Search-Based Endogenous Illiquidity^{*}

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Abstract

Illiquid asset markets have adverse effects on firms' funding conditions and investment decisions. Yet, both the micro-foundations of asset liquidity and its impact on aggregate business cycles are scarcely explored. We introduce endogenous asset liquidity into a dynamic macroeconomic model. The novelty of our approach consists in doing so via search frictions on asset markets, where liquidity is tantamount to resaleability. We show that procyclical asset liquidity and prices are both driven by investor participation on the search market. Asset illiquidity further creates a role for distinctly liquid assets as a buffer for future funding needs. Our model is able to match the flight to such liquid assets observed during recessions. Moreover, the liquidity differential between assets gives scope to non-standard monetary policy. The central bank may, for instance, control the supply of liquid assets and change the portfolio composition held by the private sector through asset purchase programs. Our analysis highlights that such policies need to be carefully designed in order to avoid crowding-out of private market participants.

Keywords: endogenous asset liquidity; search frictions; unconventional policy.

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1 Introduction

Illiquidity of financial assets arises from impediments to the sale of these assets. Empirical evidence points to pro-cyclical variation in the market liquidity of a wide range of financial assets.¹ The view that asset liquidity dries up during recessions has been reinforced by the 2007-2009 financial crisis. In this period, illiquidity problems were most pronounced in markets for corporate bonds, commercial paper, and repurchase agreements.² Variation in asset illiquidity affects the funding conditions of both financial and nonfinancial firms and has strong impacts on the macroe-conomy. U.S. nonfinancial firms, for instance, fund 35% of their total expenditures on physical capital (net of working capital) through primary (debt and equity) and secondary markets (portfolio liquidations).³ Higher liquidation costs in primary and secondary capital markets directly feed into these firms' investment and labor hiring decisions, such that liquidity fluctuation may significantly affect the business cycle.

The illiquidity of privately issued financial assets creates a role for liquid assets as an insurance against future funding constraints. Indeed, as shown in Figure 1, firms are more willing to accumulate liquid assets (i.e. money or government bonds) during recessions. We interpret this phenomenon as a flight to liquidity. Accordingly, the correlation between GDP and the liquidity share, defined as the ratio of liquid assets to GDP, is highly negative (-0.72 post 1984). The hedging capacity of liquid assets gives scope to unconventional monetary policy which can control their supply and change the portfolio composition held by firms.

We seek to jointly explain the procyclicality of asset liquidity (and prices), as well as the countercyclical flight to liquid assets in the corporate sector. To that end, we develop a dynamic stochastic general equilibrium model in which aggregate shocks generate endogenous fluctuations in asset liquidity. In this framework, asset liquidity is measured by the fraction of privately issued financial assets that is resaleable in a given period. The model shows how the variation in liquidity interacts with the funding conditions of firms over the business cycle. Our framework makes three contributions: It (i) provides a tractable search-based approach to endogenous asset market liquidity in a general equilibrium setting, (ii) generates pro-cyclical fluctuations in both

³While debt and equity issuance cover 75.67% of this financing gap, sales of liquid reserves make up a sizeable 20.74% and illiquid reserves 3.69%. For further details see Ajello (2012).

¹Studies by Huberman and Halka (2001), Chordia et al. (2001) and Chordia et al. (2005) assert that market liquidity is time-varying and highly correlated across asset classes such as bonds and stocks in the US. This observation implies that common factors drive liquidity. Moreover, the observed time variation is pro-cyclical.

²Using data that includes the majority of secondary over-the-counter corporate bond transactions in the US, Dick-Nielsen et al. (2012) identify a break in the market liquidity of corporate bonds at the onset of the subprime crisis. The liquidity component of spreads of all but AAA rated bonds increased and turnover rates declined, making refinancing on the corporate bond market more difficult. Similarly, Anderson and Gascon (2009) report that commercial paper (CP), which is largely traded on a search market with dealers as match-makers, experienced illiquidity. At the same time, money market mutual funds, the main investors in the CP market, shifted away from commercial paper to government securities reflecting both a flight-to-safety and a flight-to-liquidity motive. Finally, as emphasized by Gorton and Metrick (2012), the repo market has registered strongly increasing haircuts during the crisis. These are attributed largely to concerns about the market liquidity of securities, i.e. claims on private sector cash flows, which are used as underlying collateral in repo agreements.



Figure 1: Cyclical component of liquidity share (nominal liquid assets / nominal GDP) and real GDP as percent deviations from the trend. Liquid assets consisit of all liabilities of the federal government. Shaded areas denote NBER recessions dates. Source: U.S. Flow of funds

asset liquidity and asset prices, (iii) suggests that endogenous aggregate demand fluctuations driven by matching efficiency shocks account for the countercyclical liquidity share in the data, rather than productivity (supply) shocks.

Our model extends a standard New Keynesian business cycle model with search frictions in asset markets. The economy is populated by four types of agents: a household sector whose members are either entrepreneurs (sellers of financial assets) or workers (investors), intermediate goods producers, final goods producers, and the government. Households make consumption, labor, and investment choices. Intermediate goods producers rent capital from households and hire labor to produce. Final goods producers assemble intermediate inputs to final consumption goods. The government can affect the supply of fully liquid government bonds (money) and may purchase claims to private assets.

Household members hold claims on payoffs from assets (including other households' and their own) on their balance sheets, which we interpret as a catch-all for privately issued assets such as commercial paper, corporate bonds, bank loans, and mortgages. Each member receives an idiosyncratic type shock to become an entrepreneur or a worker. Entrepreneurs face investment opportunities. In order to take full advantage of these, entrepreneurs liquidate their financial asset portfolios. However, the private claims on their balance sheets may not be readily saleable, i.e. illiquid, because they have to be offered on a search market. As a consequence, entrepreneurs cannot fully finance investments by outside funding.

Moreover, search for appropriate counterparties is assumed to be costly for investors and sellers

alike. These costs are reflected in the asset price that is determined in a bargaining process between buyers and sellers.⁴ This market structure intends to capture the features of OTC markets, in which a large fraction of corporate bonds, asset-backed securities and private equity is traded.⁵ Alternatively, we consider our framework as a reduced-form approach towards modelling financial intermediation. In this reading, the search market structure captures the matching process between savers (investors) and the corporate sector through intermediaries.⁶ As in the search-market interpretation, a friction arises from the inefficiencies of the matching process. We conjecture that this primitive friction captures an essential feature of financial intermediation such that our results may also shed light on the effects of explicitly modelling a financial sector.

We consider two types of exogenous disturbances: aggregate productivity shocks and shocks to the matching efficiency between buyers and sellers. In both cases, the mechanism that allows the model to endogenously generate pro-cyclical asset liquidity and prices is driven by the participation of buyers in the search market. Negative aggregate productivity shocks, for instance, decrease the return to capital and make investment into capital goods less attractive. This crowds out investors from the search market. Negative matching efficiency shocks (combined with nominal frictions), on the other hand, exert a negative income effect while making investment into liquid assets more attractive. This effect also reduces the incentive for investors to engage in costly search effort. In either case, there are fewer matches, which endogenously decreases the volume of successful transactions. Hence, the resaleability of financial claims drops. The reduced willingness of buyers to participate in the market will also be reflected in declining asset prices. Lower asset liquidity and prices are, thus, tantamount to a tighter resaleability constraint. This liquidity-effect restricts the amount of funding available to entrepreneurs and, thereby, aggregate investment.

While procyclical asset liquidity and prices are consistent with both productivity and matching efficiency shocks, only the latter induce a flight to government bonds and thus a countercyclical liquidity share. In contrast, persistent TFP shocks lower the return to capital both today and in the future. Hence, investors have no incentive to hedge against future illiquidity of private financial assets by hoarding liquid assets. Matching efficiency shocks, on the other hand, do not deteriorate the quality of investment projects either today or tomorrow. Therefore, investors smooth their purchases of financial assets, such that the hedging service provided by liquid assets increases. Accordingly, the liquidity share in their portfolios increases.

To our knowledge, we are the first to incorporate a search-based financial asset market into a dynamic macroeconomic model. The tractability of our approach derives from the convenient setup of the bargaining process, which takes advantage of optimality conditions from the households' portfolio choice problems. This modelling strategy allows us to easily solve for the dynamics after aggregate shocks.

Finally, our search-based approach differs from an information-based approach to asset illiquid-

⁴This setup borrows heavily from the labour search literature.

⁵See Dick-Nielsen et al. (2012), Bao et al. (2011), Anderson and Gascon (2009).

⁶See Haan et al. (2003).

ity. In the latter, assets are either sold because of low quality or low cash flow. As income shrinks in bad economic conditions, more high-quality assets may be sold to smooth consumption. This effect would improve asset liquidity, i.e. make it counter-cyclical. Generally, information-based approaches focus on the selling pressures of claims holders. In the search-based approach, on the other hand, asset liquidity is driven by the market participation of both buyers and sellers.

