with the growth process.

Does our preference class have something to say about business-cycle analysis? We cannot identify any immediate substantive implications, but it is clear that our model can be amended with shocks and transformed to a stationary one that can be analyzed just like in the RBC and NK literatures. The preference class consistent with hours falling at a constant, but low, rate is a bit different than the standard one. From the perspective of a particular case—the MaCurdy constant-Frisch elasticity functional form—one can admit an arbitrarily low elasticity of intertemporal substitution of consumption, though only if the Frisch elasticity is then also very low.

Other areas where the new preference class may be interesting to entertain include asset pricing and public finance. For asset pricing—as we showed in the paper—it is possible to have attitudes toward risk behave qualitatively differently, and possibly more in line with data, than using standard balanced-growth preferences. These same features would potentially also help explain portfolio-choice patterns across wealth groups. For public finance, the sustainability of government programs, such as social security, and debt service in the future depend greatly on how hours worked will develop (along, of course, with the development of productivity).

We hope to address some of these applications in future work.

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A.1 Appendix A: Proof

We now present the proof of Theorem 1.

Proof. The proof starts by stating and proving two lemmata, one characterizing the marginal rate of substitution (MRS) function between c and h and one characterizing the curvature with respect to consumption: the relative risk aversion (RRA) function. The proof then uses these lemmata to derive the final characterization. Because the proof will involve a large number of auxiliary functions that are either functions of $hc^{\frac{\nu}{1-\nu}}$ or of h, we economize somewhat on notation by sometimes denoting $hc^{\frac{\nu}{1-\nu}}$ by x and by systematically letting f_i be a function of x whereas m_j is a function of h (where i and j are indices for the different functions we will define). A sequence of constants will also appear; they are denoted A_k, accordingly, from $k = 1$ and on.

We now proof the first lemma.

Proof. Because λ is arbitrary, we can set it in (8) so that $c\lambda^{1-\nu} = 1$. This delivers

$$-\frac{u_2(1, hc^{\frac{\nu}{1-\nu}})}{u_1(1, hc^{\frac{\nu}{1-\nu}})} = wc^{-\frac{1}{1-\nu}}.$$

Evaluating (8) at $\lambda = 1$ we obtain $-\frac{u_2(c,h)}{u_1(c,h)} = w$. Inserting this expression, we thus obtain

$$\frac{u_2(c, h)}{u_1(c, h)} = c^{\frac{1}{1-\nu}} \frac{u_2(1, hc^{\frac{\nu}{1-\nu}})}{u_1(1, hc^{\frac{\nu}{1-\nu}})}. \tag{A.1}$$

Now identifying $v_1(x)$ as $\frac{u_2(1,x)}{u_1(1,x)}$, where $x = hc^{\frac{\nu}{1-\nu}}$, gives the result in Lemma 1. ∎

It follows from Lemma 1 and u being twice continuously differentiable that v_1 is continuously differentiable.

Proof. The second first-order condition, (9), holds for all λ so it can be differentiated with respect to λ and then evaluated at $\lambda = 1$ and divide by (9) again to yield

$$(1-\nu)c\gamma^{1-\nu}\frac{u_{11}\left(c\gamma^{1-\nu}, h\gamma^{-\nu}\right)}{u_1\left(c\gamma^{1-\nu}, h\gamma^{-\nu}\right)} - \nu h\gamma^{-\nu}\frac{u_{12}\left(c\gamma^{1-\nu}, h\gamma^{-\nu}\right)}{u_1\left(c\gamma^{1-\nu}, h\gamma^{-\nu}\right)} = (1-\nu)c\frac{u_{11}\left(c, h\right)}{u_1\left(c, h\right)} - \nu h\frac{u_{12}\left(c, h\right)}{u_1\left(c, h\right)}. \tag{A.2}$$

