

# Job Polarization and Structural Change\*

Zsófia L. Bárány<sup>†</sup> and Christian Siegel<sup>‡</sup>

August 2013

[PRELIMINARY AND INCOMPLETE]

## Abstract

Job polarization is a widely documented phenomenon in developed countries since the 1980s: employment has been shifting from middle to low- and high-income workers, while average wage growth has been slower for middle-income workers than at both extremes. We document 1) that polarization has started as early as the 1950s in the US, and 2) that this process is closely linked to the shift from manufacturing to services. Based on these observations we propose a structural change driven explanation for polarization. Productivity growth through raising national income leads to a partial marketization of home production, and a disproportionate increase in the demand for high-end (luxury) services. To attract more workers into the low- and high-skilled services, the wages in these two sectors have to grow at a faster pace than in the middle.

**JEL codes:** E24, J22, O41

**Keywords:** Job Polarization, Structural Change, Home Production

---

\*We thank seminar participants at Trinity College Dublin and UC Berkeley for useful comments.

<sup>†</sup>Sciences Po. Email: zsofia.barany@sciences-po.org

<sup>‡</sup>University of Exeter. Email: c.siegel@exeter.ac.uk

# 1 Introduction

The polarization of employment and wages in recent decades both in the US and several European countries has generated a lot of interest in empirical labor economics (Autor, Katz, and Kearney (2006), Goos and Manning (2007), and Goos, Manning, and Salomons (2009)). This phenomenon, besides the relative growth of wages and employment at the top end of the earnings distribution, also entails the relative growth of wages and employment at the bottom.

Two popular explanations suggested in the empirical literature are the routinization hypothesis, and the consumption hypothesis. The routinization hypothesis relies on the assumption that information and computer technologies (ICT) substitute for middle-skill and hence middle-earnings (routine) occupations, whereas they complement the high-skilled and high-earnings (abstract) occupations (Autor, Levy, and Murnane (2003), Autor, Katz, and Kearney (2006), Michaels, Natraj, and Van Reenen (2010), Goos, Manning, and Salomons (2011), Autor and Dorn (2012)). On the other hand, the consumption hypothesis suggests that as the income of high-earners increases, their demand for low-skilled service jobs increases as well, leading to a spillover to the lower end of the wage distribution (Manning (2004), Mazzolari and Ragusa (2007)).

The contribution of our paper is twofold. First, empirically we document that polarization has started as early as in the 1950s in the US. We also show that the decline in middle-skill occupations is partly driven by the structural shift from manufacturing to services. Second, based on these observations we propose a structural change driven explanation for polarization. In our dynamic general equilibrium model workers with heterogeneous ability select which sector to work in. As technology improves, the demand for both high- and low-skilled services increases disproportionately, which leads to a reallocation of labor from middle-earning manufacturing jobs to high-earning and low-earning service jobs. However, to attract more workers into these two sectors, the wages in these two sectors have to grow at a faster pace than in the middle. Finally, we calibrate the model and quantitatively assess the contribution of structural change – driven by both non-homothetic preferences and unbalanced technological progress – to the polarization of wages and employment.

Using US Census data for the period 1950-2000 and American Community Survey (ACS) data for 2008, we analyze the employment share changes and log real wage changes. This analysis reveals some novel facts. We find that wage polarization started as early as 1950, and has been present since then. Real wage growth has slowed down significantly since the 1950s, and wage polarization became less pronounced over time. As for employment polarization, we find that the employment share of high-wage workers has been expanding throughout the period at the expense of middle-wage earners, and at the beginning of the sample also at the expense of low-wage earners. Using the shift-share decomposition method (as in Acemoglu and Autor (2011)), we show that a significant part of the employment share changes was driven by sectoral shifts rather than by occupational shifts.

In the model, there are two types of consumption good: manufacturing and high-end service goods. Preferences over these two goods are non-homothetic: high-end services are luxury goods, which are

only demanded at high enough income levels. Additionally a subsistence level of home production is required, which can be self-produced or acquired on the market. We model low-skilled service jobs as substitutes for household production. As technology progresses in the consumption goods sector, the employment and production structure of the economy changes: as households gradually become wealthier, the demand for high-skilled services rises over-proportionally due to the non-homotheticity of preferences. This disproportionate demand rise puts an upward pressure on prices and wages in the high-skilled service sector. Consequently, more people acquire education and work in high-end service jobs. As wages increase, more and more people decide to hire low-skilled household workers, instead of doing the housework themselves. This puts an upward pressure on wages at the bottom-end of the distribution, increasing the supply of these workers. Due to the non-homotheticity of preferences, the timing of the expansion of the high- and the low-skilled service sector can be different. While the non-homotheticity is relatively important, the desire to expand the consumption of high-skilled services dominates that of low-skilled services. This can be a potential explanation of the low-skilled service expansion becoming more pronounced later on in the data.

This paper builds on and contributes to the literature both on polarization and on structural change. To our knowledge, these two phenomena until now have been studied separately. However, according to our analysis of the data, polarization of occupations and structural change are closely linked to each other, and according to our model, industrial shifts can lead to polarization.

The polarization literature focuses on occupations, and documents that abstract (high-skilled non-routine) occupations, and manual (low-skilled non-routine) occupations have been expanding in terms of employment at the expense of routine occupations, and that the average wage growth in the first two broad occupational categories has been faster than in the latter (Autor, Levy, and Murnane (2003), Autor, Katz, and Kearney (2006), Goos and Manning (2007), Goos, Manning, and Salomons (2009), Acemoglu and Autor (2011), Autor and Dorn (2012)). It has been argued that much of the expansion of low-skilled occupations is driven by the expansion of low-skilled service jobs (Autor and Dorn (2012)). The routinization hypothesis suggests, that it is ICT that leads to the substitution of routine workers by machines, and which complements abstract workers. The displaced routine workers find either abstract or manual jobs, increasing the employment share of these occupations (Autor, Levy, and Murnane (2003), Autor, Katz, and Kearney (2006), Michaels, Natraj, and Van Reenen (2010), Goos, Manning, and Salomons (2011)). The routinization hypothesis, linked to ICT, is potentially a convincing explanation for the employment share patterns after the mid-1980s, but it does not provide a mechanism through which the relative wage of manual workers compared to routine workers can increase. It also cannot provide an explanation for the patterns observed before ICT could have taken effect. The consumption spill-over argument, on the other hand, provides an explanation for the expansion of demand for low-skilled service jobs, taking as given the increased income at the top end of the distribution (Manning (2004), Mazzolari and Ragusa (2007)). We borrow this view of low-skilled service jobs, incorporate it into a dynamic general equilibrium model, and show how the growth in low-skilled services is gener-

ated by technological progress.

The structural change literature has documented for several countries that as income increases resources are shifted away from agriculture and from manufacturing towards services (Kuznets (1957), Maddison (1980)). In particular the employment share of manufacturing has been declining since the 1950s, while the employment share of services has been increasing. The literature has identified two economic forces that lead to structural transformation: preferences and technology. The preferences explanation relies on changes in aggregate income, which if preferences for the output of different sectors are not homothetic lead to a reallocation of resources across sectors (Caselli and Coleman II. (2001), Kongsamut et al (2001)). The technology explanation assumes that productivity growth is different across sectors, which with regular preferences leads to a shift of labor into the lower growth sector (Ngai and Pissarides (2007), Acemoglu and Guerrieri (2008)). The consensus seems to be that both mechanisms together are needed to explain the patterns observed in the data (Buera and Kaboski (2009), Herrendorf et al (2009), Boppart (2011)). Several papers have also established the importance of home production for structural transformation (Ngai and Pissarides (2008), Buera and Kaboski (2009), (2012a), (2012b)). In our model we rely on both mechanisms, and incorporate home production.

We extend the structural change literature in two ways. First, we allow heterogeneous workers to endogenously sort into different skill groups and into jobs in different sectors. In the presence of differential productivity or demand growth, the optimal education decisions naturally change. Moreover, as the supply of skills and the sorting into sectors change, the relative wages and prices are affected. Therefore, we can analyze the effects of structural change on relative sectoral wages, which is not usual in models of structural change.<sup>1</sup>

Second, based on the job polarization phenomenon we distinguish between two types of services: low- and high-skilled. This is an important distinction, due to both the way they enter the utility of agents and the way they are produced. We believe that consumers enjoy these services in different ways. We model low-skilled services as substitutes for household production. In the data we do not find a clear pattern for the total amount of household production – the combined hours of home production and low-skilled service workers. Therefore, we assume that there is a fixed demand for household production, which implies an upper limit on the demand for low-skilled services. The demand for high-skilled services, on the other hand, can increase without bounds. In terms of production, high-skilled services need specialized (educated) workers, while low-skilled services can be supplied by anyone.

