Second-Best Environmental Taxation In Dynamic Models Without Commitment

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July 10, 2013

WORK IN PROGRESS

Abstract

This paper analyzes the interaction between optimal environmental regulation and fiscal policy in a setting in which a benevolent government has to raise revenue through distortionary taxes on labor and capital income, and is unable to commit to future tax rates. The main question I aim to answer is how the presence of these non-environmental taxes affects the social cost of pollution, that is, the price that the government must impose on emissions in order to internalize the pollution externality. Similar to previous studies focusing on static models with distortionary labor income taxation, and in contrast to a first-best world with lump-sum taxation, I show that the optimal Markov-perfect pollution tax is in general not at the Pigouvian level, i.e. does not equal marginal pollution damage, due to the presence of additional costs and benefits of fossil fuel use in the second-best setting. In the main quantitative exercise, I analyze the interaction between distortionary fiscal policy and environmental regulation for the case of climate change, using a simple climate-economy model where the state of the climate not only affects the production process, but has also direct utility impacts. I derive the government's generalized Euler equations and apply a projection method to compute a stationary Markov-perfect equilibrium. I show that the carbon tax path chosen under commitment is time-inconsistent, which follows from the fact that the optimal pollution tax depends on the non-environmental tax structure. Moreover, I compare the time path for the Markov-perfect carbon tax with both the first-best outcome and the Pigouvian tax schedule. I find that the optimal tax in 2010 is only about 3.5 percent lower than in first-best. More interestingly, and in contrast to previous studies, it is very close to its Pigouvian level, hence the climate damages are fully internalized. This result is sensitive to the persistency of the carbon stock, which is determined by the rate of decay of carbon in the atmosphere.

1 Introduction

The most fundamental result in environmental economics states that in order to optimally correct a pollution externality, a benevolent social planner or government must introduce regulation that equates the marginal (net) private benefit of emitting a pollutant to the marginal social cost. If the planner chooses to impose a price on the pollutant, rather than directly regulating emissions by a command-and-control approach, the optimal tax is equal to the social cost evaluated at the efficient pollution

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level (Kolstad, 2000). Hence, when designing environmental policy, it is elementary to "know" (or at least have a good estimate for) the social cost of pollution[1].

In many studies that consider a first-best world, the social cost of pollution is equal to the marginal damage caused by emitting an additional unit of the pollutant. The optimal tax that internalizes the pollution damage is called a "Pigouvian" tax. Hence, at the first-best margin, the marginal private benefit of pollution is equal to the marginal damage.

In a more realistic setting, a government has to raise revenue in order to finance expenditures on public goods. If it cannot resort to lump-sum taxation, the first-best allocation is not feasible, even if the government is assumed to be benevolent.

This has an effect on optimal environmental taxation. It is a well-known result that in the presence of non-environmental, distortionary taxes, for example a linear tax on labor income, a tax implemented to regulate a pollution externality must be set optimally *below* its Pigouvian level[2]. In other words, the pollution damage is not fully internalized and hence the social cost of polluting is less than the marginal damage. This was first shown by Bovenberg and de Mooij (1994) and has been analyzed by in a number of studies following their seminal paper, in particular Bovenberg and Goulder (1996). The canonical model used in this literature focuses on a static setting with labor taxation, which distorts the intratemporal consumption-leisure margin. In such a framework, under certain conditions, a price on pollution exacerbates this distortion by decreasing the labor supply and thus eroding the tax base. The optimal emission tax must account for this additional welfare cost of pollution reduction in second-best. This is known as the "tax-interaction effect".

A generalization of this model considers optimal environmental taxation in an intertemporal framework. This is of interest for several reasons. First, while a static model is useful for the case of an externality caused by the emission of a pure flow pollutant[3], many environmental problems - first and foremost climate change - have an inherently dynamic nature: they are caused by a stock pollutant, which accumulates over time. Hence, there is a dynamic relationship between emissions and environmental quality (or pollution damages). Moreover, intertemporal models allow individuals to postpone or bring forward consumption through saving or borrowing. Hence, these models typically feature one or more assets, for example in the form of physical or human capital or bonds, and thus a second tax base. When analyzing dynamic fiscal policy, it is realistic to assume that a government can impose taxes on income from both labor and asset holdings. In contrast, a static model cannot account for a tax that distorts the household's saving decision.

Finally, a dynamic model is necessary if one is interested in whether or not environmental taxation in second-best is "time-consistent", in the sense that the optimal tax rate announced today for any point in the future (contingent on the realization of economic or other shocks) is still optimal once this

[1] In the case of climate change, an Interagency Working Group of the US government recently published a report determining the "social cost of carbon" (SCC). They define the SCC as "an estimate of the monetized damages associated with an incremental increase in carbon emissions in a given year. It is intended to include (but is not limited to) changes in net agricultural productivity, human health, property damages from increased flood risk, and the value of ecosystem services." (IWG, 2010)

[2] For this result to hold, certain conditions have to be satisfied, most importantly a direct effect of environmental quality on utility and a positive wage elasticity of labor supply. Compare the discussion in section 3.1.

[3] Throughout this paper, a *flow* pollutant means that environmental quality depends only the flow rather than the stock of a pollutant, for example in the air or atmosphere. Put differently, a unit of emission decays fully within one period. In contrast, in case of a *stock* pollutant, there is some persistency over time, that is, a rate of decay less than unity.

realization occurs. Hence, no future government will have an incentive to reoptimize, i.e. to set a tax rate different from the one announced in the current period. This is true for a Pigouvian tax in first best.

When considering non-environmental taxes on capital and labor income, a "Ramsey equilibrium" that presumes that the government can commit to a sequence of future tax rates is in general time-inconsistent, i.e. in absence of such a commitment device, the government would reoptimize in each period (Klein et al., 2008)[4]. Hence, the question is how the lack of commitment affects optimal second-best environmental taxes.

Dispensing with the commitment assumption has the additional benefit of allowing a framework in which taxes on capital income are endogenously non-zero in the long-run. As the well-known result by Judd (1985) and Chamley (1986) states, in a Ramsey model with commitment, capital taxes in steady state - and, under certain conditions, on the transition path - are optimally zero. Hence, when analyzing optimal pollution taxes in such a model, the focus is once again on the interaction between environmental and labor taxes and the distorted intratemporal margin (Barrage, 2012). Positive capital taxes, on the other hand, allow for environmental regulation in the presence of a distorted intertemporal consumption-savings margin.

In this paper, I compute the social cost of pollution using a dynamic neoclassical growth model with energy and environmental quality to analyze optimal taxation in a second-best setting without commitment. The main theoretical question is whether or not this framework gives rise to a tax-interaction effect, and how this effect differs qualitatively from the characterization in the static setting by Bovenberg and Goulder (1996) and others. Put differently, I am interested in how fiscal policy affects environmental taxation in a dynamic setting with capital as an additional (and possibly the only) tax base, and how the emission tax relates to the marginal damage of pollution.

I then apply the model to analyze the quantitative relevance of distortionary taxation and the tax-interaction effect in the context of climate change. That is, I compute the time path for the global optimal carbon tax in a world with a distortionary tax on labor and capital income, which are set optimally, and where the government is unable to commit to future tax rates. I compare the optimal tax policy in second best with 1) the first-best outcome, i.e. when the government has access to lump-sum taxation, in order to quantify the effect of distortionary taxes on the social cost of carbon; 2) the corresponding Pigouvian tax, which fully internalizes the marginal damage of emitting carbon, to see how relevant the tax-interaction effect is empirically; and 3) the equilibrium in a setting in which the government has access to a commitment device. The latter experiment allows me to check for the time-consistency of carbon taxes in the presence of distortionary taxation.

These comparisons are relevant when trying to determine the social cost of carbon. Many studies estimate the social cost of carbon in a first-best setting (cp. for example IWG, 2010), often without taking other government activities into account. However, if the second-best equilibrium under lack of commitment is very different from the first-best outcome, these estimates may be biased, and one should account for the effects of distortionary taxation when computing the cost of emitting carbon. Moreover, if the tax-interaction effect is quantitatively meaningful and the optimal tax is sufficiently different from the marginal emission damage in equilibrium, disregarding this effect and focusing on

[4]Under certain assumptions, however, Ramsey equilibria can be time-consistent. Compare, for example, Azzimonti-Renzo et al. (2006).

the Pigouvian tax when designing climate policy may not be justified.

The main findings of this paper are the following: first, in a dynamic model with a distortionary tax on capital income and lack of commitment, the optimal pollution tax rate is in general not at the Pigouvian level. This is a generalization of the Bovenberg-Goulder result. Already in the simplest dynamic setting, in a two-period model with exogenous labor supply and no tax on labor income, there is an additional benefit of fossil fuel use in second best, which decreases the social cost of pollution: increasing fuel use - and thus reducing public goods provision - leads to an increase in output, and hence in the households' income, which in turn allows them to transfer more resources to the next period. This mitigates the intertemporal distortion present in this economy: households expect a positive tax on capital in the future, and hence reduce their savings. By letting fuel use and thus pollution rise beyond the first-best margin, the government can counteract this effect by increasing the household's available resources. This has positive first-order effect on welfare.

In other words, as in the static setting, the current government has an incentive to "underprovide" public goods, i.e. to not fully internalize the pollution damage. Note, however, that the mechanism is different: in the case of labor taxation, increasing fuel use affects the return to labor, which may have a positive effect on labor. In the case with capital, underproviding the environmental public good does not impact the return to current, but rather to *past* saving, which affects current saving through a rise in income.

In a finite-horizon setting with more than two periods and in an infinite-horizon, I identify further additional costs and benefits of fuel use, due to the impact of current emissions on future savings and labor decisions, through both the future capital stock, and, in the case of a stock pollutant, the future pollutant stock. None of those are present in the static model.

Second, the quantitative exercise yields an initial carbon tax (in 2010) which is only around 3.5% below the first-best, i.e. the tax which would be optimal if lump-sum taxation were feasible. By 2105, this deviation increases to around 15%. More interestingly, the second-best carbon tax is almost at its Pigouvian level - in fact, it exceeds it slightly - meaning that the climate damages are fully internalized. This result is in contrast to both the theoretical result for a static setting (Bovenberg and Goulder, 2002), as well as to recent empirical findings in the context of climate change (Barrage, 2012). I show that the tax-interaction effect is less prevalent for a sufficiently persistent pollutant[5], due the different second-best costs and benefits of carbon emissions canceling out. One conclusion from this is that the tax-interaction effect may be negligible when designing climate policy, and that the Pigouvian tax is a good approximation to the true optimal tax rate.

Finally, the time-consistent carbon tax is in general different from the tax path under commitment. In other words, in contrast to a first-best setting, the optimal tax chosen under commitment is not time-consistent, in the sense that the government has an incentive to implement a different tax rate than previously announced. This time-inconsistency is due to the interaction with other taxes: the level of the Markov-perfect tax depends on the non-environmental tax system. If these taxes are not time-consistent, for example in case of a zero-capital tax, neither is the corresponding carbon tax.

Methodologically, this paper is closely related to Klein et al. (2008) and Martin (2010), who analyze the standard neoclassical growth model without environmental quality. As these papers, I focus on

[5]CO_2, once emitted in the atmosphere, stays there for a long time. In the most simple model of the carbon cycle, a annual rate of depreciation of around 1% is considered appropriate (Gerlagh and Liski, 2012).

computing stationary Markov-perfect equilibria. Analogous to Klein et al. (2008), I derive the current government's generalized Euler equations, which are weighted sums of intertemporal and intratemporal wedges that the government trades off with each other. In order to solve for the steady state of this system of equations, Klein et al. (2008) use a local perturbation method. However, in the context of a long-run problem like climate change, the transition path to the steady state is of much more interest than the steady state itself. Hence, I use a global projection method which allows me to compute a more precise approximation of the policy functions[6].

An extension of Klein et al. (2008) in this paper is the analysis of a *stock* of a public good, rather than a pure flow. Note that while the application here is with respect to an environmental public good, the analysis would be similar to the case of the stock of a non-environmental public good. For example, one could think of infrastructure like public roads and buildings as a persistent public good, i.e. expenditures today matter for the stock tomorrow.

In the context of climate change, numerous studies have used "Integrated Assessment Models" (IAM) to compute the optimal carbon tax. Prominent examples are the DICE model (Nordhaus, 2008) or the WITCH model (Bosetti et al., 2006). Golosov et al. (2011) feature a climate-economy model more along the lines of modern macroeconomics. However, like most other studies, these models focus on a first-best economy without explicitly modeling government expenditures on non-environmental public goods.

Gerlagh and Liski (2012) consider a different source of time inconsistency, namely hyperbolic discounting. They compute Markov-perfect optimal carbon prices in a setting without distortionary taxes, but where the government is again unable to commit to future policies.

Closest related to this paper is Barrage (2012). She also analyzes optimal climate policy in the presence of distortionary fiscal policy. However, in contrast to the analysis below, she assumes that the government is able to commit to future tax rates. In other words, she focuses on pollution taxes as part of a commitment equilibrium, with or without a zero capital tax[7]. She finds that the baseline scenario without a capital tax, the optimal carbon tax is initially around 20% below the Pigouvian tax, which in turn is almost at the first-best level. In other words, under her assumptons, the optimal carbon tax is lower than in my model without commitment.

The remainder of this paper is structured as follows. Section 2 presents the model. In section 3, to illustrate the main mechanisms at play, I analyze a simple two-period model. In section 4, I derive the generalized Euler equations in an infinite-horizon setting for a stock pollutant. Section 5 contains the main quantitative exercise, the computation of the social cost of carbon. Section 6 concludes.

2 The Model

In this section, I introduce a simple dynamic framework in which I analyze second-best environmental taxation. Consider the standard neoclassical growth model, extended by "fossil fuel" or "energy" and "environmental quality". Fuel is used as a factor of production, in addition to capital and labor. Using fuel causes emissions of a pollutant. Hence, the amount of fuel used in production determines

[6] As a robustness check, I verify the results using value function iteration.

[7] Capital taxes can be *temporarily* positive, due to an upper bound that binds for a finite number of periods. Alternatively, a permanently positive capital tax is exogenously given.

environmental quality, which affects both the utility function of the representative household - as in the static second-best literature following Bovenberg and de Mooij (1994) - and the productivity of the representative firm, as, for example, in Golosov et al. (2011). Producers do not take into account how their decisions affect environmental quality, hence pollution represents an externality.

More formally, the representative household's per-period utility is given by $u(c, 1-h, g, s)$, where c denotes private consumption of a final good, h hours worked, g public consumption and s is an indicator of environmental quality. The latter two are not chosen by the household, hence they represent public goods. u is increasing in its first three arguments, and *decreasing* in s. In other words, a higher s - which will henceforth be interpreted as the stock of the pollutant associated with fuel use - corresponds to lower environmental quality.

Note in contrast to many papers in public finance, I have assumed here that the public consumption good is valued by the household, hence the amount provided is a choice variable of the government or planner. This assumption is important for the infinite-horizon version of the model in sections 4 and 5 for technical reasons[8]. For the two-period model in 3, it is not essential, hence I will simplify the analysis by assuming exogenous government expenditures.

The consumption good is produced with a constant returns to scale technology, represented by a production function f, which uses as inputs capital, labor and fuel, denoted by m. Moreover, s does not only affect utility, but has also an impact on the production process. "Net" output - taking environmental damages into account - is given by $y = F(k, h, m, s) = \mathcal{F}(f(k, h, m), s)$.

I will follow Golosov et al. (2011) and assume that s enters the production function multiplicatively:

$$\mathcal{F}(f(k, h, m), s) = [1 - d(s)]f(k, h, m), \tag{2.1}$$

where the "damage function" d decreases in environmental quality, $d_s < 0$, and $0 \leq d(s) \leq 1$.