1.1 Related Literature

Our work is closely related to Kiyotaki and Moore (2012) (henceforth, KM), who motivate financial assets' market liquidity by an exogenous constraint on the resaleability of private paper. Their basic idea is that disinvestment takes time (due to unmodelled frictions), such that only some fraction of asset holdings can be sold in a given period to finance new investment. Money or government bonds, on the other hand, can be readily sold when needed and thus provide a liquidity service. A "spectrum of returns" emerges as a result of differences in asset liquidity. As a consequence, the irrelevance result of Wallace (1981), which stipulates that non-standard open-market operations in private assets are irrelevant for prices and allocations, no longer holds. In fact, the portfolio composition between liquid and illiquid assets in the private sector affects the equilibrium. To the extent that open-market operations can change this composition by controlling the supply of liquid assets they have real effects. Del Negro et al. (2011) analyse the stabilizing potential of such non-standard liquidity policy after an exogenous fall in liquidity. With standard monetary policy constrained by the "Zero Lower Bound", the authors show that liquidity injections effectively dampen the liquidity shortfall.

However, Shi (2012) demonstrates that in these models exogenous liquidity shocks induce asset price booms (in real consumption good terms), because the liquidity shock essentially amounts to a supply shock on equity. Demand for equity, on the other hand, does not decrease substantially as investment projects' quality does not change. This finding highlights the need for a theory of endogenous asset liquidity.

Asset illiquidity may further interact with financing constraints to induce delays in asset sales as in Cui (2013). This interaction prolongs shocks to the financing conditions of the private sectors and results in countercyclical productivity dispersion.

Treatment of illiquidity in these papers abstracts from feedback effects between macroeconomic conditions and asset market liquidity. In particular, it cannot account for endogenous cyclical variations in asset illiquidity. The search literature, on the other hand, provides a natural theory of endogenous liquidity. It has been applied to a wide range of markets such as housing (Wheaton, 1990; Ungerer, 2012), bank loans (Haan et al., 2003; Wasmer and Weil, 2004) as well as OTC markets for asset-backed securities, corporate bonds, US federal funds, private equity or real estate (Duffie et al., 2005, 2007; Ashcraft and Duffie, 2007; Lagos et al., 2009; Feldhutter, 2011). The bottom line of this research is that search frictions can explain substantial variation in a range of measures of liquidity, such as bid-ask spreads, trade volume (market depth) and trading

delays. However, the majority of this research does not consider the adverse feedback of illiquid asset markets on the macroeconomy.

In a number of related studies, endogenous asset market liquidity has been motivated with information frictions. Eisfeldt (2004) develops a partial equilibrium model with adverse selection in asset markets. The model shows how investment and trading volume are amplified if asset liquidity endogenously varies with productivity. More recently, Guerrieri and Shimer (2012) provide a dynamic adverse selection model in an environment of exogenous fundamental asset returns. Their work focusses on the impact of adverse selection in asset markets on asset liquidity and prices, but does not consider feedback effects on production and employment. To account for such feedback effects, Kurlat (2012) extends KM with endogenous resaleability through adverse selection. Also in Bigio (2011), dispersion shocks to capital quality endogenously decrease the liquidity of private assets due to information asymmetries. Such shocks translate into substantial fluctuations in hours, investment and output when private assets are used as collateral in working capital loan contracts. However, these models do not consider alterative assets with different information properties, such as government bonds. In Eisfeldt and Rampini (2009), entrepreneurs endogenously accumulate liquid assets in the form of a risk-free store of value to fund investment opportunities. Because liquidity is accumulated out of retained earnings, the supply of liquidity correlates positively with productivity; there is no secondary market for private assets, however, and no alternative liquid asset the supply of which can be controlled exogenously. Our approach differs from these papers, in that we motivate endogenous liquidity differentials across assets and preserve the role of a liquid asset as a lubricant of financial flows as in KM.

The rest of the paper is organized as follows: Section 2 presents the model and characterizes the equilibrium of our economy. Section 3 discusses impulse responses and policy experiments. In section 4, we conclude and outline avenues for further research.

2 The Model

Environment. Time is discrete and with infinite horizon. The economy comprises four sectors: households, intermediate goods producers, final goods producers, and the government. The members of each representative household are either entrepreneurs or workers. The key deviation from the New Keynesian dynamic stochastic general equilibrium model consists in search frictions afflicting the sale of equity claims from these household members. Government bonds, on the other hand, are assumed to be fully liquid assets which can be traded freely on a spot market.

Timing. Each period is split into four phases: households' decisions, production, investment and consumption. At the beginning of each period, aggregate exogenous state variables are realized and the government policy rules are set. Then a representative household specifies policy rules for each household member, taking into account production, investment and consumption at later stages. After the decision is made, production takes place, entrepreneurs and workers meet in the asset market to trade assets, invest according to the rules specified by the household, and finally consume when they are back to the household.

Government conducts conventional monetary policy via the control of the nominal interest rate; fiscal policy via taxes on households and unconventional policy through purchasing or selling assets issued by prviate agents. Government behavior potentially changes asset prices and thus affects individual decisions.

2.1 Households

Representative household structure. The economy comprises a continuum of representative households with a unit measure of members each. Each period, household members receive an idiosyncratic shock that determines their type in the middle of the period. With probability χ household members become entrepreneurs, or equity sellers (type s), and with probability $(1 - \chi)$ they become workers, or equity buyers (type b). Entrepreneurs have productive investment opportunities, while workers earn income by supplying their labour. Type shocks are *i.i.d.* across members and through time. By the law of large numbers, each household thus consists of a fraction χ of entrepreneurs and a fraction $(1 - \chi)$ of workers. Both groups are temporarily separated such that resources cannot be re-allocated among household members during the period. Only at the very end of each period, both types come together again to share their consumption goods as well as their accumulated assets. This implies that all members enter the next period with an equal share of the household's assets.⁷

2.1.1 A Representative Household

Preferences. The household determines entrepreneurs' and workers' choices in order to maximize

$$\mathbb{E}_{t} \sum_{h=0}^{\infty} \beta^{t+h} \left[u(C_{t+h}) - \frac{\mu}{1+\nu} L_{t+h}^{1+\nu} \right]$$
(1)

Note that since both types of agents lump their consumption goods together at the end of the period, the household optimizes over household-wide consumption C_t . The full decision problem is developed in Section 2.1.3.

Portfolio. Physical capital is held by households and lent to intermediate goods producers. Thus, capital earns a return. There is a claim to the return of every unit of capital, which is either retained by households or sold to outside investors. In addition, households invest into risk-less government bonds. Hence, at the onset of each period households have a portfolio of government bonds, equity claims on other households' return on capital and own physical capital. These assets are financed by equity claims issued on the return to own physical capital and net worth. This financing structure gives rise to the beginning-of-period balance sheet in Table 1.

⁷The representative household structure with temporarily separated agents has been introduced in Lucas (1990) and applied to the KM framework in Shi (2012) and Del Negro et al. (2011).

Table 1: Household's Balance Sheet

liquid bonds	B_t	equity issued	$q_t S_t^I$
other's equity	$q_t S_t^O$		
capital stock	$q_t K_t$	net worth	$q_t S_t + B_t$

Portfolio adjustments are affected by search frictions: claims to private cash flows are sold on an over the counter (OTC) market. Here, only some fraction of offered assets is matched to appropriate buyers, such that some claims remain unmatched.⁸ We assume that an identical fraction of previously uncommitted returns to own physical capital, i.e. $(K_t - S_t^I)$, can be mortgaged. This simplification ensures that both types of assets not only yield the same return, but are equally liquid. They will thus yield the same price on the search market and can be treated as perfect substitutes, such that we only need to keep track of net equity, defined as



2.1.2 Household Members

A typical household member j is endowed with equity $s_{j,t}$ and bonds $b_{j,t}$. There exists a search market for purchasing and selling equity. Buyers exert costly search effort $e_{j,t}^b$ to acquire new or old equity. On the search market, each unit of buyers' effort results in $\phi_{b,t} \in [0, 1]$ matched purchases at unit cost κ_b . Accordingly, the individual buyer expects to purchase an amount

$$m_{j,t} = -\phi_{b,t} e^b_{j,t}$$

of matched assets on the search market. Sellers, on the other hand, decide which fraction $e_{j,t}^s \in [0, 1]$ of their total assets to put up for sale. These assets consist of existing equity claims on other households' capital stock and their own capital stock, $s_{j,t}$, plus new equity to be issued, i_t . Each unit of sellers' assets that are offered on the search market is matched with a buyer with probability $\phi_{s,t} \in [0, 1]$. Entrepreneurs, thus expect to sell

$$m_{j,t} = \phi_{s,t} e^s_{j,t} \left((1-\delta) s_{j,t} + i_{j,t} \right)$$

matched claims on the search market. The associated selling costs are κ_s per unit of listed assets. Note that the respective matching probabilities $\phi_{b,t}$ and $\phi_{s,t}$ are taken as given. These probabilities will be determined by the equilibrium on the search market. Denote the transaction price for the matched asset in the search market as q_t , which will be determined in the search market structure

⁸The exact structure of the search market is detailed in Section 2.1.2.

in Section 2.2. Importantly, a representative household takes the prices as given when choosing the search effort $e_{j,t}^b$ and the fraction of assets to post on the market $e_{j,t}^s$.