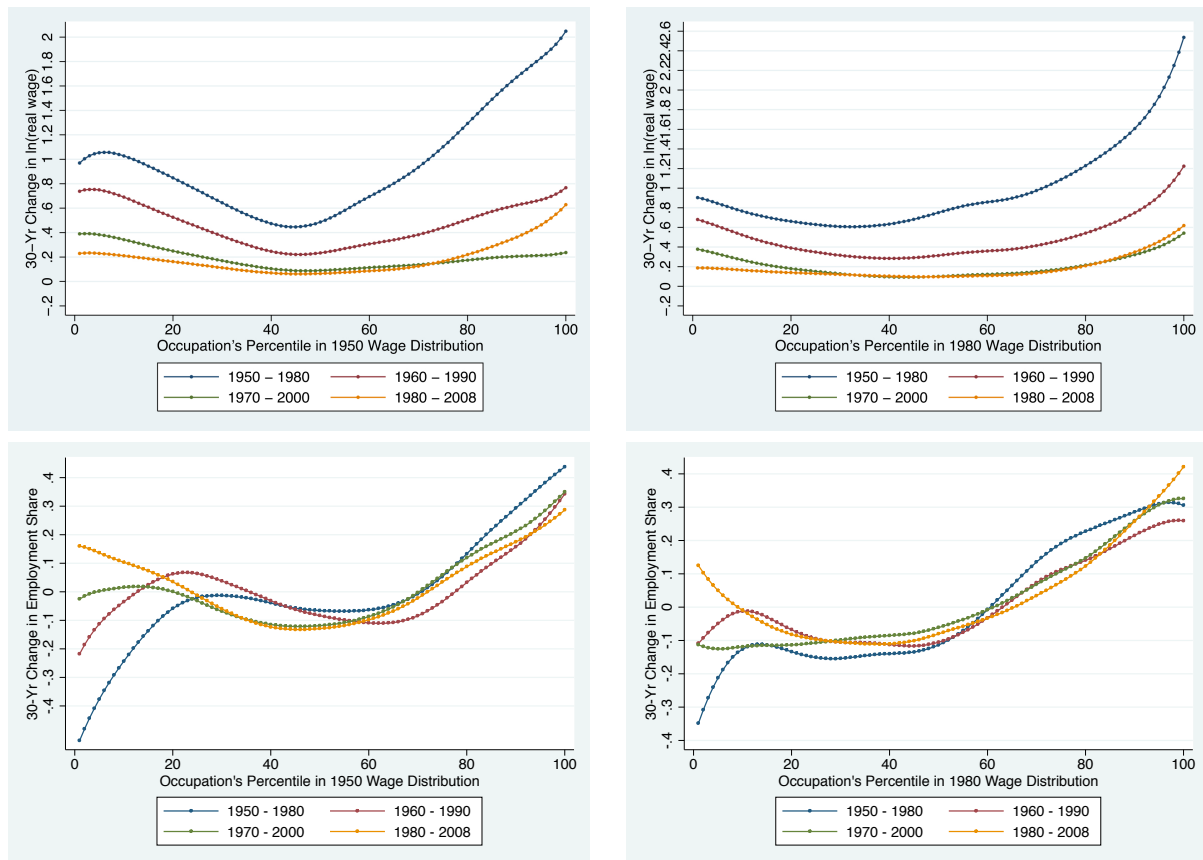
## 2 Polarization in the data

In the empirical literature, polarization is mostly represented in terms of occupations. Following the methodology used in Autor, Katz, and Kearney (2006), Acemoglu and Autor (2011), and Autor and Dorn (2012), we plot the smoothed changes in employment shares and log real wages for a balanced

---

<sup>1</sup>A notable exception is Caselli and Coleman II. (2001).

Figure 1: Wage and employment polarization



Notes: Wages are calculated from US Census data for 1950, 1960, 1970, 1980, 1990, 2000 and American Community Survey (ACS) for 2008. Balanced occupation categories (185 of them) were defined by the authors based on Meyer and Osborne (2005) and Autor and Dorn (2012). The bottom two panels show the 30-year change in employment shares (calculated as hours supplied rather than persons), and the top two panels show the 30-year change in log hourly real wages (again labor supply weighted). In the left panels occupations are ranked based on their 1950 average wage, whereas in the right panels they are ranked according to their 1980 average wage.

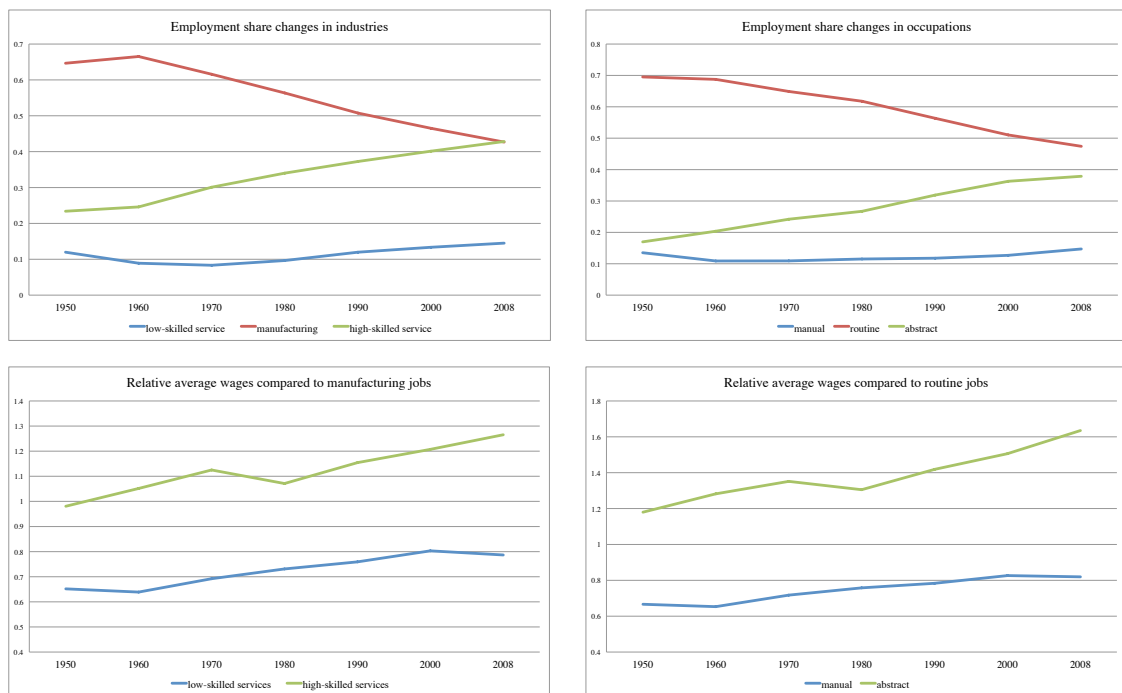
panel of occupation categories ranked according to their 1950 and 1980 mean wages. The novelty in these graphs is that we show these patterns going back until 1950, whereas most analyses look at data from only 1980 onwards. The top row of Figure 1 show that there has been real wage polarization in all 30-year periods. This polarization is present whether the occupations are ranked according to their 1950 or 1980 mean wages. The polarization of real wages is most pronounced in the first two 30-year intervals, but it is clearly discernible in the following ones as well from the slight U-shape of the smoothed changes. The picture is more mixed in terms of employment polarization (the bottom row of Figure 1): employment polarization is most pronounced in the last period (1980-2008), but it seems to be present even in the earlier decades.<sup>2</sup>

These graphs – in line with the literature – plot the change in raw employment shares and in raw log real hourly wages. These changes also include the potential effects of the changing gender, age and race composition of the labor force. These graphs also do not directly relate to the explanations put

<sup>2</sup>This does not necessarily hold for decade-by-decade analysis. Typically in some decades the top gains, whereas in others the bottom gains, but it is never the middle that grows the most in terms of employment shares. See graphs in Appendix.

forward in the literature, as they show the employment share and wage changes for occupations ranked according to their mean wage, not based on their routinizability. Therefore we classify the occupation groups into the following categories: manual, routine, and abstract (as in Acemoglu et al (2011)), and show the patterns for these three broad categories. We also classify industries into three categories: low-skilled services, which are substitutes for household production; manufacturing; and high-skilled services, which are luxury goods. Manufacturing industries are as in the structural change literature (Buera et al (2012a), Kongsamut et al (2001), Ngai et al (2008), Herrendorf et al (2009)). We classify industries to be low-skilled services if they can be viewed as substitutes for household production.

