# The Long-Run Phillips Curve and Optimal Inflation under Downward Nominal Wage Rigidity* 

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#### Abstract

We study the implications for optimal average inflation when there is both a role for money as a medium of exchange and when nominal wages are downwardly rigid. The model also features transaction costs, as in Dotsey, King, and Wolman (1999), and a non-Walrasian labor market with search frictions as in Trigari (2009). The introduction of downward nominal wage rigidities into a model with flexible wages can be decomposed into two effects; first, introducing (symmetric) wage adjustment frictions and, second making them asymmetric. Productivity growth is important for the level of inflation and also affects the size of the effect of the asymmetric wage friction. Without productivity growth, symmetric wage adjustment frictions leads to a yearly inflation rate of approximately $1.0 \%$, while introducing an asymmetry on top of this increases the inflation rate by an additional $0.7 \%$. With productivity growth, inflation is almost a percent lower and the effect of adding asymmetric wage frictions is also somewhat smaller - about $0.5 \%$. Overall, we find an optimal inflation rate of about $0-2$ percent.


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## 1 Introduction

A robust empirical finding is that money wages do not fall to any significant degree during an economic downturn. A large number of studies report substantial downward nominal wage rigidity in the U.S. as well as in Europe and Japan. ${ }^{1}$ Overall, the evidence points towards a sharp asymmetry in the distribution of nominal wage changes around zero. That is, money wages rise but they seldom fall. Recently, SchmittGrohe and Uribe (2010) raised the puzzle that most central banks targets an annual inflation rate of two percent, whereas current monetary models implies an optimal inflation rate that is usually negative. ${ }^{2}$

This paper analyzes the long-run Phillips curve and optimal monetary policy in a model with asymmetric nominal wage rigidities. Specifically, we focus on how the slope of the long-run Phillips curve and the optimal inflation rate is affected by downward nominal wage rigidities and aggregate productivity growth.

To this end, we develop a DSGE model that can account for several important factors in determining the optimal inflation rate. To capture the Friedman argument for deflation, to avoid inefficient economizing in money balances, we introduce a transaction cost (as in Khan, King, and Wolman (2003)). To include the Tobin argument for a positive rate of inflation in order to grease the wheels of wage formation in the presence of downward nominal wage rigidity (see Tobin, 1972), we introduce price- and wage-setting frictions. ${ }^{3}$ Since our ultimate aim is to study the optimal inflation rate, it is important to allow optimal price- and wage-setting decisions to depend on the inflation rate. In order to do so we model price- and wage-setting decisions as state dependent. Specifically, price setting follows Dotsey, King, and Wolman (1999), while wage setting is based on a modified version of the bargaining model in Holden (1994). Beside costs stemming from the potential break up of the firm/worker match when initiating bargaining under disagreement, firms and workers also face a fixed costs of disagreement, such as disruptions in business relationships and deteriorating management-employee relationships. ${ }^{4}$ Another key feature of the model

[^1]is that, consistently with empirical evidence, work proceed at the old contract, if no party credibly can threaten with disagreement. Moreover, since the fixed disagreement costs need not be identical for workers and firms, this opens up for downward nominal wage rigidities as a rational outcome. Finally, to provide a scope for a surplus to be bargained over, the model features a search-matching labor market akin to the model of Trigari (2009) and Christoffel, Kuester, and Linzert (2009).

To parametrize the distribution of disagreement costs in the model, we use a minimum distance estimation approach to match the nominal wage change distribution implied by the model to the empirical nominal wage change distribution observed in U.S. micro data. The estimated model yields a distribution of wage changes that captures the main features of the empirical wage change distribution. A key feature of our model that allows the model to fit the micro data with any precision is the introduction of firmlevel heterogeneity in terms of productivity, as well as, aggregate productivity growth. The first feature is needed to capture the large variance of the distribution of nominal wage changes in the data and the second feature is needed to capture the fact that nominal wages increase more than the inflation rate on average. The introduction of these two features also has implications for the optimal inflation rate via effects through the steady state wage distribution. ${ }^{5}$

Two related papers are Kim and Ruge-Murcia (2009), and Fagan and Messina (2009). They analyze the effects on the optimal inflation target from downward wage rigidity. Both these papers rely on asymmetric adjustment costs in wages as in Rotemberg (1982) to generate downward wage rigidities. It thus becomes key for the planner to avoid these costs in the design of optimal policy. We take a different stand on the underlying reason for downward wage rigidities. We think of this friction as stemming from disagreement costs and the implied effects on the threat points in the wage bargaining. Since disagreement will not occur in equilibrium, these costs are of no direct consequence for the planner when designing optimal policy. Though indirectly, via the effect on nominal wage formation through private sector behavior, these costs will affect the design of optimal policy. Importantly, we model wage dispersion explicitly and thus capture the associated inefficiencies that are due to suboptimal levels of output across firms and workers. This strategy also implies that we can match the model to micro data, which allows us to put additional empirical discipline on the analysis. The paper by Fagan and Messina (2009) also uses micro data when estimating their model, but in contrast to them, we allow for inflation to affect price- and wage-setting frequencies, a role for money as a medium of exchange. The study by Kim and Ruge-Murcia (2009) also contains the latter two of these features, but relies only on macro data for estimation and

[^2]evaluation. Moreover, both these papers lack productivity growth.
We find that the optimal annual rate under downward nominal wage rigidities is $0.8 \%$, as compared to a deflation rate of about $0.6 \%$ when wages are flexible. The optimal annual inflation rate found here is larger than in the baseline monetary models discussed in Schmitt-Grohe and Uribe (2010) where the optimal inflation rate is at most around zero. The introduction of downward nominal wage rigidities into a model with flexible wages can be decomposed into two effects. First, wage adjustment frictions are introduced and second, the frictions become asymmetric. Then the effect of introducing asymmetric wage adjustment frictions is about $0.5 \%$; with symmetric frictions, the optimal inflation rate is about $0.3 \%$. Without productivity growth, the result changes substantially. Then the optimal inflation rate with symmetric frictions is about $1.0 \%$, while the rate is about $1.7 \%$ under downward nominal wage rigidities. Thus, allowing for productivity growth reduces the optimal inflation rate by almost a percent and also reduces the effect of downward nominal wage rigidities somewhat. The reason is that productivity growth can perform the same effect of inflation; erosion of real wages, which is more important under asymmetric wage frictions. However, we also show that varying the degree of flexibility in wage formation has large effects on this conclusion. Specifically, letting new hires wages to be perfectly flexible leads to an optimal annual deflation rate of around $0.5 \%$.

This paper is organized as follows, in section 2, we outline the model. In section 3, the optimal policy is described, in section 4 the calibration of the model is presented and in section 5 the results are presented. Finally, section 6 concludes.