To simplify the exposition, I assume that there is no scarcity problem and so m is available at "infinite" capacity. Hence I abstract from the Hotelling problem of how to optimally extract a finite resource[9]. Moreover, let the *private* marginal cost of fuel use be a constant κ. This could be interpreted, for example, as a constant per-unit extraction cost. More realistically, one could let the cost be a function of the resources left in the ground, or model energy production as a separate production sector that uses labor and possibly capital as in Golosov et al. (2011) or Barrage (2012).

In addition, using fuel has a social cost: it causes pollution, which negatively affects environmental quality. Very generally, let $s_t = q(\mathbf{m}^t)$, where $\mathbf{m}^t = \{m_t, m_{t-1}, ..., m_{t-T}\}$ denotes the history of past emissions back to period $t - T$, and $\partial q/\partial m_j > 0$. In other words, the current *flow* of the pollutant affects its *stock* now and in the future, and hence has an impact on future environmental quality. Throughout the paper, I will refer to such a pollutant as a "stock pollutant". A prototypical example are greenhouse gases, first and foremost carbon dioxide (CO_2), which stays in the atmosphere for a very long time horizon.

In contrast, if current emissions affect environmental quality only in the current period, but not in the future, one deals with a flow pollutant. Formally, this implies that $q(\mathbf{m}^t) = q(m_t)$. Since environmental quality is a type of public good, this case is similar to the one analyzed in Klein et al. (2008) for valued government expenditures. In contrast to their model, however, environmental quality

[8]Compare the discussion in the appendix, section A.3, for details.

[9]In the context of climate change, fuel should be interpreted as coal rather than (conventional) oil or gas

interacts with the production side of the economy, both by affecting productivity and through the usage of fuel.

In the analysis below, I will focus on the general case of a stock pollutant, not only because it is the relevant assumption when modeling climate change. Simplifying the model to the flow pollutant case can be useful at times since it allows for closed-form solutions.

The representative household maximizes lifetime utility, subject to its budget constraint, and taking price and tax sequences as given:

$$\max_{\{c_t,h_t,k_{t+1}\}_{t=0}^T} \sum_{t=0}^{T} \beta^t u(c_t, 1-h_t, g_t, s_t) \tag{2.2}$$

where $T \leq \infty$, subject to its budget constraint:

$$c_t + k_{t+1} \leq [1 + (1-\tau_t^k)(r_t - \delta)]k_t + (1-\tau_t^h)w_t h_t. \tag{2.3}$$

r and w denote the factor prices of capital and labor, respectively, while τ^k and τ^h are the corresponding linear tax rates. δ is the rate of capital depreciation. Solving this problem yields two standard optimality conditions, one intertemporal (consumption-savings), one intratemporal (consumption-leisure).

The government's budget constraint reads:

$$g \leq \tau^k(r-\delta)k_t + \tau^h wh + \tau^m m, \tag{2.4}$$

where τ^m denotes a per-unit tax on emissions. Note that throughout this paper, I assume that the government has to balance its budget. In other words, it can neither borrow from nor lend to households. The latter assumption is crucial, as pointed out by Azzimonti-Renzo et al. (2006). They show that if the government were allowed to accumulate assets, they would be able to dispense with distortionary taxation after a finite number of periods. Hence, even in the absence of commitment, a government would set a zero tax rate on capital income in the long-run. The intuition for this result is that the government could confiscate all income in the first period, and then lend households every period and accumulate assets over time. After sufficiently many periods, the government's wealth would be large enough to finance the public good without resorting to distortionary taxation.

Azzimonti-Renzo et al. (2006) also argue that this mechanism does not apply in an overlapping generations (OLG) setting where agents have finite lives, since there exists an endogenous lower bound to taxing an "old" generation. In this paper, instead of using an OLG model, I make the accumulation of assets unfeasible by imposing a balanced budget. In a sense, this can be considered a "short-cut" that allows me to use a simpler setting and facilitates the analysis below.

Finally, the economy's resource constraint is given by:

$$c + g + i + \kappa m = \mathcal{F}(f(k,h,m), s). \tag{2.5}$$

where i denotes investment.

3 Finite Horizon

In this section, to illustrate the main mechanisms at work, I contrast the well-known result derived by Bovenberg and de Moij (1994) and others using a static framework with a two-period model with capital, and derive the Markov-perfect equilibrium without commitment. Throughout the section, for simplicity, assume that the public good is not valued by the household, hence its per-period utility $u(c, l, s)$ depends on private consumption, leisure and environmental quality. Thus, government expenditures are exogenously given.

3.1 The One-Period Model Revisited

Consider the case where $T = 0$, i.e. a static version of the dynamic model introduced above. Hence, investment is zero and the capital stock is given. Environmental quality is a flow variable. While this static model is not appropriate to analyze long-term effects on the climate - greenhouse gases are a stock rather than a flow pollutant - one can think of other pollution externalities, for example an effect on air quality.

Start by defining two "wedges", that is, distortions of the first-best margins. Let ω_{LL} denote the "labor-leisure wedge" and ω_{Env} the "environmental wedge", respectively. They are given by:

$$\omega_{LL} \equiv u_c F_h - u_l \tag{3.1}$$

$$\omega_{Env} \equiv u_c(F_m - \kappa) + q_m[u_s + u_c F_s]. \tag{3.2}$$

The first-best equilibrium, when lump-sum taxes are available, is characterized by both wedges being zero. For the environmental wedge, this just implies that the marginal benefit of increasing emissions net of private cost - the first term in (3.2) - in terms of utility equals the marginal social cost, i.e. the marginal environmental damage, captured by the second term.

In a decentralized equilibrium where the government has access to a lump-sum tax, the government can implement the first-best allocation by imposing a per-unit tax τ^m on emissions. In equilibrium, producers use fuel such that $F_m = \tau^m + \kappa$, hence the *Pigouvian* tax is given by

$$\tau^m = -q_m \left[\frac{u_s}{u_c} + F_s \right] \equiv \tau^p. \tag{3.3}$$

Government expenditures are financed through the revenue from the pollution tax, $\tau^m m$, and through a lump-sum tax.

Now consider a second-best setting, in which the government can only use the pollution tax and a proportinal tax τ^h on labor income to raise government revenue. Bovenberg and Goulder (1996) have shown that the optimal pollution price in this case is given by

$$\tau^m = -q_m \left[\frac{u_s}{u_c} \frac{1}{\eta} + F_s \right], \tag{3.4}$$

where η is the marginal cost of public funds, which measures the social cost of public consumption in terms of private consumption: raising one dollar in public revenue requires a reduction of η dollars in private consumption[10].

[10] Formally, let γ be the Lagrange multiplier associated with the government's budget constraint in period 0. Then, the marginal rate of public funds (MCPF) is defined as

$$\eta = \frac{dc}{dg} = \frac{\gamma}{u_c}.$$

In words, the MCPF is the ratio of the government's shadow cost of raising revenue, and the marginal utility of private consumption (Bovenberg and Goulder, 2002). If g enters the household's utility function, $\eta = u_g/u_c$.

If the government has access to a lump-sum tax, the MCPF equals unity. Then, (3.3) and (3.4) are identical, yielding the first-best outcome with a Pigouvian tax, $\tau^m = \tau^p$. In contrast, if the government has to resort to distortionary taxes on production factors, $\eta > 1$[11]. This is due to the fact that providing the public consumption good is more costly than in first-best since it is financed through linear taxes on labor, which erodes the tax base if $\epsilon_{h,w} > 0$.

In this case, the optimal pollution tax is below the Pigouvian level, $\tau^m < \tau^p$. τ^m represents an implicit tax on labor, since it reduces the real return to labor. Hence, it further decreases labor supply and thus the tax base, thereby causing a first-order welfare loss. In other words, it enhances the distortion caused by the labor tax (Bovenberg and Goulder, 1996), and this additional cost of providing environmental quality (or equivalently of reducing pollution) increases with η. Put differently, in the second-best case, it is optimal to have higher than first-best pollution, since decreasing pollution is more costly than in first-best[12].

As noted by Bovenberg and Goulder (2002), one can see that this result hinges on the assumption that the household derives utility from environmental quality. If this was not the case, and thus if $u_s = 0$, the optimal pollution level would be identical in both settings. In other words, the externality is not fully internalized only if pollution has a negative effect on utility[13].

An alternative way of formalizing this result is deriving a "generalized Euler equation", following Klein et al. (2008). The government's problem in this economy is given by

$$\max_{m,h} u(F(h,m) - g - \kappa m, 1-h, q(m))$$

s.t. to the household's optimality condition, $u_c F_h (1-\tau^h) - u_l = 0$ and its budget constraint, $c = (1-\tau^h) F_h h$[14]. The two constraints can be consolidated to the *implementability constraint*:

$$0 = u_c\big(F(h,m) - g, 1-h, q(m)\big)[F(h,m) - g] - u_l\big(F(h,m) - g, 1-h, q(m)\big)h \equiv \eta(h,m). \quad (3.5)$$

Define the function $\mathcal{H}$ implicitly by

$$\eta(\mathcal{H}(m), m) = 0. \quad (3.6)$$

In words, for a given resource use m, $\mathcal{H}(m)$ gives the household's optimal labor supply, i.e. the labor choice that satisfies its optimality condition. In a sense, it is the household's "best response" to the fossil fuel use announced by the government (assuming within-period commitment). Using $\mathcal{H}$, the

[11] In particular, if $u_{cl} = 0$, one can solve for η as

$$\eta = \left(1 - \epsilon_{h,w} \frac{\tau^h}{1-\tau^h}\right)^{-1},$$

where $\epsilon_{h,w}$ denotes the (uncompensated) wage elasticity of labor supply. Note from this expression that η exceeds unity only if both $\tau^h > 0$ - there are positive labor taxes - and $\epsilon_{h,w} > 0$, that is, labor supply increases with the wage. Compare also Bovenberg and Goulder (1996, 2002) for a more detailed exposition.

[12] Note that this is a net effect, accounting for revenue recycling. In other words, the social cost of providing environmental quality is higher than in first-best, even though its provision creates revenue, which can be used to decrease the distortionary labor tax.

[13] Intuitively, in the case of contemporaneous productivity damages only, a marginal increase in fuel use from the first-best level leaves output - and hence the wage - unchanged. This is not the case if there are direct damages to utility.

[14] If both the household's budget constraint and resource constraint, $c = F(h,m) - g - \kappa m$, are satisfied, so is the government's budget constraint, $g \leq \tau^h F_h h + (F_m - \kappa)m = \tau^h F_h h + \tau_m m$.

government's problem can be more compactly written as:

$$\max_{m} u(F(\mathcal{H}(m), m) - g - \kappa m, 1 - \mathcal{H}(m), q(m)). \tag{3.7}$$

Taking the derivative of (3.7) w.r.t. m gives the following optimality condition:

$$\underbrace{u_c(F_m - \kappa) + q_m[u_s + u_c F_s]}_{\omega_{Env}} + \mathcal{H}_m \underbrace{(u_c F_h - u_l)}_{\omega_{LL}} = 0. \tag{3.8}$$

This shows that in equilibrium, the government trades off wedges. In first-best, since both wedges are zero, (3.8) holds. In second-best, if $\tau^h > 0$, $\omega_{LL} > 0$, hence the environmental wedge cannot be zero unless $\mathcal{H}_m = 0$. More precisely, if $\mathcal{H}_m > 0$ ($\mathcal{H}_m < 0$), ω_{Env} must be negative (positive) in order for (3.8) to be satisfied.

Expression (3.8) illustrates that there is in general an interaction between the emission tax and the non-environmental tax, in the sense that in the presence of a distortionary tax on labor income, environmental quality is not provided at the first-best margin. Moreover, whether or not the pollution externality is less than fully internalized, i.e. whether or not $\omega_{Env} < 0$, depends on the sign of $\mathcal{H}_m$.

For terminology, note that (3.8) contains the derivative of the policy function for labor, hence, following Klein et al. (2008), I refer to it as a *generalized* Euler equations.

In the appendix, I show that if the utility function is weakly separable in environmental quality, and hence if $u_{cs} = u_{ls} = 0$,

$$\mathcal{H}_m = (F_m - \kappa + F_s q_m) \frac{\epsilon_{h,w}}{F_h[1 - \tau(1 + \epsilon_{h,w})]}. \tag{3.9}$$

This expression gives some necessary conditions for $\mathcal{H}_m \neq 0$. First, a change in the net wage must affect the labor supply decision. In other words, if the uncompensated wage elasticity of labor, $\epsilon_{h,w}$, is zero[15], so is $\mathcal{H}_m$. Moreover, note that in the special case where environmental quality affects only productivity, but not utility, the environmental wedge is given by $\omega_{Env} = u_c(F_m - \kappa + q_m F_s)$. Hence, if $\omega_{Env} = 0$, this implies that $\mathcal{H}_m = 0$, so (3.8) holds with a fully internalized pollution externality.

The sign of $\mathcal{H}_m$ is in general ambiguous. One empirically relevant case is summarized in the following proposition:

Proposition 1 *In the static model with a positive tax τ^h on labor income, if $F_m - \kappa + F_s q_m > 0$ and*

$$0 < \epsilon_{h,w} < \frac{1 - \tau^h}{\tau^h},$$

labor supply increases in fossil fuel use, and hence the environmental wedge is negative:

$$\mathcal{H}_m > 0 \quad \rightarrow \quad \omega_{Env} < 0.$$

In words, the pollution externality is not fully internalized if a higher fuel use has a positive effect - net of the pollution damage - on output, and if the uncompensated wage elasticity of labor is positive but sufficiently small compared to the labor tax rate.

[15] In particular, this is not the case with an additive-separable, logarithmic utility function. In this case, when the wage increases, the labor supply remains unchanged, since the income and the substitution effect cancel out.

In this case, using fuel has an additional benefit in second-best, apart from the usual benefit of increasing consumption: it raises labor supply, which leads to a first-order welfare gain, as $\omega_{LL} > 0$. This reduces the marginal social cost of pollution, so that optimal fuel use, relative to consumption, increases compared to the first-best case in which this additional benefit does not materialize[16]. Put differently, a pollution tax below the Pigouvian level can be interpreted as a subsidy to fuel use.

In the next section, in a two-period model with capital, I show that one can derive a generalized Euler equation equivalent to (3.8) for the case of a setting with a zero labor tax, and hence capital as the only tax base. However, as I will outline below, the mechanism is slightly different than in the static case. In particular, current fuel has no effect on the return to current savings, but instead changes the amount of resources available to the household.

3.2 A Two-Period Model With Capital

Let $T = 1$, so the representative household lives for two periods and can postpone consumption by saving into an asset, referred to as capital. Moreover, the government can impose a tax on the returns to capital. As it is well-known, there is a time-inconsistency problem associated with capital taxation: since the tax base - the current capital stock - is inelastic, the capital tax is *ex-post* non-distortionary and the government treats it as a lump-sum tax. However, tomorrow's capital tax rate will affect today's saving decision and hence tomorrow's tax base. In other words, it is *ex-ante* distortionary.

Suppose that in the current period, the government announces a capital tax rate for next period that accounts for this distortion, for example a zero rate. In the following period, in absence of a commitment technology, the government has an incentive to reoptimize and to impose a higher tax rate than previously announced[17]. In this sense, tax rates set under the assumption of commitment are not time-consistent.

There are two ways to deal with the time-inconsistency problem inherent in the structure of the model: one is to assume that the government has access to a commitment device and thus is not able to reoptimize in the second period. This is the assumption that the seminal work by Judd (1985) and Chamley (1986) builds upon. Alternatively, one can abstract from any commitment mechanism and require the government to make time-consistent choices at each point in time.

In doing so, I will focus on Markov-perfect equilibria. The basic idea of this equilibrium concept is that only current payoff-relevant states, but not the history of states and actions, matter for a player's action choice.