Figure 2: Polarization for industries and occupations



Notes: Employment shares (in terms of hours) are calculated from the same data as in Figure 1. Wages are the residuals from regressing log hourly wages on age, age squared, race, gender, and a dummy for foreign born. For details of the industry and occupation classification see text and the appendix.

Figure 2 shows the patterns of polarization both in terms of employment shares and wages for the above defined occupations and industries between 1950 and 2008. The patterns are strikingly similar between the graphs generated using industries (the left panels) and occupations (right panels). The top two panels show clear employment polarization both in terms of industries and occupations. The middle earning group (manufacturing/routine occupations) lost significantly in terms of employment share, the top (high-skilled services/abstract) gained, and the bottom (low-skilled services/manual) initially shrank, but then expanded. The bottom two panels show the change in average industry (or occupation) log hourly wage change in the given decade compared to the mean log hourly wage change in manufacturing (or routine occupations). In most decades average wages in low- and high-skilled services improved relative to manufacturing, while manual and abstract occupations also improved

relative to routine occupations. Exceptions are the first and last decade, and the period between 1970-1980, when the high-skilled services and the abstract occupations lost. This is the decade, when there was a secular compression in the skill premium. Most of this is probably driven by that.

This striking similarity in the employment share and average wage path of the three broad industry and occupation classifications can be understood when considering the employment shares of the occupation categories in the three industry categories and vice versa. The top panel in Table 1 shows the employment shares in each industry-occupation cell. It can be seen that the largest numbers are on the diagonal, implying there is a tight correspondence between industries and occupations. The middle panel shows for each industry the fraction of employment coming from manual, routine and abstract occupations, while the bottom panel shows the opposite averaged between 1950-2008. The majority of workers in low-skilled services are in manual occupations, the majority in manufacturing are in routine occupations, and the majority in high-skilled services are in abstract occupations. The opposite is also true: the majority of routine workers are employed in manufacturing, and the majority of abstract workers are employed in the high-skilled service industry.

Table 1: Overlap in employment between industry and occupation

	low-skilled service	manufacturing	high-skilled service	total
manual	5.90	1.26	5.17	12.33
routine	3.31	44.61	11.78	59.70
abstract	1.89	9.05	17.04	27.97
total	11.09	54.92	33.99	100
manual	53.66	2.30	15.49	-
routine	30.07	80.71	35.66	-
abstract	16.27	16.98	48.84	-
total	100	100	100	-
manual	48.37	10.55	41.08	100
routine	5.80	74.67	19.53	100
abstract	6.44	34.72	58.84	100

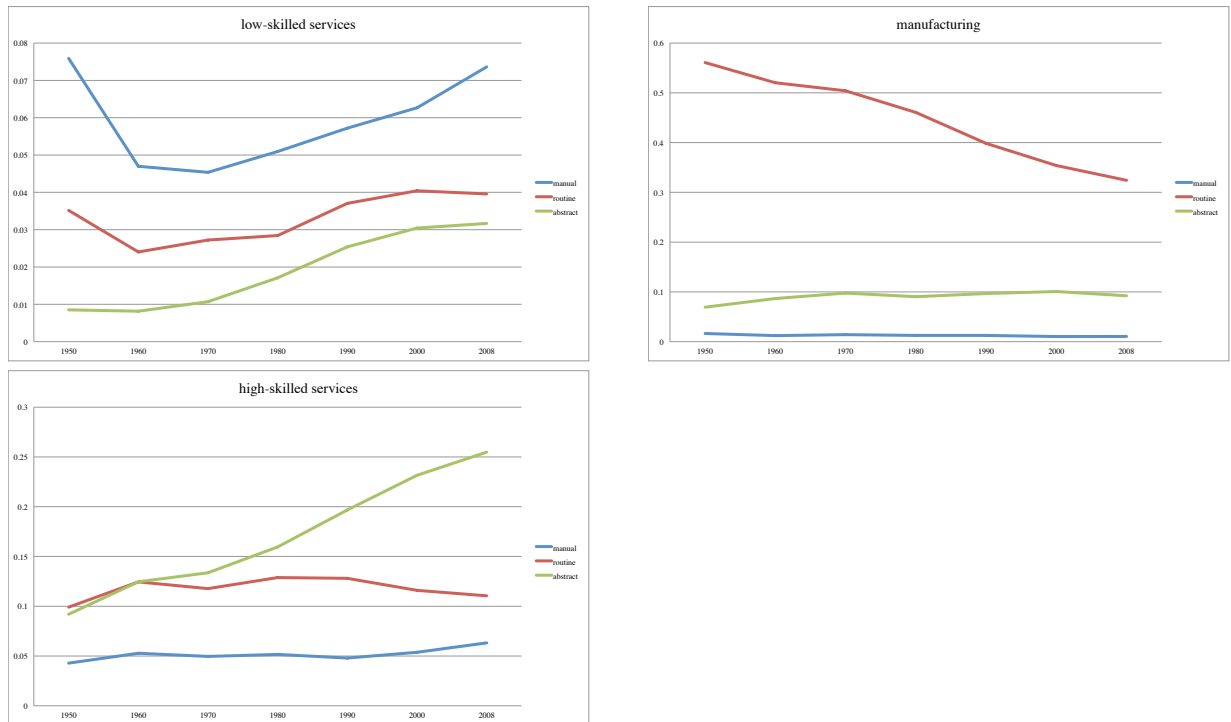
Notes: Employment shares (in terms of hours) as percents are calculated from the same data as in Figure 1. Industry and occupation classification same as in Figure 2. The top panel shows the employment shares in each of the occupation-industry cells. The middle panel shows within each industry the employment share of different occupations, while the bottom panel shows within each occupation group the employment share of different industries.

It is informative to look at the changes in the employment shares in the various industry-occupation cells. Figure 3 shows that the employment share only declined in the routine-manufacturing cell, whereas the routine-high-skilled cell's employment share was stable, while the routine-low-skilled cell's employment share increased between 1950-2008. Therefore it seems the decline in routinizable occupations is intrinsically linked to the decline in the manufacturing sector.

### 3 Model setup

Time is infinite and discrete. The demographic structure is a perpetual youth overlapping generations model, as in Blanchard (1985). Individuals are heterogeneous in their innate ability.

Figure 3: The path of employment shares for industry-occupation cells



Notes: Employment shares (in terms of hours) are calculated from the same data as in Figure 1. Industry and occupation classification same as in Figure 2.

Every individual has to decide whether to acquire education or not. Those who acquire education become high-skilled. In the calibration we identify the high-skilled as having attended college. Those who opt out from education remain low-skilled. Workers with high and low skills are employed in different sectors, and produce different goods. The high-skilled work in high-skilled services, whereas the low-skilled work either in manufacturing or in low-skilled services, which can be substitutes for home production.

Households derive utility from consuming high-skilled services and manufacturing goods. Services are luxury products in the sense that as income increases the households spend an increasing fraction of their total consumption budget on these services. Each household needs to meet a home production requirement, which can be produced at home, at a utility loss, or can be bought on the market from the low-skilled service workers.

The economy is in a decentralized equilibrium at all times: individuals make educational decisions and sectoral choices to maximize their lifetime income, the representative household maximizes its utility by optimally allocating its income between low-skilled services, manufacturing goods and high-skilled services. Production is perfectly competitive, wages and prices are such that all markets clear. We analyze the role of technological progress and non-homothetic preferences in explaining the observed wage and employment dynamics since the 1950s.



### 3.1 Sectors and production

There are three sectors in the model: high-skilled services ( $S$ ), manufacturing ( $M$ ), and low-skilled services ( $L$ ). High-skilled services and manufacturing goods are produced in perfect competition.