## 2 The Economic Environment

The basic framework shares many elements of standard DSGE models. There is a monopolistically competitive intermediate goods sector where producers set prices facing a known random (periodically) fixed cost of price adjustment as in Dotsey, King, and Wolman (1999). Thus, for tractability, we assume that prices have a finite duration of at most $J$ periods. There is also a wholesale sector that uses capital and labor to produce an input for the intermediate-goods sector. ${ }^{6}$ The input is sold on a perfectly competitive market. The wholesale sector rents capital on a competitive capital market and post vacancies on a search and matching labor market. Wages are bargained between workers and the firm following a slightly modified version of Holden (1994). In the model, the parties bargain every period. Each bargaining round starts with one of the parties making a bid, then the other party responds yes or no. If

[^3]the response is no, there is a choice whether to continue bargaining in good faith and post a counter offer or enter into disagreement. If the latter choice is made, there is a probability that the match breaks down and the wage is determined in a standard Rubinstein-Ståhl fashion. Moreover, in case a party initiate bargaining under disagreement, both parties face their own known fixed disagreement cost (randomly drawn at the beginning of each period). This cost may be due to deteriorating firm/worker and customer relationships. ${ }^{7}$ In case none of the parties chooses to bargain under disagreement, but being unable to settle on a new wage, work continues according to the old contract. If the disagreement cost is sufficiently high, it is not credible for a party to threaten with disagreement in order to achieve a new wage contract. Instead, the outcome will be to continue to work according to the old contract already in place, thus endogenizing nominal wage rigidity. To capture the downward nominal wage rigidity observed in micro data it is required that firms, on average, face higher disagreement cost. As with prices, we assume that wage contracts last for at most $J^{w}$ periods.

In order to introduce complete consumption insurance, we assume that there is a representative family as in Merz (1995). Finally, notation is simplified by assuming a flexible price retail sector that repacks the intermediate goods in accordance with consumer preferences and sells them to consumers on a competitive market.

### 2.1 Retail firms

We follow Erceg, Henderson, and Levin (2000) and Khan, King, and Wolman (2003) and assume a competitive retail sector that buy intermediate goods and sell a composite final good. The composite good is combined from intermediate goods in the same proportions as households would choose. Given intermediate goods output levels $Y_{t}^{j}$ produced by intermediate goods firms $j$, the amount of the composite $\operatorname{good} Y_{t}$ is

$$
\begin{equation*}
Y_{t}=\left[\sum_{j=0}^{J-1} \omega_{t}^{j}\left(Y_{t}^{j}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{1}
\end{equation*}
$$

where $\sigma>1$ and $\omega^{j}$ is the share of retail firms producing $Y_{t}^{j}$ at price $P_{t}^{j}$. The price $P_{t}$ of one unit of the composite good is

$$
\begin{equation*}
P_{t}=\left[\sum_{j=0}^{J-1} \omega_{t}^{j}\left(P_{t}^{j}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} . \tag{2}
\end{equation*}
$$

[^4]
### 2.2 Intermediate goods firms

The intermediate goods firms optimally choose whether to change prices, given a menu cost $c_{p}$ of changing prices. Adjustment costs are drawn from a cumulative distribution function $G_{P}$. Let the probability of adjusting prices in a given period be denoted by $\alpha_{t}^{j}$, given that the firm last adjusted it's price $j$ periods ago. We assume that there is some $J>1$ such that $\alpha^{J}=1$.

### 2.2.1 Prices

Given that an intermediate goods firm last reset prices in period $t-j$, the maximum remaining duration of a price contract is $J-j$, where $J$ is the maximum duration of a price contract and $\alpha_{t}^{j}$ is the adjustment probability $j$ periods after the price was last reset. The intermediate goods firms buys a homogeneous input from the wholesale firms at the real price $p_{t}^{w}$. Finally, the average or expected (real) adjustment cost, in terms of aggregate output, is given by

$$
\begin{equation*}
\Xi_{j, t}=\frac{1}{\alpha_{t}^{j}} \int_{0}^{G_{P}^{-1}\left(\alpha_{t}^{j}\right)} x d G_{P}(x) \tag{3}
\end{equation*}
$$

Note that the upper bound is given by the maximum menu cost $c_{p}$ that induces price adjustment, i.e., the $c_{p}$ that solves $\alpha_{t}^{j}=G_{P}\left(c_{p}\right)$. As in Khan, King, and Wolman (2003), but extended as in Lie (2010) to allow for state-dependent pricing, an intermediate producer chooses the optimal price $P_{t}^{0}$ so that

$$
\begin{align*}
v_{t}^{0}= & \max _{P_{t}^{0}}\left[\frac{P_{t}^{0}}{P_{t}}-p_{t}^{w}\right] Y_{t}^{0}+E_{t} \Lambda_{t, t+1} \beta\left(\alpha_{t+1}^{1} v_{t+1}^{0}+\left(1-\alpha_{t+1}^{1}\right) v_{t+1}^{1}\left(\frac{P_{t}^{0}}{P_{t+1}}\right)\right)  \tag{4}\\
& -E_{t} \Lambda_{t, t+1} \beta p_{t+1}^{w} \alpha_{t+1}^{1} \Xi_{1, t+1}
\end{align*}
$$

where

$$
\begin{equation*}
Y_{t}^{j}=\left(\frac{P_{t}^{j}}{P_{t}}\right)^{-\sigma} Y_{t} \tag{5}
\end{equation*}
$$

where $P_{t}$ is the aggregate price level, $\beta$ the discount factor and $\Lambda_{t, t+1}$ the ratio of Lagrange multipliers in the problem of the consumer tomorrow and today (i.e., relative value of consumption today versus
tomorrow). The values $v_{t}^{j}$ evolve according to

$$
\begin{align*}
v_{t}^{j}\left(\frac{P_{t}^{j}}{P_{t}}\right)= & {\left[\frac{P_{t}^{j}}{P_{t}}-p_{t}^{w}\right] Y_{t}^{j}+E_{t} \Lambda_{t, t+1} \beta\left(\alpha_{t+1}^{j+1} v_{t+1}^{0}+\left(1-\alpha_{t+1}^{j+1}\right) v_{t+1}^{j+1}\left(\frac{P_{t}^{j}}{P_{t+1}}\right)\right) }  \tag{6}\\
& -E_{t} \Lambda_{t, t+1} \beta p_{t+1}^{w} \alpha_{t+1}^{j+1} \Xi_{j+1, t+1}, \\
v_{t}^{J-1}\left(\frac{P_{t}^{J-1}}{P_{t}}\right)= & {\left[\frac{P_{t}^{J-1}}{P_{t}}-p_{t}^{w}\right] Y_{t}^{J-1}+E_{t} \Lambda_{t, t+1} \beta v_{t+1}^{0}-E_{t} \Lambda_{t, t+1} \beta p_{t+1}^{w} \Xi_{J, t+1} . }
\end{align*}
$$

Note that the term within the square brackets is just the firm's per unit profit in period $t+k$, given that prices were last reset in period $t$.

The first-order condition to the problem (4) is

$$
\begin{equation*}
\left[(1-\sigma) \frac{P_{t}^{0}}{P_{t}}+\sigma p_{t}^{w}\right] Y_{t}^{0} \frac{1}{P_{t}}+E_{t} \Lambda_{t, t+1} \beta\left(\left(1-\alpha_{t+1}^{1}\right) D_{1} v_{t}^{1}\left(\frac{P_{t}^{0}}{P_{t+1}}\right) \frac{1}{P_{t+1}}\right)=0 \tag{7}
\end{equation*}
$$

where, noting that $P_{t+j}^{j}=P_{t}^{0}$, the derivative $D_{1} v_{t}^{1}$ can be computed by using

$$
\begin{align*}
D_{1} v_{t}^{j}\left(\frac{P_{t}^{j}}{P_{t}}\right) & =\left[(1-\sigma) \frac{P_{t}^{j}}{P_{t}}+\sigma p_{t}^{w}\right] \frac{Y_{t}^{j}}{P_{t}}+E_{t} \Lambda_{t, t+1} \beta\left(\left(1-\alpha_{t+1}^{j+1}\right) D_{1} v_{t+1}^{j+1}\left(\frac{P_{t}^{j}}{P_{t+1}}\right) \frac{1}{P_{t+1}}\right), \\
D_{1} v_{t}^{J-1}\left(\frac{P_{t}^{J-1}}{P_{t}}\right) & =\left[(1-\sigma) \frac{P_{t}^{J-1}}{P_{t}}+\sigma p_{t}^{w}\right] \frac{Y_{t}^{J-1}}{P_{t}} . \tag{8}
\end{align*}
$$

