Macro II

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Labor - leisure trade-off – the RBC model

- Any reasonable model of business cycles arguably needs fluctuations in labor use.
- One way; search – it takes time to find a job/worker and varying amounts of job-creation - destruction – Mortensen-Pissarides – Shimer Puzzle.
- A principally different way, assume perfect labor markets and add a labor/leisure choice.
- Later, we will add frictions and imperfections to this model -> the New Keynesian model.

A growth model with a labor-leisure choice

- Problem of planner/representative household in period 0:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, L_t\right)$$
$$\text{s.t. } C_t + K_{t+1} = F\left(K_t, N_t, Z_t\right) + (1-\delta) K_t \forall t \geq 0$$
$$N_t = 1 - L_t$$
$$K_0 \text{ given.}$$

- Z_t is a stochastic productivity shock, perhaps autocorrelated and non-stationary.
- Decisions are taken every period after seeing the current period shock. Solution is a set of decision rules – choice variables $\{C_t, K_{t+1}, L_t\}$ as functions of state variables $\{Z_t, K_t\}$.

Planner solution vs. Markets

- Here, no market imperfections. Planner solution equals a decentralized equilibrium where:
  - households supply labor,rent capital to a competitive factor market and decide how much to save and invest to maximize PDV of utility (dynamic problem?)
  - firms rent capital and labor and produce with a CRS production technology to maximize profits in every period. (dynamic problem?)
  - Firms owned by households (profits?)

Optimality conditions

- Replace N_t with $1 - L_t$. Lagrange objective in period t then: $E_t \sum_{s=0}^{\infty} \beta^s \{U(C_{t+s}, L_{t+s})$ $+\lambda_{t+s}(F(K_{t+s}, 1 - L_{t+s}, Z_{t+s}) + (1-\delta) K_{t+s} - C_{t+s} - K_{t+s+1})\}$
- First-order conditions for L_t; $U_L(C_t, L_t) - \lambda_t F_N(K_t, 1 - L_t, Z_t) = 0$
- For K_{t+1}; $-\lambda_t + E_t \beta \lambda_{t+1} (F_K(K_{t+1}, 1 - L_{t+1}, Z_{t+1}) + 1 - \delta) = 0$
- For C_t; $U_C(C_t, L_t) - \lambda_t = 0$
- Define $R_{t+1} \equiv F_K(K_{t+1}, 1 - L_{t+1}, Z_{t+1}) + 1 - \delta$.
- Condition for K_{t+1} gives $U_C(C_t, L_t) = E_t \beta U_C(C_{t+1}, L_{t+1}) R_{t+1}$ (Euler equation, intertemporal tradeoff). Note expectation of product!
- Define $w_t \equiv F_N(K_t, 1 - L_t, Z_t)$
- Condition for L_t gives $\frac{U_L(C_t, L_t)}{U_C(C_t, L_t)} = w_t$.(Intratemporal tradeoff)

Balanced growth

- Over time, we know productivity (Z_t) has grown (very much). Over long periods:
  - wages have increased a lot,
  - but interest rate and labor supply has not changed (much).
  - capital has grown parallel to wages, consumption and output (*balanced growth)*
- Puts requirements on utility functions.

1: Constant labor supply in balanced growth

- Suppose the wage increases by a factor X, and consumption also increases by the same factor X. Use intratemporal FOC. Then, if U_L/U_C *increases* by a factor X, labor supply should be unchanged.
- Mathematically, $\frac{U_L(XC,L)}{U_C(XC,L)} = Xw \forall X$.
- If this is the case, we can have a balanced growth path where C and w grow over time at the same rate while L is constant.
- This is satisfied if (in fact iff, KPR 88, BK 19) utility is of the form $U(C, L) = U(Cv(L)) \Rightarrow \frac{U_L(C,L)}{U_C(C,L)} = C\frac{v'(L)}{v(L)}$
- Then is $\frac{U_L(C,L)}{U_C(C,L)}$ proportional to something only depending on L, with a proportionality factor C.

2: Constant interest rate in balanced growth

- In a steady state with constant interest rate, and constant growth rate g of consumption the Euler equation is

$$\frac{U_C(C,L)}{U_C((1+g)C,L)} = \beta R.$$

- For this to be true for all C, we need a function with constant intertemporal elasticity of substitution utility function (equivalently, CRRA). The only class of functions satisfying this are of the form $U(C,L) = u(Cv(L)) = \frac{\sigma (Cv(L))^{\frac{\sigma-1}{\sigma}}-1}{\sigma-1}$,or with $\sigma = 1$, $U(C,L) = \ln C + v(L)$.
- The parameter $\sigma > 0$ measures how much consumption needs to change (in percent) per percentage change in marginal utility i.e., $-\left(\frac{d\ln(U_c)}{d\ln c}\right)^{-1} = \sigma$. With time-additive utility, this is also the inverse of the coefficient of relative riskaversion. Why is σ determining both things? (Kreps & Porteus. Epstein & Zin)