The only input in high-skilled service production is high-skilled labor:

$$Y_s = A_s N_s, \quad (1)$$

where  $A_s$  is productivity and  $N_s$  is the total amount of efficiency units of labor hired in sector  $S$  for production. Sector  $S$  firms are price takers, therefore the wage per efficiency unit of labor has to satisfy:

$$w_s = \frac{\partial p_s Y_s}{\partial N_s} = p_s A_s. \quad (2)$$

Manufacturing goods are produced low-skilled manufacturing workers:

$$Y_m = A_m N_m, \quad (3)$$

where  $A_m$  is productivity,  $N_m$  is the total amounts of efficiency units of labor hired in sector  $M$ . Sector  $M$  firms are also price takers, the wage per efficiency unit of labor in sector  $M$  has to satisfy:

$$w_m = \frac{\partial p_m Y_m}{\partial N_m} = p_m A_m. \quad (4)$$

Note that the wage of a worker with  $a$  efficiency units of labor working in sector  $i \in \{M, S\}$  is  $w_i a$ .

The low-skilled service sector provides home production hours for households. We assume that each worker is equally talented in providing home production services, i.e. efficiency units of labor do not matter here, it is only the raw amount of hours that a worker can provide, which is crucial. Total amount of low-skilled services provided on the market:

$$Y_l = L_l, \quad (5)$$

where  $L_l$  is the raw units of labor (total amount of people) working in the low-skilled service sector. Note that since everyone has the same amount of raw labor, this implies that everyone working in the low-skilled service sector has the same earnings. The unit wage in this sector in equilibrium has to be such that the total amount demanded of low-skilled services is equal to the amount supplied.

### 3.2 Labor supply and demand for goods

Time is infinite and discrete, indexed by  $t = 0, 1, 2, \dots$ . The economy is populated by a continuum of individuals who survive from one period to the next with probability  $\lambda$ , and in every period a new

generation of measure  $1 - \lambda$  is born. Individuals are heterogeneous in their innate ability (efficiency units of labor),  $a$ , which is drawn at birth from a time invariant distribution  $f(a)$ . These assumptions imply that both the size of the population, and the distribution of abilities are constant over time.

### 3.2.1 Choice of education and sector of work

Every agent has to decide at birth whether to acquire education or not. Agents choose their education in order to maximize their lifetime income. Acquiring education grants access to the high-skilled service sector ( $S$ ), and the cost is twofold: there is a tuition fee, and a time cost. The tuition fee is  $w_s k$ , which is proportional to the sectoral unit wage,  $w_s$ . This assumption is made in order to have a steady state, and it is meant to represent the assumption that somebody working in the same occupation has to train the newly entering individual. There is potentially a study time, during which the worker does not earn any wages. We assume that the study time is less than one period, i.e.  $\psi \in [0, 1]$ . We calibrate the length of one period in order for this assumption to be reasonable. Those who acquire education have to work in the sector  $S$  in the remaining part of the first period. (This is assumed both for simplicity, and because we think of the education partly as training on the job.)

Each agent in every period of his life has to decide which sector to work in, taking as given his education, his efficiency units of labor, and the sectoral unit wages. Agents choose their sector to maximize per period income. Agents with education can work in sector  $S$ ,  $M$ , or  $L$ . Agents without education can freely choose between sector  $M$  and  $L$ .<sup>3</sup>

For workers without education it is optimal to work in sector  $M$  if

$$w_m(t)a \geq w_l(t) \quad \Leftrightarrow \quad a \geq \frac{w_l(t)}{w_m(t)} \equiv a_{lm}(t). \quad (6)$$

Note that in period  $t$  all non-educated workers with  $a < a_{lm}(t)$  work in sector  $L$ .

It is not optimal for an educated worker to work in sector  $M$  if

$$w_s(t)a \geq w_m(t)a \quad \Leftrightarrow \quad w_s(t) \geq w_m(t).$$

Therefore in period  $t$  if  $w_m(t) > w_s(t)$  all the workers who are qualified to work in sector  $S$  work in sector  $M$  instead. This also implies that there are no sector  $S$  workers available to train the potential new entrants into sector  $S$ . Therefore in any such period there is no  $S$  production. This can be only part of an equilibrium, if the demand for good  $S$  is zero and if none of the newborns wishes to acquire sector  $S$  education, which happens, if average income is relatively low.

Education  $S$  workers do not work in sector  $L$  if

$$w_s(t)a \geq w_l(t) \quad \Leftrightarrow \quad a \geq \frac{w_l(t)}{w_s(t)} \equiv a_{ls}(t) \quad (7)$$

---

<sup>3</sup>In the steady state and the transition that we consider agents with education always optimally work in the sector  $S$ .

Also note that in period  $t$  any  $S$  worker with  $a < a_{ls}(t)$  works in sector  $L$ .

Given the assumptions on sector choices the expected discounted lifetime income of a worker with education  $S$ , efficiency units of labor  $a$ , born at time  $t$  can be written as:

$$V_s(a, t) = (1 - \psi)w_s(t)a - w_s(t)k + E_t \sum_{i=1}^{\infty} (\beta\lambda)^i \max\{w_s(t+i)a, w_m(t+i)a, w_l(t+i)\}.$$

Expected discounted lifetime income of a worker without education, with efficiency units of labor  $a$ , born at time  $t$  is:

$$V_n(a, t) = E_t \sum_{i=0}^{\infty} (\beta\lambda)^i \max\{w_m(t+i)a, w_l(t+i)\}.$$

First of all note that in both cases ( $N$ ,  $S$ ) the expected present value of lifetime income is piecewise linear in the ability,  $a$ , of the individual. This implies that between any two options, the optimal decision rule can be summarized by a set of cut-off ability levels, which determine ranges of abilities,  $a$ , where one decision is optimal, while for other ability levels, the other decision is optimal.

**Lemma 1.** *The optimal educational choice in period  $t$  can be described by a cutoff ability level  $a_s(t)$ , defined as the solution of:*

$$V_s(a_s(t), t) = V_n(a_s(t), t). \quad (8)$$

*For individuals born in period  $t$  with  $a < a_s(t)$  it is optimal to remain low-skilled, while for all individuals born in period  $t$  with  $a \geq a_s(t)$  it is optimal to acquire education.*

*Proof.* It is optimal to acquire education if the following difference is positive:

$$\begin{aligned} V_s(a, t) - V_n(a, t) &= \underbrace{(1 - \psi)w_s(t)a - w_s(t)k - \max\{w_m(t)a, w_l(t)\}}_{\text{first period gain or loss}} \\ &+ \underbrace{\sum_{i=1}^{\infty} (\beta\lambda)^i \max\{w_s(t+i)a - \max\{w_m(t+i)a, w_l(t+i)\}, 0\}}_{\geq 0 \text{ and increasing in } a}. \end{aligned}$$

In the Appendix we show that this function is continuous, negative for  $a = 0$ , and crosses zero only once. □

Deriving the labor supply in period  $s$  of a cohort born in period  $0 < t \leq s$  is straightforward given Lemma 1, and given the unit wages in each sector in period  $s$ .

**Corollary 1.** *The cohort born in period  $t > 0$  supplies labor to each sector in period  $s$  in the following way.*

1. If the period  $s$  unit wages satisfy  $w_m(s) \leq w_s(s)$ , and  $a_s(t) \leq a_{lm}(s)$ , then

$$L_l(t, s) = (1 - \lambda)\lambda^{s-t} \int_0^{\max\{a_{ls}(s), a_s(t)\}} f(a) da \quad (9)$$

$$N_m(t, s) = 0 \quad (10)$$

$$N_s(t, s) = (1 - \lambda)\lambda^{s-t} \int_{\max\{a_{ls}(s), a_s(t)\}}^{\infty} a f(a) da \quad (11)$$

2. If the period  $s$  unit wages satisfy  $w_m(s) \leq w_s(s)$ , and  $a_{lm}(s) < a_s(t)$ , then

$$L_l(t, s) = (1 - \lambda)\lambda^{s-t} \int_0^{a_{lm}(s)} f(a) da \quad (12)$$

$$N_m(t, s) = (1 - \lambda)\lambda^{s-t} \int_{a_{lm}(s)}^{a_s(t)} a f(a) da \quad (13)$$

$$N_s(t, s) = (1 - \lambda)\lambda^{s-t} \int_{a_s(t)}^{\infty} a f(a) da \quad (14)$$

