The role of technology in saving the climate: Boundless or rebound?

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Abstract

How to break the global trend of rising CO_2 emissions from fossil-fuel consumption? We build a general equilibrium model of long-run economic development, and test the power of different versions of the model to explain historical observations. The results suggest that subsidizing energy efficiency will be ineffective due to endogenous shifts in consumption patterns, while subsidies to development of clean technologies may be either futile or unnecessary. The only policy which can achieve the goal is one in which the relative price of fossil fuels—and fossil-based products—increases.

Keywords: climate change, natural resources, technology, directed technological change, rebound, lock-in

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1. Introduction

How to break the global trend of rising CO_2 emissions from fossil-fuel consumption? The standard answer is to raise the price of such emissions through taxes or quotas. However, this is politically difficult for a number of reasons; for instance, energy taxes on consumers are frequently unpopular and may be regressive, and energy taxes on producers risk driving energy-intensive industries abroad and hence having only a limited effect on global emissions; furthermore, politicians fear that such taxes will have a negative impact on economic growth and employment. Under these circumstances there is an intensive search for other solutions, such as the promotion through subsidies or tax-breaks of energy-efficient technology, and the promotion of alternatives to fossil fuel, such as nuclear power or renewables. Can such policies be effective substitutes for making firms pay for each unit of pollution they emit? In order to answer this question we abstract from problems of negotiation and agreements, and build a general equilibrium model of economic growth and resource use in the long run, treating the global economy as a single decentralized market. How has this economy developed historically with regard to natural resources, and what can this development teach as about the likely effect of future policies?

Traditionally, the literature on growth and resources in the long run has focused on adaptation to future scarcity. In this literature one mechanism has received by far the most attention:

DHSS Given a lower supply of resources, the quantity of capital may increase to compensate.

Intuitively, more expensive (capital-intensive) production equipment may be more efficient w.r.t. resource use. This mechanism is the focus of the Dasgupta–Heal–Solow–Stiglitz (DHSS) model, an extension of the neoclassical growth model to account for

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the role of scarce resources in production. Developed in 1974—see Dasgupta and Heal (1974), Solow (1974a,b), and Stiglitz (1974)—it remains important today; for instance, it is the standard model used in integrated assessment models such as Nordhaus' RICE model (see Nordhaus, 2010). Nevertheless, we argue that although the DHSS mechanism may explain cross-country differences (see e.g. Atkeson and Kehoe, 1999), it is not capable of explaining long-run trends. A model which allows only substitution across a fixed menu of capital goods is not sufficient in a long-run context with economic growth and radical changes in the technologies (and hence capital goods) which are available.

Prior to the development of the DHSS model, Solow (1973)—in an essay where he is unconstrained by the need for mathematical formalism—sets out three mechanisms as key to the adaptation of the economy to resource scarcity. He does not rank the mechanisms in importance, but ironically the DHSS mechanism—Solow's focus in subsequent quantitative modelling—is not mentioned at all. The mechanisms are:

1. Increase—through technological change—resource efficiency in production of one or more product categories;
2. Substitute on the consumption side away from goods which are intensive in the resource, towards other goods;
3. Substitute on the production side away from processes which are intensive in the resource, towards other processes.

We claim, following Solow, that these three mechanisms are key to understanding how the economy adapts in the long-run to changes in resource availability—whether increases or decreases in scarcity—and to policy measures regarding mineral or energy resources. Furthermore, they interact with one another and must therefore be analysed together; for instance, if Mechanism 2 is important, it will tend to negate the effect on resource demand of Mechanism 1, since increases in resource efficiency reduce the price of resource-intensive goods, leading to substitution towards such goods in consumption.

What analysis can we find of these mechanisms in the literature, either together or in isolation? The answer is that they are rarely analysed explicitly in quantitative models, and even more rarely tested in isolation, let alone in combination.

Regarding Mechanism 1, there is of course an enormous literature on resource efficiency. However, how seriously has the long-run process of efficiency increase been analysed at the aggregate level, and how thoroughly have the models been tested? The answer seems to be, scarcely at all. In the context of long-run economic growth driven by efficiency increases, resource efficiency must increase more rapidly than the efficiency of other inputs if there is to be downward pressure on resource use. (If for instance resource efficiency increases more slowly than labour efficiency then—ceteris paribus—resource use per capita will increase.) A change in the relative productivity of different inputs is known as directed technological change (henceforth DTC). The methodology for modelling DTC was to a great extent developed by Acemoglu (see for instance Acemoglu, 2002), and the large majority of papers incorporating DTC have used variations of Acemoglu's approach. In the resource and energy literature, by far the most common approach has been to assume that these relative productivities develop independently, and that if a constant number of researchers (or in some cases a constant proportion of final-good production) is devoted to factor-specific R&D, the growth rate of productivity of that factor will be constant, irrespective of technological progress in other areas. The result is that a permanent increase in research effort directed towards a particular factor leads to a permanent increase in the growth rate of knowledge augmenting that factor, and (for instance) a permanent drop in the flow rate of energy will have no long-run effect on the economy in terms of growth rate and allocation of resources; after a period of extra investment in energy-augmenting knowledge, everything returns to normal. Examples of authors using this form of knowledge production function are Smulders and de Nooij (2003), Gerlagh (2008), Fischer and Newell (2008), and Gans (2012), following a tradition started by Kennedy (1964) in his analysis of the growth of technology augmenting capital and labour.

Consider now Mechanism 2, substitution on the consumption side. The idea that substitutability between goods of differing resource-intensity may play an important role in determining long-run resource demand has been around for a long time. Jevons (1865) argued that future scarcity of coal would be exacerbated, not alleviated, by innovations increasing the efficiency of technologies based on coal use (Mechanism 1), due to both increasing demand for consumption goods which are coal-intensive (Mechanism 2) and a switch to the use of coal as an input, substituting for other inputs (Mechanism 3). The idea that resource-intensive goods are highly substitutable in consumption for non-resource-intensive goods has been discussed more recently by energy and ecological economists (see for instance Binswanger, 2001, and citations), and it has been noted—following Jevons—that such substitutability will tend to negate the downward pressure on resource consumption exerted by efficiency increases, since such increases lead to a decline in the relative price of resource-intensive goods. This process has been named the rebound effect. This discussion has not been picked up in the general economics literature, perhaps due to a focus on one-sector models in which Mechanism 2 is ruled out by definition. Nevertheless, Knittel (2011) finds highly relevant evidence of efficiency increases combined with substitution towards resource-intensive consumption categories in the U.S. automobile industry.

Regarding Mechanism 3, there is no question that there is a very high degree of substitutability between different resource inputs. For instance, electricity from renewable sources or nuclear fission is highly substitutable for electricity from fossil-fuel burning, and different materials—iron and aluminium, bricks and concrete—are frequently substitutable for one another. The key question is instead the degree to which the relative demand for such substitutable inputs may change over time as a result of changes in relative productivity; DTC is thus at the heart of the matter again. Theoretically, two extreme cases present themselves. In the first case DTC is exogenous or even non-existent: growth in the productivities of substitutable input factors is determined exclusively by overall technological progress, and is thus independent of factor-specific R&D. This is for instance the assumption made by Acemoglu and Guerrieri (2008)—investigating the reasons for the constant shares of capital and labour over time—who assume that capital- and labour-augmenting technology both grow at constant, exogenous rates. The opposite extreme is to assume—as do Acemoglu et al. (2012)—that at any instant the relative growth rates of alternative factor-augmenting technologies are purely a function of relative factor-specific investments. Since relative investments follow relative factor shares—a result demonstrated by Hart (2013)—the result of this assumption is that the productivity of the initially dominant input increases, hence the dominance of that input increases, and the economy is 'locked in' to use of that input.[1] Such lock-in sounds bad, but actually this is a highly optimistic scenario, as shown by Acemoglu et al. (2012): In an economy locked in to use of a dirty input, a regulator can achieve a transition to an alternative clean input simply by promoting investment in the clean alternative over a limited period, up to the point at which the clean alternative becomes cheaper than the dirty one; from that point onwards no further policy interventions are necessary since increasing returns send the economy towards a 'clean corner'.

Continuing with Mechanism 3, there are of course more general alternatives to the two extremes above, alternatives in which directed technological change is endogenous but the relative growth rates of factor-augmenting knowledge stocks are not just a function of relative investment rates; for instance, progress may be a decreasing function of the current knowledge stock relative to general knowledge. It is hard to argue

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[1] The idea is an old one—see for instance Arthur (1989) among others—and the idea that we are 'locked in' to fossil-fuel use by history dates at least to Unruh (2000).

against the claim that the productivity of an input must in some way be linked to endogenous investments in R&D into that input; on the other hand, it is obvious that such research may build on general knowledge, not just existing input-specific knowledge. The idea that stocks of knowledge augmenting different inputs grow independently of one another was criticized by Nordhaus (1973), who argued that the relative ease of innovation augmenting different inputs—in his words, the shape of the innovation possibility function (IPF)—is likely to change over time. The argument is echoed by Acemoglu (2002), who discusses the IPF in his concluding remarks, highlighting the need for more research. This need remains acute, as argued by Hart (2013).