Note that in this setting, the current goverment plays a game with its successor[18]. In a two-period model, this just implies that the current government (in period 0) will take into account the optimal behaviour (response) of period 1's government when solving its problem. While the current government cannot directly choose next period's policies, it can affect them indirectly by choosing the economy's state variables, here the stocks of capital and the pollutant, in the following period.

[16] In first-best, where $\omega_{LL} = 0$, a marginal change in hours worked does not affect welfare.

[17] More precisely, if tax rates are unbounded from above, it would be optimal to impose rates such that the tax revenue covers all government expenditures

[18] Since governments in different periods are identical, the current government actually plays a game "against itself". That is, even though I have the same government making the decisions in both periods, it must be treated as different players, due to the lack of commitment. Equivalently, announcements in the current period about how the government will behave in the following period are not credible.

In order to facilitate comparison with the static case, assume that the stock of the pollutant s evolves in the following way:

$$s_0 = q_0(m_0), \quad s_1 = q(s_0, m_1). \tag{3.10}$$

That is, s_t denotes the pollutant stock *at the end* of period t.

Similarly in the static setting above, the environmental wedge in period 0 is defined as the sum of the direct marginal benefit and cost of pollution, now taking into account stock effects:

$$\omega_{Env} \equiv u_c(F_m - \kappa) + q_m[u_s + u_c F_s] + \beta q_m q_s' \left[u_s' + u_c' F_s'\right] = 0. \tag{3.11}$$

Moreover, let ω_{CS} denote the "consumption-savings wedge", i.e. the distortion of the consumption-savings margin, defined as:

$$\omega_{CS} \equiv -u_c + \beta u_c'[F_k' + 1 - \delta] = 0. \tag{3.12}$$

In first best, when the government has access to lump-sum taxation, both ω_{Env} and ω_{CS} are zero.

To analyze the second best with distortionary taxes, I derive again the government's generalized Euler equation[19]. I solve the model using backwards induction. In the second period, the government's problem can be written compactly as

$$\max_{h_1, m_1} u[F(k_1, h_1, m_1, q(s_0, m_1)) - g_1, 1 - h_1, q(s_0, m_1)]. \tag{3.14}$$

Note that this is identical to the problem of a social planner that takes the capital stock and the stock of the pollutant as given. Although the government does raise a tax on capital income, this tax is not distortionary, but equivalent to imposing a lump-sum tax: in period 1, the investment decision has been made and the capital stock is sunk. Due to the presence of this lump-sum tax, the labor tax is optimally zero. Thus, for a given capital stock, the optimal pollution tax in $t = 1$ is determined as in first-best such that $\omega_{Env} = 0$.

Let $M(k_1, s_0)$ and $H(k_1, s_0)$ denote the policy rules that solves the government's problem in period 1. The government in period 0 takes this function as given when solving its problem. In game-theoretic terms, it is the response functions of the follower (the future government) in this sequential game, which the leader (the current government) must take into account when optimizing. Note that government expenditures are financed by the revenue generated from the tax on capital income and from the pollution tax, hence the tax rate on capital in $t = 1$ is given by

$$T(k_1, s_0) = \frac{g_1 - F_m[k_1, M(k_1, s_0), H(k_1, s_0), q(s_0, M(k_1, s_0))]}{F_k[k_1, M(k_1, s_0), H(k_1, s_0), q(s_0, M(k_1, s_0))]k_1}. \tag{3.15}$$

[19] Alternatively, as before, one can compute the optimal environmental tax as a function of the MCPF. In case of a flow pollutant, it is straightforward to solve for the same expression for the optimal tax in period 0 as in the static model with labor taxation above:

$$\tau_0^m = -q_m \left[F_s + \frac{u_s}{u_c}\frac{1}{\eta}\right] = \tau_0^{p,P} + \frac{\tau_0^{p,U}}{\eta}, \tag{3.13}$$

with the MCPF given by $\eta = 1 - \xi \frac{u_{cc}}{u_c}$, where ξ denotes the Lagrange multiplier associated with the intertemporal implementability constraint (3.16). Thus, the MCPF is computed differently than in the static model. Note that $\eta > 1$ if $\xi > 0$, i.e. the MCPF is greater than unity as long as the implementability constraint has a positive shadow value.

In the following, I assume that this tax is always positive, i.e. government expenditures are large enough so that they cannot be financed by the revenue from the emission tax alone. To simplify notation, let $M(k_1, s_0) = M_1$, $H(k_1, s_0) = H_1$ and $T(k_1, s_0) = T_1$.

In the first period, assume that the government can impose a tax only on capital income, but not on labor income. Hence, there is only an *intertemporal* distortion in this economy, caused by the positive capital tax in period 1.

In $t = 0$, the government solves the following problem:

$$\max_{k_1, m_0, h_0} u[F(k_0, h_0, m_0, q_0(m_0)) - \kappa m_0 - g_0 - k_1, 1 - h_0, q_0(m_0)] \\ + \beta u\left[F(k_1, H_1, M_1, q(q_0(m_0), M_1)) - \kappa M_1 - g_1, 1 - H_1, q(q_0(m_0), M_1)\right]$$

subject to an implementability constraint, which now combines the household's budget constraint with the household's intertemporal optimality condition:

$$0 \geq \beta u_c\big(F_h(1)H_1 + (1 - T_1)F_k(1)k_1, 1 - H_1, q(q_0(m_0), M_1)\big) \\ \cdot \big(F_k(k_1, H_1, M_1, q(q_0(m_0), M_1))(1 - T_1\big) - u_c\big(F_h(0)h_0 + (1 - \tau_0)F_k(0)k_0, 1 - h_0, q_0(m_0)\big).$$

Using the definition of T_1 above, and a similar definition for the tax in period 0, one can rewrite this as:

$$0 \geq \beta u_c\big(F(k_1, H_1, M_1, q(q_0(m_0), M_1)) - g_1 - \kappa M_1, 1 - H_1, q(q_0(m_0), M_1)\big) \\ \cdot \big(F_k(1)(1 - T_1\big) - u_c\big(F(0) - g_0 - \kappa m_0 - k_1, 1 - h_0, q_0(m_0)\big) \equiv \epsilon(k_1, m_0), \tag{3.16}$$

where I have omitted the arguments of the production function.

Let $\mathcal{K}(m_0)$ denote the decision rule implicitly defined by the solution to (3.16):

$$\epsilon(\mathcal{K}(m), m) = 0. \tag{3.17}$$

Similar to the static case, $\mathcal{K}$ can be interpreted as the household's response function to the current fossil fuel choice. Then, the government's problem can be rewritten as

$$\max_{m_0} u[F(k_0, m_0, h_0, q_0(m_0)) - \kappa m_0 - g_0 - \mathcal{K}(m_0), 1 - h_0, q_0(m_0)] \\ + \beta u\left[F(\mathcal{K}(m_0), M_1, H_1, q(q_0(m_0), M_1) - \kappa M_1 - g_1, q(q_0(m_0), 1 - H_1, M_1\right]. \tag{3.18}$$

where $M_1 = M(\mathcal{K}(m_0), s_0)$ etc.

Taking the derivative with respect to m_0, I get again a linear combination of wedges, analogous to (3.8) in the static setting with labor taxation:

$$\omega_{Env} + \mathcal{K}_m \omega_{CS} = 0. \tag{3.19}$$

This equation shows that, as in the one-period model, optimal environmental policy interacts with fiscal policy. From the household's optimality condition, it follows that the consumption-savings wedge ω_{CS} is positive for $T_1 > 0$. This implies that as long as current saving is affected by current fuel use, $\mathcal{K}_m \neq 0$, emissions are not at the Pigouvian level. Moreover, the sign of environmental wedge depends on the sign of $\mathcal{K}_m$: if current savings increase (decrease) in current fuel use, wedge is negative (positive), i.e. the pollution tax is below (above) its Pigouvian level.

The intuition for (3.19) is similar to the static case above. First, note that since households understand that capital will be taxed in the following period and thus their return to savings will be lower, they consume more and save less than in first-best.

Then, if $\mathcal{K}_m > 0$, by increasing fossil fuel use and thus"underproviding" the environmental public good, i.e. by not fully internalizing the pollution damage, the first-period government can increase current savings. This has a first-order welfare gain in second-best since $\omega_{CS} > 0$, and hence the discounted marginal increase in utility due to more consumption in the subsequent period is higher than the marginal utility loss due to less consumption today.

It follows that using fossil fuel has an additional benefit which is not present in first-best. Hence, the optimal emission tax - i.e. the social cost of polluting - does not fully internalize the utility damage caused by the pollution externality, and hence the margin between private consumption and environmental quality is distorted compared to the first-best. This leads to a higher pollution level, relative to consumption. In other words, there is an incentive for the government to subsidize fossil fuel use, i.e. decrease the tax on emissions below the Pigouvian level.

In the case with labor taxation, fossil fuel use affected the labor supply through a change in the return to labor. Here, the mechanism is slightly different. The return to current savings is determined by next period's fuel use, which is not directly connected to current fuel use, at least in case of a flow pollutant. Instead, more fuel use today affects the amount of resources available to the household by increasing today's capital income, i.e. the return to past saving.

Note that this result is analogous to Klein et al. (2008), who consider only not-environmental public consumption. In general, underproviding a public good today dampens underinvestment and thus mitigates the intertemporal distortion caused by the positive tax on capital income.

Whether or not the externality is fully internalized depends on the sign on $\mathcal{K}_m$, which in turn depends on the signs of the derivatives of ϵ. In general, these are ambiguous. However, one can derive an analytical result for two special cases.

Proposition 2 *Consider the case of a flow pollutant, that is, $s_0 = q(m_0)$ and $s_1 = q(m_1)$, and assume that the utility function depends on consumption, but not on environmental quality. Then,*

$$\mathcal{K}_m = 0 \quad \rightarrow \quad \omega_{Env} = 0.$$

Proof $\mathcal{K}(m_0)$ is implicitly defined by $\epsilon(\mathcal{K}(m_0), m_0) = 0$. Differentiating w.r.t. m_0 gives

$$\mathcal{K}_m = -\frac{\epsilon_m}{\epsilon_k'}. \tag{3.20}$$

Taking the derivative of ϵ w.r.t. m_0 yields:

$$\epsilon_m = -u_{cc}(0)[F_m(0) - \kappa + F_s(0)q_m].$$

Assume that the environmental wedge is zero, $\omega_{Env} = 0$. From (3.11) with $u_s = 0$, it follows that $\epsilon_m = 0$ and hence $\mathcal{K}(m) = 0$, which is consistent with the equilibrium condition (3.19).

□

This proposition is an alternative way of showing that in the absence of direct utility damages, the externality is fully internalized even in the presence of distortionary taxes, as mentioned above.

Proposition 3 *Consider the case of a flow pollutant, that is,* $s_0 = q(m_0)$ *and* $s_1 = q(m_1)$*. Then, if* $M_k = 0$*,* $T_k \geq 0$ *and* $u_{cs} \leq 0$*,*

$$\mathcal{K}_m > 0 \quad \rightarrow \quad \omega_{Env} < 0.$$

Proof As before, I have that

$$\mathcal{K}_m = -\frac{\epsilon_m}{\epsilon'_k}. \tag{3.21}$$

Taking derivatives of ϵ and using $M_k = 0$ yields:

$$\epsilon_m = -u_{cc}(0)[F_m(0) + F_s(0)q_m] - u_{cs}(0)q_m,$$

and

$$\epsilon_{k'} = u_{cc}(0) + \beta\{u_c(1)[1 - T(1)]F_{kk}(1) + u_c(1)[-T_k(1)]F_k(1) + u_{cc}(1)[1 - T(1)]F_k(1)^2.$$

If consumption and environmental quality are weak complements ($u_{cs} \leq 0$), $\epsilon_m > 0$. Moreover, if current savings weakly increase the future tax rate ($T_k(1) \geq 0$), $\epsilon_{k'} < 0$. Hence, (3.21) implies $\mathcal{K}_m > 0$.

□

This proposition can be illustrated by a simple example that allows for a closed-form solution, which is done in the appendix.

In general, the assumptions used in Proposition 3 do not hold, and the sign of $\mathcal{K}_m$ is ambiguous, even in the simple case of a flow pollutant. In particular, a change in current savings affects in general future public good provision ($M_k \neq 0$) as well as future capital taxes ($T_k \neq 0$), which in turn may change the household's expectation of next period's return to capital and hence affect its current savings behavior. As discussed by Klein et al. (2008), this may imply that the government has an incentive to further decrease current public goods provision, thus exacerbating the distortion between private and public consumption.

When considering a stock pollutant, even with the very restrictive assumptions made in Proposition 3 it is no longer possible to make an unambiguous statement about the sign of $\mathcal{K}_m$. In addition to the assumptions made in Proposition 2, assume that $M_s(k_1, s_0) = 0$ and $T_s \geq 0$. That is, a change in the initial pollutant stock does not affect the amount of fossil fuel used in the second period, while a higher stock weakly increases the tax rate on capital. Then, the derivative of ϵ w.r.t. current fuel use m_0,

$$\epsilon_m = -u_{cc}(0)[F_m(0) + F_s(0)q_m] - u_{cs}(0)q_m + \beta q_m q'_s u'_c(1)[(1 - T(1))F'_{ks} + F'_k(-T'_s(1))], \tag{3.22}$$

The first two terms are strictly positive, while the third is strictly negative.

For illustration, consider the special case where the two terms in (3.22) cancel out, hence $\mathcal{K}_m = 0$. In this case, there is full internalization of the externality even in second-best. The intuition is that there are two counteracting effects of increasing m. On the one hand, it increases current output and hence current consumption; consumption smoothing then implies that the household moves some of the additional resources to the next period, hence current savings increase. On the other hand, more fossil fuel use today implies a higher stock tomorrow, which decreases tomorrow's output (and tomorrow's capital tax rate) and hence the return to current savings. Hence, in this case, there is no "second-best benefit" of raising pollution above the Pigouvian level.

Since the sign of $\mathcal{K}_m$, and hence the direction of the tax interaction effect is in general ambiguous, it is informative to solve the two-period model above numerically. For a range of reasonable parameter values, I find that $\mathcal{K}_m > 0$, which by (3.19) implies that the marginal damage of pollution is not fully internalized. Put differently, there seems to be an additional second-best benefit of using fossil fuel.

To summarize, this section has illustrated that in two-period setting without commitment, environmental regulation interacts with fiscal policy, in the sense that a distortion of the non-environmental margins affects the environmental margin. Moreover, under very special conditions, one can show analytically, under special assumptions, and numerically that the optimal pollution tax is below the Pigouvian level, hence the externality is not fully internalized. In the following sections, I will turn to infinite-horizon models. While increasing the number of periods gives rise to additional dynamic effects, i.e. adds more terms to the equation 3.19, the basic logic of this expression will still apply throughout the rest of the paper.

4 Infinite Horizon

In this section and the next, I consider infinite-horizon versions of the simple model outlined above. I derive the government's generalized Euler equations and show that the result obtained in the previous section in the two-period economy carries over to the longer horizon: in dynamic models without commitment, environmental policy interacts with distortionary taxation and the Markov-perfect pollution tax is not at its Pigouvian level. In addition to the second-best effects of emissions present in the two-period model above, I identify further benefits and cost from current fuel use due to an effect on future public goods provision.

In the next section, in a simple climate-economy model, I conduct a quantitative analysis with the goal of computing the social cost of carbon, which is equal to the optimal time-consistent (global) carbon tax under distortionary taxation, and comparing it to both a first-best time path - i.e. if lump-sum taxes were available - and the marginal climate damages. I find that in this setting, the second-best carbon tax is below, but very close to the Pigouvian level.