This equation has to hold for all γ (and consequently one must adjust R, but R does not appear in the equation). Moreover, it has to hold for all c and h; it has to hold for all h because Assumption 1 allows any w and hence any h (given an arbitrary c). Given this, by setting γ so that $c\gamma^{1-\nu} = 1$ we can state (A.2) as

$$(1-\nu)\frac{u_{11}(1, hc^{\frac{\nu}{1-\nu}})}{u_1(1, hc^{\frac{\nu}{1-\nu}})} - \nu hc^{\frac{\nu}{1-\nu}}\frac{u_{12}(1, hc^{\frac{\nu}{1-\nu}})}{u_1(1, hc^{\frac{\nu}{1-\nu}})} = (1-\nu)c\frac{u_{11}(c,h)}{u_1(c,h)} - \nu h\frac{u_{12}(c,h)}{u_1(c,h)},$$

which holds for all c and h. We conclude that the right-hand side of equation (A.2) only depends on $hc^{\frac{\nu}{1-\nu}}$, i.e., we can write

$$(1-\nu)c\frac{u_{11}(c,h)}{u_1(c,h)} - \nu h\frac{u_{12}(c,h)}{u_1(c,h)} = f_1(hc^{\frac{\nu}{1-\nu}}), \tag{A.3}$$

where f_1 is then defined by the expression on the left-hand side of equation (A.2). Differentiating (10) with respect to c gives

$$\frac{u_{12}(c,h)u_1(c,h) - u_{11}(c,h)u_2(c,h)}{u_1(c,h)^2} = \frac{c^{\frac{\nu}{1-\nu}}v_1(x)}{1-\nu} + \frac{\nu c^{\frac{1}{1-\nu}}v_1'(x)hc^{\frac{\nu}{1-\nu}-1}}{1-\nu} \equiv c^{\frac{\nu}{1-\nu}}f_2(x),$$

where we used the notation $x = hc^{\frac{\nu}{1-\nu}}$ and the last equality simply defines a new function f_2. Then, again using the characterization of the MRS function to replace $\frac{u_2(c,h)}{u_1(c,h)} = c^{\frac{1}{1-\nu}}v_1(hc^{\frac{\nu}{1-\nu}})$, we obtain

$$\frac{u_{12}(c,h)}{u_1(c,h)} - \frac{u_{11}(c,h)}{u_1(c,h)}c^{\frac{1}{1-\nu}}v_1(x) = c^{\frac{\nu}{1-\nu}}f_2(x),$$

and hence

$$\frac{hu_{12}(c,h)}{u_1(c,h)} = \frac{u_{11}(c,h)}{u_1(c,h)}hc^{\frac{1}{1-\nu}}v_1(x) + hc^{\frac{\nu}{1-\nu}}f_2(x) = \frac{cu_{11}(c,h)}{u_1(c,h)}xv_1(x) + xf_2(x).$$

This expression can be combined with equation (A.3) to conclude that $\frac{-cu_{11}(c,h)}{u_1(c,h)}$ must be a function only of x; we call this function v_2.[9] ■

[9] The function $v_2(x)$ is thus defined by

$$-(1-\nu)v_2(x) + \nu\left(v_2(x)xv_1(x) - xf_2(x)\right) = f_1(x),$$

which straightforwardly offers a solution (that will depend on v_1, f_1, and f_2).

We will now combine the information in Lemmata 1 and 2 to complete our proof. We do this in two steps. First we analyze the case with $\nu \neq 0$ and then the case with $\nu = 0$. Note that the case with $\nu = 0$ is already discussed in King, Plosser and Rebelo (1988).

The strategy of the proof is very similar in the two cases. First, we integrate the RRA function in Lemma 2 with respect to c to obtain a functional form for u_1. As we integrate with respect to c, an unknown function of h appears. Then, by differentiating the obtained function for u_1 with respect to h we arrive at an expression that can be compared to a restriction on $\frac{u_{12}}{u_1}$ found in the proof of Lemma 2. This comparison gives us some additional restrictions on the unknown function of h. Thus, since the proof of Lemma 2 uses Lemma 1, we are in effect making sure that the functional form we arrive at is consistent with both our lemmata. Having arrived at a form for u_1, we again integrate to deliver a candidate for u. Due to the integration a new unknown function of h again appears, but we can again restrict this function by differentiating our candidate u with respect to h and comparing the result to Lemma 1. This, then, delivers our final functional form.