Finally we take a closer look at three recent papers on innovation and climate change policy: Fischer and Newell (2008), Gans (2012), and Acemoglu et al. (2012). Fischer and Newell (2008) develop a two-period partial equilibrium model which is therefore not capable of analysing the kinds of long-run effects—such as lock-in and path dependence—which are a crucial part of our focus here. Gans (2012) focuses on general-equilibrium effects, showing (in the context of the model) that energy taxes may *reduce* energy-augmenting technological progress, since they cause the absolute size of the energy sector to shrink. Unfortunately all the results can be traced back to the assumption of a Cobb–Douglas aggregate production function in which the factor share of energy is fixed. Given this assumption—not discussed—anything reducing overall production will also reduce the absolute size of the energy sector, driving down research incentives. However, we know that in the short run the Cobb–Douglas formulation is a very poor description of the energy sector, since the short-run elasticity of substitution between energy and other inputs is very low, hence a binding cap on fossil-fuel use would in fact raise both the factor share of energy and the absolute level of factor payments to energy, raising research incentives. Cobb–Douglas may then emerge at the long-run aggregate level as a result of the process of directed technological change, as we demonstrate below (and also shown by for instance Kennedy (1964)). Finally, Acemoglu et al. (2012) build a general equilibrium model closest in spirit to ours. However, there is a unique final good, hence no scope for Mechanism 2. Furthermore, there is nothing corresponding to Mechanism 1, since overall resource or energy efficiency is not a variable in the model; there are only factor-specific efficiency levels. Thus the focus is exclusively on Mechanism 3, where they assume that the knowledge stocks grow independently of one another, thus generating path dependence. The plausibility of this assumption is not evaluated.

We build a model which encompasses Solow's three mechanisms, and evaluate the mechanisms based on historical data. The essence of the model is as follows. Agents consume two aggregate products, based on labour and resources respectively; the aggregates are made up of production by individual firms each producing unique products based on product-specific technologies; resource-intensive firms may choose between alternative (substitute) resource inputs.

The key to the parameterization of the model is twofold: Firstly, the degree to which productivity of different inputs is a function of factors other than cumulative investment in knowledge related specifically to those inputs, and secondly consumers' elasticity of substitution between the labour- and resource-intensive aggregates. If productivity is purely a function of cumulative investment then Mechanisms 1 and 3 are strong, while if there is a high elasticity of substitution on the consumption side then Mechanism 2 is strong. All of the parameterizations are consistent with historical data concerning global resource prices and aggregate consumption rates. However, they predict different observations with regard to historical changes in final-good consumption patterns and growth in resource productivity, and with regard to historical cases where one resource has substituted for or supplanted another. We use these differences to evaluate the plausibility of the parameterizations and hence draw tentative policy conclusions.

We conclude there has been—over the last 100 to 200 years—very rapid growth in energy-augmenting knowledge for particular products, combined with very high growth rates in consumption of these products; the overall result has been that primary energy consumption has tracked global product. The data thus suggest that there has been a powerful rebound effect at the aggregate level, and the policy conclusion is therefore that policy measures to boost energy-efficiency R&D are likely to have at best a moderate downward effect on global energy consumption, even if they are highly successful in boosting efficiency. Regarding substitution between resource inputs, we find persuasive evidence that such substitution has occurred rapidly in the past, driven not by policy but by underlying technological progress—in the language of Nordhaus (1973), changes in the innovation possibility function. That is, there is a natural progression from one resource to another where the later resources are intrinsically more productive but also demand a higher level of underlying technology before they can be used. We thus argue against path dependence and chance as key factors in the succession from one technology to another, and raise the possibility that regulatory intervention to boost investment in knowledge augmenting a clean resource will frequently be either unnecessary (if the fundamentals are favourable to it) or futile (if they are unfavourable). As above, the only successful policy is likely to be one which raises the price of dirty resource inputs.

The remainder of the paper is structured as follows. In Section 2 we set out stylized facts about growth and resource use with which we require our model to be consistent. In Section 3 we set out the basic model, and analyse the economy without substitute resource inputs, and in Section 4 we extend the model and focus instead on substitution between resource inputs. In Section 5 we discuss further extensions to the model. Section 6 concludes.

2. Evidence about growth and resources

Here we discuss observations about long-run growth and resource demand which may be used to support or oppose economic models of the growth process. First we briefly consider the growth process, then we consider non-renewable resources and growth. We claim that growth is driven by continual transformation of the production system rather than by growth in availability of any particular factor. We can verify this in many ways, where perhaps the simplest is by consideration of the composition of consumption over time. Jones (2005) shows that steady growth has left U.S. GDP/capita more than 10 times higher today than in 1870; has there also been steady growth in consumption rates of individual products? Consider car ownership. In 1870 there were no cars. Car ownership subsequently grew rapidly, but between 1970 and 2008 it was constant at 0.44 per capita.[2] Meanwhile, ownership of home computers and mobile phones was effectively zero in 1970, whereas today it is more-or-less universal. Now, do we consume cars and smartphones today because we work longer hours, or have saved up more capital, but with the same skills and the same machines as we had in 1870? Obviously not: It is the arrival of new knowledge and new products which drives long-run growth.

Concerning long-run resource demand, we present evidence supporting the following two stylized facts.

SF1 The prices of resources—when broadly defined to include substitutes in the same category—have tended to be constant, while consumption has tracked global product. The long-run factor share of resources has thus remained constant.

SF2 There may be very large shifts in the shares of substitutable resources; in particular, when a new resource appears on the global market there may be an initial period during which its factor share increases rapidly before levelling off in the long run.

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[2] Sources: Population data from US census, and car-ownership data from the Bureau of Transportation Statistics.

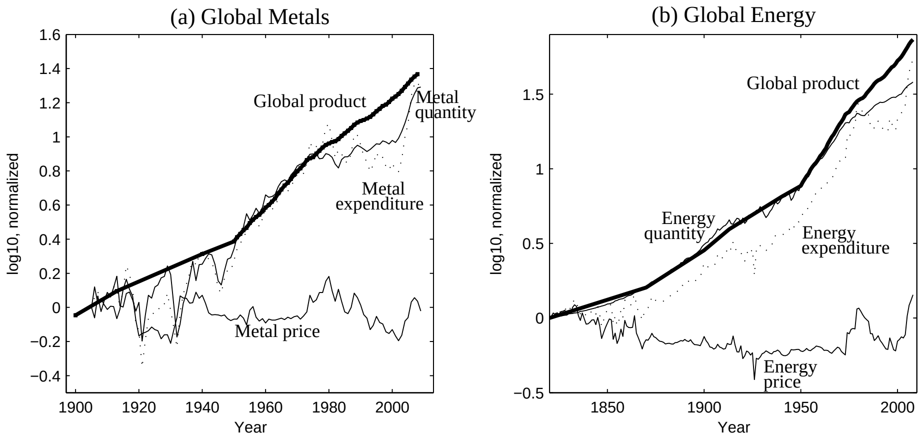

Figure 1: Long-run growth in consumption and prices, compared to growth in global product, for (a) Metals, and (b) Primary energy from combustion.

Note: Global product data from Maddison (2010). Metals: Al, Cr, Cu, Au, Fe, Pb, Mg, Mn, Hg, Ni, Pt, Rare earths, Ag, Sn, W, Zn. All metals data from Kelly and Matos (2012). Energy: Coal, oil, natural gas, and biofuel. Fossil quantity data from Boden et al. (2012). Oil price data from BP (2012). Coal and gas price data from Fouquet (2011); note that these data are only for average prices in England; we make the (heroic) assumption that weighted average global prices are similar. Biofuel quantity data from Maddison (2003). Biofuel price data from Fouquet (2011); again, we assume that the data are representative for global prices, and we extrapolate from the end of Fouquet's series to the present assuming constant prices. The older price data is gathered from historical records and is not constructed based on assumptions about elasticities of demand or similar. Sensitivity analysis shows that the assumptions are not critical in driving the results.

Regarding SF1, consider Figure 1. In Figure 1(a)—similar to that in Hart (2013)—we show data for an aggregate of the 17 most important metals measured by factor expenditure (not including uranium). The results are striking: Over a period of more than 100 years, growth in global consumption of metals almost exactly tracks growth in global product, whereas the real price is almost exactly constant. The result is that the share of metals in global product is also constant, since expenditures (the product of price and quantity) track global product. In Figure 1(b) we see the same result for global primary energy supply through combustion. Again, quantity tracks global product while the weighted energy price is almost constant. After a slight decline in the early 19th century, the factor share of energy remains almost constant, although in the short run it is tightly linked to price changes.[3]

Figure 1 hides large degrees of substitution between resources in the same category: SF2. For example, the factor share of aluminium has risen dramatically, while the share of lead has fallen; in the energy sector, as hinted at above, the share of oil has rocketed. In Figure 2 we illustrate global trends for two pairs of substitutable resources, where one of each pair is established at the start of the time period (coal, iron) and the other is emerging (crude oil, aluminium). From the initial point, in both cases the relative price of the emerging resource falls steeply over the first 20 years, and subsequently relative prices are rather constant. On the expenditure side, in both cases the established resource maintains its share of global product over the period, whereas the share of the emerging resource rises rapidly for around 50 years, and then more-or-less retains its share of global product subsequently. Note that the relative levels of the two expenditure curves are meaningful in the figure, so expenditure on oil overtakes coal in 1950, whereas aluminium catches up with iron ore at around the same time.

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[3] Note that in Figure 1 we show log-normalized data to emphasize relative growth rates, but what is the *level* of the factor share of resources compared to labour and capital? Based on the above data sources, the factor share of resources is significant but not large. For instance, the factor share of crude oil in the global economy in 2008 was 3.6 percent, whereas the factor share of the 17 major metals was just 0.7 percent. Note also that what we present as factor costs do not all go to the factor in a general equilibrium sense: Payments to capital and labour make up a large part of the costs of mineral and energy resources.

