Macro II

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New Keynesian Model

1. A reasonable avenue to make a more realistic business cycle model is to take seriously that prices and perhaps wages are not continously adjusted.
2. To talk about price stickyness, we need to allow some price-setting power – monopolistic competition.
3. Different monopolistic firms requires different goods with potential for price dispersion.
4. Otherwise, our model will build on the RBC model, i.e., being a stochastic general equilibrium model with forward looking rational agents.
5. Have become the central modeling approach in e.g., central banking.

Bonds instead of capital

- As before, we assume a representative household that maximizes

$$E_t \sum_{s=0} \beta^s U\left(C_{t+s}, L_{t+s}\right)$$

- In order to allow monetary policy to affect intertemporal tradeoff, we introduce government bonds, B_t but disregard capital. Not difficult to reintroduce. Often with investment friction. Budget constraint of individual is then

$$\text{s.t. } P_t C_t + Q_t B_t = B_{t-1} + W_t\left(1 - L_t\right) + T_t, \forall t \geq 0$$

  where Q_t is the price of one-period nominal bond that pays one unit period $t+1$ and T_t is a lump-sum transfer (firm profits, taxes...)
- In contrast to above, we now think of C_t as a basket/index of differentiated goods $C(i), i \in [0,1]$,

$$C_t \equiv \left(\int_0^1 C_t\left(i\right)^{1-\frac{1}{\varepsilon}} di\right)^{\left(1-\frac{1}{\varepsilon}\right)^{-1}},$$

- where $\varepsilon > 0$ determines how substitutable the goods are. Continous

Constructing a price index

- In the budget constraint, we used an aggregate price index, P_t. Can we construct that from the underlying prices $P_t(i)$?
- Consider the problem of minimizing the cost of getting a given amount of aggregate consumption $\bar{C}_t$

$$\min_{\{C_t(i)\}_{i=0}^{1}} \int_0^1 P_t(i)\, C_t(i)\, di - \lambda_t \left(\left(\int_0^1 C_t(i)^{1-\frac{1}{\varepsilon}}\, di \right)^{\left(1-\frac{1}{\varepsilon}\right)^{-1}} - \bar{C}_t \right)$$

- FOC for $C_t(i)$

$$\begin{aligned} & P_t(i) \\ = \;& \lambda_t \left(1-\frac{1}{\varepsilon}\right)^{-1} \left(\int_0^1 C_t(i)^{1-\frac{1}{\varepsilon}}\, di \right)^{\left(1-\frac{1}{\varepsilon}\right)^{-1}-1} \left(1-\frac{1}{\varepsilon}\right) C_t(i)^{-\frac{1}{\varepsilon}} \\ = \;& \lambda_t \left(\int_0^1 C_t(i)^{1-\frac{1}{\varepsilon}}\, di \right)^{\left(1-\frac{1}{\varepsilon}\right)^{-1}-1} C_t(i)^{-\frac{1}{\varepsilon}} \end{aligned}$$

Constructing a price index:2

- Note that $:\left(\int_0^1 C_t(i)^{1-\frac{1}{\varepsilon}} di\right)^{\left(1-\frac{1}{\varepsilon}\right)^{-1}-1} = C_t^{\frac{1}{\varepsilon}}$ by definition, giving $P_t(i) = \lambda_t C_t^{\frac{1}{\varepsilon}} C_t(i)^{-\frac{1}{\varepsilon}}$.
- What is λ_t in $P_t(i) = \lambda_t C_t^{\frac{1}{\varepsilon}} C_t(i)^{-\frac{1}{\varepsilon}}$?
- It is the minimized cost of increasing aggregate consumption by one unit, i.e., λ_t is the price index P_t. Thus, $P_t(i) = \lambda_t C_t^{\frac{1}{\varepsilon}} C_t(i)^{-\frac{1}{\varepsilon}}$ gives

$$\begin{aligned} P_t(i) &= P_t C_t^{\frac{1}{\varepsilon}} C_t(i)^{-\frac{1}{\varepsilon}} \\ \left(\frac{P_t}{P_t(i)}\right)^{\varepsilon} &= \frac{C_t(i)}{C_t} \end{aligned}$$

- One percent change in the relative price of good i, leads to ε percent decline in relative demand for that good.
- What happens with budget shares of different goods when prices increase if $\varepsilon = 1$, lower than one, higher than one?