Now we can define the budget constraint of our typical household member j along with laws of motion for the equity position. Denote the nominal price level as P_t and the nominal bond return as R_t (which is pre-determined). c_{jt} , l_{jt} , and i_{jt} are consumption, working hours and investment, respectively. Finally, let τ_t be the lump-sum taxes on household members. Accordingly, j's budget constraint and equity evolution read

$$c_{j,t} + \kappa_b e_{j,t}^b + \kappa_s e_{j,t}^s \left((1-\delta)s_{j,t} + i_{j,t} \right) + i_{j,t} + \frac{b_{j,t+1}}{P_t} + \tau_t = w_t l_{j,t} + r_t s_{j,t} + q_t m_t + R_t \frac{b_{j,t}}{P_t}$$

$$s_{j,t+1} = (1 - \delta)s_{j,t} + i_{j,t} - m_{j,t}$$

Taking into account the specific functions of workers and entrepreneurs, we separate their budget constraints and equity evolutions.

A worker. If j is a worker without investment opportunity, then

$$i_{j,t} = 0, \quad e^b_{j,t} \ge 0, \quad e^s_{j,t} = 0$$

Substituting out expected purchases on the search market, $m_{j,t}$, the evolution of equity becomes

$$s_{j,t+1} = (1-\delta)s_{j,t} + \phi_{b,t}e^b_{j,t}$$

while the flow-of-funds constraint for a worker simplifies to

$$c_{j,t} + \kappa_b e^b_{j,t} + \frac{b_{j,t+1}}{P_t} + \tau_t = w_t l_{j,t} + r_t s_{j,t} - q_t \phi_{b,t} e^b_{j,t} + R_t \frac{b_{j,t}}{P_t}.$$
(2)

An entrepreneur. If j is an entrepreneur with investment opportunity, then

$$i_{j,t} > 0, \quad e_{j,t}^s \ge 0, \quad e_{j,t}^b = 0$$

Similarly to the worker, we substitute out sales $m_{j,t}$, to retrieve the evolution of equity as

$$s_{j,t+1} = \left(1 - \phi_{s,t} e_{j,t}^s\right) \left[(1 - \delta)s_{j,t} + i_{j,t}\right]$$

and the flow-of-funds constraint as

$$c_{j,t} + \kappa_s e_{j,t}^s \left((1-\delta)s_{j,t} + i_{j,t} \right) + \left(1 - \phi_{s,t} e_{j,t}^s q_t \right) i_{j,t} + \frac{b_{j,t+1}}{P_t} + \tau_t = r_t s_{j,t} + \phi_{s,t} e_{j,t}^s q_t (1-\delta)s_{j,t} + R_t \frac{b_{j,t}}{P_t}$$

For convenience, one can subsitute out $i_{j,t}$ from the evolution of equity to express the flow-of-funds

constraint of an entrepreneur in terms of end of period equity $s_{j,t+1}$:

$$c_{j,t} + \frac{1 - \phi_{s,t} e_{j,t}^s \left(q_t - \frac{\kappa_s}{\phi_{s,t}} \right)}{\left(1 - \phi_{s,t} e_{j,t}^s \right)} s_{j,t+1} + \frac{b_{j,t+1}}{P_t} + \tau_t = r_t s_{j,t} + (1 - \delta) s_{j,t} + R_t \frac{b_{j,t}}{P_t}.$$
(3)

The effect of sales on the search market is indirectly accounted for through the average price q_t . In particular, this price affects the down-payment $\frac{1-\phi_{s,t}e_{j,t}^s\left(q_t-\frac{\kappa_s}{\phi_{s,t}}\right)}{1-\phi_{s,t}e_{j,t}^s}$ that entrepreneurs have to undertake to accumulate one unit of new equity.

2.1.3 Households' Problem

It is instructive to consider the aggregation of household members to workers and entrepreneurs. Let $j \in \{b, s\}$ indicate the household member being a worker (buying equity) or an entrepreneur (selling equity), and define aggregate variables as $X_{b,t} \equiv (1 - \chi) x_{b,t}$ and $X_{s,t} \equiv \chi x_{s,t}$. To simplify notation, we also switch to the recursive formulation for ease of later exposition, i.e., let x and x'denote x_t and x_{t+1} . We aggregate the budget constraints of individual workers and entrepreneurs, (2) and (3), to

$$C_b + \kappa_b E_b + \frac{B'_b}{P} + \mathcal{T}_b = wL + rS_b - \phi_b q_b E_b + R\frac{B_b}{P}$$

$$\tag{4}$$

$$C_s + \frac{1 - \phi_s e^s \left(q - \frac{\kappa_s}{\phi_s}\right)}{\left(1 - \phi_s e^s\right)} S'_s + \frac{B'_s}{P} + \mathcal{T}_s = rS_s + (1 - \delta)S_s + R\frac{B_s}{P}$$
(5)

where the equity positions of workers and entrepreneurs evolve according to

$$S'_b = (1 - \delta)S_b + \phi_b E_b \tag{6}$$

$$S'_{s} = (1 - \phi_{s}e^{s})\left((1 - \delta)S_{s} + I\right)$$
(7)

Let Γ be the vector of aggregate state variables, the evolution of which is taken given by the household. Once we proceed to the equilibrium definition, $\Gamma \equiv (K, z_a, z_{\phi})$ where K is the total capital stock, z_a is the productivity, and z_{ϕ} is the matching efficiency in the search market.⁹ Let $V(S, B; \Gamma)$ be the value of a household with equity S and bond B, given the collection of aggregate state variables Γ . Then the value satisfies the Bellman equation:

Problem 1:

$$V(S, B; \Gamma) = \max_{E_b, E_s, S'_b, S'_s, B'_b, B'_s} u(C_b + C_s) - \frac{\mu}{1 + \nu} L^{1 + \nu} + \beta E \left[V(S', B'; \Gamma') | \Gamma \right], \quad s.t.$$
$$S_b = (1 - \chi) S, \quad S_s = \chi S$$
$$B_b = (1 - \chi) B, \quad B_s = \chi B$$

⁹The stochastic processes for the exogenous state variables are defined below.

$$S' = S'_b + S'_s \quad B' = B'_b + B'_s$$

and (4), (5), (6), (7)

Next, we further simplify the household problem by reducing the budget constraints of both types to a single household-wide constraint. When $\frac{1-\phi_s e^s(q-\frac{\kappa_s}{\phi_s})}{(1-\phi_s e^s)} < q$,¹⁰ entrepreneurs' downpayment on equity accumulation is lower than the market price that workers need to pay for new equity. We ensure that this condition holds in our calibration, such that households should have entpreneurs spend their entire net worth on creating new equity. In this case, entrepreneurs do not bring back consumption goods and liquid bonds to the household, such that consumption smoothing and precautionary bond holding are entirely delegated to workers. Hence,

Lemma 1:

For $1 + e^s \kappa_s < q$, we have $C_s = 0$ and $B'_s = 0$.

Therefore, entrepreneurs' budget constraint can be simplified to

$$\frac{1 - \phi_s e^s \left(q - \frac{\kappa_s}{\phi_s}\right)}{\left(1 - \phi_s e^s\right)} S'_s + \chi \tau = \left(r + (1 - \delta)\right) S_s + R \frac{B_s}{P} \tag{8}$$

Combining (7) and (8), we can back out entrepreneurs' future equity as

$$S'_{s} = \frac{(r + (1 - \delta))S_{s} + R\frac{B_{s}}{P} - \chi\tau}{q_{r}} = \chi \frac{(r + (1 - \delta))S + R\frac{B}{P} - \tau}{q_{r}}$$
(9)

where

$$q_r \equiv \frac{1 - \phi_s e^s q_s}{1 - \phi_s e^s}, \quad q_s \equiv q - \frac{\kappa_s}{\phi_s}$$

 q_r can be intrepreted as the down-payment and q_s is the effective selling price for one unit of assets. Hence, entpreneurs' end-of-period equity equals their entire networth over the down-payment q_r . If $q_s > 1$, as will be the case in our calibration, then $q_r < 1$ and entrepreneurs will effectively be able to leverage their net worth. The down-payment q_r also captures the effect of search costs on equity accumulation: higher search costs decrease the effective sales price, which increases the down-payment that in turn depresses accumulated equity. Therefore, the entrepreneurs' ability to leverage will be lower if search costs are higher.