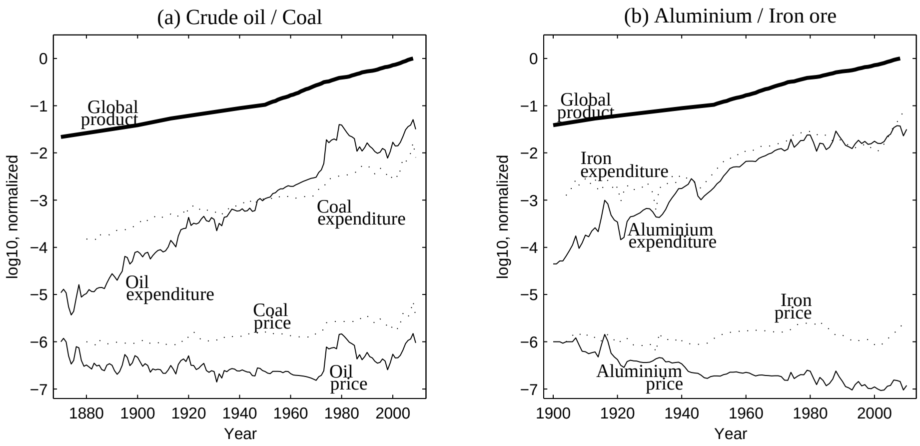

Figure 2: Long-run growth in prices and factor expenditure, compared to growth in global product, for crude oil and aluminium.

Note: Prices normalized to start at −6. Oil and aluminium expenditures normalized to end at −1.5. Coal and iron expenditures correct relative to oil and aluminium. Data sources as for Figure 1, except that we have taken coal price data from Slade (1982) and BP (2012), with a gap from 1977 to 1986. To talk of a global coal price prior to the 1980s is problematic due to limited global trade; however, there is no apparent reason to doubt the picture of rather constant prices up to the 1970s. Fouquet (2011) shows broadly similar price trends, although the level is higher in Fouquet's data, which could be because he consumer prices rather than pithead prices, and due to higher extraction costs in the U.K.

Finally, elasticity of demand. Figures 1 and 2 show clearly that the short-run price elasticity of demand for resources is small (at least for the three resources studied): Drastic price changes lead to small changes in quantity in the short run. Long-run price elasticity of demand is, on the other, not directly observable from the data, although the rather constant levels of expenditure on metals and energy in the very long run are suggestive of a long-run demand elasticity close to one. However, we should be very careful when drawing such conclusions; in the long run many different factors can affect demand, not just relative prices.

3. The model with a single resource: Directed and Rebound economies

In this section we develop the model with a single resource input. We parameterize the model in two ways—denoted the *Directed* and *Rebound* economy—which are both are consistent with SF1. We begin with the model fundamentals, then we present aspects specific to the respective economies, and finally we discuss evidence. We reject the Directed economy as a description of the energy sector based on its predictions regarding energy-augmenting knowledge, and we present evidence on shifting consumption patterns broadly consistent with the Rebound economy.

3.1. Model fundamentals: Endowments, consumers, and producers

At the start of period t there are Q_l agents endowed with one unit of labour each, infinite stocks of a homogeneous open-access resource, and a set of firm-level knowledge stocks $\mathbf{K}_{t-1}$ which are free for all to build on.

The representative consumer in the model has a linear utility function in the aggregate good Y, the price of which is normalized to unity. We therefore have a constant discount factor per period β, determined by the consumers' rate of time preference.

The aggregate good Y is made up of two further aggregate products, Y_l and Y_r, produced by labour and resources (either energy or mineral) respectively. We have

$$Y = \left[\alpha_l Y_l^\rho + \alpha_r Y_r^\rho\right]^{1/\rho}, \tag{1}$$

where ρ is a parameter between 0 (Cobb–Douglas) and $-\infty$ (Leontief) ($1/(1-\rho)$ is the elasticity of substitution between the goods), and α_l and α_r are parameters. By comparing marginal productivities we can obtain

$$\frac{p_l Y_l}{p_r Y_r} = \frac{\alpha_l}{\alpha_r}\left(\frac{Y_l}{Y_r}\right)^{\rho}, \tag{2}$$

where p_l and p_r are the prices of the aggregate products. For the purpose of illustration we can think of the two categories (labour- and resource-intensive) as services and manufactures respectively.

Each aggregate is in turn made up of a unit mass of individual products:

$$Y_l = \left(\int_0^1 y_{li}^{\eta} \mathrm{d}i\right)^{1/\eta}, \qquad Y_r = \left(\int_0^1 y_{rj}^{\eta} \mathrm{d}j\right)^{1/\eta}.$$

So in symmetric equilibrium $Y_l = y_l$, where y_l is production by the representative firm in sector L. Below we show how the mass of products is an endogenous outcome of the model, based on a zero-profit condition in production.

Now to production. Aggregate services Y_l are produced exclusively using labour L, while aggregate manufactures Y_r are produced exclusively using resources R. At the start of a given period, economic agents have the option of starting firms i or j, using inputs L and R respectively, and making products y_{li} and y_{rj}; each firm produces a single, distinct product with its own technology. The production functions for products i and j are

$$y_{li} = \gamma_l k_{li} q_{li}; \tag{3}$$

$$y_{rj} = \gamma_r k_{rj} q_{rj}. \tag{4}$$

Here γ_l and γ_r are productivity indices for the respective inputs, k_{li} is the quality of the firm's ideas applied to augmenting input L in production of good i (with the corresponding interpretation for k_{rj}); and q_{li} is the quantity of factor L hired by firm i (with the corresponding interpretation for q_{rj}). Note that if y_{li} has units *widgets per year*, and q_{li} has units *workers*, then γ_l has units *widgets per year per worker per idea* and k_{li} has units *ideas*. Furthermore, we denote $\gamma_l k_{li} q_{li}$ and $\gamma_r k_{rj} q_{rj}$ as the quantities of the *augmented* inputs.[4]

The physical inputs L and R (labour and the resource) are hired/bought by firms on competitive markets, and we have the restrictions (assuming that all available inputs are used) that

$$\int_0^1 q_{li} \mathrm{d}i = q_l = Q_l, \quad \int_0^1 q_{rj} \mathrm{d}j = q_r = Q_r, \tag{5}$$

where q_l and q_r are quantities used by the representative firm in a symmetric equilibrium, and Q_l and Q_r are the total quantities of the inputs.

The resource is extracted by perfectly competitive firms using final goods as inputs, with the following function linking aggregate extraction inputs X_r and aggregate extraction Q_r:

$$Q_r = X_r / w_r. \tag{6}$$

The resource price is thus w_r. Use (2) and the fact that revenue equals costs for the aggregate products to write

$$\frac{w_l q_l}{w_r q_r} = \frac{\alpha_l}{\alpha_r}\left(\frac{y_l}{y_r}\right)^{\rho}. \tag{7}$$

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[4] Note that the parameters γ_l and γ_r are important when considering knowledge spillovers between sectors. Under these circumstances we want to know the number or quality of the ideas possessed by firm i, k_{li}, not the productivity of the input $\gamma_l k_{li}$.

Now consider investment. Consider an agent planning to start a firm making product i in period t. (The problem for firm j is symmetric.) There is free entry and we assume Nash equilibrium, so in equilibrium each firm which enters will make zero profit and be unable to make higher profits by changing its choices. To enter, the agent must first hire research labour quantity z_{lit} (or z_{rjt} in the other sector) to design the product and manufacturing process, which determines k_{lit} (or k_{rjt}). This knowledge is built on the set of all existing knowledge in the economy, $\mathbf{K}_{t-1}$:[5]

$$k_{lit} = F_l(\mathbf{K}_{t-1}) z_{lit}^{\phi} / \zeta_l; \quad k_{rjt} = F_r(\mathbf{K}_{t-1}) z_{rjt}^{\phi} / \zeta_r. \tag{8}$$

Here ϕ is a parameter between 0 and 1, ζ_l and ζ_r are positive parameters, and F_l and F_r are non-decreasing functions of each of their arguments, and homogeneous of degree one. The investment good—research labour—is hired on perfect markets, and available in an exogenous quantity Z, hence we have the restriction[6]

$$\int_0^1 z_{li} \mathrm{d}i + \int_0^1 z_{rj} \mathrm{d}j = Z. \tag{9}$$

Return to firm i in period t. Since firms only last one period, we can set up a simple, static, profit-maximization problem. The firm maximizes revenue minus the cost of research and production inputs, hence we have

$$\max_{q_{lit}, z_{lit}} \pi_{it} = p_{lit} y_{lit} - w_{lt} q_{lit} - w_{zt} z_{lit}, \tag{10}$$

where w_l is the market price of production labour, and w_z is the market price of research labour. Note also that p_{lit} is a decreasing function of y_{lit}, y_{lit} is a linear function of k_{lit} and q_{lit}, and k_{lit} is an increasing function of z_{lit}, as specified above. The first-order conditions in q_{lit} and z_{lit} yield (dropping the time subscript) that

$$w_l q_{li} = \eta p_{li} y_{li}; \tag{11}$$

$$w_z z_{li} = \eta \phi p_{li} y_{li}. \tag{12}$$

In other words, firm i spends a fraction η of its revenue on hiring labour, and a fraction $\eta\phi$ of its revenue on research; total costs are a fraction $\eta(1+\phi)$ of total revenue.