The exact price index

- Use $\left(\frac{P_t}{P_t(i)}\right)^{\varepsilon} = \frac{C_t(i)}{C_t}$ in aggregate expenditure; $P_t C_t =$

$$\int_0^1 P_t\left(i\right) C_t\left(i\right) di = \int_0^1 P_t\left(i\right) \left(\frac{P_t}{P_t\left(i\right)}\right)^{\varepsilon} C_t di = C_t P_t^{\varepsilon} \int_0^1 P_t\left(i\right)^{1-\varepsilon} di.$$

- Dividing by C_t, gives $P_t = P_t^{\varepsilon} \int_0^1 P_t\left(i\right)^{1-\varepsilon} di$, or

$$P_t = \left(\int_0^1 P_t\left(i\right)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}$$

- This is an exact price index, defining the minimized cost per unit of aggregate consumption.

Discussion

Price index $P_t = \left(\int_0^1 P_t(i)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}$

- Note that;
- it is *H1.*
- the larger is ε, the more dispersion reduces the price index.
  - Suppose one third of prices are 1, 2 and 3, respectively. The price level is then $P_t = \left(\int_0^{1/3} 1^{1-\varepsilon} di + \int_{1/3}^{2/3} 2^{1-\varepsilon} di + \int_{2/3}^{1} 3^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}} = \left(\frac{1^{1-\varepsilon}+2^{1-\varepsilon}+3^{1-\varepsilon}}{3}\right)^{\frac{1}{1-\varepsilon}}$
  - Consider three cases, $\varepsilon = 0.01, 2$ and 100.
  - With $\varepsilon = 0.01$, $P_t = 1.998$, i.e., almost the average price.
  - With $\varepsilon = 2$, $P_t = 1.636$
  - With $\varepsilon = 100$, $P_t = 1.011$, close to the minimum price.
  - Explain!

Using the price index

- We can now conveniently treat the consumer problem in two stages;
  1. given distribution of prices, mininize cost of consuming a given consumption level. Gives P_t.
  2. decide how much to work, consume and save.
- In many applications we can forget about the first step.
- But recall that relative price differences have welfare costs.

Individual aggregate decisions - labor supply

- Given the two stage decision problem, the second yields optimality conditions as in RBC-model.

$$\begin{aligned} \frac{U_L(C_t, L_t)}{U_C(C_t, L_t)} &= \frac{W_t}{P_t} \\ U_C(C_t, L_t) &= \beta E_t\left[\frac{P_t}{Q_t P_{t+1}} U_C(C_{t+1}, L_{t+1})\right] \end{aligned}$$

  where $\frac{P_t}{Q_t P_{t+1}}$ is the real gross interest rate.
- Let us use a utility function in terms of consumption and *disutility of labor* $1 - N_t$. $U(C_t, 1 - N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \phi\frac{N_t^{1+\varphi}}{1+\varphi}$ where φ measures how inelastic labor supply is.
- Take log of the intratemporal condition $\frac{W_t}{P_t} = \frac{\phi N_t^{\varphi}}{C_t^{-\sigma}}$ and let lower case variables denote logs and dropping the constant $\ln\phi$

$$w_t - p_t = \sigma c_t + \varphi n_t$$

Individual aggregate decisions - NK Euler

- The Euler equation $1 = \beta E_t \left[\frac{P_t}{Q_t P_{t+1}} \left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \right]$ can be written,

$$1 = E_t \left(\exp \left(-\rho + i_t - \pi_{t+1} - \sigma \Delta c_{t+1} \right) \right)$$

where $\rho \equiv -\ln \beta \approx 1 - \beta$,
$i_t \equiv -\ln Q_t \approx 1 - Q_t$, $\pi_{t+1} \equiv \ln P_{t+1} - \ln P_t \approx \frac{P_{t+1}}{P_t} - 1$.
- Note that in a perfect foresight balanced growth path state with constant inflation and constant consumption growth γ, we have $-\rho + i - \pi - \sigma\gamma = 0$.
- First-order Taylor approximation around this BGP

$$\begin{aligned} & \exp \left(-\rho + i_t - \pi_{t+1} - \sigma \Delta c_{t+1} \right) \\ \approx \; & 1 + \left(i_t - i \right) - \left(\pi_{t+1} - \pi \right) \\ & -\sigma \left(\Delta c_{t+1} - \gamma \right) \\ = \; & 1 + i_t - \pi_{t+1} - \sigma \Delta c_{t+1} - i + \pi + \sigma\gamma \\ = \; & 1 + i_t - \pi_{t+1} - \sigma \Delta c_{t+1} - \rho \end{aligned}$$

- According to the Euler equation the expected value of this should be