For book keeping, we back out investment $I = \frac{S'_s}{(1-\phi_s)} - (1-\delta)\chi S$ as

$$I = \chi \frac{[r + (1 - \delta)\phi_s q_s] S + R\frac{B}{P} - \tau}{1 - \phi_s q_s}$$
(10)

Using the solution for S'_s in (9), together with the individual budget constraints (4) and (5),

¹⁰Or, equivalently, $1 + e^s \kappa_s < q$.

 $C = C_b + C_s$, $S = S_s + S_b$, and $B = B_s + B_b$, delivers the aggregate budget constraint for the whole household

$$C + q_b S' + \frac{B'}{P} + \left[(1 - \chi) + \chi \frac{q_b}{q_r} \right] \tau = wL + \left[(1 - \chi)(r + (1 - \delta)q_b) + \chi(r + (1 - \delta))\frac{q_b}{q_r} \right] S + \left[(1 - \chi) + \chi \frac{q_b}{q_r} \right] \frac{RB}{P}$$
(11)

where

$$q_b \equiv q + \frac{\kappa_b}{\phi_b} \tag{12}$$

We interpret q_b as the effective price for purchasing one unit of equity. The aggregate budget constraint (11) takes all constraints of individual household members into account. It expresses the household portfolio choice problem in a convenient form: the household's resources consist of wage payments, equity returns and the resale value of equity, as well as the bond value, which takes into account the liquidity service provided by bonds to entrepreneurs; theses resources are spent on consumption, taxes and a saving portfolio that consists of new equity and new bonds. When there are no search frictions, $\kappa_b = \kappa_s = 0$, we will prove that $q = q_s = q = 1$, and the budget constraint collapses to the standard household budget constraint in a real business cycle (RBC) model. The deviation from the underlying RBC structure is, thus, entirely due to search frictions.

The household's problem, Problem 1, can now be simplified to

Problem 2:

$$V(S, B; \Gamma) = \max_{S', B'_r, L, E_s} u(C) - \frac{\mu}{1+\nu} L^{1+\nu} + \beta E\left[V(S', B'; \Gamma')\right] \quad s.t.$$
(11)

We derive the first order necessary conditions to establish optimality from the household perspective. As standard in the portfolio choice optimization problem, the first order conditions are necessary and sufficient. The FOC for labor is

$$u'(C)w = \mu L^{\nu} \tag{13}$$

The FOC for next periods' equity S' is

$$u'(C) q_b = \beta E [V_s(S', B'; K', z')]$$

which, using the envelope condition, can be expressed in the form of a standard asset pricing formula as

$$E\left[\frac{\beta u'\left(C'\right)}{u'\left(C\right)}r'_{S}\right] = 1\tag{14}$$

where $r'_{S} = \left[(1-\chi) \left(r' + (1-\delta)q'_{b} \right) + \chi \left(r' + (1-\delta) \right) \frac{q'_{b}}{q'_{r}} \right] / q_{b}$. The FOC for next period's bond B'is

 $\frac{u'(C)}{P} = \beta E \left[V_B \left(S', B'_r; K', z' \right) \right]$

which yields another asset pricing formula

$$E\left[\frac{\beta u'\left(C'\right)}{u'\left(C\right)}r'_{B}\right] = 1\tag{15}$$

where $r'_B = \left[(1-\chi) + \chi \frac{q'_b}{q'_*} \right] \frac{PR'}{P'}$. When $\kappa_b = \kappa_s = 0$ and $q_b = q_s = q = 1$, equity can be sold frictionlessly and the bond loses its liquidity value. In this case, the FOC collapses to the standard consumption Euler equation $u'(C) = \beta E \left[u'(C') \frac{PR'}{P'} \right]$. Intuitively, as long as $\kappa_b, \kappa_s > 0$, bonds will provide a liquidity service so that their rate of return will generally be lower. This finding is one of our main analytical results:

Proposition 1:

Suppose $\kappa_b, \kappa_s > 0$ then $q_b > q_r$, so that nominal bonds have value and provide liquidity services. When, on the other hand, $\kappa_b, \kappa_s = 0$, nominal bonds can be entirely replaced by equity.

Finally, the FOC for the fraction of assets to be posted on the search market yields a corner solution $e^s = 1$ as long as the equilibrium price $q_s > 1$.

Lemma 2:

 $E_s = \chi e^s = \chi$ and the definitions of prices are changed to

$$q_r \equiv \frac{1 - \phi_s q_s}{1 - \phi_s}, \quad q_s \equiv q - \frac{\kappa_s}{\phi_s} \tag{16}$$

2.2Search and Matching in the Equity Market

Matching between buyers and sellers takes place in a decentralized market. Buyers engage in aggregate search effort denoted E_b where

$$E_b = \frac{S'_b - (1 - \delta)S_b}{\phi_b} = \frac{S' - S'_s - (1 - \delta)S_b}{\phi_b}$$
(17)

$$= \frac{S' - \chi \frac{(r+(1-\delta))S + R_r B_r - \tau}{q_r} - (1-\delta)(1-\chi)S}{\phi_b}$$
(18)

and sellers put all their assets on sale

 $A_{s} = (1 - \delta) \chi S + I$

Without government intervention, aggregate effort is $E_B = E_b$ and aggregate assets on sale are $A_S = A_s$. The number of aggregate matches M is determined by the matching function

$$M \equiv \xi e^{z_{\phi}} A_S^{\gamma} E_B^{1-\gamma}$$

where $\gamma \in (0, 1)$ is the elasticity of matches w.r.t posted assets, and $\xi e^{z_{\phi}}$ measures matching efficiency. Matching efficiency evolves according to the stochastic process

$$z'_{\phi} = \rho_{\phi} z_{\phi} + \epsilon_{\phi} \tag{19}$$

where $0 < \rho_{\phi} < 1$ and ϵ_{ϕ} is normal with mean 0 and variance σ_{ϕ}^2 . The endogenous rate at which buyers encounter matching sellers (purchase rate) is

$$\phi_b \equiv \frac{M}{E_B} = \xi e^{z_\phi} \left(\frac{E_B}{A_S}\right)^{-\gamma} \tag{20}$$

while the endogenous sales rate is

$$\phi_s \equiv \frac{M}{A_S} = \xi e^{z_\phi} \left(\frac{E_B}{A_S}\right)^{1-\gamma} \tag{21}$$

Next, we show how the search market asset price q is determined in the bargaining process. Household members come to bargain on behalf of the household's interest. We first denote v^b and v^s as the value of each individual buyer and seller. Notice that the household has already decided on search effort e^b and the fraction of assets to post in the search market e^s , and all the contingent plans on consumption, labor supply, investment and bond accumulations when individuals come to bargain. One can write v^b and v^s as

Problem 3:

$$v^{b}(m_{j}, e^{b}_{j}, s_{j}, b_{j}, l_{j}; \Gamma) = u(C_{b} + C_{s}) - \frac{\mu}{1 + \nu} L^{1 + \nu} + E[V(S', B'; \Gamma')] \quad s.t.$$

$$c_{j} + \kappa_{b}e^{b}_{j} + \frac{b'_{j}}{P} + \tau_{j} = wl_{j} + rs_{j} + qm_{j} + R\frac{b_{j}}{P}$$

$$s'_{j} = (1 - \delta)s_{j} - m_{j}$$

$$C_{b} + C_{s} = \int c_{i}di, \quad L = \int l_{i}di$$

$$S' = \int s'_{i}di, \quad B' = \int b'_{i}di$$

Problem 4:

$$v^{s}(m_{j}, e^{s}_{j}, s_{j}, b_{j}, l_{j}; \Gamma) = u(C_{b} + C_{s}) - \frac{\mu}{1 + \nu} L^{1+\nu} + E[V(S', B'; \Gamma')] \quad s.t.$$

$$c_{j} + q_{r}s'_{j} + \frac{b'_{j}}{P} + \tau_{j} = rs_{j} + (1 - \delta)s_{j} + R\frac{b_{j}}{p}$$

$$s'_{j} = \frac{1 - \phi_{s}}{\phi_{s}}m_{j}^{1-\lambda}$$

$$C_{b} + C_{s} = \int c_{i}di, \quad L = \int l_{i}di$$

$$S' = \int s'_{i}di, \quad B' = \int b'_{i}di$$

The key assumption of our individual bargaining framework is that any buyers and sellers interact at the margin. By this we mean that buyers' outside option is buying one asset less and sellers' outside option is selling one asset less. Therefore, the surplus for both buyers and sellers is the respective marginal value of an assets. The buyers' surplus is the marginal value to the household of an additional unit of matches for buyers, i.e.,

$$-v_m^b = -u'(C)q + \beta E \left[V_s\left(S', B'; \Gamma'\right) |\Gamma\right] \frac{\partial S'}{\partial s'_j} \frac{\partial s'_j}{\partial \left(-m_j\right)}$$
$$= -u'(C)q + \beta E \left[V_s\left(S', B'; \Gamma'\right) |\Gamma\right]$$