Since there is free entry profits must be zero, hence $\eta(1+\phi) = 1$. To see how this can be an endogenous outcome, let the mass of firms (previously assumed to be one) be a variable, n_l. Furthermore, let $\eta = \eta_l$, and assume that: (*i*) η_l is an increasing function of n_l; (*ii*) when $n_l = 0$, $\eta_l = 0$; and (*iii*) when $n_l \to \infty$, $\eta_l \to 1$. Then there is some value of n_l for which $\eta_l(1+\phi) = 1$, and this is the mass of firms which will form in equilibrium. We normalize this equilibrium mass of firms to one, in both sectors.

Now assume symmetry such that we can drop subscripts i and j and instead consider prices and quantities of the representative firm in each sector. Take equations 11 and 12 and the corresponding expressions for firm j to show that

$$\frac{z_l}{z_r} = \frac{w_l q_l}{w_r q_r}; \tag{13}$$

Relative investment rates in factor-augmenting knowledge are in proportion to the relative shares of the factors.[7]

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[5] Note that the set $\mathbf{K}_{t-1}$ may include L-augmenting knowledge, R-augmenting knowledge, and basic knowledge or knowledge about *general purpose technologies*.

[6] Note that technically, in the model, we make investment simultaneous with production and consumption, hence the discount rate plays no role in the model. We could easily assume that researchers were paid using final goods produced in the preceding period, which would raise the cost of research by a factor $1/\beta$. If we also made the number of researchers endogenous this would make the growth rate a decreasing function of the rate of time preference.

[7] Note that this is in accordance with the theoretical model of Hart (2013).

Equation 13—given the knowledge production functions 8—is the key result determining the evolution of knowledge stocks and hence the economy, and this evolution is our main interest in this paper. Nevertheless, we should check that there is a unique equilibrium in the economy each period, as defined below. We do this in Appendix Appendix A.1.

Definition 1. *An equilibrium in the economy is defined as an allocation of resources in which the markets for the resource, production labour, and research labour clear, and each production firm makes zero profits while optimizing its use of inputs given its choice of product. Furthermore, each firm is satisfied with its choice of product given the choices made by the other firms.*

3.2. The Directed and Rebound economies

To fully characterize the development of the economy we must specify the knowledge production functions (8) and set parameters. The key to the two economies—Directed and Rebound—lies in the knowledge production function and the parameter ρ determining the elasticity of substitution between labour- and resource-intensive goods Y_l and Y_r. The problem is how to explain SF1: Why, when the price of resources falls relative to labour, does the factor share of resources not fall?

Economy 1a. Directed. *Mechanism 1 is strong while Mechanism 2 is weak, hence SF1 is explained by the failure the resource efficiency of production k_r to rise, despite the rise in labour productivity k_l. In this economy, subsidies to research into increasing k_r will lead to aggregate reductions in resource use.*

Economy 1b. Rebound. *Mechanism 2 is strong, hence it will negate potential resource savings from Mechanism 1, and SF1 is explained by substitution towards resource-intensive goods y_r. In this economy, declining resource consumption can only be achieved by raising the relative price of resource inputs.*

In the Directed economy, the knowledge production functions (8) are specified such that the knowledge stocks of the representative firms k_l and k_r grow independently, while the elasticity of substitution between Y_l and Y_r is low, i.e. ρ is large and negative. Consider equations 1, 3 and 4. Assume symmetry and focus on the representative firms to obtain the overall production function

$$Y = [\alpha_l(\gamma_l k_l q_l)^\rho + \alpha_r(\gamma_r k_r q_r)^\rho]^{1/\rho} . \tag{14}$$

Since ρ is large and negative we thus have low short-run elasticity of substitution between L and R, thus accounting for the very low short-run elasticity of demand for resources observed in the data. How then can this model yield the unit long-run elasticity which is also observed? The key is directed (or biased) technological change.

To specify our model of DTC, we first specify how individual firms' knowledge aggregates to an overall stock of general knowledge. To do so we define stocks of L- and R-augmenting knowledge as follows:

$$K_{lt} = \left(\int_0^{n_l} k_{lit}^\xi \mathrm{d}i\right)^{1/\xi} ; \quad K_{rt} = \left(\int_0^{n_r} k_{rjt}^\xi \mathrm{d}j\right)^{1/\xi} . \tag{15}$$

Here ξ is a parameter greater than or equal to one. If $\xi = 1$ there is no overlap between firms' knowledge, so general knowledge is the sum of firm knowledge; as $\xi \to \infty$, overlap becomes perfect and general knowledge is just the most knowledgeable firm's knowledge. Recall also that n_l and n_r are both equal to one in equilibrium, and that we assume symmetry. Thus, in equilibrium, $K_{lt} = k_{lt}$ and $K_{rt} = k_{rt}$.

Having defined these knowledge stocks, we now need to specify how next-period firm-level knowledge builds on these stocks. In the Directed economy we assume independent knowledge stocks as defined by Hart (2013):

Definition 2. Independent knowledge stocks. *Knowledge stocks are independent when the production functions for firm knowledge, (8), can be written*

$$k_{lit+1} = F_l(K_{lt}) z_{lit+1}^{\phi} / \zeta_l; \quad k_{rjt+1} = F_r(K_{rt}) z_{rjt+1}^{\phi} / \zeta_r.$$

Since $k_{lt} = K_{lt}$ in equilibrium—and total research inputs Z are constant—the functions F_l and F_k must be linear, and (8) becomes, in equilibrium,

$$k_{lt+1} = k_{lt} z_{lt+1}^{\phi} / \zeta_l; \quad k_{rt+1} = k_{rt} z_{rt+1}^{\phi} / \zeta_r. \tag{16}$$

Combine these equations with the production functions 3 and 4, and with the condition on relative investments 13, to obtain

$$\frac{k_{lt+1}/k_{lt}}{k_{rt+1}/k_{rt}} = \frac{\alpha_l^{\phi}/\zeta_l}{\alpha_r^{\phi}/\zeta_r} \left(\frac{\gamma_l k_{lt} q_{lt}}{\gamma_r k_{rt} q_{rt}} \right)^{\rho\phi}. \tag{17}$$

Now use this to prove the following proposition.

Proposition 1. *Assume that the labour force grows exogenously by a constant factor* θ_{ql} *per period, while the resource price grows exogenously by a constant factor* θ_{wr} *per period. Then there exists a stable balanced growth path (b.g.p.) along which* q_r *and* w_l *grow at constant rates such that* $\theta_{wl}\theta_{ql} = \theta_{wr}\theta_{qr}$*, and the factor share of resources is constant.*

PROOF. Assume the economy is on a b.g.p., implying that the growth rates of k_l and k_r are constant (although not necessarily equal to one another). Then (17) implies that $k_l q_l/(k_r q_r)$ is constant, implying in turn that the growth rates of the augmented inputs $\gamma_l k_l q_l$ and $\gamma_r k_r q_r$ are equal. Since the augmented input factors grow at equal rates their shares are constant, hence the growth rates of factor costs are equal: $\theta_{wl}\theta_{ql} = \theta_{wr}\theta_{qr}$. Finally, to show that the b.g.p. exists and is stable consider equation 17: Firstly, for any q_l and q_r we can always find a level of k_l/k_r to obtain any desired growth in the relative knowledge stocks; secondly, the relative growth of the knowledge stocks is a strictly decreasing function of the augmented inputs.

This model thus perfectly explains SF1—i.e. unit long-run elasticity of substitution between resources and other inputs—while also accounting for inelastic short-run demand. The mechanism is straightforward. On a b.g.p., augmented inputs both grow at the overall growth rate of production, and returns to each factor also grow at this rate. So if w_r is constant then q_r must grow at the overall growth rate, and if w_r starts to rise then growth in q_r will fall back.

Now to the Rebound economy. In this specification there is a unit elasticity of substitution between the final goods Y_l and Y_r (i.e. $\rho = 0$). Thus, by l'Hôpital's rule,

$$Y = Y_l^{\alpha_l} Y_r^{\alpha_r}. \tag{18}$$

Then it follows straightforwardly that

$$p_l y_l/(p_r y_r) = w_l q_l/(w_r q_r) = \alpha_l/\alpha_r = z_l/z_r.$$

Hence the factor shares of labour and the resource are fixed; the assumption of Cobb–Douglas utility leads directly to the fixed-share property evident in Figure 1.[8]

Turning to DTC, note that since factor shares are fixed in the Rebound economy, the relative investment rate z_l/z_r will also be fixed (equation 13), hence knowledge stocks k_l and k_r will grow at the same rate in the long run if the knowledge production functions are symmetric, whatever the model of DTC. For instance, we could assume a DTC model in which knowledge stocks are locked together by construction, or a model in which they grow completely independently (as in the Directed economy); in either case, the model would be consistent with the stylized fact of constant factor share.

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[8] Note that the model cannot now explain the low short-run demand elasticity for resources, but the addition of capital (putty–clay) would generate this result without affecting the results in which we are primarily interested.

3.3. Evidence from the energy sector

We now consider evidence from the energy sector regarding the Directed and Rebound economies (1a and 1b), in particular lighting, power generation, and transport; has k_r remained constant (hence falling relative to k_l), or has y_r risen rapidly relative to y_l?

Light is a convenient product category for analysis since light is a consumption good which is rather homogeneous and unchanging over very long timescales, and the energy-efficiency of its production is easily measured. We therefore begin there. Fouquet and Pearson (2006) study light production and consumption in the U.K. over seven centuries. They conclude that the efficiency of light production in the U.K. (measured by lumen produced per watt of energy used) increased 1000-fold from 1800 to 2000; the productivity of labour in the UK over the same period rose by a factor of 12–15 (estimates vary). So, far from lagging behind labour productivity the productivity of light production has far (*far*) outstripped it; this is powerful evidence against the Directed economy. Regarding consumption of light, over the same period Fouquet and Pearson estimate that the price of fuel for lighting fell by a factor of 5, while consumption rose by a factor of 7000. This very large rise in consumption—if attributable to the price elasticity of demand—indicates an elasticity of approximately 0.7, compared to an elasticity of 1 which would be exactly consistent with the Rebound economy. On the other hand, if substitution between product categories were irrelevant then the elasticity should be zero. The data thus support the relevance of the Rebound economy, if not the exact parameterization.