Thus, optimal pricing behavior is fully characterized by expressions (7) and (8).
We model price adjustment probabilities as in Dotsey, King, and Wolman (1999) and others. Thus, adjustment probabilities are chosen endogenously by the firm and is one if $c_{p}<\frac{v_{t}^{0}-v_{t}^{j}}{p_{t}^{w}}$ and zero if $c_{p}>$ $\frac{v_{t}^{0}-v_{t}^{j}}{p_{t}^{u}}$. Adjustment costs are drawn from a cumulative distribution function $G_{P}$ and the share of firms among those that last adjusted the price $j$ periods ago that adjusts the price today is given by

$$
\begin{equation*}
\alpha_{t}^{j}=G_{P}\left(\frac{v_{t}^{0}-v_{t}^{j}}{p_{t}^{w}}\right) . \tag{9}
\end{equation*}
$$

Moreover the shares of firms with duration $j$ since the last price change is denoted by $\omega_{t}^{j}$. For $j \geq 1$ we have

$$
\begin{equation*}
\omega_{t}^{j}=\left(1-\alpha_{t}^{j}\right) \omega_{t-1}^{j-1}, \tag{10}
\end{equation*}
$$

and, for $j=0$,

$$
\begin{equation*}
\omega_{t}^{0}=\sum_{j=1}^{J-1} \alpha_{t}^{j} \omega_{t-1}^{j-1} \tag{11}
\end{equation*}
$$

Assume that $G_{P}$ follows a beta distribution, i.e., the probability density is $g_{P}=\frac{1}{\beta\left(l_{p}+1, r_{p}+1\right)} x^{l_{p}}(1-x)^{r_{p}}$.

### 2.3 Households

Households have preferences

$$
\begin{equation*}
E_{t} \sum_{r=t}^{\infty} \beta^{r-t}\left[u\left(c_{r}\right)+n_{r} \kappa^{L} \frac{\left(1-\bar{h}-h_{r}^{c}\right)^{1-\phi}}{1-\phi}+\left(1-n_{r}\right) \kappa^{L} \frac{\left(1-h_{r}^{c}\right)^{1-\phi}}{1-\phi}\right] \tag{12}
\end{equation*}
$$

where $\bar{h}$ denotes the workers' hours worked at a wholesale firm, $c_{t}$ consumption, $n_{t}^{j_{w}}$ the number of employees in wage cohort $j_{w}$ and $n_{t}$ aggregate employment. Wealth accumulation of the family is given by

$$
\begin{equation*}
M_{t}+\frac{B_{t+1}}{R_{t}}+\theta_{t+1}\left(F_{t}-Z_{t}\right) \geq \theta_{t} F_{t}+B_{t}-D_{t}+\mathcal{W}_{t} \tag{13}
\end{equation*}
$$

where $P_{t}$ is the price level, $M_{t}$ is money holdings and, $B_{t}$ bonds, $\theta_{t+1}$ is the share of intermediate product firms, $F_{t}$ the value of firms (measured on a pre-dividend basis $\left.F_{t}-Z_{t}\right)^{8}$ and $Z_{t}$ nominal dividends, $\varpi_{t}$ is wealth at the start of time $t, R_{t}$ is the one period nominal interest rate, $b_{t}$ denotes one period nominal bonds and where

$$
\begin{equation*}
\mathcal{W}_{t}=\int_{0}^{1} E_{t} W_{i t} d i+\left(1-n_{t}\right) b_{r}-T_{t}, \tag{14}
\end{equation*}
$$

with $b_{r}$ representing the value of home production. Moreover, $W_{i t}$ denotes the households nominal wage and $T_{t}$ lump-sum taxes. Each family own an equal share of all firms and of the aggregate capital stock. Finally, note that $1-n_{t}$ is equal to the unemployment rate. In real term, letting $m_{t}=\frac{M_{t}}{P_{t}}$ denote real money balances, $b_{t}$ real bond holdings, $f_{t}$ and $z_{t}$ the real value of the firm and dividends, respectively, wealth accumulation is

$$
\begin{equation*}
m_{t}+\frac{b_{t+1}}{R_{t}}+\theta_{t+1}\left(f_{t}-z_{t}\right) \geq \theta_{t} f_{t}+\frac{b_{t}}{1+\pi_{t}}-\frac{d_{t}}{1+\pi_{t}}+\frac{\mathcal{W}_{t}}{P_{t}} . \tag{15}
\end{equation*}
$$

Agents purchases goods using either money or credit. Using credit requires paying a stochastic fixed time cost. This cost is good-specific and is realized after the family has decided on the amount of a product to buy but before choosing between credit or money as means of payment. Here, credit is defined as a one-period interest rate free loan that needs to be repaid in full the next period. Families then choose to use credit as long as the gain, $R_{t} c_{t}$, is larger than the cost of credit. ${ }^{9}$ Letting $\xi_{t}$ denote the fraction

[^5]of goods purchased using credit, we have $d_{t+1}=\xi_{t} \bar{p}_{t} c_{t}$ where $\bar{p}_{t}$ is the price of the consumption good. Furthermore,
\[

$$
\begin{equation*}
m_{t}=\left(1-\xi_{t}\right) \bar{p}_{t} c_{t} \tag{16}
\end{equation*}
$$

\]

The family's first-order conditions with respect to $c_{t}$ and $\xi_{t}$ are, using that $\bar{p}_{t}=\left(1+R_{t}\right)$,

$$
\begin{align*}
& c_{t}:  \tag{17}\\
& \xi_{t}: u_{c}\left(c_{t}\right)=\lambda_{t}\left(1+R_{t}\left(1-\xi_{t}\right)\right) \\
& c_{t}=\left[n_{t} \kappa^{L}\left(1-\bar{h}-h_{t}^{c}\right)^{\phi}+\left(1-n_{t}\right) \kappa^{L}\left(1-h_{t}^{c}\right)^{\phi}\right] F^{-1}\left(\xi_{t}\right)
\end{align*}
$$

where $F^{-1}\left(\xi_{t}\right)$ is the realization of the credit cost in terms of time.
Using the envelope theorem and the first-order condition with respect to $b_{t}$ we can write the household Euler equation as

$$
\begin{equation*}
\frac{\lambda_{t}}{R_{t}}=\beta E_{t} \frac{\lambda_{t+1}}{1+\pi_{t+1}} . \tag{18}
\end{equation*}
$$

### 2.4 Search and matching

In each period wholesale firm $i$ post $v_{i t}$ vacancies and employs $n_{i t}$ workers. The aggregate number of vacancies is denoted $v_{t}$ is and aggregate employment is denoted $n_{t}$. As in Christoffel, Kuester, and Linzert (2009), the number of unemployed workers is

$$
\begin{equation*}
u_{t}=1-n_{t} . \tag{19}
\end{equation*}
$$

We assume that the number of matches $m_{t}$ is given by the following constant-returns matching function

$$
\begin{equation*}
m_{t}^{a}=\sigma_{m} u_{t}^{\sigma_{a}} v_{t}^{1-\sigma_{a}}, \tag{20}
\end{equation*}
$$

where $u_{t}$ is unemployment and $v_{t}$ the number of vacancies. The probability that a worker is matched to a firm

$$
\begin{equation*}
s_{t}^{a}=\frac{m_{t}^{a}}{u_{t}} \tag{21}
\end{equation*}
$$

and the probability that a vacancy is filled is

$$
\begin{equation*}
q_{t}=\frac{m_{t}^{a}}{v_{t}} \tag{22}
\end{equation*}
$$

amount the next period. To see this, combine the first-order condition with respect to $\xi(17)$ together with the Euler equation (18), below.