Similarly, the sellers' surplus is the marginal value to the household of an additional match for entrepreneurs

$$v_m^s = \left[-u'(C)q_r + \beta E \left[V_s \left(S', B'; \Gamma' \right) |\Gamma \right] \frac{\partial S'}{\partial s'_j} \right] \frac{\partial s'_j}{\partial m_j}$$
$$= \frac{1 - \phi_s}{\phi_s} \left[-u'(C)q_r + \beta E \left[V_s \left(S', B'; \Gamma' \right) |\Gamma \right] \right]$$

The price of a unit of assets in the search market q is determined via Nash bargaining between a buyer and a seller, i.e. agents bargain over q to maximize

$$\omega \ln \left(v_m^s \right) + (1 - \omega) \ln \left(-v_m^b \right)$$

where ω is the bargaining weight of sellers. The sufficient and necessary FOC yields

$$= \frac{\frac{\omega}{\frac{1-\phi_s}{\phi_s} \left[-u'(C)q_r + \beta E\left[V_s\left(S', B'; \Gamma'\right) |\Gamma\right]\right]}{1-\omega}}{\frac{1-\omega}{-u'(C)q + \beta E\left[V_s\left(S', B'; \Gamma'\right) |\Gamma\right]}}$$

To simplify, the solution can be expressed as

Lemma 3:

The search market bargaining price is

$$q = 1 + \kappa_s + \frac{\kappa_b}{\phi_b} \left(\frac{\phi_s}{1 - \omega} - 1 \right).$$
(22)

The bargaining solution links the asset price q to the degree of asset resaleability ϕ_s and the probability of meeting a matching seller ϕ_b . Moreover, the bargaining solution (22) establishes an intuitive link between asset illiquidity and asset prices: When assets become harder to sell, prices also fall. This relation can be summarized as our main theoretical result:

Proposition 2:

The search market price q correlates positively with asset resaleability ϕ_s (i.e. $\frac{\partial q}{\partial \phi_s} > 0$) and negatively with search market "tightness" from the buyer perspective ϕ_b (i.e. $\frac{\partial q}{\partial \phi_b} < 0$).

Intuitively, with less search efforts from the buyer side, ϕ_b increases because fewer buyers are competing for the assets that are up for sale; at the same time, the resaleable fraction of the assets, ϕ_s , decreases because fewer buyers imply fewer matches. The withdrawal of buyers from the market and the drop in market depth (i.e. the number of matches) reduce the overall surplus from matching. Accordingly, the increase of ϕ_b and the decrease of ϕ_s in (22) depress the bargained asset price. Our simple framework is, thus, suitable to deliver both decreasing liquidity and falling equity prices.

When there are neither search costs nor posting costs, i.e. $\kappa_s = 0$ and $\kappa_b = 0$, the search market price will go to q = 1, because there is no asset supply shortage. In this case (and in the absence of price stickiness) the economy collapses to the frictionsless RBC framework.

Corollary 1:

When $\kappa_s = 0$ and $\kappa_b = 0$

$$q_s = q_b = q = 1.$$

2.3 The Government

Following Del Negro et al. (2011) the government conducts fiscal policy, conventional monetary policy, and unconventional credit policy. We discuss the policies one by one in the following.

Fiscal policy is financed mainly through taxing and issuing government bonds. To focus on

monetary policy, we do not allow the tax to vary, i.e.,

$$\tau = \bar{\tau} \tag{23}$$

Conventional monetary policy consists of the central bank setting the nominal interest rate or money stock growth rate rule. The two types of conventional policy we consider are

(1) Constant supply of liquid assets

$$B' = B_g \quad \text{or} \quad R' = 1 \tag{24}$$

R' = 1 means that liquid assets bear no nominal interest so that we can view them as fiat money. (2) Feedback interest rate rule

$$R' = \max\left\{R\pi^{\psi_{\pi}}, 0\right\} \tag{25}$$

where $\pi = \frac{P}{P_{-1}}$ and $\psi_{\pi} > 1.^{11}$

Unconventional policy corresponds to government purchases or sales of private paper (denoted by S'_q) in the search market as a function of the liquidity of private paper

$$S'_g = \min\left\{K\psi_k\left(\frac{\phi_s}{\bar{\phi}_s} - 1\right), 0\right\}$$
(26)

where $\bar{\phi}_s$ is the steady state of ϕ_s .

Our approach to modelling such policy is intended to ensure that unconventional interventions affect the economy exclusively through their impact on asset market prices. In particular, unconventional policy does not directly relax any agents' resaleability constraint. Also, when the government intervenes, it has to respect the search market structure.

The key question regarding our modelling strategy for unconventional policy is at what price the government can intervene. The full solution would require modelling the valuation of asset purchases or sales from the government perspective to obtain a bargainning solution. We do not opt for this, since optimal policy design is beyond the scope of the present paper. Instead, we let the government solely use the bargaining price from the market.¹² This choice is equivalent to assuming that the government does not pay search costs to intervene on the market. Therefore, the aggregate inputs to the search market aggregator, effort and assets for sale, are modified to be

$$E_B = E_b + \frac{S'_g - (1 - \delta) S_g}{\phi_b}$$

¹¹We proceed with the first specification of conventional monetary policy, i.e. with the case of fiat money. Details for the case of interest-bearing bonds are available from the authors upon request. The qualitative results are robust to either specification.

¹²We make this assumption on two grounds: One, to mimick the actual purchases of public and private assets by central banks in response to the Great Recession; and two, to maintain tractability of the search market structure in the absence of optimal policy considerations. Our results should thus be interpreted as pointing to a lower bound of the effectiveness of government interventions.

$$A_S = A_s + \frac{S'_g - (1 - \delta) S_g}{\phi_s}$$

Although we allow the government sector to have a better technology for entering the search market than private agents, we still view this approach as helpful for examining the mechanism of unconventional policy. In particular, our modelling strategy

(1) maintains the matching and bargaining framework even with government interventions,

(2) avoids the problem of different prices for asset sales and purchases from the government perspective (see equation (27)),

(3) still allows the market price, q, to react to policy which potentially changes demand and supply.

As per the government's budget constraint (27), asset purchases are financed to a large extent through the issuance of liquid assets. Therefore, even given technological constraints, the government can potentially correct externalities in the economy with liquidity frictions by supplying liquid assets. By affecting asset prices, such policy feeds into private portfolio choices between liquid, risk-free assets and private paper. Via these, unconventional interventions ultimately affect asset prices, search effort and asset liquidity (ϕ_s) in the search market.

Fiscal policy interacts with monetary policy and both policies interact with the real economy. These interactions determine the price level P and the supply of nominal liquid assets B'. Let the real bond be $B_r \equiv B/P$ and let the supply of nominal liquid assets be $B' = \eta B$ where η is the growth rate of liquid assets. We can write the government budget constraint as

$$qS'_{g} + RB_{r} + G = \tau + (r + (1 - \delta)q)S_{g} + \eta B_{r}$$
(27)

where G corresponds to government spending. To have a more tractable equilibrium definition later, we work with real bonds as the liquidity measure. The inflation rate can, accordingly, be written as

$$\pi = \frac{P}{P_{-1}} = \frac{\eta_{-1}B_{r,-1}}{B_r} \tag{28}$$

which is used in the household maximization problem, i.e., (15)

2.4 Intermediate Goods and Final Goods Producers

The production phase is divided into two sub-stages. During the first, a continuum of intermediate goods producers indexed by $i \in [0, 1]$ assembles differentiated intermediate goods Y_i using capital and labor as inputs. Each intermediate producer i operates in an environment of monopolistic competition. During the second stage, consumption goods are produced by perfectly competitive final goods producers from intermediate goods. Both intermediate and final goods producers are owned by the households.

Final goods producers. Consumption goods producers combine a continuum of intermediate

products $Y_i, i \in [0, 1]$ according to

$$Y = \left[\int_0^1 Y_i^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}$$

where $\theta \in (0, \infty)$ is the elasticity of substitution of inputs in production.