Case with $\nu \neq 0$: note that the characterization of the RRA function in Lemma 2 can be restated as

$$\frac{\partial \log u_1(c,h)}{\partial \log(c)} = -v_2\left(\exp\left(\log(h) + \frac{\nu}{1-\nu}\log(c)\right)\right).$$

This equation can be integrated straightforwardly with respect to $\log(c)$ to arrive at

$$u_1(c,h) = f_3(hc^{\frac{\nu}{1-\nu}})m_1(h), \tag{A.4}$$

where f_3 is a new function of x and m_1 is an arbitrary function of h.[10]

[10] The integration delivers an expression for $\log u_1(c,h)$ as a function of $\log x$ plus a function of h. The latter function can only be a function of h since c was integrated over. The function of $\log x$ can be rewritten as a function of x. Equation (A.4) is then obtained after raising e to the left- and right-hand sides of this equation and f_3 and m_1 are defined accordingly.

Now observe that it follows from the proof of Lemma 2 that also $\frac{u_{12}(c,h)h}{u_1(c,h)}$ can be written as a function of x alone: it equals $-v_2(x)xv_1(x) + xf_2(x)$. We use this fact to further restrict the function m_1. In particular, by taking derivatives in equation (A.4) with respect to h, multiplying by h, and dividing by u_1, we obtain an expression for $\frac{u_{12}(c,h)h}{u_1(c,h)}$ that can be written as

$$f_4(hc^{\frac{\nu}{1-\nu}}) + \frac{m_1'(h)h}{m_1(h)},$$

where f_4 is defined by $f_4(x) \equiv f_3'(x)x/f_3(x)$. For the consistency of these two expressions for $\frac{u_{12}(c,h)h}{u_1(c,h)}$—the one just stated, and the arbitrary function of x given above $(-v_2(x)xv_1(x) + xf_2(x))$—it must be that $\frac{m_1'(h)h}{m_1(h)}$ is a constant.[11] Hence, $m_1(h) = A_1h^\kappa$ for some constants A_1 and κ, i.e., it is isoelastic. Using this fact in (A.4) gives

$$u_1(c,h) = f_3(hc^{\frac{\nu}{1-\nu}})A_1h^\kappa. \tag{A.5}$$

Since $\nu \neq 0$, the expression on the right-hand side can equivalently be written $f_5(h^{\frac{1-\nu}{\nu}}c)h^\kappa$, by defining $f_5(x) = A_1f_3(x^{\frac{1-\nu}{\nu}})$. Therefore, (A.5) can be easily integrated with respect to c to deliver

$$u(c,h) = f_6(hc^{\frac{\nu}{1-\nu}})h^{\kappa-\frac{1-\nu}{\nu}} + m_2(h), \tag{A.6}$$

where f_6 is the new function that results from the integration of f_5 over c and m_2 is an arbitrary function of h (as the integration was over c). With the aim of further restricting m_2, we can express u_2 as

$$u_2(c,h) = u_1(c,h)c^{\frac{1}{1-\nu}}v_1(x) = f_3(x)A_1h^\kappa c^{\frac{1}{1-\nu}}v_1(x) = f_7(hc^{\frac{\nu}{1-\nu}})h^{\kappa-\frac{1}{\nu}}, \tag{A.7}$$

where we have used the characterization of the MRS function in Lemma 1, (A.5), and finally the definition $f_7(x) \equiv f_3(x)A_1x^{\frac{1}{\nu}}v_1(x)$. Thus, we can now check consistency

[11] If $\frac{m_1'(h)h}{m_1(h)}$ would depend on h, consistency could not be fulfilled for any given combination of c and h.

by taking the derivative of u with respect to h in (A.6) and comparing with (A.7). The derivative becomes

$$\left(\kappa - \frac{1-\nu}{\nu}\right) f_6(x)h^{\kappa-\frac{1}{\nu}} + c^{\frac{\nu}{1-\nu}} f_6'(x)h^{\kappa-\frac{1-\nu}{\nu}} + m_2'(h) \equiv f_8(x)h^{\kappa-\frac{1}{\nu}} + m_2'(h),$$