Moreover, by comparing the Markov-perfect carbon tax to the outcome under commitment, I will show that in the presence of distortionary taxes, the optimal pollution price is in general not time-consistent. That is, the tax schedule set by a government which had access to a commitment device is different than the one chosen under lack of commitment. Note that this time inconsistency is due to the interaction between environmental and non-environmental taxes: as seen above, the optimal pollution price depends on the optimal tax structure. If other taxes are time-inconsistent - for example a positive tax on labor income in a scenario where separate tax rates on labor and capital are feasible - so is the environmental tax.

To keep the exposition simple, I assume that current emissions affect environmental quality only in the future, but not today[20], and the stock follows a simple recursive law of motion: $s_{t+1} = q(s_t, m_t)$.

[20] I could have assumed here that current emissions do affect both current and future utility or producitivity. However, the simpler case of future impacts only seems more relevant for climate change, where damages occur with a considerable

4.1 First-best

In the presence of lump-sum taxation, the first-best equlibrium is characterized by the following set of equations:

$$\omega_{CS} \equiv u_c - \beta u_c'[F_k' + 1 - \delta] = 0 \tag{4.1}$$

$$\omega_{LL} \equiv u_l - u_c F_h = 0 \tag{4.2}$$

$$\omega_{PG} \equiv u_g - u_c = 0 \tag{4.3}$$

$$\omega_{Env} \equiv u_c(F_m - \kappa) + \beta q_m \left[u_s' + u_c' F_s' - \frac{q_s'}{q_m'} u_c'(F_m' - \kappa) \right] = 0. \tag{4.4}$$

As before, I define wedges for the consumption-savings margin (ω_{CS}), the labor-leisure margin (ω_{LL}), and the environmental margin (ω_{Env}). In addition, since public consumption is endogenous, there is a wedge the public-private good margin, denoted by ω_{PG}. The first-best equilibrium is characterized by all these wedges being simultaneously zero.

4.2 Second-best with commitment

Turning to the second-best setting without lump-sum taxes, start by assuming that the government has access to a technology that allows it to commit to all future tax rates. If the government can impose separate tax rates on income from labor and capital, as opposed to a general income tax, we know from the seminal work by Chamley (1986) and Judd (1985) that capital taxes are zero in the long run (steady state). Moreover, if per-period utility is separable in consumption and leisure, and has a constant intertemporal elasticity of substitution with respect to consumption, capital taxes are zero as of the second period.

The household takes the sequences of before-tax factor prices and taxes, $\{\tau_t^k\}$ and $\{\tau_t^h\}$, as given. Solving its problem gives rise to the usual optimality conditions:

$$u_c(t) - \beta u_c(t+1)[1 + (1 - \tau_{t+1}^k)(r_{t+1} - \delta)] = 0 \tag{4.5}$$

$$u_l(t) - u_c(t)(1 - \tau_t^h) w_t = 0. \tag{4.6}$$

The government's problem can be written as

$$\max_{c_t, k_{t+1}, s_{t+1}, h_t, g_t, m_t, \tau_{t+1}^k, \tau_t^h} \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - h_t, g_t, s_t)$$

s.t.

$$c_t + g_t + k_{t+1} \leq F(k_t, h_t, m_t, s_t) + (1 - \delta) k_t \tag{4.7}$$

$$g_t \leq F_m(t) m_t + \tau_t^h F_h(t) h_t + \tau_t^k (F_k(t) - \delta) k_t, \tag{4.8}$$

$$s_{t+1} \geq q(s_t, m_t) \tag{4.9}$$

as well as (4.5) and (4.6), with $w_t = F_h(t)$ and $r_t = F_k(t)$. That is, the government maximizes its objective function, lifetime utility, subject to the resource constraint, its budget constraint and the household's optimality conditions.

time lag.

In the spirit of the so-called "primal approach" (Ljungqvist and Sargent, 2004) I can substitute (4.5) and (4.6) in (4.7) to eliminate taxes from the problem:

$$\begin{aligned} g_t \leq\ & F_m(t)m_t + \left(1 - \frac{u_l(t)}{u_c(t)}\right) F_h(t)h_t \\ & + \left[1 - (F_k(t) - \delta)^{-1} \left(\frac{u_c(t)}{\beta u_c(t+1)} - 1\right)\right] (F_k(t) - \delta)k_t \end{aligned}$$

for $t \geq 1$. Using the resource constraint and the fact that $F_m m + F_k k + F_h h = F(k, h, m)$, this implementability constraint can be written as

$$u_l(t)h_t + \beta^{-1}u_c(t-1)k_t \leq u_c(t)(c_t + k_{t+1}). \tag{4.10}$$

This formulation applies to the case in which the government has access to two separate tax rates levied on labor and capital income, respectively. Below, I will consider two additional cases. First, I restrict the labor tax to be non-positive: $\tau_t^h \leq 0$. This gives rise to the additional constraint

$$u_c(t)F_h(t) \leq u_l(t). \tag{4.11}$$

Second, I consider a total income tax, i.e. the government taxes labor and capital income with the same rate. In this case, the additional constraint reads:

$$F_h(t)[u_c(t-1) - \beta u_c(t)] - \beta[F_k(t) - \delta]u_l(t) = 0. \tag{4.12}$$

Taking first-order conditions gives rise to a system of n non-linear equations in n unknown variables for each period that characterizes the "Ramsey equilibrium". It is straight forward to solve for the steady state and for the transition path, by guessing that the steady state will be reached after T periods and hence that $k_T = k_{T-1}$ etc.

4.3 Second-best without commitment

Next, I relax the assumption of a commitment device, and instead look for the time-consistent stationary Markov-perfect equilibrium in this economy[21]. The analysis is a straight forward extension of Klein et al. (2008), adding a second public good, environmental quality, which in contrast to the other good affects not only utility, but also the production process. Moreover, in the general case, it is the stock rather than the flow of this public good that matters. Hence, compared Klein et al. (2008), there is an second state state variable in addition to capital, namely the current pollutant stock s.

Capital income tax only For ease of exposition, I start by defining the Markov-perfect equilibrium for the case of capital taxes only, where taxes on labor income are exogenously set to zero. In other words, the household's intratemporal consumption-leisure margin is assumed to be undistorted. This may seem a very restrictive assumption. As Martin (2010) shows, however, this is a short-cut for a more general setting in which the government has ex-ante access to two separate taxes, one on capital

[21] An alternative approach would be to look for all sustainable equilibria, along the lines of Phelan and Stacchetti (2001) or Reis (2011).

and one on labor income. Assuming that labor taxes are bounded to be non-negative[22], the equilibrium features a zero tax rate on labour income. This is intuitive: recall that taxes on capital are ex post non-distortionary, and thus can be considered as de facto lump-sum taxes. Hence, in the presence of such a tax, assuming that it is unbounded, it cannot be optimal to have a positive distortionary tax on labor income.

A Markov-perfect equlibrium is defined as a value function v, differentiable policy functions ψ and ϕ, a savings function n^k and a labor-supply function n^h such that for all k and s, $\psi(k,s)$, $\phi(k,s)$, $n^k(k,s)$ and $n^h(k,s)$ solve

$$\max_{k',s',h,g,m} u(F(k,h,m,s)+(1-\delta)k-g-k',1-h,g,s)+\beta v(k',s'), \tag{4.13}$$

subject to

$$\begin{aligned}&u_c[\mathcal{C}(k,k',h,g,m,s),1-h,g,s]\\ &-\beta u_c'[\mathcal{C}(k',n^k(k',s'),p(k',s'),\psi(k',s'),\phi(k',s'),s'),1-n^h(k',s'),\psi(k',s'),s']\\ &\cdot\Big\{1+[1-T(k',n^h(k',s'),\psi(k',s'),\phi(k',s'),s')][F_k(k',n^h(k',s'),\phi(k',s'),s')-\delta]\Big\}=0,\end{aligned} \tag{4.14}$$

and $s'=q(s,m)$, and where

$$\mathcal{C}(k,k',h,g,m,s)=F(k,h,m,s)+(1-\delta)k-g-k', \tag{4.15}$$

and

$$T(k,h,g,m,s)=\frac{g-F_m(k,h,m,s)m}{(F_k(k,h,m,s)-\delta)k}. \tag{4.16}$$

Moreover, for all k,

$$\begin{aligned}v(k,s)=&u[\mathcal{C}(k,h(k,s),n^h(k,s),\psi(k,s),\phi(k,s),s),1-n^h(k,s),\psi(k,s),s)]\\ &+\beta v(n^k(k,s),q(s,\phi(k,s))).\end{aligned} \tag{4.17}$$

As outlined above, the current government plays a game against its successor, possibly itself. Following the one-stage deviation principle, the current government's strategy constitutes an equilibrium if it maximizes its objective function, subject to all relevant constraints, taking the strategies of the other player - the future government - as given. In other words, assuming that the future government chooses policies according to the equilibrium decision rules, it must be optimal for the current government to follow the same policy functions.

When solving the model I follow Klein et al. (2008). Denote the left hand side of (4.14) as $\eta(k,s,k',h,g,m)$. Define the function $\mathcal{K}(k,s,g,m)$ implicitly as

$$\eta(k,s,\mathcal{K}(k,s,g,m),h,g,m)=0. \tag{4.18}$$

As in the two-period model above, $\mathcal{K}$ can be interpreted as the household's response function to the current government's policy choice, assuming that future governments follow the equilibrium policies:

[22] In other words, the government is not allowed to subsidize labor. Martin (2010) shows that subsidizing labor is actually optimal in a setting with unrestricted separate tax rates on labor and capital. However, this seems to be less relevant empirically than zero labor taxes.

it gives the household's optimal savings level if the current governments set expenditures g and a pollution tax that results in emission level m. In equilibrium, $\mathcal{K}(k, s, \psi(k), \phi(k)) = n^k(k, s)$.

Solving the government's problem, taking $\mathcal{K}$ as given, results in the following system of optimality conditions that characterize the stationary Markov-perfect equilibrium[23]:

$$u_c - \beta u_c'[1 + (1 - T(k', s', h', g', m'))(F_k' - \delta)] = 0 \tag{4.19}$$

$$u_l - u_c F_h = 0 \tag{4.20}$$

$$u_g - u_c + \mathcal{K}_g[-u_c + \beta u_c'(F_k' + 1 - \delta)] - \beta \mathcal{K}_g \frac{\mathcal{K}_k'}{\mathcal{K}_g'}(u_g' - u_c') = 0 \tag{4.21}$$

$$\begin{aligned} u_c F_m + \beta q_m \left[u_s' + u_c' F_s' - \frac{q_s'}{q_m'} u_c' F_m'\right] + \mathcal{K}_m[-u_c + \beta u_c'(F_k' + 1 - \delta)] \\ -\beta(u_g' - u_c')\left[\mathcal{K}_m \frac{\mathcal{K}_k'}{\mathcal{K}_g'} + q_m \frac{\mathcal{K}_s'}{\mathcal{K}_g'} - q_m \frac{q_s'}{q_m'} \frac{\mathcal{K}_m'}{\mathcal{K}_g'}\right] = 0 \end{aligned} \tag{4.22}$$

where the latter two equations are again generalized Euler equations. Using the wedges defined in (4.1)-(4.4) above, I can write (4.21) and (4.22) as a linear combination of wedges:

$$\omega_{PG} + \mathcal{K}_g \omega_{CS} - \beta \mathcal{K}_g \frac{\mathcal{K}_k'}{\mathcal{K}_g'} \omega_{PG}' = 0 \tag{4.23}$$

$$\omega_{Env} + \mathcal{K}_m \omega_{CS} - \beta \omega_{PG}' \mathcal{K}_m \frac{\mathcal{K}_k'}{\mathcal{K}_g'} - \beta \omega_{PG}' \frac{q_m}{\mathcal{K}_g'}\left[\mathcal{K}_s' - \frac{q_s'}{q_m'} \mathcal{K}_m'\right] = 0. \tag{4.24}$$

As before, the government trades off wedges in equilibrium. In first-best, as $\omega_{CS} = \omega_{PG} = \omega_{env} = 0$, (4.23) and (4.24) are satisfied. If the government has to resort to distortionary taxation, the household's optimality condition (4.19) implies that if $T(k', s', h', g', m') > 0$, the consumption-savings wedge is positive. Then, unless $\mathcal{K}_m = 0$, (4.23) and (4.24) imply that ω_{PG} and ω_{env} cannot be zero at the same time: in optimum, neither public good is provided at the first-best margin. Recall that for the environmental public good, the result that $\omega_{env} \neq 0$ just says that the social cost of pollution is not equal to the marginal damage.

Before further interpreting (4.23) and (4.24), note that all elements in these expressions are endogenous and solved for simultaneously when computing the equilibrium. Moreover, for most of them it is not possible to determine the sign analytically. Hence, the question if and how the interaction with distortionary fiscal policy affects the optimal pollution tax and whether the social cost of pollution is below or above the pollution damage is a quantitative one, and requires to numerically solve for the equilibrium. This is done in the next section, in a simple model of climate change.

Here, for the sake of the argument, I make assumptions regarding the signs of the derivatives of $\mathcal{K}_m$. In particular, assume that $\mathcal{K}$ increases in k and m, and decreases in g and in s, and that the non-environmental good is underprovided, $\omega_{PG} > 0$. These assumptions are consistent with (4.23), the GEE for the non-environmental public good. Moreover, as I will show in the next section, these signs are what I find when numerically solving the calibrated model.

Closer inspection of (4.24) then shows that there are additional second-best effects which were not present in the two-period model above. Consider the effect of a marginal increase in current fuel use.

[23] In the appendix, section A.3, I derive the generalized Euler equations for the more general case with a total income tax, as discussed below. For the case of a capital income tax only, the derivation is completely analogous, but with $\omega_{LL} = \omega_{LL}' = 0$.

As in first best, it has the usual marginal benefit - increase in current consumption - and marginal damage, captured by ω_{Env}. In addition, it increases current savings by $\mathcal{K}_m$. As in the finite-horizon model above, the second term in 4.24 captures the effect that more fuel use today allows the household to move more resources to the next period, which increases utility in second best. Call this the "direct second-best flow effect" of pollution.

Moreover, a higher capital stock tomorrow due to more fuel use today leads to more savings tomorrow, assuming that $\mathcal{K}_k' > 0$. This, in turn, dampens the intertemporal distortion *in the future* and hence allows the future government to increase provision of current public goods. Recall that the reason why public goods are underprovided is that the government wants to give the household more resources to move to the future. But doing so in the current period increases savings in the future, thereby reducing the need to provide additional resources in the subsequent period. In other words, underproviding public goods today mitigates the desire of the successive government to underprovide public goods tomorrow. This is an additional benefit of current fuel use.

In (4.24), this effect is captured by the third term. Note that it was also present in the model by Klein et al. (2008), hence does not depend on the public good being persistent. Hence, I will refer to this as the "indirect second-best flow effect". Under the assumptions made above, this term is positive, and hence reinforces the direct second-best flow effect. Hence, in absence of any other effects of current fuel use - and in particular when considering a flow pollutant - (4.24) implies that $\omega_{Env} < 0$, i.e. the pollution damage is not fully internalized.

The fourth term in (4.24) captures the effect of a change in current fuel use on the future pollutant stock, and thus on future savings and future public good provision. This happens via two channels: first, a higher stock tomorrow may directly affect future output and household income, and hence future savings. This is reflected in the first term inside the brackets. Moreover, a higher stock also changes tomorrow's optimal fuel use, which in turn affects output and hence the savings decision as well.

To see what this "second-best stock effect" implies for the sign of ω_{Env}, recall that I assumed that in equilibrium, $\mathcal{K}_m > 0$, $\mathcal{K}_k > 0$, $\mathcal{K}_g < 0$ and $\mathcal{K}_s < 0$. With only a flow pollutant, these assumptions imply that the social cost of pollution is not at the Pigouvian level. With a stock pollutant, this is no longer true in general: while the second and the third term of (4.24) are positive, the fourth term, however, is negative, given that $q_s > 0$ and $q_m > 0$ by definition of the function q. This implies that the sign of the first term is now ambiguous, and possibly positive. In other words, there can be an equilibrium that features an undistorted environmental margin (i.e. $\omega_{Env} = 0$), despite all other margins being distorted, or even an *overprovision* of environmental quality.