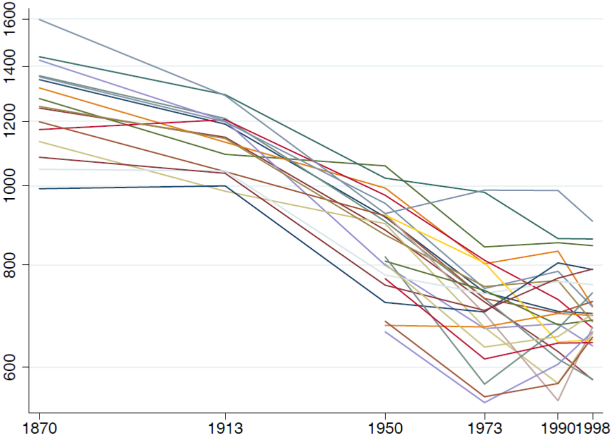
What happens with CARA?

- Assume constant *absolute risk aversion*, $U = -\frac{e^{-\sigma C}}{\sigma} + v(L)$, with $U_C = e^{-\sigma C_t}$.Then

$$\frac{U_C(C)}{U_C((1+g)C)} = \frac{e^{-\sigma C}}{e^{-\sigma(1+g)C}} = e^{\sigma Cg}$$

- In words, as the level of consumption increases, the rate of interest required to support a constant growth rate g *increases.*
- This is since with a CARA utility, the ratio of the marginal utilities between two consumption levels depends on the *difference* between them, not the ratio as with CRRA.
- Therefore, with CARA utility, constant interest rate can support growth that is constant in absolute value (linear, not exponential growth), something that doesn't seem in accordance with empirics.

Is labor supply really constant in the long run?



- Figure 1: **Yearly hours worked per capita 1870–1998**

Generalization

- Boppart and Krusell (JPE 2019), show that iff utility is of the form

$$U(C,1-N) = \frac{\sigma\left(Cv\left(NC^{\frac{\varphi}{1-\varphi}}\right)\right)^{\frac{\sigma-1}{\sigma}}-1}{\sigma-1},$$

$$U(C,1-N) = \ln(C)+\ln\left(v\left(NC^{\frac{\varphi}{1-\varphi}}\right)\right), \text{for } \sigma=1,$$

- we can have a (generalized) balanced growth path, now with a constant growth rate in hours worked. Here the argument of v,i.e., $NC^{\frac{\varphi}{1-\varphi}}$ is constant. When consumption increases N falls if $\varphi \in (0,1)$.
- The standard case with constant labor supply is $\varphi = 0$.
- Note also that I use the specification of BK 19, letting the $v(.)$ function depend on N rather than L. Then, $v()$ is utility of labor, decreasing and convex function that captures the disutility of labor. As we see, if φ is positive, an increase in consumption increase the disutility of labor as well as the marginal disutility of labor, inducing less work.

Shocks and labor supply

- Key task of the RBC model is to be able to produce variations in labor supply.
- Consider the log-case. Then, the intertemporal Euler condition is $\frac{1}{C_t} = \beta E_t \frac{R_{t+1}}{C_{t+1}}$and the intratemporal $v'(L_t) = \frac{w_t}{C_t}$.
- If a permanent technological shock changes wages and consumption by the same proportion (like along a balanced growth path), the RHS of intratemporal is unchanged and so should therefore labor supply be. Remember that this is by construction!
- On the other hand, a *temporary* technological should shift w_t *more* than C_t proportionally, since individuals want to smooth consumption. Therefore, a temporary shock should affect labor supply more the more temporary it is.
- What happens to labor supply response of temporary productivity shock if there is no possibility to save?

Role of labor elasticity

- Suppose $U(C_t, L_t) = \ln C_t + v(L_t) = \ln C_t + \frac{\nu}{\nu-1}\phi L_t^{\frac{\nu-1}{\nu}}$, then the intratemporal condition is

$$\frac{U_L}{U_C} = \frac{\phi L_t^{-\frac{1}{\nu}}}{\frac{1}{C_t}} = w_t \Rightarrow C_t = \frac{w_t L_t^{\frac{1}{\nu}}}{\phi}$$

- Use in Euler with log utility,
$1 = \beta E_t \left[R_{t+1} \frac{U'(C_{t+1})}{U'(C_t)} \right] = \beta E_t \left[R_{t+1} \frac{C_t}{C_{t+1}} \right];$

$$1 = \beta E_t \left[R_{t+1} \frac{w_t}{w_{t+1}} \left(\frac{L_t}{L_{t+1}} \right)^{\frac{1}{\nu}} \right]$$

- Again, if w_t is high relative to w_{t+1}, we should expect leisure to be relatively low (work a lot) in period t. How much depends on labor supply elasticity, i.e., ν.
- Microevidence suggests low ν, macro a higher value. Problem? What about variations in R_{t+1}?