where the equality comes from collecting terms and defining a new function f_8 accordingly. For consistency, thus, this expression has to equal $f_7(x)h^{\kappa-\frac{1}{\nu}}$ for all x and h. This is possible if and only if $m_2'(h) = A_2h^{\kappa-\frac{1}{\nu}}$, where A_2 is a constant. Concentrating first on the case where $\kappa - \frac{1}{\nu} \neq -1$, we obtain $m_2(h) = \left(1+\kappa-\frac{1}{\nu}\right)^{-1} A_2h^{1+\kappa-\frac{1}{\nu}} + A_3 \equiv A_4h^{1+\kappa-\frac{1}{\nu}} + A_3$. The constant A_3 can be set arbitrarily as it does not affect choice. The second term in (A.6) can thus be merged together with the first term using factorization and we can write $u(c,h)$ as $f_9(x)h^{1+\kappa-\frac{1}{\nu}} + A_3$, with $f_9(x) \equiv f_6(x) + A_4$. Now note that $h^{1+\kappa-\frac{1}{\nu}} = x^{1+\kappa-\frac{1}{\nu}}c^{-\frac{\nu}{1-\nu}\left(1+\kappa-\frac{1}{\nu}\right)}$, so that $u(c,h)$ can be written as $f_9(x)x^{1+\kappa-\frac{1}{\nu}}c^{-\frac{\nu}{1-\nu}\left(1+\kappa-\frac{1}{\nu}\right)} + A_3$. Now define $v(x) \equiv \left((1-\sigma)f_9(x)x^{1+\kappa-\frac{1}{\nu}}\right)^{\frac{1}{1-\sigma}}$ and $\sigma \equiv \kappa\frac{\nu}{1-\nu}$ and we conclude that we can write $u(c,h) = \frac{(cv(x))^{1-\sigma}-1}{1-\sigma}$ (where A_3 has been set to $-1/(1-\sigma)$).

In the special case where $1+\kappa = 1/\nu$, we obtain from equation (A.6) that $u(c,h) = f_6(hc^{\frac{\nu}{1-\nu}}) + m_2(h)$, but we also see from the arguments above that $m_2(h)$ has to equal $A_2\log h + A_5$, where A_5 is again an arbitrary constant. Since (given $\nu \neq 0$) we can write $\log h = \log x - \frac{\nu}{1-\nu}\log c$, our candidate u can be rewritten as $u(c,h) = f_6(x) - A_2\frac{\nu}{1-\nu}\log(c) + A_2\log(x) + A_5$. The constant A_5 can be set to zero and we can write $u(c,h) = -A_2\frac{\nu}{1-\nu}\left[\log(c) - \frac{1-\nu}{A_2\nu}f_6(x) - \frac{1-\nu}{\nu}\log(x)\right]$. The factorized constant can be normalized to -1 (as it does not affect choice), and we can then define $\log v(x) \equiv f_6(x) + \frac{1-\nu}{\nu}\log x$, an arbitrary function; this concludes the case $1+\kappa = 1/\nu$. Hence we obtain the utility function

$$u(c,h) = \begin{cases} \frac{\left(c\cdot v(hc^{\frac{\nu}{1-\nu}})\right)^{1-\sigma}-1}{1-\sigma} & \text{if } \sigma \neq 1 \\ \log(c) + \log v(hc^{\frac{\nu}{1-\nu}}), & \text{if } \sigma = 1. \end{cases}$$

Utility is strictly decreasing in h and strictly increasing in c as long as $v'(x) < 0$ and $v(x) > -\frac{\nu}{1-\nu}v'(x)x$.

Case with $\nu = 0$: in this case we can rewrite the RRA function in Lemma 2 as

$$\frac{\partial \log u_1(c,h)}{\partial \log c} = -v_2(h). \tag{A.8}$$

We can integrate this equation with respect to $\log c$ to obtain

$$\log u_1(c,h) = -v_2(h)\log c + m_3(h), \tag{A.9}$$

where m_3 is an arbitrary function, given that we integrated over c. Differentiating with respect to h then gives

$$\frac{u_{12}(c,h)}{u_1(c,h)} = -v_2'(h)\log c + m_3'(h). \tag{A.10}$$

From the proof of Lemma 2 we know that $\frac{u_{12}(c,h)}{u_1(c,h)}$ must be possible to write as a function of h alone (recall that $\nu = 0$). From this we conclude that we must have $v_2'(h) = 0$, i.e., the only version of equation (A.9) that is possible is $\log u_1(c,h) = -\sigma \log c + m_3(h)$, where σ is a constant. Using this fact and raising e to both sides of (A.9) then delivers