Light production is a convenient sector within which to measure efficiency, but it is not very large. Now we turn to the production of motive power from fossil fuels, a very large sector; we do not consider consumption since motive power is typically not a consumption good *per se*.[9] Historically this concerns the efficiency of steam engines, while over the last century we must consider electric power generation and the internal combustion engine. Regarding steam engines, sources such as Hills (1993) suggest that their efficiency in generating power from coal inputs increased steadily from their invention in the early 1700s up to 1900, and by a factor of around 20 over the entire period; this growth in efficiency is again more rapid than the growth in labour productivity over the same period. Subsequently, the efficiency of coal-fired power stations has continued to increase but at a declining rate; see for instance Yeh and Rubin (2007) for detailed evidence.

Finally, transport. Here the situation is complicated by the fact that the cost of personal transport is not just financial, it is also measured in time. Furthermore, transport varies in quality as well as quantity; faster is, ceteris paribus, better. The result is that rising income is correlated with more rapid forms of transport, and a greater distance travelled per person–year, but not with more time spent travelling. Schäfer (2006) shows that world travel (in terms of person–kilometres travelled per year) has grown approximately in proportion to the increase in global product, which would on the surface fit the picture of no substitution between consumption categories, consistent with the Directed economy. On the other hand, he argues there are simultaneous shifts from public transport to light-duty vehicles to high-speed transport modes (such as flying), shifts which are encouraged by efficiency improvements in the transport sector, yet simultaneously drive up overall energy consumption in that sector. So, efficiency improvements in (for example) passenger flight since the 1920s have clearly driven massive increases in energy consumption within that sector. Finally, Knittel (2011) analyses technological change and consumption patterns in the U.S. automobile industry, and shows that—for a vehicle of fixed characteristics in terms of weight and engine power—fuel economy would have increased by 60 percent over the period 1980–2006

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[9] Note that this includes electricity generation, since the key step in generating electricity from fossil fuels is the generation of motive power.

due to technological change; this is approximately on a par with increases in labour productivity, and thus not consistent with the DTC story. Furthermore, he also shows that actual average fuel economy increased by just 15 percent, the difference being due to countervailing increases in the weight and power of vehicles. Thus we have efficiency improvement leading to a fall in unit costs of energy services and hence an increase in consumption of these services: Rebound.

4. The model with a substitute resources: Lock-in and Fundamentalist economies

In this section we focus on substitution between alternative resource inputs. We require the model to be consistent with SF2 of Section 2, i.e. that there may be large shifts in the shares of substitutable resources, and that when a new resource appears on the global market there may be an initial period during which its factor share increases rapidly before levelling off in the long run. We parameterize the model in two ways —denoted the *Lock-in* and *Fundamentalist* economies—and we show that only the Fundamentalist economy is consistent with SF2.

4.1. The model of the resource sector

The key to the model of this section is substitution between alternative resource inputs. In focusing on this substitution we simplify the relationship between the labour-intensive and resource-intensive sectors; in effect we build a partial-equilibrium model of the resource sector, nested in the overall general equilibrium model. This simplifies the model greatly, at the expense of the loss of second-order effects. We are interested in the first-order effects of competition between alternative resources, not the (small) knock-on effects on overall growth and wages.

We assume that the resource-intensive sector is small in the sense that neither the wage to researchers nor the growth rate of general knowledge is affected by changes in the resource-intensive sector. Furthermore, we make the simplest possible assumption about the utility function that is reasonably consistent with the evidence presented above; that is, we assume Cobb–Douglas. Thus we have (from equation 1)

$$Y = y_l^{\alpha_l} y_r^{\alpha_r},$$

where $\alpha_l \gg \alpha_r$. Furthermore, since the labour-intensive sector is very large compared to the resource-intensive sector, we assume that its development is unaffected by changes in the resource-intensive sector: It is assumed to be on a b.g.p. along which y_l, w_l, and w_z all grow by a constant factor θ per period. Finally, we approximate the growth rate of Y as being equal to the growth rate of y_l. Returns to the resource sector also grow by θ per period, since $p_r y_r = \alpha_r Y$. And because of the equal returns, Y/w_z is constant; we define $Y/w_z = \bar{Y}$.[10]

Now assume two coexisting resources, D and C. (Think of them as *dirty* and *clean*, or in the case of energy *fossil* and *renewable*.) The resources are imperfect substitutes in a CES production function—the elasticity of substitution is $1/(1-\varepsilon)$, where $\varepsilon \in (0,1)$ —hence both are used by each firm in sector R. For the representative firm we have

$$y_r = \left[(\gamma_d k_d q_d)^{\varepsilon} + (\gamma_c k_c q_c)^{\varepsilon}\right]^{1/\varepsilon}. \tag{19}$$

The two resource inputs are produced (or extracted) according to equations analogous to (6), hence their prices w_c and w_d are exogenous to the production sector. Putting this information together we can set up the representative firm's optimization problem as follows:

$$\max_{q_d, q_c, z_d, z_c} \pi = p_r \left[(\gamma_d k_d q_d)^{\varepsilon} + (\gamma_c k_c q_c)^{\varepsilon}\right]^{1/\varepsilon} - (w_d q_d + w_c q_c) - w_z (z_d + z_c), \tag{20}$$

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[10] Given the growth factor θ we have $\bar{Y} = (\zeta_l \theta)^{1/\phi}/(\eta\phi)$.

subject to knowledge production functions which are analogous to (8). Note that since the price of research labour w_z is determined in the labour-intensive sector, w_z in equation 20 is exogenous to the behaviour of resource-intensive firms and there is no restriction on $z_c + z_d$.

Regarding the static problem (the allocation of production inputs for given k_c and k_d) first-order conditions in w_c and w_d yield $w_c q_c = \eta p_r y_r^{1-\varepsilon}(\gamma_c k_c q_c)^{\varepsilon}$, and similarly for the dirty input. Use these two equations to show that

$$w_c q_c + w_d q_d = \eta \alpha_r \bar{Y} w_z \quad \text{and} \tag{21}$$

$$\frac{w_c q_c}{w_d q_d} = \left(\frac{\gamma_c k_c / w_c}{\gamma_d k_d / w_d} \right)^{\varepsilon/(1-\varepsilon)}. \tag{22}$$

We thus have two equations in the two unknowns (q_c and q_d). Consider now the dynamic problem, and first-order conditions in z_c and z_d, which yield

$$z_c + z_d = \eta \phi \alpha_r \bar{Y}; \tag{23}$$

$$\frac{z_c}{z_d} = \frac{w_c q_c}{w_d q_d}. \tag{24}$$

Again, relative investments are equal to relative factor shares. And, again, we need to check for uniqueness, which we do separately for the two economies below.

4.2. The Lock-in and Fundamentalist economies

The next step is to specify the knowledge production functions and set parameters. However, this time we have set $\rho = 0$ in both cases, and the key is the specification of the knowledge growth equations. The economies are as follows.

Economy 2a. Lock-in. *Knowledge stocks augmenting alternative inputs grow independently—Definition 2—hence a new substitute resource is typically unable to enter without regulatory assistance.*

Economy 2b. Fundamentalist. *Relative productivities of alternative resources change as a result of underlying technological progress rather than endogenous changes in relative investment.*

In the Lock-in economy we assume—as with Directed—independent knowledge stocks:

$$k_{djt+1} = K_{dt} z_{djt+1}^{\phi} / \zeta_d; \quad k_{cjt+1} = K_{ct} z_{cjt+1}^{\phi} / \zeta_c. \tag{25}$$

General knowledge stocks K_d and K_c are built up in an equivalent way to K_l and K_r above (equation 15), hence in equilibrium $k_d = K_d$ and $k_c = K_c$.

For given prices w_d and w_c, and imposing the restriction $\varepsilon(1+\phi) < 1$,[11] we have (from equation 24) the following change in the relative stocks of knowledge of the representative firm:

$$\frac{k_{dt+1}/k_{ct+1}}{k_{dt}/k_{ct}} = \left[\left(\frac{\zeta_c}{\zeta_d} \right)^{(1-\varepsilon)/(\varepsilon\phi)} \frac{\gamma_d k_{dt} / w_{dt+1}}{\gamma_c k_{ct} / w_{ct+1}} \right]^{\varepsilon\phi/[1-\varepsilon(1+\phi)]}. \tag{26}$$

Note first that this equation shows that for given k_{ct} and k_{dt}, and with prices known and exogenous, k_{ct+1} and k_{dt+1} are uniquely determined. Second, note that for constant relative prices of the resource inputs the system is unstable, since growth in k_d/k_c is an increasing function of k_d/k_c. That is, for constant relative prices the ratio of the quantities demanded will approach either zero or infinity, depending on the starting

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[11] If the restriction does not hold neither does equation 26, and the economy goes straight to a corner.

point; in the case of energy, if the starting point is ‘fossil dominated’ then the economy will carry on heading for a ‘fossil corner’ in which there is zero demand for renewable energy because technologies are not adapted to use it. Put another way we have path-dependence, in the sense that the resource with an early lead ends up being dominant; chance drives the dynamics.