3. If  $w_m(s) > w_s(s)$ , then

$$L_l(t, s) = (1 - \lambda)\lambda^{s-t} \int_0^{a_{lm}(s)} f(a) da \quad (15)$$

$$N_m(t, s) = (1 - \lambda)\lambda^{s-t} \int_{a_{lm}(s)}^{\infty} a f(a) da \quad (16)$$

$$N_s(t, s) = 0 \quad (17)$$

*Proof.* It is easy to see that we have covered each possible configuration of cutoffs. See Appendix for details.  $\square$

**Corollary 2.** *The effective labor supply in period  $s$  of workers born in period  $s$ :*

$$L_l(s, s) = (1 - \lambda) \left( \int_0^{\min\{a_s(s), a_{lm}(s)\}} f(a) da \right); \quad (18)$$

$$N_m(s, s) = (1 - \lambda) \int_{\min\{a_s(s), a_{lm}(s)\}}^{a_s(s)} a f(a) da; \quad (19)$$

$$N_s(s, s) = (1 - \psi_s)(1 - \lambda) \int_{a_s(s)}^{\infty} a f(a) da - k(1 - \lambda) \int_{a_s(s)}^{\infty} f(a) da. \quad (20)$$

*Proof.* Using the fact that education takes time, the labor supply by those born in period  $t$  is a fraction  $(1 - \psi)$  of their labor supply otherwise. On top of this, those who acquire education require training, in the amount  $k$  for each individual. This amount of labor is used for their training, rather than for production, therefore they reduce the effective labor supply by this amount per person.  $\square$

**Assumption 1.** *We assume that the economy starts in period 0, with a total one unit of raw labor, who made education decisions according to  $0 < a_s(0) < \infty$ .*

Note that this assumption is equivalent to assuming that the economy from the start of time ( $t = -\infty$ ) has been in a steady state where the optimal educational decision is given by cutoffs  $0 < a_s(0) < \infty$ , and whatever change (if any) happened in period 1 was unanticipated by all agents born until period 0.

**Corollary 3.** *Assume that the period  $s$  unit wages satisfy  $w_m(s) \leq w_s(s)$ . The cohort born in period  $t = 0$  supplies labor to each sector in period  $s$  in a similar way as in Corollary 1 except the multiplier is  $\lambda^{s+1}$  instead of  $(1 - \lambda)\lambda^s$  which would be indicated by applying Corollary 1 to  $t = 0$ .*

Given the labor supplies by each cohort in Corollaries 1 and 2 the total effective labor supply in period  $s$  is given by:

$$L_l(s) = \sum_{j=0}^s L_l(s-j, s) \quad (21)$$

$$N_m(s) = \sum_{j=0}^s N_m(s-j, s) \quad (22)$$

$$N_s(s) = \sum_{j=0}^s N_s(s-j, s) \quad (23)$$

### 3.2.2 Demand for consumption goods and low-skilled services

We use the 'big family' assumption, that is we assume that there is a representative consumer in each period, which represents all individuals alive in that period. The consumer cannot borrow or lend, and maximizes the following utility in each period, subject to the following constraints:

$$\begin{aligned} \max_{c_m, c_s, h} \ln & \left( \underbrace{\theta_m c_m^{\frac{\varepsilon-1}{\varepsilon}} + \theta_s (c_s + \gamma_s)^{\frac{\varepsilon-1}{\varepsilon}}}_C \right)^{\frac{\varepsilon}{\varepsilon-1}} - \phi \frac{h^\nu}{\nu} \\ \text{s.t.} \quad & p_m c_m + p_s c_s + w_l \underbrace{(\bar{h} - h)}_{=l} \leq \underbrace{w_s N_s + w_m N_m + w_l L_l}_{\equiv m} \quad (\lambda_0) \\ & 0 \leq c_s \ (\mu_s), 0 \leq c_m \ (\mu_m), 0 \leq h \ (\mu_l), h \leq \bar{h} \equiv \frac{\bar{x}}{A_h} \ (\mu_h) \end{aligned}$$

Given labor supplies to the different sectors, total disposable income is  $m$ , which can be used for consumption goods and low-skilled services (excluding tuition fees).

The utility of the consumer is non-homothetic in manufacturing goods ( $c_m$ ) and high-skilled service goods ( $c_s$ ). We assume that  $\gamma_s > 0$ , which is equivalent to assuming that  $c_s$  is a luxury good, i.e. the household will only demand a positive amount if it is sufficiently rich.

The total number of raw hours needed for household work is  $\bar{h}$ , which can vary over time. (Specifically we assume that  $A_h \bar{h} = \bar{x}$ , where  $\bar{x}$  is a constant. So if  $A_h$  increases over time, then the necessary number of total home production hours declines.) Doing the home production causes disutility to the household, which is increasing ( $\phi > 0$ ) in the number of hours spent on home production, and

potentially convex ( $\nu > 1$ ). The household can hire a low-skilled service worker to do some or all of the home production.

The following Corollary summarizes the solution to the consumer's problem.

**Corollary 4.** *The following cases can arise:*

1. *At the lowest income levels ( $m < \min\{m_{12a}, m_{12b}\}$ ) the optimal choices are:*

$$h = \bar{h} \quad (24)$$

$$c_s = 0 \quad (25)$$

$$c_m = \frac{m}{p_m} \quad (26)$$

*In this case  $\mu_h > 0, \mu_l = 0, \mu_m = 0, \mu_s > 0$ .*

2. *For higher income levels ( $m \in [\min\{m_{12a}, m_{12b}\}, \min\{m_{2a3}, m_{2b3}\})$ ,*

*(a) if  $m \in [m_{12a}, m_{2a3})$  and  $m_{12a} < m_{12b}$  the optimal choices are described by*

$$c_s = 0 \quad (27)$$

$$h = \left( \frac{w_l \theta_m}{p_m \phi} \right)^{\frac{1}{\nu-1}} \left( \theta_m c_m^{\frac{\varepsilon-1}{\varepsilon}} + \theta_s \gamma_s^{\frac{\varepsilon-1}{\varepsilon}} \right)^{-\frac{1}{\nu-1}} c_m^{-\frac{1}{\varepsilon(\nu-1)}} \quad (28)$$

$$c_m = \frac{m - w_l(\bar{h} - h)}{p_m} \quad (29)$$

*In this case  $\mu_h = 0, \mu_l = 0, \mu_m = 0, \mu_s > 0$ .*

*(b) if  $m \in [m_{12b}, m_{2b3})$  and  $m_{12b} < m_{12a}$ , then the optimal choices are:*

$$h = \bar{h} \quad (30)$$

$$c_m = \left( \frac{p_m \theta_s}{p_s \theta_m} \right)^{-\varepsilon} (c_s + \gamma_s) \quad (31)$$

$$c_s = \frac{m + p_s \gamma_s}{p_m \left( \frac{p_m \theta_s}{p_s \theta_m} \right)^{-\varepsilon} + p_s} - \gamma_s \quad (32)$$

*In this case  $\mu_h > 0, \mu_l = 0, \mu_m = 0, \mu_s = 0$ .*

3. *For the highest income levels, if  $m > \min\{m_{2a3}, m_{2b3}\}$  the optimal choices are:*

$$c_s = \left( \frac{p_m \theta_s}{p_s \theta_m} \right)^{\varepsilon} c_m - \gamma_s \quad (33)$$

$$h = \left( \frac{w_l \theta_m}{p_m \phi} \right)^{\frac{1}{\nu-1}} \left( \theta_m + \theta_s \left( \frac{p_m \theta_s}{p_s \theta_m} \right)^{\varepsilon-1} \right)^{-\frac{1}{\nu-1}} c_m^{-\frac{1}{\nu-1}} \quad (34)$$

$$c_m = \frac{m - p_s c_s - w_l(\bar{h} - h)}{p_m} \quad (35)$$

In this case  $\mu_h = 0, \mu_l = 0, \mu_m = 0, \mu_s = 0$ .