$$u_1(c,h) = c^{-\sigma} m_4(h), \tag{A.11}$$

where $m_4(h) = \exp(m_3(h))$. Integrating (A.11) with respect to c we can write

$$u(c,h) = \begin{cases} \frac{(cv(h))^{1-\sigma}-1}{1-\sigma} + m_5(h) & \text{if } \sigma \neq 1 \\ m_4(h)\log(c) + \log v(h) & \text{if } \sigma = 1; \end{cases} \tag{A.12}$$

here, in the first equation $-1/(1-\sigma) + m_5$ is another function (of h) that appears because of the integration over c and $v(h)$ is defined from $\frac{v(h)^{1-\sigma}-1}{1-\sigma} = m_4(h)$, whereas in the second equation $\log v$ is the function that appears due to the integration.

We will now, along the lines of the case where $\nu \neq 0$, show that m_4 and m_5 will have to have very specific forms. We look at each in turn. So in the case with $\sigma \neq 1$, combine (A.11) with Lemma 1 to write

$$u_2(c,h) = c^{1-\sigma} v_1(h) m_3(h). \tag{A.13}$$

This can be contrasted with the result of differentiating (A.12) with respect to h, an operation that yields

$$u_2(c,h) = c^{1-\sigma} v(h)^{-\sigma} v'(h) + m_5'(h).$$

Since these last two equations both have to hold for all c and h, it must be that $m_5'(h) = 0$, i.e., that $m_5(h)$ is a constant (which can be abstracted from).

Turning to the case where $\sigma = 1$, along the same lines we again derive two expressions for u_2 and check consistency. Combining (A.11) with Lemma 1 one obtains that u_2 cannot depend on c. Differentiating the second line of (A.12) with respect to h, however, delivers a function of c unless $m_4(h)$ is a constant; as it does not affect choice, we set this constant to 1.

This is our final characterization and we have now reproduced the statement in our main theorem. In summary, in the $\sigma \neq 1$ case we obtain $u(c,h) = \frac{(cv(h))^{1-\sigma}-1}{1-\sigma}$ and in the $\sigma = 1$ case we obtain $\log(c) + \log v(h)$. This completes the proof for the case $\nu = 0$. To ensure that $u(c,h)$ is strictly decreasing in h we need $v'(x) < 0$. ■

B.1 Appendix B: Additional figures

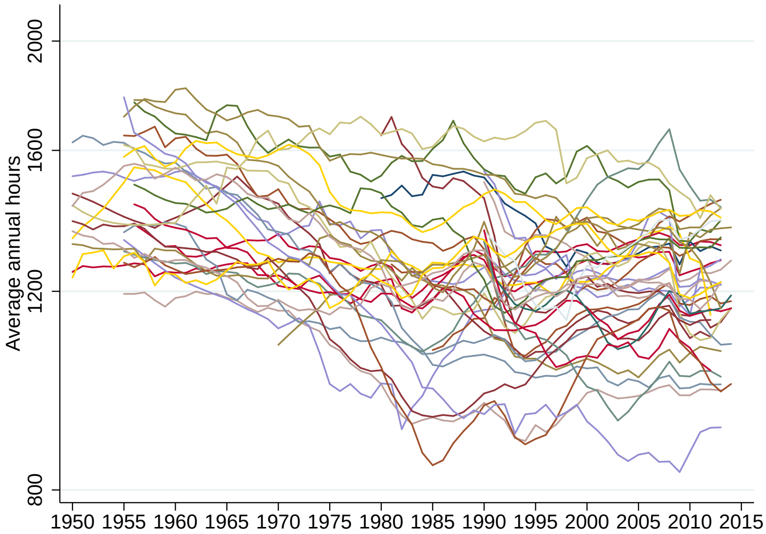


Figure B.1: **Average annual hours per capita aged 15–64, 1950–2015**

Notes: Source: GGDC Total Economy Database for total hours worked and OECD for the data on population aged 15–64. The figure is comparable to the ones in Rogerson (2006). The sample includes 37 countries. Regressing the logarithm of hours worked on time and country fixed effects gives slope coefficient of -0.00336. The R^2 of the regression is 0.64.