In the Fundamentalist economy we introduce a new distinction between knowledge stocks k_c and k_d and productivities a_c and a_d, and the dynamics are driven by the difference between resource-specific parameters $\bar{k}_c$ and $\bar{k}_d$, which represent the degree of technological sophistication required to make use of each resource. Production of y_c is a function of input productivity a_c,

$$y_c = \gamma_c a_c q_c,$$

and input productivity is a function of input-related knowledge k_c and $\bar{k}_c$,

$$a_c = \max\left\{(k_c - \bar{k}_c)^{1-\omega_c} k_c^{\omega_c}, 0\right\},$$

?? change in accordance with presentation! ?? where $\omega_c \in (0,1)$. Symmetric expressions apply for input D. For simplicity we completely short-circuit the process of DTC by assuming that there is only one type of investment z_r, and it boosts both types of knowledge equally. Since k_c and k_d are equal we define $k_{ct} = k_{dt} = k_{rt}$, and in equilibrium

$$k_{rt+1} = k_{rt} z_{rt+1}^{\phi} / \zeta_r. \tag{27}$$

In this economy there will therefore be no path-dependence or lock-in: In equation 23 we now have z_r on the LHS instead of $z_c + z_d$ (and equation 24 no longer applies). Hence $z_r = \eta \phi \alpha_r \bar{Y}$, and k_c and k_d will both grow at the growth rate of labour-augmenting knowledge in the long run.

The dynamics of resource productivity are as follows. If $k_c < \bar{k}_c$ the productivity of input C is zero; technology is too primitive to make any use of the input. However, since k_c rises at a constant rate θ then at some point we have $k_c = \bar{k}_c$, and the productivity of the input rises above zero beyond this point. The initial rate of increase will be very large, approaching θ asymptotically from above. The rate of approach will depend on ω_c; in the limit as ω_c approaches one, a_c jumps straight to k_c as soon as $k_c > \bar{k}_c$. The productivity of resource D will follow a similar pattern, but the timing will be different if $\bar{k}_c \neq \bar{k}_d$.

4.3. *Evidence from resource data*

Here we compare the ability of the two economies—Lock-in and Fundamentalist—to explain the data presented in Figure 2, concluding that the data is easily explicable within the Fundamentalist economy, and not within the Lock-in economy.

Consider first the Lock-in economy. Assume that there is a well-established resource (coal, iron) and then a rival, substitute resource emerges onto the market (oil, aluminium). According to the Lock-in economy these ‘new’ resources should be unable to gain a foothold in the market, since knowledge specific to these resources must start at zero (since they have not existed in the past), in which case not only is their productivity zero, but also the productivity of investments in factor-specific knowledge is zero; there are no giants on whose shoulders to stand, and the factor-specific knowledge never gets off the ground. If a new resource were somehow to successfully enter the market it should then take over *completely* from the original resource; the economy should head towards a new corner. All of these predictions are at odds with the data presented in Figure 2.

Consider now the Fundamentalist economy. In Figure 2 we have pairs of resources which are substitutes, and in which one of each pair requires simpler technology than the other for its utilization. The former (more basic) resources are coal and iron, the latter (more advanced) resources are oil and aluminium; for instance, oil emerged later than coal because it requires more advanced technology to extract, transport, and utilize oil compared to coal. According to this economy, technologies for utilizing both coal and oil do not develop in a vacuum, based only on factor-specific investment; instead they build on general knowledge in—for instance—mathematics, physics, and chemistry, upon which (in turn) general knowledge in engineering is built. Technologies crucial for industrial-scale use of coal—such as the steam engine—build on less advanced technology than those which allow the advantages of oil to be captured, which include technologies for refinement, and the internal combustion engine. Similarly, use of aluminium requires more advanced technology than the use of iron.

Denoting the former (simpler) resource by C and the latter by D, we thus have—in the model—$\bar{k}_c < \bar{k}_d$. Hence resource C is adopted first, and there is a period of rapid productivity growth before the growth rate in k_c approaches the overall growth rate. The economy is on this path at the start of the time period in Figure 2. Eventually k_d approaches $\bar{k}_d$, and there is a similar period of rapid growth in the productivity of resource D, and hence a rapid rise in the factor share of that resource. In the long run there is a transition towards a new balanced growth path along which the shares of the resources are constant, a function of the ratio of their intrinsic price-adjusted productivities, $\gamma_d/w_d/(\gamma_c/w_c)$, and the elasticity of substitution $1/(1-\varepsilon)$ (equation 24).

4.4. Evidence from patent data

According to the Directed economy, the production function for knowledge must have the characteristic of independent knowledge stocks (or something close to it); i.e. there should not exist powerful links between knowledge stocks which greatly increase the productivity of investment in sectors which are far behind the technology frontier in the economy. Arguments for such links appealing to intuition date at least to Nordhaus (1973), but what about hard evidence?

Evidence regarding the production function for knowledge can be found in studies of patent data. Very direct evidence can be found in Trajtenberg et al. (1992), who study patent citations and show that patenting firms cite patents both within their own three-digit industry, but also outside it.[12] Popp (2002) provides less direct evidence, in that he finds evidence for diminishing returns to investment in specific technologies over time; a rise in energy-prices induces a surge in patenting activity within energy sectors such as wind or solar, but the effect peters out rather rapidly, and Popp concludes that there is 'fishing out' of knowledge within specific energy-related sectors. Within our DTC model the interpretation differs in a crucial respect; it is not some constant stock of potential knowledge that is fished out, rather the surge of research raises the level of sector-specific knowledge relative to general knowledge, reducing spillovers from the general knowledge stock and hence reducing the productivity of further energy-augmenting research. Intuitively, we can think of discoveries of potential relevance to energy-augmentation being made frequently in other (much larger) research sectors; for instance, think of the invention of the computer. It takes time and research effort for the benefits of these discoveries to be incorporated in the energy sector. If there is a surge of research in the energy sector then initially there will be many potentially useful technologies available which have not yet been applied in that sector, but over time these 'low-hanging fruits' will be picked and the productivity of energy-augmenting research will fall.

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[12] They score patents as follows: Within 3-digit scores zero, within 2-digit scores 0.33, within 1-digit scores 0.67 and outside 1-digit scores 1. The average score is 0.31.

5. Discussion

In this section we first discuss policy implications if the Rebound and Fundamentalist economies are taken as accurate descriptions of the actual global economy. We go on to discuss the characteristics that more realistic models might have, and what the effect of these changes would be on the policy implications.

5.1. *Rebound (hence not Directed) and Fundamentalist (hence not Lock-in)*

The policy implications of the Rebound and Fundamentalist economies are straightforward. In the Rebound economy, raising the level of resource-augmenting knowledge will have no effect on resource demand, since there will be a corresponding increase in demand for the resource-intensive good, and resource consumption will be unchanged. Consider for instance the air travel industry. According to the Rebound economy, raising the fuel-efficiency of airplanes will lead to more flying and no fall in total fuel use. This is by contrast to the Directed economy, in which policies which lead to an increase in energy efficiency k_r will lead to an almost equally large fall in energy consumption q_r.[13]

In the Fundamentalist economy relative knowledge levels are fixed so there is no role for boosting clean technology relative to dirty. Assume for instance that solar PV (electricity from photovoltaic cells) is the best clean alternative. According to the Fundamentalist economy fossil energy's early dominance over solar PV is a function of the fundamental properties of the resources; solar PV was intrinsically more expensive than fossil power in this period, due either to underlying cost and productivity parameters, or because solar PV requires a higher level of technology than was available at this time. If the latter explanation is key it could be that the costs of solar PV will fall below fossil costs over time, but according to the model it is overall technological progress—rather than technology subsidies—which will be key to this process. This is by contrast to the Lock-in economy, according to which the fact that we chose fossil fuels rather than solar PV during early industrialization is a historical accident which has left us locked in to fossil-fuel use. It is then urgent to promote solar PV because the economy is currently moving in the wrong direction, further and further into a fossil corner. Assume that a regulator can boost k_c (solar PV productivity), for instance through research subsidies. Given a sufficient boost—such that the term inside the square brackets in equation 26 is less than one—then she can tip the economy over to a point where solar PV dominates fossil fuel in that its factor share is larger, implying greater investment—and faster growth—in k_c than in k_d, hence the cost advantage of solar grows over time and the economy moves towards a clean (solar) corner. The model thus mirrors the results of Acemoglu et al. (2012) under equivalent supply assumptions.[14]

In both the Rebound and Fundamentalist economies the only effective policy instruments to reduce fossil-fuel consumption are ones that raise the relative price of fossil-fuel consumption; in the first case the price should be raised relative to the wage, in the second case it should be raised relative to the price of the clean energy input. These conclusions are oversimplified, based as they are on extreme parameterizations of the model, but on the other hand it should be borne in mind that existing recommendations, highlighting the role of efficiency improvements and path dependence, are also based on extreme cases, i.e. single-sector models with no role for rebound effects, and DTC models with independent knowledge stocks and hence little or no role for technology fundamentals. Furthermore, our analysis suggests that the latter extremes are in conflict with the evidence.

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[13] Note however that even in the Directed economy, subsidies to energy efficiency may be problematic, as they are likely to crowd out private investment. If such subsidies have an effect, the energy share will fall and hence private investment in energy-augmenting knowledge will fall.

[14] From a theoretical standpoint, social benefits to research subsidies may be very large because such subsidies crowd in private investment in the socially preferable technology.