Finally, a match is broken with probability $1-\rho$.

### 2.5 Wage determination

In the model, the parties bargain every period. Each bargaining round starts with one of the parties making a bid, then the other party responds yes or no. If the response is no, there is a choice whether to continue bargaining in good faith and post a counter offer or enter into disagreement. If the latter choice is made, there is a probability that the match breaks down and the wage is determined in a standard Rubinstein-Ståhl fashion. Moreover, in case a party initiate bargaining under disagreement, both parties face their own known fixed disagreement cost (randomly drawn at the beginning of each period). As in Holden (1994), this cost may be due to deteriorating firm/worker and customer relationships. In case none of the parties chooses to bargain under disagreement, but are unable to settle on a new wage, work continues according to the old contract. If the disagreement cost is sufficiently high, it is not credible for a party to threaten with disagreement in order to achieve a new wage contract. Instead, the outcome will be to continue to work according to the old contract already in place, and the model thus generates nominal wage rigidities as a rational endogenous outcome.

Note that there is no disagreement in equilibrium, and hence the equilibrium disagreement costs is zero. Thus, in contrast to price adjustment costs, this cost neither enter resource constraints nor firm/worker value functions. Moreover, this cost is of no direct concern to the planner, although it affects the optimal solution indirectly through its impact on private sector behavior.

Wholesale firms bargains with workers with some positive probability $\alpha_{t}^{j_{w}}$ in the $j_{w}$ 'th period following the last renegotiation. Though, in the model at hand, these probabilities are endogenous. Wage adjustment probabilities are given by $\alpha_{t}^{j_{w}}$ with $\alpha_{t}^{J_{w}}=1$ for some $J_{w}>1$. In the model, these adjustment probabilities may depend on whether wages increase or decrease.

### 2.6 Value functions

The wholesale firm $i$ use capital $k_{i t}$ and labor $h_{i t}$ as inputs to produce output $y_{i t}$ using a constant returns technology

$$
\begin{equation*}
y_{i t}=a_{i t} h_{i t} . \tag{23}
\end{equation*}
$$

where $a_{i t}=e^{\gamma_{r} t} \varepsilon_{i t}^{a}$ with $\gamma_{r}$ being the growth of technology and $\varepsilon_{i t}^{a}$ an idiosyncratic shock. Also, let $A$ denote the set of productivity levels. For simplicity, however, we will suppress the idiosyncratic productivity dimension in the notation in what follows.

The value for the family of a worker at wholesale firm $i$ is in period $t$ is, letting $\vartheta\left(a_{t+1}, a_{t}\right)$ denote the transition probability from productivity state $a_{t}$ to $a_{t+1}$,

$$
\begin{align*}
V_{t}^{j_{w}}\left(w_{t}^{j_{w}}, a_{t}\right)= & w_{t}^{j_{w}} \bar{h}+\kappa^{L} \frac{\left(1-\bar{h}-h_{r}^{c}\right)^{1-\phi}}{1-\phi}+\beta \sum_{a_{t+1} \in A} E_{t} \Lambda_{t, t+1} \vartheta\left(a_{t+1}, a_{t}\right) \\
& \times\left[\left(\rho \alpha_{t+1}^{j_{w}+1}\left(w_{t+1}^{j_{w}+1}, a_{t+1}\right) V_{t+1}^{0}\left(w_{t+1}^{0}, a_{t+1}\right)+(1-\rho) U_{t+1}\right)\right.  \tag{24}\\
& \left.+\left(\rho\left(1-\alpha_{t+1}^{j_{w}+1}\left(w_{t+1}^{j_{w}+1}, a_{t+1}\right)\right) V_{t+1}^{j_{w}+1}\left(w_{t+1}^{j_{w}+1}, a_{t+1}\right)+(1-\rho) U_{t+1}\right)\right]
\end{align*}
$$

where $w_{t}^{j_{w}}=\frac{W_{t}^{j_{w}}}{P_{t}}$ is the real wage and $\bar{h}$ hours that are fixed. The value when being unemployed is

$$
\begin{equation*}
U_{t}=b_{r}+\kappa^{L} \frac{\left(1-h_{r}^{c}\right)^{1-\phi}}{1-\phi}+\beta E_{t} \Lambda_{t, t+1}\left(s_{t+1}^{a} V_{x, t+1}+\left(1-s_{t+1}^{a}\right) U_{t+1}\right) . \tag{25}
\end{equation*}
$$

In the model, we let the share of new hires that get a rebargained wage be a free parameter. Thus, letting $\omega_{t}^{j_{w}}\left(w_{t+1}^{j_{w}+1}, a_{t}\right)$ denote the share of workers with wage $w_{t}^{j_{w}}$ and productivity $a_{t}$, we have

$$
\begin{align*}
V_{x, t}= & s^{n e w} \sum_{a_{t} \in A} \omega^{e r g}\left(a_{t+1}\right) V_{t}^{0}\left(w_{t}^{0}, a_{t}\right)  \tag{26}\\
& +\left(1-s^{n e w}\right) \sum_{j_{w}=0}^{J_{w}-1} \sum_{a_{t} \in A} \omega_{t}^{j_{w}}\left(w_{t+1}^{j_{w}+1}, a_{t}\right) V_{t}^{j_{w}}\left(w_{t}^{j_{w}}, a_{t}\right),
\end{align*}
$$

is the average value of employment an $s^{\text {new }}$ is the share getting new rebargained wages. Then the bargaining surplus (defined by $j_{w}=0$ ) for the worker is, as usual in bargaining models with a probability of match breakdown, given by the difference between the value of employment and unemployment

$$
\begin{equation*}
H_{t}^{j_{w}}\left(w_{t}^{j_{w}}, a_{t}\right)=V_{t}^{j_{w}}\left(w_{t}^{j_{w}}, a_{t}\right)-U_{t} \tag{27}
\end{equation*}
$$

and hence, the value of an additional employee for the household can then be written as

$$
\begin{align*}
H_{t}^{j_{w}}\left(w_{t}^{j_{w}}, a_{t}\right)= & w_{t}^{j_{w}} \bar{h}+\kappa^{L} \frac{\left(1-\bar{h}-h_{r}^{c}\right)^{1-\phi}}{1-\phi}-b_{r}-\kappa^{L} \frac{\left(1-h_{r}^{c}\right)^{1-\phi}}{1-\phi}+\beta \sum_{a_{t+1} \in A} E_{t} \Lambda_{t, t+1} \vartheta\left(a_{t+1}, a_{t}\right) \\
& \times\left[\left(\rho \alpha_{t+1}^{j_{w}+1}\left(w_{t+1}^{j_{w}+1}, a_{t+1}\right) H_{t+1}^{0}\left(w_{t+1}^{0}, a_{t+1}\right)-s_{t+1}^{a} H_{x, t+1}\right)\right.  \tag{28}\\
& \left.+\left(\rho\left(1-\alpha_{t+1}^{j_{w}+1}\left(w_{t+1}^{j_{w}+1}, a_{t+1}\right)\right) H_{t+1}^{j_{w}+1}\left(w_{t+1}^{j_{w}+1}, a_{t+1}\right)-s_{t+1}^{a} H_{x, t+1}\right)\right]
\end{align*}
$$