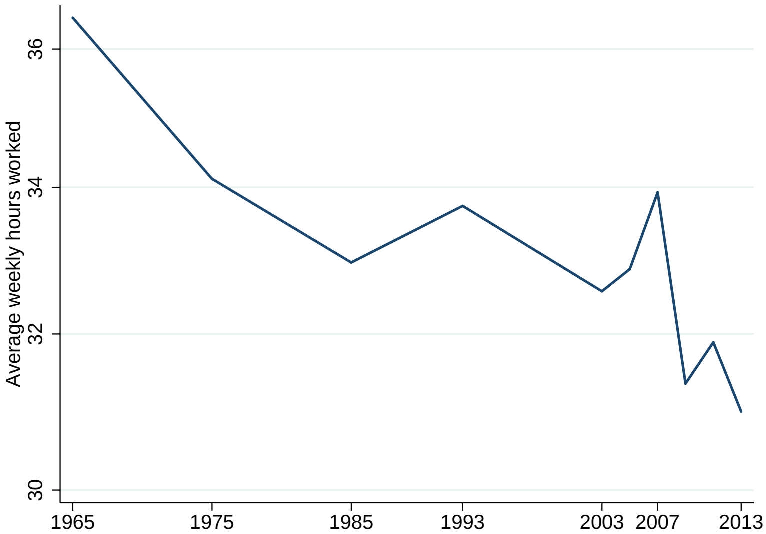


Figure B.2: **U.S. time used survey: Weekly hours worked**

Notes: Source: ATUS, following the methodology in Aguiar and Hurst (2007). The sample contains all non-retired, non-student individuals at age 21–65. For the years 1965–2003 the series is comparable to Aguiar and Hurst (2007) Table II and is updated till 2013 using the same methodology. Regressing the logarithm of hours worked on time gives slope coefficient of -0.0024.

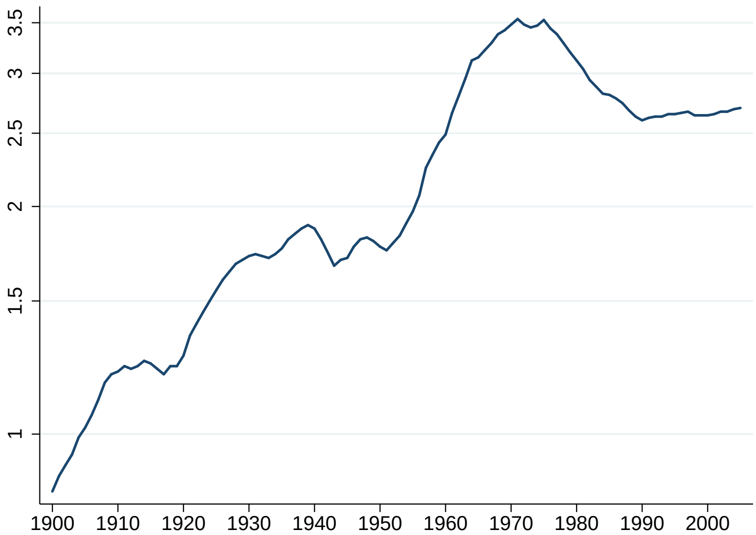
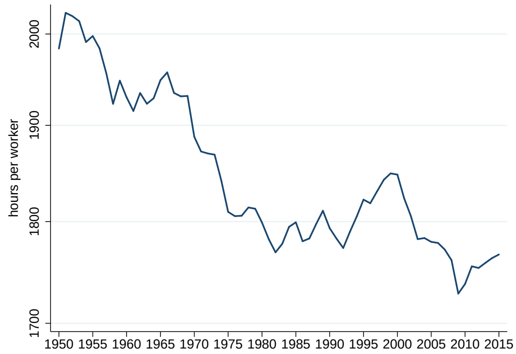
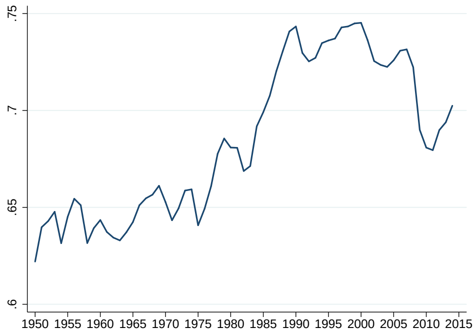


Figure B.3: **Weekly hours of School (including class and homework) per population aged 14+, 1900–2005**

Notes: Source: Ramey and Francis (2009).



