An example

$$\begin{bmatrix} \dot{c}(t) \\ \dot{k}(t) \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} c(t) \\ k(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Eigenvalues are -1 and 2 and the eigenvectors are:

$$\left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\} \leftrightarrow -1,$$

$$\left\{ \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\} \leftrightarrow 2.$$

Thus,

$$\mathbf{B}^{-1} = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

The steay state is

$$\begin{bmatrix} c^{ss} \\ k^{ss} \end{bmatrix} = -\begin{bmatrix} 0 & -2 \\ -1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{3}{2} \\ \frac{1}{2} \end{bmatrix}$$

and using (119) we find the solution,

$$\begin{aligned} \begin{bmatrix} c(t) \\ k(t) \end{bmatrix} &= \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} e^{-t} & 0 \\ 0 & e^{2t} \end{bmatrix} \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix} \begin{bmatrix} c(0) - \frac{3}{2} \\ k(0) - \frac{1}{2} \end{bmatrix} \\ &\quad + \begin{bmatrix} \frac{3}{2} \\ \frac{1}{2} \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{3}\left(2e^{-t} + e^{2t}\right) & \frac{2}{3}\left(e^{-t} - e^{2t}\right) \\ \frac{1}{3}\left(e^{3t} - e^{2t}\right) & \frac{1}{3}\left(e^{-t} + e^{2t}\right) \end{bmatrix} \begin{bmatrix} c(0) - \frac{3}{2} \\ k(0) - \frac{1}{2} \end{bmatrix} + \begin{bmatrix} \frac{3}{2} \\ \frac{1}{2} \end{bmatrix} \end{aligned}$$

Alternatively, we can using the formula (109), then we have

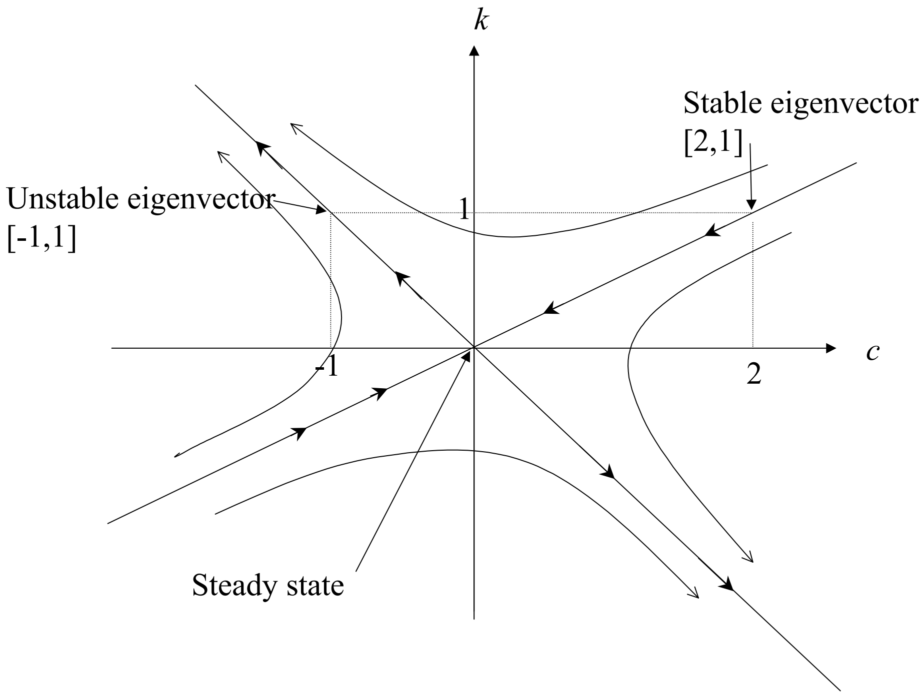
$$\begin{aligned} \begin{bmatrix} c(t) \\ k(t) \end{bmatrix} &= \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} e^{-t} & 0 \\ 0 & e^{2t} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} \frac{3}{2} \\ \frac{1}{2} \end{bmatrix} \\ &= \begin{bmatrix} \left(c_1 2e^{-t} - c_2 e^{2t}\right) \\ \left(c_1 e^{-t} + c_2 e^{2t}\right) \end{bmatrix} + \begin{bmatrix} \frac{3}{2} \\ \frac{1}{2} \end{bmatrix} \end{aligned}$$

and we can then solve for the constants c_1 and c_2. Suppose we have an end-condition saying that the the solution must converge to the steady state. Then, we are intetersted in the stable saddle-path only and c_2 must be zero and the solution is

$$\begin{bmatrix} c(t) \\ k(t) \end{bmatrix} = \begin{bmatrix} 2e^{-t}c_1 \\ 1e^{-t}c_1 \end{bmatrix} + \begin{bmatrix} \frac{3}{2} \\ \frac{1}{2} \end{bmatrix}$$

As we see, the ratio between the *deviations from the steady state* is given by the elements of the stable eigenvector.

We can now draw this. Let us draw the phase diagram around the steady state (i.e., the origo in the graph is the steady state $[3/2, 1/2]$. First, we draw the eigenvectors, then the arrows indicating movement.



Linear system with one stable root and one unstable.