where

$$
H_{x, t}=V_{x, t}-U_{t} .
$$

For the firm wholesale firm, the value of an additional employee is

$$
\begin{align*}
J_{t}^{j_{w}}\left(w_{t}^{j_{w}}, a_{t}\right)= & p_{t}^{w} a_{t} \bar{h}-w_{t}^{j_{w}} \bar{h} \\
& +\beta \sum_{a_{t+1} \in A} \Lambda_{t, t+1} \vartheta\left(a_{t+1}, a_{t}\right) \alpha_{t+1}^{j_{w}+1}\left(w_{t+1}^{j_{w}+1}, a_{t+1}\right)\left(\rho J_{t+1}^{0}\left(w_{t+1}^{0}, a_{t+1}\right)\right)  \tag{29}\\
& +\beta \sum_{a_{t+1} \in A} \Lambda_{t, t+1} \vartheta\left(a_{t+1}, a_{t}\right)\left(1-\alpha_{t+1}^{j_{w}+1}\left(w_{t+1}^{j_{w}+1}, a_{t+1}\right)\right) \rho J_{t+1}^{j_{w}+1}\left(w_{t+1}^{j_{w}+1}, a_{t+1}\right) .
\end{align*}
$$

A firm that last renegotiated wages $j$ periods ago can credibly disagree if the gain from adjusting the wage

$$
d J_{t}^{j_{w}}\left(w_{t}^{j_{w}}, a_{t}\right)=J_{t}^{0}\left(w_{t}^{0}, a_{t}\right)-J_{t}^{j_{w}}\left(w_{t}^{j_{w}}, a_{t}\right),
$$

is larger that the disagreement cost. Similarly, the worker disagree if

$$
d H^{j_{w}}\left(w_{t}^{j_{w}}, a_{t}\right)=H_{t}^{0}\left(w_{t}^{0}, a_{t}\right)-H_{t}^{j_{w}}\left(w_{t}^{j_{w}}, a_{t}\right),
$$

is larger that the workers disagreement cost.

Wage determination Wages are determined in bargaining between firms and the household member employed by the firm. Akin to Holden (1994), if it is not credible to threaten with disagreement the parties settle on the previous periods wage. If it is credible to threaten with disagreement the wage is determined in a standard Rubinstein-Ståhl barging game. Since there is equivalence between the standard noncooperative approach in Rubinstein (1982) and the Nash bargaining approach, we use the latter method. The nominal wage $W_{i t}^{0}$ is then chosen such that is solves the following problem

$$
\begin{equation*}
\max _{W_{i t}^{0}}\left(H_{t}^{0}\left(w_{t}^{0}, a_{t}\right)\right)^{\varphi}\left(J_{t}^{0}\left(w_{t}^{0}, a_{t}\right)\right)^{1-\varphi}, \tag{30}
\end{equation*}
$$

and $\varphi$ denotes the bargaining power of workers. The first-order condition with respect to the nominal wage $W_{i t}^{0}$ corresponding to (30) is

$$
\begin{equation*}
\varphi J_{t}^{0}\left(w_{t}^{0}, a_{t}\right) D_{W} H_{t}^{0}\left(w_{t}^{0}, a_{t}\right)+(1-\varphi) H_{t}^{0}\left(w_{t}^{0}, a_{t}\right) D_{W} J_{t}^{0}\left(w_{t}^{0}, a_{t}\right)=0 \tag{31}
\end{equation*}
$$

where $D_{W} H_{t}^{0}\left(w_{t}^{0}, a_{t}\right)$ and $D_{W} J_{t}^{0}\left(w_{t}^{0}, a_{t}\right)$ are computed using expressions (28) and (29).

### 2.6.1 Adjustment probabilities

The disagreement costs for the firm follow the cumulative distribution function $G^{J}:\left[0, \mathcal{B}^{J}\right] \rightarrow[0,1]$ and the disagreement cost of workers follow the cumulative distribution function $G^{H}:\left[0, \mathcal{B}^{H}\right] \rightarrow[0,1]$ with upper bounds $\mathcal{B}^{J}$ and $\mathcal{B}^{H}$, respectively. The adjustment probabilities are given by

$$
\begin{equation*}
\alpha_{t}^{j_{w}}\left(w_{t}^{j_{w}}, a_{t}\right) \tag{32}
\end{equation*}
$$

and depend on both $G^{J}\left(d J_{t}^{j_{w}}\left(w_{t}^{j_{w}}, a_{t}\right)\right)$ and $G^{H}\left(d H_{t}^{j_{w}}\left(w_{t}^{j_{w}}, a_{t}\right)\right)$. A detailed description on how there are computed are given in the appendix.

### 2.6.2 The hiring decision and employment flows

Firms chooses it's hiring so that the hiring cost of an additional employee is equal to the value. Thus, letting $q_{t}=\frac{m_{t}^{a}}{v_{t}}$ denote the probability of filling a vacancy, hiring is determined by

$$
\begin{align*}
\kappa_{t}= & s^{n e w} q_{t} \beta \sum_{a_{t+1} \in A} E_{t} \Lambda_{t, t+1} \omega^{e r g}\left(a_{t+1}\right) J_{t+1}^{0}\left(w_{t+1}^{0}, a_{t+1}\right)  \tag{33}\\
& +\left(1-s^{n e w}\right) q_{t} \beta \sum_{j_{w}=0}^{J_{w}-1} \sum_{a_{t+1} \in A} E_{t} \omega_{t+1}^{j_{w}+1}\left(w_{t+1}^{j_{w}}, a_{t+1}\right) \Lambda_{t, t+1} J_{t+1}^{j_{w}}\left(w_{t+1}^{j_{w}}, a_{t+1}\right)
\end{align*}
$$

where the expectation is taken across all firms, $\omega^{\operatorname{erg}}\left(a_{t+1}\right)$ is the (ergodic) probability of entering in cohort $a_{t+1}, \omega_{t}^{j_{w}}$ is the share in cohort $j_{w}$ in period $t$ with productivity $a_{t+1}$ and wage $w_{t+1}^{j_{w}}$. Below, we both analyze the case when $s^{n e w}$ is calibrated according to empirical evidence and when wages for new hires are fully flexible.