The cutoff income levels are defined by the following equations: The cutoff  $m_{12a}$  is the income level, where the supply of  $h$  exactly equals  $\bar{h}$  according to case 2a. We can find the  $c_m$  for which  $h = \bar{h}$  under case 2a. Then

$$m_{12a} = p_m c_m (h = \bar{h}). \quad (36)$$

The cutoff  $m_{12b}$  is the income level, where the demand for  $c_s$  is exactly zero according to case 2b:

$$m_{12b} = p_m \left( \frac{p_m \theta_s}{p_s \theta_m} \right)^{-\varepsilon} \gamma_s. \quad (37)$$

The cutoff  $m_{2a3}$  is defined as the income level where the demand for  $c_s$  is exactly zero under case 3 (therefore it can only follow case 2a). Using that  $c_s = 0$  we get that  $c_m = \left( \frac{p_m \theta_s}{p_s \theta_m} \right)^{-\varepsilon} \gamma_s$ . Using this value we can express the optimal  $h$  under case 3, and can calculate the income level that way:

$$m_{2a3} = p_m \left( \frac{p_m \theta_s}{p_s \theta_m} \right)^{-\varepsilon} \gamma_s + w_l \left( \bar{h} - \left( \frac{w_l \theta_m}{p_m \phi} \right)^{\frac{1}{\nu-1}} \left( \theta_m + \theta_s \left( \frac{p_m \theta_s}{p_s \theta_m} \right)^{\varepsilon-1} \right)^{-\frac{1}{\nu-1}} \left( \frac{p_m \theta_s}{p_s \theta_m} \right)^{-\frac{\varepsilon}{\nu-1}} \gamma_s^{-\frac{1}{\nu-1}} \right). \quad (38)$$

The cutoff  $m_{2b3}$  is the income level where the supply of  $h$  is exactly  $\bar{h}$  under case 3 (therefore it can only follow case 2b). Using that  $h = \bar{h}$  we can express  $c_m = \left( \frac{w_l \theta_m}{p_m \phi} \right) \left( \theta_m + \theta_s \left( \frac{p_m \theta_s}{p_s \theta_m} \right)^{\varepsilon-1} \right)^{-1} \bar{h}^{-(\nu-1)}$ , which therefore gives the value of  $c_s$ . The income that allows the purchase of that bundle is:

$$m_{2b3} = \left( p_m + p_s \left( \frac{p_m \theta_s}{p_s \theta_m} \right)^{\varepsilon} \right) \left( \frac{w_l \theta_m}{p_m \phi} \right) \left( \theta_m + \theta_s \left( \frac{p_m \theta_s}{p_s \theta_m} \right)^{\varepsilon-1} \right)^{-1} \bar{h}^{-(\nu-1)} - p_s \gamma_s. \quad (39)$$

*Proof.* The Lagrangian of the problem is:

$$\mathcal{L}(c_m, c_s, h) = \ln C^{\frac{\varepsilon}{\varepsilon-1}} - \phi \frac{h^\nu}{\nu} - \lambda (p_m c_m + p_s c_s + w_l \underbrace{(\bar{h} - h)}_{=l} - m) + \mu_s c_s + \mu_m c_m + \mu_l h - \mu_h (h - \bar{h})$$

The first order conditions are:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial c_m} &= \frac{1}{C} \theta_m c_m^{-\frac{1}{\varepsilon}} - \lambda p_m + \mu_m = 0 \\ \frac{\partial \mathcal{L}}{\partial c_s} &= \frac{1}{C} \theta_s (c_s + \gamma_s)^{-\frac{1}{\varepsilon}} - \lambda p_s + \mu_s = 0 \\ \frac{\partial \mathcal{L}}{\partial h} &= -\phi h^{\nu-1} + \lambda w_l - \mu_h + \mu_l = 0 \end{aligned}$$

To solve the household's problem we have to consider which constraints can be binding for which income levels. First notice, that the non-negativity constraint on  $h$  never binds due to the specification of the disutility from housework, if prices  $(p_m, p_s)$  and wages  $(w_l)$  are positive and finite. Since the disutility from housework goes to zero as  $h \rightarrow 0$ , while the utility from consumption only goes to zero

as consumption goes to infinity, for finite income levels the household is always better off by increasing  $c$  than reducing  $h$  to zero. The constraint on the non-negativity of  $c_m$  can also never be binding, since the marginal utility from consumption at  $c_m = c_s = 0$  is infinity, while the marginal disutility of labor is a positive, finite number. Therefore  $\mu_m = 0$  and  $\mu_l = 0$  always hold. By considering the other possibilities the statement of the Corollary follows.  $\square$

For the purpose of the definition of a decentralized equilibrium, we define the following implicit functions, which summarize the optimal decisions of the household:

$$c_m = f_m(m, p_m, p_s, w_l) \quad (40)$$

$$c_s = f_s(m, p_m, p_s, w_l) \quad (41)$$

$$l = \bar{h} - h = f_h(m, p_m, p_s, w_l) \quad (42)$$

## 4 Competitive Equilibrium

A competitive equilibrium is a sequence of cutoff education abilities  $\{a_s(t)\}_{t=1}^{\infty}$ , labor supply abilities  $\{a_{lm}(t), a_{ls}(t)\}_{t=1}^{\infty}$ , wages  $\{w_l(t), w_m(t), w_s(t)\}_{t=1}^{\infty}$ , prices  $\{p_m(t), p_s(t)\}_{t=1}^{\infty}$ , given the path of productivities  $\{A_h(t), A_m(t), A_s(t)\}_{t=0}^{\infty}$  and initial education decisions  $a_s(0)$  which satisfy:

1.  $a_s(t)$  is the solution to equation (8);
2.  $a_{lm}(t), a_{ls}(t)$  are defined as in (6), and (7);
3. the unit wage rates are such that the market for  $L$  (from (5) and (42)),  $M$  (4) and  $S$  (2) labor clears;
4.  $p_m$  and  $p_s$  are such that the market for  $M$  (from (3) and (40)) goods and  $S$  goods (from (1) and (41)) clears.

The economy is always in a competitive equilibrium, where newborns choose their education optimally, and older cohorts choose their sector of work optimally. Firms maximize their profits. Markets clear.

For any initial condition, there is a unique stationary competitive equilibrium, which features constant education and sector-of-work decisions. In a steady state the following holds for the education and sector-of-work decisions:

$$a_{lm} = \frac{w_l}{w_m};$$

$$a_s = \frac{k}{\left(1 - \frac{w_m}{w_s}\right) \frac{1}{1-\beta\lambda} - \psi}.$$



In the steady state the effective and raw labor supplies are given by:

$$\begin{aligned}
L_l &= \int_0^{a_{lm}} 1dF(a); \\
N_m &= \int_{a_{lm}}^{a_s} adF(a); \quad L_m = \int_{a_{lm}}^{a_s} 1dF(a); \\
N_s &= (\lambda + (1 - \lambda)(1 - \psi)) \int_{a_s}^{\infty} adF(a) - (1 - \lambda)k \int_{a_s}^{\infty} 1dF(a); \quad L_s = \int_{a_s}^{\infty} 1dF(a).
\end{aligned}$$

Therefore across different steady states it holds that  $L_l$  increases with  $w_l/w_m$ , and  $N_s(L_s)$  increases with  $w_s/w_m$ . This is similar to some kind of polarization: if the relative unit wages increase at the top and at the bottom, then the raw labor supplies move the same way. However, in reality we do not observe the relative unit wages, but we observe the relative average wages.

## 5 Calibration (Incomplete)

All parameters are time-invariant, and the only exogenous change over time is productivity growth. A model period is set to 10 years. We therefore set  $\beta = 0.95^{10} = 0.5987$ , and to match the average potential working age from 18 to 65, we set  $\lambda = 1 - 1/4.3 \approx 0.7674$ . We assume that a worker deciding to acquire education has to dedicate 4 years for this, and set therefore  $\psi = 0.4$ .

We estimate the preference function of the household using data on expenditures and price indices from NIPA for the years 1950-2006 (similarly to Herrendorf, Rogerson, and Valentinyi (2009)). Relying on our classification of industries, we classify expenditures as low-skilled services, manufacturing and high-skilled services. We generate price indices from the expenditure data and the quantity indices available. We calculate expenditure shares and use these along with the prices indices in a GMM estimation of  $\theta_m, \theta_s, \varepsilon, \nu, \bar{x}, \phi$ . We assume that the productivity in the home production sector is constant.

We calibrate the remaining parameters so that our model replicates key moments of the US economy in 1950. These moments are the relative average industry wages and the industry employment shares.

For productivity growth we rely on Herrendorf, Rogerson, and Valentinyi (2009), who estimate a labor-augmenting technological progress, with annual growth rates of 2% for manufacturing, and 1.1% for the service sector.