5.2. *Generalizations and their implications*

Here we briefly discuss two possible generalizations of the model, the first of which is to relax the assumption of constant elasticity of substitution between labour- and resource-intensive goods. There is some evidence that demand for resource-intensive goods rises rapidly early in the development process, only to slow down in a process of 'dematerialization' in advanced economies. The reason this does not show up in the global data would then be that while growth in resource demand has slowed in the OECD, it has accelerated in (for instance) China and India. So an alternative to the Rebound economy (1b) would be one in which the income elasticity of demand for resource-intensive goods is a decreasing function of income. If this holds then efficiency increases will have some downward effect on resource consumption, and when all countries have reached the 'dematerialization' stage—a distant prospect—the growth rate of resource demand will decline without the need for policy measures. More generally, in a more sophisticated model the rebound effect may vary from one sector to another, and its overall strength or importance may be lower.

The second generalization would be to introduce a model of directed technological change with links between knowledge stocks, rather than the special cases of independent knowledge stocks and no DTC which we model. The standard model of links, which I denote 'constant elasticity knowledge dependence', is presented by (for instance) Acemoglu (2002). Unfortunately, however, it suffers from some of the same problems as the model of independent stocks. In particular, it remains incapable of explaining how a new resource takes over from an old if the new resource enters the market with zero resource-specific knowledge. As a simple alternative we propose a model based on spillovers of knowledge between sectors. (For much more analysis of this model and related models see Hart (2012).) Now we have (compare to 25 and 27)

$$k_{djt+1} = (K_{dt} + \sigma G_t) z^{\phi}_{djt+1} / \zeta_r; \quad k_{cjt+1} = (K_{ct} + \sigma G_t) z^{\phi}_{cjt+1} / \zeta_r. \tag{28}$$

Here σ is a positive parameter less than one, G_t represents general knowledge in the economy, and we have set $\zeta_c = \zeta_d = \zeta_r$, since there is no reason *a priori* why research should be more productive (in terms of ideas per researcher) in one sector compared to another, other than due to the existing knowledge stocks. We analyse the lock-in model where (25) is replaced by (28) in Appendix Appendix A.2, and conclude that the lock-in property disappears in this model: for given underlying parameters there is a unique b.g.p. and no path dependence. If a DTC model based on equation 28 were added to the Fundamentalist economy the result would be that the lag in adoption of new (more technologically demanding) energy sources would be greater, and temporary technology subsidies would likely be welfare-improving since they reduce the lag, but there would still be no lock-in or path dependence.

6. Conclusions

Recall that the underlying question addressed in the paper is how to break the global trend of rising CO_2 emissions from fossil-fuel consumption. To answer this question we investigate two pairs of economies within our long-run growth model: Directed/Rebound and Lock-in/Fundamentalist. Our analysis supports the Rebound and Fundamentalist economies, and the policy conclusions are stark: Subsidizing, or otherwise promoting, energy efficiency may yield minimal CO_2 reductions; subsidizing alternative energy technologies may be futile if the underlying technology is uncompetitive, and unnecessary if the underlying technology is competitive; and the only surefire way to reduce emissions may be to raise the price of fossil fuels. Meanwhile, policy papers building their conclusions on models consistent with the Directed and Lock-in economies should be treated with caution in the absence of good evidence backing up their assumptions concerning directed technological change and substitution on the consumption side.

These conclusions concord with those of authors such as Fischer and Newell (2008), whose analysis is based on a two-period partial equilibrium analysis which does not give full rein to the mechanisms proposed by, for instance, Acemoglu et al. (2012). On the other hand, they contrast with a lot of the more theoretically oriented literature. Regarding Directed/Rebound, it is common to simply assume a one-sector economy and thus assume away the possibility of substitution on the consumption side. Having made this first, crucial step, DTC emerges unchallenged as the explanation for the long-run constancy of factor shares, whether it is with regard to capital and labour (Kennedy, 1964) or energy (Smulders and de Nooij, 2003). Regarding Lock-in/Fundamentalist, Acemoglu et al. (2012) place the DTC mechanism at the centre of their paper, and hence they find that the factor shares of substitute inputs are fully path dependent (in the main variant of the model), in the sense that the economy heads to a corner solution in which one input dominates completely, and the dominant input is determined by chance (or the order of appearance of the inputs) rather than fundamentals.

Finally, regarding the rebound effect, note that the idea that substitution on the consumption side—rather than DTC—may be the key explanation for constancy of factor shares is not new. Acemoglu and Guerrieri (2008) show exactly that for the case of capital and labour; the constant shares of capital and labour are attributable not to DTC with independent knowledge stocks, as famously argued by Kennedy (1964), but rather to consumers substituting between consumption categories of differing capital intensity.

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Appendix A. Appendix – For online publication

Appendix A.1. Solution of general model with one resource input

Here we demonstrate the existence of a unique solution the general model of Section 3. Assume that in period $t-1$ we have the set of knowledge $\mathbf{K}_-$. We aim to find all of the endogenous variables in period t. Our solution strategy is first to assume that quantities used by the representative firms, q_l and q_r, are exogenously given, and to find relative investments and relative prices. Using this solution we then confirm that there exists a unique solution for the quantity of resource q_r and the price of labour w_l given that w_r and q_l are exogenously given.

First take equation 13 and substitute in, firstly the production functions (3) and (4), and secondly the knowledge production functions (8); note that we drop the time subscript. Hence

$$\frac{z_l}{z_r} = \frac{\alpha_l}{\alpha_r}\left(\frac{\gamma_l q_l}{\gamma_r q_r}\right)^{\rho}\left(\frac{F_l(\mathbf{K}_-)z_l^{\phi}/\zeta_l}{F_r(\mathbf{K}_-)z_r^{\phi}/\zeta_r}\right)^{\rho}. \tag{A.1}$$

Now rearrange to derive

$$\left(\frac{z_l}{z_r}\right)^{1-\rho\phi} = \frac{\alpha_l}{\alpha_r}\left(\frac{\gamma_l q_l}{\gamma_r q_r}\right)^{\rho}\left(\frac{F_l(\mathbf{K}_-)/\zeta_l}{F_r(\mathbf{K}_-)/\zeta_r}\right)^{\rho}. \tag{A.2}$$

Equation A.2—together with the restriction on total research labour—gives us a unique solution for z_{lt} and z_{rt}, for given quantities q_{lt} and q_{rt} and knowledge stocks $\mathbf{K}_-$. Note also that equation A.2 shows that z_l/z_r is a decreasing function of q_l/q_r as long as $\rho < 0$.

Given the above—the unique solutions for z_l and z_r, the initial knowledge set $\mathbf{K}_-$, and the knowledge production functions—we know k_l and k_r, and hence we can solve for all the other variables in the model. Here we focus on the resource price w_r. From the first-order conditions we have $w_r q_r = \eta p_r y_r$, and hence by substituting for p_r and y_r we can obtain

$$w_r q_r = \eta\alpha_r Y^{1-\rho}(\gamma_r k_r q_r)^{\rho}. \tag{A.3}$$

Substitute for Y—using equations 1, 3 and 4—and rearrange to obtain

$$w_r = \left[\frac{\alpha_l}{\alpha_r}\left(\frac{\gamma_l k_l q_l}{\gamma_r k_r q_r}\right)^{\rho} + 1\right]^{(1-\rho)/\rho}\eta\alpha_r^{1/\rho}\gamma_r k_r. \tag{A.4}$$

Now return to equation A.2, and substitute the expression for z_l/z_r back into the knowledge production functions to derive

$$\frac{k_l}{k_r} = \left(\frac{F_l(\mathbf{K}_-)/\zeta_l}{F_r(\mathbf{K}_-)/\zeta_r}\right)\left[\left(\frac{\alpha_l}{\alpha_r}\right)^{1/\rho}\frac{\gamma_l q_l}{\gamma_r q_r}\left(\frac{F_l(\mathbf{K}_-)/\zeta_l}{F_r(\mathbf{K}_-)/\zeta_r}\right)\right]^{\rho\phi/(1-\rho\phi)}. \tag{A.5}$$

and hence

$$\frac{k_l}{k_r} = \left(\frac{F_l(\mathbf{K}_-)/\zeta_l}{F_r(\mathbf{K}_-)/\zeta_r}\right)^{1/(1-\rho\phi)}\left[\frac{\alpha_l}{\alpha_r}\left(\frac{\gamma_l q_l}{\gamma_r q_r}\right)^{\rho}\right]^{\phi/(1-\rho\phi)}. \tag{A.6}$$

Substitute this into A.4 to obtain

$$w_r = \left[\frac{\alpha_l}{\alpha_r}\left(\frac{\gamma_l q_l}{\gamma_r q_r}\right)^{\rho}\left(\frac{F_l(\mathbf{K}_-)/\zeta_l}{F_r(\mathbf{K}_-)/\zeta_r}\right)^{\rho/(1-\rho\phi)}\left[\frac{\alpha_l}{\alpha_r}\left(\frac{\gamma_l q_l}{\gamma_r q_r}\right)^{\rho}\right]^{\rho\phi/(1-\rho\phi)} + 1\right]^{(1-\rho)/\rho}\eta\alpha_r^{1/\rho}\gamma_r k_r. \tag{A.7}$$

and hence

$$w_r = \left\{ \left[\frac{\alpha_l}{\alpha_r} \left(\frac{\gamma_l q_l}{\gamma_r q_r} \right)^{\rho} \right]^{1/(1-\rho\phi)} \left(\frac{F_l(\mathbf{K}_-)/\zeta_l}{F_r(\mathbf{K}_-)/\zeta_r} \right)^{\rho/(1-\rho\phi)} + 1 \right\}^{(1-\rho)/\rho} \eta \alpha_r^{1/\rho} \gamma_r k_r. \tag{A.8}$$

By inspection of (A.8), as $q_r \to \infty$, $w_r \to 0$, and as $q_r \to 0$, $w_r \to \bar{w}_r$, where $\bar{w}_r$ is finite and positive. Furthermore, as long as Y_l and Y_r are complements ($\rho < 0$), w_r is unambiguously decreasing in q_r across the allowed range of q_r (i.e. when q_r is positive). Hence as long as w_r is in the domain $(0, \bar{w}_r)$ then q_r is a function of w_r as well as w_r being a function of q_r.[15] The implication of this is as follows. In the baseline model described above we treat q_l and q_r as exogenous, and w_r is then endogenous. Given the result above, we can also treat the resource price w_r as being exogenous, in which case there is a unique quantity of resources q_r extracted, which is endogenous to the model.