Let P_i be the nominal price for the intermediate goods. Recall that the nominal price level for final goods is P, then profit maximization of the final goods producer implies a (downward-sloping) demand function for each intermediate good i

$$Y_i = \left(\frac{P_i}{P}\right)^{-\theta} Y$$

From the zero profits condition for final goods producers, the aggregate price level for final goods can be expressed as

$$P = \left[\int_0^1 P_i^{1-\theta} di\right]^{\frac{1}{1-\theta}}$$

Intermediate goods producers. Each interemdiate firm has access to a constant-returns-to-scale (CRS) technology for producing output from capital and labor. Firm i rents capital k_i (at rental rate r) and employs labor l_i in a competitive labor market (at real wage w) to produce

$$Y_i = e^{z_a} k_i^{\alpha} l_i^{1-\alpha}$$

where $\alpha \in (0, 1)$ and z_a follows

$$z_a' = \rho_a z_a + \varepsilon_a \tag{29}$$

where ε_a is a normally distributed random variable with mean zero and standard deviation σ_a . Intermediate goods producers face menu costs in adjusting their relative prices.¹³ Accordingly, *i*'s real current-period profit is

$$\begin{aligned} \frac{\Pi_i}{P} &= \frac{P_i}{P} Y_i - m_{c,i} Y_i - \frac{\zeta}{2} \left(\frac{P_i}{\bar{\pi} P_{i,-1}} - 1 \right)^2 Y \\ &= \left(\frac{P_i}{P} \right)^{1-\theta} Y - \left(\frac{P_i}{P} \right)^{-\theta} m_{c,i} Y - \frac{\zeta}{2} \left(\frac{P_i}{\bar{\pi} P_{i,-1}} - 1 \right)^2 Y \end{aligned}$$

where we have substituted the individual demand function facing producer i, $m_{c,i} = \frac{rk_i + wl_i}{Y_i}$ is the marginal costs of producing Y_i , ζ measures the magnitude of price adjustment costs, and π is the steady state gross inflation rate. Taking the cost-minimizing factor inputs k_i and l_i as given,

¹³Conceptually, we follow Rotemberg (1982) in introducing quadratic price adjustment costs to make pricesetting a dynamic probem. Our particular specification of adjustment cost is adopted from Ireland (2004).

intermediate goods producer *i* sets his price $P_{i,t}$ in order to solve

$$\max_{P_{i,t}} E_t \sum_{s=0}^{\infty} \Lambda_{t+s} \frac{\prod_{i,t+s}}{P_t}$$

where $\Lambda_{t+s} = \frac{\beta u'(C_{t+s})}{u'(C_t)}$ is the stochastic disoucnt factor of the households. The first-order conditions for this problem associated with each producer are, in recursive notation,

$$(\theta - 1) \left[\frac{P_i}{P} \right]^{-\theta} \left(\frac{Y}{P} \right) = \theta \left[\frac{P_i}{P} \right]^{-\theta - 1} m_{c,i} \left(\frac{Y}{P} \right) - \zeta \left(\frac{P_i}{\bar{\pi} P_{i,-1}} - 1 \right) \left(\frac{Y}{\bar{\pi} P_{i,-1}} \right) + \zeta E \left[\left(\frac{\beta u' \left(C' \right)}{u' \left(C \right)} \right) \left[\frac{P'_i}{\bar{\pi} P_i} - 1 \right] \left(\frac{Y'}{P_i} \right) \left(\frac{P'_i}{\bar{\pi} P_i} \right) \right]$$

If $\zeta = 0$, i.e. without price adjustments costs, the model collapses to the case of monopolistic competition. Then, relative prices are set at a constant mark-up $\frac{\theta}{\theta-1}$ over *nominal* marginal costs

$$P_i = \frac{\theta}{\theta - 1} P m_{c,i}$$

We look for a symmetric equilibrium in which all intermediate goods producers set the same price, such that we can drop type subscript i and $P_i = P \ \forall i \in [0, 1]$. The first-order conditions then collapse to

$$(\theta - 1)Y = \theta m_c Y - \zeta \left(\frac{\pi}{\bar{\pi}} - 1\right) \left(\frac{\pi}{\bar{\pi}}\right) Y + \zeta E \left[\frac{\beta u'(C')}{u'(C)} \left(\frac{\pi'}{\bar{\pi}} - 1\right) \left(\frac{\pi'}{\bar{\pi}}\right) Y'\right]$$
(30)

This expression is the New Keynesian Phillips Curve.

Finally, we obtain the rate of return on capital and the wage rate from the cost-minimizing choices of capital and labor that are necessary to produce a given amount of intermediate goods Y_i .

$$\min_{k_i, l_i} rk_i + wl_i - m_{c,i} \left(Y_i - e^{z_a} k_i^{\alpha} l_i^{1-\alpha} \right)$$

where the Lagrange multiplier $m_{c,i}$ represents the marginal costs of producing Y_i . Again imposing symmetry and aggregating over individual choices yields

$$w = m_c \left(1 - \alpha\right) e^{z_a} \left(\frac{K}{L}\right)^{\alpha} = m_c F_L(K, L)$$
(31)

$$r = m_c \alpha e^{z_a} \left(\frac{K}{L}\right)^{\alpha} = m_c F_K(K, L)$$
(32)

2.5 Equilibrium Definition

The recursive competitive equilibrium is a mapping $(S, S_g, R, z_a, z_{\phi}) \rightarrow (S', S'_g, R', z'_a, z'_{\phi})$, with associated consumption and investment choices $\{C, I\}$, capital and labor inputs $\{K, L\}$, marginal costs $\{m_c\}$, sales and purchase rates on the asset market $\{\phi_s, \phi_b\}$, government policy rules $\{\tau, R', S'_g\}$, real liquid assets and their growth rate $\{B_r, \eta\}$, asset, goods and factor prices $\{q, q_b, q_s, q_r, w, r, \pi\}$, and laws governing the evolution of (z_a, z_{ϕ}) . In particular, the mapping satisfies

1. Individual Optimality:

Given prices $\{q, q_b, q_s, q_r, w, r, \pi\}$ and search market characteristics $\{\phi_b, \phi_s\}$, the policy functions solve the representative household's and intermediate good producers' optimization problems; i.e., (13)-(15) are satisfied (replacing $\frac{p}{p'}$ by using 28) with investment *I* defined in (10); similarly, intermediate good producers' optimality conditions (30)-(32) are satisfied.

2. Government policy rules:

Fiscal policy obeys (23), conventional monetary policy follows either (24) or (25), and unconventional monetary policy is conducted according to (26).

- 3. Market clearing¹⁴:
 - (a) Households' budget constraint (11) is statisfied.
 - (b) Government's budget constraint (27) is satisfied.
 - (c) Search market clears:

$$\phi_s = (\phi_b)^{\frac{\gamma-1}{\gamma}} \left(\xi e^{z_\phi}\right)^{\frac{1}{\gamma}}$$

The search market price q is determined by Nash bargaining according to (22). while effective equity prices and the downpayment $\{q_b, q_s, q_r\}$ are given by definitions (12) and (16).

(d) Equity market clears:

$$S + S_q = K$$

(e) Capital market clears:

$$K' = (1 - \delta)K + I$$

4. Exogenous matching efficiency and productivity evolve according to (19) and (29), respectively.

¹⁴Labor market clearing $L = (1 - \chi)l$ is implicitly assumed.

Note that the goods market clearing condition is implied by combining household members' budget constraints and the government budget constraint. This yields the aggregate resource constraint

$$C + I + G + \kappa_b E_b + \kappa_s ((1 - \delta)\chi S + I) = \left[1 - \frac{\zeta}{2} \left(\frac{\pi}{\bar{\pi}} - 1\right)^2\right] Y$$

3 Numerical Results

We choose those parameters that are not related to the search market by following a conventional calibration found in the literature (Table 2). To pin down the search-market related parameters, we choose four targets in the long run steady state: Tobin's q, the liquidity share, the resaleable fraction of assets, and finally the purchase and selling price difference.¹⁵ The average Tobin's q in the U.S. ranges from 1.1 to 1.21 according to Compustat data. From the flow of funds data, the liquidity share, defined as the total real liquid assets over real GDP is about 40% in the data. Finally, since our illiquid assets represent all assets that are not government bonds, we choose the price spread to be relatively high as 6%.

We have only four targets, although there are five parameters that relate to the search market, i.e. ξ , κ_b , κ_s , γ , and ω . However, given our targets the matching efficiency parameter ξ is a function of γ . Therefore, it is sufficient to determine four parameters. Without loss of generality, we set $\gamma = 0.5$. The baseline calibration can be found in Table 2 in the appendix.

After determining the steady state, we log-linearize the system around the deterministic steady state to approximate the solution to the nonlinear model. Then we solve the linear rational expectation model with a persistent productivity and matching efficiency process and analyze unconventional government interventions in asset markets.

3.1 Technology Shock

Responses to a negative technology shock are displayed in Figure 2. In order to keep the analysis simple, we shut off unconventional policy and focus on the case of fiat money, i.e. conventional monetary policy follows (24).

A negative technology shock depresses the productivity of capital and, thus, its value to the household. Accordingly, the total match surplus to be bargained over on the search market shrinks. This makes search for investment into entrepreneurs less attractive and the search effort exerted by workers drops. As a consequence the overall number of matches decreases, which is reflected in the sharp drop in the resaleable fraction ϕ_s . A negative TFP shock thus triggers an endogenous decline in the liquidity of financial assets issued by private agents. As demonstrated in our analytical result (22), the fall in demand on the asset market also translates into a lower bargained asset price. The lower resale value of financial assets depresses entrepreneurs' net worth. This is equivalent to a tighter financing constraint (10), which translates into a strong fall in investment.

¹⁵See Table 3.

