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Problem set 1.

1. Solve $x_t = \frac{1}{2}x_{t-1} + \frac{1}{2^t}$

2. Solve $x_t = \frac{1}{2}x_{t-1} + \frac{1}{3^t}$

3. Solve $x_t = \dfrac{t+1}{t+2}x_{t-1} + 1$. *Hint:* define z_t by $x_t = \dfrac{z_t}{t+2}$.

4. Reduce the system
$$x_{t+1} = 1.5x_t + 2y_t - 4.5$$
$$y_{t+1} = 0.25x_t + y_t - 0.25$$
to a second order difference equation in x_t. Solve this second order equation. Then solve the first of the the given equations for y_t.

   The solutions (x_t, y_t) features a saddle point. Write down the solutions which converges to a stationary solution as $t \to \infty$.

5. You take a loan of k dollars at a monthly interest rate r. Let a be the monthly payment rate. Write down the appropriate difference equation for x_t, the loan balance outstanding in the t:th month, and use this to show that the loan is paid off after m months if
$$a = \frac{rk}{1-(1+r)^{-m}}$$

6. Consider the differential equation $\dot{x}(t) = x^2 - 4x + h$ where h is a constant. For $h = 1$ there are two stationary solutions; determine these. Which stationary solution is stable? For which initial conditions $x(t_0)$ will the solution $x(t)$ converge to the stable stationary solution when $t \to \infty$? Draw a diagram in a t, x plane of various solutions to the differential equation.

   There is a threshold h^* such that if $h > h^*$ there exists no stationary solution. Determine h^*. What happens to the solutions when $h > h^*$?

7. Solve the differential equation $\dot{x}(t) = te^{-x}$, $x(0) = 0$.