(a) Hours per worker



(b) Participation rate

Figure B.4: Hours per worker and participation rate in the post-war U.S.

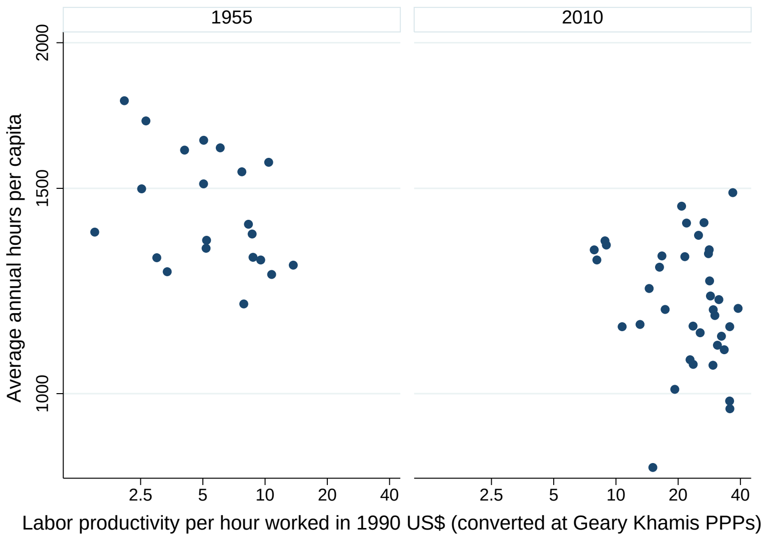
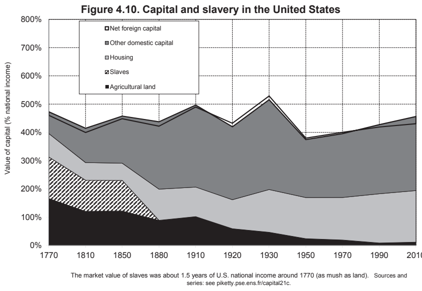
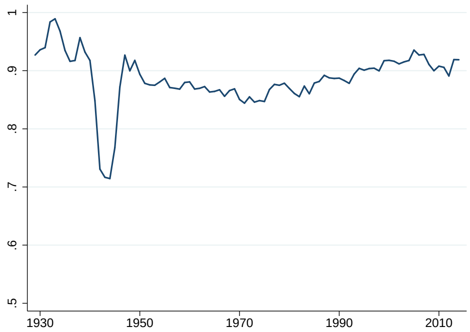


Figure B.5: **Hours worked vs. labor productivity**

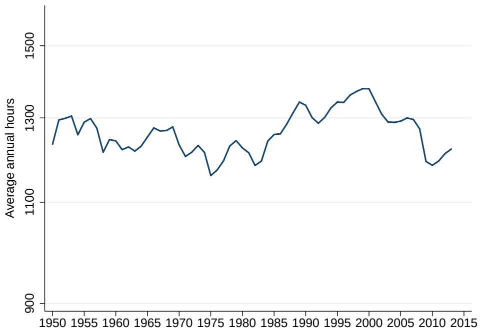
Source: GGDC Total Economy Database for total hours worked and labor productivity and OECD for the data on population aged 15–64. The figure shows the scatter plot between labor productivity and hours worked for the year 1955 and 2010.



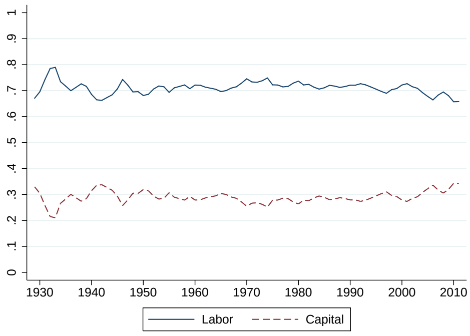
(a) Capital-output ratio



(b) Consumption-income ratio



(c) Hours worked



(d) Factor income shares

Figure B.6: Additional balanced-growth facts