Importantly, liquidity (resaleability) of financial assets is endogenously generated through the features of the search market. In the absence of search frictions, these liquidity effects would not occur. In the RBC world, a negative TFP shock primarily affects the demand for capital goods and thus reduces the optimal level of investment. In contrast, entrepreneurs in our framework are financing constrained. Therefore, investment falls more strongly than in the RBC benchmark.



Figure 2: Impulse responses after a negative productivity shock (one percent).

However, technology shocks fail to generate the empirically observed countercyclical response of the liquidity share in the economy. This finding is somewhat surprising as one would expect the liquidity service provided by money (bonds) to become more valuable to the household when the liquidity of private assets declines. This effect is compensated by three factors: First, the technology shock decreases available resources, such that households are exposed to a negative income effect that lowers their demand for any asset. Second, in view of the persistence of productivity shocks even future investment will be unattractive. Hence, the incentive to hedge against persistent illiquidity is weak. And third, as is standard in the New Keynesian literature, prices (and inflation) increase in response to the negative TFP shock. As the nominal price level reacts sluggishly real factor prices cannot adjust flexibly and the marginal costs of production increase. This translates into inflationary pressures, which contribute to the strong decline in the real value of money (bonds) and, hence, the liquidity share.¹⁶

These results warrant two key observations: One, endogenizing asset resaleability helps reconciling declining liquidity with falling asset prices. Two, even in the presence of endogenous

 $^{^{16}{\}rm The}$ liquidity share even drops in the absence of the price effect due to nominal rigidities. See Figure 5 in the appendix.



Figure 3: Impulse responses after a negative matching efficiency shock (one percent).

procyclical asset liquidity, technology shocks cannot account for the observed countercyclical behaviour of the liquidity share.

3.2 Matching Efficiency Shock

In Kiyotaki and Moore (2012) and Shi (2012) asset resaleability, i.e. ϕ_s , is exogenous. The focus of their analysis is a negative shock to the exogenous resaleability. The problematic feature of such a liquidity shock is that it acts like a supply shock: When ϕ_s drops, the supply of assets up for sale shrinks. However, productivity of capital is not affected by the shock such that the demand for private assets does not fall. Therefore, asset prices boom - a counter-factual phenomonon in recessions. In contrast, our framework is capable of generating an initial pro-cyclical response of asset prices even with exogenous liquidity shocks.

To mimic the exogenous liquidity shock scenario, we present the dynamics after a pure liquidity shock, which we capture by a negative shock to the matching efficiency process z_{ϕ} . In our framework this shock acts like a demand shock in the New Keynesian model: Initially, it directly reduces the number of matches on the search market due to technological reasons. This implies less investment and lower production and resources in the future without any decrease in total factor productivity. Anticipating this, price-setting intermediate goods firms expect lower marginal costs, or equivalently higher mark-ups due to a smaller scale of production. They react by reducing their prices to increase relative demand for their products. But they can do so only sluggishly due to adjustment costs. Hence, marginal costs fall today, the mark-up increases and factor rents decrease. This triggers a strong fall in employment and production, leaving households with less resources.

At this point our search-based endogenous liquidity framework is critical to translate the negative income effect into a sufficient decline in demand on the asset market. Households prefer workers to substitute into consumption and liquid government bonds (flight to liquidity), rather than spending more resources in costly search on the asset market. Consequently, search effort (tantamount to asset demand) shrinks and asset resaleability ϕ_s drops unequivocally both due to the exogenous shock and the endogenous withdrawal of buyers. The decline in the purchase rate, on the other hand, exhibits a hump-shaped response. The initially less pronounced drop precisely reflects the aforementioned endogenous decline in search effort.

In sum, the negative demand effect dominates the negative supply effect associated with the exogenous decrease in matching efficiency, at least initially. Accordingly, the asset price falls.¹⁷ Our model demonstrates that both endogenous liquidity and nominal price rigidities are necessary to generate this pro-cyclical response of asset prices after pure liquidity shocks.¹⁸

Note that the matching efficiency shock also increases the hedging value of money (bonds). As mentioned, total factor productivity is unaffected by the matching efficiency shock. Therefore, future investment remains attractive. To take advantage of future investment opportunities, households want to hedge against the persistent illiquidity of privately issued assets by expanding their government bond holdings. This motive increases demand for the liquid asset today, which drives up the liquidity share in line with the data.

3.3 Unconventional Policy

Next, we investigate whether government purchases of claims to private assets may have real effects. To illustrate this point, we set $\psi_k < 0$ so that the government starts to purchase private equity when the resaleability of assets is below the steady state level. We focus on aggregate matching efficiency shocks as the source of the disturbance.

Recall that the government respects all technological constraints in the economy. Figure 4 displays the sensitivity of the model dynamics to asset purchases by the public sector on the search market. The steady state of the economy is the same as that of the previous economy without unconventional interventions (i.e. where the government holds no private assets). Once the matching efficiency shock hits, unconventional policy is activated according to rule (26).

The policy succeeds in reducing the need for liquidity in the private sector, which can be inferred from the price of liquid asset that decreases more strongly. This effect is due to the increased supply of government bonds (money), which we force the government to use in order to

¹⁷Figure 6 displays the transition to the new steady state after a permanent negative matching efficiency shock. This exercise reveals that the positive over-shooting of the asset price is due to the temporary nature of the shock. Hence, the greater the persistence of the shock, the more protracted the decline in the asset price.

¹⁸In contrast to our result, nominal frictions in the KM model would exercerbate the equity price boom as discussed in Shi (2012). This further supports our claim that endogenous liquidity frictions are key to generating lower asset prices.

purchase private financial assets. On the other hand, the intervention has one major drawback: When the government is constrained to purchase assets on the search market it crowds out private buyers. This effect dominates the liquidity provision by the government, such that the net impact on investment and output is negative. These results suggest that asset purchases could be more effective if conducted on a centrally cleared market or targeted at the balance sheets of intermediating institutions.

4 Conclusion

We illustrate how asset liquidity (or the resaleable fraction of financial assets) can co-move with output through asset markets with search frictions, and how liquidity can have feedback effects on consumption, investment, and employment. Investor participation in this market drives the liquidity and price of financial assets. Liquidity of these assets is important for financing new investment. Therefore, aggregate shocks that affect the incentive of households to purchase financial assets also affect the financing conditions of firms through endogenous liquidity fluctuations.

Our model matches the procyclicality of asset liquidity (and prices) observed in the data. Moreover, it shows that matching efficiency rather than productivity shocks can explain a countercyclical share of liquid assets relative to GDP. The endogenous nature of asset liquidity is key to match this set of stylized facts. Implicitly, we also corroborate the finding in the previous literature that purely exogenous liquidity shocks lead to higher asset prices in recessions.

Importantly, we consider the matching framework as a shortcut for modelling financial in-



Figure 4: Impulse responses after a negative productivity shock (one percent). Blue line: responses under no unconventional policy. Red Line: responses under unconventional policy.

termediation. Financial intermediaries help channel funds from investors to firms which need outside funding. Although we do not explicitly model intermediaries' balance sheets and borrowing/lending decision in our framework, the structure of financial intermediation is similar to the process of matching investors and entrepreneurs with investment opportunities. In light of our results we conjecture that any explicit search-based theory of financial intermediation also needs to incorporate some degree of product market imperfections (e.g. nominal rigidities) in order to account for the aforementioned stylized facts.

Our framework also shows that open market operations in the form of asset purchase programs can potentially have real effects by easing liquidity frictions. However, such policies need to be carefully designed in order to avoid crowding-out of private market participants. Building on this result, future research could focus on the optimal mix of conventional, unconventional and fiscal policy measures in the presence of illiquid asset markets. The interactions among secondary asset markets, policy responses, and nominal price levels can shed light on important policy debates.

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Appendices

A Equilibrium Conditions

Given the aggregate state variable

$$(S, S_g, R; z_a, z_\phi)$$

we are solving for

$$(S', S'_g, R', \eta, B_r, C, I, L, K, mc, \Pi, \phi_b, \phi_s, q, q_r, q_b, q_s, r, w, \tau, \pi)$$

together with the exogenous law of motion of (z_a, z_{ϕ}) . Since there are 21 variables in total, one needs 21 equations.