Appendix A.2. The extended model with links between knowledge stocks

Here we analyse the economy based on the lock-in economy, but with (25) is replaced by (28):

$$k_{d\,jt+1} = (K_{dt} + \sigma G_t) z_{d\,jt+1}^{\phi} / \zeta_r; \quad k_{c\,jt+1} = (K_{ct} + \sigma G_t) z_{c\,jt+1}^{\phi} / \zeta_r.$$

Assume an initial situation in which fossil energy D dominates completely since it is so cheap compared to renewable. Then we have

$$z_d = \eta \phi \alpha_r \bar{Y},$$

and we have (after substituting in the definition of $\bar{Y}$)

$$\frac{k_{dt+1}}{k_{dt}} = \left(1 + \sigma \frac{G_0}{k_{dt}/\theta^t} \right) \alpha_r^{\phi} \theta \frac{\zeta_l}{\zeta_r}.$$

The economy thus converges to a balanced growth path along which G/k_d is constant,

$$\frac{G}{k_d} = \frac{1}{\sigma} \left(\frac{1}{\alpha_r^{\phi}} \frac{\zeta_r}{\zeta_l} - 1 \right),$$

and the distance of fossil-augmenting knowledge from the frontier is a decreasing function of σ and α_r (the fossil share).

Assume now that C enters the fray. We then have (from equation 24)

$$\frac{z_d}{z_c} = \left(\frac{\gamma_d k_d / w_d}{\gamma_c k_c / w_c} \right)^{\varepsilon/(1-\varepsilon)},$$

and from (28) we have

$$\frac{k_d}{k_c} = \frac{k_{d-} + \sigma G_-}{k_{c-} + \sigma G_-} \left(\frac{z_d}{z_c} \right)^{\phi},$$

where the subscript minus indicates values from the previous period (which are known), as above. From these two equations we can solve immediately for k_d/k_c,

$$\frac{k_d}{k_c} = \left[\left(\frac{k_{d-} + \sigma G_-}{k_{c-} + \sigma G_-} \right)^{1-\varepsilon} \left(\frac{\gamma_d / w_d}{\gamma_c / w_c} \right)^{\varepsilon\phi} \right]^{1/[1-\varepsilon(1+\phi)]},$$

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[15] Note that if Y_l and Y_r are substitutes then the term in curly brackets is decreasing in q_r but k_r is increasing in q_r so we cannot say a priori whether or not q_r is a function of w_r.

noting the restriction that $\varepsilon(1+\phi) < 1$. So we see that if for instance we start with $k_{c-} = 0$ then k_c will be positive as long as w_c is finite.

To understand the properties of the economy better we must investigate more deeply. Combine the production functions for knowledge with the equation for total resource-augmenting research (23), $z_c + z_d = \eta\phi\alpha_r\bar{Y}$, to obtain

$$\left(\frac{k_d}{k_{d-}+\sigma G_-}\right)^{1/\phi} + \left(\frac{k_c}{k_{c-}+\sigma G_-}\right)^{1/\phi} = \eta\phi\alpha_r\bar{Y}.$$

Rearrange to obtain an expression for k_c, and substitute this expression into the equation for k_d/k_c above to yield

$$\frac{k_d}{\left[\eta\phi\alpha_r\bar{Y} - \left(\frac{k_d}{k_{d-}+\sigma G_-}\right)^{1/\phi}\right]^{\phi}(k_{c-}+\sigma G_-)} = \left[\left(\frac{k_{d-}+\sigma G_-}{k_{c-}+\sigma G_-}\right)^{1-\varepsilon}\left(\frac{\gamma_d/w_d}{\gamma_c/w_c}\right)^{\varepsilon\phi}\right]^{1/[1-\varepsilon(1+\phi)]},$$

and hence

$$\frac{k_d}{\left[\eta\phi\alpha_r\bar{Y}\,(k_{d-}+\sigma G_-)^{1/\phi} - k_d^{1/\phi}\right]^{\phi}} = \left[\frac{k_{d-}+\sigma G_-}{k_{c-}+\sigma G_-}\cdot\frac{\gamma_d/w_d}{\gamma_c/w_c}\right]^{\varepsilon\phi/[1-\varepsilon(1+\phi)]} = \Omega^{\varepsilon\phi/[1-\varepsilon(1+\phi)]}.$$

Note the definition of Ω in the final equation, and rearrange to yield

$$k_d = (\eta\phi\alpha_r\bar{Y})^{\phi}\,(k_{d-}+\sigma G_-)\left(\frac{\Omega^{\varepsilon/[1-\varepsilon(1+\phi)]}}{1+\Omega^{\varepsilon/[1-\varepsilon(1+\phi)]}}\right)^{\phi}.$$

By symmetry we also have

$$k_c = (\eta\phi\alpha_r\bar{Y})^{\phi}\,(k_{c-}+\sigma G_-)\left(\frac{1}{1+\Omega^{\varepsilon/[1-\varepsilon(1+\phi)]}}\right)^{\phi}.$$

Since Ω is defined in terms of knowns, these two equations define the evolution of the system. But where do they lead? It turns out that they lead to a unique stable b.g.p. when relative prices are constant. To see this, first redefine the variables as k_c/G and k_d/G. Then

$$\Omega = \frac{(k_d/G)_- + \sigma}{(k_c/G)_- + \sigma}\cdot\frac{\gamma_d/w_d}{\gamma_c/w_c},$$

and

$$\theta(k_d/G) = (\eta\phi\alpha_r\bar{Y})^{\phi}\,((k_d/G)_- + \sigma)\left(\frac{\Omega^{\varepsilon/[1-\varepsilon(1+\phi)]}}{1+\Omega^{\varepsilon/[1-\varepsilon(1+\phi)]}}\right)^{\phi}.$$

$$\theta(k_c/G) = (\eta\phi\alpha_r\bar{Y})^{\phi}\,((k_c/G)_- + \sigma)\left(\frac{1}{1+\Omega^{\varepsilon/[1-\varepsilon(1+\phi)]}}\right)^{\phi}.$$

Rearrange and use the fact that $\bar{Y} = (\zeta_l\theta)^{1/\phi}/(\eta\phi)$ to obtain

$$\frac{(k_d/G)}{(k_d/G)_-} = \zeta_l\alpha_r^{\phi}\left(1+\sigma(k_d/G)_-^{-1}\right)\left(\frac{\Omega^{\varepsilon/[1-\varepsilon(1+\phi)]}}{1+\Omega^{\varepsilon/[1-\varepsilon(1+\phi)]}}\right)^{\phi}. \tag{A.9}$$

$$\frac{(k_c/G)}{(k_c/G)_-} = \zeta_l\alpha_r^{\phi}\left(1+\sigma(k_c/G)_-^{-1}\right)\left(\frac{1}{1+\Omega^{\varepsilon/[1-\varepsilon(1+\phi)]}}\right)^{\phi}. \tag{A.10}$$

To show that there exists a b.g.p. we simply need to show (i) that for any allowed value of k_c/G there is a value of k_d/G such that k_d/G is constant, and (ii) the corresponding result with the roles of k_c and k_d reversed. Given the restriction that $\varepsilon(1+\phi) < 1$, the result follows straightforwardly from noting that the right hand side of (A.9) is monotonically decreasing in k_d/G, and that at the limits (when k_d/G approaches zero and infinity) it is greater than one and less than one respectively. The corresponding result applies to (A.10).[16]

The above result, though important, is not particulary enlightening. To get a simpler result assume that the energy sector is small enough such that researchers build overwhelmingly on general knowledge rather than existing energy-specific knowledge. Then it is straightforward to show that the long-run factor shares of the two resources are

$$\frac{w_d q_d}{w_c q_c} = \left(\frac{\gamma_d/w_d}{\gamma_c/w_c}\right)^{\varepsilon/[1-\varepsilon(1+\phi)]}.$$

So the equilibrium shares are in proportion to the intrinsic productivities of the resources, magnified according to the degree of substitutability between them and the elasticity of knowledge to research effort. That is, a resource with a high ratio of intrinsic productivity to price will take a large share. How large that share is will increase in the elasticity of substitution between the resources; if the resources are easily substitutable then a resource with a small intrinsic productivity advantage will have a large long-run market share. Furthermore, it will increase in the responsiveness of knowledge to investment; if this elasticity is high then a small amount of extra investment will lead to a large knowledge increase, again strengthening the advantage of the dominant input.

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[16] Note that we need to apply the restriction that $\alpha_r \zeta_l < 1$, which implies that knowledge growth in the resource sector is not fixed at a level higher than overall growth in the economy, which it could be due to a combination of the high factor share of resources and the low productivity of labour-augmenting research.