## 6 Quantitative Results (Incomplete)

We simulate the transition path of the model to its steady state. The economy is not in a steady state initially, and workers have perfect foresight of the exogenous path of productivity growth. We study the endogenous response of employment and wages. Our baseline is, as in the data, that productivity growth is higher in manufacturing than in services. Taking estimates by Herrendorf, Rogerson, and Valentinyi (2009) we let  $M$ 's productivity grow by 2% per annum and  $S$ 's productivity grow by 1.1%.

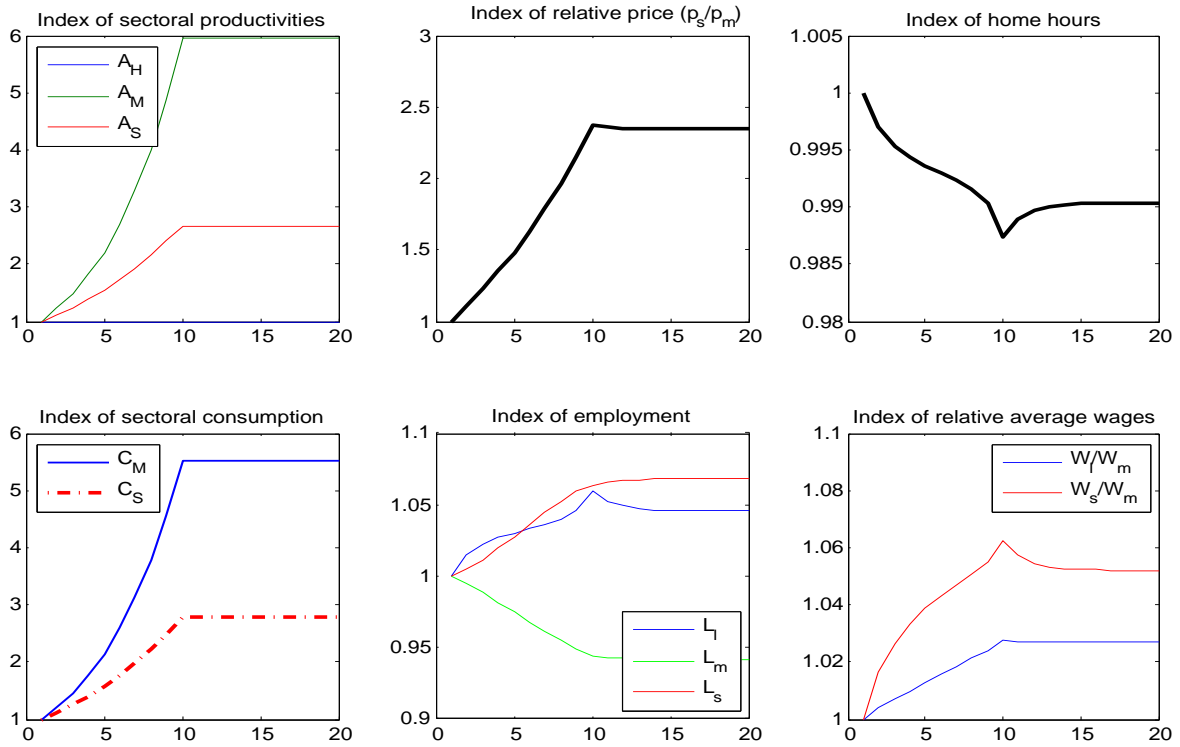


Figure 4: Transition of the Benchmark Model

Figure 4 plots the resulting transition. As productivity growth implies a rise in national income, consumption increases and hours spent on home production decrease. Productivity growth in manufacturing makes workers redundant since the demand for these goods does not rise as much as productivity due to the non-homotheticity of preferences. Those with very low abilities that used to work in manufacturing, now sort into low-skilled services. Whereas some higher ability workers who formerly would have remain low-skilled and worked in manufacturing, now acquire education and provide their labor in sector  $S$ . To understand the effect on wages, notice that since high-skilled services are luxury commodities, their consumption increases despite their rising relative price. To satisfy demand, employment in the  $S$  sector has to increase, and firms have to pay higher unit wages to attract workers. As a consequence, the cutoff ability of workers sorting into  $S$  falls, which tends to decrease the average wage paid in the sector. However, the effect of higher unit wages is the dominating one, and on average  $S$  workers wages increase relative to  $M$ . Hence, both wages and employment at the top-end of the distribution increase. Moreover, with national income the demand for services substituting for home production increases, time spent on home production falls, and instead the employment of  $L$  labor increases. As the demand for  $L$  services increase, also the wage for  $L$  workers rises, in order to pull workers into the low-skilled services sector. As a consequence, employment and wages rise at either end of the distribution.

To disentangle the role of differential productivity growth from the non-homotheticity of preferences, we conduct a counterfactual analysis: What would be the transition if productivity growth was