Since there has been a significant controversy in the literature whether the wages of newly hired workers are more flexible than for incumbent workers, we find it important to motivate this assumption. Micro-data studies, summarized in Pissarides (2009), seem to indicate that newly hired workers wages are substantially more flexible than incumbents wages. However, answering the question wether newcomers wages are more cyclical than incumbents wages is associated with severe identification problems. Especially, the studies summarized in Pissarides (2009) generally fail to control for effects stemming from variations in the composition of firms and match quality over the cycle. It might thus be that the empirical evidence just reflect that workers move from low wage firms (low quality matches) to high wage firms (high quality matches) in boom periods and vice versa in recessions. The approach taken by e.g. Gertler and Trigari (2009) to address this issue is introduce a job-specific fixed effects in a regression of individual
wages on the unemployment rate and the interaction of the unemployment rate and dummy variable indicating if the tenure of the worker is short. This should control for composition effects in workers, firms and match quality. The problem, however, is that the interaction effect is then only identified with the within-match variation. It answers the question wether wages for workers with short tenure responds more to cyclical factors than wages for workers with longer tenure after that the worker has already been hired. Albeit an interesting question in itself, it is not the question at hand. Thus, existing micro-data studies can only takes us so far. If we instead turn to survey evidence, like Bewley (1999), Bewley (2007) for the U.S. and the study performed within the Eurosystem Wage Dynamics Network (WDN) covering about 17,000 firms in 17 European countries, we see strong evidence of that the wages of new hires are tightly linked to those of incumbents. As reported by Galuscak et al., 2010, about $80 \%$ percent of the firms in the WDN survey respond that internal factors (like the internal pay structure) are the more important factor driving wages of new hires rather than external or market conditions. Finally, turning to the macro evidence, De Walque, 2009, develops a DSGE model that allows for a separate analysis of the flexibility of new and incumbent workers wages via different probabilities of being able to negotiate the wage. Estimates of this model relying on the European AWM database (presented in the final report of the WDN; see Several (2009)), indicate that new hires negotiate their wage in the same proportion as incumbents, in line with the survey evidence. Thus, all in all, we view the assumption underlying (33) as the natural baseline. However, we also explore the implications of modeling new hires wages as perfectly flexible.

Finally, the employment flow between categories $n_{t}^{j_{w}}$ is given by
$n_{t}^{0}\left(a_{t}\right)=\sum_{j_{w}=1}^{J_{w}} \sum_{a_{t-1} \in A} \vartheta\left(a_{t}, a_{t-1}\right) \rho \alpha_{t-1}^{j_{w}-1}\left(w_{t-1}^{j_{w}-1}, a_{t-1}\right) n_{t-1}^{j_{w}-1}\left(a_{t-1}\right)+\left(s^{n e w} \omega^{\text {erg }}\left(a_{t}\right)+\left(1-s^{n e w}\right) \frac{n_{t}^{0}\left(a_{t}\right)}{n}\right) m_{t}^{a}$,
and, for $j_{w}>0$,

$$
\begin{equation*}
n_{t}^{j_{w}}\left(a_{t}\right)=\sum_{a_{t-1} \in A} \vartheta\left(a_{t}, a_{t-1}\right) \rho\left(1-\alpha_{t-1}^{j_{w-1}}\left(w_{t-1}^{j_{w}-1}, a_{t-1}\right)\right) n_{t-1}^{j_{w}-1}\left(a_{t-1}\right)+\left(1-s^{n e w}\right) \frac{n_{t}^{j_{w}}\left(a_{t}\right)}{n} m_{t}^{a} \tag{35}
\end{equation*}
$$

## 3 Policy

We model policy in two ways. First, we analyze the model when policy is governed by a Taylor rule

$$
\frac{R_{t}}{\bar{R}}=\left(\frac{\pi_{t}}{\bar{\pi}}\right)^{r_{\pi}}\left(\frac{y_{t}^{e}}{\bar{y}^{e}}\right)^{r_{y}}
$$

where $y_{t}^{e}$ denotes output in efficiency units.
Second, we solve for the optimal policy. In the model we have several distortions. First, there is imperfect competition in the product market. There is also a distortion due to transactions costs in the final goods market. Furthermore, there are relative price and relative wage distortions. Finally, there are distortions in the hiring decision on the labor market.

The policymaker maximizes (12) subject to the constraints (7), (8), (18) the resource constraint, equating supply with demand ${ }^{10}$

$$
\begin{equation*}
\sum_{j=0}^{J-1} \omega_{t}^{j} y_{t}^{j}-\sum_{j=0}^{J-1} \omega_{t}^{j} \alpha_{t}^{j} \Xi_{j, t}=\sum_{j=0}^{J-1} \omega_{t}^{j}\left(p_{t}^{j}\right)^{-\sigma}\left[c_{t}+m_{t}^{a} v_{t}-\left(1-n_{t}\right) b_{t}\right] \tag{36}
\end{equation*}
$$

the flow equation of prices

$$
\begin{equation*}
p_{t}^{j}=\frac{p_{t-1}^{j-1}}{1+\pi_{t}} \tag{37}
\end{equation*}
$$

expressions $(2),(4),(6),(9),(17),(19),(21),(28),(29),(31),(32),(33),(34),(35)$ and the flow equation of wages

$$
\begin{equation*}
w_{t}^{j}=\frac{w_{t-1}^{j-1}}{1+\pi_{t}} . \tag{38}
\end{equation*}
$$

## 4 Calibration

For our numerical exercises, we assume that $u\left(c_{t}\right)=\log c_{t}$. The calibration of the deep parameters are presented in Table 2. We set the quarterly discount factor to 0.9945 and average quarterly productivity growth to 1.0043 to generate a real interest rate of $4 \%$. To model the idiosyncratic productivity process, we use a three-state Markov chain with a ratio between the min and the max state of 0.54 . The value of $b_{r}$ implies a replacement rate (the ratio of home production value to the average wage) of around 0.6 We set the bargaining power to $\varphi=0.5$ implying symmetrical bargaining. For the job separation rate $1-\rho$, we follow Gertler, Sala, and Trigari (2008) and set $\rho=0.895$. The values for the parameters in the credit cost distribution is taken from Lie (2010). We assume that the distribution of menu costs in price setting follow the beta distribution with parameters as in Lie (2010), i.e., letting $g_{P}=\frac{1}{\beta(l, r)} x^{l-1}(1-x)^{r-1}$ denote the probability density function we have $l=2.1, r=1.0$ and upper

[^6]See Christoffel, Kuester, and Linzert (2009).

Table 1: Baseline Calibration of the Model

| Parameters |  |  |  |
| :--- | :--- | :--- | :--- |
| $\beta$ | 0.9945 | $b_{r}$ | 0.35 |
| $\sigma$ | 10 | $\rho$ | 0.9 |
| $\phi$ | 1 | $\sigma_{a}$ | 0.6 |
| $\varphi$ | 0.5 | $\sigma_{\mu}$ | 0.37 |
| $\bar{\varepsilon}^{a}$ | 1.0043 | $\bar{h}$ | 0.2 |
| $\rho_{a}$ | 0.95 | $\kappa$ | 0.024 |
| $\kappa^{L}$ | 1.27 | $r_{\pi}$ | 1.5 |
| $r_{y}$ | 0.1 | $s^{\text {new }}$ | 0 |

bound 0.015 . The disagreement costs for firms and workers when bargaining over wages also follow the beta distribution $g^{H}=\frac{1}{\beta\left(l_{H}, r_{H}\right)} x^{l_{H}-1}(1-x)^{r_{H}-1}$ for workers and $g^{J}=\frac{1}{\beta\left(l_{J}, r_{J}\right)} x^{l_{J}-1}(1-x)^{r_{J}-1}$ for firms. We set $l_{H}=1.6, r_{H}=1$ and $l_{J}=1.6, r_{J}=1$.