Put differently, if future savings decrease as a consequence of a higher pollutant stock, then using more fuel in the current period has an additional cost, since it exacerbates the distortion *in the future*, which induces the subsequent government to reduce public good provision. This cost may partly or even fully offset the second-best benefits of fuel use, and hence move the social cost of pollution closer to or even above the marginal pollution damage.

Total income tax Next, I want to consider a setting in which both the intertemporal consumption-savings margin and the intratemporal labor-leisure margin are distorted simultaneously, i.e. where I have positive tax rates on both labor and capital income. Empirically, such a setting appears much more realistic than assuming that the government finances its expenditures using only a tax on capital income. As discussed above, however, this outcome cannot occur if the government can set different tax rates on labor and capital, given that the capital income tax is not bounded from above.

There are several ways to modify the model such that one would get positive tax rates on both production factors in equilibrium[24]. The simplest approach, following Klein et al. (2008), is to assume a "total income tax", that is, restricting the government to impose the same tax rate on both labor and capital income.

Define the best-response function $\mathcal{H}(k, g, m, s)$ for the household's labor choice. The generalized Euler equation with respect to fuel use then reads[25]:

$$\begin{aligned} & \omega_{Env} + \mathcal{K}_m \omega_{CS} - \beta \omega'_{PG} \mathcal{K}_m \frac{\mathcal{K}'_k}{\mathcal{K}'_g} - \beta \omega'_{PG} \left[q_m \frac{\mathcal{K}'_s}{\mathcal{K}'_g} - q_m \frac{q'_s}{q'_m} \frac{\mathcal{K}'_m}{\mathcal{K}'_g} \right] \\ & \quad + \mathcal{H}_m \omega_{LL} + \beta \omega'_{LL} \mathcal{K}_m \left[\mathcal{H}'_k - \frac{\mathcal{K}'_k}{\mathcal{K}'_g} \mathcal{H}'_g \right] \\ & \quad + \beta \omega'_{LL} q_m \left[\mathcal{H}'_s - \frac{\mathcal{K}'_s}{\mathcal{K}'_g} \mathcal{H}'_g - \frac{q'_s}{q'_m} \mathcal{H}'_m + \frac{q'_s}{q'_m} \frac{\mathcal{K}'_m}{\mathcal{K}'_g} \mathcal{H}'_g \right] = 0 \end{aligned} \tag{4.25}$$

The first four terms are the same as before. In addition, a change in current fossil fuel affects both current and future labor supply, which has non-zero impacts on utility in second-best. The first of the additional terms, $\mathcal{H}_m \omega_{LL}$, captures the same effect of current fuel use on labor supply as in the static model with labor taxation, described in section 3.1 above.

The second additional term reflects the second-best effect of an increase in current fuel use on tomorrow's labor-leisure wedge through a change in the future capital stock, which in turn affects tomorrow's labor and savings decision. The former, direct effect is captured by the term $\mathcal{H}'_k$. The latter effect, in turn, impacts the level of public goods provision and taxation, which also affects labor supply, as is illustrated by the term $\frac{\mathcal{K}'_k}{\mathcal{K}'_g} \mathcal{H}'_g$.

Similarly, the last term in (4.25) captures the impact of current fuel use on the future distortion via tomorrow's pollutant stock. Again, this affects the tomorrow's labor supply both directly and through its effect on future savings and hence public goods provision and taxation, as before ($\mathcal{H}'_s - \frac{\mathcal{K}'_s}{\mathcal{K}'_g} \mathcal{H}'_g$). In addition, a higher pollutant stock induces the future government to put a stricter limit on emissions, all else equal. A change in fuel use then affects future labor supply and hence tomorrow's labor-leisure wedge in an analogous way as changes in the capital and pollutant stock ($-\frac{q'_s}{q'_m} \mathcal{H}'_m + \frac{q'_s}{q'_m} \frac{\mathcal{K}'_m}{\mathcal{K}'_g} \mathcal{H}'_g$).

Taking stock To summarize, in this section I have shown that the result from the simple two-period model, namely that there is a tax-interaction effect in a dynamic model without commitment, carries over to the infinite-horizon case. In this setting, I have identified the second-best benefits and costs of emissions that affect the social cost of pollution.

Note that this section has given no indication about how important the tax-interaction effect is. As argued above, this is a quantitative question. As an application, I will therefore compute the optimal Markov-perfect carbon tax in a calibrated climate-economy model, and compare it to the outcome in first best, under commitment, and when internalizing only the pollution damage.

[24]Martin (2010) considers an exogenous upper bound on the capital tax, as well as making the utilization rate of capital endogenous. The former is somewhat unsatisfying since it leaves the origin of the bound unmodeled. The latter changes slightly the logic of the mechanism.

[25]Compare section A.3 in the appendix for the derivation.

5 Application: Climate Change

In this section, I use the framework outlined above to compute the social cost of carbon in a global Markov-perfect equilibrium. When modeling climate change, I resort to a number of simplifying assumptions. First, to keep the number of states low, in the baseline model I restrict the number of state variables to two[26], namely the economy's capital stock and the atmospheric concentration of CO_2. Note that state-of-the-art Integrated Assessment Models (IAM) typically feature more variables summarizing the state of the climate (Nordhaus, 2008; Cai et al., 2013). Increasing the number of state variables is be an important extension of the analysis below.

Second, I look for *stationary* Markov-perfect equilibria. That is, I assume that the economy converges to a balanced growth path (BGP) in the long-run. This in turn requires three features: first, there is no substitute for fossil fuel, hence it is used forever, but is constant along the BGP. Moreover, labor- and of energy-augmenting technical change occur at the same rate. Third, I assume that all carbon in the atmosphere depreciates with a positive rate, i.e. no share of carbon remains in the atmosphere in infinity.

Finally, the model is deterministic, hence I abstract from all uncertainty related to the climate or economic development. These assumptions restrict the empirical validity of the quantitative predictions made about optimal carbon taxes. Nevertheless, the model can be used to illustrate climate policy under different assumptions regarding the tax regime. In this sense, while the absolute levels of the carbon tax given below should be not be taken as precise estimates or policy recommendations, they are informative as to whether and how tax policy matters for environmental regulation.

5.1 Carbon Cycle

When analyzing climate change, one has to model how emissions translate into changes in atmospheric carbon concentration and temperature (the carbon cycle) and how temperature changes map into damages to utility and productivity. The carbon cycle is commonly modeled as a function that maps current and past global CO_2 emissions, plus a vector of initial carbon concentrations, into average global temperature change in period t, relative to the pre-industrial level (Barrage, 2012):

$$T_t = \mathcal{F}(\mathbf{s}_0, m_0, m_1, ..., m_t). \tag{5.1}$$

The dimension of $\mathbf{s}_0$ depends on the number of carbon "reservoirs". For example, the DICE model (Nordhaus, 2008) features three reservoirs, the atmosphere, the upper oceans and biosphere, and the deep oceans with mixing between them. Other studies use a carbon cycle model with only atmospheric concentration (Golosov et al., 2011; Gerlagh and Liski, 2012).

A second important modeling choice is what share of carbon emitted into the atmosphere is - eventually - absorbed by other reservoirs, and what share remains there forever. Golosov et al. (2011) assume that 20% of emitted carbon stays in the atmosphere for infinity, while around 48% leave within one model period. The remainder depreciates at a rate of 2.3% per ten-year period.

[26]In an extension, to check the robustness of the result obtained in the baseline model, I consider the model with a "temperature lag", i.e. I assume that the impact of a higher forcing, resulting from a higher carbon stock, on global mean temperature does not occur instantaneously, but instead that the temperature adapts gradually to higher forcing. In this setting, there is a third state variable, namely the current global mean temperature.

Gerlagh and Liski (2012) model the the atmosphere as a number of "boxes", each representing a share of the atmospheric CO_2 concentration, where box i is characterized by the carbon depreciation rate η_i. The total carbon in the atmosphere is the sum over all boxes:

$$s_{i,t} = (1-\eta_i)s_{i,t-1} + a_i m_{t-1} \tag{5.2}$$
$$s_t = \sum_i s_{i,t}, \tag{5.3}$$

where $\sum_i a_i = (1+\mu)^{-1}$ and μ is the share of emissions that leave the atmosphere within a short time. Gerlagh and Liski (2012) note that three boxes appear to be sufficiently precise to capture the important dynamics of the carbon cycle.

As a benchmark model, in order to hold the number of state variables small, I use (5.2) with one box to model the carbon accumulation process. This allows me to limit the model to only two variable, capital and the atmospheric carbon stock. The latter evolves recursively, according to the simple law of motion:

$$s_{t+1} = (1-\eta)s_t + \zeta m_t, \tag{5.4}$$

where η denotes the rate depreciation of carbon in the atmosphere.

To model the relationship between atmospheric carbon concentration and global temperature change, consider the following simple mapping, used by Gerlagh and Liski (2012):

$$T_t = T_{t-1} + \epsilon(\varphi(s_t) - T_{t-1}), \tag{5.5}$$

where the function $\varphi(s_t)$ gives the *long-run* increase in global mean surface temperature for a given atmospheric carbon stock s_t. A commonly used functional form is[27]:

$$\varphi(s_t) = \phi \frac{\log\left(\frac{s_t}{\bar{s}}\right)}{\log 2}, \tag{5.6}$$

where $\bar{s} = 581GtC$ is the preindustrial CO_2 concentration in the atmosphere. The parameter ϕ denotes the climate sensitivity, defined as the increase in global mean temperature if the atmospheric carbon concentration doubles and usually set around 3.

The parameter ϵ gives the adjustment speed, that is, how much of the overall gap between current and long-run temperature change will be closed between the current and the next period. Golosov et al. (2011) use a special case of (5.5), with $\epsilon = 1$. This implies that the full impact of a higher atmospheric carbon stock on the temperature occurs "instantaneously" (i.e. within the same model period). Instead, the more general mapping by Gerlagh and Liski (2012) allows for an adjustment lag with respect to the temperature change.

Below, in the baseline model, I use the specification with $\epsilon = 1$, and hence the change in mean global temperature is given by $T_t = \varphi(s_t)$. To check for robustness, in section 5.5.3, I then relax the assumption of instantaneous adjustment ($\epsilon < 1$).

[27] This specification of the law of motion for temperature change is a special case of the more general approach in the DICE model. In particular, I abstract from an impact of the temperature in the lower oceans on the surface temperature. The function $\varphi(s_t)$ comes from the commonly used expression for radiative forcing, multiplied with a parameter capturing the temperature change for a unit of forcing (Nordhaus, 2008; Barrage, 2012).

5.2 Utility

Following Klein et al. (2008), I assume a per-period utility function with constant elasticities of substitution, both between consumption and leisure, and between the consumption-leisure composite and the non-environmental public good. As it is standard in the environmental economics literature, I let preferences between private consumption and environmental quality be additively separable:

$$u(c,l,g,s) = \frac{[(1-\alpha_g)(\alpha_c c^\rho + (1-\alpha_c)l^\rho)^{\frac{\psi}{\rho}} + \alpha_g g^\psi]^{\frac{1-\nu}{\psi}} - 1}{1-\nu} + \varsigma(T), \tag{5.7}$$

where ς is a convex function and T denotes the increase in global mean temperature. In words, the function ς denotes the direct impact on utility due to negative In the baseline calibration, I let $\nu \to 1$, $\rho \to 0$ and $\psi \to 0$, hence the per-period utility function is separable in all arguments[28]. Moreover, I assume a quadratic utility damage due to the externality:

$$u(c,l,g,s) = [(1-\alpha_g)\alpha_c \ln c + (1-\alpha_g)(1-\alpha_c)\ln l + \alpha_g \ln g] - \frac{\alpha_s}{2}T^2. \tag{5.8}$$

Note that this specification is convenient since it allows for the existence of a balanced growth path (King et al., 2002) if fossil fuel is used forever.

5.3 Production

Following Nordhaus (2008) and Golosov et al. (2011), damages to productivity enter the production function multplicatively:

$$F(k,h,m,T) = (1-d(T))f(k,h,m),$$

where $d(T)$ denotes the "damage function". A commonly used specification is:

$$d(T) = (1+\gamma T^2)^{-1}\gamma T^2. \tag{5.9}$$

For the gross production function, I assume a nested CES:

$$f(k,h,m) = \left[(1-\xi)\left(k^\theta (A^h h)^{1-\theta}\right)^{\frac{\sigma-1}{\sigma}} + \xi (A^m m)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}. \tag{5.10}$$

As mentioned above, along a BGP, labor- and energy-augmenting productivity, represented by A^h and A^m, respectively, grow at the same rate.

Of course, this is not the only possible specification when using a CES structure. It is used, for example, in the WITCH integrated assessment model (Bosetti et al., 2006) and by Hassler et al. (2011). Note the specific structure assumed here: the elasticity of substitution between capital and labor is unity, hence they enter in a Cobb-Douglas production function. This assumption is made mainly for simplicity: a more general model would allow for elasticities of substitution greater or smaller than one. The elasticity of substitution between this capital-labor composite and energy is denoted by σ. This is commonly assumed to be between zero and one; in the baseline, I will set $\sigma = 0.5$ as in the WITCH model.

Parameter	Value	Source
Preferences		
β	.985	Nordhaus (2010)
α_c	.29	Calibration
α_g	.12	Calibration
α_s	.018	Calibration, sensitivity
Production		
δ	.08	Standard
θ	.36	Standard
ξ	.03	Hassler et al. (2012)
σ	.5	WITCH
κ	.6	Calibration, Golosov et al. (2012)
γ	.017	Barrage (2012)
Climate		
ϕ	3	Standard
η	.01	Gerlagh and Liski (2012), sensitivity
ζ	.8	Gerlagh and Liski (2012)
$\bar{s}$	581 GtC	Golosov et al. (2012)

Table 1: Baseline Calibration

5.4 Calibration and Solution

The following table lists the parameter settings for the baseline calibration.

Regarding the calibration of the parameters γ and α_s, Barrage (2012) finds that production damages account for about 74% of total output damages if $T = 2.5°C$. From this, she calibrates $\gamma = 0.00172$. I fix this value, and then choose α_s to target an first-best temperature increase of $3°C$, as in Golosov et al. (2011). I conduct some sensitivity analysis by varying the value for α_s and check for the robustness of the results.

The parameters α_c and α_g are calibrated such that the share of time worked and the ratio of government expenditures to output take realistic value, with $g/y \approx 0.2$ and $h \approx 0.27$.

I solve for the Markov-perfect equilibrium by approximating the policy functions n^k, n^h, ψ and ϕ with Chebyshev polynomials and applying the collocation method[29]. Compared to Klein et al. (2008), who use a perturbation method to solve for the steady state, this global method allows me to compute the transition path to the steady state. Given the long-run nature of climate change, the steady state is only of little interest.

[28]Note that (5.8) violates the axioms suggested by Weitzman (2010) for a utility function with climate damages, since the consumption risk aversion (or the inverse of the intertemporal elasticity of substitution) is unity.

[29]A detailed description of the algorithm can be found in the appendix. I check the validity of the results using value function iteration (VFI).