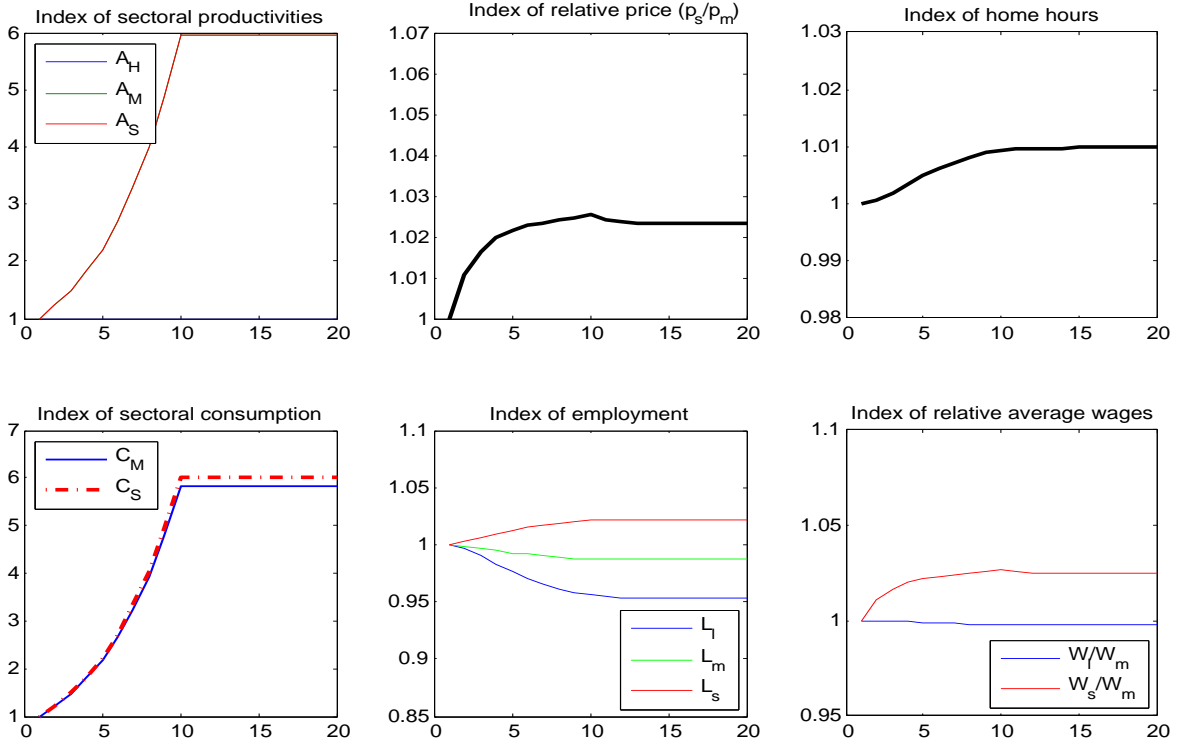


Figure 5: Transition with equal productivity growth

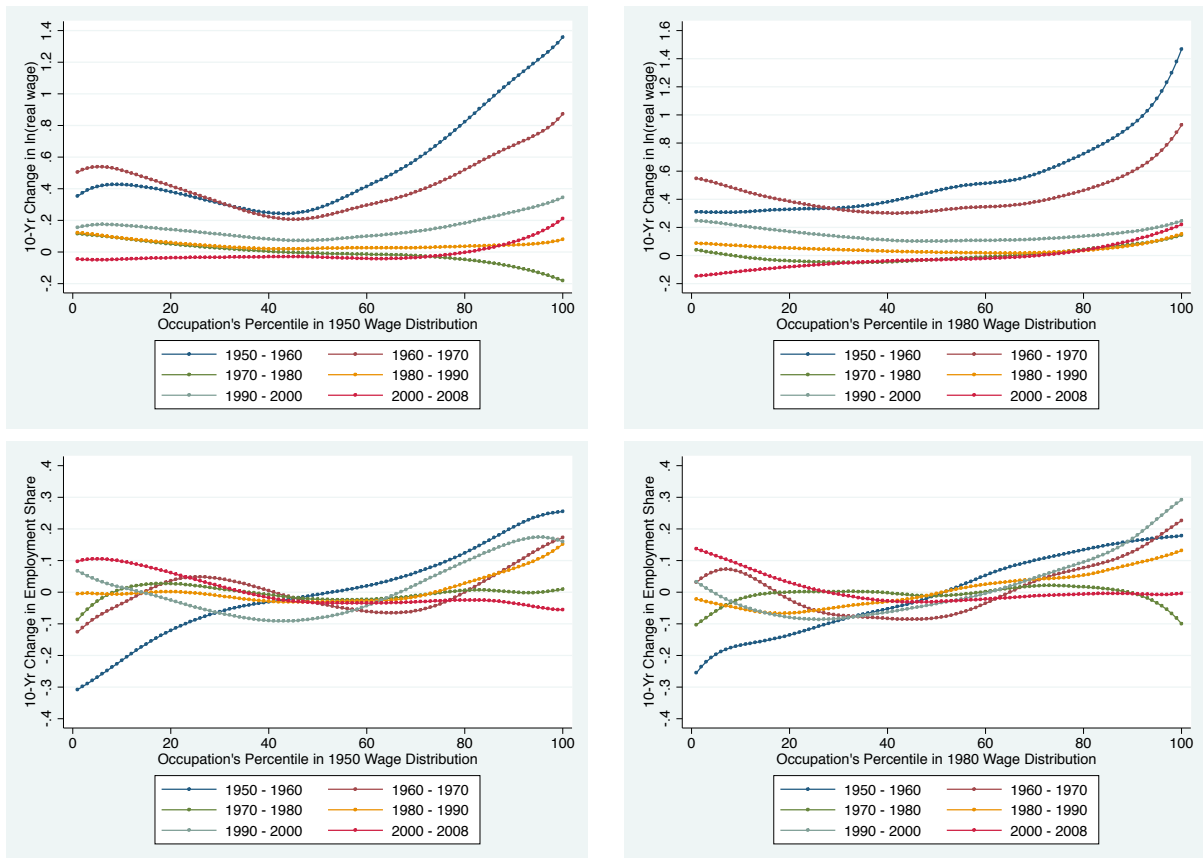
equal in the two market consumption sectors? Therefore we take our baseline model, but feed in productivity growth common to  $M$  and  $S$ . In particular we feed in a growth rate of 2% per annum. Under this counterfactual simulation, shown in figure 5, the model still implies rising employment in the  $S$  sector, together with rising relative average wages in sector  $S$  compared to sector  $M$ . However, the model does not generate a rise in  $L$  sector employment and wages relative to  $M$ . We conclude from this counterfactual analysis that higher productivity growth in manufacturing, that displaces workers in that sector, is important in understanding the simultaneous polarization of wages and employment.

## References

- ACEMOGLU, D., AND D. AUTOR (2011): "Chapter 12 - Skills, Tasks and Technologies: Implications for Employment and Earnings," vol. 4, Part B of *Handbook of Labor Economics*, pp. 1043 – 1171. Elsevier.
- ACEMOGLU, D., AND V. GUERRIERI (2008): "Capital Deepening and Nonbalanced Economic Growth," *Journal of Political Economy*, 116(3), pp. 467–498.
- AUTOR, D. H., AND D. DORN (2012): "The Growth of Low Skill Service Jobs and the Polarization of the U.S. Labor Market," *The American Economic Review*.
- AUTOR, D. H., L. F. KATZ, AND M. S. KEARNEY (2006): "The Polarization of the U.S. Labor Market," *The American Economic Review*, 96(2), pp. 189–194.
- AUTOR, D. H., F. LEVY, AND R. J. MURNANE (2003): "The Skill Content of Recent Technological Change: An Empirical Exploration," *The Quarterly Journal of Economics*, 118(4), pp. 1279–1333.
- BLANCHARD, O. J. (1985): "Debt, Deficits and Finite Horizons," *Journal of Political Economy*, 93(2), 223–247.
- BOPPART, T. (2011): "Structural Change and the Kaldor Facts in a Growth Model with Relative Price Effects and Non-Gorman Preferences," Working paper no. 2, University of Zurich Department of Economics.
- BUERA, F. J., AND J. P. KABOSKI (2009): "CAN TRADITIONAL THEORIES OF STRUCTURAL CHANGE FIT THE DATA?," *Journal of the European Economic Association*, 7(2-3), 469–477.
- (2012a): "The Rise of the Service Economy," *American Economic Review*, 102(6), 2540–69.
- (2012b): "Scale and the origins of structural change," *Journal of Economic Theory*, 147(2), 684 – 712, [Issue in honor of David Cass](#).
- CASELLI, F., AND W. J. COLEMAN II. (2001): "The U.S. Structural Transformation and Regional Convergence: A Reinterpretation," *Journal of Political Economy*, 109(3), pp. 584–616.
- GOOS, M., AND A. MANNING (2007): "Lousy and Lovely Jobs: The Rising Polarization of Work in Britain," *The Review of Economics and Statistics*, 89(1), 118–133.
- GOOS, M., A. MANNING, AND A. SALOMONS (2009): "Job Polarization in Europe," *The American Economic Review*, 99(2), pp. 58–63.
- (2011): "Explaining job polarization: the roles of technology, offshoring and institutions," Open Access publications from Katholieke Universiteit Leuven urn:hdl:123456789/331184, Katholieke Universiteit Leuven.

- HERRENDORF, B., R. ROGERSON, AND A. VALENTINYI (2009): "Two Perspectives on Preferences and Structural Transformation," Working Paper 15416, National Bureau of Economic Research.
- KONGSAMUT, P., S. REBELO, AND D. XIE (2001): "Beyond Balanced Growth," *The Review of Economic Studies*, 68(4), pp. 869–882.
- KUZNETS, S. (1957): "Quantitative Aspects of the Economic Growth of Nations: II. Industrial Distribution of National Product and Labor Force," *Economic Development and Cultural Change*, 5(4), pp. 1–111.
- MADDISON, A. (1980): "Economic Growth and Structural Change in the Advanced Countries," in *Western Economies in Transition: Structural Change and Adjustment Policies in Industrial Countries*, ed. by I. Leveson, and J. W. Wheeler, pp. 41–65. London: Croom Helm.
- MANNING, A. (2004): "We Can Work It Out: The Impact of Technological Change on the Demand for Low-Skill Workers," *Scottish Journal of Political Economy*, 51(5), 581–608.
- MAZZOLARI, F., AND G. RAGUSA (2007): "Spillovers from high-skill consumption to low-skill labor markets," *The Review of Economics and Statistics*, (0).
- MEYER, P. B., AND A. M. OSBORNE (2005): "Proposed Category System for 1960-2000 Census Occupations," Working Papers 383, U.S. Bureau of Labor Statistics.
- MICHAELS, G., A. NATRAJ, AND J. VAN REENEN (2010): "Has ICT Polarized Skill Demand? Evidence from Eleven Countries over 25 years," Working Paper 16138, National Bureau of Economic Research.
- NGAI, L. R., AND C. A. PISSARIDES (2007): "Structural Change in a Multisector Model of Growth," *The American Economic Review*, 97(1), pp. 429–443.
- (2008): "Trends in hours and economic growth," *Review of Economic Dynamics*, 11(2), 239 – 256.
- RAMEY, V. A. (2009): "Time spent in home production in the twentieth-century United States: new estimates from old data," *Journal of Economic History*, 69(1), 1.

Figure 6: Wage and employment polarization II.



Notes: Wages are calculated from US Census data for 1950, 1960, 1970, 1980, 1990, 2000 and American Community Survey (ACS) for 2008. Balanced occupation categories (185 of them) were defined by the authors based on Meyer and Osborne (2005) and Autor and Dorn (2012). The bottom two panels show the 10-year change in employment shares (calculated as hours supplied rather than persons), and the top two panels show the 10-year change in log hourly real wages (again labor supply weighted). In the left panels occupations are ranked based on their 1950 average wage, whereas in the right panels they are ranked according to their 1980 average wage.