To find the bounds of the distribution, we fit the dispersion of yearly wage changes in the model (given the yearly average inflation rate of $2.56 \%$ during the period 1993 - 1997) to the (average) empirical dispersion of yearly wage changes in the US during the period 1993 - 1997 using a minimum distance estimator. ${ }^{11}$ The time period is chosen since it represents a period with stable inflation close to two percent. This procedure yields parameters $\mathcal{B}^{H}=0.1075$ for and for firms $\mathcal{B}^{J}=0.3004$. When imposing a symmetry restriction, we find the upper bounds to equal $\mathcal{B}^{H}=\mathcal{B}^{J}=0.1507$. Moreover, the maximum length of a wage (price) contract is set to 6 (9) quarters. ${ }^{12}$

Figure 1 (2) illustrates the model (empirical) distribution of nominal wage changes for stayers. Comparing the two figures, we see that the model captures key features of the empirical wage distribution fairly well. For example, the spike at zero nominal wage change and the peak around $5 \%$ as well as the absence of any substantial mass on nominal wage cuts.

## 5 Results

In figure 3, we plot the dynamics of the model when policy is governed by a Taylor rule. ${ }^{13}$ We first plot the response of output and inflation to a one and three standard deviation (positive and negative)

[^7]

Figure 1: The nominal wage change distribution implied by the model.


Figure 2: Empirical distribution of nominal wage changes in the US during the period 1993-1997


Figure 3: Impulse responses for inflation and productivity to a productivity shock.
productivity shock, respectively. To get a better idea of the difference between positive and negative shocks, we mirror the negative shock through the x -axis.

The difference in the output response between negative and positive shocks is fairly small, although it is slightly bigger for the three standard deviation shock. The response of inflation is bigger, especially in the three standard deviation case, where the impact response of a negative shock is almost twice as large as for a positive shock.

### 5.1 The Long-run Phillips Curve

In the model, there is a long-run trade-off between inflation and output, i.e., the Long-run Phillips curve has a positive slope. Inflation erodes wages that don't change, in turn increasing vacancy creation and affecting output. Figure 4 below illustrates how output varies with inflation, depending on the symmetry of wage adjustment.

With productivity growth, the slope of the long-run Phillips curve is about the same with symmetric and asymmetric wage frictions. On the other hand, without productivity growth, the slope of the long-run Phillips curve is flatter with asymmetric wage frictions.


Figure 4: The Long-run Phillips curve

### 5.2 Optimal Inflation

The trade-off between output and inflation indicated in figure 4 indicates that the planner can use inflation to affect welfare. Specifically, since the Hosios condition need not hold, the planner can use inflation to affect wages and in turn reduce inefficiencies due to search and matching frictions; for a detailed discussion see Carlsson and Westermark (2012).

To analyze the effects of downward nominal wage rigidity, we compare the optimal inflation rate to the optimal rate in a model where these rigidities are not present. Moreover, it is interesting to try to distinguish between the effects of just adding (symmetric) wage setting frictions from the effect of adding asymmetries, i.e., downward nominal wage rigidity. We do this by also looking at third model; a model with sticky wages but symmetric adjustment probabilities (averaging parameters of the two disagreement cost distributions). Finally, we analyze the case with flexible wages for new entrants.

Table 2: Yearly optimal inflation rate under the Ramsey policy

|  | Asymmetric wage frictions | Symmetric wage frictions | Flexible wages |
| :--- | :--- | :--- | :--- |
| Baseline | 0.77 | 0.28 | -0.60 |
| Zero productivity growth | 1.69 | 1.02 | -0.13 |
| $s^{\text {new }}=0.5$, growth | 0.14 | -0.11 | -0.60 |
| $s^{\text {new }}=0.5$, no growth | 0.93 | 0.51 | -0.13 |
| Flex wages for new hires | -0.52 | -0.60 | -0.60 |

We find that the optimal annual inflation rate under downward nominal wage rigidities is about $0.8 \%$
in the baseline calibration. The optimal annual inflation rate found here is larger than in the baseline monetary models discussed in Schmitt-Grohe and Uribe (2010) where the optimal inflation rate is at most around zero. The difference between the optimal inflation rate under downward nominal wage rigidities with a model where wage frictions are symmetric is almost $0.5 \%$. When looking at a model without productivity growth, the difference is substantial, with the optimal inflation rate under downward nominal wage rigidities being almost a percentage point larger than in the model with productivity growth. Also, the difference between asymmetric and symmetric wage frictions increases a little, to about $0.7 \%$.

The increase in inflation relative to a model with flexible wages and price adjustment frictions is slightly above one percentage unit. The introduction of downward nominal wage rigidities into a model with flexible wages can be decomposed into two effects. First, wage adjustment frictions are introduced and second, the frictions become asymmetric. The effects of introducing wage adjustment frictions stands for slightly more than half of the entire inflation effect when there is productivity growth. Asymmetric frictions increases inflation by about $0.5 \%$. On the other hand, without productivity growth, introducing symmetric wage frictions increases inflation by about $1.1 \%$ and asymmetric wage frictions increases inflation by an additional $0.7 \%$. Thus, introducing productivity growth leads to a somewhat smaller effect on inflation of introducing downward nominal wage rigidities. The reason is that productivity growth gives a similar effect; given that wages are nominally fixed, productivity growth decreases the real marginal costs for firms pay, making it less important to use inflation to erode wages. ${ }^{14}$

Next, we experiment by letting newly hired workers become flexible. We then find an optimal annual deflation rate of around $0.5 \%$. Thus the treatment of the wage flexibility of newly hired workers has an impact on the optimal policy prescription.

## 6 Concluding Discussion

We develop a DSGE model where there is a role for money as a medium of exchange, as well as, when declining nominal wages might not be a viable margin for adjustment. To capture the Friedman argument, we introduce a transaction cost (as in Schmitt-Grohe and Uribe (2004)). To include the Tobin argument, we introduce price- and wage-setting friction. Since our ultimate aim is to study the optimal inflation rate, it is important to allow optimal price- and wage-setting decision to depend on the inflation rate. To this end, both price and wage decisions are modeled as state dependent. Price-setting frictions are

[^8]introduce as in Dotsey, King, and Wolman (1999)). Wage setting is based on the bargaining model in Holden (1994), where downward nominal wage rigidities can arise as a rational outcome. Finally, the model feature a search-matching labor market akin to the model of Christoffel, Kuester, and Linzert (2009). To parametrize the distribution of wage adjustment costs in the model, we use a minimumdistance estimation approach to match the nominal wage change distribution implied by the model to the empirical nominal wage change distribution observed in U.S. micro data. The estimated model yields a distribution of wage changes that captures the overall shape of the empirical wage distribution. An important feature that allows the model to fit the data with any precision, is the introduction of firm-level heterogeneity in terms of productivity, as well as, aggregate productivity growth.

The model response of output to a productivity shock is fairly symmetric, while the response of inflation displays larger asymmetries, with a bigger response for negative shock, especially when shocks are large. Furthermore, we establish the existence of an upward-sloping Long-run Phillips curve. The reason for this is that an increase in inflation leads to a larger erosion of nominal wages and hence lower real wages, in turn increasing vacancy creation, employment and output.

We find that the optimal annual rate under downward nominal wage rigidities is $0.8 \%$. The optimal annual inflation rate found here is larger than in the baseline monetary models discussed in SchmittGrohe and Uribe (2010) where the optimal inflation rate is at most around zero. The effect of introducing asymmetric wage adjustment frictions is small, though; with symmetric frictions, the optimal inflation rate is about $0.3 \%$. Without productivity growth, the result changes substantially. Then the optimal inflation rate with symmetric frictions is about $1.0 \%$, while the rate is about $1.7 \%$ under downward nominal wage rigidities. Thus, allowing for productivity growth substantially reduces the optimal inflation rate and also mitigates the effect of downward nominal wage rigidities. The reason is that productivity growth can perform the same effect of inflation; erosion of real marginal costs. However, we also show that the flexibility of the wage formation has an effect on this conclusion. Letting new hires wages to be perfectly flexible leads to an optimal annual deflation rate of around $0.5 \%$..