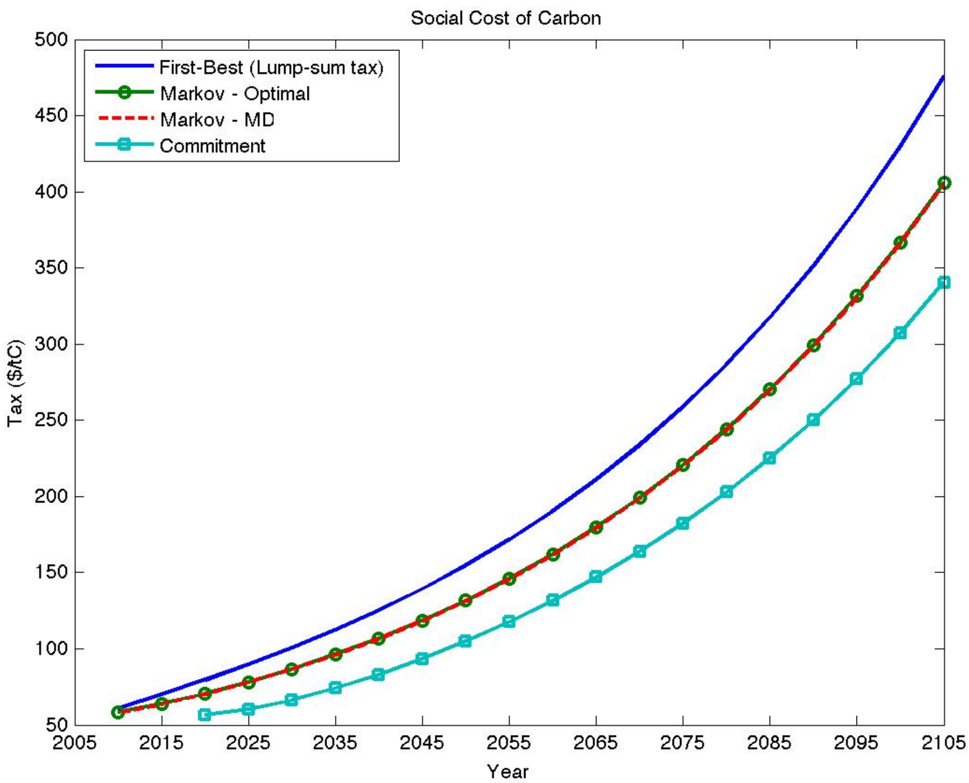

Figure 1: Optimal carbon tax - baseline

5.5 Results

5.5.1 Baseline Model

Figure 1 shows the time path of the optimal carbon price under a total income tax for the first best, the equilibrium with commitment and the Markov-perfect equilibrium, as well the marginal climate damage in the latter case. Table 2 contains the numerical resilts. In first best, the tax schedule starts at \$60.5/tC in 2010 and rises to \$475.5/tC in 2105. In the presence of distortionary taxation, in contrast, the carbon tax increases from about \$58.5/tC (2010) to \$405.8/tC (2105). Hence, comparing the different settings for fiscal policy, the optimal carbon tax is initially only about 3.5% lower in second-best. This gap is increasing over time, to about 16% in 2105. Moreover, the optimal carbon tax in a Markov-perfect equilibrium is very close to the corresponding Pigouvian level, i.e. the marginal damage caused by emissions. In fact, it slightly exceeds it by 0.9% in the baseline setting.

Comparing these results with similar studies, the initial carbon tax rates are close to Barrage (2012), who finds \$67/tC in 2015 for the first-best scenario (compared to \$70/tC here) and \$55/tC for the second-best case (\$64/tC here). In 2105, however, the differences are more pronounced: for the second-best case, Barrage (2012) finds an optimal tax rates of \$541/tC.

Figure 2 displays the analogous graph for the case of a tax only on capital income. Note that first, the absolute level of the social cost of carbon is lower than in the previous scenario, amounting to \$46.8/tC in 2010 and \$271.1/tC in 2105. Moreover, the relative deviation between the optimal tax

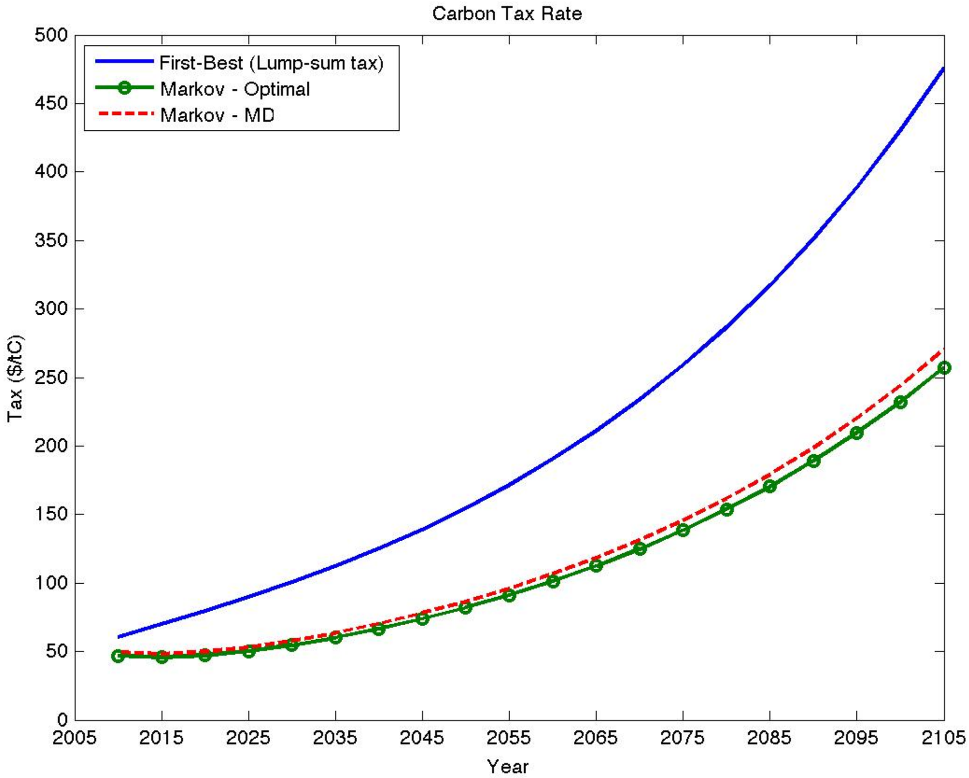

Figure 2: Optimal carbon tax - capital tax only

and the marginal damage is around 5% throughout the whole time span.

What is the intuition for these results? First note that due to the distortions to both the consumption-savings and the consumption-labor margin, the second-best equilibria both with and without commitment feature lower output and lower consumption than the first-best outcome. This, in turn, reduces the damages from climate change, and hence the absolute level of both the Pigouvian and the optimal carbon tax under distortionary taxation. In the capital-tax-only setting, due to the smaller tax base, the capital tax rate and hence the size of the intertemporal distortion is considerably higher than in the baseline model, which further decreases output and consumption, and hence the monetary value of climate damages.

Second, the equilibrium with commitment features very high carbon taxes in the initial periods, while in the long run tax rates are lower than in both the first-best and the Markov-perfect equilibrium. Note that in the long run, the commitment outcome features a higher total income tax and hence lower output and consumption than the other settings (Klein et al., 2008), which implies lower climate damages and hence reduced carbon taxes.

Third, the Markov-perfect carbon tax is very close to the marginal emission damage. To understand this result, recall the generalized Euler equation derived above. For ease of exposition, consider the setting with only a capital income tax:

$$\omega_{Env} + \mathcal{K}_m \omega_{CS} - \beta \omega'_{PG} \mathcal{K}_m \frac{\mathcal{K}'_k}{\mathcal{K}'_g} - \beta \omega'_{PG} \frac{q_m}{\mathcal{K}'_g} \left[\mathcal{K}'_s - \frac{q'_s}{q'_m} \mathcal{K}'_m \right] = 0. \tag{5.11}$$

	$\$/GtC$		τ^m/τ^p		τ^k	
Year	2010	2105	2010	2105	2010	2105
Baseline	58.4	405.7	1.009	1.003	0.191	0.181
CIT	46.8	257.1	0.946	0.952	0.964	0.704
$\alpha_g = 0.17$	56.1	388.3	1.008	1.001	0.270	0.262
$\alpha_g = 0.07$	60.6	423.1	1.005	1.002	0.085	0.075
$\eta = 0.005$	74.5	517.0	1.006	1.001	0.176	0.164
$\eta = 0.025$	35.3	227.3	1.024	1.017	0.215	0.212
$\eta = 0.05$	22.7	130.5	1.066	1.071	0.230	0.231

Table 2: Results

Note that from the definition of ω_{Env}, we have

$$\frac{\omega_{Env}}{\tau_p u_c} = \frac{(F_m - \kappa) + u_c^{-1}\beta q_m \left[u'_s + u'_c F'_s - \frac{q'_s}{q'_m} u'_c F'_m\right]}{\tau_p} = \frac{\tau_m - \tau_p}{\tau_p},$$

where τ_p denotes the Pigouvian level, i.e. the marginal emission damage. Then, dividing (5.11) by $\tau_p u_c$ and rearranging gives an expression for the relative deviation of the second-best carbon tax from the Pigouvian level, Δ_τ[30]:

$$\Delta_\tau \equiv \frac{\tau_p - \tau_m}{\tau_p} = \underbrace{\frac{\mathcal{K}_m \omega_{CS}}{\tau_p u_c}}_{X_1} + \underbrace{\frac{(-\beta \omega'_{PG} \mathcal{K}_m \frac{\mathcal{K}'_k}{\mathcal{K}'_g})}{\tau_p u_c}}_{X_2} + \underbrace{\frac{-\beta \omega'_{PG} \frac{q_m}{\mathcal{K}'_g} \left[\mathcal{K}'_s - \frac{q'_s}{q'_m} \mathcal{K}'_m\right]}{\tau_p u_c}}_{X_3}. \tag{5.12}$$

That is, Δ_τ can be broken down into three terms. X_1 captures the direct second-best flow effect: if $\mathcal{K}_m > 0$, current fuel use increases current savings, which mitigates the intertemporal distortion. X_2 represents the indirect second-best flow effect: if it is positive, the increase in current savings due to current fuel use leads to higher savings in the future, mitigating the future underprovision of public goods. X_3 finally captures the second-best stock effect: current fuel use increases the pollutant stock in the subsequent period, which affects future savings, thus possibly exacerbating the distortion in the future.

Table 3 quantifies these effects. First note that in the baseline setting, X_1 and X_2 are positive, implying that there are additional second-best benefits of fuel use, which decrease the social cost of carbon. That is, if disregarding other effects, the government would have an incentive to increase current emissions and hence to subsidize fuel use beyond the first-best margin. In this case, the social cost of carbon would be around 5.4% below the marginal damage, due to the mechanisms described above.

[30] In case of a general income tax, one can derive an analogous expression using (4.25):

$$\Delta_\tau \equiv \frac{\tau_p - \tau_m}{\tau_p} = \underbrace{\frac{\mathcal{K}_m \omega_{CS}}{\tau_p u_c}}_{X_1} + \underbrace{\frac{(-\beta \omega'_{PG} \mathcal{K}_m \frac{\mathcal{K}'_k}{\mathcal{K}'_g})}{\tau_p u_c}}_{X_2} + \underbrace{\frac{-\beta \omega'_{PG} \frac{q_m}{\mathcal{K}'_g} \left[\mathcal{K}'_s - \frac{q'_s}{q'_m} \mathcal{K}'_m\right]}{\tau_p u_c}}_{X_3}$$
$$+ \underbrace{\frac{\mathcal{H}_m \omega_{LL}}{\tau_p u_c}}_{X_4} + \underbrace{\frac{\beta \omega'_{LL} \mathcal{K}_m \left[\mathcal{H}'_k - \frac{\mathcal{K}'_k}{\mathcal{K}'_g} \mathcal{H}'_g\right]}{\tau_p u_c}}_{X_5} + \underbrace{\frac{\beta \omega'_{LL} q_m \left[\mathcal{H}'_s - \frac{\mathcal{K}'_s}{\mathcal{K}'_g} \mathcal{H}'_g - \frac{q'_s}{q'_m} \mathcal{H}'_m + \frac{q'_s}{q'_m} \frac{\mathcal{K}'_m}{\mathcal{K}'_g} \mathcal{L}'_g\right]}{\tau_p u_c}}_{X_6} = 0.$$

	τ^m/τ^p	2010					
Year	2010	X1	X2	X3	X4	X5	X6
Baseline	1.009	0.015	0.039	-0.062	-0.040	-0.045	0.085
CIT	0.946	0.145	0.266	-0.357	-	-	-
$\alpha_g = 0.17$	1.008	0.027	0.072	-0.112	-0.031	-0.083	0.119
$\alpha_g = 0.07$	1.005	0.005	0.012	-0.020	-0.026	-0.014	0.039
$\eta = 0.005$	1.006	0.014	0.039	-0.064	-0.022	-0.045	0.072
$\eta = 0.025$	1.024	0.012	0.029	-0.044	-0.107	-0.034	0.121
$\eta = 0.05$	1.066	0.002	0.006	-0.009	-0.219	-0.006	0.160

Table 3: Tax interaction effect

However, X_3 is negative. That implies that the second-best stock effect results in an additional cost of fuel use that *decreases* the welfare gain from higher emissions. Hence, the stock effect goes in the opposite direction than the flow effects. Note this is qualitatively true for both the baseline setting with a total income tax and the capital-tax-only scenario. Intuitively, there is a quantitative difference: due to the higher capital tax in the latter case, the consumption-savings and the public-consumption wedge increase. As a consequence, X_1, X_2 and X_3 increase in absolute value, while keeping their signs. However, the increase in the former two is less than cancelled out by the decrease in X_3.

In the baseline model, there are additional effects due to the distortion of the labor margin, represented by X_4 - X_6. Note that X_4, capturing the effect of fossil fuel use on the labor supply discussed above in the static model, is negative. This implies that in equilibrium a marginal increase in fuel use *decreases* the labor supply, i.e. $\mathcal{H}_m < 0$. Intuitively, in the model with both capital and labor, more fuel use increases the return to either production factor. If the increase in capital income is sufficiently high, the household reduces labor supply, despite a higher wage. In other words, the income effect will dominate the substitution effect. In this case, since this exacerbates the intratemporal distortion, it is a second-best cost of fuel use, raising the social cost of carbon and hence mitigating the government's incentive to increase emissions.

Next, X_5 captures the effect of an increase in current fuel use and hence a higher capital stock tomorrow (given that $\mathcal{K}_m > 0$) on tomorrow's labor supply, both directly and through the effect of more public spending and hence a higher tax rate. Formally, these two channels are captures by the term $\mathcal{H}'_k - \frac{\mathcal{K}'_k}{\mathcal{K}'_g}\mathcal{H}'_g$, where, in the baseline setting, I find that $\mathcal{H}'_k > 0$ and $\mathcal{H}'_g < 0$. Overall, this term and hence X_5 is negative, implying that a higher capital stock tomorrow leads a decrease in labor supply. Again, this exacerbates the intratemporal distortion in the future, and hence is a second-best cost of current emissions.

Finally, X_6, capturing the effect of higher pollutant stock on the future labor supply through the different channels described above, gets his sign from the term

$$\mathcal{H}'_s - \frac{\mathcal{K}'_s}{\mathcal{K}'_g}\mathcal{H}'_g - \frac{q'_s}{q'_m}\mathcal{H}'_m + \frac{q'_s}{q'_m}\frac{\mathcal{K}'_m}{\mathcal{K}'_g}\mathcal{L}'_g > 0,$$

which is positive in the baseline equilibrium. Here, increasing current fossil fuel and hence tomorrow's pollutant stock has a positive effect on tomorrow's labor supply, and hence mitigates the intratemporal distortion in the future. This is a second-best benefit of resource use, which decreases the social cost of carbon. Note that quantitatively, the latter two terms, reflecting the effect of tomorrow's fuel use on the labor supply, dominate the direct effects of a higher pollutant stock. In other words, what drives the sign here is the fact that more pollution today causes tomorrow's government to use less energy and hence pollute less. This reduces the household's income, which has two effects: first, it induces the household to increase its labor supply (since again the income effect dominates the substitution effect). Second, it reduces savings, and hence enhances the government's desire to underprovide the public goods. The resulting decrease in the tax rate further increases labor supply.