1. Given the aggregate state and the price functions, the policy functions solve the representative household's optimisation problem

$$u'(C)w = \mu L^{\nu}$$

$$u'(C) = \beta E \left[u'(C') \left[(1-\chi) + \chi \frac{q'_b}{q'_r} \right] \frac{R'B'_r}{\eta B_r} \right]$$
$$u'(C) q_b = \beta E \left[u'(C') \left[(1-\chi) \left(r' + (1-\delta)q'_b \right) + \chi \left(r' + (1-\delta) \right) \frac{q'_b}{q'_r} \right] \right]$$
$$I = \chi \frac{\left[r + (1-\delta)\phi_s q_s \right] S + R\frac{B}{p} - \tau}{1 - \phi_s q_s}$$

(a) Intermediate Goods Producer

$$\pi(\pi - 1) = Y \frac{\theta}{\zeta} \left[m_c - \frac{\theta - 1}{\theta} \right] + \mathbb{E} \frac{\beta u'(C')}{u'(C)} (\pi' - 1)\pi'$$

$$r = m_c e^z F_K(K, L)$$
$$w = m_c e^z F_L(K, L)$$
$$\Pi = Y - (rK + wL) - \frac{\zeta}{2} (\pi - 1)^2$$

2. Policy: (a). Fiscal Policy:

$$\tau = \bar{\tau}$$

(b). Unconventional Monetary Policy: Purchasing rule:

$$S'_g = \max\left\{K\psi_k\left(\frac{\phi_s}{\bar{\phi}_s} - 1\right), 0\right\}$$

(c). Conventional Monetary Policy: (nominal interest rate)

$$R' = R\left(\pi\right)^{\psi}$$

3. Price definition: The replacement cost of equity is defined as

$$q_r \equiv \frac{1 - \phi_s q_s}{1 - \phi_s}$$

The effective purchasing price is defined as

$$q_b \equiv q + \frac{\kappa_b}{\phi_b}$$

The effective selling price is defined as

$$q_s \equiv q - \frac{\kappa_s}{\phi_s}$$

4. Household budget constraint

$$C + q_b S' + \eta B_r + \left[(1 - \chi) + \chi \frac{q_b}{q_r} \right] \tau = wL + \left[(1 - \chi)(r + (1 - \delta)q_b) + \chi(r + (1 - \delta))\frac{q_b}{q_r} \right] S + \left[(1 - \chi) + \chi \frac{q_b}{q_r} \right] RB_r + \Pi$$

5. Government budget constraint: (Note: used to pin-down η)

$$qS'_{g} + RB_{r} + G = \tau + (r + (1 - \delta)q)S_{g} + \eta B_{r}$$

where $\eta \equiv \frac{B'}{B}$, $B_r \equiv \frac{B}{p}$ and

$$\pi = \frac{p}{p_{-1}} = \eta_{-1} \frac{B_{r,-1}}{B_r}$$

6. Search Market Clearing

$$\phi_s = \phi_b^{\frac{\gamma-1}{\gamma}} \left(\xi e^{z_\phi}\right)^{\frac{1}{\gamma}}$$
$$q = 1 + \kappa_s + \frac{\kappa_b}{\phi_b} \left(\frac{\phi_s}{1-\omega} - 1\right).$$

7. Equity Market Clearing:

$$S + S_q = K$$

8. Capital Market Clearing:

$$K' = (1 - \delta)K + I$$

9. Exogenous shocks

$$z'_{a} = \rho_{a}z + \epsilon'_{a}$$
$$z'_{\phi} = \rho_{\phi}z_{\phi} + \epsilon'_{\phi}$$

B Steady State

In steady state, any variable X = X'. The solution strategy is to guess (K, L, B_r, ϕ_b) and to express all other variables as functions of these. Given (K, L, B_r, ϕ_b) , we have

$$\begin{aligned} z_a' &= \rho_a z + \epsilon_a' \to z_a = 0 \\ \\ z_{\phi}' &= \rho_{\phi} z_{\phi} + \epsilon_{\phi}' \to z_{\phi} = 0 \\ \\ \eta &= 1 \\ \\ \pi &= 1 \end{aligned} \\ S_g' &= \max \left\{ K \psi_k \left(\frac{\phi_s}{\phi_s} - 1 \right), 0 \right\} \to S_g = 0 \\ \\ \tau &= \bar{\tau} \end{aligned} \\ \\ qS_g' + RB_r + G &= \tau + (r + (1 - \delta) q) S_g + \eta B_r \to R = \eta + \frac{\tau - G}{B_r} \\ \\ S + S_g &= K \to S = K \end{aligned}$$

$$m_c = \frac{\theta - 1}{\theta}$$

 $r = m_c e^z F_K(K, L)$

$$\begin{split} w &= m_c e^z F_L(K,L) \\ \Pi &= K^{\alpha} L^{1-\alpha} - (rK + wL) \\ u'(C)w &= \mu L^{\nu} \rightarrow C = \left[\frac{w}{\mu L^{\nu}}\right]^{1/\sigma} \\ u'(C) &= \beta E \left[u'(C')\left[(1-\chi) + \chi \frac{d_b}{q'_r}\right] \frac{R'B'_r}{\eta B_r}\right] \rightarrow \frac{q_b}{q_r} = \frac{\frac{\pi}{\beta R} - (1-\chi)}{\chi} \\ u'(C) q_b &= \beta E \left[u'(C')\left[(1-\chi)(r' + (1-\delta)q'_b) + \chi(r' + (1-\delta))\frac{q'_b}{q'_r}\right]\right] \\ \rightarrow q_b &= \frac{\beta \left\{(1-\chi)r + (r + (1-\delta))\left[\frac{\pi}{\beta R} - (1-\chi)\right]\right\}}{1-\beta(1-\chi)(1-\delta)} \\ q_r &= \frac{q_b}{\frac{q_b}{q_r}} = \frac{\beta \chi \left\{(1-\chi)r + (r + (1-\delta))\left[\frac{\pi}{\beta R} - (1-\chi)\right]\right\}}{[1-\beta(1-\chi)(1-\delta)]\left[\frac{\pi}{\beta R} - (1-\chi)\right]} \\ K' &= (1-\delta)K + I \rightarrow I = \delta K \\ \phi_s &= \phi_b^{\frac{\gamma-1}{\gamma}}(\xi)^{\frac{1}{\gamma}} \\ q_b &= q + \frac{\kappa_b}{\phi_b} \rightarrow q = q_b - \frac{\kappa_b}{\phi_b} \end{split}$$

$$q_s \equiv q - \frac{\kappa_s}{\phi_s} \to q_s = q - \frac{\kappa_s}{\phi_s}$$

To check for consistency of the initial guess, we solve the following five equations:

$$C + q_b S + \eta B_r + \left[(1 - \chi) + \chi \frac{q_b}{q_r} \right] \tau = wL + \left[(1 - \chi)(r + (1 - \delta)q_b) + \chi(r + (1 - \delta))\frac{q_b}{q_r} \right] S$$
$$+ \left[(1 - \chi) + \chi \frac{q_b}{q_r} \right] RB_r + \Pi$$
$$I = \chi \frac{\left[r + (1 - \delta)\left(\phi_s q_s + \lambda\left(1 - \phi_s q_s\right)\right)\right] S + R\frac{B}{p} - \tau}{1 - \phi_s q_s}$$

$$q_r \equiv \frac{1 - \phi_s q_s}{1 - \phi_s}$$
$$q = 1 + \kappa_s + \frac{\kappa_b}{\phi_b} \left(\frac{\phi_s}{1 - \omega} - 1\right).$$

C Tables

Table 2: Baseline calibration

	Parameter	Value	Target/Source
Preferences			_ ,
Households' discount factor	β	0.99	standard
Relative Risk aversion	σ	1	log utility
Utility weight on leisure	μ	2.7	working time: 30%
Inverse Frisch elasticity of labour supply	ν	1	Del Negro et al. (2011)
Mass of entrepreneurs	χ	0.05	Shi (2012)
Intermediate goods production	, .		
Capital share of output	α	0.33	standard
Elasticity of substitution	θ	6	Faia (2010)
Price adjustment costs	ζ	29	Faia (2010)
Capital goods production	Ŭ		
Depreciation rate of capital	δ	0.025	standard
Search and Matching			
Matching efficiency	ξ	0.411	sales rate
Buyer search costs	κ_b	0.055	bid-ask spread
Seller search costs	κ_s	0.011	liquidity share
Supply sensitivity of matching	γ	0.5	endogenous
Bargaining weight of sellers	ω	0.883	Tobin's q
Government			
Government consumption	g	0	exogenous
Sensitivity of unconv. policy rule	$ar{\psi}_{m{k}}$	-0.1	exogenous
Shock processes			
Persistence, productivity shock	ρ_z	0.9225	standard
Std. dev., productivity shock	σ_z	0.01	1%
Persistence, liquidity shock	$ ho_{\phi}$	0.9225	exogenous
Std. dev., liquidity shock	σ_{ϕ}	0.01	1%

Notes: The model is calibrated for quarterly data.

Table 3: Selected Moments: Data vs. Model

Moment	Concept	Data	Model
Search Market Sales Rate	ϕ_s	24.7%	24.7%
Search Market Bid-ask Spread	$\frac{q_b - q_s}{q_b}$	1.56%	5.4%
Tobin's q	$\overset{r_{0}}{q}$	1.05 - 1.21	1.1
Liquidity Share	$\frac{B}{Y}$	52.07%	52.07%

Notes: to be completed



Figure 5: Impulse responses after a negative productivity shock (one percent) with monopolistic competition only.



Figure 6: Transition paths after a permanent negative matching efficiency shock (one percent).