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## Appendix

This appendix briefly describes wage adjustment probabilities and the optimal policy problem stated in section 3. For detailed derivations, see the accompanying technical appendix.

## A Wage Adjustment Probabilities

The fraction of firms that disagree is

$$
\begin{array}{cc}
1 & \text { if } \mathcal{B}^{J}<d J_{t}^{j_{w}}\left(w_{t}^{j_{w}}, a_{t}\right) \\
G^{J}\left(d J_{t}^{j_{w}}\left(w_{t}^{j_{w}}, a_{t}\right)\right) & 0 \leq d J_{t}^{j_{w}}\left(w_{t}^{j_{w}}, a_{t}\right) \leq \mathcal{B}^{J} \\
0 & d J_{t}^{j_{w}}\left(w_{t}^{j_{w}}, a_{t}\right)<0
\end{array}
$$

Similarly, the fraction of workers that has an incentive to disagree to force a renegotiation of the wage contract is

$$
\begin{array}{cc}
1 & \text { if } \mathcal{B}^{H}<d H_{t}^{j_{w}}\left(w_{t}^{j_{w}}, a_{t}\right) \\
G^{H}\left(H_{t}^{j_{w}}\left(w_{t}^{j_{w}}, a_{t}\right)\right) & 0 \leq d H_{t}^{j_{w}}\left(w_{t}^{j_{w}}, a_{t}\right) \leq \mathcal{B}^{H} \\
0 & d H_{t}^{j_{w}}\left(w_{t}^{j_{w}}, a_{t}\right)<0
\end{array}
$$

The adjustment probabilities are then

$$
\alpha_{t}^{j_{w}}\left(w_{t}^{j_{w}}, a_{t}\right)=1
$$

if if $\mathcal{B}^{J}<d J_{t}^{j_{w}}\left(w_{t}^{j_{w}}, a_{t}\right)$ or if $\mathcal{B}^{H}<d H_{t}^{j_{w}}\left(w_{t}^{j_{w}}, a_{t}\right)$,

$$
\begin{aligned}
\alpha_{t}^{j_{w}}\left(w_{t}^{j_{w}}, a_{t}\right)= & G^{J}\left(d J_{t}^{j_{w}}\left(w_{t}^{j_{w}}, a_{t}\right)\right)+G^{H}\left(d H_{t}^{j_{w}}\left(w_{t}^{j_{w}}, a_{t}\right)\right) \\
& -G^{H}\left(d H_{t}^{j_{w}}\left(w_{t}^{j_{w}}, a_{t}\right)\right) G^{J}\left(d J_{t}^{j_{w}}\left(w_{t}^{j_{w}}, a_{t}\right)\right),
\end{aligned}
$$

if $0 \leq d J_{t}^{j_{w}}\left(w_{t}^{j_{w}}, a_{t}\right) \leq \mathcal{B}^{J}$ and $0 \leq d H_{t}^{j_{w}}\left(w_{t}^{j_{w}}, a_{t}\right) \leq \mathcal{B}^{H}$,

$$
\alpha_{t}^{j_{w}}\left(w_{t}^{j_{w}}, a_{t}\right)=G^{J}\left(d J_{t}^{j_{w}}\left(w_{t}^{j_{w}}, a_{t}\right)\right),
$$

if $0 \leq d J_{t}^{j_{w}}\left(w_{t}^{j_{w}}, a_{t}\right) \leq \mathcal{B}^{J}$ and $d H_{t}^{j_{w}}\left(w_{t}^{j_{w}}, a_{t}\right)<0$,

$$
\alpha_{t}^{j_{w}}\left(w_{t}^{j_{w}}, a_{t}\right)=G^{H}\left(d H_{t}^{j_{w}}\left(w_{t}^{j_{w}}, a_{t}\right)\right),
$$

if $d J_{t}^{j_{w}}\left(w_{t}^{j_{w}}, a_{t}\right)<0$ and $0 \leq d H_{t}^{j_{w}}\left(w_{t}^{j_{w}}, a_{t}\right) \leq \mathcal{B}^{H}$ and zero otherwise.


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[^1]:    ${ }^{1}$ The empirical evidence ranges from studies using data from personnel files presented in Altonji and Devereux (2000), Baker, Gibbs, and Holmstrom (1994), Fehr and Goette (2005), and Wilson (1999), survey/register data in Altonji and Devereux (2000), Akerlof, Dickens, and Perry (1996), Dickens et. al., 2007, Fehr and Goette (2005), Holden and Wulfsberg (2008), Kuroda and Yamamoto (2003a, 2003b) to interviews or surveys with wage setters like Agell and Lundborg (2003), and Bewley (1999), just to mention a few.
    ${ }^{2}$ An exception is Kim and Ruge-Murcia (2009) who finds an optimal inflation rate of about 0.4 percent in a model with downward nominal wage rigidity. However, including productivity growth would, almost certainly push this figure substantially below zero. See Amano et. al., 2009, for the effect of productivity growth on optimal inflation. For a detailed overview of the literature, see Schmitt-Grohe and Uribe (2010).
    ${ }^{3}$ Another reason for a positive steady state inflation rate is to avoid the non-negativity constraint in nominal interest rates to bind too frequently, see e.g. Billi and Kahn (2008).
    ${ }^{4}$ We thus modify the bargaining set up in Holden (1994) by assuming that disagreement can lead to a break up of the firm/worker match rather than a conflict period. This generates a bargaining formulation that is in line with standard search-matching models used in the macro literature (see e.g. Trigari, 2009 and others)

[^2]:    ${ }^{5}$ The effect of aggregate productivity growth has been studied previously by Amano et. al., 2009, finding a negative impact on the optimal steady state inflation rate.

[^3]:    ${ }^{6}$ For simplicity, we abstract from capital accumulation, though.

[^4]:    ${ }^{7}$ Note that there is no disagreement in equilibrium, and hence the equilibrium disagreement cost is zero. Thus, in contrast to state-dependent pricing, these cost neither enter resource constraints nor firm/worker value functions.

[^5]:    ${ }^{8}$ Note that the net cost of buying a unit of claims is $F_{t}-Z_{t}$.
    ${ }^{9}$ That is, the real discounted net gain of placing the transaction amount in a bond for a period and repay the transaction

[^6]:    ${ }^{10}$ Note that, since adjustment costs is in terms of aggregate output, the left-hand side is total output, net of these costs. The right-hand side consists of the weighted sum across firm demand $\left(p_{t}^{j}\right)^{-\sigma} y_{t}^{d}$ with

    $$
    y_{t}^{d}=c_{t}+m_{t}^{a} v_{t}-\left(1-n_{t}\right) b_{r}
    $$

[^7]:    ${ }^{11}$ The micro data on wages is collected from the the Panel Study of Income Dynamics and is corrected from measurement errors as described by Dickens et. al., 2007?.
    ${ }^{12}$ The difference in contract length between prices and wages is due to the increase in the computational burden of increasing the maximum length of wage contracts.
    ${ }^{13}$ When dynamics are computed adjustment probabilities are restricted to be striclty interior by using an approximation of a step function.

[^8]:    ${ }^{14}$ Note that the result for flexible wages is very much in line with Lie (2010).