Overall, in the baseline setting, the second-best benefits and costs cancel out almost exactly, leading to an environmental margin very close to the first best. In other words, the social cost of carbon is almost equal to the marginal climate damage. Note that this result is different from what Barrage (2012) finds in a similar framework without commitment. In her model, the optimal second-best is initially about 20% below the Pigouvian tax.

5.5.2 Sensitivity Analysis

To check the robustness of this result, the remainder of tables 2 and 3 contains the findings for changes to the baseline calibration. First, I increase (reduce) the preference weight of the public consumption good. Intuitively, when the taste for public goods rises (falls), so does the revenue requirement and hence the capital tax rate. For a higher (lower) distortion, output and consumption, and hence climate damage fall (rise), which is reflected in a slightly lower (higher) social cost of carbon. However, Δ_τ remains almost unchanged, since the increase (decrease) in X_1, X_2 and X_6 is again almost completely canceled out by an increase (decrease) in the absolute values of X_3, X_4 and X_5.

In lines 4-6, I change the annual rate of CO_2 depreciation in the atmosphere, η, which is 0.01 in the baseline calibration. In absolute terms, the carbon tax decreases (rises) when this rate increases (falls). The climate damage caused by a unit of fuel emitted today is less severe for a higher depreciation rate, hence the optimal carbon tax is lower. This also affects the capital tax rate: for a higher rate of decay, the government generates less green revenue and hence must impose a higher tax on income, which exacerbates the distortions in the economy.

With regard to the tax-interaction effect, note that only X_4 and X_6 increase in absolute value as η rises, although with different signs. For example, when $\eta = 0.05$, the size and direction of the tax-interaction effect is almost completely determined by the sum of these two terms. At the same time, X_1 is almost zero, which reflects a very small effect of fossil fuel use on savings, i.e $\mathcal{K}_m \approx 0$.

To understand these results, first note that as η decreases, the effect of an increase in current fossil fuel use on the net wage is less pronounced, for two reasons. First, the increase in the gross wage becomes smaller, since more fuel is used for a higher η, which reduces the marginal product. Second, due to the higher income tax, a larger share of the gross wage is taxed away. Below a certain threshold for η, a marginal increase in fuel use *reduces* the net wage, which in turn leads to a fall in the labor supply. This dampens the rise in total income, so $\mathcal{K}_m$ is smaller. If it total income decreases following an increase in fuel use, the effect on current savings may even be negative.

This describes the outcome for $\eta = 0.05$. Here, $\mathcal{K}_m$ is very small, due to an almost zero increase in total income. As a consequence, X_1, X_2 and X_3, all of which depend $\mathcal{K}_m$ tend towards zero. On the other hand, $\mathcal{H}_m$ is negative and different from zero, due to a negative effect of increased fuel use on the net wage. The same is true for $\mathcal{H}'_m$ in the subsequent period, which mainly drives the increase in X_6.

Note, however, that X_6 decreases in η, hence there are two counteracting effects on X_6. This explains why X_4 increases (in absolute value) more strongly than X_6. Overall, for $\eta = 0.05$, the second-best costs of fuel use outweigh the benefits, and hence the social cost of carbon *exceeds* the marginal climate damages in equilibrium.

5.5.3 Temperature Lag

So far I have assumed that a change in radiative forcing translate into a higher temperature instantaneously, i.e. one period. More realistically, temperature adjusts to a higher forcing with a lag (Nordhaus, 2008; Gerlagh and Liski, 2012). That is, let the global mean temperature change evolve according to the following law of motion:

$$T_t = T_{t-1} + \epsilon(\phi \frac{\log\left(\frac{s_t}{\bar{s}}\right)}{\log 2} - T_{t-1}), \tag{5.13}$$

where the adjustment speed ϵ is less than unity. I follow Gerlagh and Liski (2012) who set $\epsilon = 0.186$ for ten-year periods[31].

Note that in this case, since there is no longer a one-to-one mapping between atmospheric carbon concentration and temperature change, there is one additional state variable, namely temperature change in the previous period.

Figure 3 shows the time path for social cost of carbon in first and second best, as well as the second-best marginal emission damage. First note that in all scenarios, the optimal tax is lower than in the baseline setting: the second-best (first-best) tax starts at \$34.1 (\$33.6) in 2010 and increases to \$254.3 (\$294.9) in 2105. Due to the adjustment lag, a large share of climate damages occur at later periods than in the baseline, which implies that the cost of carbon decreases, due to discounting.

Moreover, the result found in the baseline model that the tax-interaction effect is quantitatively irrelevant, is robust to the lower adjustment speed: as before, the social cost of carbon is almost equal to the marginal emission damage, and hence second-best benefits and costs of resource use cancel out.

6 Conclusion

In this paper, I have analyzed optimal environmental regulation in a dynamic framework in the presence of distortionary non-environmental taxes and under lack of commitment. Most importantly, I have characterize the tax-interaction effect in this setting: by deriving the generalized Euler equation of the current government, I have identified a number of additional costs and benefits that arise due to the interaction between environmental policy and distortionary taxation and that are not present in first best. Hence, in general, the social cost of pollution is not at the Pigouvian level, i.e. does not equal marginal pollution damage.

Moreover, by computing the Markov-perfect emission tax in a model of climate change, I have quantified the impact of distortionary taxation and the tax-interaction effect on the social cost of carbon. My main findings are that the second-best carbon tax path is different from the optimal policies both in the presence of lump-sum taxes, and under commitment. Moreover, it is very close to

[31]Since I calibrate my model to five-year periods, I set $\epsilon = 0.1$, which corresponds approximately to an annual adjustment rate of 0.02, used by Gerlagh and Liski (2012): $1 - 0.0204^{10} = 0.186$.

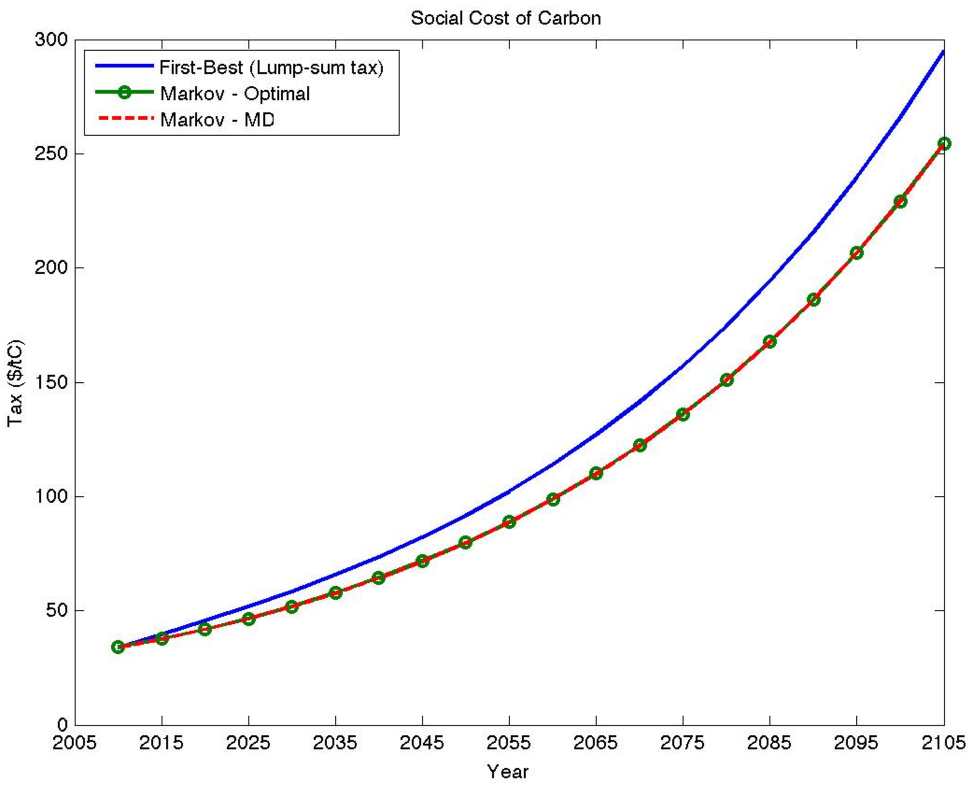

Figure 3: Optimal carbon tax with a temperature lag

the Pigouvian level, hence for my calibration, the climate damages to utility and productivity are fully internalized.

It should be noted, however, that the results from this quantitative exercise likely suffer from the simplicity of the climate-economy model used here. In particular, there are two dimensions in which the model should be improved and which are currently work in progress. First, as discussed above, the climate model is considerably simple, in the sense that it is stationary, features only two states and does not address uncertainty. A more realistic climate model would increase the number of carbon “boxes” in the atmosphere, each with a different rate of carbon depreciation, to at least up to three (Gerlagh and Liski, 2012). Moreover, one should divide the model period into two phases: a finite number of periods in which fossil fuel is used, and an infinite-horizon phase without fossil fuel use, during which the climate approaches a stready state. This is the modeling strategy for example in Cai et al. (2013).

Second, throughout this paper I have considered a one-country model of fiscal and climate policy. In reality, there is no global government, and taxes are set on the national level. Since emissions in one country affect each other country, there is scope for strategic interaction. Hence, it may be interesting to extend the analysis above to a multi-country setting, in which a country, when choosing its policies, takes the behavior of all other countries as given. In other words, this would result in a model with both lack of commitment *within* a country, as well as lack of coordination *across* countries.

Finally, fiscal policy usually features different tax rates on labor and capital income, hence the assumption of a total income tax seems too restrictive. Recall that this assumption allowed me to get a positive tax rate for both types of income. There are other ways to obtain this outcome in equilibrium, as discussed above. In particular, one could get an *endogenous* upper bound on the capital tax by assuming a two-country setting, with or without strategic interaction, but with capital mobility. That is, if one allows capital to be reallocated between countries *after* the tax rates have been announced, this would put an upper bound on the capital tax and hence may require a positive labor tax. This extension is currently work in progress.

References

Azzimonti-Renzo, M., P. Sarte, and J. Soares (2006). Optimal policy and (the lack of) time inconsistency: Insights from simple models.

Barrage, L. (2012). Optimal dynamic carbon taxes in a climate-economy model with distortionary fiscal policy. Mimeo, Yale University.

Bosetti, V., C. Carraro, M. Galeotti, E. Massetti, and M. Tavoni (2006). A world induced technical change hybrid model. *The Energy Journal* (Special Issue), 13–38.

Bovenberg, A. and R. de Mooij (1994). Environmental levies and distortionary taxation. *The American Economic Review 84*(4), 1085–1089.

Bovenberg, A. and L. Goulder (1996). Optimal environmental taxation in the presence of other taxes: general-equilibrium analyses. *The American Economic Review 86*(4), 985–1000.

Bovenberg, A. and L. Goulder (2002). Environmental taxation and regulation. *Handbook of public economics 3*, 1471–1545.

Cai, Y., K. L. Judd, and T. S. Lontzek (2013). The social cost of stochastic and irreversible climate change. *NBER Working Paper 18704*.

Chamley, C. (1986). Optimal taxation of capital income in general equilibrium with infinite lives. *Econometrica: Journal of the Econometric Society*, 607–622.

Gerlagh, R. and M. Liski (2012). Carbon prices for the next thousand years. *CESifo Working Paper Series*.

Golosov, M., J. Hassler, P. Krusell, and A. Tsyvinski (2011). Optimal taxes on fossil fuel in general equilibrium. *NBER Working Paper 17348.*

IWG (2010). Social cost of carbon for regulatory impact analysis under executive order 12866. Interagency Working Group on Social Cost of Carbon (IWG), United States Government. http://www.whitehouse.gov/sites/default/files/omb/inforeg/for-agencies/Social-Cost-of-Carbon-for-RIA.pdf.

Judd, K. L. (1985). Redistributive taxation in a simple perfect foresight model. *Journal of public Economics 28*(1), 59–83.

King, R. G., C. I. Plosser, and S. T. Rebelo (2002). Production, growth and business cycles: Technical appendix. *Computational Economics 20*(1), 87–116.

Klein, P., P. Krusell, and J. Rios-Rull (2008). Time-consistent public policy. *The Review of Economic Studies 75*(3), 789–808.

Kolstad, C. D. (2000). *Environmental economics*, Volume 15. Oxford university press Oxford.

Ljungqvist, L. and T. Sargent (2004). *Recursive macroeconomic theory.* MIT press.

Lontzek, T., Y. Cai, and K. Judd (2012). Tipping points in a dynamic stochastic iam.

Martin, F. (2010). Markov-perfect capital and labor taxes. *Journal of Economic Dynamics and Control 34*(3), 503–521.

Nordhaus, W. (2008). *A question of balance: Weighing the options on global warming policies.* Yale University Press.

Phelan, C. and E. Stacchetti (2001). Sequential equilibria in a ramsey tax model. *Econometrica 69*(6), 1491–1518.

Reis, C. (2011). Taxation without commitment. *Economic Theory*, 1–24.

Weitzman, M. L. (2010). What is the "damage function" for global warming - and what difference does it make? *Climate Change Economics 1*(01), 57–69.

A Appendix

A.1 Static Model: $\mathcal{H}_m$

To get to an expression for $\mathcal{H}_m$, write the government's problem as

$$\max_{m,h} u(F(h,m)-g, 1-h, q(m)) \tag{A.1}$$

s.t. to the implementability constraint

$$\eta(h,m) \equiv u_c\big(F(h,m)-g,1-h,q(m)\big)[F(h,m)-g] - u_l\big(F(h,m)-g,1-h,q(m)\big)h = 0. \tag{A.2}$$

Define $\mathcal{H}(m)$ implicitly by $\eta(\mathcal{H}(m),m)$. Taking the derivative w.r.t. m gives

$$\eta_h\mathcal{H}_m + \eta_m = 0 \quad \to \mathcal{H}_m = -\frac{\eta_m}{\eta_h}. \tag{A.3}$$

Taking derivatives of η gives

$$\begin{aligned}
\mathcal{H}_m = -\frac{\eta_m}{\eta_h} &= -\frac{u_{cc}cF_m + u_cF_m - u_{cl}F_mh}{u_{cc}cF_h - u_{cl}c + u_cF_h - u_{cl}F_hh + u_{ll}h - u_l} \\
&= -F_m\frac{u_{cc}c + u_c - u_{cl}h}{u_{cc}\tilde{w}h(\tilde{w}+\tau F_h) - u_{cl}(c+F_hh) + (u_cF_h - u_l) + u_{ll}h} \\
&= -F_m\frac{u_{cc}c + u_c - u_{cl}h}{\tau F_hu_{cc}c + u_{cc}\tilde{w}^2h - u_{cl}(c+\tau F_hh + c) + \tau u_cF_h + u_{ll}h} \\
&= -F_m\frac{u_{cc}c + u_c - u_{cl}h}{\tau F_h(u_{cc}c - u_{cl}h + u_c) + h(u_{cc}\tilde{w}^2 - 2u_{cl}\tilde{w} + u_{ll})} \\
&= -F_m\frac{\frac{u_{cc}c+u_c-u_{cl}h}{u_{cc}\tilde{w}^2-2u_{cl}\tilde{w}+u_{ll}}}{\left(F_h - \frac{c}{h}\right)\frac{u_{cc}c+u_c-u_{cl}h}{u_{cc}\tilde{w}^2-2u_{cl}\tilde{w}+u_{ll}} + h},
\end{aligned} \tag{A.4}$$

where I have repeatedly used $c = (1-\tau)F_hh = \tilde{w}h$. Note a special case following from (A.4): if $u_{cl} = 0$ and the utility is given by $u(c,l,s) = \log(c) + v1(l) + v2(s)$, we get

$$u_ccc + c - u_{cl}h = 0 \quad \to \ \mathcal{H}_m = 0. \tag{A.5}$$

Next, note that from the household's intratemporal optimality condition, taking the net wage $\tilde{w}$ as given, I can derive the partial-equilibrium response in the labor supply to an increase in the wageby totally differentiating:

$$u_c\big(\tilde{w}h, 1-h, q(m)\big)\tilde{w}h - u_l\big(\tilde{w}h, 1-h, q(m)\big) = 0. \tag{A.6}$$

$$\begin{aligned} d\tilde{w}[\tilde{w}u_{cc}h + u_c - u_{cl}h] + dh[\tilde{w}^2 u_{cc} - \tilde{w}u_{cl} - u_{cl}\tilde{w} + u_{ll}] &= 0 \\ \frac{dh}{d\tilde{w}} &= -\frac{u_{cc}c + u_c - u_{cl}h}{u_{cc}\tilde{w}^2 - 2u_{cl}\tilde{w} + u_{ll}} \end{aligned} \tag{A.7}$$

Hence, the (uncompensated) wage elasticity of labor supply is given by:

$$\epsilon_{h,w} = -\frac{dh}{d\tilde{w}}\frac{\tilde{w}}{h} = -\frac{u_{cc}c + u_c - u_{cl}h}{u_{cc}\tilde{w}^2 - 2u_{cl}\tilde{w} + u_{ll}}\frac{\tilde{w}}{h}. \tag{A.8}$$

Substituting in (A.4), we get

$$\mathcal{H}_m = (F_m + F_s q_m)\frac{\epsilon_{h,w}\frac{h}{\tilde{w}}}{-\left(F_h - \frac{c}{h}\right)\epsilon_{h,w}\frac{h}{\tilde{w}} + h} = (F_m + F_s q_m)\frac{\epsilon_{h,w}}{(1-\tau)F_h - \tau F_h \epsilon_{h,w}},$$

and hence

$$\mathcal{H}_m = (F_m + F_s q_m)\frac{\epsilon_{h,w}}{F_h[1 - \tau(1 + \epsilon_{h,w})]}. \tag{A.9}$$

A.2 Analytical Example

This section follows the example given by Martin (2010) in a model without fuel and environmental quality. Let

$$u(c, g, s) = \alpha_c \ln c + \alpha_g \ln g - \frac{\alpha_m}{2}m^2.$$

This functional form accounts for an increasing marginal damage of the pollution stock. Note that in order to get a closed-form solution, I now assume that public good consumption is endogenous, as it enters the utility function. This does not change the results of the model above qualitatively. Moreover, assume a Cobb-Douglas production function:

$$f(k, m, h) = k^\theta m^\gamma h^{1-\theta-\gamma}, \tag{A.10}$$

where $h = 1$ is given.

In a first-best setting with lump-sum taxes, optimal fuel use can be computed as:

$$m_0 = \sqrt{\frac{\gamma}{\alpha_m}(\alpha_c + \alpha_g)(1 + \beta\theta)} \tag{A.11}$$

$$m_1 = \sqrt{\frac{\gamma}{\alpha_m}(\alpha_c + \alpha_g)}, \tag{A.12}$$

Note that for the particular functional forms used here, equilibrium fuel depends only on parameters and not on the economy's capital stock. The same is true for the capital tax rate in period 1, hence the assumptions in Proposition 3 are satisfied.

The government's problem in period 0 reads:

$$\max_{c_0,k_1,g_0,m_0} u(c_0,g_0,m_0)+\beta u(c_1,g_1,m_1) \tag{A.13}$$

subject to

$$k_1 = \upsilon_0(k_0^\theta m_0^\gamma - g_0) \tag{A.14}$$
$$c_0 = (1-\upsilon_0)(k_0^\theta m_0^\gamma - g_0) \tag{A.15}$$
$$c_1 = \rho_1 k_1^\theta m_1^\gamma \tag{A.16}$$
$$g_1 = (1-\rho_1)k_1^\theta m_1^\gamma \tag{A.17}$$
$$m_1 = \sqrt{\alpha_m^{-1}\gamma(\alpha_c+\alpha_g)}. \tag{A.18}$$

where ρ_1 and υ_0 are functions only of parameters. The first two equations follow from the household's Euler equation and the resource constraint in period 0. The remaining four equations are the solutions to the government's problem in period 1.

When solving this problem (see appendix), I find that the margin between consumption of the private good and both of the public goods is distorted. That is, $u_g = u_c\Omega_0$, and

$$u_m + \Omega_0 u_c f_m = 0, \tag{A.19}$$

where

$$\Omega_0 = \frac{\beta\theta(\alpha_c+\alpha_g)+\alpha_c}{\beta(\theta+\gamma)(\alpha_c+\alpha_g)+\alpha_c-\beta\alpha_g}. \tag{A.20}$$

Note that under the particular functional forms chosen here, fuel use is at its first-best level, as given by (A.11). Hence, there is no underprovision of the environmental public good in absolute terms. However, with $\Omega_0 > 1$, private consumption exceeds its first-best level, implying that savings must be lower. This confirms that the fuel price is below its Pigouvian level: in first-best, a higher consumption would optimally require a higher environmental quality (or equivalently less pollution). Hence, the fact that fuel consumption remains constant while consumption increases, compared to the first-best outcome, implies that pollution exceeds its Pigouvian level. In other words, the public good environmental quality is underprovisioned relative to private consumption (as is public consumption).

A.3 Derivation of the GEE in the Infinite-horizon Model

A.3.1 Markov Equilibrium - GIT

Let the equilibrium policy functions for saving, hours worked, fuel use and government expenditures be denoted by $n^k(k,s)$, $n^h(k,s)$, $\psi(k,s)$ and $\phi(k,s)$, respectively.

Define the residual functions that capture the household's optimality conditions when set to zero:

$$\begin{aligned}&\eta(k,s,g,m,k',h) = u_c[\mathcal{C}(k,k',h,g,m,s),1-h,g,s]\\ &-\beta u_c'[\mathcal{C}(k',n^k(k',s'),p(k',s'),\psi(k',s'),\phi(k',s'),s'),1-n^h(k',s'),\psi(k',s'),s']\\ &\cdot\Big\{1+[1-T(k',n^h(k',s'),\psi(k',s'),\phi(k',s'),s')][F_k(k',n^h(k',s'),\phi(k',s'),s')-\delta]\Big\},\end{aligned} \tag{A.21}$$

and

$$\epsilon(k,s,g,m,k',h) = \frac{u_l[\mathcal{C}(k,k',h,g,m,s),1-h,g,s]}{u_c[\mathcal{C}(k,k',h,g,m,s),1-h,g,s]} - F_h(k,h)[1-T(k,h,g,m,s)]. \tag{A.22}$$

That is, $\eta(k,s,g,m,k',h) = 0$ gives the household's Euler equation and $\epsilon(k,s,g,m,k',h) = 0$ the household's labor-leisure condition.

Define functions $\mathcal{K}(k,s,g,m)$ and $\mathcal{H}(k,s,g,m)$ implicitly by

$$\eta(k,s,g,m,\mathcal{K}(k,s,g,m),\mathcal{H}(k,s,g,m)) = 0 \tag{A.23}$$

$$\epsilon(k,s,g,m,\mathcal{K}(k,s,g,m),\mathcal{H}(k,s,g,m)) = 0. \tag{A.24}$$

Using those functions and

$$\mathcal{C}(k,s,k',h,g,m) = F(k,h,m,s) + (1-\delta)k - g - k',$$

write the government's problem in t compactly as:

$$\max_{k',s',h,g,m} u(\mathcal{C}(k,s,k',h,g,m),1-h,g,s) + \beta v(k',s'),$$

s.t.

$$\begin{aligned} s' &= q(s,m) \\ k' &= \mathcal{K}(k,s,g,m) \\ h &= \mathcal{H}(k,s,g,m). \end{aligned}$$

or

$$\max_{g,m} u[\mathcal{C}(k,s,\mathcal{K}(k,s,g,m),\mathcal{H}(k,s,g,m),g,m),1-\mathcal{H}(k,s,g,m),g,s] + \beta v[\mathcal{K}(k,s,g,m),q(s,m)], \tag{A.25}$$

where

$$\begin{aligned} v(k,s) =& u[\mathcal{C}(k,h(k,s),n^h(k,s),\psi(k,s),\phi(k,s),s),1-n^h(k,s),\psi(k,s),s] \\ &+ \beta v(n^k(k,s),q(s,\phi(k,s))). \end{aligned} \tag{A.26}$$

Taking f.o.c. yields:

$$-u_c(\mathcal{K}_g - F_h\mathcal{H}_g + 1) - u_l\mathcal{H}_g + u_g + \beta v'_k\mathcal{K}_g = 0 \tag{A.27}$$

$$-u_c[\mathcal{K}_m - F_h\mathcal{H}_m - (F_m - \kappa)] - u_l\mathcal{H}_m + u_m + [\beta v'_k\mathcal{K}_m + v'_s q_m] = 0. \tag{A.28}$$

.

The derivatives of the value function $v(k,s)$ with respect to k and s read:

$$v_k = u_c[F_k + 1 - \delta - \mathcal{K}_k + F_h\mathcal{H}_k] - u_l\mathcal{H}_k + \beta v'_k\mathcal{K}_k, \tag{A.29}$$

and

$$v_s = u_c[F_s - \mathcal{K}_s + F_h\mathcal{H}_s] - u_l\mathcal{H}_s + u_s + \beta[v'_k\mathcal{K}_s + v'_s q_s]. \tag{A.30}$$

Note that at this point, one can see why I need a second *endogenous* public good in addition to environmental quality. If government expenditures were exogenous, I would have only one first-order condition above, namely (A.28). However, in order to substitute for v_k' and v_s' in (A.29) and (A.30), I need two equations; (A.28) alone would not be sufficient. From (A.27) and (A.28):

$$\beta v_k' = \frac{1}{\mathcal{K}_g}\left[u_c(\mathcal{K}_g - F_h\mathcal{H}_g + 1) + u_l\mathcal{H}_g - u_g\right] \tag{A.31}$$

$$\begin{aligned}\beta v_s' &= \frac{1}{q_m}\left[u_c(\mathcal{K}_m - F_h\mathcal{H}_m - (F_m - \kappa)) + u_l\mathcal{H}_m - \frac{\mathcal{K}_m}{\mathcal{K}_g}\left[u_c(\mathcal{K}_g - F_h\mathcal{H}_g + 1) + u_l\mathcal{H}_g - u_g\right]\right]\\ &= -\frac{1}{q_m}\left[u_c(F_h\mathcal{H}_m + (F_m - \kappa)) - u_l\mathcal{H}_m + \frac{\mathcal{K}_m}{\mathcal{K}_g}\left[u_c(1 - F_h\mathcal{H}_g) + u_l\mathcal{H}_g - u_g\right]\right]\end{aligned} \tag{A.32}$$

Inserting (A.31) in (A.29) and updating gives:

$$v_k' = u_c'(F_k' + 1 - \delta) + \mathcal{H}_k'(u_c'F_h' - u_l') - \frac{\mathcal{K}_k'}{\mathcal{K}_g'}\left[u_g' - u_c' + \mathcal{H}_g'(u_c'F_h' - u_l')\right] \tag{A.33}$$

Similarly, (A.32) in (A.30) yields:

$$\begin{aligned}v_s' =& u_s' + u_c'F_s' - \frac{q_s'}{q_m'}u_c'(F_m' - \kappa) + (u_g' - u_c')\left[-\frac{\mathcal{H}_s'}{\mathcal{H}_g'} + \frac{q_s'}{q_m'}\frac{\mathcal{H}_m'}{\mathcal{H}_g'}\right]\\ &+ (u_c'F_h' - u_l')\left[\mathcal{H}_s' - \frac{\mathcal{K}_s'}{\mathcal{K}_g'}\mathcal{H}_g' - \frac{q_s'}{q_m'}\mathcal{H}_m' + \frac{q_s'}{q_m'}\frac{\mathcal{K}_m'}{\mathcal{K}_g'}\mathcal{H}_g'\right]\end{aligned} \tag{A.34}$$

Finally, substituting (A.33) for v_k' in (A.27) gives:

$$\begin{aligned}&u_g - u_c + \mathcal{H}_g(u_cF_h - u_l) + \mathcal{K}_g[-u_c + \beta u_c'(F_k' + 1 - \delta)]\\ &+ \beta\mathcal{K}_g\left\{\mathcal{H}_k'(F_h'u_c' - u_l') - \frac{\mathcal{K}_k'}{\mathcal{K}_g'}\left[\mathcal{H}_g'(F_h'u_c' - u_l') + u_g' - u_c'\right]\right\} = 0.\end{aligned}$$

Moreover, substituting (A.34) for v_s' in (A.28) yields:

$$\begin{aligned}&u_c(F_m - \kappa) + \beta q_m\left[u_s' + u_c'F_s' - \frac{q_s'}{q_m'}u_c'(F_m' - \kappa)\right] + \mathcal{H}_m(u_cF_h - u_l)\\ &+ \mathcal{K}_m[-u_c + \beta u_c'(F_k' + 1 - \delta)] + \beta(u_g' - u_c')\left[-\mathcal{K}_m\frac{\mathcal{K}_k'}{\mathcal{K}_g'} - q_m\frac{\mathcal{K}_s'}{\mathcal{K}_g'} + q_m\frac{q_s'}{q_m'}\frac{\mathcal{K}_m'}{\mathcal{K}_g'}\right]\\ &+ \beta\mathcal{K}_m(u_c'F_h' - u_l')\left[\mathcal{H}_k' - \frac{\mathcal{K}_k'}{\mathcal{K}_g'}\mathcal{H}_g'\right] + \beta(u_c'F_h' - u_l')q_m\left[\mathcal{H}_s' - \frac{\mathcal{K}_s'}{\mathcal{K}_g'}\mathcal{H}_g' - \frac{q_s'}{q_m'}\mathcal{H}_m' + \frac{q_s'}{q_m'}\frac{\mathcal{K}_m'}{\mathcal{K}_g'}\mathcal{H}_g'\right] = 0\end{aligned}$$

Using the wedges defined above, these two equation can be written as

$$\omega_{PG} + \mathcal{K}_g\omega_{CS} - \beta\mathcal{K}_g\frac{\mathcal{K}_k'}{\mathcal{K}_g'}\omega_{PG}' + \mathcal{H}_g\omega_{LL} + \beta\omega_{LL}'\mathcal{K}_g\left[\mathcal{H}_k' - \frac{\mathcal{K}_k'}{\mathcal{K}_g'}\mathcal{H}_g'\right] = 0,$$

and

$$\begin{aligned}&\omega_{Env} + \mathcal{K}_m\omega_{CS} - \beta\omega_{PG}'\mathcal{K}_m\frac{\mathcal{K}_k'}{\mathcal{K}_g'} - \beta\omega_{PG}'\left[q_m\frac{\mathcal{K}_s'}{\mathcal{K}_g'} - q_m\frac{q_s'}{q_m'}\frac{\mathcal{K}_m'}{\mathcal{K}_g'}\right]\\ &\quad + \mathcal{H}_m\omega_{LL} + \beta\omega_{LL}'\mathcal{K}_m\left[\mathcal{H}_k' - \frac{\mathcal{K}_k'}{\mathcal{K}_g'}\mathcal{H}_g'\right]\\ &\quad + \beta\omega_{LL}'q_m\left[\mathcal{H}_s' - \frac{\mathcal{K}_s'}{\mathcal{K}_g'}\mathcal{H}_g' - \frac{q_s'}{q_m'}\mathcal{H}_m' + \frac{q_s'}{q_m'}\frac{\mathcal{K}_m'}{\mathcal{K}_g'}\mathcal{H}_g'\right] = 0.\end{aligned}$$